



# Time Series Analysis and Forecasting

## Chapter 4: SARIMA Models



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## Outline

- Motivation
- Seasonality in Time Series
- Seasonal Differencing
- The SARIMA Model
- Seasonal ACF and PACF Patterns
- Estimation and Diagnostics
- Forecasting with SARIMA
- Case Study: Airline Passengers
- Summary
- Quiz


## Motivating Example: Seasonality Is Everywhere



charts/ch4\_motivation\_seasonal.pdf

- Retail sales exhibit clear **annual patterns**: December peaks, January troughs
- Standard ARIMA models cannot capture these **repeating seasonal cycles**
- Ignoring seasonality leads to systematic forecast errors

## Understanding Seasonal Components



charts/ch4\_motivation\_decomposition.pdf

- ▣ Seasonal time series = **Trend** + **Seasonal Pattern** + **Residuals**
- ▣ Decomposition helps visualize each component separately
- ▣ SARIMA models capture both trend dynamics and seasonal behavior

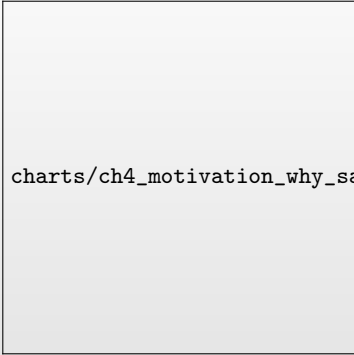
## Real-World Application: Monthly Patterns



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- Energy demand shows strong **monthly seasonality**
  - ▶ Heating cycles in winter, cooling cycles in summer
- Pattern repeats predictably each year with slight weather variations
- Utility companies use SARIMA forecasts for capacity planning

## Why Do We Need SARIMA?



charts/ch4\_motivation\_why\_sarima.pdf

- ▣ **Left:** Seasonal ACF patterns — spikes at lags 12, 24, 36 reveal annual cycle
- ▣ **Right:** ARIMA residuals still show seasonal autocorrelation (incomplete model)
- ▣ **SARIMA solution:** Adds seasonal AR/MA terms to capture periodic patterns

## What We'll Learn Today

### Concepts

- ▣ Identifying seasonal patterns
- ▣ Seasonal differencing operator
- ▣  $SARIMA(p, d, q)(P, D, Q)_s$  notation
- ▣ The famous “Airline Model”
- ▣ Model selection for seasonal data

### Skills

- ▣ Diagnose seasonality from ACF/PACF
- ▣ Determine seasonal period  $s$
- ▣ Choose  $(P, D, Q)$  seasonal orders
- ▣ Implement SARIMA in Python/R
- ▣ Forecast seasonal time series

### Key Insight

SARIMA = ARIMA applied at **two frequencies**: the regular (short-term) and seasonal (long-term) levels

## What is Seasonality?


### Definition 1 (Seasonality)

A time series exhibits **seasonality** when it shows regular, periodic fluctuations that repeat over a fixed period  $s$  (the seasonal period).

### Common Seasonal Periods

- Monthly data:  $s = 12$  (annual cycle)
- Quarterly data:  $s = 4$  (annual cycle)
- Weekly data:  $s = 52$  (annual) or  $s = 7$  (weekly pattern)
- Daily data:  $s = 7$  (weekly pattern)

## Seasonality: Visual Illustration



charts/ch4\_def\_seasonality.pdf

### Seasonal Periods

Left: Monthly data with  $s = 12$  (annual cycle). Right: Quarterly data with  $s = 4$ . The pattern repeats every  $s$  periods — this regularity is exploited by SARIMA models.

## Examples of Seasonal Data

### Economic Series

- ▣ Retail sales (holiday peaks)
- ▣ Tourism (summer/winter)
- ▣ Agricultural production
- ▣ Energy consumption
- ▣ Employment (hiring cycles)


### Other Domains

- ▣ Weather/temperature
- ▣ Website traffic
- ▣ Hospital admissions
- ▣ Transportation usage
- ▣ Electricity demand

### Why It Matters

Ignoring seasonality leads to biased forecasts and invalid inference!

