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Scheduling problems under learning effects: classification and cartography

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Traditionally, the processing times of jobs are assumed to be fixed and known throughout the entire process. However, recent empirical research in several industries has demonstrated that processing times decline as workers improve their skills and gain experience after doing the same task for a long time. This phenomenon is known as learning effects. Recently, several researchers have devoted a lot of effort on scheduling problems under learning effects. Although there is increase in the number of research in this topic, there are few review papers. The most recent one considers solely studies on scheduling problems with learning effects models prior to early 2007. For that, this paper focuses on reviewing the most recent advances in this field. First, we attempt to present a concise overview of some important learning models. Second, a new classification scheme for the different model of scheduling under learning effects is proposed and discussed. Next, a cartography showing the relation between some well-known models is proposed. Finally, our viewpoints and several areas for future research are provided.

Keywords: learning effects; scheduling problems; classification; cartography; survey

1. Introduction

Generally, for most manufacturing systems, job processing times are assumed to be known and constant over time. However, this hypothesis is not suitable for many realistic situations where the employees and the machines execute the same task repeatedly; they learn how to perform more efficiently. Several research studies have confirmed that human performance improves with reinforcement or frequent repetitions (Yelle 1979).

Therefore, the processing time of a job is shorter if it is scheduled later in a sequence. This phenomenon is known as the 'learning effect'. Lately, learning effect was first applied in manufacturing systems more than 70 years ago (Wright 1936). Here, the proposed formula showed that a given operation is subject to a 20% productivity improvement each time the production quantity doubles. After the success of Wright' s formula, many other forms of learning models have been proposed in order to represent the learning effect as realistic as possible such as the Stanford-B model, DeJong model, S-curve model and plateau model.

Recently, learning effect phenomenon has received increasing attention in the framework of scheduling. Biskup (1999) and Cheng and Wang (2000) were among the pioneers that investigated the learning effect in scheduling problems. Learning effects are important when humans perform significant task in scheduling environments, for example, software update, setup or removal between jobs, machine operations, maintenance and/or equipment replacement. According to Biskup (1999), the time needed to produce a unit decreases, usually following a negative exponential curve, as the number of repetitions of job. Hence, the actual processing time of job i if scheduled in position r is given by:

$$P_{ir} = P_i r^a \quad i, r = 1, \dots, n \tag{1}$$

where $P_{i,r}$ is the actual processing time of job i, $a < 0$ is a constant learning index, P_i is the normal processing time (i.e. the processing time without any learning effects) of job i and n is the total number of jobs. Cheng and Wang (2000) propose another learning formula in which the actual processing time of a job i is modeled as follow:

$$P_{ir} = a_i - v_i . \min\{n_i, n_{0i}\} \quad i, r = 1, \dots, n \tag{2}$$

where a_i is the nominal processing time of job i , v_i is the learning effect coefficient, n_i indicates the number of jobs processed before job i in the schedule and n_{0i} indicates a threshold level at which the learning effect on job i plateaus. As the production environments will keep changing constantly, several learning effect models have been proposed to make the formulation of problem as realistic as possible. For that, this paper focuses on reviewing the most recent advances in this field. We first

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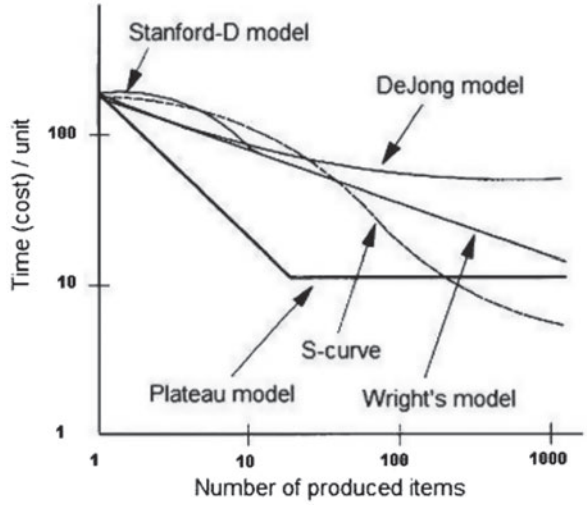

Figure 1. Different learning curve models in logarithm scale (Adapted from Badiru (1992)).

pointed out briefly several learning theories. Section 2 proposes our classification scheme and a comprehensive survey of scheduling problems under different learning effects. In Section 3, we present our cartography showing the relation between the existing learning effects models. Finally, Section 4 contains some gaps, directions and perspectives for future research.

2. Learning theory and manufacturing systems

The basic learning phenomenon consists on that, *most manufacturing systems learn and improve over time*. Therefore, learning takes place as firms and employees perform a task over and over; if workers repeat similar processes over time, they learn how to perform more efficiently. Hence, learning effects affirm that the time needed to produce a single unit decreased at a uniform rate. The uniform rate of learning (90, 80, 70%, etc.) was restrictedly related to the manufacturing process being observed. Wright's model describes the learning of a new product 'start-up phase'. It is based on the idea that the decreases of time needed to produce a single unit can be continued forever. However, in most real-world cases, the decrease cannot continue indefinitely and eventually saturation would be expected to take place. As a result of this saturation, production reaches a kind of steady state where the time needed to produce a single unit remains constant. For that, there have been several geometric versions of learning effects to model different manufacturing processes. Figure 1 shows a comparison of different learning curve.

In order to present briefly the well-known learning model, we show in Table 1 the formula, features and some example of manufacturing system for each of them. In Table 1, we use the following parameters:

C_x = Cost (or direct labor hours) required to produce the Xth unit
C_1 = Cost of the first unit
X = the cumulative unit number
b = the learning index
B = previous experience at the start of the process
= the number of units produced prior to first unit
M = incompressibility factor

From this table, we remark that each model is only more or less approximate to the reality. For more exhaustive information on learning curve models and its application, the readers can refer to the recent literature review proposed by Anzanello and Fogliatto (2011).

Lately, researchers have used the concept of learning curve as a tool to formulate future strategies (Amit 1986). Nevertheless, few works have focused on evaluating the impact of learning effects on scheduling problems. For this reason, over the last several years, the literature of scheduling under learning effects is expanding rapidly. Several models have been proposed for different scheduling problems. The formulation of the learning effect used in connection with scheduling

Table 1. Learning effect models.

Learning effects model	Formula	Features	Example of Manufacturing process
The log-linear model Wright (1936)	$C_x = C_1(X^b)$	The simplest and most common equation	it applies to a wide variety of processes
The S-curve model Carr (1946)	$MC_x = C_1\left[M + (1-M)(X+B)^b\right]$	Describing learning when both machinery intervention occur, and the first cycles of operation demand in-depth analysis	it applies to manufacturing systems characteristics by both machinery and manual operations
The Stanford-B model Asher (1956)	$C_x = C_1(X + B^b)$	Used to model processes where workers start out more productively than the asymptote predicts	Used to model airframe production and mining
The DeJong model De Jong (1957)	$MC_x = C_1\left[M + (1-M)\,X^{-b}\right]$	Used to model processes where a portion of the process cannot improve	Used in factories where the assembly line ultimately limits improvement
The plateau model Baloff (1971)	$C_x = C_1(X^{-b})$	Assumes that production cost reaches a steady state at which point cost levels off.	Electric power industry, steel industry

problems has been debated by many researchers and practitioners. However, there has been widespread use of the log-linear learning curve in this field. The next section presents our classification scheme to the most well-known model in the literature.

3. Scheduling problems under learning effects

In the scheduling context, the learning effect is presented by job processing times described by non-increasing functions dependent on the experience of the processor. Different kinds of scheduling under learning effects model exist in literature. Biskup (2008) classifies the existing model into two main approaches: the first one is the position-based learning effects in which experience depends on the number of previously processed jobs. Alternatively, the second approach called the sum-of-processing time-based learning effects in which experience depends on the sum-of-the processing time of all jobs already processed. Moreover, in the same paper, the author provides the main features of manufacturing systems in which learning effects has a major impact such as a high level of human activities, innovation and shorter product life cycle. In fact, the above classification considers solely studies on learning effect models prior to early 2007. That's why we propose an updated and enriched scheme for the various approaches presented in Figure 2.

3.1 *Position-based learning effects*

The first observation by Wright was that if the workers repeat similar tasks, then the time needed to perform a task decreases with the number of repetitions of this task. Motivated by this observation, *position-based learning model* has been proposed for the case that the actual processing of the job is mainly machine-driven and has (near to) no human interference. Regarding Equation (1), the time needed to execute an operation decreases by the number of repetitions, meaning that learning is primarily based on the repetition of processing time-independent tasks such as set-ups and reading data. Biskup (1999) shows that the single machine scheduling problem with the consideration of learning effects remains polynomially solvable for two objectives, namely minimising the deviation from a common due date and minimising the flow times. Using the same formula, many studies investigate several scheduling problems such as Mosheiov (2001a, 2001b), Bachman and Janiak (2004), Zhao, Zhang, and Tang (2004), Lee, Wu, and Sung (2004), Lee and Wu (2004), Chen, Wu, and Lee (2006), Koulamas and Kyparisis (2007), Gordon et al. (2008), Icsler, Toklu, and Celik (2012), Yin, Wu et al. (2012), Hosseini and Tavakkoli-Moghaddam (2013) and Yin et al. (2014). Gordon and Strusevich (2009) investigate the single-machine problem and due date problem.

