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STAT 230
Section 3, Group 40

Helicopter Experiment

Introduction

What gives a paper helicopter its fastest descent time? Perhaps a heavier paper type will cause the helicopter to have a slower descent time than a helicopter made out of a lighter option. Additionally, perhaps a longer wing length will result in a slower descent time compared to a shorter wing length due to the larger surface area. In other words, will wing length and/or material affect a paper helicopter descent time? By review of this report, the reader will find which factors, when altered, significantly affect the descent time of a paper helicopter.

Design

We would like to evaluate two possible factors: material and wing length. Our material factor will have try two different paper types: copy paper and construction paper. We will test wing length at 1.5 inches and 2.5 inches. The response variable we are measuring for is the time it takes for a helicopter to fall from the second floor to the first floor in the atrium of the campus library. This will be recorded in seconds.

Procedure: The experimental unit for this study is an individual paper helicopter. Five helicopters were made for each combination of factors, totalling 20 paper helicopters. That is, five helicopters made of copy paper with 1.5 inch wings, five made of copy paper with 2.5 inch wings, five made of construction paper with 1.5 inch wings, and five made of construction paper with 2.5 inch wings. All 20 helicopters are to be made with quarter sheets of paper (4.25 x 5.5 in.), should use the same type of paper clip, have a tail length of 2 inches, a tail width of 1 and 7/16th inches (See figure 1.1 below). In random order, the 20 helicopters will be dropped from the same height--choose a location with a clear drop where the helicopter will not be interrupted during its descent. Be careful while constructing the helicopters to not get any paper cuts. Additionally, choose a safe location to drop your helicopter from, perhaps somewhere with a railing to prevent falling.

We did a practice experiment in which we had larger dimensions for the helicopter and only 3 replicates. It took us only about an hour total to make the helicopters and perform our experiment. Our results yielded little variance and a low power value. We did a power test on our results and realized that to have a large power value, we needed about 10 replicates. As a result, we decided to make our dimensions smaller in hopes to see more variance and make more replicates in order to increase our power. We settled for 5 replicates for each treatment with smaller dimensions, which took about two hours to make; the experiment itself took about 30 minutes. The cost of each replicate was negligible in our decision (we already possessed almost all the supplies needed for each replicate; we only needed to purchase a stack of construction paper for about $3.00).