## Example: Airline Passengers Data



charts/ch4\_airline\_data.pdf

- ▣ Monthly international airline passengers (1949–1960)
- ▣ Clear **upward trend** and **growing seasonal amplitude**
- ▣ Summer peaks reflect vacation travel patterns

## Visualizing Seasonal Patterns



charts/ch4\_seasonal\_boxplot.pdf

- Box plot reveals consistent seasonal pattern across years
- July–August show highest passenger counts (summer travel)
- November–February show lowest counts (winter months)

## Deterministic vs Stochastic Seasonality

### Deterministic Seasonality

Fixed seasonal pattern:  $Y_t = \sum_{j=1}^s \gamma_j D_{jt} + \varepsilon_t$   
where  $D_{jt}$  are seasonal dummies.

**Properties:**

- Pattern constant over time
- Removed by regression

### Stochastic Seasonality

Evolving seasonal pattern:  $\Delta_s Y_t = Y_t - Y_{t-s}$   
exhibits dependence structure.

**Properties:**

- Pattern evolves over time
- Requires seasonal differencing

## Detecting Seasonality


### Visual Methods (Primary Approach)

- ▣ **Time series plot** – look for repeating patterns
- ▣ **Seasonal boxplot** – compare distributions across seasons
- ▣ **ACF plot** – spikes at seasonal lags ( $s, 2s, 3s, \dots$ )

### ACF Signature of Seasonality

- ▣ Strong spikes at lags  $s, 2s, 3s, \dots$  indicate seasonal pattern
- ▣ Slow decay at seasonal lags  $\Rightarrow$  stochastic seasonality (needs differencing)
- ▣ Quick cutoff at seasonal lags  $\Rightarrow$  deterministic seasonality (use dummies)

## ACF Reveals Seasonal Structure



charts/ch4\_acf\_seasonality.pdf

- **Slow decay** at all lags indicates non-stationarity (trend)
- **Spikes at lags 12, 24, 36** confirm seasonal pattern ( $s = 12$ )
- Slow decay at seasonal lags  $\Rightarrow$  needs seasonal differencing  $(1 - L^{12})$

## The Seasonal Difference Operator

### Definition 2 (Seasonal Difference)

The **seasonal difference operator**  $\Delta_s$  is defined as:

$$\Delta_s Y_t = (1 - L^s) Y_t = Y_t - Y_{t-s}$$

where  $L^s Y_t = Y_{t-s}$  is the seasonal lag operator.

### Examples

- Monthly data ( $s = 12$ ):  $\Delta_{12} Y_t = Y_t - Y_{t-12}$   
Compares each month to the same month last year
- Quarterly data ( $s = 4$ ):  $\Delta_4 Y_t = Y_t - Y_{t-4}$   
Compares each quarter to the same quarter last year

## Seasonal Difference: Visual Illustration



charts/ch4\_def\_seasonal\_diff.pdf

### Effect of Seasonal Differencing

Left: Original series with clear seasonal pattern. Right: After  $\Delta_{12} = (1 - L^{12})$ , seasonal pattern is removed. Year-over-year comparison eliminates seasonal effects.

## Proof: Seasonal Differencing Removes Deterministic Seasonality

**Claim:** If  $Y_t = \mu_t + \varepsilon_t$  where  $\mu_t = \mu_{t-s}$  (periodic mean), then  $\Delta_s Y_t$  removes the seasonal mean.