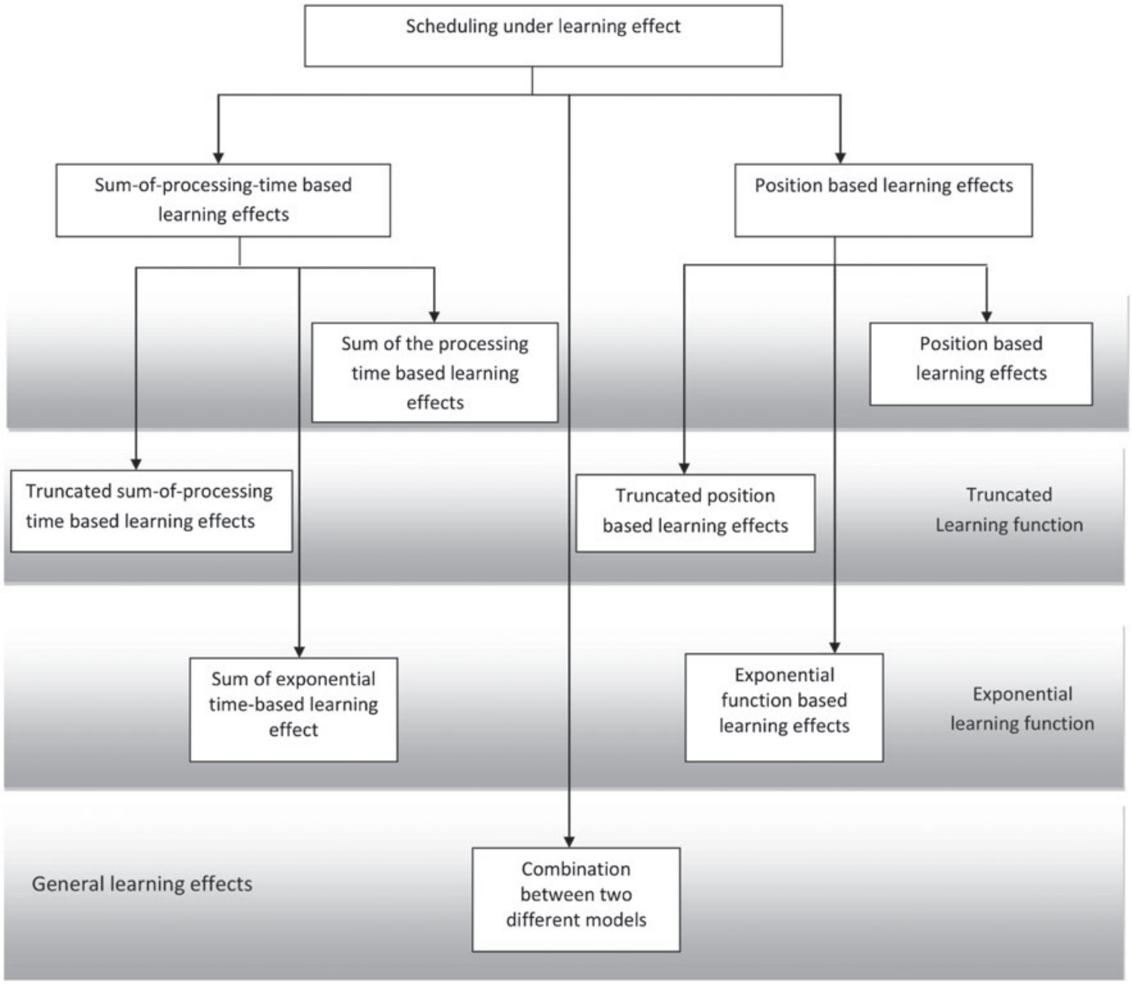

Figure 2. Classification scheme for learning effects model used on scheduling problems.

They focus on the objective function that involves the cost of changing the due dates, a possible penalty for the total earliness of the scheduled jobs and the total penalty for discarding jobs. Hosseini and Tavakkoli-Moghaddam (2013) develops two meta-heuristics for solving a two-machine flow shop scheduling problem that minimises bi-objectives, namely the total idle time and the mean deviation from a common due data. Yin et al. (2014) addresses a single-machine total tardiness scheduling problem with arbitrary release dates and position-dependent learning effects simultaneously. They present a Mixed Integer Programming model (MIP) to find the optimal solution for small size problems and a branch-and-bound (B&B) strategy with several dominance properties for medium-to-large-size problem instances, and four Marriage-in-honey-Bees Optimisation algorithms (MBO) are developed to derive near-optimal solutions for the problem. Shiau et al. (2015) propose a B&B and genetic algorithms to solve a two-agent scheduling problem in a two-machine permutation flow shop with learning effects. The objective is to minimise the total completion time of the jobs from one agent, given that the maximum tardiness of the jobs from the other agent cannot exceed a bound.

Furthermore, there are many which consider the flow shop problems under this phenomena (Wang and Xia 2005; Wang 2006; Pargar and Zandieh 2012; Vahedi-Nouri, Fattahi, and Ramezanian; 2013a, 2013b) and identical parallel machine problems (Mosheiov 2001a; Huang, Wang, and Ji 2014; Lin 2014). Further details can be found in Table 2.

Motivated by several observations from manufacturing systems, Equation (1) has been modified by many studies. For example, Lee and Wu (2004) considered that learning takes place on each machine separately. This assumption is very common for multiple-machine scheduling with learning considerations. Slightly different from Equation (1), the learning effect can be formalised as follows:

$$P_{ijr} = P_{ij} r^{a} \quad i, r = 1, \ldots, n \tag{3}$$

with $j = 1, \ldots, m$ being the machine index and p_{ijr} being the processing time of job i if it is scheduled in position r on machine j. The authors developed a dominance property, a strong lower bound and a branch-and-bound algorithm to solve the two-machine flow shop problem with the goal to minimise total completion time. Mosheiov and Sidney (2003) assumes that learning effects differ from each job. For that, the authors propose a job-dependent learning effects as follows:

$$P_{ir} = P_i r^{a_i} \quad i, r = 1, \ldots, n \tag{4}$$

with a_i is a job-dependent negative parameter. Under this model, Vahedi Nouri, Fattahi, and Ramezanian (2013a) propose a hybrid firefly-simulated annealing algorithm for the flow shop problem with flexible maintenance activities.

Another observation from real manufacturing considers that if the production process itself is 100% machine-driven, it might be more appropriate to assume that learning takes place during the set-up time of the machine only. For that, Koulamas and Kyparisis (2007) assumes that the normal processing time p_i consists of a set-up time s_i and a production time v_i as shown:

$$P_{ir} = s_{ir} + v_i \qquad \text{with} \quad s_{ir} = s_i r^a \quad i, r = 1, \ldots, n \tag{5}$$

For the same observation, Pargar and Zandieh (2012) consider the bi-criteria hybrid flow shop scheduling with set-up times. They propose a mathematical model and meta-heuristic to minimise the weighted sum of total tardiness and makespan problem. Several researches consider another real-life situation, in which any delay in processing a job may result in an increasing effort to accomplish the job. Otherwise, the actual processing time of a job is modelled as increasing function of its starting time due to deterioration effects. In this way, Wang (2006) propose a new formula as follows:

$$P_{ir} = (P_i + wt)r^a \quad i, r = 1, \ldots, n \tag{6}$$

with t the starting time and w the amount of increase in processing time per unit delay in its starting time. The results on several single-machine and flow shop problems under this formula can be found in Table 2. Other research is inspired from Equation (6) such as Wang (2007), Wang and Cheng (2007), Kuo and Yang (2011) and Yin et al. (2015).

For example, Yin et al. (2015) consider the same model to solve the single-machine scheduling problem with deterioration model. The objective is to find the optimal schedule such that the makespan or total completion time is minimised. They prove that both problems are solvable in O(n log n) time. Taking into account the same assumptions, Wu, Lee, and Liou (2013) propose a branch-and-bound algorithm (B&B) and four heuristic algorithms to solve single-machine scheduling with two competing agents. The objective is to minimise the total weighted completion time of jobs from the first agent, given that the number of tardy jobs of the second agent is zero. Wang, Huo, and Ji (2014) consider deteriorating jobs and resource allocation into single-machine group technology scheduling problems. They proved that the problems to minimise the total weighted completion time and the makespan remain polynomially solvable under certain conditions. For the same problem, Pan et al. (2014) propose a new learning formula where learning effect depends not only on job position, but also on the position of the corresponding job group. Under the same model, Xu et al. (2016) show that the makespan and the total completion time problems remain polynomially optimal solvable under the proposed model. They propose some heuristics and a genetic algorithm (GA) to search for approximate solutions in order to minimise the total completion time for re-entrant permutation flow shop scheduling with learning effects. Table 2 presents the more recent problems under the position-based learning effects.

Although their successful, position-based learning effects have shortcomings which neglect the processing times of jobs already produced. For this reason, another approach called sum-of-processing time-based learning effects was proposed.

