DSC 255R

1. b. $\|x-z\|_1 = \sum_{i=1}^{m} |x_i - z_i|$

$\sum_{i=1}^{4} |x_i - z_i| = \begin{bmatrix} -2 \\ 0 \\ -2 \\ 0 \end{bmatrix} \sum_{i=1}^{n} \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix} = \boxed{4}$

mixed up the order

a. $\sqrt{(-2)^2 + 0^2 + (-2)^2 + 0^2} = \sqrt{8} = \boxed{2\sqrt{2}}$

c. $\|x\|_\infty$ $\quad L_\infty = \max_i |x_i - z_i|$

$= \max \begin{bmatrix} |-2| \\ |0| \\ |-2| \\ |0| \end{bmatrix} = \boxed{2}$

2. $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

a. $\|x\|_1 = 1+2+3 = \boxed{6}$

b. $\|x\|_2 = \sqrt{1^2 + 2^2 + 3^2} = \boxed{\sqrt{14}}$

c. $\|x\|_\infty = \max_i |x_i| = \boxed{3}$

3. $d(x,y) \geq 0$ ✓ all distances are nonnegative

$d(x,y) = 0$ iff $x = y$ ✓ true for all entries, it's only equal to 0 when x and y are the same.

$d(x,y) = d(y,x)$ ✓ symmetric for all $x, y \in \{A, B, C, D\}$

$d(x,z) \leq d(x,y) + d(y,z)$

✗ x, $d(A, D) \geq d(A, C) + d(A, D)$

$5 \geq 1+2$

This distance function is not a metric because it violates the triangle inequality condition.

4. $d(p, q) = \sum_{x \in X} p(x) \log_2 \frac{p(x)}{q(x)}$

$= \frac{1}{2}\log_2(1/2 / 1/4) + \frac{1}{4}\log_2(1/4 / 1/4) + \ldots$

$+ 1/16 \log_2(\frac{1}{16} / 1/6)$

$= 1/2 + 0 + 1/8 \log_2 3/4 + \frac{1}{16}\log_2 3/8 + \frac{1}{16}\log_2 3/8$

$\approx \boxed{0.2712}$

5. a. classification
b. regression
c. regression
d. classification

6. a. $\cancel{E(X)^2} \ \cancel{+E(X)} \ E(X^2) - E(X)^2$

$E(X) = 0 \quad E(X^2) = 1$

$Var(X) = \boxed{1}$

b. $E(X) = x \quad Var(X) = 0$

c. $X \in \{0, 1\} \quad X = 1$ w/ $p = 1/4$

$E(X) = \cancel{\frac{1 \cdot 1/4 + 0 \cdot 3/4}{2}} = \boxed{\frac{1}{4}} \quad E(X^2) = \cancel{\frac{1/16}{2}} = \cancel{1/32} \ \frac{1}{16}$

$\cancel{Var(X) = 1/32}$

$Var(X) = \frac{1}{4} - \frac{1}{16} = \boxed{3/16}$

7. a. $\mathrm{Cov}(X,Y) = E\big((X-E(X))(Y-E(Y))\big)$
$= E(XY) - E(X)E(Y)$

$E(X) = -1(0+0+\frac{1}{3}) + 0(\frac{1}{3}) + 1(\frac{1}{3}) = 0$

$E(Y) = \cancel{-1(1/6+1/6) + 0(\frac{1}{3}) + 1(\frac{1}{6}+1/6)}$

$-1(1/3) + 0 + 1 \cdot 1/3 = 0$

$E(XY) = 1 \cdot -1 \cdot (1/3) + 1 \cdot -1 \cdot (\frac{1}{3}) = -2/3$

$\mathrm{Cov}(X,Y) = -2/3 - 0 = \boxed{-2/3}$

b. $\mathrm{corr}(X,Y) = \dfrac{\mathrm{cov}(X,Y)}{\mathrm{std}(X)\,\mathrm{std}(Y)}$

$\mathrm{Var}(X) = \frac{1}{3} + \frac{1}{3} = 2/3$ $\quad \mathrm{std}(X) = \sqrt{2/3}$

$\mathrm{Var}(Y) = 2/3$ $\quad \mathrm{std}(Y) = \sqrt{2/3}$

$\mathrm{corr}(X,Y) = \dfrac{-2/3}{\sqrt{2/3}\cdot\sqrt{2/3}} = \boxed{-1}$

8. a. $P(X=x, Y=y) = P(X=x) \cdot P(Y=y)\ \forall x,y$

$P(X=-1, Y=-1) = 1/6$

$P(X=-1) \cdot P(Y=-1) = 1/3 \cdot 1/3 = 1/9$

$\boxed{\text{X and Y are not independent.}}$

b. $\mathrm{Cov}(X,Y)$

$E(XY) - E(X)E(Y)$

$E(XY) = \sum_{x,y} xy \cdot P(X=x, Y=y)$

$= -1 \cdot -1 \cdot \frac{1}{6} + \ldots + 1 \cdot 1 \cdot \frac{1}{6}$

$= 0$

$E(X) = -1 \cdot (1/6 + 1/6) + 0 + 1 \cdot (1/6 + 1/6) = 0$

$E(Y) = 0$ $\quad \mathrm{Cov}(X,Y) = 0$

X and Y are uncorrelated