



Time Series Analysis and Forecasting

Seminar 3: ARIMA Models



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Seminar Outline

Today's Activities:

1. **Review Quiz** — Checking understanding of ARIMA concepts
2. **True/False Questions** — Conceptual checks
3. **Practice Problems** — Calculations with ARIMA
4. **Worked Examples** — Real-world applications
5. **Discussion Questions** — Practical applications
6. **AI-Assisted Exercises** — Human vs. AI modeling

Quiz 1: Integration Order

Question

A time series Y_t requires two differences to become stationary. What is its order of integration?

- A) $I(0)$
- B) $I(1)$
- C) $I(2)$
- D) Cannot be determined

Quiz 1: Integration Order

Question

A time series Y_t requires two differences to become stationary. What is its order of integration?

Answer: $C - I(2)$

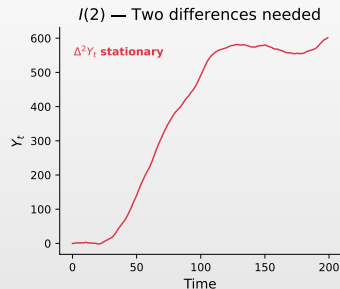
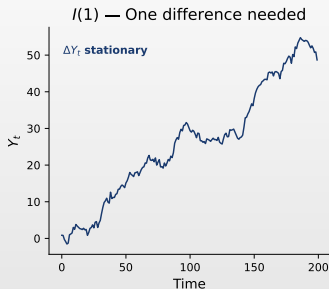
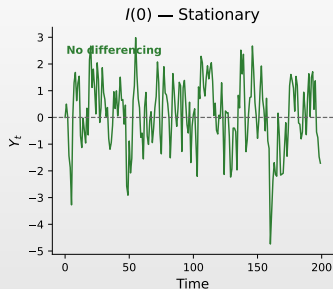
Definition: $Y_t \sim I(d)$ if $\Delta^d Y_t$ is stationary but $\Delta^{d-1} Y_t$ is not.

Example: If Y_t follows $\Delta^2 Y_t = \varepsilon_t$, then:

- ☐ $\Delta Y_t = \Delta Y_{t-1} + \varepsilon_t$ (still has unit root)
- ☐ $\Delta^2 Y_t = \varepsilon_t$ (white noise, stationary)

Real-world: Price levels may be $I(2)$ when inflation itself is non-stationary.

Visual: Integrated Processes



$I(0)$: stationary. $I(1)$: one difference needed. $I(2)$: two differences needed to become stationary.

TSA_ch3_def_integrated



Quiz 2: Random Walk Properties

Question

For a random walk $Y_t = Y_{t-1} + \varepsilon_t$ with $\text{Var}(\varepsilon_t) = \sigma^2$, what is $\text{Var}(Y_t)$?

- A) σ^2
- B) $t \cdot \sigma^2$
- C) σ^2/t
- D) $\sigma^2/(1 - \phi^2)$

 TSA_ch3_rw_variance

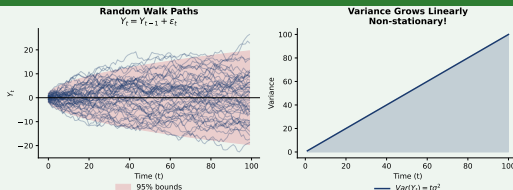


Quiz 2: Random Walk Properties

Question

For a random walk $Y_t = Y_{t-1} + \varepsilon_t$ with $\text{Var}(\varepsilon_t) = \sigma^2$, what is $\text{Var}(Y_t)$?

Answer: $B - t \cdot \sigma^2$



Proof: $Y_t = \sum_{i=1}^t \varepsilon_i \Rightarrow \text{Var}(Y_t) = t\sigma^2$ (grows linearly \Rightarrow non-stationary!)

Quiz 3: ADF Test Hypotheses

Question

In the Augmented Dickey-Fuller test, what is the null hypothesis?

- A) The series is stationary
- B) The series has a unit root
- C) The series has no autocorrelation
- D) The series is normally distributed

Quiz 3: ADF Test Hypotheses

Question

In the Augmented Dickey-Fuller test, what is the null hypothesis?

