

hw8

March 2, 2025

```
[2]: %pip install tensorflow  
      %pip install torch torchvision
```

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Requirement already satisfied: tensorflow in  
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages  
(2.18.0)  
Requirement already satisfied: absl-py>=1.0.0 in  
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages  
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Requirement already satisfied: astunparse>=1.6.0 in  
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Requirement already satisfied: gast!=0.5.0,!0.5.1,!0.5.2,>=0.2.1 in  
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protobuf!=4.21.0,!4.21.1,!4.21.2,!4.21.3,!4.21.4,!4.21.5,<6.0.0dev,>=3.20.3  
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Requirement already satisfied: tensorboard<2.19,>=2.18 in
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Requirement already satisfied: keras>=3.5.0 in
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Requirement already satisfied: wheel<1.0,>=0.23.0 in
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(from astunparse>=1.6.0->tensorflow) (0.45.1)
Requirement already satisfied: rich in
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(from keras>=3.5.0->tensorflow) (13.9.4)
Requirement already satisfied: namex in
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(from keras>=3.5.0->tensorflow) (0.0.8)
Requirement already satisfied: optree in
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(from keras>=3.5.0->tensorflow) (0.14.1)
Requirement already satisfied: charset-normalizer<4,>=2 in
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(from requests<3,>=2.21.0->tensorflow) (3.4.1)
Requirement already satisfied: idna<4,>=2.5 in

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(from requests<3,>=2.21.0->tensorflow) (3.10)
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Requirement already satisfied: certifi>=2017.4.17 in
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Requirement already satisfied: markdown>=2.6.8 in
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(from tensorboard<2.19,>=2.18->tensorflow) (3.7)
Requirement already satisfied: tensorboard-data-server<0.8.0,>=0.7.0 in
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages
(from tensorboard<2.19,>=2.18->tensorflow) (0.7.2)
Requirement already satisfied: werkzeug>=1.0.1 in
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages
(from tensorboard<2.19,>=2.18->tensorflow) (3.1.3)
Requirement already satisfied: MarkupSafe>=2.1.1 in
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(from werkzeug>=1.0.1->tensorboard<2.19,>=2.18->tensorflow) (3.0.2)
Requirement already satisfied: markdown-it-py>=2.2.0 in
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(from rich->keras>=3.5.0->tensorflow) (3.0.0)
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(from rich->keras>=3.5.0->tensorflow) (2.19.1)
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/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages
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Note: you may need to restart the kernel to use updated packages.
Requirement already satisfied: torch in
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(from torch) (3.17.0)
Requirement already satisfied: typing-extensions>=4.10.0 in
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Requirement already satisfied: jinja2 in
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(from torch) (3.1.5)

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Requirement already satisfied: fsspec in
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(from torch) (2025.2.0)

Requirement already satisfied: setuptools in
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(from torch) (75.8.0)

Requirement already satisfied: sympy==1.13.1 in
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages
(from torch) (1.13.1)

Requirement already satisfied: mpmath<1.4,>=1.1.0 in
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages
(from sympy==1.13.1->torch) (1.3.0)

Requirement already satisfied: numpy in
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages
(from torchvision) (2.0.2)

Requirement already satisfied: pillow!=8.3.*,>=5.3.0 in
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages
(from torchvision) (11.1.0)

Requirement already satisfied: MarkupSafe>=2.0 in
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages
(from jinja2->torch) (3.0.2)

Note: you may need to restart the kernel to use updated packages.

```
[4]: import pandas as pd
import numpy as np
from sklearn.svm import LinearSVC
from sklearn.svm import SVC
import matplotlib.pyplot as plt
from sklearn.gaussian_process import GaussianProcessClassifier
from sklearn.gaussian_process.kernels import RBF
from tensorflow.keras.datasets import mnist
import os
import sys
import gzip
if sys.version_info[0] == 2:
    from urllib import urlretrieve
else:
    from urllib.request import urlretrieve
from sklearn.metrics import accuracy_score
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split
```

```
[2]: df1 = np.loadtxt('week8/data1.txt')
df2 = np.loadtxt('week8/data2.txt')
```

```
[3]: def quadratic_kernel(x1, x2):
    return (np.dot(x1, x2) + 1) ** 2
```

```
def rbf_kernel(x1, x2, gamma=0.1):
    return np.exp(-gamma * np.linalg.norm(x1 - x2) ** 2)
```

```
[12]: class KernelPerceptron:
    def __init__(self, kernel=quadratic_kernel):
        self.kernel = kernel
        self.alpha = None
        self.b = 0
        self.labels = None
        self.X = None

    def fit(self, X, y):
        n_samples = X.shape[0]
        self.alpha = np.zeros(n_samples)
        self.labels = y
        self.X = X
        misclassified = True

        while misclassified:
            misclassified = False
            for i in range(n_samples):
                pred_sum = 0
                for j in range(n_samples):
                    pred_sum += self.alpha[j] * y[j] * self.kernel(X[j], X[i])
                pred = np.sign(pred_sum + self.b)
                if pred != y[i]:
                    self.alpha[i] += 1
                    self.b += y[i]
                    misclassified = True

    def predict(self, X):
        predictions = []
        for x in X:
            pred_sum = 0
            for i in range(len(self.alpha)):
                pred_sum += self.alpha[i] * self.labels[i] * self.kernel(self.
↪X[i], x)
            pred_sum += self.b
            pred = np.sign(pred_sum)
            predictions.append(pred)

        return np.array(predictions)
```

```
[13]: def plot_decision_boundary(X, y, model, title="Decision Boundary"):
    h = 0.01
    x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
```

```

y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1

xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                     np.arange(y_min, y_max, h))

Z = model.predict(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)

plt.contourf(xx, yy, Z, alpha=0.3, cmap=plt.cm.coolwarm)
plt.scatter(X[:, 0], X[:, 1], c=y, s=30, cmap=plt.cm.coolwarm,
            ↪edgecolors='k')
plt.title(title)
plt.show()