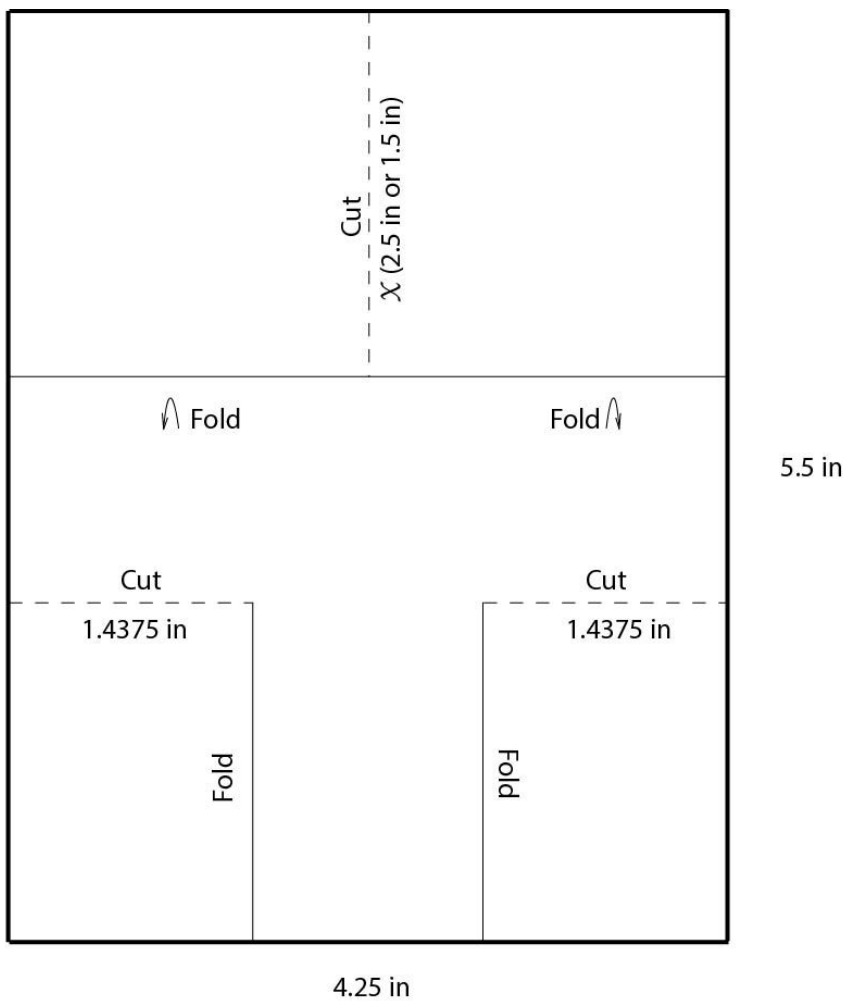

Figure 1.1.: Diagram to Construct Paper Helicopter.

Table 1
Randomized Order of Helicopter Experimental Combinations

Run	Experiment	Factor 1 Level: Material	Factor 2 Level: Wing Length (in.)
1	17	Construction	2.5

2	14	Construction	1.5
3	4	Copy	1.5
4	20	Construction	2.5
5	15	Construction	1.5
6	5	Copy	1.5
7	3	Copy	1.5
8	12	Construction	1.5
9	2	Copy	1.5
10	10	Copy	2.5
11	11	Construction	1.5
12	9	Copy	2.5
13	8	Copy	2.5
14	19	Construction	2.5
15	7	Copy	2.5
16	18	Construction	2.5
17	13	Construction	1.5
18	16	Construction	2.5
19	6	Copy	2.5
20	1	Copy	1.5

Perform the Experiment

We performed our experiment on Thursday, November 15th, around 6 in the afternoon. We met in the HBLL and all three of us participated in constructing the paper helicopters (we made the mistake of not constructing our helicopters in a random order). The helicopters took us approximately two hours to make, and performing the actual experiment only took about 15 minutes. We dropped the helicopters between the 1st and 2nd floor in the library, making sure we dropped from a safe height where there was a railing. Derik dropped the helicopters, Daniel stood by with the check sheet to state the order of experiments, and Savannah stayed on the first floor with a timer to record the descent time.

Figure 1.2.: Derik and Daniel Dropping Helicopters.

Figure 1.3.: Derik and Daniel Dropping Helicopters

Figure 1.4.: Helicopters After Landing.

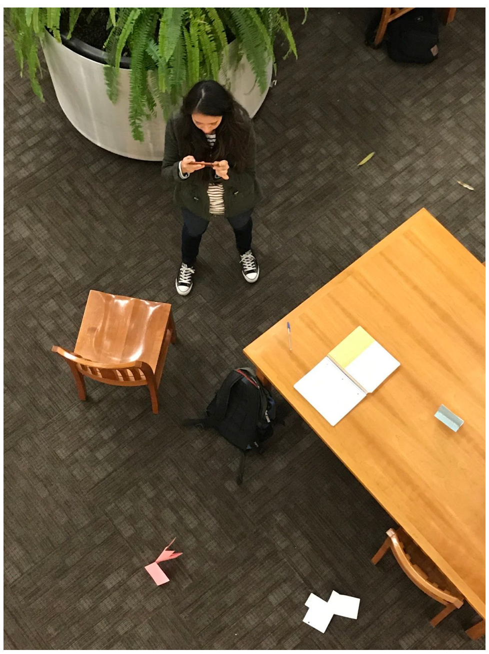

Figure 1.5.: Savannah Timing the Helicopters' Descent.

Experiment Results

Table 2
Helicopter Descent Experiment Data

Run	Material	Wing Length (in.)	Time (s)

1	Construction	2.5	4.03
2	Construction	1.5	3.08
3	Copy	1.5	3.16
4	Construction	2.5	4.35
5	Construction	1.5	3.30
6	Copy	1.5	2.88
7	Copy	1.5	3.16
8	Construction	1.5	3.04
9	Copy	1.5	3.05
10	Copy	2.5	3.72
11	Construction	1.5	3.25
12	Copy	2.5	4.19
13	Copy	2.5	3.75
14	Construction	2.5	3.89
15	Copy	2.5	3.93
16	Construction	2.5	4.02
17	Construction	1.5	3.20
18	Construction	2.5	3.78
19	Copy	2.5	3.8
20	Copy	1.5	2.91

