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Basic BP ANNs

with

Daniel L. Silver, Ph.D.

Andy McIntyre, Ph.D.

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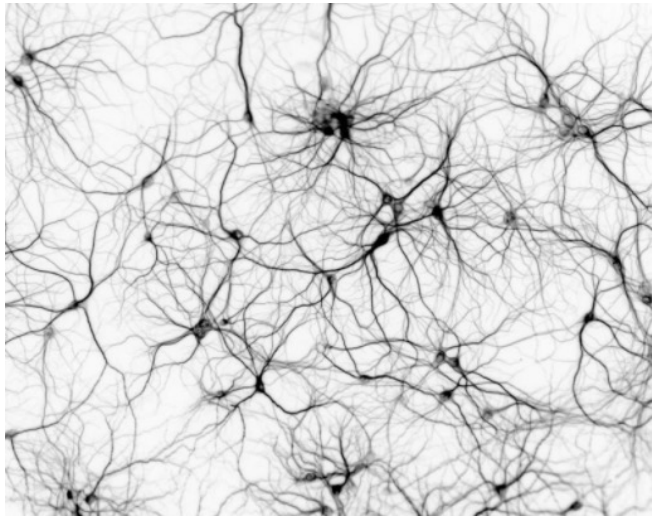
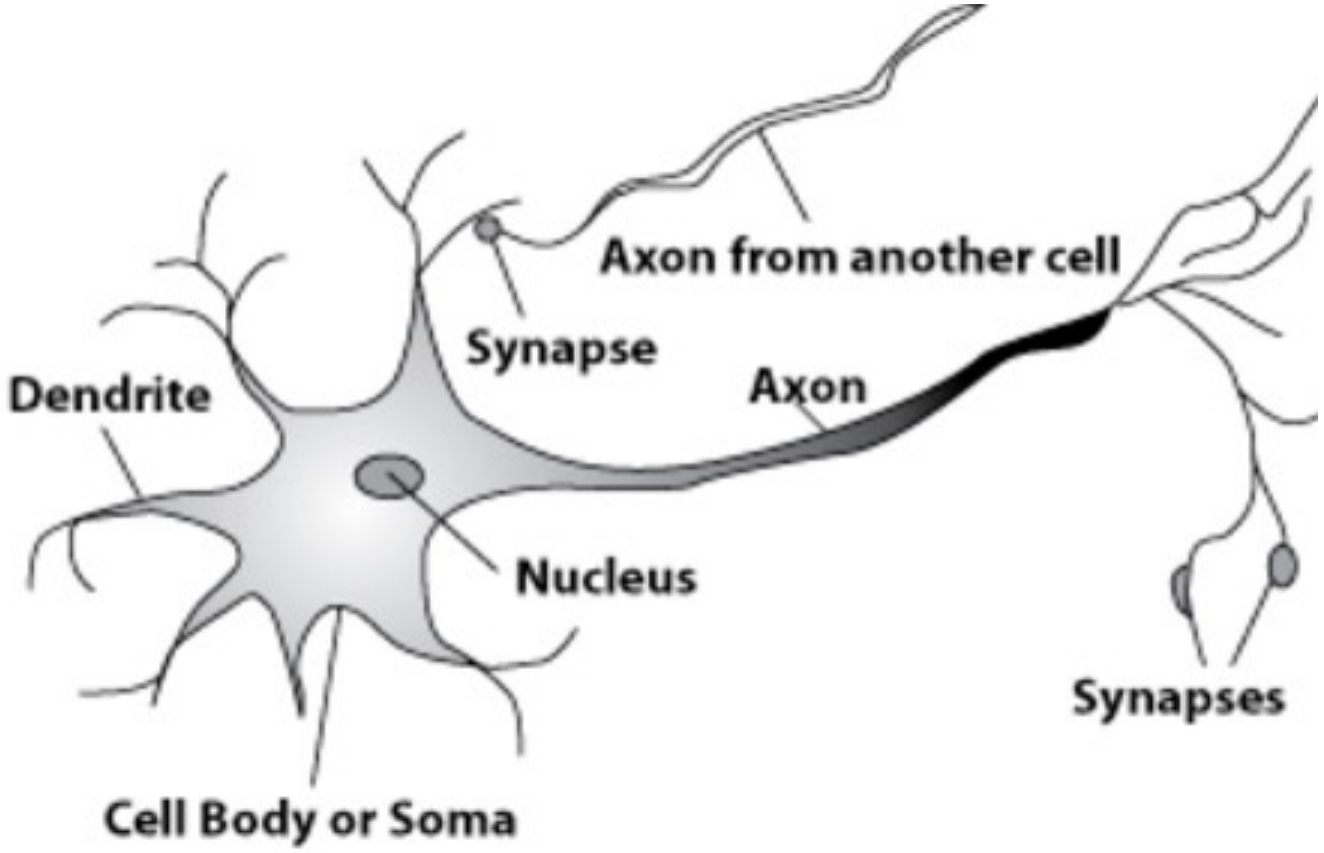
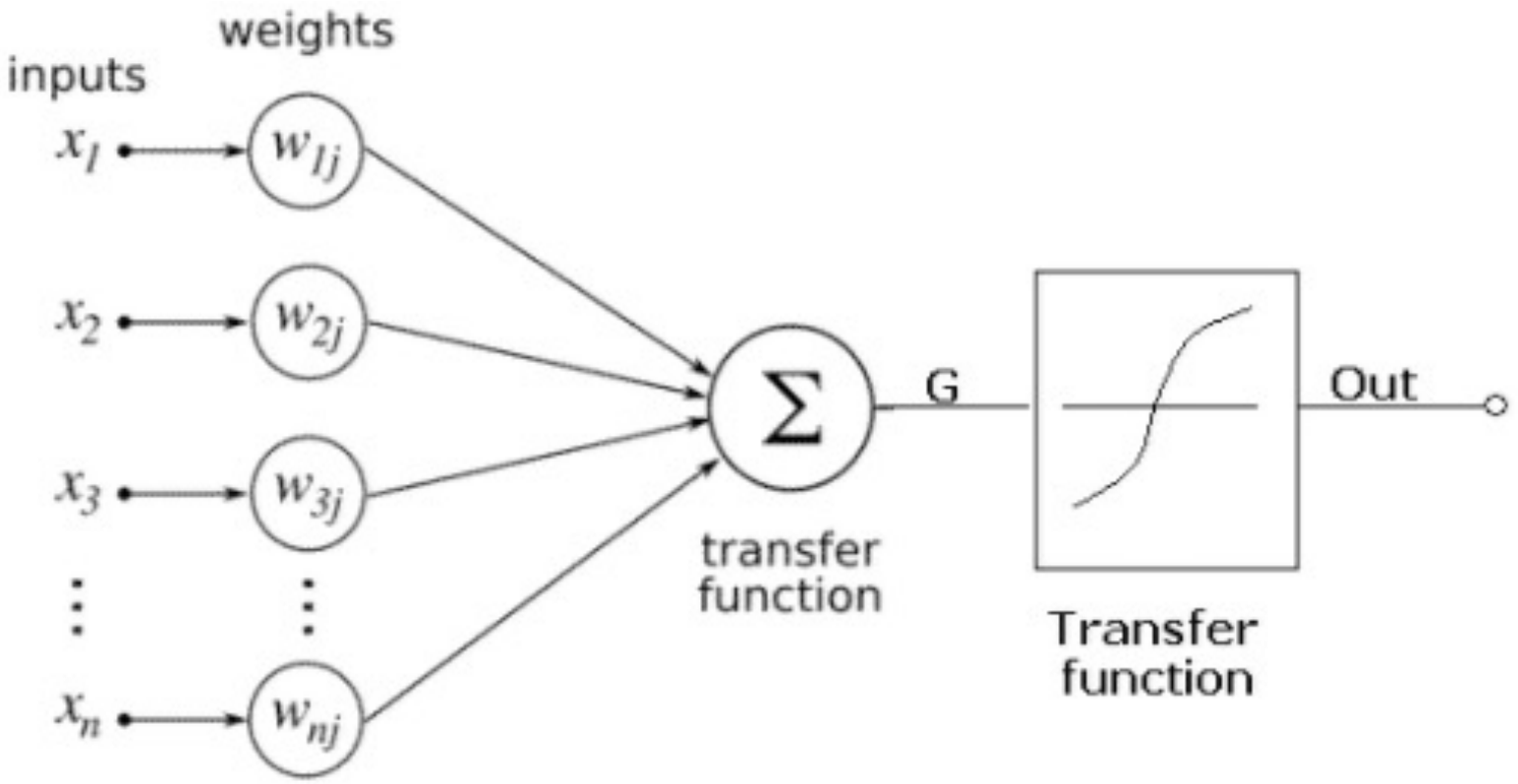
From Biological to Artificial Neurons





