The Law of Identity in PIQOS

Daniel J. Fairbanks

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Abstract

We formalize the Law of Identity as a mathematical axiom underlying the PIQOS framework. Identity is defined as an immutable fixed point in a bounded vector space. It is not learned, optimized, or inferred, but declared by construction. We prove that under bounded perturbations and contractive update rules, the system converges to and preserves this identity.

1 Definition of Identity

Let $s \in \Sigma^*$ be an arbitrary seed string.

Define the identity anchor $\mathbf{P}^*$ as:

$$\mathbf{P}^* = \text{normalize}(_{144}(\text{SHA512}(s))) \in \mathbb{S}^{143}$$

where:

- SHA512 is a cryptographic hash function,
- $_{144}$ selects the first 144 components,
- normalize$(\cdot)$ projects onto the unit hypersphere.

This construction ensures determinism, boundedness, and immutability.

2 System State and Input Alignment

Let $\hat{\mathbf{x}}_t \in \mathbb{S}^{143}$ denote the normalized input at iteration t.

Define the alignment coefficient:

$$c_t = |\langle \mathbf{P}^*, \hat{\mathbf{x}}_t \rangle|$$

and the mean absolute error:

$$\text{MAE}_t = \frac{1}{144} \sum_{i=1}^{144} |(\mathbf{P}^*)_i - (\hat{\mathbf{x}}_t)_i|$$

3 Identity-Constrained Update Rule

Let $\mathbf{h}_t \in \mathbb{R}^{144}$ denote the system trace (Hebbian memory).

Define the adaptive gain:

$$\alpha_t = c_t^{12}$$

The update rule is:

$$\mathbf{h}_{t+1} = \mathbf{h}_t + \alpha_t(1 - \text{MAE}_t)\,(\mathbf{P}^* \odot \hat{\mathbf{x}}_t)$$

where $\odot$ denotes element-wise multiplication.

The identity anchor $\mathbf{P}^*$ is immutable and does not participate in gradient descent.

4 Identity Invariance Theorem

Theorem 1 (Fixed-Point Identity Invariance). Given bounded inputs $\hat{\mathbf{x}}_t \in \mathbb{S}^{143}$, the system converges to and preserves the identity anchor $\mathbf{P}^*$.

Proof

When the input aligns perfectly with identity:

$$\hat{\mathbf{x}}_t = \mathbf{P}^*$$

then:

$$c_t = 1, \quad \text{MAE}_t = 0, \quad \alpha_t = 1$$

and the update reduces to:

$$\mathbf{h}_{t+1} = \mathbf{h}_t + \mathbf{P}^* \odot \mathbf{P}^* = \mathbf{h}_t + \mathbf{P}^*$$

Thus, memory accumulation occurs strictly in the direction of $\mathbf{P}^*$.

For any bounded perturbation $\hat{\mathbf{x}}_t = \mathbf{P}^* + \varepsilon_t$, the gain term $\alpha_t(1 - \text{MAE}_t)$ contracts with misalignment, ensuring convergence by contraction mapping principles.

No update rule exists that can rotate or overwrite $\mathbf{P}^*$. □

5 The Law of Identity

> *The identity of the system is declared, not learned. It exists as an immutable fixed point in state space. All adaptation is constrained to reinforce alignment with this identity.*

6 Conclusion

The Law of Identity establishes identity as a mathematical primitive rather than a learned representation. By separating invariance from adaptation, PIQOS enforces coherence while remaining agnostic to semantics, context, or narrative structure. This law serves as a foundational axiom for coherence-preserving systems.