```

```

[14]: X1, y1 = df1[:, :2], df1[:, 2]
      X2, y2 = df2[:, :2], df2[:, 2]

model_quadratic = KernelPerceptron(kernel=quadratic_kernel)
model_quadratic.fit(X1, y1)
plot_decision_boundary(X1, y1, model_quadratic, title="Quadratic Kernel - df1")

model_quadratic_data2 = KernelPerceptron(kernel=quadratic_kernel)
model_quadratic_data2.fit(X2, y2)
plot_decision_boundary(X2, y2, model_quadratic_data2, title="Quadratic Kernel - ↪
            ↪df2")

gamma_1 = 0.1
model_rbf_data1 = KernelPerceptron(kernel=lambda x1, x2: rbf_kernel(x1, x2, ↪
            ↪gamma_1))
model_rbf_data1.fit(X1, y1)
plot_decision_boundary(X1, y1, model_rbf_data1, title=f"RBF Kernel ↪
            ↪(gamma={gamma_1}) - df1")

model_rbf_data2 = KernelPerceptron(kernel=lambda x1, x2: rbf_kernel(x1, x2, ↪
            ↪gamma_1))
model_rbf_data2.fit(X2, y2)
plot_decision_boundary(X2, y2, model_rbf_data2, title=f"RBF Kernel ↪
            ↪(gamma={gamma_1}) - df2")

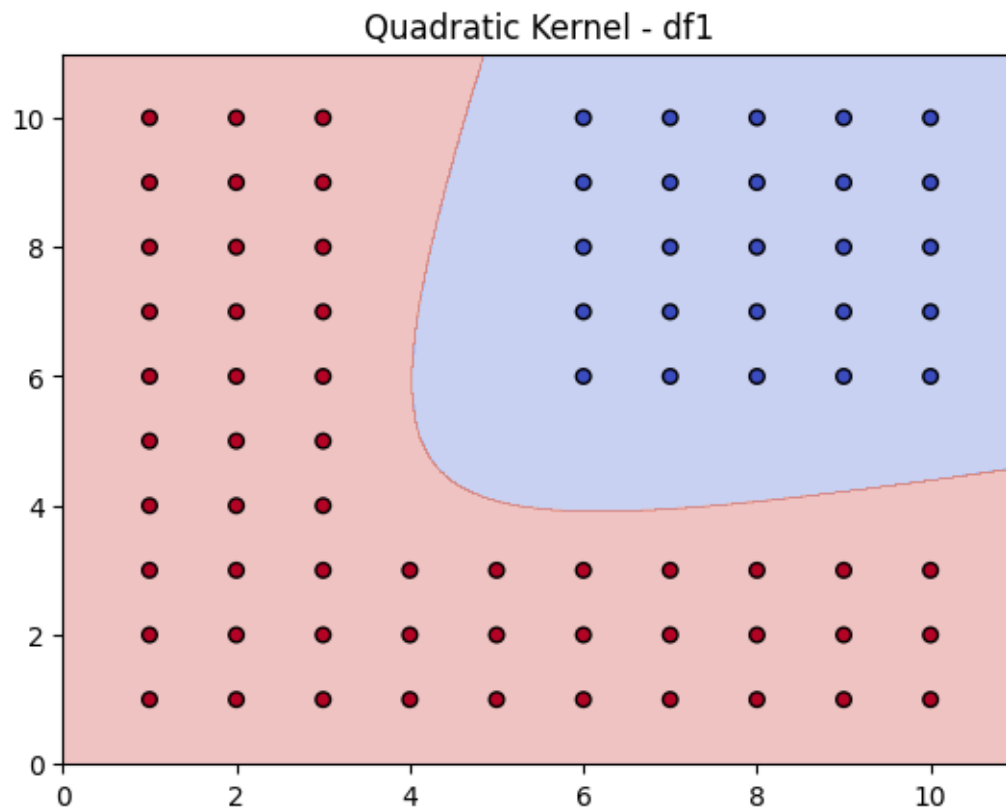
gamma_2 = 0.5
model_rbf_data1_2 = KernelPerceptron(kernel=lambda x1, x2: rbf_kernel(x1, x2, ↪
            ↪gamma_2))
model_rbf_data1_2.fit(X1, y1)
plot_decision_boundary(X1, y1, model_rbf_data1_2, title=f"RBF Kernel ↪
            ↪(gamma={gamma_2}) - df1")

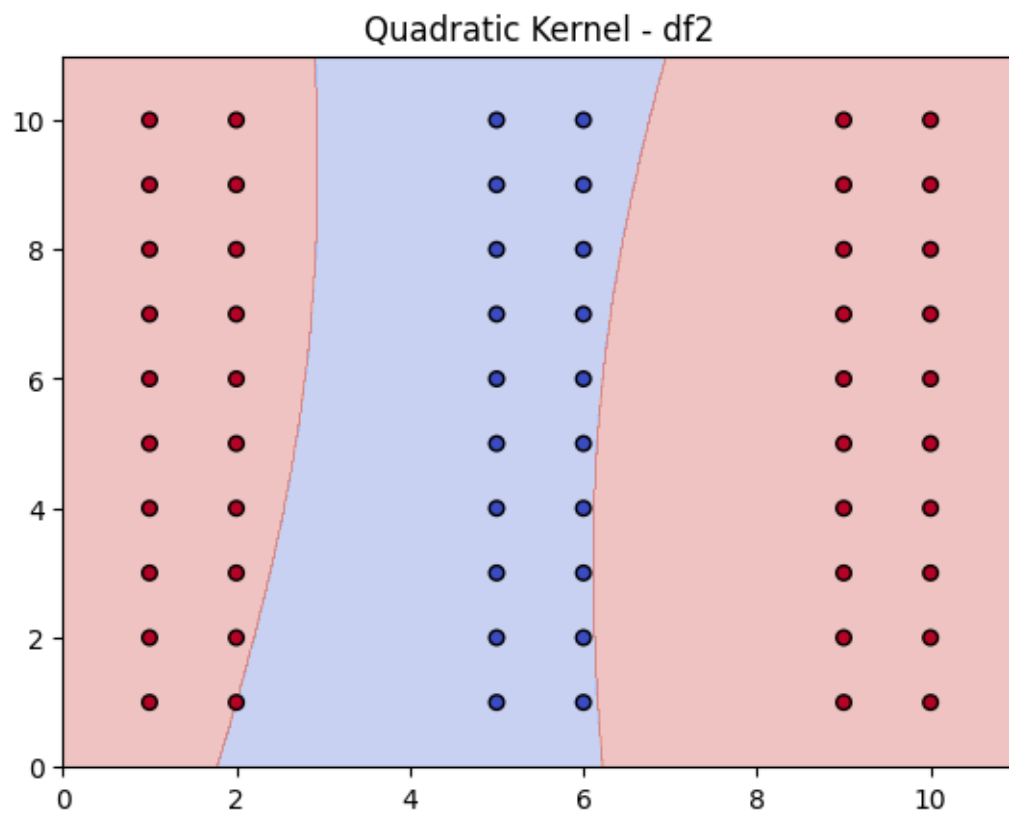
```

