HW #3

1. a. $\mu = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ Cov Matrix: $\begin{pmatrix} 1 & 0.25 \\ 0.25 & 1 \end{pmatrix}$

$Cov(X_1, X_2) = E(XY) - E(X)E(Y)$

$Corr(X, Y) = \frac{Cov(X, Y)}{SD(X)\,SD(Y)}$

$-0.5 = \frac{Cov(X_1, X_2)}{1 \cdot 0.5} \Rightarrow Cov(X_1, X_2) = 0.25$

Cov Matrix Σ: $\begin{pmatrix} 1 & -0.25 \\ -0.25 & 1 \end{pmatrix}$

$\mu = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 1 & 0.25 \\ 0.25 & 1 \end{pmatrix}$

b. $\mu = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

2. One cause may be that the training set contains data that is labelled mostly with the class +, so when it generates a prediction it will be biased heavily to the + class.

We could also be using the wrong distributions for our generative model, so the data may not be following the assumptions of the model.

3. $\max_j \pi_j P_j(x)$

$\pi_1 = 0.33$ $\pi_2 = 0.39$ $\pi_3 = 0.28$

$P_1 = N(13.7, 0.20)$ $P(2) = N(12.3, 0.28)$

$P(3) = N(13.2, 0.27)$

a. class 2

b. $.39 \cdot .55 = 0.2145$ $0.28 \cdot 0.55 < 0.2145$

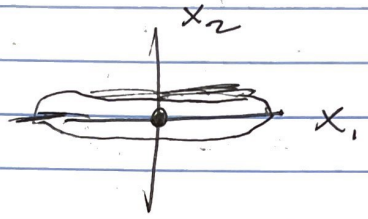
class 2

$.28 \cdot .8 = .204$ $.39 \cdot .3 = .117$ class 3

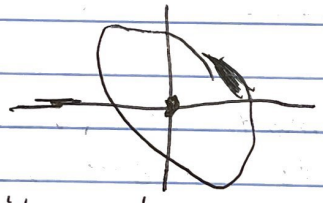
3. d. class 1
e. class 1

4. $N(\mu, \Sigma)$

a. $\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\Sigma = \begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix}$



b. $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 1 & -0.75 \\ -0.75 & 1 \end{bmatrix}$



5. $\mathbb{R}^2$, unit vectors orthogonal to $[1, 1]$

$x + y = 0$ $\quad \vec{v} = \begin{bmatrix} x \\ -x \end{bmatrix}$ $\quad \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad x + y = 0$

$x = -y$

$\sqrt{x^2 + y^2} = 1$

$\sqrt{x^2 + (-x)^2} = \sqrt{x^2 + x^2} = \sqrt{2x^2} = \sqrt{2}|x| = 1$

$|x| = \frac{1}{\sqrt{2}} \Rightarrow x = \pm \frac{1}{\sqrt{2}}$

$v_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$ $\quad v_2 = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

$v_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\quad v_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

6. $x_1 \cdot x = x_1^2 + \ldots + x_d^2 = ||x||^2$

$||x||^2 = 25$ $\quad ||x|| = 5$

$S = \{x \in \mathbb{R}^d : ||x|| = 5\}$

This set contains all points in $\mathbb{R}^d$ that are 5 units away from the origin

7. $f(x) = 2x_1 - x_2 + 6x_3$ w. x for $x \in \mathbb{R}^3$

$w = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

8. AB : 10×20

A : 10×30 B : 30×20

9. $x^{(1)}, \ldots, x^{(n)} \in \mathbb{R}^d$

a. X : $n \times d$

b. XX^T

$n \times d$ $d \times n$ $n \times n$

XX^T : $n \times n$

c. $(XX^T)_{ij} = x_i x_j$

10. $x = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ $x^T x$, xx^T

$x^T x = \begin{bmatrix} 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = 35$ $x^T x = 35$

$xx^T = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 9 & 15 \\ 5 & 15 & 25 \end{bmatrix}$ $xx^T =$

11. $f: \mathbb{R}^3 \to \mathbb{R}$ $f(x) = 3x_1^2 + 2x_1x_2 - 4x_1x_3 + 6x_3^2$

$x^T M x = \sum_{ij} M_{ij} x_i x_j$

$M = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$

$2x_1x_2$ $1+1=2$

$-4x_1x_3$ $M_{13} = M_{31} = -2$

$\begin{bmatrix} 3 & 2 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

12. $A = \text{diag}(1, 2, 3, 4, 5, 6, 7, 8)$

a. $|A| = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8$

$= \boxed{40320}$

1 3
336 5
1680
8 6720
3
20160
40320

b. $A^{-1} = \text{diag}(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8})$

$$\begin{bmatrix} 1 & & 0 \\ & 1/2 & \\ & & \ddots \\ 0 & & 1/8 \end{bmatrix}$$