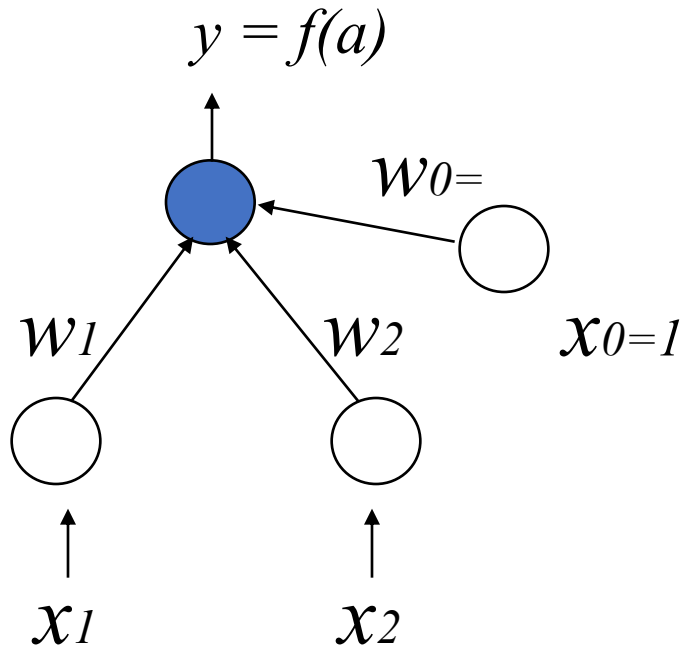
From Biological to Artificial Neurons



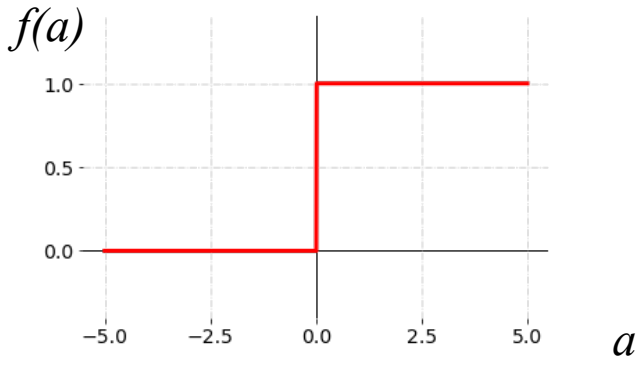
Learning in a Simple Neuron

Logical OR Function

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1



where f(a) is the step function, such that: f(a)=1, a > 0
f(a)=0, a <= 0



Learning in a Simple Neuron

Perceptron Learning Algorithm:

1. **Initialize weights**
2. **Present a pattern and target output**
3. **Compute output :**
4. **Update weights :**

$$y = f[\sum_{i=0}^{2} w_i x_i]$$

$$w_i(t+1) = w_i(t) + \Delta w_i$$

Repeat starting at 2 until acceptable level of error

This is an online method of computing a regression = fitting a function to a set of data points

Learning in a Simple Neuron

Widrow-Hoff or Delta Rule for Weight Modification

Where: $w_i(t+1) = w_i(t) + \Delta w_i$; $\Delta w_i = \eta d x_i(t)$

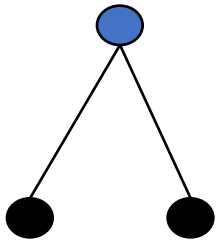
η = *learning rate* (*o* < η <= *1*), typically set = *0.1*

d = *error signal* = desired output - network output. = $t - y$

Fundamentally a gradient descent method of search

What is an artificial neuron doing when it learns?

