DSC 255R

1. b. $\|x-z\|_1 = \sum_{i=1}^{m} |x_i - z_i|$

$$\sum_{i=1}^{4} |x_i - z_i| = \begin{bmatrix} -2 \\ 0 \\ -2 \\ 0 \end{bmatrix} \overset{\text{mixed up the order}}{\rightarrow} \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\sum_{i=1}^{4} = 4$$

a. $\sqrt{(-2)^2 + 0^2 + (-2)^2 + 0^2} = \sqrt{8} = 2\sqrt{2}$

c. $\|x\|_\infty$ $\quad L\infty = \max_i |x_i - z_i|$

$= \max \begin{bmatrix} |-2| \\ |0| \\ |-2| \\ |0| \end{bmatrix} = 2$

2. $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

a. $\|x\|_1 = 1+2+3 = 6$

b. $\|x\|_2 = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

c. $\|x\|_\infty = \max_i |x_i| = 3$

3. $d(x,y) \geq 0$ ✓ all distances are nonnegative

$d(x,y) = 0$ iff $x = y$ ✓ true for all entries, it's only equal to 0 when x and y are the same

$d(x,y) = d(y,x)$ ✓ symmetric for all $x, y \in \{A, B, C, D\}$

$d(x,z) \leq d(x,y) + d(y,z)$

$\forall x, d(A,B) \geq d(A,C) + d(A,D)$

$5 \geq 1+2$ ✗

This distance function is not a metric because it violates the triangle inequality condition.

4. $d(p,q) = \sum_{x \in X} p(x) \log_2 \frac{p(x)}{q(x)}$

$= \frac{1}{2}\log_2(1/2 / 1/4) + \frac{1}{4}\log_2(1/4 / 1/4) + \ldots$
$+ 1/16 \log_2(\frac{1}{16} / 1/6)$

$= 1/2 + 0 + 1/8 \log_2 3/4 + \frac{1}{16}\log_2 3/8 + \frac{1}{16}\log_2 3/8$

≈ 0.2712

5. a. classification
b. regression
c. regression
d. classification

6. a. $E(X^2) - E(X)^2$
$E(X) = 0 \quad E(X^2) = 1$
$Var(X) = 1$

b. $E(X) = x \quad Var(X) = 0$

c. $X \in \{0, 1\}$ $X = 1$ w/ $p = 1/4$
$E(X) = 1 \cdot 1/4 + 0 \cdot 3/4 = \frac{1}{4}$ $E(X^2) = \frac{1}{4} = \frac{1}{16}$

$Var(X) = \frac{1}{4} - \frac{1}{16} = 3/16$

7. a. $Cov(X,Y) = E(X - E(X))(Y - E(Y))$
$= E(XY) - E(X)E(Y)$

~~1/9~~ ~~1/9~~

$E(X) = -1(0+0+\frac{1}{3}) + 0(\frac{1}{3}) + 1(\frac{1}{3}) = 0$

~~$E(Y) = -1(1/6 + 1/6) + 0(\frac{1}{3}) + 1(1/6 + 1/6)$~~

$= -1(1/3) + 0 + 1 \cdot 1/3 = 0$

$E(XY) = 1 \cdot -1 \cdot (1/3) + 1 \cdot -1 \cdot (\frac{1}{3}) = -2/3$

$Cov(X,Y) = -2/3 - 0 = \boxed{-2/3}$

b. $corr(X,Y) = \frac{cov(X,Y)}{std(X)\,std(Y)}$

$Var(X) = \frac{1}{3} + \frac{1}{3} = 2/3 \qquad std(X) = \sqrt{2/3}$

$Var(Y) = 2/3 \qquad std(Y) = \sqrt{2/3}$

$corr(X,Y) = \frac{-2/3}{\sqrt{2/3} \cdot \sqrt{2/3}} = \boxed{-1}$

8. a. $P(X=x, Y=y) = P(X=x) \cdot P(Y=y) \quad \forall x, y$

$P(X=-1, Y=-1) = 1/6$

$P(X=-1) \cdot P(Y=-1) = 1/3 \cdot 1/3 = 1/9$

X and Y are not independent.

