



# Time Series Analysis and Forecasting

## Chapter 10: Comprehensive Review



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## Learning Objectives

By the end of this chapter, you will be able to:

- ▣ Apply the complete forecasting workflow from data to evaluation
- ▣ Select appropriate models based on data characteristics
- ▣ Evaluate forecast accuracy using proper metrics and cross-validation
- ▣ Integrate knowledge from all previous chapters in practice

## Outline

Forecasting Methodology

Case Study 1: Bitcoin Volatility (GARCH)

Case Study 2: Sunspot Cycles (Fourier)

Case Study 3: Unemployment (Prophet)

Case Study 4: Multivariate Analysis (VAR)

Synthesis and Guidelines

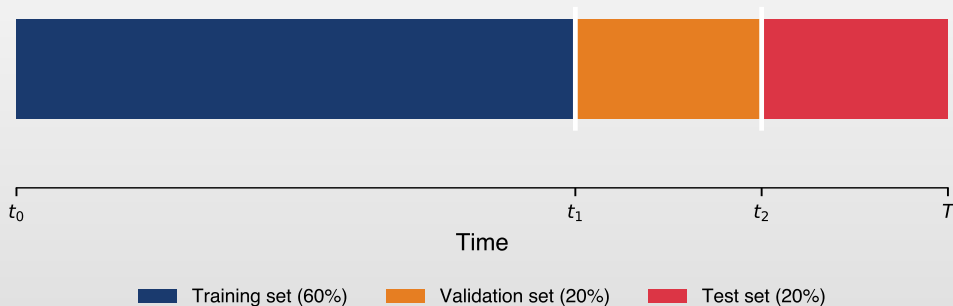
AI Use Case

Quiz

Summary

## Train/Validation/Test Framework

Train / Validation / Test Split



## The Scientific Approach to Forecasting

### Research Question

How do we **rigorously evaluate** forecast performance while avoiding overfitting?

### The Fundamental Problem

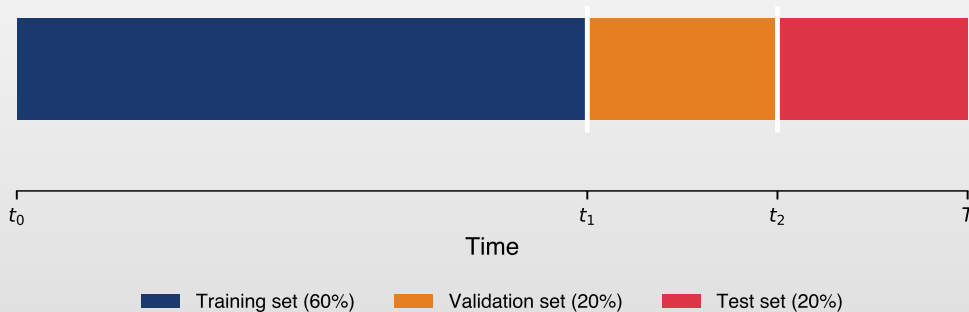
- In-sample fit  $\neq$  Out-of-sample performance
- Models can “memorize” training data without learning patterns
- **Solution:**
  - ▶ Proper train/validation/test methodology

### Key Principle

“The test set must remain **untouched** until final evaluation.”  
— Standard practice in machine learning and econometrics

## Train/Validation/Test Framework

### Train / Validation / Test Split



## Evaluation Metrics

### Definition 1 (Forecast Error Metrics)

Let  $y_t$  be actual,  $\hat{y}_t$  forecast:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_t (y_t - \hat{y}_t)^2}, \quad \text{MAE} = \frac{1}{n} \sum_t |y_t - \hat{y}_t|, \quad \text{MAPE} = \frac{100\%}{n} \sum_t \left| \frac{y_t - \hat{y}_t}{y_t} \right|$$

### When to Use Each

- ▣ **RMSE**: Penalizes large errors
- ▣ **MAE**: Robust to outliers
- ▣ **MAPE**: Scale-independent (%)

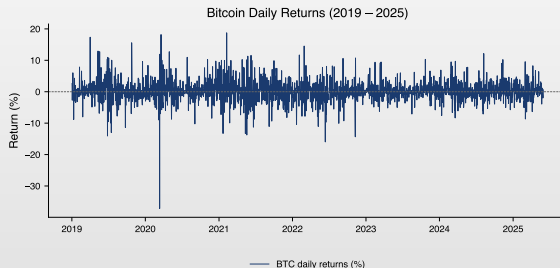
### Caution

- ▣ MAPE undefined when  $y_t = 0$
- ▣ Compare on **same** test set
- ▣ Report **out-of-sample** metrics

## Bitcoin: Volatility Clustering

### Observation

- Large returns follow large returns, small follow small—**volatility clustering**





## Bitcoin: Problem Statement

### Research Question

Can we forecast Bitcoin's **volatility** using GARCH models?

