Chapter 1: Introduction to Time Series

Seminar

Seminar Outline

Today's Activities:

1. **Quick Review** – Key concepts recap
2. **Multiple Choice Quizzes** – Test your understanding
3. **True/False Questions** – Conceptual checks
4. **Calculation Exercises** – Hands-on practice
5. **Python Exercises** – Coding practice
6. **Discussion Questions** – Critical thinking

Key Formulas to Remember

Decomposition:

- Additive: $X_t = T_t + S_t + \varepsilon_t$
- Multiplicative: $X_t = T_t \times S_t \times \varepsilon_t$

Exponential Smoothing:

- SES: $\hat{X}_{t+1} = \alpha X_t + (1 - \alpha)\hat{X}_t$
- Holt: adds trend b_t
- HW: adds seasonal S_t

Stationarity:

- $\mathbb{E}[X_t] = \mu$ (constant)
- $\text{Var}(X_t) = \sigma^2$ (constant)
- $\text{Cov}(X_t, X_{t+h}) = \gamma(h)$

Random Walk:

- $X_t = X_{t-1} + \varepsilon_t$
- $\text{Var}(X_t) = t\sigma^2$ (grows!)

Key Concepts Summary

Concept	Key Point	When to Use
Additive decomp.	Constant seasonal amplitude	Stable variance
Multiplicative decomp.	Seasonal grows with level	Increasing variance
SES	Level only (α)	No trend, no seasonality
Holt	Level + Trend (α, β)	Trend, no seasonality
Holt-Winters	Level + Trend + Seasonal	Trend and seasonality
ADF Test	H_0: unit root	Test for non-stationarity
KPSS Test	H_0: stationary	Confirm stationarity
Differencing	Remove stochastic trend	Random walk, unit root
Regression	Remove deterministic trend	Linear/polynomial trend

[QUIZ] Quiz 1: Time Series Basics

Question: Which of the following is NOT a characteristic of time series data?

A. Observations are ordered in time
B. Consecutive observations are typically correlated
C. Observations are independent and identically distributed
D. The data has a natural temporal ordering

Think about it before moving to the next slide...

[QUIZ] Quiz 1: Answer

Question: Which is NOT a characteristic of time series data?

A. Observations are ordered in time ×
B. Consecutive observations are typically correlated ×
C. **Observations are independent and identically distributed ✓**
D. The data has a natural temporal ordering ×

Explanation

Time series observations are typically **dependent** (autocorrelated), not independent. The assumption of i.i.d. observations is fundamental to cross-sectional analysis but violated in time series. This temporal dependence is what makes time series analysis unique and requires specialized methods.

[QUIZ] Quiz 2: Decomposition

Question: When should you use multiplicative decomposition instead of additive?

A. When the seasonal pattern has constant amplitude
B. When the variance of the series is stable over time
C. When the seasonal fluctuations grow proportionally with the level
D. When the time series has no trend component

Think about it before moving to the next slide...

[QUIZ] Quiz 2: Answer

Question: When should you use multiplicative decomposition?

- A. When the seasonal pattern has constant amplitude ×
- B. When the variance of the series is stable over time ×
- C. **When the seasonal fluctuations grow proportionally with the level ✓**
- D. When the time series has no trend component ×

Explanation

In multiplicative decomposition $X_t = T_t \times S_t \times \varepsilon_t$, the seasonal component S_t is a ratio (e.g., 1.2 means 20% above average). This means the absolute seasonal effect **scales with the level**. Use when you see "fan-shaped" patterns where variance increases with the mean.

[QUIZ] Quiz 3: Exponential Smoothing

Question: In Simple Exponential Smoothing with $\alpha = 0.9$, what happens?

A. Forecasts are very smooth and stable
B. Recent observations have very little weight
C. Forecasts react quickly to recent changes
D. The forecast is essentially a long-term average

Think about it before moving to the next slide...

[QUIZ] Quiz 3: Answer

Question: In SES with $\alpha = 0.9$, what happens?

- A. Forecasts are very smooth and stable $\times$
- B. Recent observations have very little weight $\times$
- C. **Forecasts react quickly to recent changes** $\checkmark$
- D. The forecast is essentially a long-term average $\times$

Explanation

With $\alpha = 0.9$: $\hat{X}_{t+1} = 0.9X_t + 0.1\hat{X}_t$
This means 90% weight on the most recent observation! High α values make forecasts very responsive to new data. Low α (e.g., 0.1) produces smoother, more stable forecasts that average over more history.