b. $Cov(X,Y) = E(XY) - E(X)E(Y)$

$E(XY) = \sum_{x,y} xy \cdot P(X=x, Y=y)$

$= -1 \cdot -1 \cdot \frac{1}{6} + \ldots + 1 \cdot 1 \cdot \frac{1}{6}$

$= 0$

$E(X) = -1 \cdot (1/6 + 1/6) + 0 + 1 \cdot (1/6 + 1/6) = 0$

$E(Y) = 0 \qquad Cov(X,Y) = 0$

X and Y are uncorrelated

In [201…
```
import numpy as np
from sklearn.model_selection import train_test_split
from sklearn.neighbors import KNeighborsClassifier
from sklearn.metrics import accuracy_score
from sklearn.metrics import confusion_matrix, ConfusionMatrixDisplay
import matplotlib.pyplot as plt
from sklearn.model_selection import LeaveOneOut
from sklearn.model_selection import KFold
```

In [93]:
```
labels = ['NO', 'DH', 'SL']
data = np.loadtxt('hw2/spine-data.txt', converters={6: lambda s: labels.inde
```

In [ ]:
```
#data.view()
X = data[:, :5].reshape(310, 5)
Y = data[:, 6].reshape(310, 1)
```

In [107…
```
X_train, X_test, y_train, y_test = train_test_split(X, Y, test_size = 60/310
#X_test.reshape(60,5)
#y_test.reshape(60,1)
```

In [112…
```
neighbors_1 = KNeighborsClassifier(p=1).fit(X_train, y_train)
neighbors_2 = KNeighborsClassifier(p=2).fit(X_train, y_train)
```

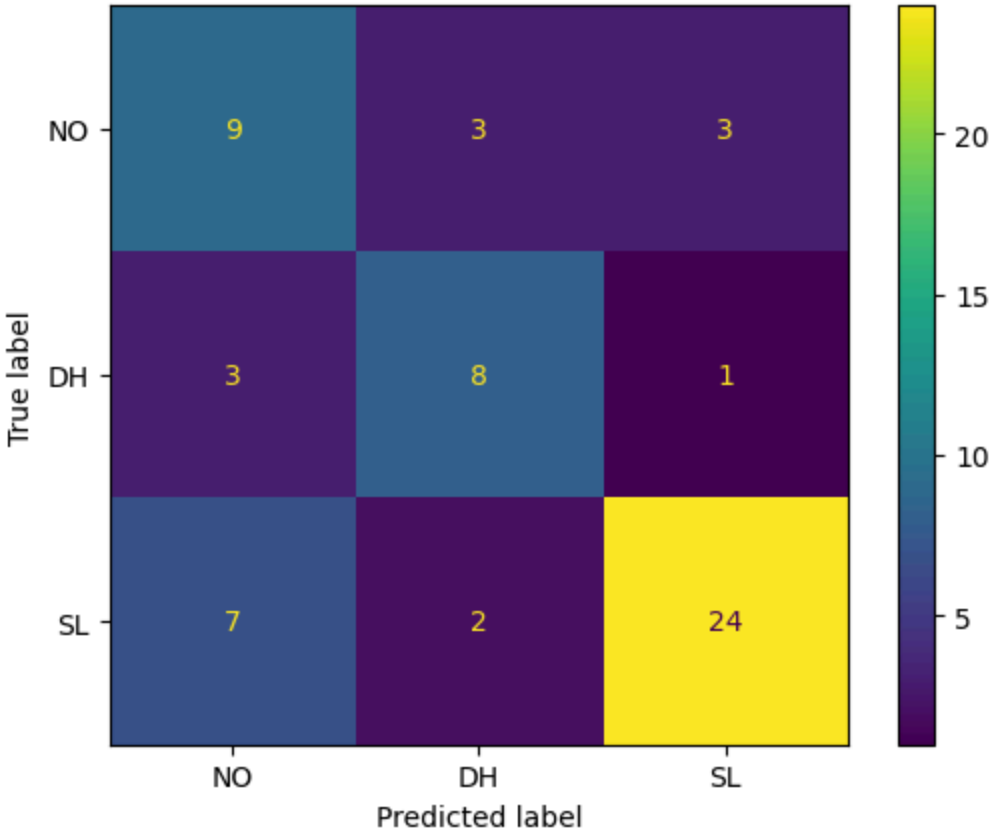
```
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packa
ges/sklearn/neighbors/_classification.py:239: DataConversionWarning: A colum
n-vector y was passed when a 1d array was expected. Please change the shape
of y to (n_samples,), for example using ravel().
  return self._fit(X, y)
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packa
ges/sklearn/neighbors/_classification.py:239: DataConversionWarning: A colum
n-vector y was passed when a 1d array was expected. Please change the shape
of y to (n_samples,), for example using ravel().
  return self._fit(X, y)
```