### Data Characteristics

- ▣ Source: Yahoo Finance (BTC-USD)
- ▣ Period: Jan 2019 – Jan 2025
- ▣ Frequency: Daily
- ▣ Observations:  $\approx 2,200$  days

### Stylized Facts

- ▣ Returns: near-zero mean
- ▣ Fat tails (kurtosis  $> 3$ )
- ▣ Volatility clustering

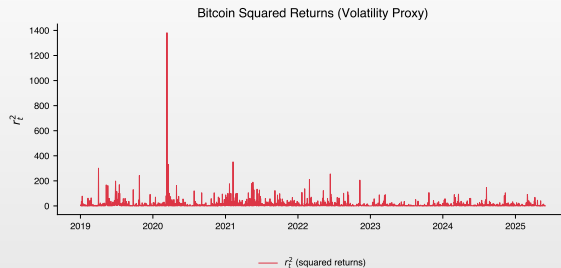
### Key Insight

Financial returns are typically:

- ▣ **Unpredictable** in mean
- ▣ **Predictable** in variance

⇒ Focus on **volatility forecasting**

## Bitcoin: Evidence for GARCH



 TSA\_ch10\_btc\_acf\_squared

## GARCH Model Specification

### Definition 2 (GARCH(p,q) Model)

Let  $r_t$  denote returns. The GARCH(p,q) model:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t \stackrel{iid}{\sim} N(0, 1)$$
$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2$$

where  $\omega > 0$ ,  $\alpha_i \geq 0$ ,  $\beta_j \geq 0$ , and  $\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1$ .

### Model Variants

- ▣ **GARCH(1,1)**: Most common
- ▣ **GJR-GARCH**: Leverage effect
- ▣ **EGARCH**: Log-variance, asymmetric

### Interpretation

- ▣  $\alpha$ : Shock impact (ARCH effect)
- ▣  $\beta$ : Volatility persistence
- ▣  $\alpha + \beta \approx 1$ : High persistence

## GARCH: Stationarity and Unconditional Variance

### Theorem 1 (Covariance Stationarity of GARCH(1,1))

If  $\alpha_1 + \beta_1 < 1$ , then  $\{\varepsilon_t\}$  is covariance stationary with:

$$\bar{\sigma}^2 = \mathbb{E}[\sigma_t^2] = \frac{\omega}{1 - \alpha_1 - \beta_1}$$

### Derivation

Take expectations of both sides of the variance equation:

$$\begin{aligned}\mathbb{E}[\sigma_t^2] &= \omega + \alpha_1 \mathbb{E}[\varepsilon_{t-1}^2] + \beta_1 \mathbb{E}[\sigma_{t-1}^2] \\ \bar{\sigma}^2 &= \omega + (\alpha_1 + \beta_1) \bar{\sigma}^2 \quad (\text{stationarity}) \\ \bar{\sigma}^2 &= \frac{\omega}{1 - \alpha_1 - \beta_1}\end{aligned}$$

### Multi-Step Forecasts Converge to $\bar{\sigma}^2$

As  $h \rightarrow \infty$ :  $\mathbb{E}_t[\sigma_{t+h}^2] \rightarrow \bar{\sigma}^2$  at rate  $(\alpha_1 + \beta_1)^h$ .

## Bitcoin: Model Selection on Validation Set

### Methodology

Fit each model on training data, evaluate on validation set.

Model	AIC	BIC	Val MAE	Selection
GARCH(1,1)	6,994.8	7,020.6	2.638	Best
GARCH(2,1)	6,993.7	7,024.6	2.640	
GJR-GARCH(1,1)	6,983.7	7,014.6	2.669	
EGARCH(1,1)	—	—	—	Failed*

\* Analytic forecasts not available for  $h > 1$

### Result

GARCH(1,1) selected based on lowest validation MAE for volatility forecasts.



## Bitcoin: Data Split and Stationarity

### Data Split

Set	Period	N
Training (70%)	2019-01 to 2023-03	1,543
Validation (20%)	2023-03 to 2024-06	441
Test (10%)	2024-06 to 2025-01	221
<b>Total</b>		<b>2,205</b>

### Stationarity Tests

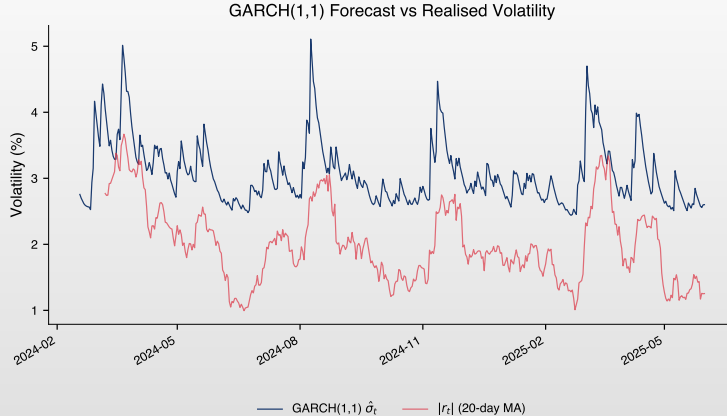
Series	ADF	Result
Prices	$p = 0.50$	Non-stationary
Returns	$p < 0.01$	Stationary

⇒ Model **returns**, not prices

### Why Stationarity Matters

GARCH requires weakly stationary input. Prices follow random walk; returns are stationary.