**Proof:** Let  $Y_t = \mu_t + \varepsilon_t$  where  $\mu_t$  has period  $s$ . Apply seasonal difference:

$$\begin{aligned}\Delta_s Y_t &= Y_t - Y_{t-s} = (\mu_t + \varepsilon_t) - (\mu_{t-s} + \varepsilon_{t-s}) \\ &= \mu_t - \mu_{t-s} + \varepsilon_t - \varepsilon_{t-s} \\ &= 0 + \varepsilon_t - \varepsilon_{t-s} \quad (\text{since } \mu_t = \mu_{t-s})\end{aligned}$$

**Properties of  $\Delta_s Y_t = \varepsilon_t - \varepsilon_{t-s}$ :**

- $\mathbb{E}[\Delta_s Y_t] = 0$  (constant mean)
- $\text{Var}(\Delta_s Y_t) = 2\sigma^2$  (constant variance)
- Autocovariance:  $\gamma(s) = -\sigma^2$ ,  $\gamma(k) = 0$  for  $k \neq 0, s$

### Result

Seasonal differencing transforms periodic seasonal pattern into MA(1) at seasonal lag.

## Combining Regular and Seasonal Differencing

### Full Differencing

For series with both trend and seasonality:

$$\Delta\Delta_s Y_t = (1 - L)(1 - L^s)Y_t$$

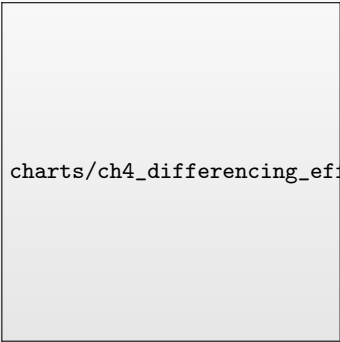
### Expansion

$(1 - L)(1 - L^s)Y_t = Y_t - Y_{t-1} - Y_{t-s} + Y_{t-s-1}$ . For monthly:  $\Delta\Delta_{12} Y_t = Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13}$

### Order of Differencing

$d$ : regular differences (trend removal);  $D$ : seasonal differences (seasonal trend removal)

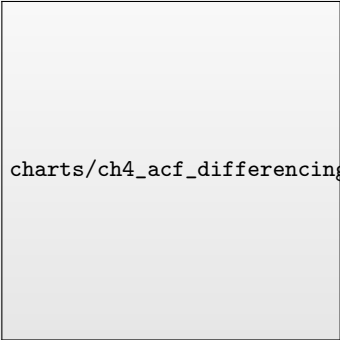
## Effect of Differencing Operations



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- ▣ Regular differencing removes trend but seasonal pattern remains
- ▣ Seasonal differencing removes seasonality but trend pattern remains
- ▣ **Both differences** needed to achieve stationarity

## ACF Before and After Differencing



charts/ch4\_acf\_differencing.pdf

- ▣ Original ACF: slow decay indicates non-stationarity
- ▣ After  $\Delta$ : seasonal spikes remain at lags 12, 24, 36
- ▣ After  $\Delta_{12}$ : trend decay remains at early lags
- ▣ After  $\Delta\Delta_{12}$ : ACF cuts off  $\Rightarrow$  **stationary**

## Seasonal Integration

### Definition 3 (Seasonally Integrated Process)

A series  $Y_t$  is **seasonally integrated** of order  $(d, D)_s$ , written  $Y_t \sim I(d, D)_s$ , if:

$$(1 - L)^d (1 - L^s)^D Y_t$$

is stationary.

### Common Cases

- $I(1, 0)_{12}$ : Regular unit root only (monthly)
- $I(0, 1)_{12}$ : Seasonal unit root only
- $I(1, 1)_{12}$ :
  - ▶ Both regular and seasonal unit roots

## SARIMA Model Definition

### Definition 4 (SARIMA( $p, d, q$ ) $\times$ ( $P, D, Q$ )<sub>s</sub>)

The **Seasonal ARIMA** model is:

$$\phi(L)\Phi(L^s)(1-L)^d(1-L^s)^DY_t = c + \theta(L)\Theta(L^s)\varepsilon_t$$

### Components

- ▣  $\phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p$ : Non-seasonal AR
- ▣  $\Phi(L^s) = 1 - \Phi_1 L^s - \dots - \Phi_P L^{Ps}$ : Seasonal AR
- ▣  $\theta(L) = 1 + \theta_1 L + \dots + \theta_q L^q$ : Non-seasonal MA
- ▣  $\Theta(L^s) = 1 + \Theta_1 L^s + \dots + \Theta_Q L^{Qs}$ : Seasonal MA
- ▣  $(1-L)^d$ :
  - ▶ Regular differencing;  $(1-L^s)^D$ : Seasonal differencing