In [133…
```
y_pred_1 = neighbors_1.predict(X_test)
y_pred_2 = neighbors_2.predict(X_test)
acc_1 = accuracy_score(y_test, y_pred_1)
acc_2 = accuracy_score(y_test, y_pred_2)
```

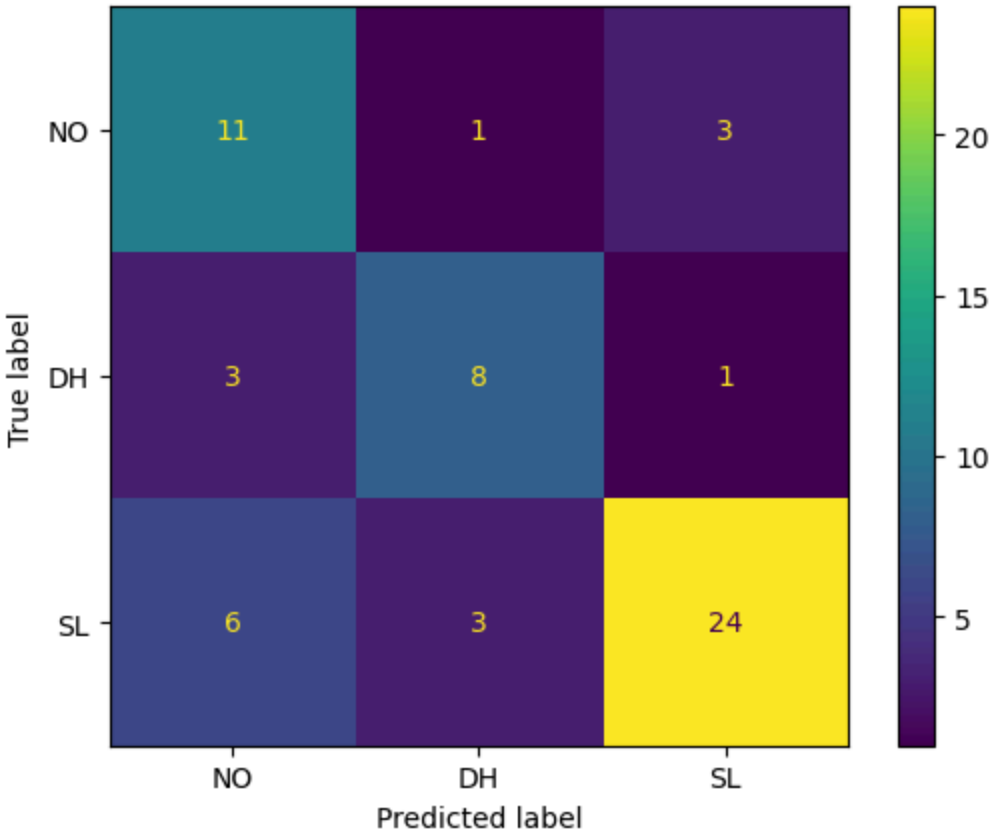
In [134…
```
cm_1 = confusion_matrix(y_test, y_pred_1)
cm_2 = confusion_matrix(y_test, y_pred_2)
print("model with l1 norm error rate: ", 1 - acc_1)
print("model with l2 norm error rate: ", 1 - acc_2)
```

```
model with l1 norm error rate:  0.31666666666666665
model with l2 norm error rate:  0.2833333333333333
```

In [129…
```
cm_1_disp = ConfusionMatrixDisplay(cm_1, display_labels=['NO', 'DH', 'SL'])
cm_1_disp.plot()
plt.show()
```



In [130…
```
cm_2_disp = ConfusionMatrixDisplay(cm_2, display_labels=['NO', 'DH', 'SL'])
cm_2_disp.plot()
plt.show()
```



In [144…
```
!pip install pandas
```

```
Requirement already satisfied: pandas in /Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages (2.2.3)
Requirement already satisfied: numpy>=1.26.0 in /Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages (from pandas) (2.2.1)
Requirement already satisfied: python-dateutil>=2.8.2 in /Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages (from pandas) (2.9.0.post0)
Requirement already satisfied: pytz>=2020.1 in /Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages (from pandas) (2024.2)
Requirement already satisfied: tzdata>=2022.7 in /Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages (from pandas) (2024.2)
Requirement already satisfied: six>=1.5 in /Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages (from python-dateutil>=2.8.2->pandas) (1.17.0)
```

In [191…
```
wine_data = np.loadtxt('hw2/wine.csv', delimiter=',')
x_wine = wine_data[:, 1:]
y_wine = wine_data[:, :1]
```