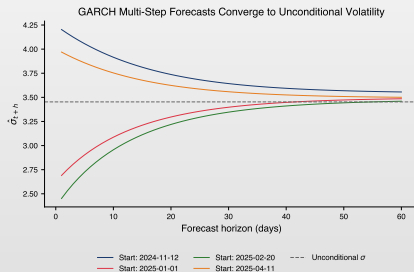
## Bitcoin: Final Test Set Evaluation



## GARCH: Multi-Step Forecasts Converge

### Key Insight

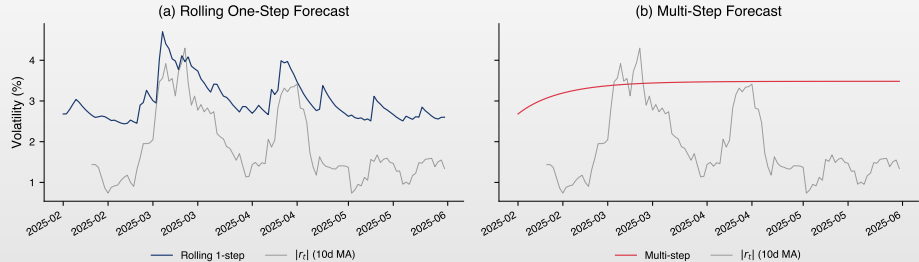
- Multi-step forecasts converge to  $\bar{\sigma}^2 = \frac{\omega}{1-\alpha-\beta}$
- Use rolling forecasts





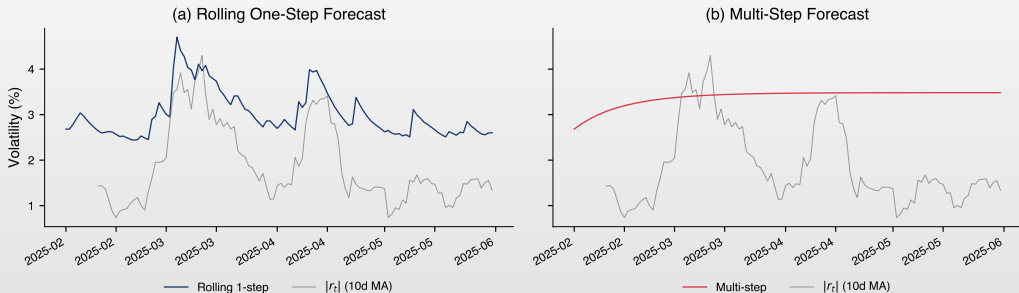
## GARCH: Rolling One-Step-Ahead Solution

Rolling vs Multi-Step GARCH Forecasts



## GARCH: Rolling One-Step-Ahead Solution

Rolling vs Multi-Step GARCH Forecasts



TSA\_ch10\_rolling\_vs\_multistep

## Bitcoin: Key Findings

### Summary

1. **Returns are stationary**; prices are not
2. **GARCH(1,1)** outperforms more complex variants
3. **High persistence** ( $\alpha + \beta = 0.93$ )
4. Volatility is **predictable** even when returns are not

### Practical Implications

- ▣ Risk management: VaR, Expected Shortfall
- ▣ Option pricing requires volatility forecasts
- ▣ Portfolio optimization with time-varying risk

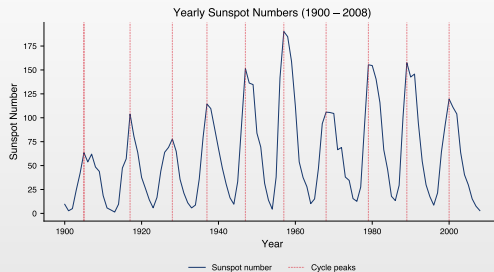
### Limitations

- ▣ GARCH assumes **symmetric** shocks
- ▣ Does not capture **jumps**
- ▣ Normal distribution may be restrictive

### Extensions

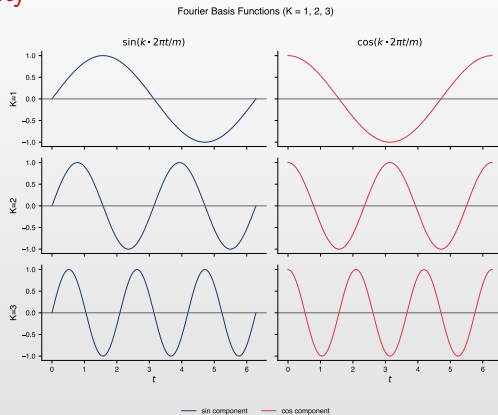
- ▣ Student-t innovations
- ▣ Realized volatility
- ▣ HAR models

