Analysis of Manasa and Factorials

Here is my analysis about the problem:

(A) To count the number of zeros, we just need to count the multiplicity of 5

Counting the number of zeros of m! means counting the multiplicity of 5 as the multiplicity of 2 must be greater than that of 5 and 10=2x5. To compute the multiplicity of 5 of *m*!, we can think in this way: multiples of 5 contribute one 5, multiples of 25 contribute two 5, multiples of 125 contribute three 5… So, we have the following formula:

$$\left[\frac{m}{5}\right]+\left[\frac{m}{5^2}\right]+\left[\frac{m}{5^3}\right]+\ldots+\left[\frac{m}{5^k}\right]\text{where } 5^k \le m \text{ and } 5^{k+1} > m$$

where [*a*] means the greatest integer less than or equal to *a*. For example, [5] = 5, [4.1] = 4, [3.9] = 3.

(B) The answer must be a multiple of 5

The multiplicity of 5 of 5*a!*, (5*a*+1)!, (5*a*+2)!, (5*a*+3)!, (5*a*+4)! are the same since 5*a*+1, 5*a*+2, 5*a*+3 and 5*a*+4 do not contribute any 5.

(C) You can start 4n+1 and then test the next number which is divisible by 5

You should find the smallest multiple of 5 which is greater than 4*n*+1 (say this is *x*), then increment *x* by 5 to get the next testing number if you need to. So, a linear search will be fine and you don't need to perform binary search.

Consider a number which is in the form of $s5^a$, the multiplicity of 5 of $s5^a$ is:

$$\sum_{r=0}^{a}\left[\frac{s5^a}{5^r}\right]=s\sum_{r=0}^{a-1}5^r=s\left(\frac{5^a-1}{5-1}\right)=s\left(\frac{5^a-1}{4}\right)$$

Also, the multiplicity of 5 *of* $s5^a+t5^b$ is $s\left(\frac{5^a-1}{4}\right)+t\left(\frac{5^b-1}{4}\right)$.

Now we represent m in quinary form:

$$m=\sum_{r=0}^{k}a_r5^r$$

Then the multiplicity of 5 of $m!$ is $\sum_{r=0}^{k}\frac{a_r(5^r-1)}{4}$ which is n

$$i.e.\sum_{r=0}^{k}\frac{a_r(5^r-1)}{4}=n$$

$$\sum_{r=0}^{k}a_r(5^r-1)=4n$$

$$m=\sum_{r=0}^{k}a_r5^r=4n+\sum_{r=0}^{k}a_r\ge 4n+1$$