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Portfolio Optimization with ROSCA option

1. Introduction

Rotating Savings and Credit Association (ROSCA) is a particular savings method which has been utilized world-wide, particularly in developing countries. It is seemingly without origin and is a method which was created when there are no formal saving options such as risk-free savings account or stock options and is said to be advantageous during inflationary periods. The method can be described as:

1. Agents form a group of participants
2. Each participant puts X amount of money into a pooled fund each time period (usually one a month)
3. Within each time period, one of the participants receives the pooled funds according to some pre-established rules excluding previous winners. This is done until all members win the pot.

Now traditionally, there is a designated leader who holds the pot and monitors this process and therefore there's always been a risk of the leader stealing the pot. As for the rules which determines who receives the pot, there are several ways of doing this, usually either member agree on a particular order, or it is randomly chosen. Another notable method is that members bid on a higher priority in the order (Besley et al , 2001). The number of periods or time horizon is simply the number of participants; this assures everyone receives the pot and that everyone pay equal amounts into the pot every period.

The disadvantage of this method compared to alternative financial products is that it is interest free, however it does temporarily increase the purchasing power of the agent. Meaning if the temporary increase in purchasing power is awarded early relative to the length of the time horizon, then interest is earned in terms of present value; in the sense that an agent would prefer the increase in consumption due to receiving the pot now rather than later. Hence, why this is not a trivial problem as many might assume given there is no interest, as there is an underlying interest implied by present value. However, if the agent is the last one to receive the pot, then it is a trivial problem, as it would have been better to just invest in the risk free or risky asset.

I will be incorporating this ROSCA method as a third option in a portfolio optimization problem. It will be competing against a risk-free asset and a risky asset. The pot winner will be randomly drawn and there will be zero risk associated with the option. The number of participants in the ROSCA naturally equals the number of time periods for the time horizon, so that for each time one of the participants (who has yet to receive the fund will receive it. For simplicity, I plan on doing my optimization problem for 2 periods.

2. Literature Review

Despite ROSCAs prevalence throughout the world, there is a very small literature written on the topic. Specifically, when it comes to analyzing the method theoretically, the work by Besley et al (2001) is most notable. In their model, they compare one's lifetimes utility while in participating in a ROSCA versus someone who saves alone or in autarky. Unlike the portfolio optimization model in this paper, their maximization problem, in continuous time, depends on agents saving for a purchase of an indivisible durable consumption good, which provides a constant flow of utility once purchased. Agents earn a constant flow of income, and do not have access to any formal savings. Their model also assumes no discounting, and their results are invariant to agent preferences. They find that the ROSCA dominates over autarky for an agent's lifetime utility.

In a follow up piece by Besley et al (1994), the authors investigate more thoroughly, asking questions such as: In what ways do the allocations they produce differ from that achieved from a formal credit market? Focusing this time more on which is pareto efficient. Their model set up is very similar to the previous paper, with the goal of savings to purchase a durable good which provides a constant flow of utils. There also exist non-durable goods, and the overall comparison of ROSCA vs formal credit market. The authors once again assume no discounting, wanting to preclude any motive for saving or borrowing apart from the desire to acquire the durable. This paper specifically differs from the previous by assuming that group members holding the indivisible good at any point time are treated as a continous variable. They find that neither a random or bidding ROSCA are efficient, and that individuals are better off with a credit market than a bidding ROSCA. However, they find that a random ROSCA may sometimes yield a higher level of expected utility than a credit market.

For the rest of the literature, it is primarily empirical evidence from developing countries. In Handa et al (1999) , they find evidence from Jamaica, that payments to the ROSCA leader significantly enhance the sustainability of the ROSCA. They also find evidence of an inverse relationship between size of ROSCA and size of contribution, as well as evidence of the funds being used for the purchase of durable goods. In Gugerty (2007), the author finds evidence from Kenya, that ROSCAs are used as a savings mechanism for commitment when using time inconsistent preferences.

3. Model

This optimization problem will be inherently stochastic since there is a random probability of receiving the pot as well as probability associated with the risky asset. Therefore, when setting up this optimization problem there are several states and their conditions that need to be noted. For simplicity, the states and there underlying returns can be summarized in the following table.

Table 1.

State A	State B	State C	State D
a_1 = 1.01 (risk free)	b_1 = 1.01	c_1 = 1.01	d_1 = 1.01
a_2 = 1.08 (risky)	b_2 = .98	c_2 = .98	d_2 = 1.08
a_3 =0.0 (no pot)	b_3 = 2.0 (win pot)	c_3 = 0.0 (no pot)	d_3 = 0.0 (win pot)

Notice for this summary table that asset 1 (illustrated by a1, b1 , c1, d1) is the risk free asset and is therefore constant in each state. Therefore, there needs to be four underlying states to account for the two outcome possibilities for asset 2 (risky) and asset 3 (ROSCA).

The return for receiving the jackpot is 2.0 because there are two participants (therefore two periods), so the pot winner is guaranteed to win two times the monthly fee (which is assumed to be the price of the asset). It should also be noted that compared to traditional assets where the payment is in the first period and the return gained in the next, with the ROSCA option it is possible for the agent to be rewarded the pot the same period its fee is paid.

Due to the inherent nature of ROSCA, if the pot is awarded in the first period, then it is guaranteed to not be awarded in the next period, and vice versa. Therefore, if the state is a or c (no jackpot), then the next state must be either b or d. To account for this transition rule, probability variables π_W and π_L are created to demonstrate whether the pot is won or lost for the first period. Probability variables will also be created for the states, in which probabilities π_a and π_c are for when the pot is won for the first period, and probabilities π_b and π_d are for if the pot is won in the second period. The initial problem can be seen to have initial consumption be based on the state w or l, representing if the pot is rewarded in the initial period.