Simple
Neural Network



Logical OR
Function

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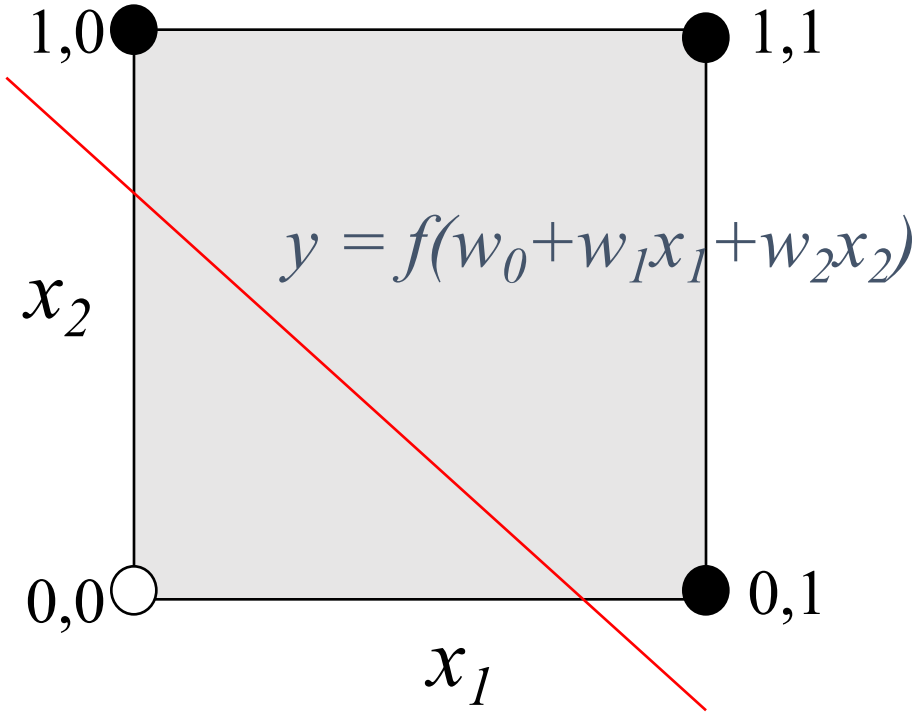
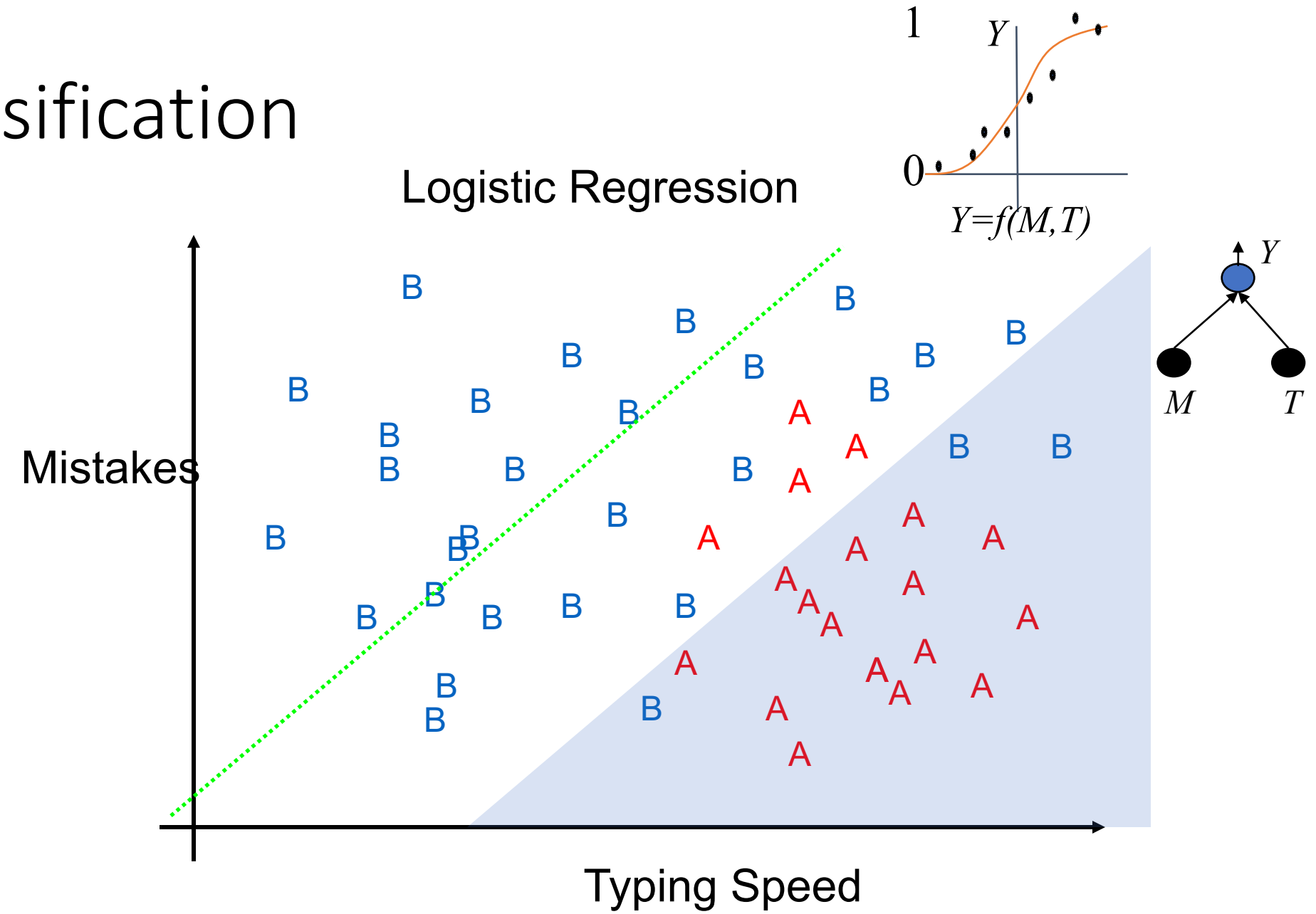