3.2 *Sum-of-processing time-based learning effects*

To overcome the shortcomings of position-based learning effects. Kuo and Yang (2006) introduce another viewpoint of learning effects i.e. *the more time you practice, the better performance you obtain*, based on *the sum-of-actual-processing times*. In many realistic settings, in which the operating processes of a job are different or the human interactions have a significant impact during the processing of the job, the processing time will add to the worker's experience and cause learning effects. For example, patient diagnosis and treatment or maintenance task whose conditions are different and there are no identical repetitions of operating process in their jobs. In such situations, there still exists a certain learning effect after operating the job due to their experiences. Therefore, the actual processing time of a job is affected by the total actual processing time of the previous job schedules. According to Kuo and Yang (2006), the actual processing time $P_{i,r}$ is defined as:

$$P_{i,r} = \left(1 + \sum_{k=1}^{r-1} P_k\right)^a P_i \tag{7}$$

Table 2. Position-based learning effects.

Problems	Solution algorithms	References
$1\left\|p_{j[r]}=p_j r^a\right\|\sum T$	random search, the tabu search and the simulated annealing	Eren and Güner (2007)
$F2\left\|p_j[r]=p_j r^a,\ d_j=d\right\|\sum\left(\alpha E_j+\beta T_j\right)$	Genetic algorithm, Tabu search, Random search	Icsler, Toklu, and Celik (2012)
$1\left\|p^x_{j[r]}=p^x_j r^a\right\|\sum W_j C^A_j : L^B_{\max}$	Branch and bound, Simulated annealing algorithms	Li and Hsu (2013)
$F_m\|p_{ijr}=p_{ij}r^{a_i}\|\sum Fj$	Heuristic	Vahedi-Nouri, Fattahi, and Ramezanian (2013b)
$P_m=P_j r^{b_j}+\alpha t\|TCT\ ADC$	Ass	Huang, Wang, and Ji (2014)
$P_m=P_j r^{b_j}+\alpha t\|TCT\ ADW$	Ass	Huang, Wang, and Ji (2014)
$1\left\|[p_r+(\alpha * t_r)]r^a\right\|\sum_{j=1}^n\left(AE_j+\beta T_j\right)$	SPT	Lin (2014)
$1\left\|\left[p_r+\left(\alpha * t_r^b\right)\right]r^a\right\|\sum_{j=1}^n\left(AE_j+\beta T_j\right)$	SPT	Lin (2014)
$1\left\|p_{j[r]}=\left(p_j+\alpha t_r\right)r^a\right\|C_{\max}$	SPT, LPT	Yin et al. (2015)
$1\left\|p_{j[r]}=\left(p_j+\alpha t_r\right)r^a\right\|\sum C_j$	SPT, LPT	Yin et al. (2015)
$F2\left\|a_{j[r]}=a_j\prod_{k=0}^{r-1}L_k, b_{j[r]}=b_j\prod_{k=0}^{r-1}L_k\right\|\sum C_j$	Branch and bound	Shiau et al. (2015)
$F2\left\|a_{j[r]}=a_j\prod_{k=0}^{r-1}L_k, b_{j[r]}=b_j\prod_{k=0}^{r-1}L_k\right\|T_{\max}$	Branch and bound	Shiau et al. (2015)
$Fm\left\|p_{j[r]}=(\alpha_j+\beta t)r^a,\alpha_{ij}=\alpha\right\|L_{\max}$	EDD	Wang and Xia (2005)
$Fm\left\|p_{j[r]}=(\alpha_j+\beta t)r^a,\alpha_{2j}=\alpha_2=\right\|C_{\max}$	non-increasing order of α_{1j}	Wang and Xia (2005)
$Fm\left\|p_{j[r]}=(\alpha_j+\beta t)r^a,\alpha_{ij}=\alpha\right\|\sum W_j C_j$	non-increasing order of w_j	Wang and Xia (2005)

where $a <= 0$ is a constant learning index.

Therefore, *the sum-of-processing time-based learning effect* takes into account the processing time of all jobs processed so far. A different sum-of-processing time-based learning effect has been proposed by Koulamas and Kyparisis (2007):

$$P_{ir}=P_i\left(\frac{\sum_{i=1}^n P_{[i]}}{\sum_{i=1}^n P_{[i]}}\right)^b \tag{8}$$

They proved that for both single-machine makespan minimisation problem and total completion time problem, an optimal sequence is given by the SPT rules. Under the same model, several studies have investigated other problems such as Wang (2010) who considered single machine. He showed that the makespan minimisation problem, the total completion time minimisation problem and the total completion time square minimisation problem can be solved by the Shortest Processing Time (SPT) rule. Kuo, Hsu, and Yang (2012) and Li et al. (2013) investigate some flow shop scheduling problems with several objective functions, namely: the total completion time, the makespan, the total weighted completion time, the total weighted discounted completion time and the sum of the quadratic job completion times. Recently, Wu, Lai, and Lee (2015) investigate a single-machine problem with both the sum-processing time-based learning and forgetting effects. They propose a model including some existing models as special cases (Kuo and Yang 2006; Koulamas and Kyparisis 2007; Janiak and Rudek 2009; Wang et al. 2009b). Table 3 shows the most well-known problems after 2007 and their solution algorithms.

3.3 *Truncated learning effects*

Although the models presented above are successful, they represent a major drawback. This drawback has been highlighted first by Cheng et al. (2009) who prove that the actual processing time of a job decline to zero precipitously as the number of jobs, or sum-of-processing times of those already processed, becomes larger. This phenomenon is at odds with reality. In fact, the learning effect should be limited. Slightly different from Kuo and Yang (2006), Cheng et al. (2009) propose another

Table 3. Sum-of-processing time-based learning effects.

Problems	Solution algorithms	References
$1\left\| p_{j[r]} = \left(1+\sum_{k=l}^{r-l} p_{i[k]}\right)^{a_i}\right\| \sum W_j C_j$	WSPT	Wang et al. (2008)
$1\left\| p_{j[r]} = \left(1+\sum_{k=l}^{r-l} p_{i[k]}\right)^{a_i}\right\| L_j$	Earliest Due date rule (EDD)	Wang et al. (2008)
$1\left\| p_{j[r]} = \left(1+\sum_{k=l}^{r-l} p_{i[k]}\right)^{a_i}\right\| \sum U_j$	Moore's Algorithm	Wang et al. (2008)
$1\left\| p_{j[r]} = \left(1+\sum_{k=1}^{r-1} \ln P_{[k]}\right)^{a} P_i\right\| C_{\max}$	SPT	Cheng et al. (2009)
$1\left\| p_{j[r]} = \left(1+\sum_{k=1}^{r-1} \ln P_{[k]}\right)^{a} P_i\right\| \sum C_j$	SPT	Cheng et al. (2009)
$1\left\| p_{j[r]} = \left(1+\sum_{k=1}^{r-1} \ln P_{[k]}\right)^{a} P_i\right\| \sum W_j C_j$	Non-decreasing order of p_i/w_i	Cheng et al. (2009)
$1\left\| p_{j[r]} = \left(1+\sum_{k=1}^{r-1} \ln P_{[k]}\right)^{a} P_i\right\| \sum T_j$	Non-decreasing order of d_i	Cheng et al. (2009)
$1\left\| p_{j[r]} = \left(1+\sum_{k=1}^{r-1} \ln P_{[k]}\right)^{a} P_i\right\| L_{\max}$	Non-decreasing order of d_i	Cheng et al. (2009)
$1\left\| p_{j[r]} = \left(1+\sum_{k=1}^{r-1} \ln P_{[k]}\right)^{a} P_i\right\| T_{\max}$	Non-decreasing order of d_i	Cheng et al. (2009)
$F_m\left\| p_{j[r]} = P_{ij}\left(1+\sum_{k=1}^{r-l} P_{i[k]}\right)^{a_i}\right\| C_{\max}$	ARB	Kuo, Hsu, and Yang (2012)
$F_m\left\| p_{j[r]} = P_{ij}\left(1+\sum_{k=1}^{r-l} P_{i[k]}\right)^{a_i}\right\| \sum C_i$	SPT	Kuo, Hsu, and Yang (2012)
$F_m\left\| p_{j[r]} = P_{ij}\left(1+\sum_{k=1}^{r-l} P_{i[k]}\right)^{a_i}\right\| \sum W_i C_i$	WSPT	Kuo, Hsu, and Yang (2012)
$F_m\left\| p_{j[r]} = P_{ij}\left(1+\sum_{k=1}^{r-l} P_{i[k]}\right)^{a_i}\right\| L_{\max}$	EDD	Kuo, Hsu, and Yang (2012)
$F_m\left\| Prmu,\ p_{j[r]} = P_{ij}\left(1+\sum_{k=1}^{r-l} P_{i[k]}\right)^{a_i}\right\| C_{\max}$	SPT	Li et al. (2013)
$F_m\left\| Prmu,\ p_{j[r]} = P_{ij}\left(1+\sum_{k=1}^{r-l} P_{i[k]}\right)^{a_i}\right\| \sum C_i$	SPT	Li et al. (2013)
$F_m\left\| Prmu,\ p_{j[r]} = P_{ij}\left(1+\sum_{k=1}^{r-l} P_{i[k]}\right)^{a_i}\right\| \sum W_j\left(1-e^{-\gamma C_j}\right)$	WDSPT	Li et al. (2013)
$F_m\left\| Prmu,\ p_{j[r]} = P_{ij}\left(1+\sum_{k=1}^{r-l} P_{i[k]}\right)^{a_i}\right\| \sum W_i C_i$	WSPT	Li et al. (2013)

function to model this observation in which the actual job processing time is a function of the sum of the logarithm of the processing times of the jobs already processed. The use of the logarithm function is to model the phenomenon: that learning as a human activity is subject to the law of diminishing return. Hence, the actual processing time is defined as:

$$P_{ir} = \left(1+\sum_{k=1}^{r-1} \ln P_{[k]}\right)^{a} P_i \tag{9}$$

Motivated by the same observation, a new approach denoted 'truncated learning effects' was proposed by Cheng et al. (2011), Wu, Yin, and Cheng (2011) and Wu et al. (2012) for the sum-of-processing time-based learning effect where the actual processing time of a job is a function that depends not only on the sum of normal processing times of the jobs already processed but also depends on a control parameter. In view of this observation, the actual processing time of a job is:

$$P_{ir} = \max\left\{\left(1+\sum_{k=1}^{r-1} P_{[k]}\right)^{a}, \beta\right\} P_i \tag{10}$$

where $a \leq 0$ is a constant learning index and β is the truncation parameter with $0 < \beta < 1$. For the same observation, Amirian and Sahraeian (2015) propose the same formula with different incompressibility factors and different degrees of learning for each job.