[QUIZ] Quiz 4: Stationarity

Question: A random walk process $X_t = X_{t-1} + \varepsilon_t$ is:

- A. Strictly stationary
- B. Weakly stationary
- C. Non-stationary because variance grows with time
- D. Stationary after adding a constant

Think about it before moving to the next slide...

[QUIZ] Quiz 4: Answer

Question: A random walk is:

- A. Strictly stationary ×
- B. Weakly stationary ×
- C. **Non-stationary because variance grows with time ✓**
- D. Stationary after adding a constant ×

Explanation

For random walk: $X_t = \sum_{i=1}^{t} \varepsilon_i$

- $\mathbb{E}[X_t] = 0$ (constant mean – OK)
- $\mathrm{Var}(X_t) = t\sigma^2$ (variance depends on t – NOT OK!)

Since variance is not constant, the process violates the stationarity condition. Solution: **differencing** gives $\Delta X_t = \varepsilon_t$ which IS stationary.

[QUIZ] Quiz 5: Unit Root Tests

Question: You run ADF and KPSS tests. ADF fails to reject H_0, and KPSS rejects H_0. What do you conclude?

- A. The series is stationary
- B. The series has a unit root (non-stationary)
- C. The results are inconclusive
- D. You need to run more tests

Think about it before moving to the next slide...

[QUIZ] Quiz 5: Answer

Question: ADF fails to reject, KPSS rejects. Conclusion?

- A. The series is stationary ×
- B. **The series has a unit root (non-stationary) ✓**
- C. The results are inconclusive ×
- D. You need to run more tests ×

Explanation

- ADF: H_0 = unit root. Fail to reject $\Rightarrow$ evidence FOR unit root
- KPSS: H_0 = stationary. Reject $\Rightarrow$ evidence AGAINST stationarity

Both tests agree: the series is **non-stationary**. You should difference the series before modeling with ARMA.

[QUIZ] Quiz 6: Forecast Evaluation

Question: Which metric is most appropriate for comparing forecast accuracy across different time series with different scales?

- A. Mean Absolute Error (MAE)
- B. Root Mean Squared Error (RMSE)
- C. Mean Absolute Percentage Error (MAPE)
- D. Mean Squared Error (MSE)

Think about it before moving to the next slide...

[QUIZ] Quiz 6: Answer

Question: Best metric for comparing across different scales?

A. Mean Absolute Error (MAE) ×
B. Root Mean Squared Error (RMSE) ×
C. **Mean Absolute Percentage Error (MAPE) ✓**
D. Mean Squared Error (MSE) ×

Explanation

$\text{MAPE} = \frac{100}{n} \sum \left| \frac{e_t}{X_t} \right|$ expresses errors as **percentages**.

- MAE, RMSE, MSE are **scale-dependent** (units of X_t)
- MAPE is **scale-independent** (always in %)
- Caveat: MAPE fails when X_t is near zero

[QUIZ] True or False?

Mark each statement as True (T) or False (F):

1. The ACF of a stationary AR(1) process decays exponentially. ________
2. White noise is always normally distributed. ________
3. Differencing can make a non-stationary series stationary. ________
4. The PACF of a MA(1) process cuts off after lag 1. ________
5. You should always use the test set for hyperparameter tuning. ________
6. Holt-Winters is appropriate for data with no seasonality. ________

Answers on next slide...

[QUIZ] True or False: Answers

1. The ACF of a stationary AR(1) decays exponentially. **TRUE**
   For AR(1): $\rho(h) = \phi^h$, which decays exponentially.
2. White noise is always normally distributed. **FALSE**
   White noise only requires zero mean, constant variance, no autocorrelation. Gaussian white noise is a special case.
3. Differencing can make a non-stationary series stationary. **TRUE**
   Differencing removes stochastic trends (unit roots).
4. The PACF of a MA(1) cuts off after lag 1. **FALSE**
   It's the ACF that cuts off for MA. PACF decays for MA processes.
5. You should always use the test set for hyperparameter tuning. **FALSE**
   Use validation set for tuning. Test set is for final evaluation only!
6. Holt-Winters is appropriate for data with no seasonality. **FALSE**
   Use Holt's method (no seasonal component) or SES for non-seasonal data.

