Nearest neighbor for handwritten digit recognition

In this notebook we will build a classifier that takes an image of a handwritten digit and outputs a label 0-9. We will look at a particularly simple strategy for this problem known as the **nearest neighbor classifier**.

To run this notebook you should have the following Python packages installed:

- `numpy`
- `matplotlib`
- `sklearn`

1. The MNIST dataset

`MNIST` is a classic dataset in machine learning, consisting of 28x28 gray-scale images handwritten digits. The original training set contains 60,000 examples and the test set contains 10,000 examples. In this notebook we will be working with a subset of this data: a training set of 7,500 examples and a test set of 1,000 examples.

In [1]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
import time

## Load the training set
train_data = np.load('MNIST/train_data.npy')
train_labels = np.load('MNIST/train_labels.npy')

## Load the testing set
test_data = np.load('MNIST/test_data.npy')
test_labels = np.load('MNIST/test_labels.npy')
```

```
Matplotlib is building the font cache; this may take a moment.
```

In [2]:
```
## Print out their dimensions
print("Training dataset dimensions: ", np.shape(train_data))
print("Number of training labels: ", len(train_labels))
print("Testing dataset dimensions: ", np.shape(test_data))
print("Number of testing labels: ", len(test_labels))
```

```
Training dataset dimensions:  (7500, 784)
Number of training labels:  7500
Testing dataset dimensions:  (1000, 784)
Number of testing labels:  1000
```

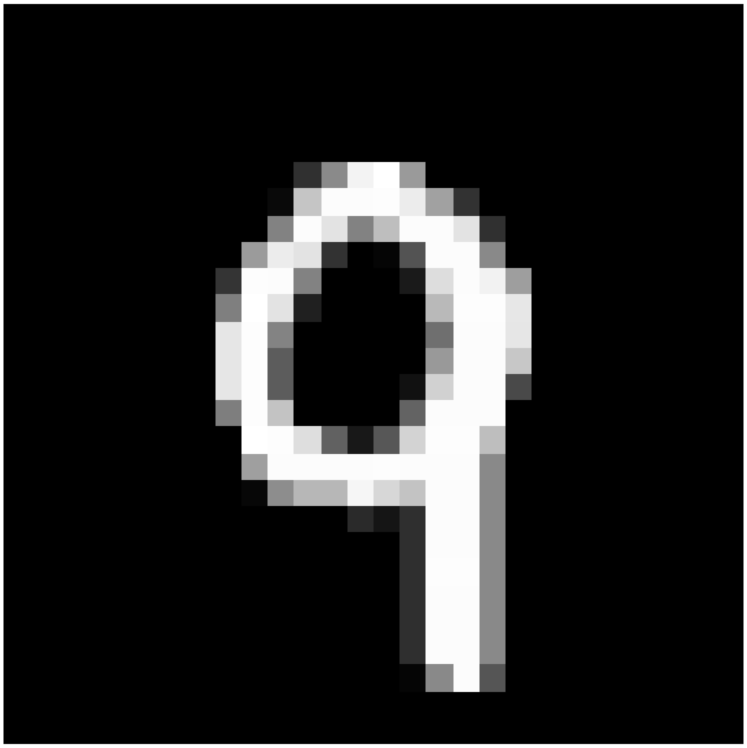
In [3]:
```
## Compute the number of examples of each digit
train_digits, train_counts = np.unique(train_labels, return_counts=True)
print("Training set distribution:")
print(dict(zip(train_digits, train_counts)))

test_digits, test_counts = np.unique(test_labels, return_counts=True)
print("Test set distribution:")
print(dict(zip(test_digits, test_counts)))
```