Answer: B – The series has a unit root

ADF regression: $\Delta Y_t = \alpha + \gamma Y_{t-1} + \sum_{j=1}^p \delta_j \Delta Y_{t-j} + \varepsilon_t$

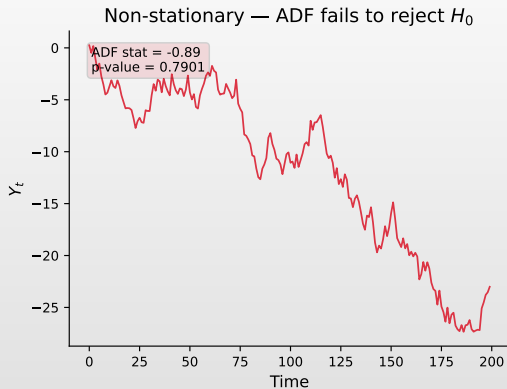
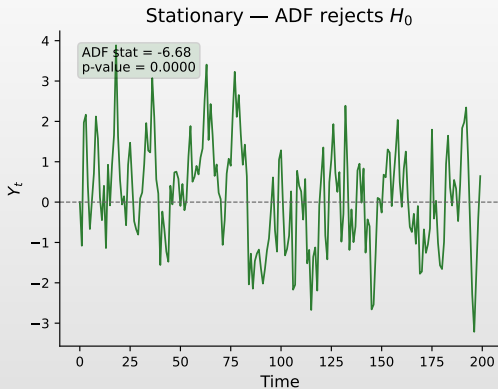
Hypotheses:

- ☐ $H_0 : \gamma = 0$ (unit root, non-stationary)
- ☐ $H_1 : \gamma < 0$ (stationary)

Decision: Reject H_0 if t -statistic $<$ critical value (e.g., -2.86 at 5%)

Note: Uses special Dickey-Fuller distribution, not standard t .

Visual: ADF Test



Left: stationary – ADF rejects unit root. Right: non-stationary – ADF fails to reject.

 TSA_ch3_def_adf

Quiz 4: ARIMA Notation

Question

What does ARIMA(2,1,1) mean?

- A) AR(2) on differenced data with MA(1) errors
- B) AR(1) with 2 differences and MA(1)
- C) MA(2) with 1 difference and AR(1)
- D) 2 lags, 1 trend, 1 seasonal component

Quiz 4: ARIMA Notation

Question

What does ARIMA(2,1,1) mean?

Answer: A – AR(2) on differenced data with MA(1) errors

ARIMA(p, d, q): $\phi(L)(1 - L)^d Y_t = \theta(L)\varepsilon_t$

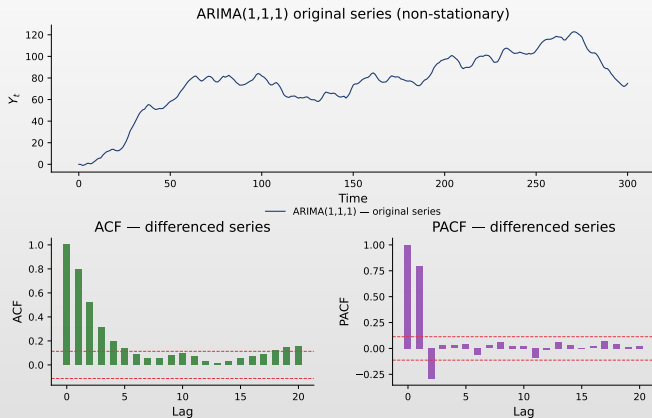
ARIMA(2,1,1) expands to:

$$(1 - \phi_1 L - \phi_2 L^2)(1 - L)Y_t = (1 + \theta_1 L)\varepsilon_t$$

Or equivalently: $(1 - \phi_1 L - \phi_2 L^2)\Delta Y_t = (1 + \theta_1 L)\varepsilon_t$

Interpretation: First difference the series, then fit ARMA(2,1) to ΔY_t .

Visual: ARIMA Process



Top: original ARIMA series. Bottom: after differencing, use ACF/PACF to identify AR and MA orders.

TSA_ch3_def_arima



Quiz 5: Difference Operator

Question

What is $(1 - L)^2 Y_t$ expanded?

- A) $Y_t - Y_{t-1}$
- B) $Y_t - 2Y_{t-1} + Y_{t-2}$
- C) $Y_t + 2Y_{t-1} + Y_{t-2}$
- D) $Y_t - Y_{t-2}$

Quiz 5: Difference Operator

Question

What is $(1 - L)^2 Y_t$ expanded?

Answer: $B - Y_t - 2Y_{t-1} + Y_{t-2}$

Binomial expansion: $(1 - L)^2 = 1 - 2L + L^2$

Applied: $(1 - L)^2 Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$ (the “change in changes”)

Quiz 6: KPSS vs ADF

Question

How does the KPSS test differ from the ADF test?

- A) KPSS tests for seasonality, ADF tests for trends
- B) KPSS has stationarity as null, ADF has unit root as null
- C) KPSS is more powerful than ADF
- D) There is no difference

 TSA_ch3_adf_kpss

Quiz 6: KPSS vs ADF

Question

How does the KPSS test differ from the ADF test?