Alternatively, Wu, Yin, and Cheng (2013) and Cheng et al. (2013) propose the truncated position-based learning effects where the actual processing time of a job is a function of its position and a control parameter. So, the actual processing time of the jobs is defined as

$$P_{ir} = \max\left\{r^{a}, \beta\right\} P_i \tag{11}$$

The use of the truncated function is to model the phenomenon that learning of a human activity is limited. Other researches inspired from these two models are Wang and Wang (2013), Wang, Wang et al. (2013) and Niu, Wang, and Yin (2014). We present in Table 4 the more well-known problems with truncated learning effects and their solution algorithms.

Table 4. Truncated learning effects.

Problems	Solution algorithms	References
$1\vert P_j = \max\left\{\left(1+\sum_{l=1}^{r-1} P_{[l]}\right)^a, \beta\right\} \sum_{j=1}^{n} W_j C_j(S)\left(1-I_j\right), \sum_{j=1}^{n} U_j(S) I_j = 0$	Branch and bound and simulated annealing algorithms	Cheng et al. (2011)
$F2\left\vert a_{jr} = a_j \max\left\{r^a, \beta\right\}, b_{jr} = b_j \max\left\{r^a, \beta\right\}\right\vert C_{\max}$	Branch and bound and Genetic algorithms	Cheng et al. (2013)
$F_m\left\vert prmu,\ p_{ijr} = p_{ij} \max\left\{r^a, \beta\right\}\right\vert C_{\max}$	SPT	Wang and Wang (2013)
$F_m\left\vert prmu,\ p_{ijr} = p_{ij} \max\left\{r^a, \beta\right\}\right\vert \sum C_j$	SPT	Wang, Zhou et al. (2013)
$F_m\left\vert prmu,\ p_{ijr} = p_{ij} \max\left\{r^a, \beta\right\}\right\vert L_{\max}$	EDD	Wang, Zhou et al. (2013)
$F_m\left\vert prmu,\ p_{ijr} = p_{ij} \max\left\{r^a, \beta\right\}\right\vert \sum W_j C_j$	WSPT	Wang, Zhou et al. (2013)
$1\left\vert p_{j[r]} = p_j \max\left\{\left(1-\frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_{[l]}}\right)^a, \beta\right\}\right\vert \sum W_j\left(1-e^{-\gamma C_j}\right)$	WDSPT	Wu, Yin, and Cheng (2011)
$1\left\vert p_{j[r]} = p_j \max\left\{\left(1-\frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_{[l]}}\right)^a, \beta\right\}\right\vert \sum C_j$	SPT	Wu, Yin, and Cheng (2011)
$1\left\vert p_{j[r]} = p_j \max\left\{\left(1-\frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_{[l]}}\right)^a, \beta\right\}\right\vert \sum W_j C_j$	WSPT	Wu, Yin, and Cheng (2011)
$1\left\vert p_{j[r]} = p_j \max\left\{\left(1-\frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_{[l]}}\right)^a, \beta\right\}\right\vert L_j$	Non-decreasing order of d_i	Wu, Yin, and Cheng (2011)
$1\left\vert p_{j[r]} = p_j \max\left\{\left(1-\frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_{[l]}}\right)^a, \beta\right\}\right\vert \sum T_j$	Non-decreasing order of d_i	Wu, Yin, and Cheng (2011)
$F2\left\vert a_{jr} = a_j \max\left\{\left(1+\sum_{l=1}^{r-l} a_{[l]}\right)^a, \beta\right\}, b_{jr} = \max\left\{\left(1+\sum_{l=1}^{r-l} b_{[l]}\right)^a\right\}\right\vert \sum C_j$	Branch and Bound and simulated annealing algorithms	Lai et al. (2014)
$F2\left\vert a_{jr} = a_j \max\left\{\left(1-\sum_{l=1}^{r-l} a_{[l]}\right)^a, \beta\right\}, b_{jr} = \max\left\{\left(1-\sum_{l=1}^{r-l} b_{[l]}\right)^a\right\}\right\vert C_{\max}$	Genetic heuristic based algorithms and Branch and bound	Wu et al. (2015)
$F2\left\vert a_{jr} = a_j \max\left\{\left(1-\sum_{l=1}^{r-l} a_{[l]}\right)^a, \beta\right\}, b_{jr} = \max\left\{\left(1-\sum_{l=1}^{r-l} b_{[l]}\right)^a\right\}\right\vert \sum C_j$	Genetic heuristic based algorithms and Branch and bound	Wu et al. (2012)
$F2\left\vert a_{ijr} = a_{ij} \max\left\{r^a, \beta\right\}\right\vert C_{\max}$	ARB	Wang and Wang (2013)
$F_m\left\vert a_{ijr} = a_{ij} \max\left\{r^a, \beta\right\}\right\vert \sum C_j$	SPT, heuristics (NEH, WY)	Wang and Wang (2013)
$F_m\left\vert a_{ijr} = a_{ij} \max\left\{r^a, \beta\right\}\right\vert \sum W_j C_j$	WSPT, Heuristics	Wang and Wang (2013)
$F_m\left\vert a_{ijr} = a_{ij} \max\left\{r^a, \beta\right\}\right\vert \sum W_j\left(1-e^{-\gamma C_j}\right)$	WDSPT, Heuristics	Wang and Wang (2013)
$F_m\left\vert a_{ijr} = a_{ij} \max\left\{r^a, \beta\right\}\right\vert \sum C_j^2$	SPT, Heuristics	Wang and Wang (2013)
$F_m\left\vert a_{ijr} = a_{ij} \max\left\{r^a, \beta\right\}\right\vert L_{\max}$	EDD, NEH, WY	Wang and Wang (2013)

3.4 *Exponential learning effect function*

The learning effects consider the learning curve as a log-linear learning curve in which $p_{j[r]} = p_j r^a$ and the learning index is a constant. Then, for a given job processed in different positions is not stable because of $\frac{p_{j,r+1}}{p_{j,r}} = (\frac{r+1}{r})^a$ Hence, the processing time of a job decreases quickly if $a \leq 0$ and far from zero which is at odds with reality. For this reason, Wang and Xia (2005) propose an exponential learning effect model in which the actual processing time defined as:

$$p_{j,r} = p_j b^{r-1} \tag{12}$$

where $0 < b < 1$ is a constant. Consequently, $\frac{p_{j,r+1}}{p_{j,r}} = b$ and the processing time decrease slowly if b is near 1. Under this model, Dolgui, Gordon, and Strusevich (2012) show that the single-machine scheduling problem with precedence constraints is solvable in polynomial time. Furthermore, many researchers have been inspired from this model such as Zhang et al. (2011)who assume that the actual processing time of job J_j depends on the starting time of its processing as well as the position of the job in the sequence. Then, the actual processing is defined as:

$$p_{j,r}(t) = p_j(1 - \beta t)\alpha^{r-1} \tag{13}$$

where α and β are parameters such that $0 < \alpha < 1, 0 < \beta < 1$. They consider the single-machine problems and they prove that the classical SPT (Shortest Processing Time) rule remains optimal when minimising the makespan or the total completion time objective functions. Other results for this formula are present on Table 5. Under the same learning formula, Huang and Wang (2014) consider the two resource-constrained single-machine group scheduling problems. They consider the exponential positional learning. Wang and Wang (2011a) propose an exponential sum-of-the-actual-processing time-based learning effect. Then, the actual processing time is defined as:

$$p_j\left(\alpha a^{\frac{\sum_{i=1}^{r-1} p_i^A}{\sum_{i=1}^{n}}} + \beta\right) \tag{14}$$

They proved that for single machine with makespan minimisation problems, the sum of completion times minimisation problem, and some special cases of the total weighted completion time minimisation problem and the maximum lateness minimisation problem can be solved in polynomial time. Ma, Shao, and Wang (2014) use the same model as Wang, Wang, and Wang (2010) for different objective functions. Some recent papers which have considered scheduling problems with exponential function-based learning effects are Wang, Wang, and Wang (2010), Bai, Wang, and Wang (2012), Wang, Hsu, and Yang (2013), Ma, Shao, and Wang (2014) and Wang and Wang (2014a). Table 5 presents some exponential learning effect models.