## SARIMA: Visual Illustration



`charts/ch4_def_sarima.pdf`

### Differencing Strategy

Progressive transformation: Original  $\rightarrow$  regular difference (removes trend)  $\rightarrow$  seasonal difference (removes seasonality)  $\rightarrow$  both. Apply minimum differencing needed to achieve stationarity.

## Proof: Multiplicative Seasonal Structure

**Why multiplicative?** Consider  $\text{SARIMA}(1, 0, 0) \times (1, 0, 0)_s$ :

$$(1 - \phi L)(1 - \Phi L^s)Y_t = \varepsilon_t$$

**Expand:**  $(1 - \phi L)(1 - \Phi L^s)Y_t = Y_t - \phi Y_{t-1} - \Phi Y_{t-s} + \phi\Phi Y_{t-s-1}$

Interpretation (Monthly,  $s = 12$ )

$Y_t$  depends on:  $Y_{t-1}$  (last month),  $Y_{t-12}$  (same month last year),  $Y_{t-13}$  (interaction).

**Parsimony:** Multiplicative form uses 2 parameters ( $\phi, \Phi$ ); additive would need 3+.

## SARIMA Notation

### Full Specification

$\text{SARIMA}(p, d, q) \times (P, D, Q)_s$  has 7 parameters to specify:

| Parameter | Meaning                         |
|-----------|---------------------------------|
| $p$       | Non-seasonal AR order           |
| $d$       | Non-seasonal differencing order |
| $q$       | Non-seasonal MA order           |
| $P$       | Seasonal AR order               |
| $D$       | Seasonal differencing order     |
| $Q$       | Seasonal MA order               |
| $s$       | Seasonal period                 |

### Example

$\text{SARIMA}(1, 1, 1) \times (1, 1, 1)_{12}$ : Monthly data with AR(1), MA(1), seasonal AR(1), seasonal MA(1), and both regular and seasonal differencing.

## Common SARIMA Models

Airline Model:  $\text{SARIMA}(0, 1, 1) \times (0, 1, 1)_s$

$(1 - L)(1 - L^s)Y_t = (1 + \theta L)(1 + \Theta L^s)\varepsilon_t$  - Classic model (Box & Jenkins, 1970)

$\text{SARIMA}(1, 0, 0) \times (1, 0, 0)_s$

$(1 - \phi L)(1 - \Phi L^s)Y_t = \varepsilon_t$  - Pure seasonal and non-seasonal AR

$\text{SARIMA}(0, 1, 1) \times (0, 1, 0)_s$

$(1 - L)(1 - L^s)Y_t = (1 + \theta L)\varepsilon_t$  - Random walk + seasonal diff + MA(1)

## ACF/PACF for Seasonal Models

### Key Insight

Seasonal models show patterns at both:

- Non-seasonal lags:  $1, 2, 3, \dots$
- Seasonal lags:  $s, 2s, 3s, \dots$

| Model      | ACF                      | PACF                     |
|------------|--------------------------|--------------------------|
| SAR( $P$ ) | Decays at $s, 2s, \dots$ | Cuts off after $P_s$     |
| SMA( $Q$ ) | Cuts off after $Q_s$     | Decays at $s, 2s, \dots$ |
| SARMA      | Decays at seasonal lags  | Decays at seasonal lags  |

## Example: Airline Model ACF/PACF

SARIMA(0, 1, 1)  $\times$  (0, 1, 1)<sub>12</sub>

After differencing  $W_t = (1 - L)(1 - L^{12})Y_t$ :  $W_t = (1 + \theta L)(1 + \Theta L^{12})\varepsilon_t$

### Expected ACF Pattern

Spikes at lag 1 ( $\theta$ ), lag 12 ( $\Theta$ ), lag 13 ( $\theta \cdot \Theta$  interaction); all other lags near zero.