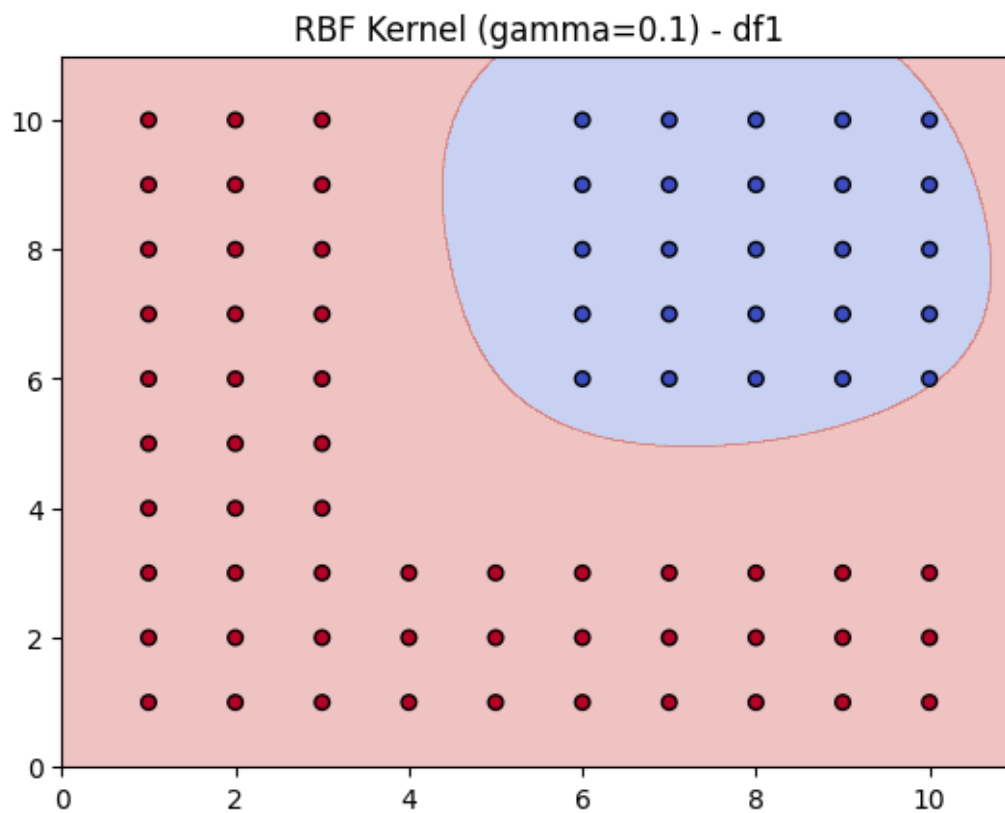
```

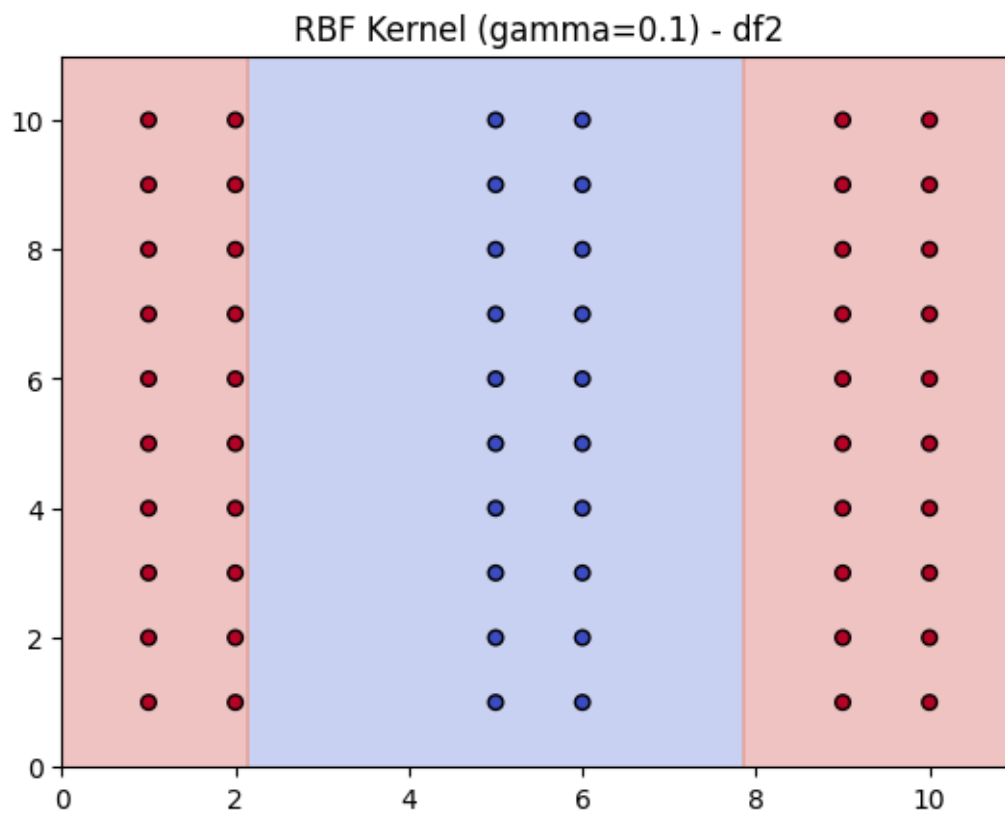
model_rbf_data2_2 = KernelPerceptron(kernel=lambda x1, x2: rbf_kernel(x1, x2,
    ↪gamma_2))