Exercise 1: Simple Exponential Smoothing

Problem: Given the following data and $\alpha = 0.3$:

t	1	2	3	4	5
X_t	10	12	11	14	13

Starting with $\hat{X}_1 = X_1 = 10$, calculate:

a) The forecasts $\hat{X}_2, \hat{X}_3, \hat{X}_4, \hat{X}_5$
b) The forecast for $t = 6$: $\hat{X}_6$
c) The forecast errors $e_t = X_t - \hat{X}_t$ for $t = 2, 3, 4, 5$
d) The MAE and RMSE

Formula: $\hat{X}_{t+1} = \alpha X_t + (1 - \alpha)\hat{X}_t$

Exercise 1: Solution

Using $\hat{X}_{t+1} = 0.3X_t + 0.7\hat{X}_t$:

t	1	2	3	4	5	6
X_t	10	12	11	14	13	?
$\hat{X}_t$	10	10	10.6	10.72	11.70	**12.09**
e_t	–	2	0.4	3.28	1.30	–

Calculations:

- $\hat{X}_2 = 0.3(10) + 0.7(10) = 10$
- $\hat{X}_3 = 0.3(12) + 0.7(10) = 10.6$
- $\hat{X}_4 = 0.3(11) + 0.7(10.6) = 10.72$
- $\hat{X}_5 = 0.3(14) + 0.7(10.72) = 11.70$
- $\hat{X}_6 = 0.3(13) + 0.7(11.70) = \mathbf{12.09}$

$\textbf{MAE} = \frac{|2|+|0.4|+|3.28|+|1.30|}{4} = 1.745 \quad \textbf{RMSE} = \sqrt{\frac{4+0.16+10.76+1.69}{4}} = 2.04$

Exercise 2: Autocovariance

Problem: For a stationary process with:

- $\mathbb{E}[X_t] = 5$
- $\gamma(0) = 4$ (variance)
- $\gamma(1) = 2$
- $\gamma(2) = 1$

Calculate:

a) The autocorrelation function $\rho(0), \rho(1), \rho(2)$

b) $\text{Cov}(X_t, X_{t-1})$

c) $\text{Corr}(X_5, X_7)$

d) If $X_t = 6$, what is $\mathbb{E}[X_{t+1}|X_t = 6]$ assuming AR(1)?

Exercise 2: Solution

a) Autocorrelations:

$$\rho(h) = \frac{\gamma(h)}{\gamma(0)}$$

- $\rho(0) = \gamma(0)/\gamma(0) = 1$
- $\rho(1) = \gamma(1)/\gamma(0) = 2/4 = 0.5$
- $\rho(2) = \gamma(2)/\gamma(0) = 1/4 = 0.25$

b) $\text{Cov}(X_t, X_{t-1}) = \gamma(1) = 2$ (by stationarity, lag 1 covariance)

c) $\text{Corr}(X_5, X_7) = \rho(|7-5|) = \rho(2) = 0.25$

d) For AR(1) with $\phi = \rho(1) = 0.5$:

$$\mathbb{E}[X_{t+1}|X_t] = \mu + \phi(X_t - \mu) = 5 + 0.5(6-5) = 5.5$$

Exercise 3: Random Walk Properties

Problem: Consider a random walk $X_t = X_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim WN(0, 4)$ and $X_0 = 100$.

Calculate:

a) $\mathbb{E}[X_{10}]$
b) $\text{Var}(X_{10})$
c) $\text{Cov}(X_5, X_{10})$
d) The 95% confidence interval for X_{100}
e) After observing $X_5 = 108$, what is your best forecast for X_6?

Exercise 3: Solution

Random walk: $X_t = X_0 + \sum_{i=1}^{t} \varepsilon_i$ with $\sigma^2 = 4$

a) $\mathbb{E}[X_{10}] = X_0 = 100$ (mean stays at starting value)

b) $\text{Var}(X_{10}) = 10 \times \sigma^2 = 10 \times 4 = 40$

c) $\text{Cov}(X_5, X_{10}) = \min(5, 10) \times \sigma^2 = 5 \times 4 = 20$

d) For X_{100}:

- $\mathbb{E}[X_{100}] = 100$, $\text{Var}(X_{100}) = 400$, $SD = 20$
- 95% CI: $100 \pm 1.96 \times 20 = [60.8, 139.2]$

e) Best forecast: $\hat{X}_6 = X_5 = 108$
(Random walk: best forecast is the last observed value)

Python Exercise 1: Load and Plot

Task: Load S&P 500 data and create a basic time series plot.