The discriminate function divides up the 0 and 1 class examples.

Classification



Limitations of Simple Neural Networks

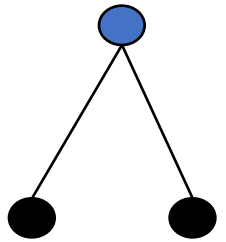
The Limitations of Perceptrons

(Minsky and Papert, 1969)

- Able to form only *linear discriminate functions*; *i.e.* classes which can be divided by a line or hyper-plane
- Most functions are more complex; *i.e.* they are *non-linear* or not *linearly separable*
- This crippled research in neural net theory for 15 years ....

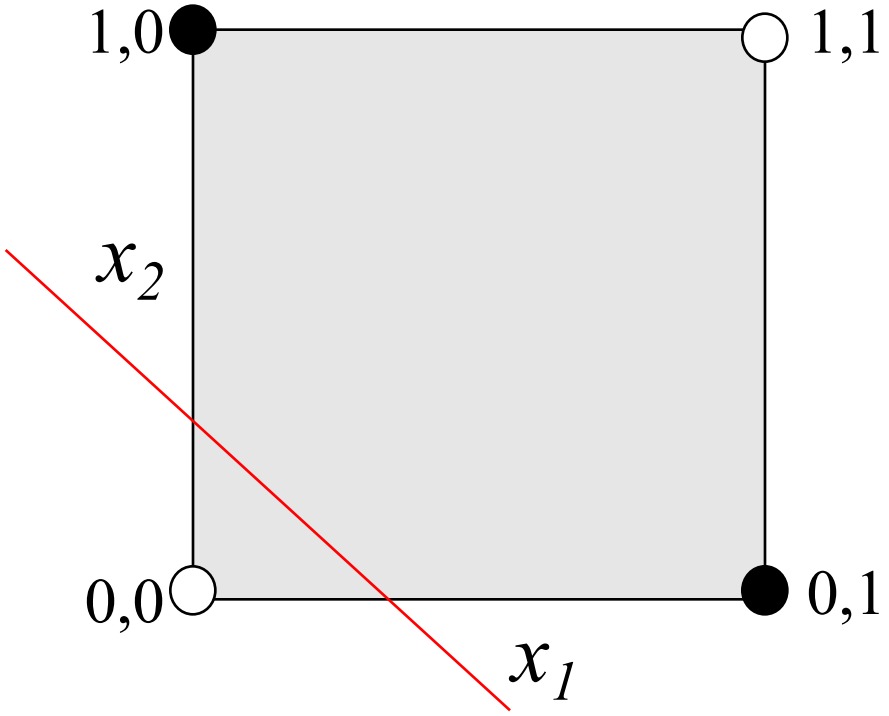
EXAMPLE

Simple
Neural Network



Logical XOR
Function

x1	*x2*	*y*
0	0	0
0	1	1
1	0	1
1	1	0

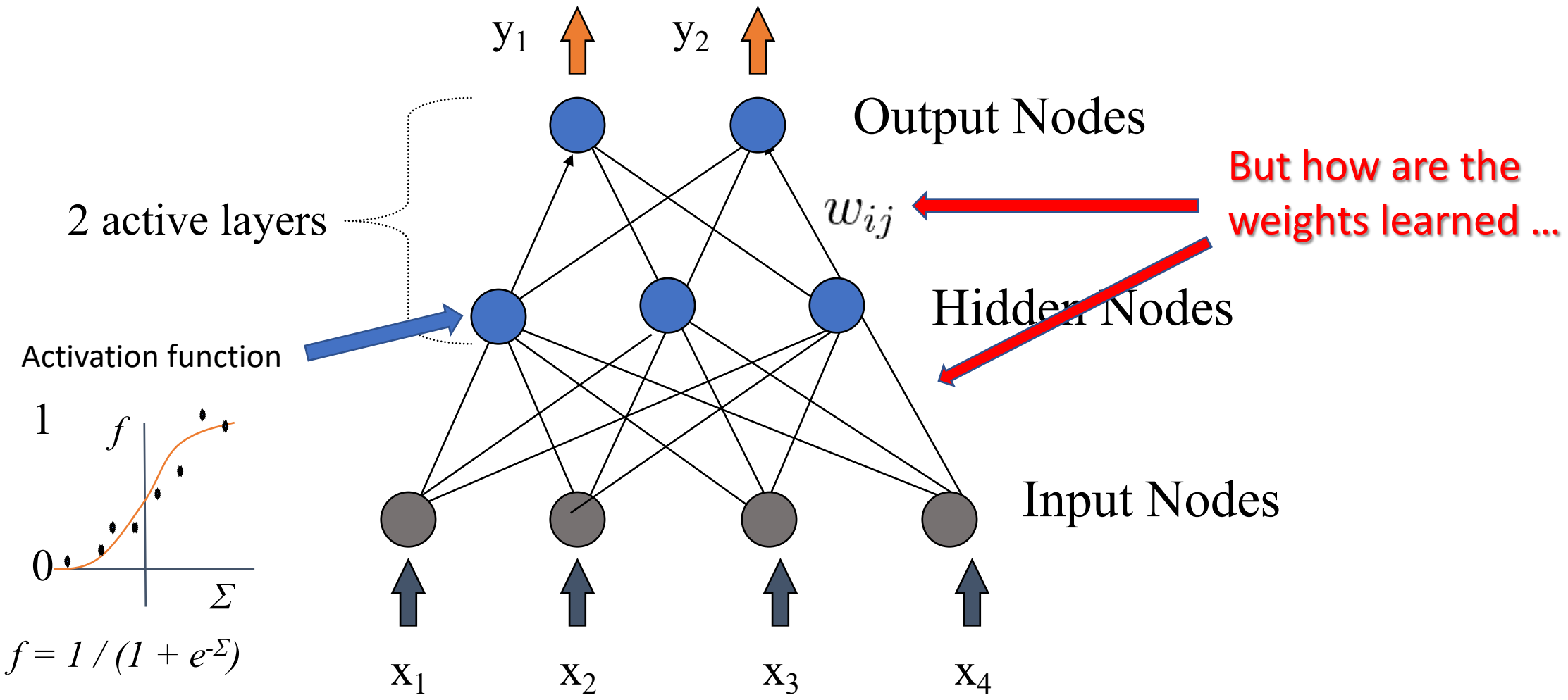


One linear discriminant function cannot divide up the 0 and 1 class examples.

TUTORIAL #1(a)

- Develop and train a simple neural network to learn the OR and XOR function (perceptron.ipynb)

Multi-layer Feed-forward ANNs

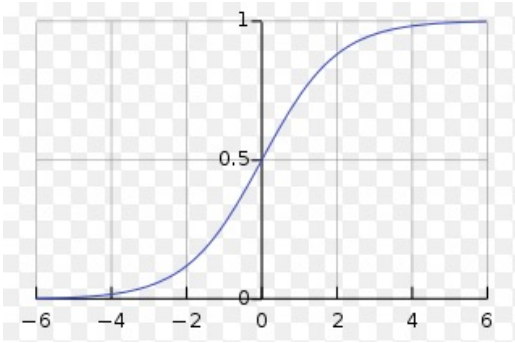


Multi-layer Feed-forward ANNs

Over the 15 years (1969-1984) research continued ...

- *hidden layer* of nodes allowed combinations of linear functions
- *non-linear activation functions* displayed properties closer to real neurons:
  - output varies continuously but not linearly
  - differentiable .... *Sigmoid* $f(a) = 1/(1 + e^{-a})$
- non-linear ANN classifier was possible
- But how are the weights learned …

$p = 1/(1+e^{-\Sigma})$



$\Sigma_{j=0..k} w_j x_j$