3.5 *Position-based and sum-of-processing time-based learning effects models*

In several real-life manufacturing systems, the learning effects can be caused by different observations simultaneously. For that, the most recent research takes into account more than one observation to model the learning effects as realistic as possible. Cheng, Wu, and Lee (2008) assume that learning effect might arise from both human and machine learning effects simultaneously. For that, they introduced the first general learning effects model which combines the position-based learning effect to model the machine learning effects and the sum-of-actual-processing time to model the human learning effects. Hence, the actual processing time of this model is inspired from Koulamas and Kyparisis (2007) to model the-sum-of-actual-processing time and from Biskup (1999) to model the position-based learning effects. Then, the actual processing time of job j if it is scheduled in the rth position in a sequence is:

$$p_{j[r]} = p_j\left(1 - \frac{\sum_{l=1}^{r-1} p_l}{\sum_{l=1}^{r-1} p_l}\right)^{a_1} r^{a_2} \tag{15}$$

where p_j is the basic processing time of job j, $p_{[l]}$ denotes the basic processing time of the job scheduled in the lth position in the sequence and a_1 and a_2 denote two learning indices with $a_1 >= 1$ and $a_2 < 0$. They show that the single-machine problems to minimise makespan and total completion time are polynomially solvable. Also, they present polynomial–time optimal solutions for some special cases of the m-machine flow shop problems to minimise makespan and total completion time. Wu and Lee (2008) consider a similar model that can be described as follows:

$$p_{j[r]} = p_j\left(1 + \frac{\sum_{l=1}^{r-1} p_l}{\sum_{l=1}^{r-1} p_l}\right)^{a_1} r^{a_2} \tag{16}$$

They obtained similar results as Cheng, Wu, and Lee (2008). Afterwards, several researches have been inspired from this idea such as Wu and Lee (2009), Wang and Wang (2011b, 2013, 2014a), Cheng, Kuo, and Yang (2014), Luo and Zhang (2015). They proved some single-machine scheduling problems and some flow shop scheduling problems can be solved in polynomial time.

Another point of view from manufacturing systems affirms that each job has a different processing complexity. For that, each job can have different human learning effects. Yang, Cheng, and Kuo (2013) introduce the first model which considers both sum-of-processing times-based and the processing complexity of the jobs already processed. Then, the actual processing time of job J_i is defined as follows:

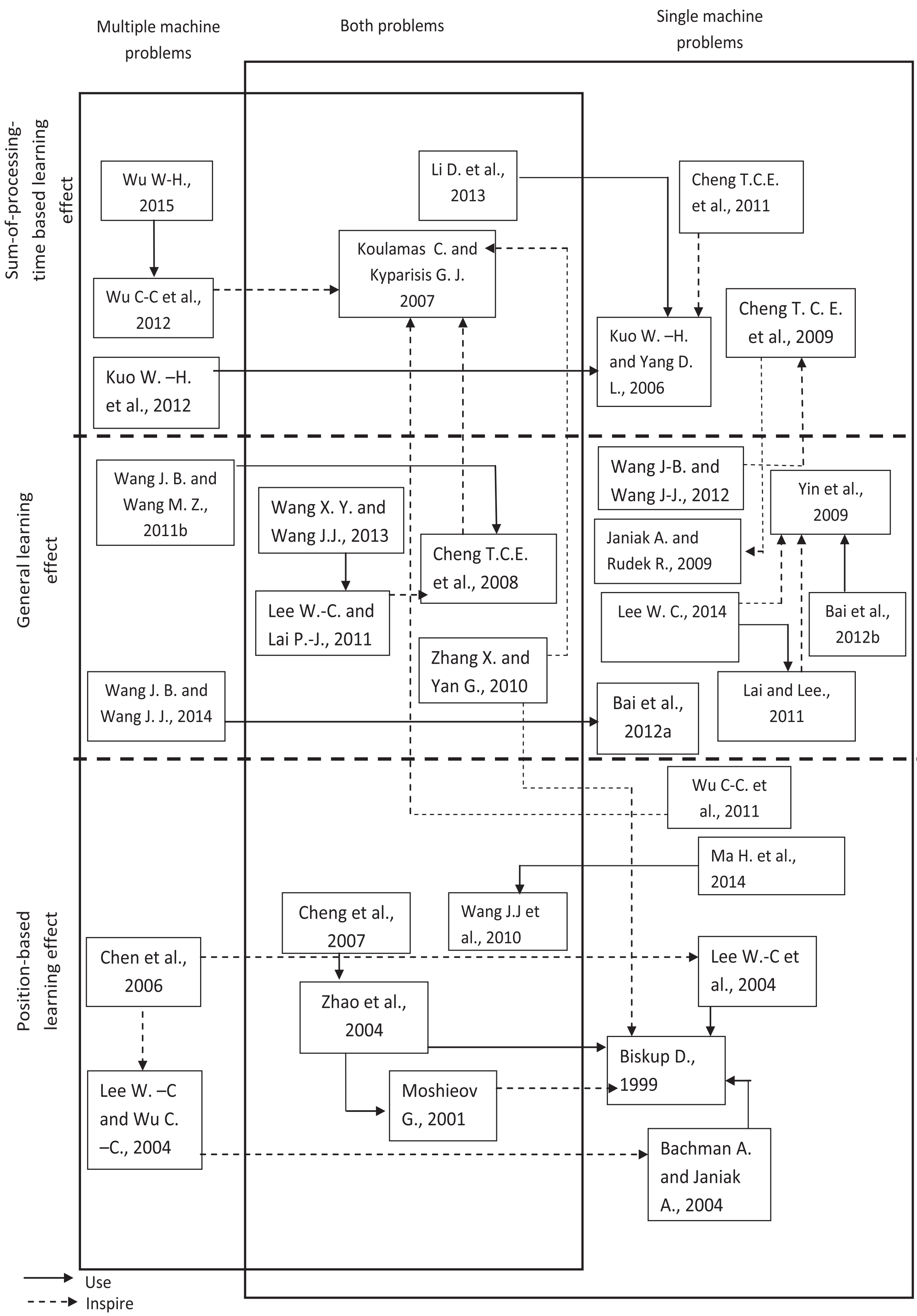

Figure 3. Cartography of some important research related to scheduling problems with learning effects.

Table 5. Exponential learning effects function.

Problems	Solution algorithms	References
$1\left\|p_{jr}^{A}=p_j\left(\alpha a^{\sum_{i=1}^{r-1}p_{[i]}}+\beta\right), S_{psd}\right\|C_{\max}$	SPT	Wang et al. (2009b)
$1\left\|p_{jr}^{A}=p_j\left(\alpha a^{\sum_{i=1}^{r-1}p_{[i]}}+\beta\right), S_{psd}\right\|\sum C_j$	SPT	Wang et al. (2009b)
$1\left\|p_{jr}^{A}=p_j\left(\alpha a^{\sum_{i=1}^{r-1}p_{[i]}}+\beta\right), S_{psd}\right\|\sum W_jC_j$	WSPT	Wang et al. (2009b)
$1\left\|p_j\left(\alpha a^{\frac{\sum_{i=1}^{r-1}p_i^A}{\sum_{i=1}^{n}}}+\beta\right)\right\|C_{\max}$	SPT	Wang and Wang (2011a)
$1\left\|p_j\left(\alpha a^{\frac{\sum_{i=1}^{r-1}p_i^A}{\sum_{i=1}^{n}}}+\beta\right)\right\|\sum C_j$	SPT	Wang and Wang (2011a)
$1\left\|p_j\left(\alpha a^{\frac{\sum_{i=1}^{r-1}p_i^A}{\sum_{i=1}^{n}}}+\beta\right)\right\|\sum C_j^{\theta}$	SPT	Wang and Wang (2011a)
$1\left\|p_{j,r}(t)=p_j(1-\beta t)\alpha^{r-1}\right\|C_{\max}$	SPT	Zhang et al. (2011)
$1\left\|p_{j,r}(t)=p_j(1-\beta t)\alpha^{r-1}\right\|\sum C_j$	SPT	Zhang et al. (2011)
$1\left\|p_{j,r}(t)=p_j(1-\beta t)\alpha^{r-1}\right\|\sum W_jC_j$	WSPT	Zhang et al. (2011)
$1\left\|p_{j,r}(t)=p_j(1-\beta t)\alpha^{r-1}\right\|L_{\max}$	EDD	Zhang et al. (2011)
$1\left\|p_{j,r}(t)=p_j(1-\beta t)\alpha^{r-1}\right\|\sum W_j\left(1-e^{-rC_j}\right)$	Heuristic (WDSPT)	Zhang et al. (2011)
$1\left\|p_j\left(\alpha a^{\frac{\sum_{i=1}^{r-1}p_i^A}{\sum_{i=1}^{n}}}+\beta\right)\right\|\sum W_jC_j$	WSPT	Ma, Shao, and Wang (2014)
$1\left\|p_j\left(\alpha a^{\frac{\sum_{i=1}^{r-1}p_i^A}{\sum_{i=1}^{n}}}+\beta\right)\right\|\sum L_j$	SPT	Ma, Shao, and Wang (2014)
$1\left\|p_j\left(\alpha a^{\frac{\sum_{i=1}^{r-1}p_i^A}{\sum_{i=1}^{n}}}+\beta\right)\right\|L_{\max}$	EDD	Ma, Shao, and Wang (2014)
$1\left\|p_j\left(\alpha a^{\frac{\sum_{i=1}^{r-1}p_i^A}{\sum_{i=1}^{n}}}+\beta\right)\right\|\sum T_j$	EDD	Ma, Shao, and Wang (2014)
$1\left\|p_j\left(\alpha a^{\frac{\sum_{i=1}^{r-1}p_i^A}{\sum_{i=1}^{n}}}+\beta\right)\right\|\sum W_j\left(1-e^{-\gamma C_j}\right)$	Heuristic (WDSPT)	Ma, Shao, and Wang (2014)

$$p_{j[r]}=p_j\left(1+\sum_{k=1}^{r-1}W_{[k]}p_k\right)^{a}r^{b} \tag{17}$$

where $a<0$ and $b<0$ are the learning indices for sum-of-processing time-based and position-based learning, respectively, $P_{[k]}$ is the normal processing time of the job scheduled in position k of a sequence and $w_{[k]}>0$ is the associated weight of the job. The weights are dependent on the processing complexity of the jobs. They consider some single-machine and flow shop scheduling problems under the proposed learning model to minimise several performance measures, such as the makespan, the total completion time, the sum of kth power of completion times and the maximum lateness.