In [192…
```
all_train_model = KNeighborsClassifier(n_neighbors=1)
all_train_model.fit(x_wine, y_wine)
```

```
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-packages/sklearn/neighbors/_classification.py:239: DataConversionWarning: A column-vector y was passed when a 1d array was expected. Please change the shape of y to (n_samples,), for example using ravel().
  return self._fit(X, y)
```

Out[192…

▾ KNeighborsClassifier

KNeighborsClassifier(n_neighbors=1)

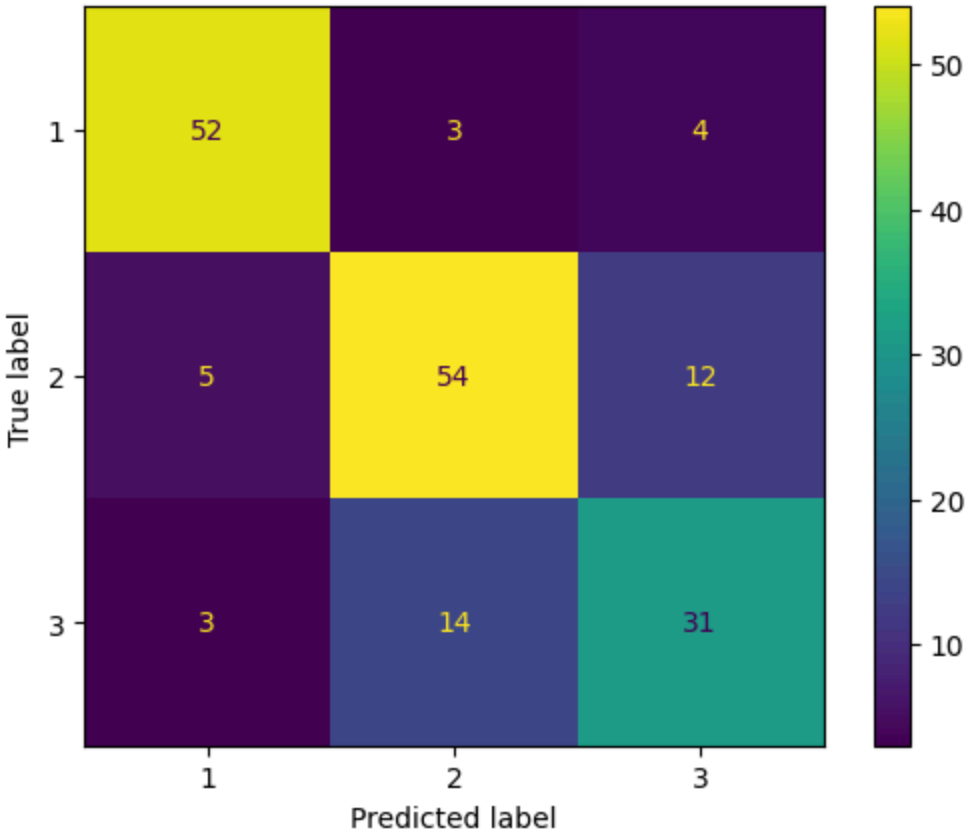
In [ ]:
```
predictions = []
actual_values = []
loov = LeaveOneOut()
loov.get_n_splits(x_wine)

for i, (train_index, test_index) in enumerate(loov.split(x_wine)):
    #print(f"Fold {i}:")
    #print(f"  Train: index={train_index}")
    #print(f"  Test:  index={test_index}")
    X_train, X_test = x_wine[train_index], x_wine[test_index]
    y_train, y_test = y_wine[train_index], y_wine[test_index]

    all_train_model.fit(X_train, y_train)

    y_pred = all_train_model.predict(X_test)

    predictions.append(y_pred[0])
    actual_values.append(y_test[0])
```

In [208…
```
print("LOOCV accuracy: ", accuracy_score(actual_values, predictions))
```

```
LOOCV accuracy:  0.7696629213483146
```