Summary Statistics

Table 3

Summary Statistics of Helicopter Experimental Combinations

Material	Wing length (in)	Mean	Standard Deviation	Length
Construction	1.5	3.174	0.111	5
Copy	1.5	3.032	0.133	5
Construction	2.5	4.014	0.214	5
Copy	2.5	3.878	0.192	5

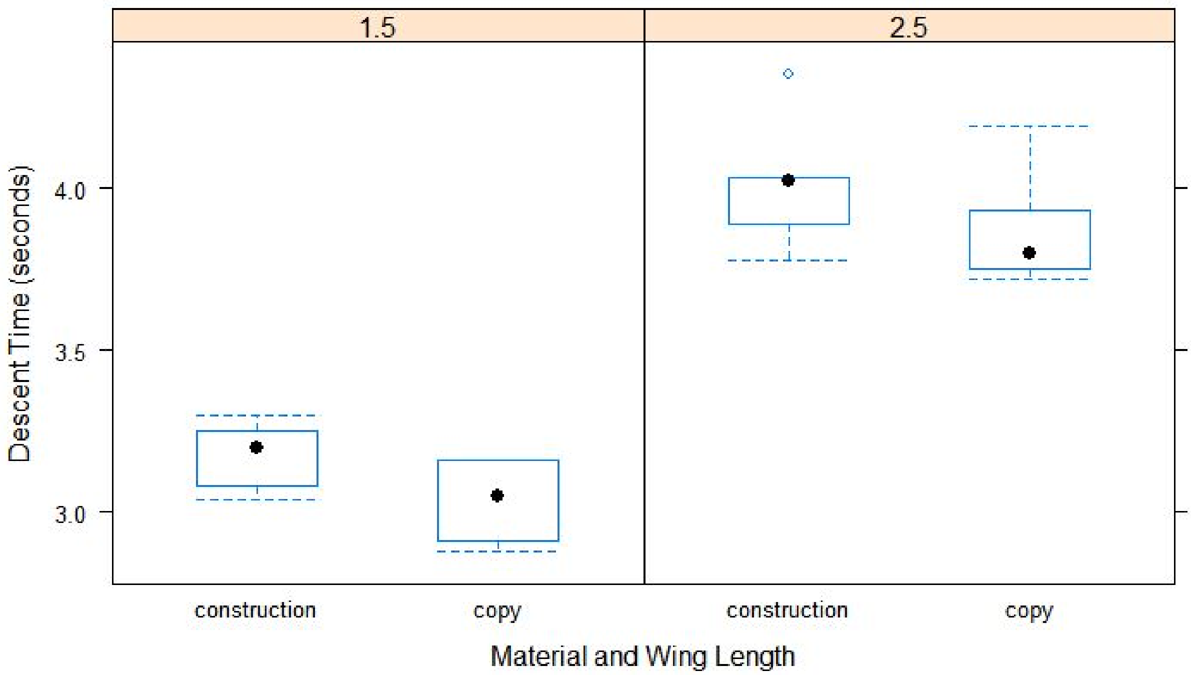

Figure 1.6.: Boxplot of Descent Time by Different Materials and Wing Lengths.

