Decision Networks

Daniel Otero Fadul

Department of Science
School of Engineering and Science

Principle of Maximum Utility

In this context, to make a decision, we will follow the *principle of maximum utility*, that is, we will choose the option that maximizes the *expected utility*.

Suppose we have evidence e. The probability of obtaining a result $\hat{s}$ given evidence e is written as

$$P(\text{result} = \hat{s}|e).$$

Principle of Maximum Utility

Let $U(a, s)$ be the *utility function*, which assigns a number to each possible combination of a and s. Then, the *expected utility* of an action a given certain evidence e is defined as

$$EU(a|e) = \sum_{s} P(\text{result} = s|e)U(a, s).$$

The principle of maximum utility states that we should choose the action $a^\star$ that maximizes $EU(a|e)$:

$$a^\star = \underset{a}{\operatorname{argmax}}\, EU(a|e).$$

Toma de Decisiones

A popular technique for decision-making is the **decision network**. This type of network combines Bayesian networks with additional nodes for actions and utilities.

- **Random nodes** (Ovals): These nodes represent random variables, as in the case of Bayesian networks. Each of these nodes is generally associated with a conditional probability distribution that is indexed by the states of its parents. In decision networks, parents can be random nodes
- **Decision nodes** (Rectangles): These nodes represent the points where you can choose between several actions.
- **Utility nodes** (Diamonds): These nodes represent the utility function. These nodes have decision nodes as parents and all the variables associated with the possible outcomes of the system that can alter the values taken by the utility function.

Toma de Decisiones

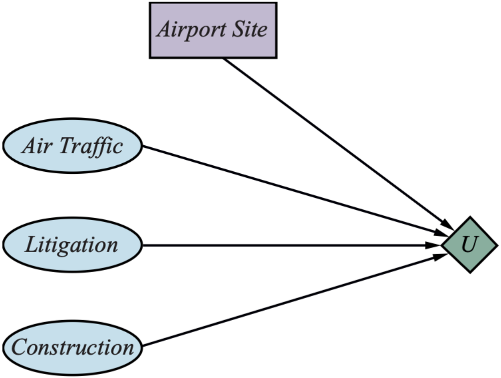

Figura: The decision network for choosing a location where an airport will be built. Image taken from [1].

Toma de Decisiones

The actions taken are selected by evaluating the decision network for each possible option of the decision node. The algorithm for evaluating the network is as follows:

- Consider the evidence of the result being analyzed.
- For each possible value of the decision node:
  - Choose the action being considered.
  - Calculate the posterior probabilities for each parent of the utility node using some inference algorithm.
  - Calculate the utility resulting from the action being evaluated.
- Obtain the action with the highest utility.

Simple Example

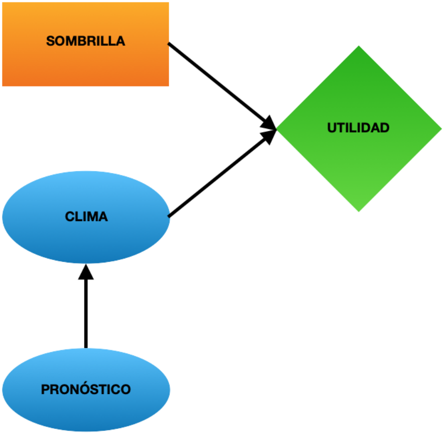

Figura: Decision network for deciding whether to take an umbrella with us or not.

Simple Example

We have the following table for the weather node:

	Sunny	**Rainy**
Good	0.65	0.35
Bad	0.3	0.7

On the other hand, for the utility function we have the following:

Weather	**Umbrella**	**Utility**
Sunny	Carry	20
Sunny	Not to carry	100
Rainy	Carry	70
Rainy	Not to carry	0

Let's assume that the forecast is good. Then,

$$\begin{aligned} EU(\text{Carry}|\text{Good}) &= \sum_s P(\text{result} = s|\text{Good})U(\text{Carry}, s) \\ &= (0.65)(20) + (0.35)(70) \\ &= 37.5. \end{aligned}$$

On the other hand,

$$\begin{aligned} EU(\text{Not to carry}|\text{Good}) &= \sum_s P(\text{result} = s|\text{Good})U(\text{Not to carry}, s) \\ &= (0.65)(100) + (0.35)(0) \\ &= 65. \end{aligned}$$

Therefore, the action that maximizes the expected utility is not to carry an umbrella.

BIBLIOGRAFÍA

1 Stuart Rusell,Peter Norvig, S. J., *"Artificial Intelligence: A Modern Approach"*, Cuarta Edición, Prentice Hall, 2020.