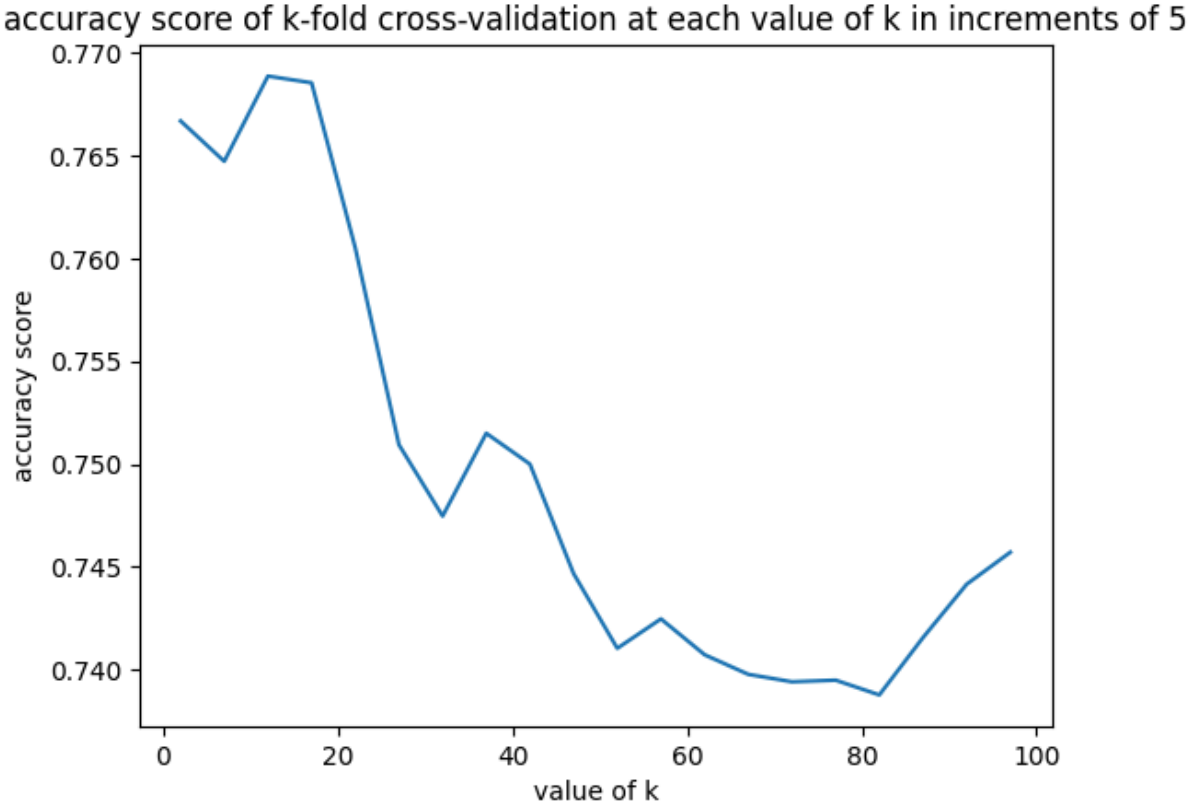
In [209…
```
all_model_cm = confusion_matrix(actual_values, predictions)
cm_disp = ConfusionMatrixDisplay(all_model_cm, display_labels=['1', '2', '3'
cm_disp.plot()
```

Out[209…
```
<sklearn.metrics._plot.confusion_matrix.ConfusionMatrixDisplay at 0x136feb1
d0>
```



In [ ]:
```
kfold = list(range(2, 100, 5))
kfold_acc = []
for i in range(2, 100, 5):
    kf = KFold(n_splits=i)
    for i, (train_index, test_index) in enumerate(kf.split(x_wine)):
        #print(f"Fold {i}:")
        #print(f"  Train: index={train_index}")
        #print(f"  Test:  index={test_index}")
        X_train, X_test = X[train_index], X[test_index]
        y_train, y_test = y[train_index], y[test_index]

        all_train_model.fit(X_train, y_train)

        y_pred = all_train_model.predict(X_test)

        predictions.append(y_pred[0])
        actual_values.append(y_test[0])

    kfold_acc.append(accuracy_score(actual_values, predictions))
```

In [214…
```
plt.plot(kfold, kfold_acc)
plt.xlabel('value of k')
plt.ylabel('accuracy score')
plt.title('accuracy score of k-fold cross-validation at each value of k in i
plt.show()
```