## Sunspots: The 11-Year Solar Cycle

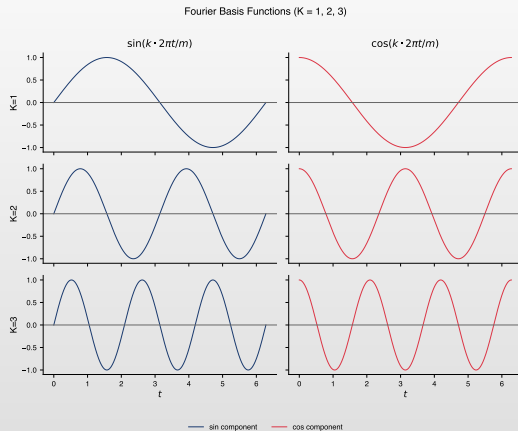


TSA\_ch10\_sunspots\_acf

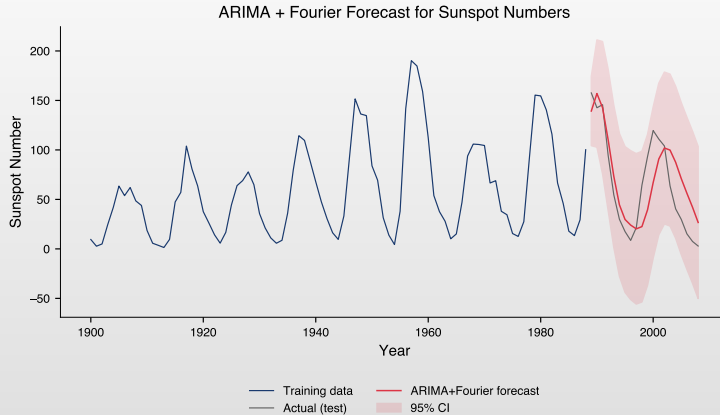
## Fourier Terms for Seasonality



## Fourier Terms for Seasonality



## Sunspots: Forecast Results



## Sunspots: Model Selection

### Methodology

Compare  $K = 1, 2, 3, 4$  Fourier harmonics on validation set.

Data Split	Set	Period	N
	Training (70%)	1900–1975	76
	Validation (20%)	1976–1997	22
	Test (10%)	1998–2008	11
	<b>Total</b>		<b>109</b>

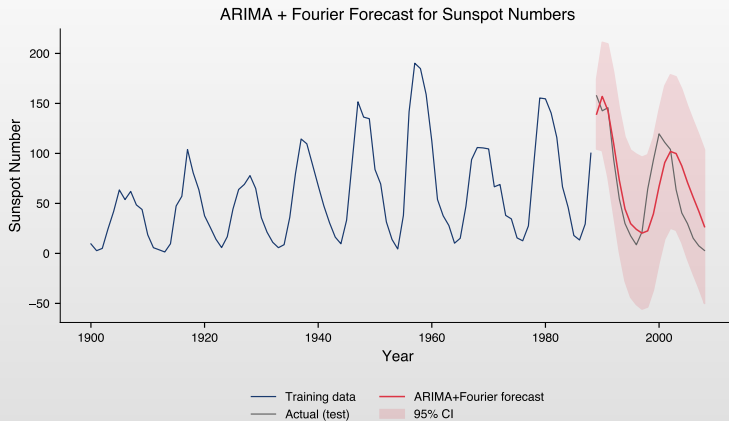
Model Comparison			
K	AIC	Val RMSE	
1	665.9	87.15	
2	668.0	86.92	
3	671.8	<b>86.81</b>	Best
4	674.5	87.93	

### Result

$K = 3$  Fourier harmonics selected (6 parameters for 11-year cycle).



## Sunspots: Forecast Results



## Sunspots: Key Takeaways

### When to Use Fourier Terms

- Seasonal period  $s$  is **long** (e.g., 11 years, 52 weeks)
- SARIMA would require too many seasonal lags
- Pattern is **smooth and periodic**
- Multiple cycles need to be captured

### Choosing $K$

Start with  $K = 1$ , increase until validation error stops improving. Too high  $K =$  overfitting.

### Fourier vs SARIMA

	Fourier	SARIMA
Long seasons	✓	×
Short seasons	OK	✓
Parameters	$2K$	Many
Flexibility	Fixed	Adaptive

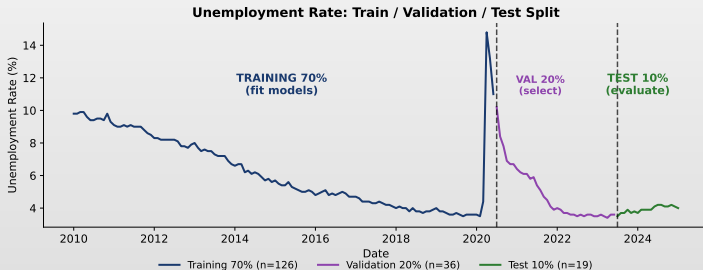
### Applications

Climate cycles, business cycles, astronomical phenomena

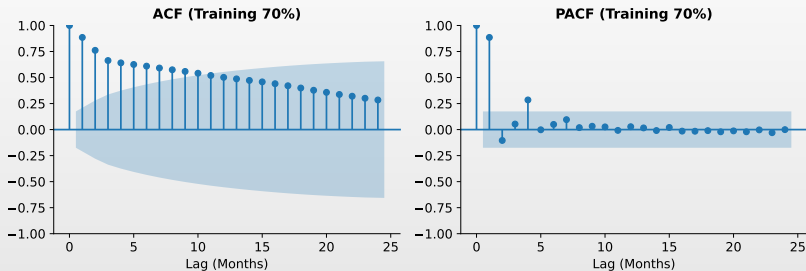
## Unemployment: Train / Validation / Test Split