Answer: B – Reversed null hypotheses

ADF Test	KPSS Test
H_0 : Unit Root	H_0 : Stationary
H_1 : Stationary	H_1 : Unit Root
Reject if $t\text{-stat} < \text{critical}$	Reject if $LM > \text{critical}$

Decision Matrix		
ADF rejects	KPSS fails to reject	→ Stationary
ADF fails to reject	KPSS rejects	→ Unit Root
Both reject	or both fail	→ Inconclusive

Strategy: Use both tests together for robust inference!



Quiz 7: Overdifferencing

Question

If $Y_t \sim I(1)$ and we compute $\Delta^2 Y_t$, what happens?

- A) We get a better stationary series
- B) We introduce artificial negative autocorrelation
- C) The variance decreases
- D) Nothing changes

 TSA_ch3_overdifferencing

Quiz 7: Overdifferencing

Question

If $Y_t \sim I(1)$ and we compute $\Delta^2 Y_t$, what happens?

Answer: B – Artificial negative autocorrelation



Diagnostic: ACF at lag 1 ≈ -0.5 signals overdifferencing. Reduce d by 1!

Quiz 8: Forecast Variance

Question

For an ARIMA(0,1,0) model (random walk), how does forecast variance behave as horizon h increases?

- A) Stays constant
- B) Decreases to zero
- C) Grows linearly with h
- D) Converges to a finite limit

Quiz 8: Forecast Variance

Question

For an ARIMA(0,1,0) model (random walk), how does forecast variance behave as horizon h increases?

Answer: C – Grows linearly with h

Random walk forecast: $\hat{Y}_{T+h|T} = Y_T$ (best forecast is current value)

Forecast error: $Y_{T+h} - \hat{Y}_{T+h|T} = \sum_{i=1}^h \varepsilon_{T+i}$

Variance:

$$\text{Var}(Y_{T+h} - \hat{Y}_{T+h|T}) = h\sigma^2$$

95% CI: $Y_T \pm 1.96\sqrt{h}\sigma$ (widens with \sqrt{h})

Quiz 9: Unit Root Test Power

Question

The ADF test has low power when:

- A) Sample size is very large
- B) The true root is close to but not equal to 1
- C) The series has no trend
- D) The series is clearly stationary

Quiz 9: Unit Root Test Power

Question

The ADF test has low power when:

Answer: B – Root close to but not equal to 1

Example: AR(1) with $\phi = 0.95$ vs random walk ($\phi = 1$) – similar ACF patterns!

Low power: High probability of Type II error (failing to reject false H_0)

Solutions: Larger samples, Phillips-Perron test, panel unit root tests

Quiz 10: ARIMA Model Selection

Question

After differencing once, the ACF shows a spike at lag 1 only, and PACF decays. The appropriate model is:

- A) ARIMA(1,1,0)
- B) ARIMA(0,1,1)
- C) ARIMA(1,1,1)
- D) ARIMA(0,2,1)

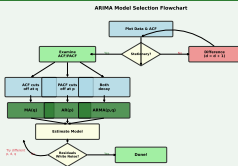
 TSA_ch3_arima_flowchart

Quiz 10: ARIMA Model Selection

Question

After differencing once, the ACF shows a spike at lag 1 only, and PACF decays. The appropriate model is:

Answer: B – ARIMA(0,1,1)



Pattern: ACF cuts off at lag 1, PACF decays \Rightarrow MA(1). Full model: ARIMA(0,1,1) = IMA(1,1)

Quiz 11: Trend Stationarity vs Difference Stationarity

Question

A trend-stationary process is made stationary by:

- A) Taking first differences
- B) Removing the deterministic trend via regression
- C) Taking second differences
- D) Applying seasonal adjustment

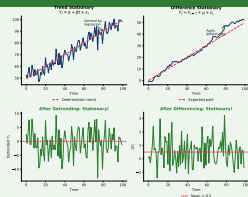
 TSA_ch3_trend_vs_diff

Quiz 11: Trend Stationarity vs Difference Stationarity

Question

A trend-stationary process is made stationary by:

Answer: B – Removing deterministic trend via regression



Trend-stationary: Detrend (shocks are temporary). **Difference-stationary:** Difference (shocks are permanent).