3.6 *General learning effects models*

In this subsection, we consider a more general learning model where the actual processing time of a job is not only a function of the total normal processing time of the job already processed, but also a function of the job's scheduled position. This observation is first studied by Yin et al. (2009). They prove that some single-machine scheduling problems and m-machine permutation flow shop problems are still polynomially solvable under the proposed model. According to this latter, the actual processing time of job J_i is defined as follows:

Table 6. General learning effects.

Problems	Solution algorithms	References
$1 \left\| p_{j[r]} = p_j \left(1 - \frac{\sum_{l=1}^{r-1} p_l}{\sum_{l=1}^{r-1} p_l}\right)^{a_1} r^{a_2} \right\| \sum C_j$	SPT	Cheng, Wu, and Lee (2008)
$1 \left\| p_{j[r]} = p_j \left(1 - \frac{\sum_{l=1}^{r-1} p_l}{\sum_{l=1}^{r-1} p_l}\right)^{a_1} r^{a_2} \right\| C_{\max}$	SPT	Cheng, Wu, and Lee (2008)
$1 \left\| p_{j[r]} = a_j - b_j \min\left\{\sum_{l=1}^{r-l} a_{[l]}, g_j\right\} \right\| C_{\max}$	Branch and Bound Heuristics	Janiak et al. (2009)
$1 \left\| p_{j[r]} = p_j \left(1 + \frac{\sum_{l=1}^{r-l} p_l}{\sum_{l=1}^{n} p_l}\right)^{a_1} r^{a_2} \right\| C_{\max}$	SPT	Wu and Lee (2008)
$1 \left\| p_{j[r]} = p_j \left(1 + \frac{\sum_{l=1}^{r-l} p_l}{\sum_{l=1}^{n} p_l}\right)^{a_1} r^{a_2} \right\| \sum C_j$	SPT	Wu and Lee (2008)
$1 \left\| p_{j[r]} = p_j \left(1 + \frac{\sum_{l=1}^{r-l} p_l}{\sum_{l=1}^{n} p_l}\right)^{a_1} r^{a_2} \right\| \sum W_j C_j$	Non decreasing order of p_i/w_i	Wu and Lee (2008)
$1 \left\| p_{jr}^A = p_j \left(\alpha a^{\sum_{i=1}^{r-1} w_i p_{[i]}} + \beta\right) b^{r-1}, S_{psd} \right\| C_{\max}$	SPT	Bai, Wang, and Wang (2012)
$1 \left\| p_{jr}^A = p_j \left(\alpha a^{\sum_{i=1}^{r-1} w_i p_{[i]}} + \beta\right) b^{r-1}, S_{psd} \right\| \sum C_j$	SPT	Bai, Wang, and Wang (2012)
$1 \left\| p_{jr}^A = p_j \left(\alpha a^{\sum_{i=1}^{r-1} w_i p_{[i]}} + \beta\right) b^{r-1}, S_{psd} \right\| \sum C_j^{\theta}$	SPT	Bai, Wang, and Wang (2012)
$1 \left\| p_{jr}^A = p_j \left(\alpha a^{\sum_{i=1}^{r-1} w_i p_{[i]}} + \beta\right) b^{r-1}, S_{psd} \right\| L_{\max}$	EDD	Bai, Wang, and Wang (2012)
$1 \left\| p_{jr}^A = p_j \left(\alpha a^{\sum_{i=1}^{r-1} w_i p_{[i]}} + \beta\right) b^{r-1}, S_{psd} \right\| \sum W_j C_j$	WSPT	Bai, Wang, and Wang (2012)
$1 \left\| p_{j[r]} = p_j q_r^{a_1} \left(C_0 + \sum_{l=1}^{r-l} \beta_{r-l} p_{[l]}\right)^{a_2} \right\| \sum W_j C_j$	Non decreasing order of p_i/w_i	Wu and Lee (2009)
$1 \left\| p_{j[r]} = p_j q_r^{a_1} \left(C_0 + \sum_{l=1}^{r-l} \beta_{r-l} p_{[l]}\right)^{a_2} \right\| C_{\max}$	SPT	Wu and Lee (2009)
$1 \left\| p_{j[r]} = p_j q_r^{a_1} \left(C_0 + \sum_{l=1}^{r-l} \beta_{r-l} p_{[l]}\right)^{a_2} \right\| \sum C_j$	SPT	Wu and Lee (2009)
$1 \left\| p_{j[r]} = p_j q_r^{a_1} \left(C_0 + \sum_{l=1}^{r-l} \beta_{r-l} p_{[l]}\right)^{a_2} \right\| T_{\max}$	EDD	Wu and Lee (2009)
$F_m \left\| p_{ij[r]} = p_{ij} q_r^{a_1} \left(C_0 + \sum_{l=1}^{r-l} \beta_{r-l} p_{[l]}\right)^{a_2} \right\| C_{\max}$	SPT	Wu and Lee (2009)
$1 \left\| p_{j[r]} = p_j q_r^{a_1} \left(1 - \sum_{l=1}^{r-l} \beta_l p_{[l]}\right)^{a_2} \right\| C_{\max}$	SPT	Wang and Wang (2011b)
$1 \left\| p_{j[r]} = p_j q_r^{a_1} \left(1 - \sum_{l=1}^{r-l} \beta_l p_{[l]}\right)^{a_2} \right\| W_j C_j$	Non decreasing order of p_i/w_i	Wang and Wang (2011b)
$F_m \left\| p_{ij[r]} = p_{ij} q_r^{a_1} \left(1 - \sum_{l=1}^{r-l} \beta_l p_{[l]}\right)^{a_2} \right\| \sum T_j$	EDD	Wang and Wang (2011b)
$1 \left\| p_j \left(1 - \frac{\sum_{l=r}^{n} p_l}{\sum_{l=1}^{n} p_l}\right)^{a} b^{(r-1)} \right\| \sum C_j$	SPT	Low and Lin (2011)
$1 \left\| p_j \left(1 - \frac{\sum_{l=r}^{n} p_l}{\sum_{l=1}^{n} p_l}\right)^{a} b^{(r-1)} \right\| \sum W_j C_j$	Non decreasing order of p_i/w_i	Low and Lin (2011)
$1 \left\| p_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_k, r\right) \right\| \sum C_j$	SPT	Lai and Lee (2011)
$1 \left\| p_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_k, r\right) \right\| \sum W_j C_j$	WSPT	Lai and Lee (2011)
$1 \left\| p_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_k, r\right) \right\| \sum L_{\max}$	EDD	Lai and Lee (2011)
$1 \left\| p_{j[r]} = p_j q_r^{a_1} \left(C - \frac{\sum_{l=1}^{r-l} \beta_l \ln p_{[l]}}{\sum_{l=1}^{n} p_{[l]}}\right)^{a_2} \right\| \sum C_j^{\theta}$	SPT	Wang and Wang (2014b)
$1 \left\| p_{j[r]} = p_j q_r^{a_1} \left(C - \frac{\sum_{l=1}^{r-l} \beta_l \ln p_{[l]}}{\sum_{l=1}^{n} p_{[l]}}\right)^{a_2} \right\| C_{\max}$	SPT	Wang and Wang (2014b)
$1 \left\| p_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_k, r\right), S_{psd} \right\| C_{\max}$	SPT	Wang and Wang (2013)
$1 \left\| p_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_k, r\right), S_{psd} \right\| \sum C_j$	SPT	Wang and Wang (2013)
$1 \left\| p_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_k, r\right), S_{psd} \right\| \sum W_j C_j$	WSPT	Wang and Wang (2013)
$1 \left\| p_{j[r]} = p_j q_r^{a_1} \left(1 - \sum_{l=1}^{r-1} \beta_r - p_{[l]}\right)^{a_2} \right\| C_{\max}$	SPT	Zhang and Yan (2010)
$1 \left\| p_{j[r]} = p_j q_r^{a_1} \left(1 - \sum_{l=1}^{r-1} \beta_r - p_{[l]}\right)^{a_2} \right\| \sum C_j$	Non decreasing order of p_i/w_i	Zhang and Yan (2010)
$1 \left\| p_{j[r]} = p_j q_r^{a_1} \left(1 - \sum_{l=1}^{r-1} \beta_r - p_{[l]}\right)^{a_2} \right\| \sum T_j$	EDD	Zhang and Yan (2010)

(*Continued*)

Table 6. (*Continued*).