Starter Code

```
import yfinance as yf
import matplotlib.pyplot as plt

# Download S&P 500 data
sp500 = yf.download('^GSPC', start='2020-01-01', end='2025-01-01')

# TODO: Plot the closing prices
# TODO: Add title, labels, and grid
# TODO: Calculate and print basic statistics
```

Questions:

1. What is the mean and standard deviation of returns?
2. Does the series appear stationary? Why or why not?

Python Exercise 2: Decomposition

Task: Perform STL decomposition on airline passengers data.

Starter Code

```
from statsmodels.tsa.seasonal import STL
import pandas as pd

# Load airline passengers
url = 'https://raw.githubusercontent.com/..../airline.csv'
airline = pd.read_csv(url, parse_dates=['Month'],
                      index_col='Month')

# TODO: Apply STL decomposition with period=12
# TODO: Plot all components
# TODO: What percentage of variance is explained by trend?
```

Hint: Use `STL(data, period=12).fit()`

Python Exercise 3: Exponential Smoothing

Task: Compare SES, Holt, and Holt-Winters on real data.

Starter Code

```
from statsmodels.tsa.holtwinters import (SimpleExpSmoothing,
    ExponentialSmoothing)

# Split data: 80% train, 20% test
train = airline[:'1958']
test = airline['1959':]

# TODO: Fit SES, Holt, and Holt-Winters
# TODO: Generate forecasts for test period
# TODO: Calculate RMSE for each method
# TODO: Which method performs best? Why?
```

Python Exercise 4: Stationarity Testing

Task: Test for stationarity using ADF and KPSS tests.

Starter Code

```
from statsmodels.tsa.stattools import adfuller, kpss

# Test S&P 500 prices
prices = sp500['Close']
returns = prices.pct_change().dropna()

# TODO: Run ADF test on prices and returns
# TODO: Run KPSS test on prices and returns
# TODO: Interpret the results

# ADF: adfuller(series)
# KPSS: kpss(series, regression='c')
```

Questions:

1. Are prices stationary? Are returns stationary?
2. Do ADF and KPSS agree?

Discussion Question 1

Scenario

You are analyzing monthly sales data for a retail company. The data shows clear seasonality (high sales in December) and an upward trend. The seasonal peaks have been getting larger over time.

Discuss:

1. Should you use additive or multiplicative decomposition? Why?
2. Which exponential smoothing method would you recommend?
3. How would you evaluate your forecast model?
4. What could go wrong if you used the wrong decomposition?

Discussion Question 2

Scenario

A colleague claims: "I ran the ADF test on my stock price data and got a p-value of 0.65, so my data is stationary and I can fit an ARMA model directly."

Discuss:

1. What is wrong with this interpretation?
2. What do the ADF hypotheses actually test?
3. What should the colleague do before fitting an ARMA model?
4. How could the KPSS test help clarify the situation?

Discussion Question 3

Scenario

You're building a forecasting model and achieve excellent results: MAPE of 2% on your dataset. Your manager is impressed and wants to deploy the model immediately.

Discuss:

1. What questions should you ask before deployment?
2. Did you use proper train/validation/test splits?
3. Could there be data leakage in your evaluation?
4. What additional diagnostics would you run?
5. How would you monitor the model in production?

Discussion Question 4

Scenario

You need to forecast daily electricity demand for the next week. The data shows: (1) strong daily patterns (peaks at 6pm), (2) weekly patterns (lower on weekends), and (3) annual patterns (higher in summer/winter).

Discuss:

1. How would you handle multiple seasonal patterns?
2. Would Holt-Winters work here? Why or why not?
3. What's the advantage of Fourier terms in this case?
4. How would you set up your train/validation/test split?

Key Takeaways from Today

1. **Time series are dependent** – not i.i.d. like cross-sectional data
2. **Choose decomposition wisely** – multiplicative when seasonal amplitude grows
3. **Understand smoothing parameters** – high α = reactive, low α = smooth
4. **Test for stationarity** – use both ADF and KPSS together
5. **Proper evaluation** – never tune on test set!
6. **Random walk is non-stationary** – variance grows with time

Next Seminar

ARMA/ARIMA model identification, estimation, and forecasting

Questions?

Good luck with the exercises!

Practice makes perfect.