```
Training set distribution:
{np.uint8(0): np.int64(750), np.uint8(1): np.int64(750), np.uint8(2): np.int
64(750), np.uint8(3): np.int64(750), np.uint8(4): np.int64(750), np.uint8
(5): np.int64(750), np.uint8(6): np.int64(750), np.uint8(7): np.int64(750),
np.uint8(8): np.int64(750), np.uint8(9): np.int64(750)}
Test set distribution:
{np.uint8(0): np.int64(100), np.uint8(1): np.int64(100), np.uint8(2): np.int
64(100), np.uint8(3): np.int64(100), np.uint8(4): np.int64(100), np.uint8
(5): np.int64(100), np.uint8(6): np.int64(100), np.uint8(7): np.int64(100),
np.uint8(8): np.int64(100), np.uint8(9): np.int64(100)}
```

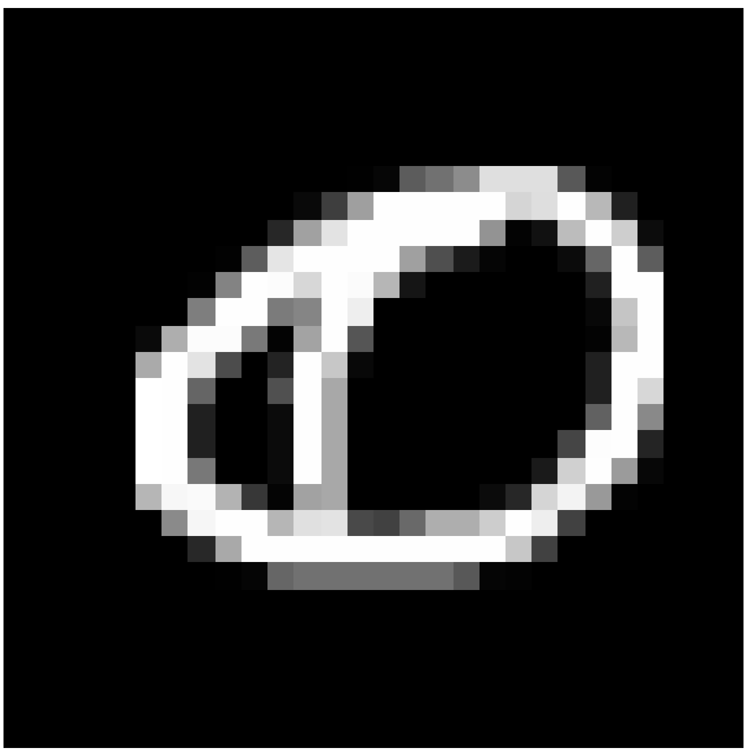
2. Visualizing the data

Each data point is stored as 784-dimensional vector. To visualize a data point, we first reshape it to a 28x28 image.

In [4]:
```
## Define a function that displays a digit given its vector representation
def show_digit(x):
    plt.axis('off')
    plt.imshow(x.reshape((28,28)), cmap=plt.cm.gray)
    plt.show()
    return

## Define a function that takes an index into a particular data set ("train"
def vis_image(index, dataset="train"):
    if(dataset=="train"):
        show_digit(train_data[index,])
        label = train_labels[index]
    else:
        show_digit(test_data[index,])
        label = test_labels[index]
    print("Label " + str(label))
    return

## View the first data point in the training set
vis_image(0, "train")

## Now view the first data point in the test set
vis_image(0, "test")
```



```
Label 9
```



```
Label 0
```

3. Squared Euclidean distance

To compute nearest neighbors in our data set, we need to first be able to compute distances between data points. A natural distance function is *Euclidean distance*: for two vectors $x, y \in \mathbb{R}^d$, their Euclidean distance is defined as

$$\|x - y\| = \sqrt{\sum_{i=1}^{d} (x_i - y_i)^2}.$$

Often we omit the square root, and simply compute *squared Euclidean distance*:

$$\|x - y\|^2 = \sum_{i=1}^{d} (x_i - y_i)^2.$$

For the purposes of nearest neighbor computations, the two are equivalent: for three vectors $x, y, z \in \mathbb{R}^d$, we have $\|x - y\| \leq \|x - z\|$ if and only if $\|x - y\|^2 \leq \|x - z\|^2$.

Now we just need to be able to compute squared Euclidean distance. The following function does so.

In [5]:
```
## Computes squared Euclidean distance between two vectors.
def squared_dist(x,y):
    return np.sum(np.square(x-y))

## Compute distance between a seven and a one in our training set.
print("Distance from 7 to 1: ", squared_dist(train_data[4,],train_data[5,]))

## Compute distance between a seven and a two in our training set.
print("Distance from 7 to 2: ", squared_dist(train_data[4,],train_data[1,]))

## Compute distance between two seven's in our training set.
print("Distance from 7 to 7: ", squared_dist(train_data[4,],train_data[7,]))
```