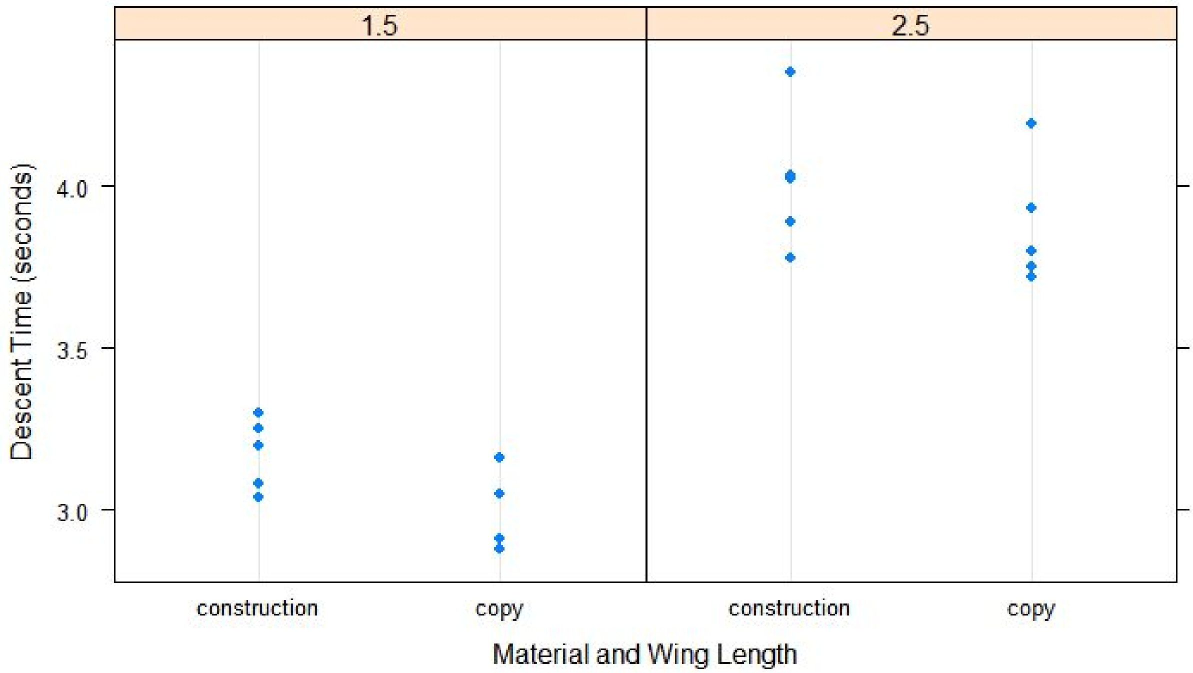

Figure 1.7.: Dotplot of Descent Time by Different Materials & Wing Lengths.

Model Specification:

This study is a basic factorial design, where all combinations of levels of factors appear as treatments. In fact, our study is crossed since we have four treatments where all the levels of one factor appear with every level of the other factor. Additionally, because we had an equal number of replicates in each group, this study is balanced. Since we are working with two factors, we will use a BF[2] model.

Using a BF[2] model, the model specification is as follows:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk},$$

where i indicates the i^{th} material (copy, construction), j indicates the j^{th} wing length (1.5 in., 2.5 in.), k indicates the k^{th} replicate of the given factor combination; and μ is equal to the grand mean time descent time across all factors, α_i is equal to the treatment effect of the i^{th} material, β_j is equal to the treatment effect of the j^{th} wing length, γ_{ij} is equal to the interaction effect of the i^{th} material and j^{th} wing length, ε_{ijk} is equal to the error of the k^{th} replicate for the i^{th} material and j^{th} wing length.

Residual Analysis:

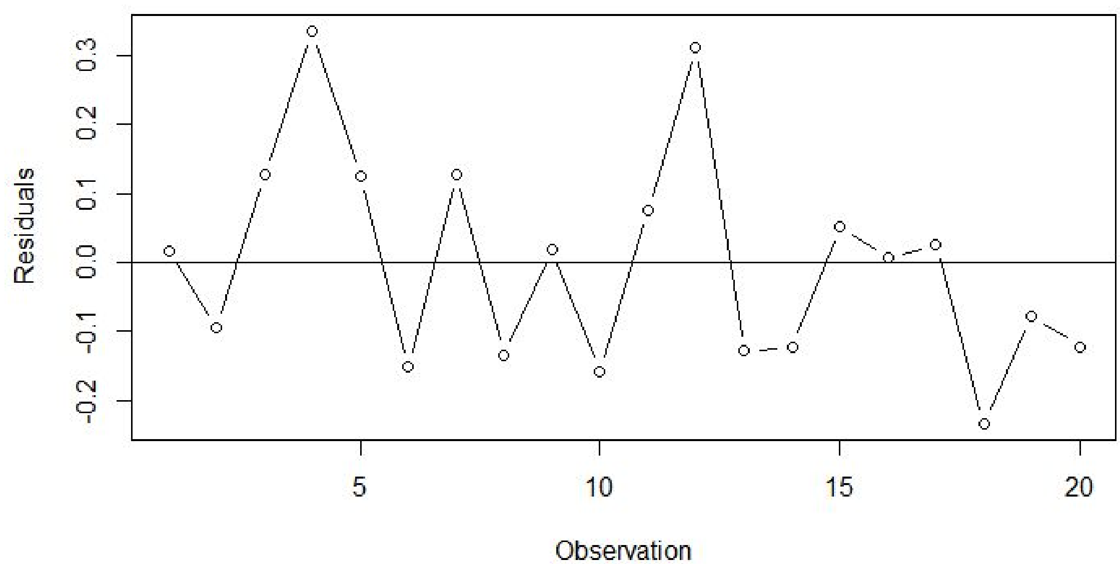

Figure 2.1.: Index Plot of Residuals.