### Expected PACF Pattern

Exponential decay at lags 1, 2, 3, ... and at lags 12, 24, 36, ...

## Model Identification Guidelines

### Step-by-Step Process

1. Examine ACF for slow decay at seasonal lags  $\Rightarrow$  seasonal differencing
2. After differencing, check ACF/PACF patterns
3. Non-seasonal behavior at lags  $1, 2, \dots, s - 1$
4. Seasonal behavior at lags  $s, 2s, 3s, \dots$

### Practical Tips

- ▣ Start with  $d \leq 1$  and  $D \leq 1$
- ▣ Usually  $P, Q \leq 2$  is sufficient
- ▣ Use information criteria (AIC, BIC) for final selection
- ▣ Auto-SARIMA algorithms can help

## Estimation Methods

### Maximum Likelihood Estimation

Standard approach for SARIMA:

- ▣ Conditional MLE (conditional on initial values)
- ▣ Exact MLE (via Kalman filter)

### Computational Considerations

- ▣ More parameters than ARIMA  $\Rightarrow$  more data needed
- ▣ Seasonal parameters estimated from lags  $s, 2s, \dots$
- ▣ Need sufficient seasonal cycles (at least 3-4 years of monthly data)

## Stationarity and Invertibility

### Stationarity Conditions

Both non-seasonal and seasonal AR polynomials must have roots outside the unit circle:

- $\phi(z) = 0 \Rightarrow |z| > 1$
- $\Phi(z^s) = 0 \Rightarrow |z| > 1$

### Invertibility Conditions

Both non-seasonal and seasonal MA polynomials must have roots outside the unit circle:

- $\theta(z) = 0 \Rightarrow |z| > 1$
- $\Theta(z^s) = 0 \Rightarrow |z| > 1$

## Diagnostic Checking

### Residual Analysis

After fitting SARIMA, check that residuals are white noise:

1. Plot residuals over time (no patterns)
2. ACF of residuals (no significant spikes)
3. Ljung-Box test at multiple lags including seasonal
4. Normality tests (Q-Q plot, Jarque-Bera)

### Important

Check ACF at **both** non-seasonal and seasonal lags!

Significant ACF at lag 12 suggests inadequate seasonal modeling.

## Model Selection Criteria

### Information Criteria

Compare competing SARIMA models using:

- ▣  $AIC = -2 \ln(L) + 2k$
- ▣  $BIC = -2 \ln(L) + k \ln(n)$
- ▣  $AICc = AIC + \frac{2k(k+1)}{n-k-1}$  (corrected for small samples)

where  $k = p + q + P + Q + 1$  (plus 1 for variance).

### Auto-SARIMA

Python's `pmdarima.auto_arima()` with `seasonal=True` automatically searches for optimal  $(p, d, q) \times (P, D, Q)_s$ .

## Point Forecasts

### Forecast Computation

SARIMA forecasts are computed recursively:

- ▣ Replace future  $\varepsilon_{T+h}$  with 0
- ▣ Replace future  $Y_{T+h}$  with forecasts  $\hat{Y}_{T+h|T}$
- ▣ Use known past values  $Y_T, Y_{T-1}, \dots$

### Seasonal Pattern in Forecasts

SARIMA forecasts naturally capture seasonality:

- ▣ Short-term: influenced by recent values
- ▣ Long-term: revert to seasonal pattern

## Forecast Intervals

### Uncertainty Quantification

$(1 - \alpha)\%$  prediction interval:

$$\hat{Y}_{T+h|T} \pm z_{\alpha/2} \sqrt{\text{Var}(e_{T+h})}$$

Variance computed from  $\text{MA}(\infty)$  representation.