```
Distance from 7 to 1:  5357193.0
Distance from 7 to 2:  12451684.0
Distance from 7 to 7:  5223403.0
```

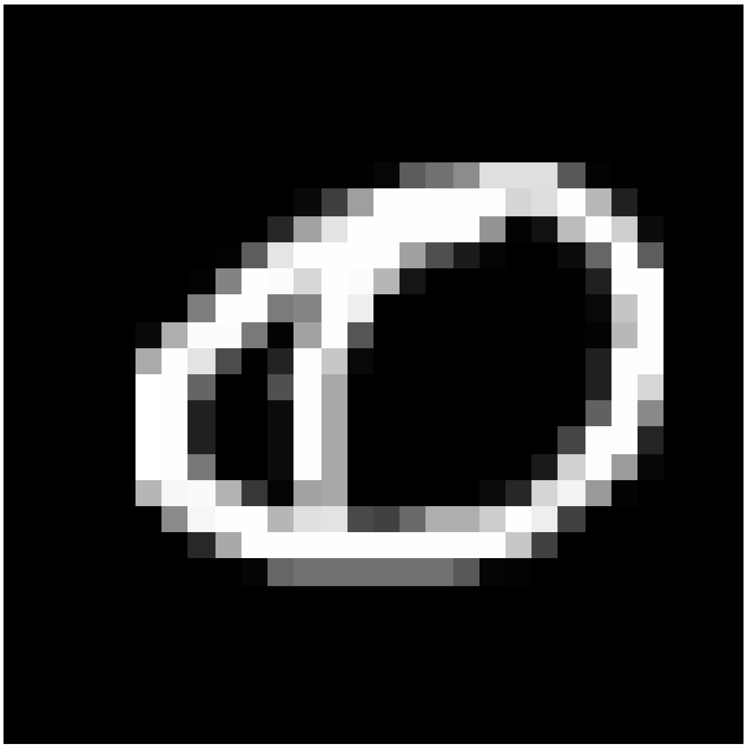
4. Computing nearest neighbors

Now that we have a distance function defined, we can now turn to nearest neighbor classification.

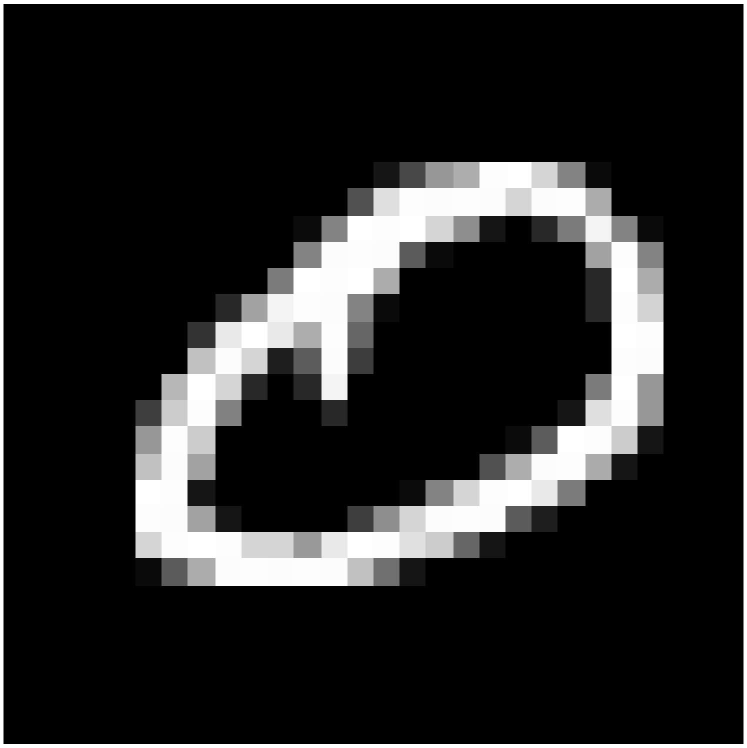
In [6]:
```
## Takes a vector x and returns the index of its nearest neighbor in train_d
def find_NN(x):
    # Compute distances from x to every row in train_data
    distances = [squared_dist(x,train_data[i,]) for i in range(len(train_lab
    # Get the index of the smallest distance
    return np.argmin(distances)