The Back-propagation Algorithm

- 1986: the solution to multi-layer ANN weight update rediscovered
- Conceptually simple - the global error is backward propagated to network nodes, weights are modified proportional to their contribution
- Most important ANN learning algorithm
- Become known as *back-propagation* because the error is sent back through the network to correct all weights

The Back-propagation Algorithm

- Like the Perceptron - calculation of error is based on difference between target and actual output:

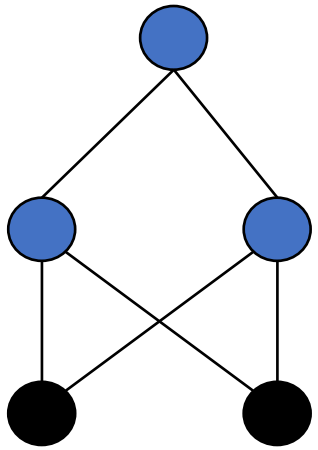
$$E = \frac{1}{2}\sum_j (t_j - o_j)^2$$

- However in BP it is the rate of change of the error with respect to each weight - the important feedback through the network
  generalized delta rule

$$\Delta w_{ij} = -\eta \frac{\delta E}{\delta w_{ij}}$$

- Relies on a continuous non-linear activation function (sigmoid) for communication between layers

The Back-propagation Algorithm



Where: $$\Delta w_{ij} = -\eta * \frac{\delta E}{\delta w_{ij}} = \eta * \delta_j * o_i$$

For output nodes:

$$\delta_j = o_j(1 - o_j)(t_k - o_j)$$

For hidden nodes:

$$\delta_i = o_i(1 - o_i) \sum_j w_{ij}\delta_j$$

The Back-propagation Algorithm

On-Line algorithm:

1. Initialize weights
2. Present a pattern and target output $o_j = f[\sum_{i=0}^{n} w_{ij} o_i]$
3. Compute output : $w_i(t+1) = w_i(t) + \Delta w_i$
4. Update weights :