### Key Properties

- Intervals widen with forecast horizon
- For  $I(1, 1)_s$  series: intervals grow without bound
- Seasonal pattern visible in point forecasts
- Uncertainty captures both trend and seasonal variation

## Long-Horizon Forecasts

### Behavior as $h \rightarrow \infty$

- ▣ Point forecasts converge to deterministic seasonal pattern
- ▣ If drift present: linear trend + seasonal pattern
- ▣ Forecast intervals continue to widen

### Practical Implication

- ▣ Short-term: SARIMA captures both level and season
- ▣ Medium-term: Good seasonal forecasts, growing uncertainty
- ▣ Long-term:
  - ▶ Mainly reflects seasonal pattern, wide intervals

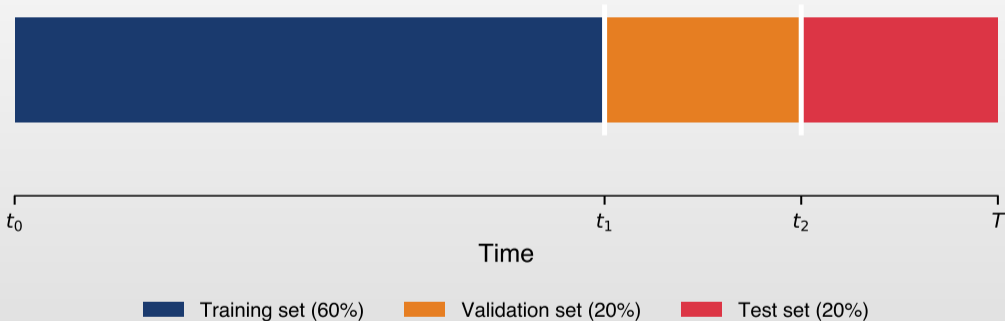
## Case Study: Airline Passengers Data

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- ▣ Classic Box-Jenkins dataset: monthly airline passengers (1949-1960)
- ▣ Clear upward trend and increasing seasonal amplitude
- ▣ Multiplicative seasonality suggests log transformation

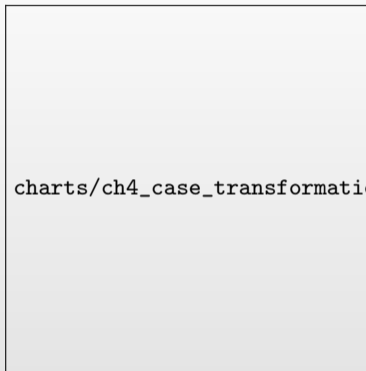
## Data Splitting Strategy

### Train / Validation / Test Split



- **Training set (70%)** — Fit model parameters
  - ▶ Estimate SARIMA coefficients ( $\phi, \theta, \Phi, \Theta$ )
  - ▶ Largest portion ensures reliable parameter estimates
- **Validation set (15%)** — Select best model

## Step 1: Transformations



charts/ch4\_case\_transformations.pdf

- ▣ Log transform stabilizes variance (multiplicative  $\rightarrow$  additive)
- ▣ First difference removes trend; seasonal difference removes seasonality
- ▣ Double-differenced series appears stationary

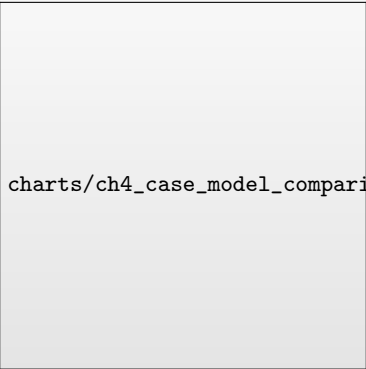
## Step 2: ACF/PACF Analysis



charts/ch4\_case\_acf\_pacf.pdf

- ACF: Significant spike at lag 1 and lag 12  $\Rightarrow$  MA(1), SMA(1)
- PACF: Exponential decay pattern confirms MA structure
- Suggests SARIMA(0, 1, 1)  $\times$  (0, 1, 1)<sub>12</sub> (airline model)

