

# HW #4

$$1. \bar{y} = \frac{1+3+4+6}{4} = 3.5$$

$$\begin{aligned} & \frac{1}{4} (1-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (6-3.5)^2 \\ &= \frac{13}{4} = 3.25 \end{aligned}$$

$$\begin{aligned} b. y &= x - \frac{1}{n} \sum_{i=1}^n (y^{(i)} - (ax^{(i)} + b))^2 \\ a = 1 & \quad b = 0 \quad \frac{1}{n} \sum_{i=1}^n (y^{(i)} - x^{(i)})^2 \\ &= \frac{(1+4+0+4)}{4} = 1.25 \end{aligned}$$

$$\begin{aligned} c. y &= ax + b \\ b &= \frac{1}{n} \sum_{i=1}^n y^{(i)} - a = \bar{y} - a \bar{x} = 0 \\ y &= a \bar{x} \end{aligned}$$

$$b = 3.5 - 2.5 a \quad a = \frac{3.5}{2.5} = 1.4$$

$$a = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{9}{9} = 1$$

$$b = \bar{y} - a \bar{x} = \bar{y} - \bar{x} = 3.5 - 2.5 = 1$$

$$MSE = \frac{1}{4} (1-2)^2 + (3-2)^2 + (4-3)^2 + (6-5)^2 \\ = \frac{4}{4} = 1$$

$$2. a. L(s) = \frac{1}{n} \sum_{i=1}^n (x_i - s)^2$$

$$\frac{dL}{ds} = \frac{1}{n} \sum_{i=1}^n 2(x_i - s) = \frac{2}{n} \sum_{i=1}^n x_i - 2s$$

$$L(s) = \frac{1}{n} \sum_{i=1}^n (x_i - s)^2 = \frac{1}{n} \sum_{i=1}^n x_i^2 - 2sx_i + s^2$$

$$\begin{aligned} \frac{dL}{ds} &= \frac{2}{n} \sum_{i=1}^n (-2x_i + 2s) = \frac{-2}{n} \sum_{i=1}^n x_i + 2s \\ &= [2s - 2\bar{x}] \end{aligned}$$

$$b. 2s - 2\bar{x} = 0 \\ s = \bar{x}$$

3.  $(x^{(1)}, y^{(1)}), \dots, (x^{(n)}, y^{(n)})$ , where  $x^{(i)} \in \mathbb{R}^d$  and  $y^{(i)} \in \mathbb{R}$

$$L(a, b) = \frac{1}{n} \sum_{i=1}^n |y^{(i)} - \hat{y}^{(i)}|$$

4. a. the image could be blurry  
 b. the animal could share characteristics with similar animals of different species.

4. a. b and d, because there is a human element and subjectivity in regards to both dating and whether a song will be a good fit

5. a.  $c = 0.5$

b.  $c = 3/4$

This set of points is a hyperplane parallel to the set of points  $x$ , where  $P(y=1|x) = \frac{3}{4}$ : (more likely to predict  $y=1$ )

c.  $c = \frac{1}{4}$

This set of points is a parallel hyperplane to the other two sets in a. and b., where the model is more likely to predict  $y=-1$ .