Problems	Solution algorithms	References
$F_m \left\| p_{j[r]} = p_j q_r^{a_1} \left(1 - \sum_{l=1}^{r-1} \beta_r - p_{[l]}\right)^{a_2}, p_{ij} = p_j \right\| C_{\max}$	SPT	Zhang and Yan (2010)
$F_m \left\| p_{j[r]} = p_j q_r^{a_1} \left(1 - \sum_{l=1}^{r-1} \beta_r - p_{[l]}\right)^{a_2}, p_{ij} = p_j \right\| \sum C_{mj}$	SPT	Zhang and Yan (2010)
$F_m \left\| p_{j[r]} = p_j q_r^{a_1} \left(1 - \sum_{l=1}^{r-1} \beta_r - p_{[l]}\right)^{a_2}, p_{ij} = p_j \right\| L_{\max}$	EDD	Zhang and Yan (2010)
$F_m \left\| p_{j[r]} = p_j q_r^{a_1} \left(1 - \sum_{l=1}^{r-1} \beta_r - p_{[l]}\right)^{a_2}, idm \right\| C_{\max}$	Non decreasing order of p_{mj}	Zhang and Yan (2010)
$FP_m \left\| p_{j[r]}^A = p_j \left(\alpha a^{\sum_{i=1}^{r-1} w_i p_{[i]}} + \beta\right) b^{r-1} \right\| C_{\max}$	Heuristics	Wang and Wang (2014a)
$FP_m \left\| p_{j[r]}^A = p_j \left(\alpha a^{\sum_{i=1}^{r-1} w_i p_{[i]}} + \beta\right) b^{r-1} \right\| \sum C_j$	Heuristics	Wang and Wang (2014a)
$FP_m \left\| p_{j[r]}^A = p_j \left(\alpha a^{\sum_{i=1}^{r-1} w_i p_{[i]}} + \beta\right) b^{r-1} \right\| \sum W_j C_j$	Heuristics	Wang and Wang (2014a)
$1 \left\| p_{j[r]} = p_j \left(1 + \sum_{k=1}^{r-1} W_{[k]} p_k\right)^a r^b \right\| C_{\max}$	Non decreasing order of p_j	Yang, Cheng, and Kuo (2013)
$1 \left\| p_{j[r]} = p_j \left(1 + \sum_{k=1}^{r-1} W_{[k]} p_k\right)^a r^b \right\| \sum C_j$	Non decreasing order of p_j	Yang, Cheng, and Kuo (2013)
$1 \left\| p_{j[r]} = p_j \left(1 + \sum_{k=1}^{r-1} W_{[k]} p_k\right)^a r^b \right\| \sum C_i^k$	Non decreasing order of p_j	Yang, Cheng, and Kuo (2013)
$1 \left\| p_{j[r]} = p_j \left(1 + \sum_{k=1}^{r-1} W_{[k]} p_k\right)^a r^b \right\| L_{\max}$	Non decreasing order of p_j	Yang, Cheng, and Kuo (2013)
$1 \left\| p_{j[r]}^A = p_j^N f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}^{A\|N}, r\right) \right\| \sum W_j C_j$	WSPT	Luo (2015)
$1 \left\| p_{j[r]}^A = p_j^N f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}^{A\|N}, r\right) \right\| C_{\max}$	SPT	Luo (2015)
$1 \left\| p_{j[r]}^A = p_j^N f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}^{A\|N}, r\right) \right\| L_{\max}$	EDD	Luo (2015)
$1 \left\| p_{j[r]}^A = \left\{ p_j \left(\alpha a^{\frac{\sum_{i=1}^{r-1} p_i^A}{\sum_{i=1}^{n}}} + \beta\right), \gamma \right\} \right\| C_{\max}$	SPT	Wang, Wang et al. (2013)
$1 \left\| p_{j[r]}^A = \left\{ p_j \left(\alpha a^{\frac{\sum_{i=1}^{r-1} p_i^A}{\sum_{i=1}^{n}}} + \beta\right), \gamma \right\} \right\| \sum C_j$	SPT	Wang, Wang et al. (2013)
$1 \left\| p_{j[r]}^A = \left\{ p_j \left(\alpha a^{\frac{\sum_{i=1}^{r-1} p_i^A}{\sum_{i=1}^{n}}} + \beta\right), \gamma \right\} \right\| \sum W_j C_j$	WSPT	Wang, Wang et al. (2013)
$1 \left\| p_{j[r]}^A = \left\{ p_j \left(\alpha a^{\frac{\sum_{i=1}^{r-1} p_i^A}{\sum_{i=1}^{n}}} + \beta\right), \gamma \right\} \right\| \sum C_j^\theta$	SPT	Wang, Wang et al. (2013)
$1 \left\| p_{j[r]}^A = \left\{ p_j \left(\alpha a^{\frac{\sum_{i=1}^{r-1} p_i^A}{\sum_{i=1}^{n}}} + \beta\right), \gamma \right\} \right\| \sum L_j$	SPT	Wang, Wang et al. (2013)
$1 \left\| p_{j[r]}^A = \left\{ p_j \left(\alpha a^{\frac{\sum_{i=1}^{r-1} p_i^A}{\sum_{i=1}^{n}}} + \beta\right), \gamma \right\} \right\| L_{\max}$	EDD	Wang, Wang et al. (2013)
$1 \left\| p_{j[r]}^A = \left\{ p_j \left(\alpha a^{\frac{\sum_{i=1}^{r-1} p_i^A}{\sum_{i=1}^{n}}} + \beta\right), \gamma \right\} \right\| \sum W_j \left(1 - e^{-\gamma C_j}\right)$	WDSPT	Wang, Wang et al. (2013)
$1 \left\| p_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_k, r\right), s_{spd} \right\| C_{\max}$	SPT	Lee (2014)
$1 \left\| p_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_k, r\right) \right\| \sum C_i$	SPT	Lee (2014)
$1 \left\| p_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_k, r\right) \right\| \sum W_i C_i$	WSPT	Lee (2014)
$1 \left\| p_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} \beta_k p_k, r\right) \right\| \sum T_i$	EDD	Lee (2014)
$1 \left\| p_{jr}^A = p_j f\left(\sum_{k=1}^{r-1} p_k\right) g(r) \right\| C_{\max}$	SPT	Yin et al. (2009)
$1 \left\| p_{jr}^A = p_j f\left(\sum_{k=1}^{r-1} p_k\right) g(r) \right\| \sum C_i$	SPT	Yin et al. (2009)
$1 \left\| p_{jr}^A = p_j f\left(\sum_{k=1}^{r-1} p_k\right) g(r) \right\| \sum C_i^k$	SPT	Yin et al. (2009)
$1 \left\| p_{jr}^A = p_j f\left(\sum_{k=1}^{r-1} p_k\right) g(r) \right\| \sum W_i C_i$	WSPT	Yin et al. (2009)

(*Continued*)

Table 6. (*Continued*).

Problems	Solution algorithms	References
$1\left\|p_{jr}^{A}=p_j f\left(\sum_{k=1}^{r-1}p_k\right)g(r)\right\|\sum L_{\max}$	EDD	Yin et al. (2009)
$1\left\|p_{jr}^{A}=p_j f\left(\frac{\sum_{k=1}^{r-1}p_{[k]}^{A}}{\sum_{k=1}^{n}p_k}\right)g(r)\right\|C_{\max}$	SPT	Yin, Xu, and Wang (2010a)
$1\left\|p_{jr}^{A}=p_j f\left(\frac{\sum_{k=1}^{r-1}p_{[k]}^{A}}{\sum_{k=1}^{n}p_k}\right)g(r)\right\|\sum C_i^k$	SPT	Yin, Xu, and Wang (2010a)
$1\left\|p_{jr}^{A}=p_j f\left(\frac{\sum_{k=1}^{r-1}p_{[k]}^{A}}{\sum_{k=1}^{n}p_k}\right)g(r)\right\|\sum W_iC_i$	WSPT	Yin, Xu, and Wang (2010a)
$1\left\|p_{jr}^{A}=p_j f\left(\frac{\sum_{k=1}^{r-1}p_{[k]}^{A}}{\sum_{k=1}^{n}p_k}\right)g(r)\right\|\sum L_{\max}$	EDD	Yin, Xu, and Wang (2010a)
$1\left\|p_{jr}=p_j f\left(\sum_{k=1}^{r-1}\alpha_k p_{[k]}\right)g(r)\right\|\sum W_iC_i$	WSPT	Yin, Liu et al. (2012)
$1\left\|p_{jr}=p_j f\left(\sum_{k=1}^{r-1}\alpha_k p_{[k]}\right)g(r)\right\|\sum L_{\max}$	EDD	Yin, Liu et al. (2012)
$1\left\|p_{jr}=p_j f\left(\sum_{k=1}^{r-1}\alpha_k p_{[k]}\right)g(r)\right\|C_{\max}$	SPT	Yin, Liu et al. (2012)
$1\left\|p_{jr}=p_j f\left(\sum_{k=1}^{r-1}\alpha_k p_{[k]}\right)g(r)\right\|\sum C_i$	SPT	Yin, Liu et al. (2012)
$1\left\|p_{jr}=p_j f\left(\sum_{k=1}^{r-1}\alpha_k p_{[k]}\right)g(r)\right\|\sum W_j\left(1-e^{-\gamma C_j}\right)$	WDSPT	Yin, Liu et al. (2012)

$$p_{jr}^{A}=p_j f\left(\sum_{k=1}^{r-1}p_k\right)g(r) \tag{18}$$

where $\sum_{k=1}^{0}p_k=0$, p_k denotes the normal processing time of the job scheduled in the kth position in the sequence. Here, it is worth noting that model Equations (3), (5), (7), (8), (10)–(12), (14)–(16) are all special cases of model Equation (18).