## Takes a vector x and returns the class of its nearest neighbor in train_d
def NN_classifier(x):
    # Get the index of the the nearest neighbor
    index = find_NN(x)
    # Return its class
    return train_labels[index]
```

In [7]:
```
## A success case:
print("A success case:")
print("NN classification: ", NN_classifier(test_data[0,]))
print("True label: ", test_labels[0])
print("The test image:")
vis_image(0, "test")
print("The corresponding nearest neighbor image:")
vis_image(find_NN(test_data[0,]), "train")
```

```
A success case:
NN classification:  0
True label:  0
The test image:
```



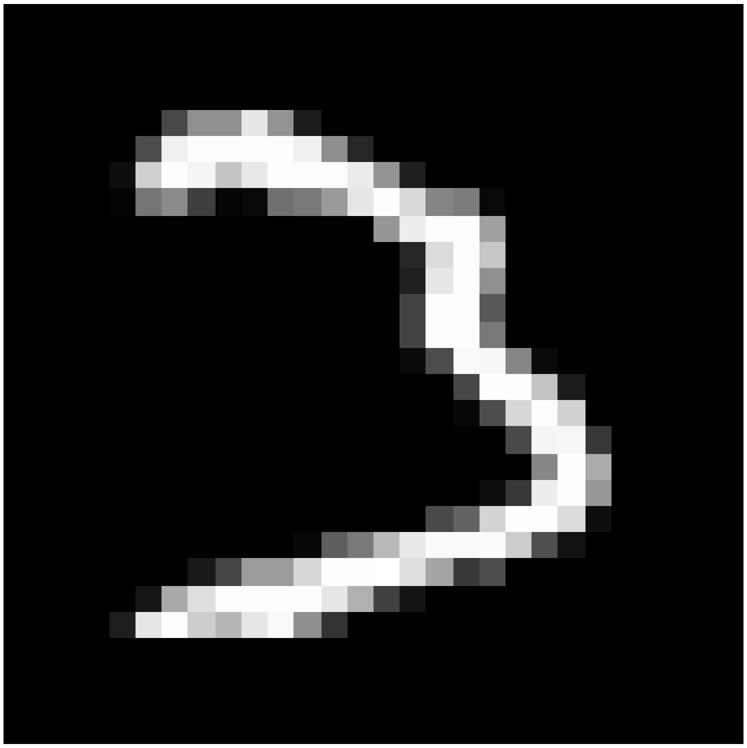
```
Label 0
The corresponding nearest neighbor image:
```



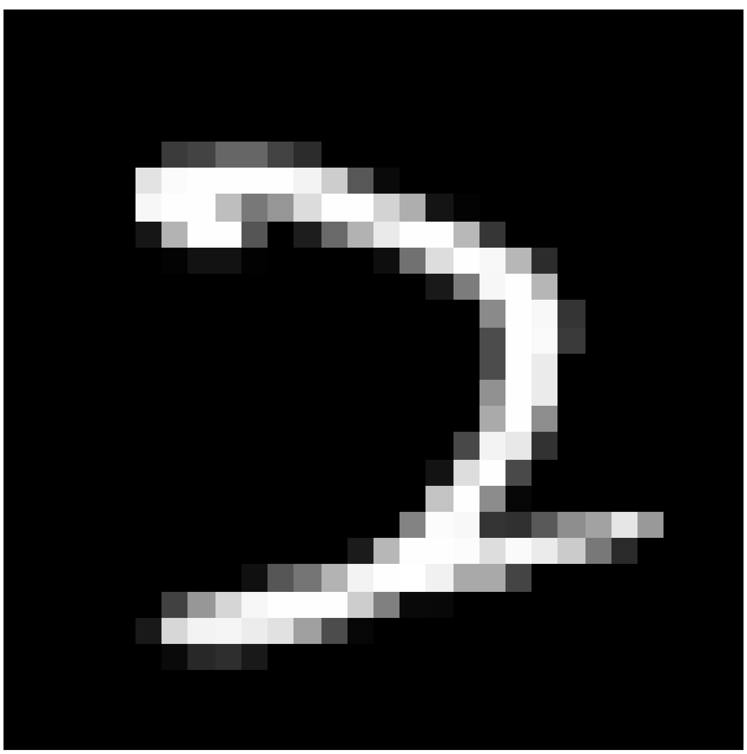
```
Label 0
```

In [8]:
```
## A failure case:
print("A failure case:")
print("NN classification: ", NN_classifier(test_data[39,]))
print("True label: ", test_labels[39])
print("The test image:")
vis_image(39, "test")
print("The corresponding nearest neighbor image:")
vis_image(find_NN(test_data[39,]), "train")
```