The initial optimization problem can be described as:

$$\max_{\{C_t,\{\emptyset_{it}\}_{i=1}^{3}\}} E_t \frac{C_t}{1-\gamma}^{1-\gamma} + \beta E_t \frac{C_t^{1-\gamma}}{1-\gamma}$$

$$\text{s.t} \sum_{i=1}^{3} \emptyset_{it} = 1$$

$$C_t = \begin{cases} w_t - \sum_{i=1}^{3} P_t \emptyset_{it} \ \ if\ state\ l \\ w_t - \sum_{i=1}^{2} P_t \emptyset_{it} + P_3 \emptyset_{3t}\ if\ state\ w \end{cases}$$

$$\pi_l + \pi_w = 1$$

$$C_{t+1} = \begin{cases} a_1\emptyset_{1t} + a_2\emptyset_{2t} - \emptyset_{3t}\ \ if\ state\ a \mid w \\ b_1\emptyset_{1t} + b_2\emptyset_{2t} + \emptyset_{3t}\ if\ state\ b \mid l \\ c_1\emptyset_{1t} + c_2\emptyset_{2t} - \emptyset_{3t}\ \ if\ state\ c \mid w \\ d_1\emptyset_{1t} + d_2\emptyset_{2t} + \emptyset_{3t}\ if\ state\ d \mid l \end{cases}$$

$$\pi_a + \pi_c = 1$$

$$\pi_b + \pi_d = 1$$

Notice for the first period consumption, the net gain is $P_3\emptyset_{3t}$ since the returns are twice the cost of payment This problem can be furthered simplified substituting the constraints into the objective and implementing the probability of each state. Prices will also be normalized to 1, meaning the cost of each asset is the same.

Simplified problem with asset returns:

$$\max_{\{\emptyset_{it}\}_{i=1}^{3}} \pi_w[\frac{(w_t - 2\emptyset_{1t} - 2\emptyset_{2t} + 1)^{1-\gamma}}{(1-\gamma)} + \beta\pi_a \frac{(2.01\emptyset_{1t} + 2.08\emptyset_{2t} - 1)^{1-\gamma}}{(1-\gamma)} + \beta(1 - \pi_a)\frac{(2.01\emptyset_{1t} + 1.98\emptyset_{2t} - 1)^{1-\gamma}}{(1-\gamma)}]$$

$$+ (1-\pi_w)[\frac{(w_t - 1)^{(1-\gamma)}}{(1-\gamma)} + \beta\pi_b \frac{(0.01\emptyset_{1t} - 0.02\emptyset_{2t} + 1)^{(1-\gamma)}}{(1-\gamma)} + \beta(1 - \pi_b)\frac{(0.01\emptyset_{1t} + 0.08\emptyset_{2t} + 1)^{(1-\gamma)}}{(1-\gamma)}]$$

Unfortunately, this problem cannot be solved analytically, and a numerical solution is needed. Therefore, I assume the risk aversion $\gamma = 2$ and discount factor $\beta = .98$. As for probability measures for each state, I assume a uniform distribution for the ROSCA draw, meaning $\pi_w = 0.5$. The assumption that probability of winning the pot in period 1 is equal to the probability of winning the pot in period 2, is not a strong assumption, given this uniform distribution is naturally unbiased.

As for what to assume for the probabilities of the underlying states, assuming a uniform distribution would naturally give more favor to the risky asset given it will have a much higher expected value. It's unfortunate that the optimal weights will naturally be biased based on these probability assumptions; therefore, in my results I will be looking at what the optimal weights are given these differences.

4. Results

With the previously mentioned parameters, the following optimal portfolio weights can be summarized in table 2, under the differing state probabilities or scenarios. For all states, the optimal weights contain a majority stake in the ROSCA option and a minority stake in the risk-free asset. These results are surprising and are possibly due to the risk aversion value being set to 2. Even under scenarios 3, where the expected value of the risky asset is much higher than the risk-free asset, one would expect investment in it. If these results are true, then this relationship has to be driven from the discount factor, which weighs present consumption over future consumption. Future research on this topic is needed, specifically under different probabilities of draws as well as different values of risk aversion and discount factor.

Table 2.

Scenarios: Differing State Probabilities	Optimal Weights
Scenario 1 $\pi_a = 0.50\ (\pi_c = 0.50)$ $\pi_b = 0.50\ (\pi_d = 0.50)$	$\emptyset_{1t}$ = .40530277, $\emptyset_{2t} =$ 0.0, $\emptyset_{3t}$ =.59469723
Scenario 2 $\pi_a = 0.30\ (\pi_c = 0.70)$ $\pi_b = 0.70(\pi_d = 0.30)$	$\emptyset_{1t} =$ 0.40218164, $\emptyset_{2t} = 0.0$, $\emptyset_{3t} =$ .59781836
Scenario 3 $\pi_a = 0.70\ (\pi_c = 0.30)$ $\pi_b = 0.30(\pi_d = 0.70)$	$\emptyset_{1t} =. 0.411181645$, $\emptyset_{2t} = 0.0$, $\emptyset_{3t} =$ .588818355
Scenario 4 $\pi_a = 0.30\ (\pi_c = 0.70)$ $\pi_b = 0.30(\pi_d = 0.70)$	$\emptyset_{1t} = 0.404155$, $\emptyset_{2t} = 0.0$, $\emptyset_{3t} =. 0.595845$
Scenario 5 $\pi_a = 0.70\ (\pi_c = 0.30)$ $\pi_b = 0.70(\pi_d = 0.30)$	$\emptyset_{1t} = 0.40803195$, $\emptyset_{2t} = 0.0$, $\emptyset_{3t} =$. 0.5996805

Works Cited

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