Quiz 12: ARIMA Invertibility

Question

ARIMA(0,1,1) with $\theta_1 = 1.2$ is:

- A) Stationary and invertible
- B) Non-stationary but invertible
- C) Non-stationary and non-invertible
- D) Stationary but non-invertible

Quiz 12: ARIMA Invertibility

Question

ARIMA(0,1,1) with $\theta_1 = 1.2$ is:

Answer: C – Non-stationary and non-invertible

Stationarity: $d = 1$ (unit root) \Rightarrow **Non-stationary**

Invertibility: MA root $z = -1/1.2 = -0.833$ is inside unit circle; $|\theta_1| > 1 \Rightarrow$ **Non-invertible**

Fix: Rewrite with $\theta^* = 1/1.2 = 0.833$ and adjust variance.

Quiz 13: Spurious Regression

Question

Regressing one random walk on another independent random walk typically shows:

- A) No significant relationship
- B) High R^2 and significant t-statistics (spuriously)
- C) Negative correlation
- D) Perfect multicollinearity

Quiz 13: Spurious Regression

Question

Regressing one random walk on another independent random walk typically shows:

Answer: B – High R^2 and significant t -statistics (spuriously)

Granger & Newbold (1974): Spurious regression phenomenon

Symptoms: High R^2 (> 0.9), significant t -stats, low Durbin-Watson ($\ll 2$), non-stationary residuals

Solutions: Difference both series, or test for cointegration

Quiz 14: Long-Run Forecast

Question

The long-run forecast from ARIMA(1,1,0) with $\phi_1 = 0.7$ converges to:

- A) Zero
- B) The unconditional mean
- C) A linear trend extrapolation
- D) The last observed value



Quiz 14: Long-Run Forecast

Question

The long-run forecast from ARIMA(1,1,0) with $\phi_1 = 0.7$ converges to:

Answer: C – A linear trend extrapolation

Model: $(1 - \phi_1 L)(1 - L)Y_t = c + \varepsilon_t$; **Long-run:** $\hat{Y}_{T+h} \approx Y_T + h \cdot \frac{c}{1-\phi_1}$

Key differences: Stationary ARMA \rightarrow mean; $I(1)$ no drift \rightarrow last value; $I(1)$ with drift \rightarrow linear

True/False Questions

Question

Determine if each statement is True or False:

1. An $I(2)$ process requires two differences to become stationary.
2. The ADF test always includes a constant term.
3. $ARIMA(0,1,0)$ is another name for a random walk.
4. Differencing a stationary series makes it “more stationary.”
5. The KPSS test has stationarity as the null hypothesis.
6. $ARIMA$ models can only capture linear patterns.

Answer on next slide...

True/False: Solutions

Answers

1. An $I(2)$ process requires two differences to become stationary. **TRUE**
 $I(d)$ means d differences needed. $I(2)$ = two unit roots.
2. The ADF test always includes a constant term. **FALSE**
You choose: no constant, constant only, or constant + trend.
3. ARIMA(0,1,0) is another name for a random walk. **TRUE**
 $(1 - L)Y_t = \varepsilon_t \Rightarrow Y_t = Y_{t-1} + \varepsilon_t$.
4. Differencing a stationary series makes it “more stationary.” **FALSE**
Over-differencing creates non-invertible MA; hurts model performance.
5. The KPSS test has stationarity as the null hypothesis. **TRUE**
KPSS: H_0 = stationary. Opposite of ADF.
6. ARIMA models can only capture linear patterns. **TRUE**
ARIMA is linear in parameters. Nonlinear patterns need GARCH, neural nets, etc.

Problem 1: Unit Root Testing

Exercise

You have quarterly GDP data for 80 quarters. The ADF test (with constant and trend) gives a test statistic of -2.85 . The 5% critical value is -3.41 .

1. What is your conclusion about stationarity?
2. What would you do next?

Problem 1: Unit Root Testing

Exercise

You have quarterly GDP data for 80 quarters. The ADF test (with constant and trend) gives a test statistic of -2.85 . The 5% critical value is -3.41 .

1. What is your conclusion about stationarity?
2. What would you do next?

Solution

1. Since $-2.85 > -3.41$, we **fail to reject** H_0 . The data appears to have a unit root (non-stationary).
2. Take the first difference ΔY_t and repeat the ADF test on the differenced series to confirm it is now stationary.

Problem 2: Model Identification

Exercise

After differencing a time series once, the ACF shows:

- ▣ Significant spike at lag 1 ($\rho_1 = 0.4$)
- ▣ All other lags insignificant

The PACF shows gradual decay.

What ARIMA model is suggested?

Problem 2: Model Identification

Exercise

After differencing a time series once, the ACF shows:

- ▣ Significant spike at lag 1 ($\rho_1 = 0.4$)
- ▣ All other lags insignificant

The PACF shows gradual decay.

What ARIMA model is suggested?