### Methodology

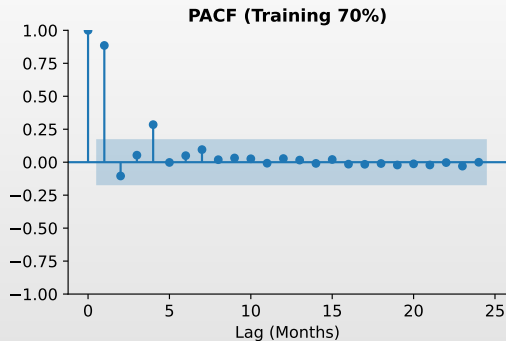
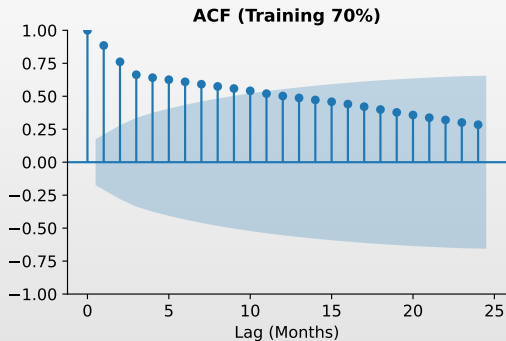
- **Training:** Fit models
- **Validation:** Select best
- **Test:** Final evaluation



## Unemployment: Preliminary Analysis

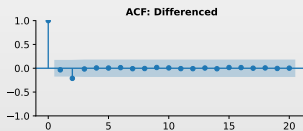
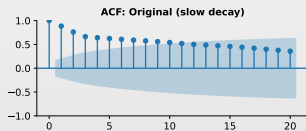
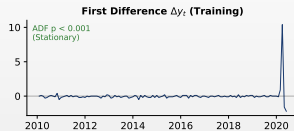
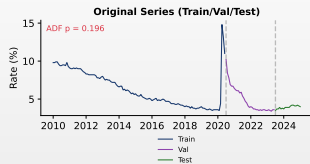


## Unemployment: Preliminary Analysis

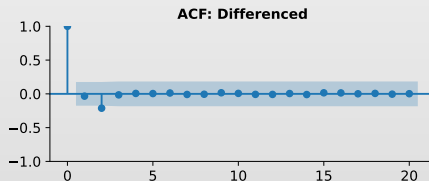
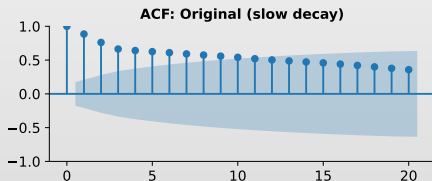
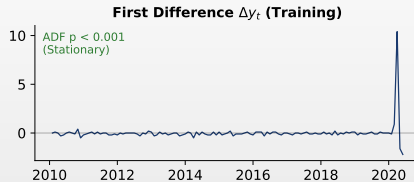
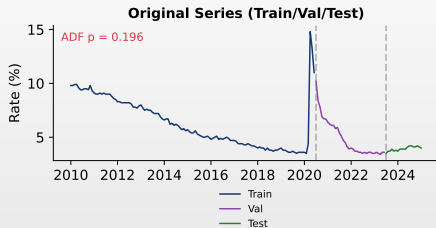


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## Unemployment: Stationarity Tests



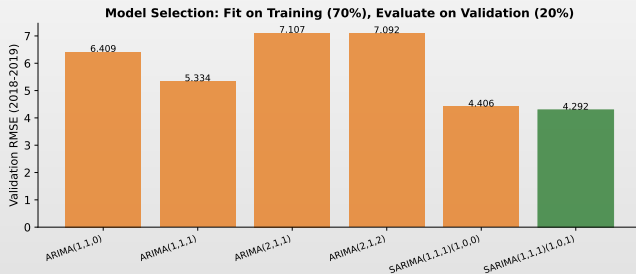
## Unemployment: Stationarity Tests



## Unemployment: Model Selection (Validation Set)

Best: SARIMA(1,1,1)(1,0,0)<sub>12</sub>

Selected by lowest validation RMSE



TSA\_ch10\_sarima\_model\_selection



## Unemployment: SARIMA Parameters

SARIMA(1,1,1)(1,0,0)<sub>12</sub> fitted on Train+Val (2010-2019)

- AR(1):  $\phi_1 = -0.86$
- MA(1):  $\theta_1 = 0.78$
- SAR(12):  $\Phi_1 = -0.08$  (n.s.)