```
A failure case:
NN classification:  2
True label:  3
The test image:
```



```
Label 3
The corresponding nearest neighbor image:
```



```
Label 2
```

5. Processing the full test set

Now let's apply our nearest neighbor classifier over the full data set.

Note that to classify each test point, our code takes a full pass over each of the 7500 training examples. Thus we should not expect testing to be very fast. The following code takes about 100-150 seconds on 2.6 GHz Intel Core i5.

In [27]:
```
## Predict on each test data point (and time it!)
t_before = time.time()
test_predictions = [NN_classifier(test_data[i,]) for i in range(len(test_lab
t_after = time.time()

## Compute the error
err_positions = np.not_equal(test_predictions, test_labels)
error = float(np.sum(err_positions))/len(test_labels)

print("Error of nearest neighbor classifier: ", error)
print("Classification time (seconds): ", t_after - t_before)
```

```
Error of nearest neighbor classifier:  0.046
Classification time (seconds):  14.424214124679565
```

6. Faster nearest neighbor methods

Performing nearest neighbor classification in the way we have presented requires a full pass through the training set in order to classify a single point. If there are N training points in $\mathbb{R}^d$, this takes $O(Nd)$ time.

Fortunately, there are faster methods to perform nearest neighbor look up if we are willing to spend some time preprocessing the training set. `scikit-learn` has fast implementations of two useful nearest neighbor data structures: the *ball tree* and the *k-d tree*.

In [26]:
```
from sklearn.neighbors import BallTree

## Build nearest neighbor structure on training data
t_before = time.time()
ball_tree = BallTree(train_data)
t_after = time.time()

## Compute training time
t_training = t_after - t_before
print("Time to build data structure (seconds): ", t_training)

## Get nearest neighbor predictions on testing data
t_before = time.time()
test_neighbors = np.squeeze(ball_tree.query(test_data, k=1, return_distance=
ball_tree_predictions = train_labels[test_neighbors]
t_after = time.time()

## Compute testing time
t_testing = t_after - t_before
print("Time to classify test set (seconds): ", t_testing)
```

```
## Verify that the predictions are the same
print("Ball tree produces same predictions as above? ", np.array_equal(test_
```

```
Time to build data structure (seconds):  0.10273623466491699
Time to classify test set (seconds):  4.070975065231323
Ball tree produces same predictions as above?  True
```

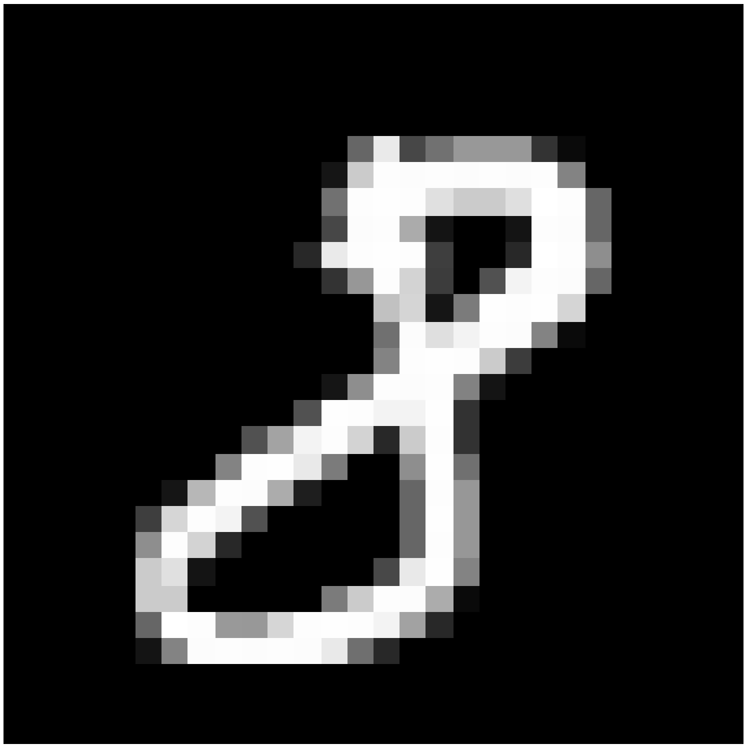
In [28]:
```
from sklearn.neighbors import KDTree