Solution

- ▣ ACF cuts off after lag 1 \Rightarrow MA(1) component
- ▣ PACF decays \Rightarrow Confirms MA structure
- ▣ Since we differenced once: $d = 1$

Suggested model: ARIMA(0,1,1) or IMA(1,1)

Problem 3: ARIMA Equation

Exercise

Write out the full equation for ARIMA(1,1,1):

$$(1 - \phi_1 L)(1 - L)Y_t = c + (1 + \theta_1 L)\varepsilon_t$$

Expand this completely in terms of Y_t , Y_{t-1} , Y_{t-2} , etc.

Problem 3: ARIMA Equation

Exercise

Write out the full equation for ARIMA(1,1,1):

$$(1 - \phi_1 L)(1 - L)Y_t = c + (1 + \theta_1 L)\varepsilon_t$$

Expand this completely in terms of Y_t , Y_{t-1} , Y_{t-2} , etc.

Solution

Expanding $(1 - \phi_1 L)(1 - L) = 1 - L - \phi_1 L + \phi_1 L^2 = 1 - (1 + \phi_1)L + \phi_1 L^2$:

$$Y_t - (1 + \phi_1)Y_{t-1} + \phi_1 Y_{t-2} = c + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

Or equivalently:

$$Y_t = c + (1 + \phi_1)Y_{t-1} - \phi_1 Y_{t-2} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

Problem 4: Forecast Calculation

Exercise

Given ARIMA(0,1,1): $\Delta Y_t = \varepsilon_t + 0.3\varepsilon_{t-1}$

At time T : $Y_T = 100$, $\hat{\varepsilon}_T = 2$, $\sigma^2 = 4$

Calculate:

1. $\hat{Y}_{T+1|T}$ (one-step forecast)
2. $\hat{Y}_{T+2|T}$ (two-step forecast)

Problem 4: Forecast Calculation

Exercise

Given ARIMA(0,1,1): $\Delta Y_t = \varepsilon_t + 0.3\varepsilon_{t-1}$

At time T : $Y_T = 100$, $\hat{\varepsilon}_T = 2$, $\sigma^2 = 4$

Calculate:

1. $\hat{Y}_{T+1|T}$ (one-step forecast)
2. $\hat{Y}_{T+2|T}$ (two-step forecast)

Solution

1. $\hat{Y}_{T+1|T} = Y_T + 0.3\hat{\varepsilon}_T = 100 + 0.3(2) = 100.6$
2. $\hat{Y}_{T+2|T} = \hat{Y}_{T+1|T} + 0.3 \cdot 0 = 100.6 + 0 = 100.6$
(Future shocks $\varepsilon_{T+1}, \varepsilon_{T+2}$ are forecast as 0)

Problem 5: Confidence Intervals

Exercise

Continuing from Problem 4, calculate the 95% forecast intervals for $\hat{Y}_{T+1|T}$ and $\hat{Y}_{T+2|T}$.
Recall: $\sigma^2 = 4$, $\theta_1 = 0.3$

Problem 5: Confidence Intervals

Exercise

Continuing from Problem 4, calculate the 95% forecast intervals for $\hat{Y}_{T+1|T}$ and $\hat{Y}_{T+2|T}$.
Recall: $\sigma^2 = 4$, $\theta_1 = 0.3$

Solution

For IMA(1,1), the MA(∞) weights are $\psi_0 = 1$, $\psi_j = 1 + \theta_1$ for $j \geq 1$.

1-step: $\text{Var}(e_{T+1}) = \sigma^2 \psi_0^2 = 4$, so $SE = 2$

$$100.6 \pm 1.96(2) = [96.68, 104.52]$$

2-step: $\text{Var}(e_{T+2}) = \sigma^2(\psi_0^2 + \psi_1^2) = 4(1 + 1.3^2) = 10.76$, $SE = 3.28$

$$100.6 \pm 1.96(3.28) = [94.17, 107.03]$$

Example: Testing for Unit Root in Stock Prices

Scenario

You have daily closing prices for a stock over 500 days. You want to determine if prices follow a random walk.

Step-by-step Approach

1. **Visual inspection:** Plot prices – likely shows trend
2. **ADF test on prices:** Expect to fail to reject H_0 (unit root)
3. **Take log returns:** $r_t = \ln(P_t/P_{t-1}) = \Delta \ln(P_t)$
4. **ADF test on returns:** Should reject H_0 (stationary)
5. **Conclusion:** Log prices are $I(1)$, returns are $I(0)$

Example: Box-Jenkins for Inflation Data

Scenario

Monthly inflation rates for 10 years. Build an ARIMA model.