We see no dominant pattern that suggests the assumption is violated (there is a slight decrease toward the end of the graph, but we did not think it seemed significant). And so, we can assume independence is satisfied (even though we made the mistake of constructing all the helicopters all at once rather than doing randomizing a complete trial--where we randomly assign a treatment, construct the helicopter, drop it and record its time, and then move onto the next trial); it is independent still most likely due to the fact that we at least randomized the order in which we dropped the helicopters.

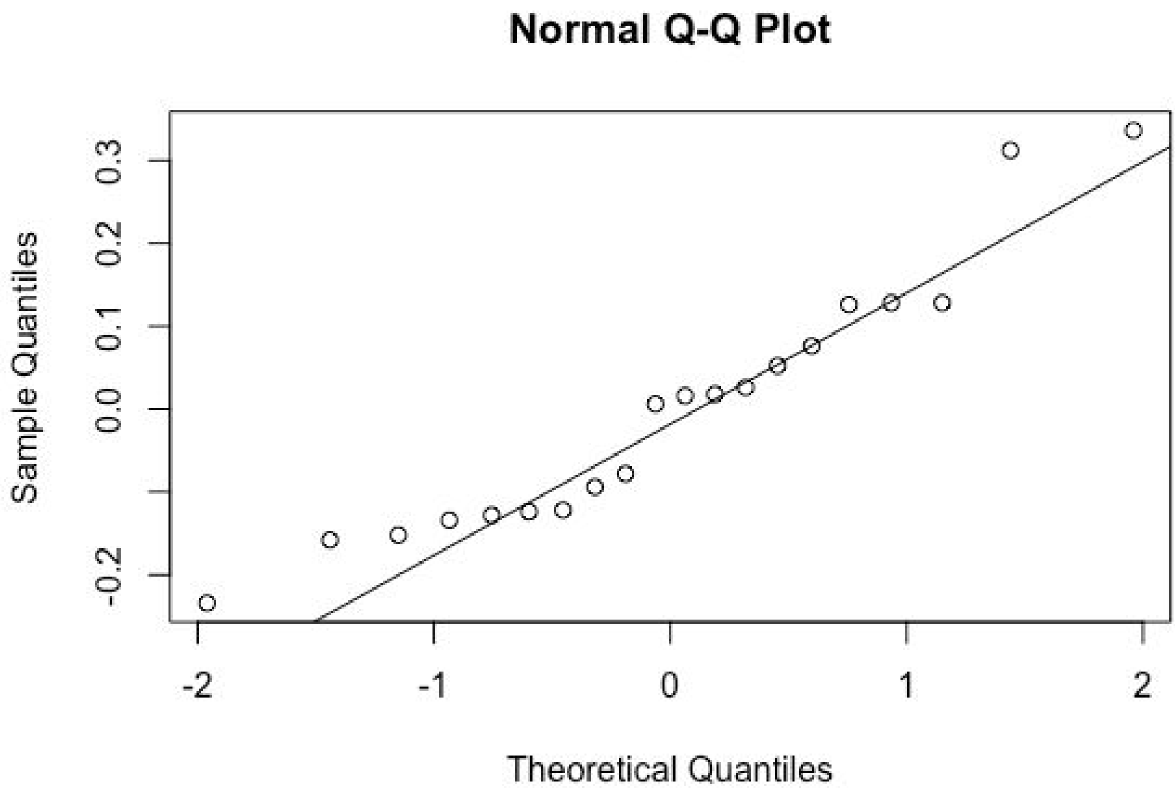

Figure 2.2.: Normal Q-Q Plot. Analyzing normality of residuals.

While the points do not follow the line exactly, they fall close enough that we can assume normality is satisfied. Additionally, referring to Table 3, the ratio between the largest and smallest standard deviation is less than two, meaning we can assume that common variance is satisfied.