**SARIMA(1,1,1)(1,0,1) - Fitted on Train+Val (85%)**

Parameter	Coef	Std Err	P-value	Sig
ar.L1	0.8423	0.2084	0.0001	***
ma.L1	-0.9540	0.1973	0.0000	***
ar.S.L12	0.0326	4.5951	0.9943	
ma.S.L12	-0.0113	4.6087	0.9980	
sigma2	0.8122	0.0608	0.0000	***

## Ljung-Box Test for Residual Autocorrelation

### Definition 3 (Ljung-Box Test)

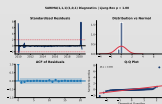
For residuals  $\hat{\varepsilon}_t$  with sample autocorrelations  $\hat{\rho}_k$ , the test statistic:

$$Q(h) = n(n+2) \sum_{k=1}^h \frac{\hat{\rho}_k^2}{n-k} \stackrel{H_0}{\sim} \chi^2(h-p-q)$$

where  $p, q$  are ARMA orders.  $H_0$ : Residuals are white noise.

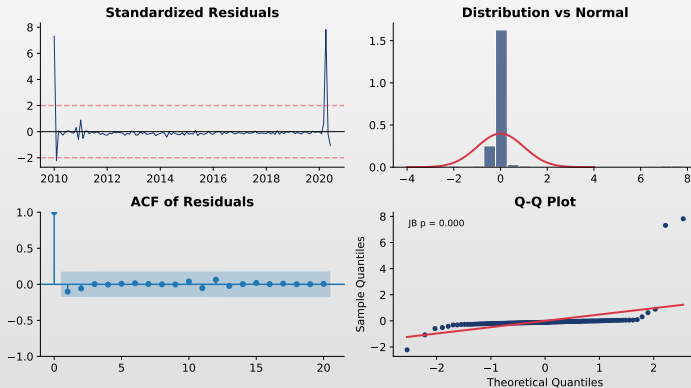
### Interpretation

- Large  $Q$  (small p-value): Reject  $H_0$ , residuals have structure
- Small  $Q$  (large p-value): Fail to reject  $H_0$ , model is adequate
- Rule of thumb: Use  $h = \min(10, n/5)$  for lag order



## Unemployment: SARIMA Diagnostics

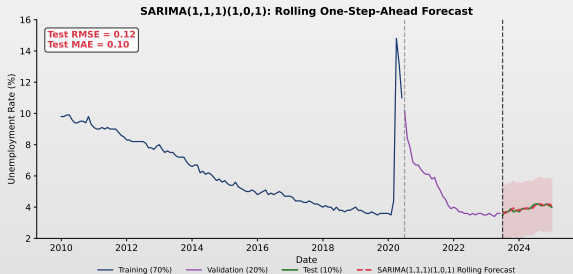
**SARIMA(1,1,1)(1,0,1) Diagnostics | Ljung-Box  $p = 1.00$**



## Unemployment: SARIMA Rolling Forecast

### Problem: Structural Break

- Rolling one-step-ahead forecast (re-estimate at each  $t$ )
- Test RMSE = 0.12**



## Prophet Model

### Definition 4 (Prophet Decomposition)

$$y_t = g(t) + s(t) + h(t) + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2)$$

where  $g(t)$  = trend,  $s(t)$  = seasonality,  $h(t)$  = holidays,  $\sigma^2$  = noise variance (estimated).

### Changepoint Detection

- Automatic location selection
- `changepoint_prior_scale` controls flexibility

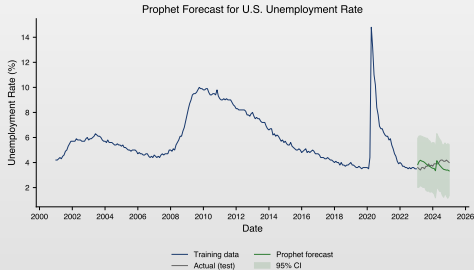
### Advantages

- Handles missing data
- Interpretable components
- Robust to outliers

## Unemployment: Prophet Forecast Results

### Key Finding

- Prophet adapts via changepoint detection
- Test RMSE = 0.58**



TSA\_ch10\_unemployment\_forecast

## Unemployment: Model Tuning

### Hyperparameter Tuning

Tune `changepoint_prior_scale` on validation set.

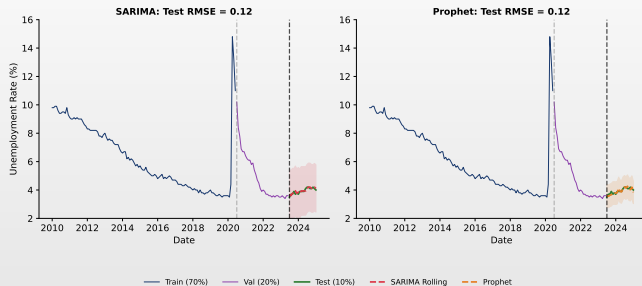
Data Split	Set	Period	N
	Training (70%)	2010-01 to 2020-06	126
	Validation (20%)	2020-07 to 2023-06	36
	Test (10%)	2023-07 to 2025-01	19
	<b>Total</b>		<b>181</b>

Scale Comparison	Scale	Val RMSE	
	0.01	4.21	
	0.05	3.89	
	0.10	<b>3.52</b>	Best
	0.30	3.67	
	0.50	3.81	