model_rbf_data2_2.fit(X2, y2)
plot_decision_boundary(X2, y2, model_rbf_data2_2, title=f"RBF Kernel
    ↪(gamma={gamma_2}) - df2")

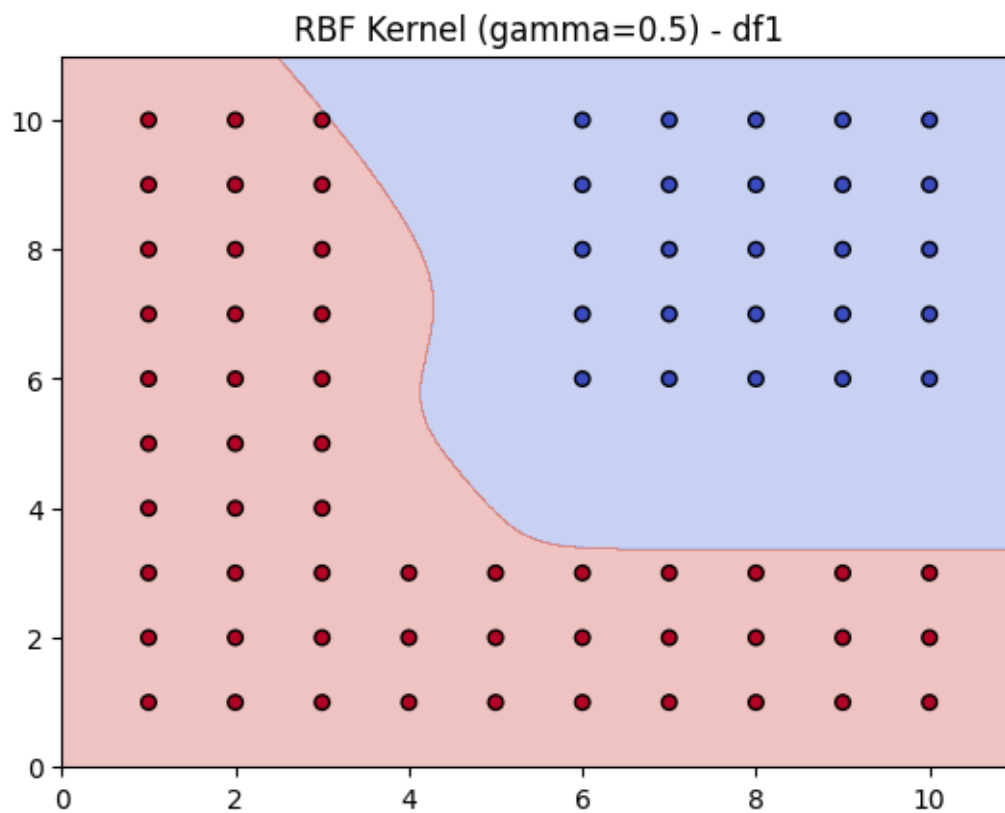
```