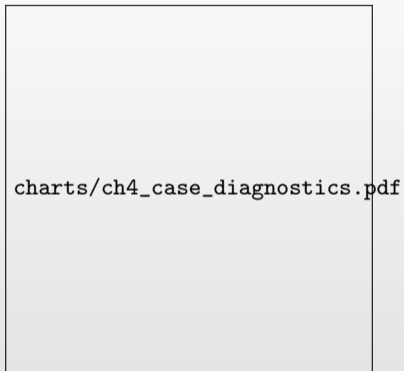
## Step 3: Model Comparison



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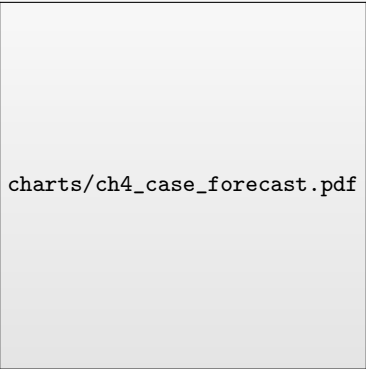
- ▣ Compare candidate SARIMA models using AIC criterion
- ▣  $\text{SARIMA}(0, 1, 1) \times (0, 1, 1)_{12}$  provides best fit (lowest AIC)
- ▣ This is the famous “airline model” identified by Box & Jenkins

## Step 4: Residual Diagnostics



- ▣ Residuals appear random with no remaining autocorrelation
- ▣ Q-Q plot shows approximate normality
- ▣ Model adequately captures both trend and seasonal structure

## Step 5: Forecasting



`charts/ch4_case_forecast.pdf`

- ▣ 24-month forecast with 95% confidence interval
- ▣ Model captures seasonal pattern and upward trend
- ▣ Prediction intervals widen appropriately with forecast horizon

## Key Takeaways

### Main Points

1. **Seasonality** is common in economic and business data
2. **Seasonal differencing**  $(1 - L^s)$  removes stochastic seasonality
3. **SARIMA**  $(p, d, q) \times (P, D, Q)_s$  extends ARIMA for seasonal data
4. **Multiplicative structure** captures seasonal-trend interactions
5. **ACF/PACF** show patterns at both regular and seasonal lags
6. **Model selection**: Use AIC/BIC or auto-SARIMA algorithms

### Next Steps

Chapter 5 will cover multivariate time series: VAR models, Granger causality, and cointegration.

## Quiz Question 1

### Question

For monthly data with annual seasonality, what is the seasonal period  $s$ ?

- (A)  $s = 4$
- (B)  $s = 7$
- (C)  $s = 12$
- (D)  $s = 52$

## Quiz Question 1: Answer

Correct Answer: (C)  $s = 12$  (12 months per year)

Common periods: Quarterly=4, Monthly=12, Weekly=52, Daily=7, Hourly=24

charts/ch4\_quiz1\_seasonal\_periods.pdf

## Quiz Question 2

### Question

What does the seasonal difference operator  $(1 - L^{12})$  do to a monthly series?

- (A) Computes  $Y_t - Y_{t-1}$  (month-to-month change)
- (B) Computes  $Y_t - Y_{t-12}$  (year-over-year change)
- (C) Computes the 12-month moving average
- (D) Removes the trend component only

## Quiz Question 2: Answer

Correct Answer: (B) Year-over-year change

$(1 - L^{12})Y_t = Y_t - Y_{t-12}$  removes the seasonal pattern by comparing same months.

charts/ch4\_quiz2\_seasonal\_diff.pdf

## Quiz Question 3

### Question

In  $\text{SARIMA}(1, 1, 1) \times (1, 1, 1)_{12}$  notation, what does the  $(1, 1, 1)_{12}$  part represent?