### Interpretation

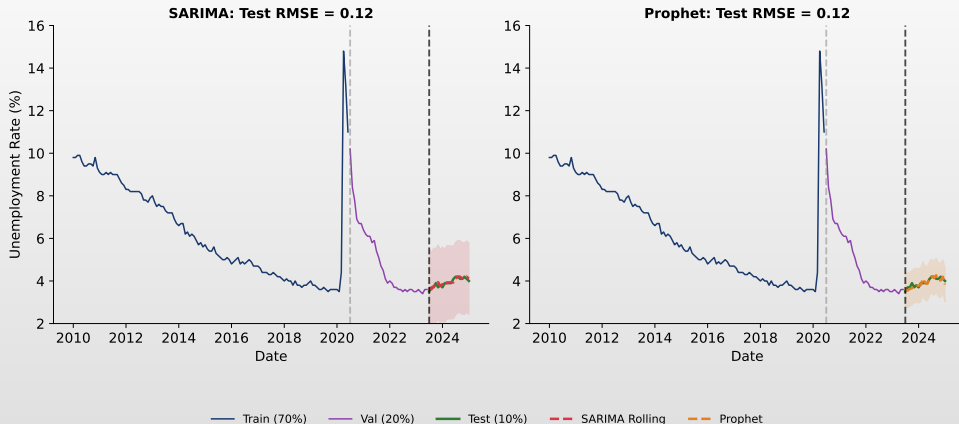
Scale = 0.10 balances flexibility (capturing COVID shock) with stability.

## Unemployment: SARIMA vs Prophet Comparison





## Unemployment: SARIMA vs Prophet Comparison



## Prophet: When to Use It

### Ideal Use Cases

- Business data with **holidays**
- **Missing values** present
- Need **interpretable** components
- Forecasts with **uncertainty bands**

### Caveat: Structural Breaks

Prophet handles breaks via changepoints, but **SARIMA outperformed** it on unemployment (0.12 vs 0.58). Always validate!

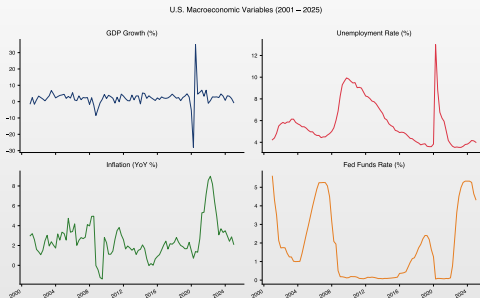
### Prophet vs ARIMA

	Prophet	ARIMA
Changepoints	✓	×
Missing data	✓	×
Holidays	✓	×
Speed	Fast	Moderate
Interpretable	✓	×

### Key Parameters

`changepoint_prior_scale`: flexibility  
`seasonality_prior_scale`: smoothness

## VAR: Multivariate Economic Data



## VAR: Multivariate Economic Data

U.S. Macroeconomic Variables (2001 – 2025)



## VAR Model Specification

### Definition 5 (Vector Autoregression VAR(p))

For  $K$  variables  $y_t = (y_{1t}, \dots, y_{Kt})'$ :

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + u_t$$

where  $A_i$  are  $K \times K$  coefficient matrices,  $u_t \sim N(0, \Sigma)$ ,  $\Sigma$  = covariance matrix.

### For Our 4-Variable System

VAR(2) has:

- ▣ 4 intercepts
- ▣  $2 \times 4 \times 4 = 32$  AR coefficients
- ▣ **36 parameters total**

### Lag Selection

Use information criteria:

- ▣ AIC: Tends to overfit
- ▣ **BIC**: More parsimonious
- ▣ Cross-validation on held-out data

## Information Criteria for Model Selection

### Definition 6 (Akaike and Bayesian Information Criteria)

For a model with log-likelihood  $\mathcal{L}$ ,  $k$  parameters, and  $n$  observations:

$$\text{AIC} = -2\mathcal{L} + 2k$$

$$\text{BIC} = -2\mathcal{L} + k \ln(n)$$

#### AIC

- Asymptotically efficient
- May overfit with small  $n$
- Minimizes prediction error

#### BIC

- Consistent (finds true model)
- Heavier penalty:  $\ln(n) > 2$  if  $n > 7$
- More parsimonious

## VAR: Lag Selection and Estimation

### BIC by Lag Order

Lag	BIC
1	-4.810
2	<b>-5.178</b> Best
3	-4.633
4	-4.614

### Data Split

Set	Period	N
Training (70%)	2001-Q1 to 2017-Q4	67
Validation (20%)	2018-Q1 to 2022-Q4	20
Test (10%)	2023-Q1 to 2025-Q1	10
<b>Total</b>		<b>97</b>

### Validation Check

VAR(2) also achieves lowest validation RMSE.

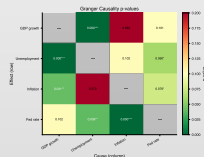
## Granger Causality: Empirical Results

### Interpretation

Each cell shows p-value for testing whether the row variable Granger-causes the column variable. Green:  $p < 0.10$ . Red: row causes column.