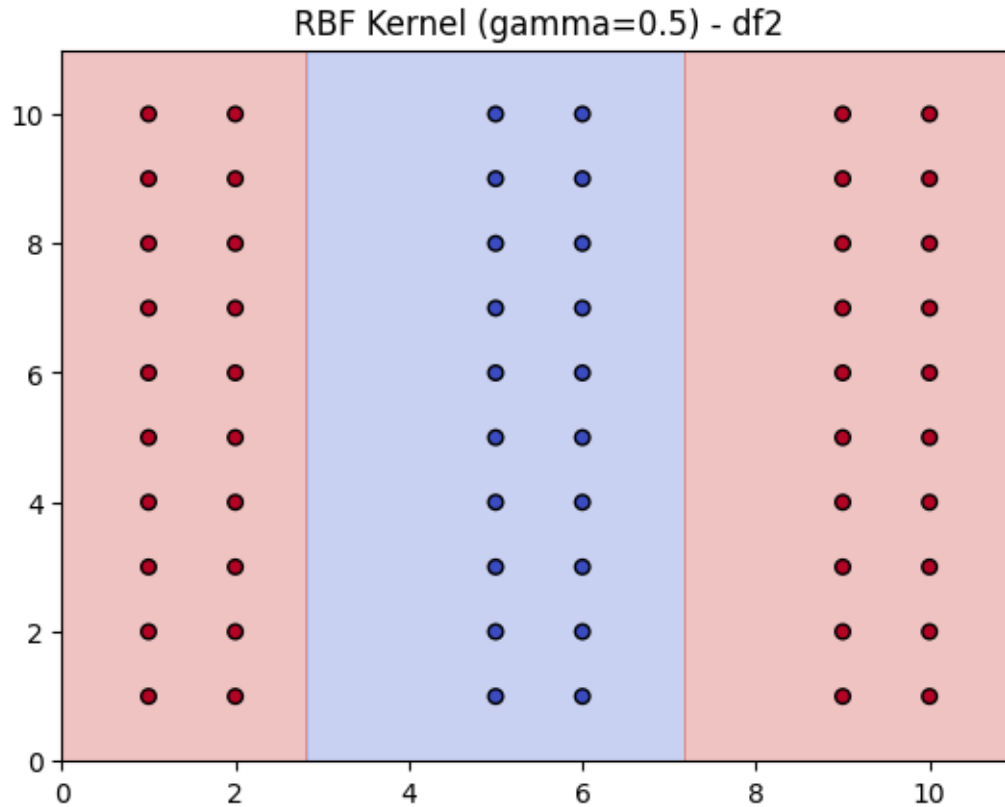










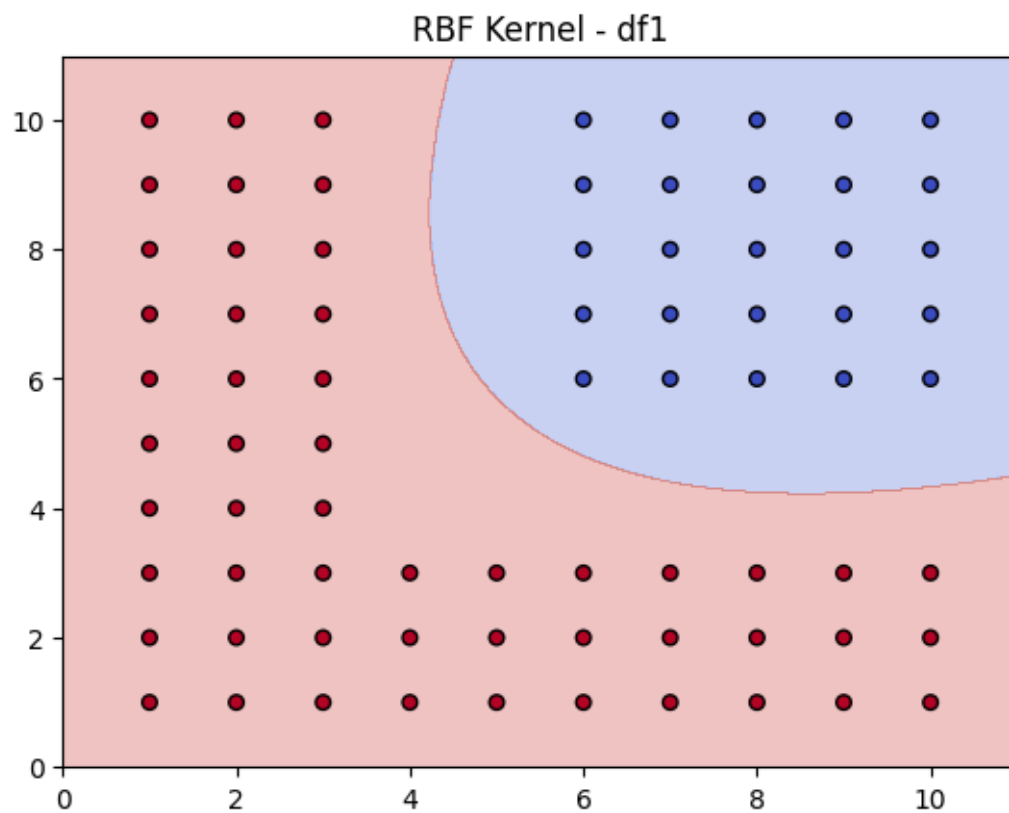


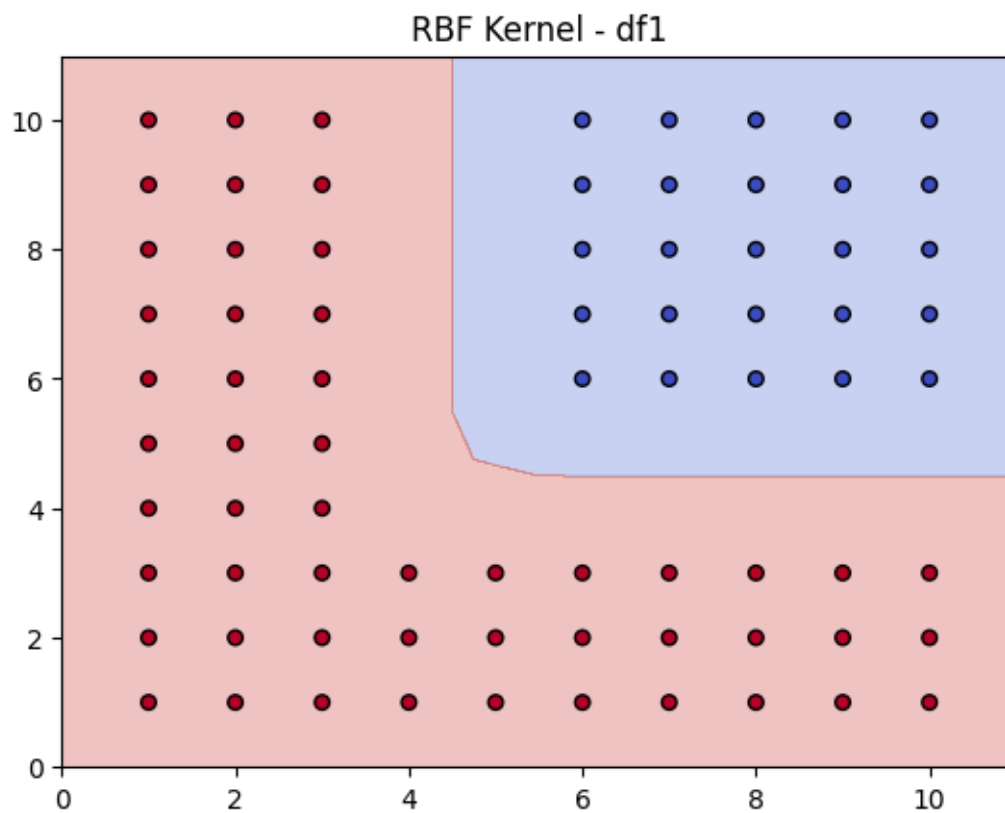
```
[11]: kernel = 1.0 * RBF(1.0)
      gpc = GaussianProcessClassifier(kernel=kernel,
      random_state=0).fit(X1, y1)
      plot_decision_boundary(X1, y1, gpc, title='RBF Kernel - df1, gamma = 1')

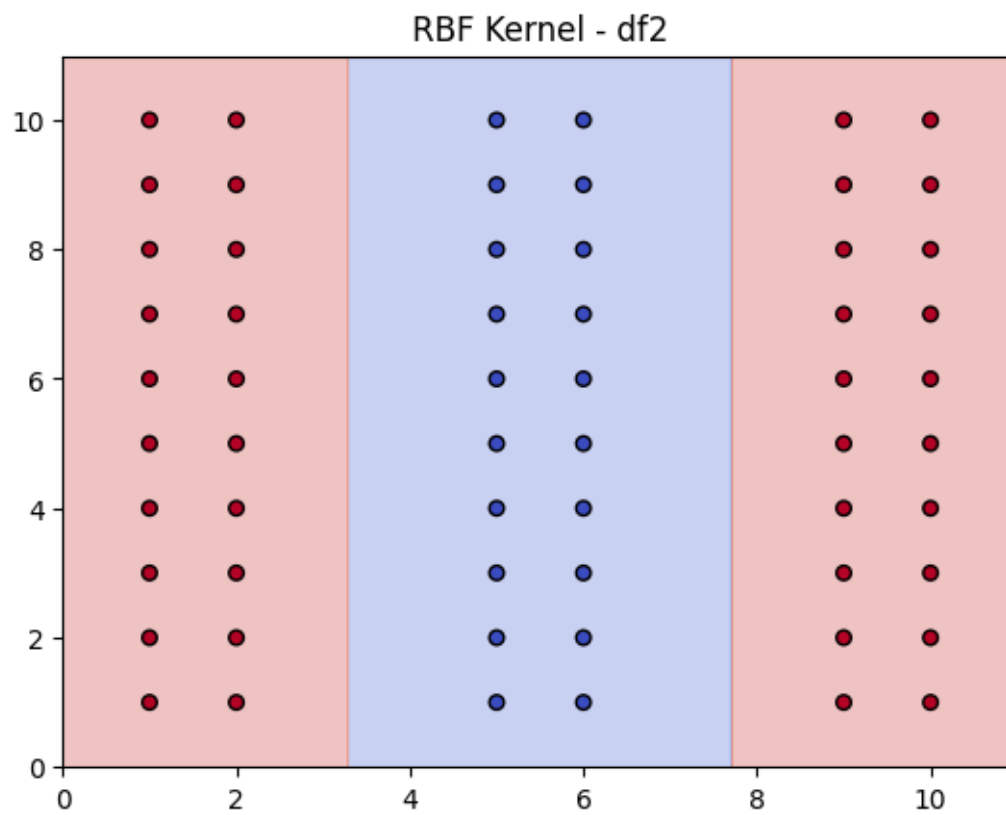
      kernel = 1.0 * RBF(0.1)
      gpc = GaussianProcessClassifier(kernel=kernel,
      random_state=0).fit(X1, y1)
      plot_decision_boundary(X1, y1, gpc, title='RBF Kernel - df1, gamma = 0.1')

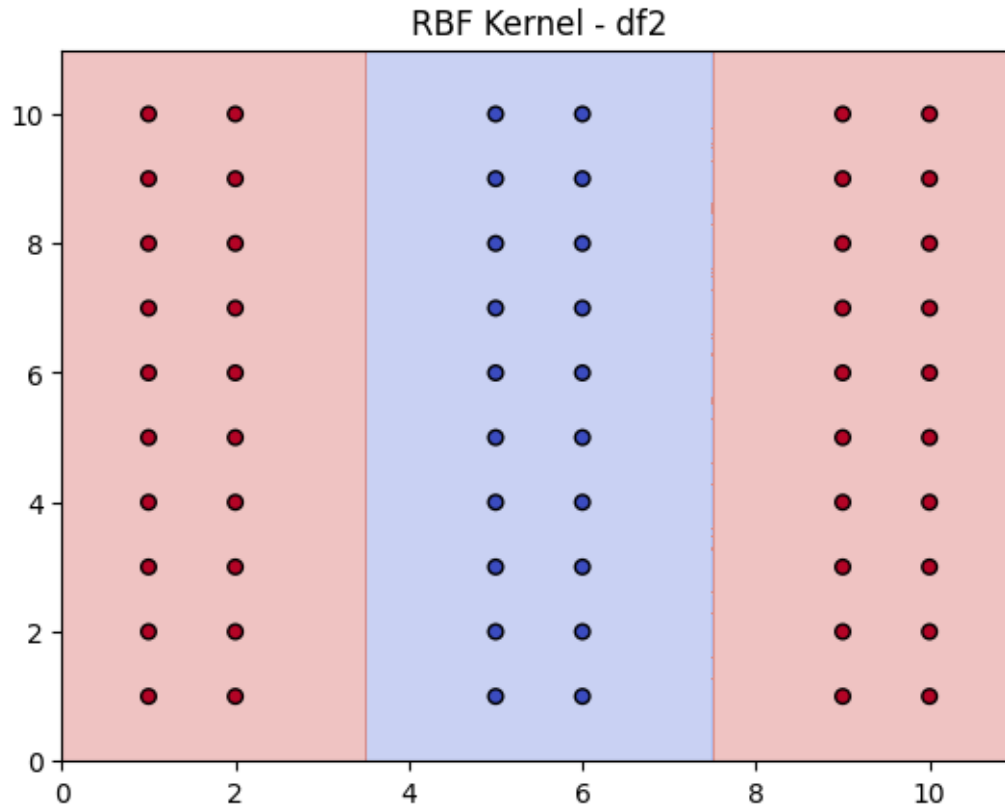
      kernel = 1.0 * RBF(1.0)
      gpc = GaussianProcessClassifier(kernel=kernel,
      random_state=0).fit(X2, y2)
      plot_decision_boundary(X2, y2, gpc, title='RBF Kernel - df2, gamma = 1')

      kernel = 1.0 * RBF(0.1)
      gpc = GaussianProcessClassifier(kernel=kernel,
      random_state=0).fit(X2, y2)
      plot_decision_boundary(X2, y2, gpc, title='RBF Kernel - df2, gamma = 0.1')
```









```
[9]: def download(filename, source='http://yann.lecun.com/exdb/mnist/'):
    print("Downloading %s" % filename)
    urlretrieve(source + filename, filename)

    # Invokes download() if necessary, then reads in images
    def load_mnist_images(filename):
        if not os.path.exists(filename):