In [226…
```
x_data = wine_data[:, 1:]
normalized_wine = (x_data-np.min(x_data))/(np.max(x_data)-np.min(x_data))
print(normalized_wine)
X = normalized_wine
y = wine_data[:, :1]
normalized_model = KNeighborsClassifier(n_neighbors=1)
#normalized_model.fit(X, y)
```

```
[[8.39350664e-03 9.40548971e-04 1.36915357e-03 ... 5.41708585e-04
  2.25612696e-03 6.33900242e-01]
 [7.78036396e-03 9.82218862e-04 1.19652116e-03 ... 5.47661426e-04
  1.94657920e-03 6.24970980e-01]
 [7.75655259e-03 1.32748367e-03 1.51202176e-03 ... 5.35755743e-04
  1.80966384e-03 7.05334341e-01]
 ...
 [7.82203385e-03 2.47042926e-03 1.26795526e-03 ... 2.73830713e-04
  8.51256347e-04 4.96984886e-01]
 [7.76250543e-03 1.46439903e-03 1.33343652e-03 ... 2.79783555e-04
  8.86973397e-04 4.99961307e-01]
 [8.33397822e-03 2.36327811e-03 1.55369165e-03 ... 2.85736396e-04
  8.75067714e-04 3.33281742e-01]]
```

In [ ]:
```
predictions = []
actual_values = []
```

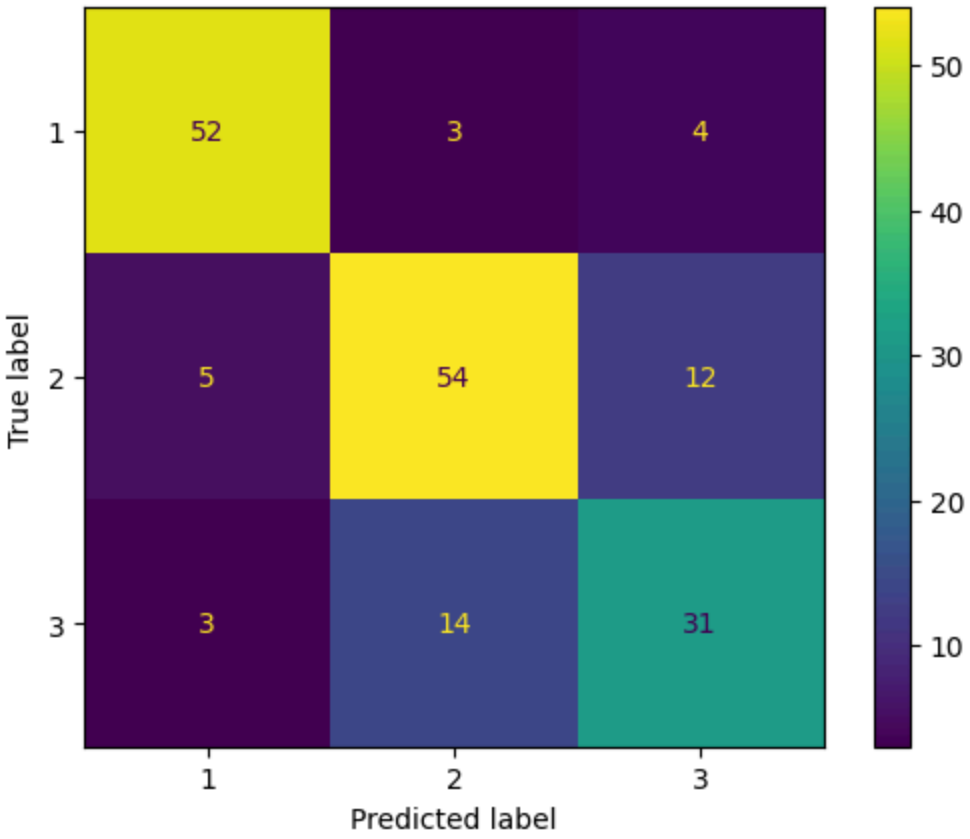
```
loov = LeaveOneOut()
loov.get_n_splits(X)

for i, (train_index, test_index) in enumerate(loov.split(X)):
    #print(f"Fold {i}:")
    #print(f"  Train: index={train_index}")
    #print(f"  Test:  index={test_index}")
    X_train, X_test = X[train_index], X[test_index]
    y_train, y_test = y[train_index], y[test_index]

    normalized_model.fit(X_train, y_train)

    y_pred = normalized_model.predict(X_test)

    predictions.append(y_pred[0])
    actual_values.append(y_test[0])
```

In [228…
```
accuracy_score(actual_values, predictions)
```

Out[228…
```
0.7696629213483146
```

In [230…
```
normalized_cm = confusion_matrix(actual_values, predictions)
cm_disp = ConfusionMatrixDisplay(normalized_cm, display_labels=['1', '2', '3
cm_disp.plot()
```

Out[230…
```
<sklearn.metrics._plot.confusion_matrix.ConfusionMatrixDisplay at 0x1580313
d0>
```



The accuracy score seems to be about the same with the normalization.