where $w_{ij} = -\eta \frac{\delta E}{\delta w_{ij}}$

Repeat starting at 2 until acceptable level of error

The Back-propagation Algorithm

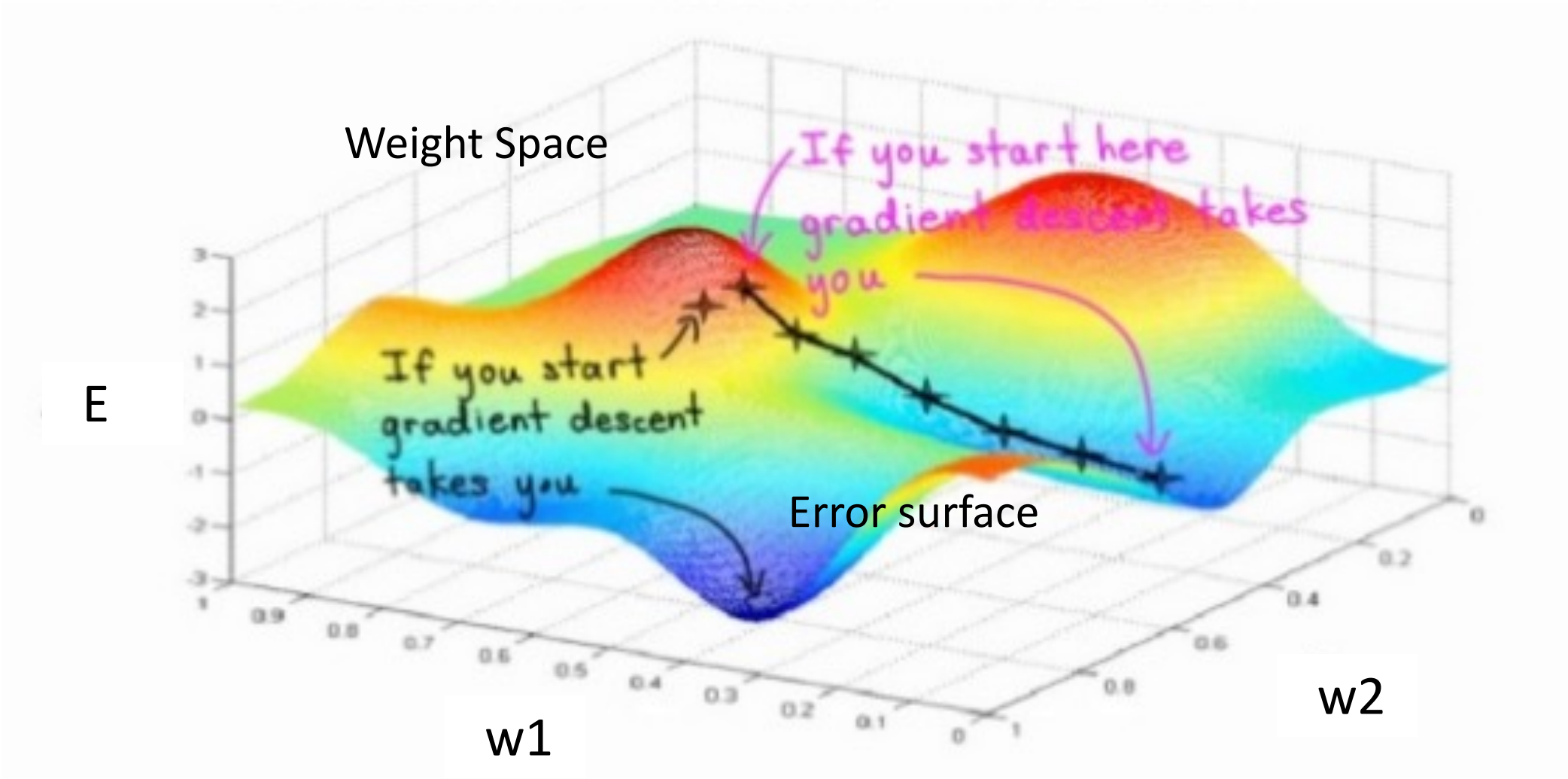
Visualizing the bp learning process:

The bp algorithm performs a *gradient descent* in weights space toward a minimum level of error using a fixed *step size* or learning rate η

The gradient is given by : $\frac{\delta E}{\delta w_{ij}}$

= rate at which error changes as weights change

Gradient Descent - Intuition



The Back-propagation Algorithm

Momentum Descent:

- Minimization can be speed-up if an additional term is added to the update equation:

$$\alpha[w_{ij}(t) - w_{ij}(t-1)]$$

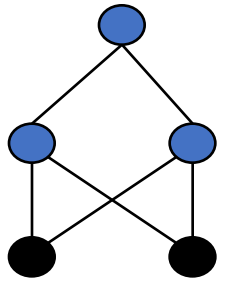
where: $0 < \alpha < 1$

$$\Delta w_{ij}(t) = \eta d_j o_i + \alpha \Delta w_{ij}(t-1)$$

- Thus:
  - Augments the effective learning rate η to vary the amount a weight is updated
  - Analogous to momentum of a ball - maintains direction
  - Rolls through small local minima
  - Increases weight update when on stable gradient