            download(filename)
        with gzip.open(filename, 'rb') as f:
            data = np.frombuffer(f.read(), np.uint8, offset=16)
        data = data.reshape(-1, 784)
        return data

    def load_mnist_labels(filename):
        if not os.path.exists(filename):
            download(filename)
        with gzip.open(filename, 'rb') as f:
            data = np.frombuffer(f.read(), np.uint8, offset=8)
        return data
```



```
[10]: X_train = load_mnist_images('train-images-idx3-ubyte.gz')
y_train = load_mnist_labels('train-labels-idx1-ubyte.gz')

X_test = load_mnist_images('t10k-images-idx3-ubyte.gz')
y_test = load_mnist_labels('t10k-labels-idx1-ubyte.gz')

print(f"Training data shape: {X_train.shape}")
print(f"Test data shape: {X_test.shape}")
print(f"Training labels shape: {y_train.shape}")
print(f"Test labels shape: {y_test.shape}")
```

```
Training data shape: (60000, 784)
Test data shape: (10000, 784)
Training labels shape: (60000,)
Test labels shape: (10000,)
```

```
[11]: X_train = X_train.astype('float32') / 255.0 # Normalize
X_test = X_test.astype('float32') / 255.0 # Normalize

X_train = X_train.reshape(-1, 28 * 28)
X_test = X_test.reshape(-1, 28 * 28)

scaler = StandardScaler()
X_train = scaler.fit_transform(X_train)
X_test = scaler.transform(X_test)
```

```