- (A) AR(1), differencing once, MA(1) at the regular level
- (B) Seasonal AR(1), seasonal differencing once, seasonal MA(1)
- (C) 12 AR terms, 12 differences, 12 MA terms
- (D) The model has 12 parameters in total

## Quiz Question 3: Answer

Correct Answer: (B)

Seasonal AR(1), seasonal differencing once, seasonal MA(1)

### SARIMA Notation Breakdown

$\text{SARIMA}(p, d, q) \times (P, D, Q)_s$ :

|               |  |
|---------------|--|
| $(p, d, q)$   | Non-seasonal: AR( $p$ ), $d$ differences, MA( $q$ )  |
| $(P, D, Q)_s$ | Seasonal: SAR( $P$ ), $D$ seasonal diffs, SMA( $Q$ ) |

For  $(1, 1, 1) \times (1, 1, 1)_{12}$ :

- Non-seasonal: AR(1), one regular difference, MA(1)
- Seasonal: SAR(1) at lag 12, one  $\Delta_{12}$ , SMA(1) at lag 12

## Quiz Question 4

### Question

The “Airline Model” is  $\text{SARIMA}(0, 1, 1) \times (0, 1, 1)_{12}$ . How many parameters need to be estimated (excluding variance)?

- (A) 1
- (B) 2
- (C) 4
- (D) 12

## Quiz Question 4: Answer

Correct Answer: (B) — 2 parameters

SARIMA(0, 1, 1)  $\times$  (0, 1, 1)<sub>12</sub>:  $(1 - L)(1 - L^{12})Y_t = (1 + \theta_1 L)(1 + \Theta_1 L^{12})\varepsilon_t$

Parameters:  $\theta_1$  (non-seasonal MA) and  $\Theta_1$  (seasonal MA), plus  $\sigma^2$ .

### Why “Airline Model”?

Box & Jenkins (1970) used this model to forecast international airline passengers. Remarkably effective for many seasonal economic series!

## Quiz Question 5

### Question

You observe significant ACF spikes at lags 12, 24, and 36 in a monthly series. What does this suggest?

- (A) The series has a unit root
- (B) The series has annual seasonality that needs seasonal differencing
- (C) The series follows an AR(36) process
- (D) The series is already stationary

## Quiz Question 5: Answer

Correct Answer: (B) Needs seasonal differencing

ACF spikes at 12, 24, 36 = stochastic seasonality. Apply  $(1 - L^{12})$  to remove it.

charts/ch4\_quiz5\_seasonal\_acf.pdf

## Quiz Question 6

### Question

After applying  $(1 - L)(1 - L^{12})$  to a monthly series, the ACF shows a significant spike only at lag 1 and lag 12. What SARIMA model is suggested?

- (A)  $\text{SARIMA}(1, 1, 0) \times (1, 1, 0)_{12}$
- (B)  $\text{SARIMA}(0, 1, 1) \times (0, 1, 1)_{12}$
- (C)  $\text{SARIMA}(1, 1, 1) \times (1, 1, 1)_{12}$
- (D)  $\text{SARIMA}(0, 1, 0) \times (0, 1, 0)_{12}$

## Quiz Question 6: Answer

Correct Answer: (B)

SARIMA(0, 1, 1)  $\times$  (0, 1, 1)<sub>12</sub> (The Airline Model)

### ACF/PACF Identification Rules





For MA processes, ACF **cuts off** after lag  $q$ :

| Pattern                  | Suggests                    |
|--------------------------|-----------------------------|
| ACF spike at lag 1 only  | MA(1) for non-seasonal part |
| ACF spike at lag 12 only | SMA(1) for seasonal part    |

Combined: MA(1)  $\times$  SMA(1) = (0,  $d$ , 1)  $\times$  (0,  $D$ , 1)<sub>12</sub>

With  $d = 1$  and  $D = 1$  (already differenced): (0, 1, 1)  $\times$  (0, 1, 1)<sub>12</sub>

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