Slightly different from Equation (18), Yin, Xu, and Wang (2010a) propose a more general model where the actual processing time of a job is based on a general function of the total actual processing times of the jobs already processed and a general function of the job's scheduled position as follows:

$$p_{jr}^{A}=p_j f\left(\frac{\sum_{k=1}^{r-1}p_k^{A}}{\sum_{k=1}^{n}p_k}\right)g(r) \tag{19}$$

They proved that for the single machine with makespan minimisation problem and the sum of the kth power of completion times, minimisation problem can be solved in polynomial time, respectively. Other researches used this model for different problems such as Yin, Xu, and Wang (2010b) consider sequence-dependent set-up time with some single-machine scheduling problems. They proved that minimising the makespan, the sum of the kth power of completion times, the total weighted completion time and the maximum lateness is polynomially solvable under certain conditions.

Inspired by Yin et al. (2009), Lai and Lee (2011) propose a new general learning effects in which the actual job processing time is a general function of the normal processing time of jobs already processed and its scheduled position. Then, the actual processing is:

$$p_{jr}^{A}=p_j f\left(\sum_{k=1}^{r-1}\beta_k p_k, r\right) \tag{20}$$

for $r=1,2,\ldots,n$, if it is scheduled in the rth position in a sequence where p_k and p_{jr}^{A} denote the normal and the actual processing times of the job j scheduled in the kth position in a sequence, and $\beta_1\leq\beta_2\leq\cdots\leq\beta_n$ which means that the weights of jobs contributing to actual processing time of a job are ordered non-decreasingly. This model has the advantage that different learning curves can be constructed easily, such as the plateau function. Yin, Xu, and Huang (2012) propose a revision of the model of Lee (2011).

Subsequently, several research studies consider Equation (20) such as Lee (2014) which uses this model in connection with deteriorating jobs and past sequence-dependent set-up time simultaneously. Also, Luo (2015) uses Equation (20) to solve multi-criteria single-machine problems. Another more general model is proposed by Yin, Liu et al. (2012) which considers the effects of position-dependent learning and time-dependent deterioration simultaneously. The results of this model are reported in Table 6.

Wang, Wang et al. (2013) proposes a single-machine scheduling problem with truncated exponential learning functions. By the truncated exponential learning functions, they mean that the actual job processing time is a function which depends not only on the total normal processing times of the jobs already processed but also on a control parameter. Table 6 presents the well-known general learning effect models.

4. Cartography

To have a global view on the most used studies in this field, we propose our cartography of some well-known models in Figure 3. Several researches used the same learning effects model for different objective functions: for example, Wu et al. (2015) used the same model as Wu et al. (2012) to solve different objective functions. Wang and Wang (2014a) use the same model Bai, Wang, and Wang (2012) to solve flow shop problem with different objective functions. Moreover, other research can inspire others. For example, Cheng, Wu, and Lee (2008) inspired from Koulamas and Kyparisis (2007). The general learning model proposed by Zhang and Yan (2010) was inspired from Koulamas and Kyparisis (2007) and Biskup (1999).

Regarding the various learning models considered in connection with scheduling context, we remark that the most of models have been inspired or constructed from several other formulas simultaneously. We note here that all of the models presented in our cartography are based on log-linear learning curve (Wright 1936). For that, we present some models under different learning curves in the following. Wang et al. (2009a) considered single-machine scheduling of deteriorating jobs under DeJong's learning model to minimise such common scheduling objectives as makespan, total completion time and maximum lateness. He showed that the problems are polynomially solvable or they are polynomially solvable under agreeable conditions. Okołowski and Gawiejnowicz (2010) propose for the first time the Dejong's learning curve in connection with parallel machine scheduling problem. The authors used the following formula:

$$P_{j,r} = P_j \left[M + (1 - M)\, r^{a} \right] \tag{21}$$

They proposed two exact algorithms: a sequential branch and bound and a parallel Branch and Bound to minimise the makespan function. Other research proposed by Ji et al. (2015) solved several single- and parallel machine problems under Dejong's model. A recent research proposed by Ji et al. (2016) considers parallel machine scheduling with deteriorating jobs and DeJong's learning effect. They prove that for the total completion time and the makespan, objectives functions are still polynomially solvable under agreeable conditions.

A more general model proposed by Lee (2011) can describe many different learning curves, such as the well-known plateau or the S-shaped phenomena, which is defined as follows:

$$P_{j,r} = P_j \prod_{r-1}^{r-1} a_l \tag{22}$$

where a_l denotes the learning impact on processing the lth position job and the $\prod_{r-1}^{r-1} a_l$ denotes the cumulative level of learning effect after processing r jobs. Here, the authors show that the single machine problem to minimise the total tardiness is polynomially solvable under certain agreeable conditions. Other models are based on plateau or S-shaped model such as Janiak and Rudek (2008) and Janiak et al. (2009).

5. Conclusions and future trends

Conventional research models have shown that learning effects have a significant impact on manufacturing systems. However, in last decades, this phenomenon has been considered in connection with scheduling problems. Due to several observations from production systems, scheduling research community has proposed several kinds of learning effects to find the optimal schedule as realistic as possible.

In this paper, we shed light on several new kinds of learning effects throughout a new classification scheme. Then, we delivered a cartography of some well-known scheduling problems under learning considerations. In the following, we intend to open new insights for further research considering gaps and future trends in this area:

- It's well surprising that most of learning models in scheduling are based on the log-linear learning curve. However, several researches about learning itself pointed out that the curve may be an 'S'-shaped function, plateau model, DeJong model or the Stanford-B model. Despite the relevance of these models in real life, there are few papers which considered them in the scheduling domain.
- How to schedule jobs on a production shop characterised by flexible routing and flexible machines, where worker learns and improves his performance by processing tasks? On the one hand, a typical example for such production

shop is the aircraft industry. On the other hand, learning curve was first discovered in aircraft industry (Wright 1936). Hence, it's so interesting to study the flexible job shop scheduling problem with learning consideration. To the best of our knowledge, there are only few researches of Tayebi Araghi, Jolai, and Rabiee (2014) which propose a hybrid meta-heuristic approach to solve the flexible job shop scheduling problem with learning effects.

- We note that the computational complexity of the general problems remains open, so resolving this issue is an interesting topic for future research.
- Another interesting research direction has been tackled by many works which consider simultaneously learning and ageing (or deterioration) effects. This latter is a phenomenon of reduction of the processor efficiency as a result of its fatigue. Consequently, job processing times may be an increasing function of job starting time. We refer the readers to the most recent survey of Janiak, Krysiak, and Trela (2011)which presents an overview of existing scheduling problems with both learning and ageing effects with some real-life applications. However, in the same paper, the authors state that both phenomena are observed in general long time horizon in repetitive systems and scheduling problems concern repetitive short horizon planning problems. For that, they consider these two phenomena as contradictory and there is no sense to consider this kind of scheduling problems. Here, at first glance, we agree with this observation. Nevertheless, much research has demonstrated that learning and/or ageing effects might be important in short-term production planning such as when calculating the optimal lot-sizes problem. Keachie and Fontana (1966) and Li and Cheng (1994). For scheduling problems, these phenomena were analysed and studied decades ago in different production systems (Womer 1979; Yelle 1979; Li and Cheng 1994). Furthermore, recent researches prove the important impact of these phenomena on such problems (Li 2016). On the one hand, modern manufacturers understand that the world's economy is undergoing a period of massive change and uncertainty. To this end, modern scheduling applications are subject to higher level of product personalisation and also to shorter product life cycle. On the other hand, learning effects are important if the production environment changes. Consequently, it's natural to consider this phenomenon on scheduling problems. In fact, all existing learning/ageing models in literature try to simulate their impact on scheduling problem (Janiak and Rudek 2009; Janiak and Rudek 2010).
- Each learning effects model is inspired from different observations on manufacturing system. For that, if several observations occur simultaneously and in the same manufacturing system, we can combine several existing models with regard to which best represents the reality. This remark may be an appropriate area that requires attention of researchers in the near future.
- It is interesting to study the multi-criteria problems of scheduling problems under learning effects to best model the reality.
- To the best of our knowledge, there are a few researches which study the impact of dynamic events such as breakdown machines, dynamic arrivals of jobs and absence of workers on the scheduling problems under learning effects. Only the research proposed by Zhang, Wu, and Zhou (2013) considered to analyse position-based learning and deterioration effects in single-machine stochastic scheduling with breakdown machine problems. Furthermore, there is the most recent research proposed by Li (2016) which considers a single-machine scheduling with random nominal processing time and/or random job-based learning effect, with the objective of minimising the total flow time and makespan. This way may be an appropriate area that requires more attention of researchers in the near future.
- Research on time-dependent scheduling problems has created a new area in the scheduling field. In fact, there are many situations where the processing time depends on the starting time of the job. In such situations, the processing time of a job may be an increasing and/or decreasing function of its starting time. Several real example are provided in Alidaee and Womer (1999). For the literature review, we refer the readers to the most recent researches in this area (Yin, Cheng, and Wu 2015).

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