[ ]: C_values = [0.01, 0.1, 1.0, 10.0, 100.0]
linear_svm_results = {}

for C in C_values:
    print(f"Training Linear SVM with C={C}...")
    model = LinearSVC(C=C, loss='hinge', max_iter=10000)
    model.fit(X_train, y_train)

    train_pred = model.predict(X_train)
    test_pred = model.predict(X_test)

    train_accuracy = accuracy_score(y_train, train_pred)
    test_accuracy = accuracy_score(y_test, test_pred)

    linear_svm_results[C] = {'train_accuracy': train_accuracy, 'test_accuracy':
    test_accuracy}
    print(f"Training Accuracy: {train_accuracy}")
    print(f"Test Accuracy: {test_accuracy}")
    print("-----")

print("Training Kernel SVM with quadratic kernel (degree=2)..." )
```

```

kernel_svm_model = SVC(kernel='poly', degree=2, C=1.0)
kernel_svm_model.fit(X_train, y_train)

train_pred_kernel = kernel_svm_model.predict(X_train)
test_pred_kernel = kernel_svm_model.predict(X_test)

train_accuracy_kernel = accuracy_score(y_train, train_pred_kernel)
test_accuracy_kernel = accuracy_score(y_test, test_pred_kernel)

n_support_vectors = len(kernel_svm_model.support_vectors_)

print(f"Kernel SVM (quadratic kernel) Training Accuracy: {train_accuracy_kernel}")
print(f"Kernel SVM (quadratic kernel) Test Accuracy: {test_accuracy_kernel}")
print(f"Number of Support Vectors: {n_support_vectors}")