EXAMPLE

Multi-layer
Neural Network



Hidden layer
of neurons

Logical XOR
Function

x1	*x2*	*y*
0	0	0
0	1	1
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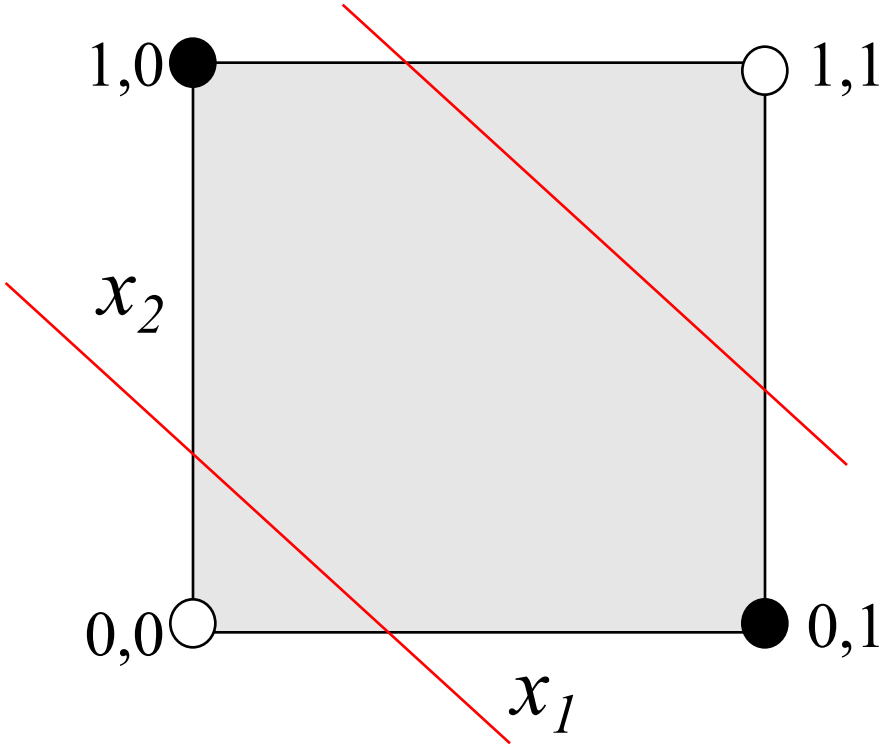
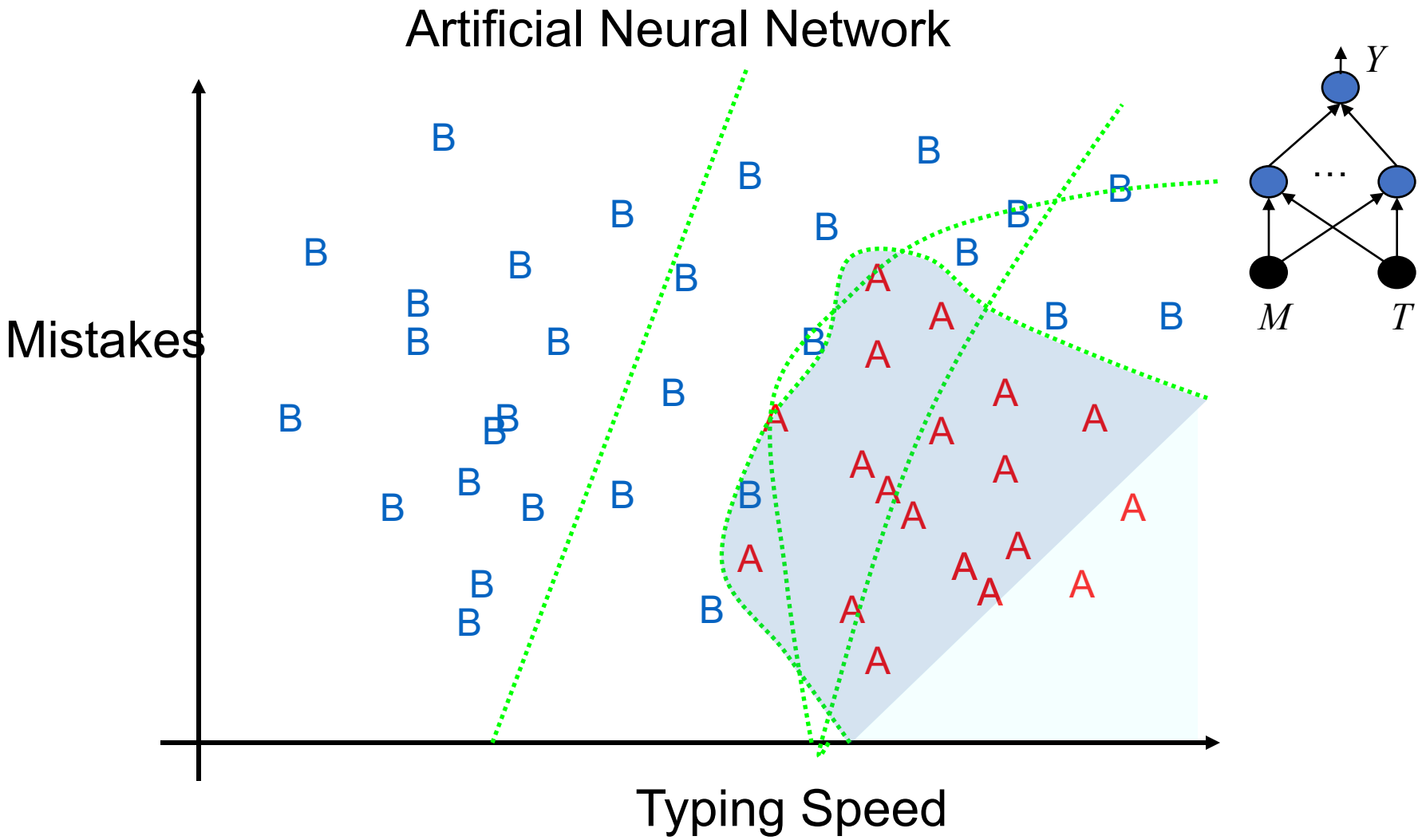