## Build nearest neighbor structure on training data
t_before = time.time()
kd_tree = KDTree(train_data)
t_after = time.time()

## Compute training time
t_training = t_after - t_before
print("Time to build data structure (seconds): ", t_training)

## Get nearest neighbor predictions on testing data
t_before = time.time()
test_neighbors = np.squeeze(kd_tree.query(test_data, k=1, return_distance=Fa
kd_tree_predictions = train_labels[test_neighbors]
t_after = time.time()

## Compute testing time
t_testing = t_after - t_before
print("Time to classify test set (seconds): ", t_testing)

## Verify that the predictions are the same
print("KD tree produces same predictions as above? ", np.array_equal(test_pr
```

```
Time to build data structure (seconds):  0.06371021270751953
Time to classify test set (seconds):  4.248404026031494
KD tree produces same predictions as above?  True
```

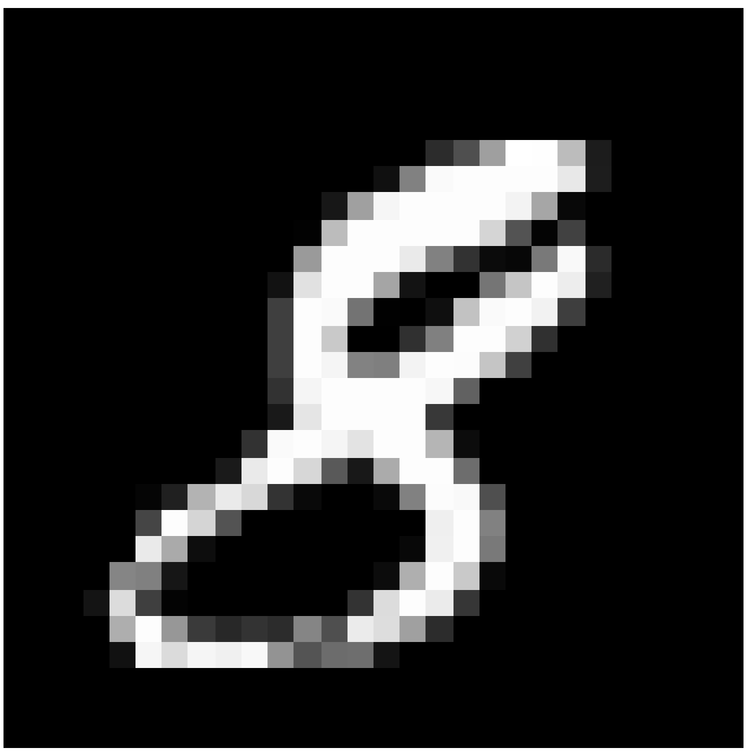
Problem 8a

This test point is classified correctly.

In [25]:
```
vis_image(99, 'test')
vis_image(find_NN(test_data[99,]), 'train')
NN_classifier(test_data[99,])
#vis_image(find_NN(train_data[99,]), 'test')
```



```
Label 8
```



```
Label 8
```

Out[25]:
```
np.uint8(8)
```

Problem 8b

The digit misclassified most often is 9, while the digit misclassified least often (not at all) is 1.

In [30]:
```
from sklearn import metrics
conf_matrix = metrics.confusion_matrix(test_predictions, test_labels)
print(conf_matrix)
```

```
[[ 99   0   0   0   0   1   0   0   2   1]
 [  0 100   1   0   0   0   0   4   0   1]
 [  0   0  94   2   0   0   0   0   1   1]
 [  0   0   1  91   0   0   0   0   1   1]
 [  0   0   0   2  97   0   0   1   1   2]
 [  1   0   0   4   0  98   1   0   0   1]
 [  0   0   0   0   0   0  99   0   1   0]
 [  0   0   3   0   0   0   0  94   1   3]
 [  0   0   1   1   0   0   0   0  92   0]
 [  0   0   0   0   3   1   0   1   1  90]]
```

Problem 8c

Printing the average image of each digit.

In [35]:
```
average_digits = np.zeros((10, 784))
for i in range(10):
    digit_images = test_data[test_labels == i]
    average_digits[i] = np.mean(digit_images, axis = 0)
    show_digit(average_digits[i])
```