Workflow

1. **Plot & test:** ADF suggests borderline – try both $d = 0$ and $d = 1$
2. **If $d = 0$:** Fit ARMA models, compare AIC
3. **If $d = 1$:** Examine ACF/PACF of ΔY_t
 - ▶ ACF: spike at lag 1, then cuts off
 - ▶ PACF: decays
 - ▶ \Rightarrow Try ARIMA(0,1,1)
4. **Estimate:** Fit ARIMA(0,1,1), check coefficients
5. **Diagnose:** Ljung-Box on residuals (want $p > 0.05$)
6. **Compare:** AIC of ARIMA(0,1,1) vs ARMA(1,1) on levels

Example: Interpreting Python Output

statsmodels ARIMA Output

```

                        ARIMA Model Results
=====
Dep. Variable:          D.y    No. Observations:   99
Model:                ARIMA(1,1,1)    AIC             285.32
                                   BIC             295.63
=====

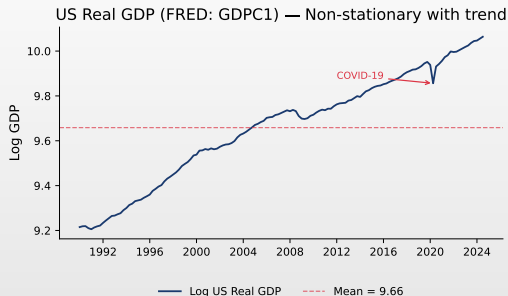
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	coef	std err	z	P> z
const	0.0521	0.048	1.085	0.278
ar.L1	0.4532	0.102	4.443	0.000
ma.L1	-0.2891	0.118	-2.450	0.014
sigma2	1.2340	0.176	7.011	0.000

Interpretation

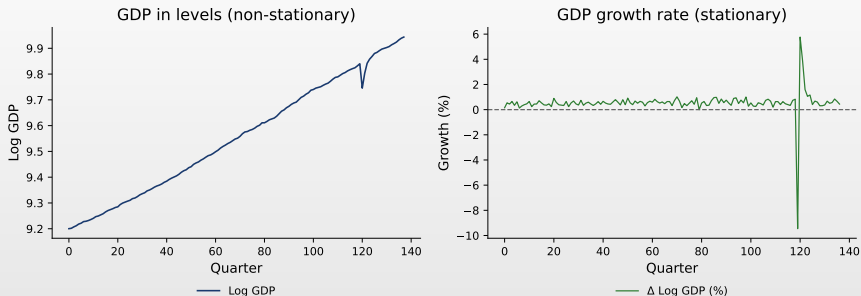
- AR coefficient (0.45) is significant, MA coefficient (-0.29) is significant
- Constant (0.052) not significant – could set $c = 0$
- Check: $|\phi_1| < 1$ (stationary), $|\theta_1| < 1$ (invertible) – OK!

Case Study: US Real GDP (1990–2024)



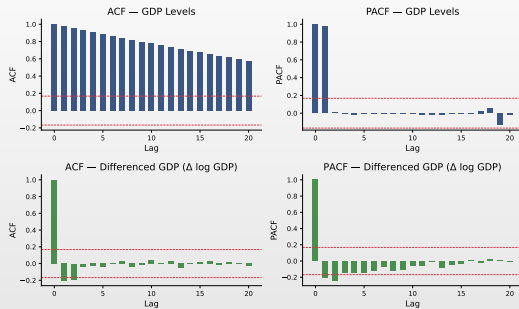
- US Real GDP in billions of 2017 dollars (quarterly data)
- Clear **upward trend** – typical of macroeconomic series
- Notable drops during recessions (2008-2009, 2020)
- Non-stationary: needs differencing before ARIMA modeling

Stationarity Through Differencing



- Left: GDP in levels – clear upward trend (non-stationary)
- Right: GDP growth rate = $\Delta \log(Y_t) \times 100$ – stationary
- First differencing of log GDP removes the stochastic trend
- Growth rate fluctuates around a constant mean ($\approx 0.6\%$ quarterly)

ACF/PACF: Levels vs Differenced



- **Top row:** ACF/PACF of GDP levels – slow decay indicates non-stationarity
- **Bottom row:** ACF/PACF of GDP growth – mostly within confidence bands
- Pattern suggests low-order ARIMA model is appropriate

ARIMA Estimation Results: US GDP Growth

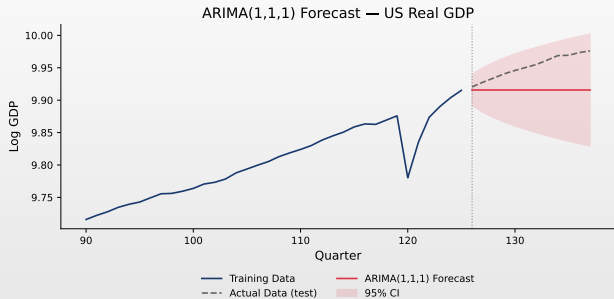
Model: ARIMA(1,1,1) on log(GDP)

Parameter	Estimate	Std. Error	z-stat	p-value
ϕ_1 (AR.L1)	0.312	0.185	1.69	0.091
θ_1 (MA.L1)	-0.087	0.203	-0.43	0.668
σ^2	0.00012	—	—	—

Interpretation

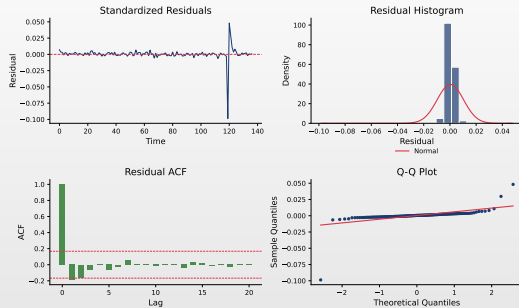
- Low-order ARIMA captures GDP dynamics reasonably well
- AR(1) coefficient positive – GDP growth shows persistence
- Alternative: simpler random walk (ARIMA(0,1,0)) often competitive

Forecast: ARIMA vs Actual



- Blue: historical training data; Green: actual test data
- Red dashed: ARIMA forecasts with 95% confidence interval
- Forecasts capture the general trend direction
- Confidence intervals widen as forecast horizon increases

Model Diagnostics: Residual Analysis



- Residuals show no systematic patterns over time
- Distribution approximately normal (histogram and Q-Q plot)
- ACF of residuals within bounds – no significant autocorrelation remaining
- Model adequately captures the data generating process

Discussion: Deterministic vs Stochastic Trends

Key Question

Why is it important to distinguish between deterministic and stochastic trends?

Discussion Points