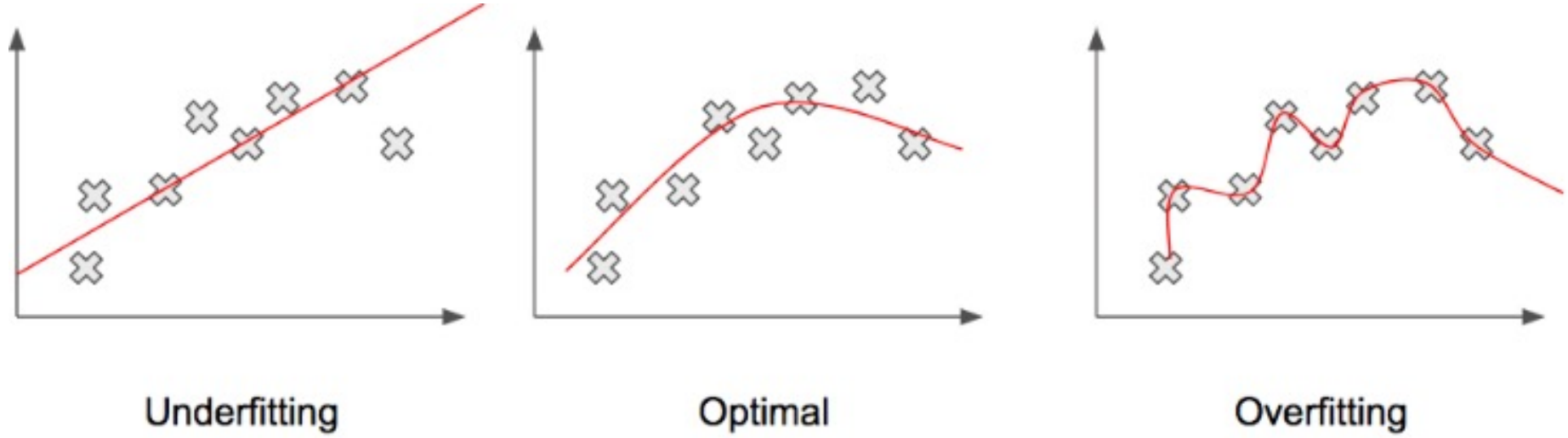
Two neurons are needed! Their combined results can produce good classification.

Classification



Generalization

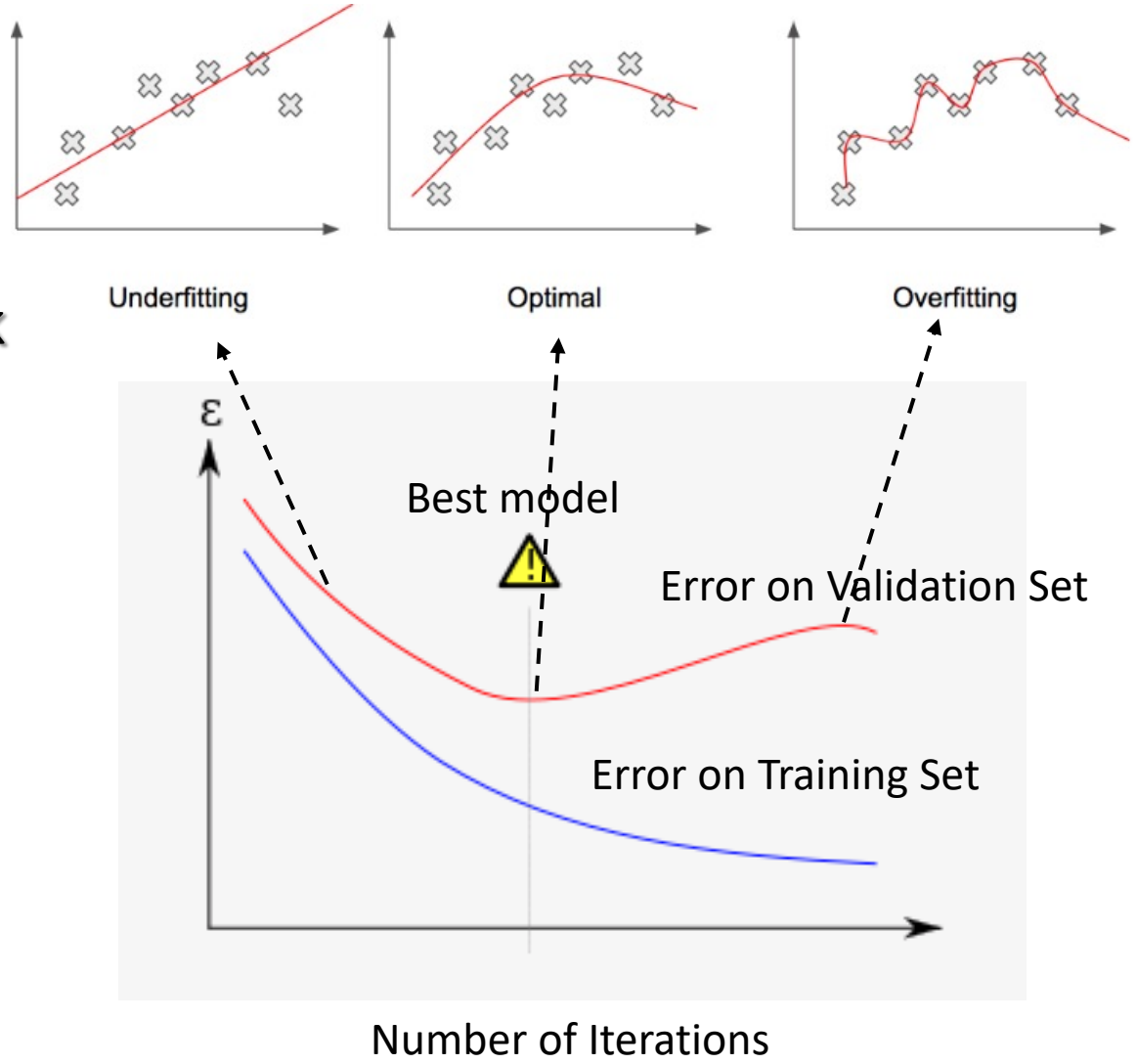
- The objective of learning is to achieve good *generalization* to new cases, otherwise just use a look-up table.
- Generalization can be defined as a mathematical *interpolation* or *regression* over a set of training points



Generalization

Preventing Over-training:

- Use a separate *validation* or *tuning set* of examples
- Monitor error on the val. set as network trains
- Stop training just prior to over-fit error occurring = *early stopping* or *tuning*
- Number of effective weights is reduced
- Most new systems have automated early stopping methods



TUTORIAL #1(b)

- Develop and train a backpropagation network to learn XOR function (mlp_bp_ann.ipynb)