ANOVA:

Table 4

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Analysis of Variance Table

Response: time
                    Df Sum Sq Mean Sq  F value    Pr(>F)
material             1 0.0966  0.0966   3.4272   0.08268 .
winglength           1 3.5532  3.5532 126.0575 5.361e-09 ***
material:winglength  1 0.0000  0.0000   0.0016   0.96862
Residuals           16 0.4510  0.0282
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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
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Inference:
In our study, the main effect of wing length is the only statistically significant effect --with a p-value of 5.361e-09, which is clearly less than .05--on the helicopter's descent time.

Table 5
Tukey multiple comparisons of means (95% family-wise confidence level)

Winglenth (in.)	Difference	Lower	Upper	P adj
2.5 - 1.5	0.843	0.679	1.007	0

On average, the helicopters with a wing length of 2.5 inches fell 0.843 seconds slower than helicopters with a wing length of 1.5 inches (95% CI: 0.679 to 1.007). With a p-value less than 0.05 and no zero in the confidence interval, we can say there is a significant difference between the descent time of helicopters with 2.5 wing lengths and those with 1.5 wing lengths.

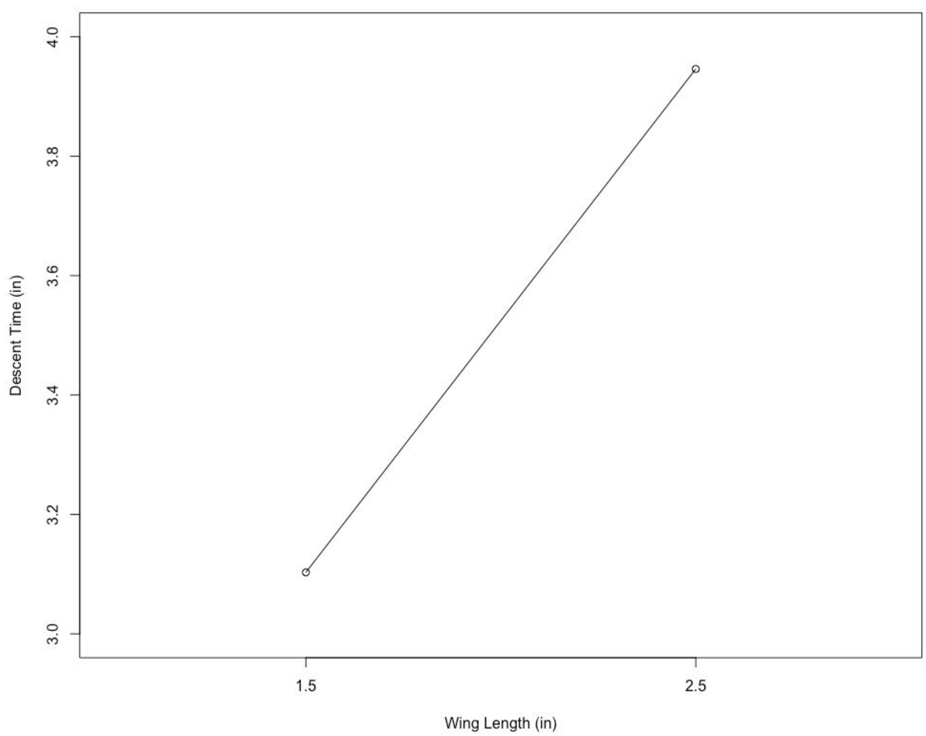

Figure 2.3.: Main Effects Plot of Wing Length (in).

Summary:

The null hypotheses of this experiment were that wing length and material have no effect on a helicopters descent time, and that there is no interaction between wing length and material. The alternative hypotheses were that wing length and/or material do have an effect on a helicopters descent time, and that there is an interaction between the two factors. After performing an ANOVA test on all factors, we found that wing length was the only factor that had a P-Value less than .05, signifying a significant effect on descent time (see Table 4). Overall, helicopters with wing lengths of 2.5 inches fell 0.843 seconds slower than helicopters with wing lengths of 1.5 inches. Based on these findings, in order to change the descent time of a paper helicopter, it is most effective to adjust the wing length. This makes sense because a longer wing will have more surface area causing more drag and a slower descent. There may be other factors such as tail length that could affect the descent time of a paper helicopter that can be explored in further research.