- ▣ **Wrong treatment consequences:**
 - ▶ Detrending a unit root \Rightarrow spurious stationarity
 - ▶ Differencing a trend-stationary \Rightarrow overdifferencing
- ▣ **Economic interpretation:**
 - ▶ Deterministic trend: shocks are temporary
 - ▶ Stochastic trend: shocks have permanent effects
- ▣ **Policy implications:**
 - ▶ Does a recession permanently lower GDP, or does the economy return to trend?

Discussion: Model Selection Criteria

Key Question

When should you use AIC vs BIC for ARIMA model selection?

Considerations

- **AIC:** Minimizes prediction error, may overfit
 - ▶ Better for forecasting
 - ▶ Tends to select larger models
- **BIC:** Consistent model selection, more parsimonious
 - ▶ Better for identifying “true” model
 - ▶ Penalizes complexity more heavily
- **Practical advice:** Report both, prefer BIC if they disagree substantially

Discussion: Limitations of ARIMA

Key Question

What are the main limitations of ARIMA models?

Discussion Points

- **Linearity:** Cannot capture nonlinear dynamics
- **Constant variance:** Assumes homoskedasticity (no GARCH effects)
- **No structural breaks:** Parameters assumed constant
- **Univariate:** Ignores relationships with other variables
- **Symmetric:** Treats positive and negative shocks equally
- **Long-horizon forecasts:** Uncertainty grows rapidly

Extensions

These limitations motivate GARCH (volatility), VAR (multivariate), regime-switching models, etc.

Key Points from Today's Seminar

What We Covered

1. **Integration and differencing:** $I(d)$ processes require d differences
2. **Unit root testing:** ADF tests H_0 : unit root; KPSS tests H_0 : stationary
3. **ARIMA(p,d,q):** Combines ARMA with differencing
4. **Model identification:** Use ACF/PACF patterns and information criteria
5. **Forecasting:** Point forecasts and growing confidence intervals

Next Seminar

Hands-on Python exercises with real economic data:

- ☐ Unit root testing with `statsmodels`
- ☐ Auto-ARIMA with `pmdarima`
- ☐ Forecasting and model diagnostics

AI in ARIMA Modeling

Context

AI tools can fit ARIMA models and generate forecasts automatically. The critical skill is **evaluating whether the methodology is correct**.

Key questions to ask about any AI-generated ARIMA analysis:

1. Did it test for unit roots **before** choosing the differencing order?
2. Is the differencing order d justified by ADF/KPSS tests?
3. Did it check for overdifferencing (ACF lag 1 ≈ -0.5)?
4. Are residuals white noise (Ljung-Box test)?
5. Do forecast confidence intervals widen appropriately with horizon?

AI Exercise 1: Critique an AI ARIMA Analysis

Scenario

You asked an AI: “Fit the best ARIMA model to this GDP data.” It returned:

- ▣ Fitted ARIMA(3,2,2) with AIC = 315.7
- ▣ No unit root test performed before differencing
- ▣ Applied $d = 2$ “to be safe”
- ▣ Ljung-Box p-value = 0.04 (reported as “close enough”)
- ▣ 10-year quarterly forecast with narrow confidence intervals

Your critique:

1. Why is choosing $d = 2$ without testing dangerous? What is overdifferencing?
2. Why is Ljung-Box $p = 0.04$ **not** acceptable at 5% level?
3. Is ARIMA(3,2,2) likely over-parameterized? What would BIC suggest?
4. What is the correct Box-Jenkins methodology that was skipped?

AI Exercise 2: Prompt Refinement for ARIMA