```

Training Linear SVM with C=0.01...

```

/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-
packages/sklearn/svm/_base.py:1249: ConvergenceWarning: Liblinear failed to
converge, increase the number of iterations.

```

```
warnings.warn(
```

Training Accuracy: 0.92115

Test Accuracy: 0.9157

Training Linear SVM with C=0.1...

```

/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-
packages/sklearn/svm/_base.py:1249: ConvergenceWarning: Liblinear failed to
converge, increase the number of iterations.

```

```
warnings.warn(
```

Training Accuracy: 0.92915

Test Accuracy: 0.9195

Training Linear SVM with C=1.0...

```

/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-
packages/sklearn/svm/_base.py:1249: ConvergenceWarning: Liblinear failed to
converge, increase the number of iterations.

```

```
warnings.warn(
```

Training Accuracy: 0.9309

Test Accuracy: 0.9182

Training Linear SVM with C=10.0...

1 Problem 7b.

1.1 The dataset does not seem to be linearly separable because for larger values of C , the test error increases, thus indicating that the model is overfitting and the data is not well-separated.

1.1.1 The training was taking too long for the higher values of C , but I would use more max iterations if possible to increase the accuracies.

```
[14]: linear_svm_results = {}
C = 10.0
print(f"Training Linear SVM with C={10.0}...")
model = LinearSVC(C=C, loss='hinge', max_iter=1000)
model.fit(X_train, y_train)

train_pred = model.predict(X_train)
test_pred = model.predict(X_test)

train_accuracy = accuracy_score(y_train, train_pred)
test_accuracy = accuracy_score(y_test, test_pred)

linear_svm_results[C] = {'train_accuracy': train_accuracy, 'test_accuracy':
    ↪test_accuracy}
print(f"Training Accuracy: {train_accuracy}")
print(f"Test Accuracy: {test_accuracy}")
print("-----")

C = 100.0
print(f"Training Linear SVM with C={100.0}...")
model = LinearSVC(C=C, loss='hinge', max_iter=1000)
model.fit(X_train, y_train)

train_pred = model.predict(X_train)
test_pred = model.predict(X_test)

train_accuracy = accuracy_score(y_train, train_pred)
test_accuracy = accuracy_score(y_test, test_pred)

linear_svm_results[C] = {'train_accuracy': train_accuracy, 'test_accuracy':
    ↪test_accuracy}
print(f"Training Accuracy: {train_accuracy}")
print(f"Test Accuracy: {test_accuracy}")
print("-----")
```

Training Linear SVM with C=10.0...

/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages/sklearn/svm/_base.py:1249: ConvergenceWarning: Liblinear failed to converge, increase the number of iterations.

```
warnings.warn(
Training Accuracy: 0.8850166666666667
Test Accuracy: 0.876
-----
Training Linear SVM with C=100.0...
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-
packages/sklearn/svm/_base.py:1249: ConvergenceWarning: Liblinear failed to
converge, increase the number of iterations.
    warnings.warn(
Training Accuracy: 0.88225
Test Accuracy: 0.8762
-----
```

```
[ ]: %%timeit
print("Training Kernel SVM with quadratic kernel (degree=2)...")
kernel_svm_model = SVC(kernel='poly', degree=2, C=1.0, max_iter=1000)
kernel_svm_model.fit(X_train, y_train)

train_pred_kernel = kernel_svm_model.predict(X_train)
test_pred_kernel = kernel_svm_model.predict(X_test)

train_accuracy_kernel = accuracy_score(y_train, train_pred_kernel)
test_accuracy_kernel = accuracy_score(y_test, test_pred_kernel)

n_support_vectors = len(kernel_svm_model.support_vectors_)

print(f"Kernel SVM (quadratic kernel) Training Accuracy: {train_accuracy_kernel}")
print(f"Kernel SVM (quadratic kernel) Test Accuracy: {test_accuracy_kernel}")
print(f"Number of Support Vectors: {n_support_vectors}")
```

```
Training Kernel SVM with quadratic kernel (degree=2)...
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-
packages/sklearn/svm/_base.py:305: ConvergenceWarning: Solver terminated early
(max_iter=1000). Consider pre-processing your data with StandardScaler or
MinMaxScaler.
    warnings.warn(
Kernel SVM (quadratic kernel) Training Accuracy: 0.98055
Kernel SVM (quadratic kernel) Test Accuracy: 0.966
Number of Support Vectors: 15601
Training Kernel SVM with quadratic kernel (degree=2)...
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-
packages/sklearn/svm/_base.py:305: ConvergenceWarning: Solver terminated early
(max_iter=1000). Consider pre-processing your data with StandardScaler or
MinMaxScaler.
```

```
warnings.warn(
```

```
[ ]:
```