Task

Iteratively improve prompts for fitting an ARIMA model to US GDP data.

Round 1 (vague): *"Fit a time series model to quarterly GDP"*

- What did the AI produce? Did it test stationarity? What's missing?

Round 2 (better): *"Test stationarity with ADF, determine differencing order, fit ARIMA using BIC, check residuals with Ljung-Box"*

- Did the AI follow the Box-Jenkins methodology correctly?

Round 3 (expert): *"Follow Box-Jenkins: (1) plot series & test ADF/KPSS, (2) difference if needed & re-test, (3) identify p, q from ACF/PACF, (4) estimate $ARIMA(p, 1, q)$, (5) Ljung-Box on residuals, (6) rolling 1-step forecast on last 20 obs with RMSE"*

- Compare results across all three rounds

AI Exercise 3: Model Selection Competition

Task

Download US Real GDP data (quarterly) using `pandas_datareader` or FRED.

Your approach (manual):

- ▣ ADF/KPSS tests → determine d
- ▣ ACF/PACF of differenced series → candidate models
- ▣ Compare AIC/BIC across ARIMA(1,1,0), ARIMA(0,1,1), ARIMA(1,1,1)
- ▣ Residual diagnostics for selected model
- ▣ Rolling 1-step forecast on last 20 observations

AI approach:

- ▣ Ask AI to “find the best ARIMA model and forecast GDP”

Compare:

- ▣ Which differencing order and model did each select?
- ▣ Compare out-of-sample RMSE; did the AI check for overdifferencing?
- ▣ **Submit:** 1-page reflection on AI strengths and weaknesses

Key Formulas Summary

Concept	Formula
Random walk	$Y_t = Y_{t-1} + \varepsilon_t$
Random walk variance	$\text{Var}(Y_t) = t\sigma^2$
ARIMA(p, d, q)	$\phi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t$
First difference	$\Delta Y_t = Y_t - Y_{t-1} = (1-L)Y_t$
Second difference	$\Delta^2 Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$
ADF regression	$\Delta Y_t = \alpha + \gamma Y_{t-1} + \sum \delta_j \Delta Y_{t-j} + \varepsilon_t$
ADF null	$H_0 : \gamma = 0$ (unit root)
RW forecast	$\hat{Y}_{T+h T} = Y_T$
RW forecast CI	$Y_T \pm z_{\alpha/2} \sqrt{h} \sigma$
AIC	$-2 \ln(\hat{L}) + 2k$
BIC	$-2 \ln(\hat{L}) + k \ln(n)$

Notation: \hat{L} = maximum of the likelihood function, k = no. of parameters, n = sample size, σ^2 = white noise variance

What's Next?

Seminar 4: SARIMA Models for Seasonal Data

- ▣ **Seasonality:** repetitive patterns at regular intervals
- ▣ **Seasonal differencing:** the $(1 - L^s)$ operator
- ▣ **SARIMA** $(p, d, q)(P, D, Q)_s$: the seasonal extension of ARIMA
- ▣ **Case study:** Airline passenger forecasting with Python

Questions?

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- ▣ Tsay, R.S. (2010). *Analysis of Financial Time Series*, 3rd ed., Wiley.
- ▣ Franke, J., Härdle, W.K., & Hafner, C.M. (2019). *Statistics of Financial Markets*, 4th ed., Springer.

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- ▣ Nielsen, A. (2019). *Practical Time Series Analysis*, O'Reilly Media.
- ▣ Petropoulos, F., et al. (2022). *Forecasting: Theory and Practice*, International Journal of Forecasting.
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Online resources and code

- ▣ **Quantlet**: <https://quantlet.com> — Code repository for statistics
- ▣ **Quantinar**: <https://quantinar.com> — Learning platform for quantitative methods
- ▣ **GitHub TSA**: https://github.com/QuantLet/TSA/tree/main/TSA_ch3 — Python code for this seminar