### Economic Findings

- Unemp  $\rightarrow$  GDP ( $p = 0.045$ ): Okun's Law
- Fed  $\rightarrow$  Inflation ( $p = 0.087$ ): Monetary policy transmission
- GDP  $\rightarrow$  Unemp: Weak evidence





## Granger Causality: Formal Definition

### Definition 7 (Granger Causality)

$X$  **Granger-causes**  $Y$  if, for some  $h > 0$ :

$$\text{MSE} \left[ \mathbb{E}[Y_{t+h} | \mathcal{F}_t^{X,Y}] \right] < \text{MSE} \left[ \mathbb{E}[Y_{t+h} | \mathcal{F}_t^Y] \right]$$

where  $\mathcal{F}_t^{X,Y}$  includes past values of both  $X$  and  $Y$ , while  $\mathcal{F}_t^Y$  includes only past  $Y$ .

### Important Caveat

Granger causality is **predictive causality**, not true causality. “ $X$  Granger-causes  $Y$ ” means  $X$  contains useful information for forecasting  $Y$ , not that  $X$  causes  $Y$  in a structural sense.

### Test Procedure

Use F-test (or Wald test) to test  $H_0$ : coefficients on lagged  $X$  are jointly zero in the  $Y$  equation.

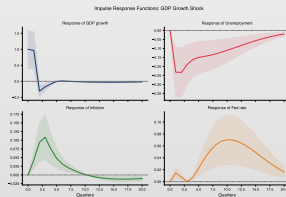
## Impulse Response Functions (IRF)

### What is IRF?

Shows how a 1-unit shock affects others over time.

### GDP Shock Effects

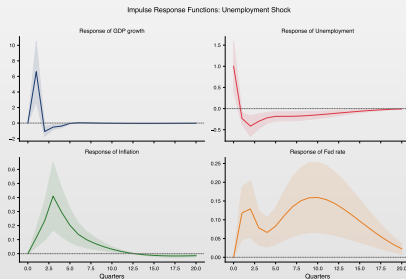
- **Unemp** ↓: Okun's Law
- **Inflation** ↑: Demand-pull
- **Fed Rate** ↑: Taylor Rule



## IRF: Unemployment Shock

## Effects

□  $\uparrow$  Unemp  $\Rightarrow$   $\downarrow$  GDP,  $\downarrow$  Infl, Fed cuts rates

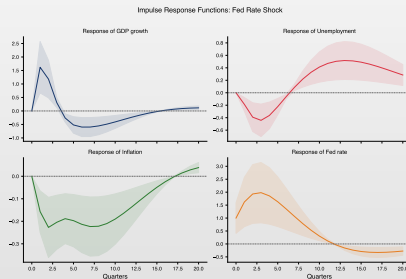


TSA\_ch10\_irf\_unemp\_shock

## IRF: Fed Rate Shock

## Monetary Policy

☐ Rate hike  $\Rightarrow$  GDP  $\downarrow$ , Unemp  $\uparrow$ , Infl  $\downarrow$

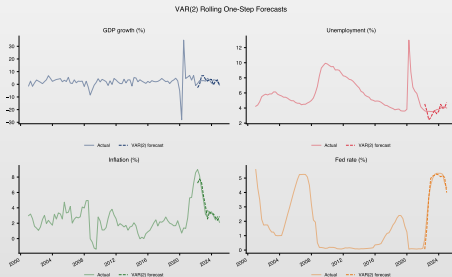


 TSA\_ch10\_irf\_fed\_shock

## VAR: Forecast (Train/Val/Test)

### Rolling One-Step-Ahead Forecast

- VAR captures GDP-Unemployment dynamics
- COVID shock visible in test period



## VAR: Test Set Results

### Test Set Performance by Variable

Variable	RMSE	MAE	Dir. Acc.
GDP Growth	1.33	0.99	50%
Unemployment	0.64	0.52	50%
Inflation	1.56	1.12	60%
Fed Rate	2.59	2.45	80%
<b>Average</b>	<b>1.53</b>	<b>1.27</b>	<b>60%</b>

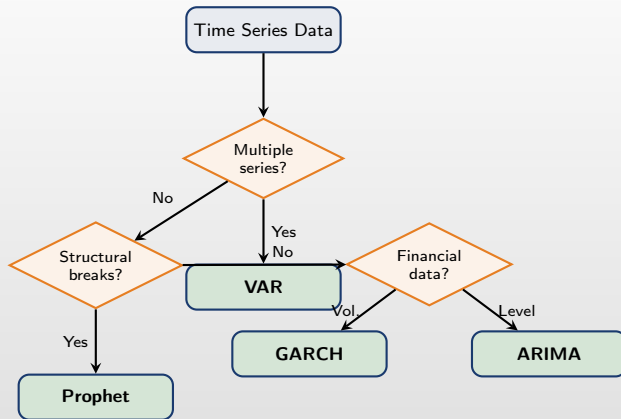
### Strengths

- Cross-variable dynamics
- Good directional accuracy

### Limitations

- Many parameters
- Sensitive to lag selection

## Model Selection Framework



## Summary: Model Comparison

### Results

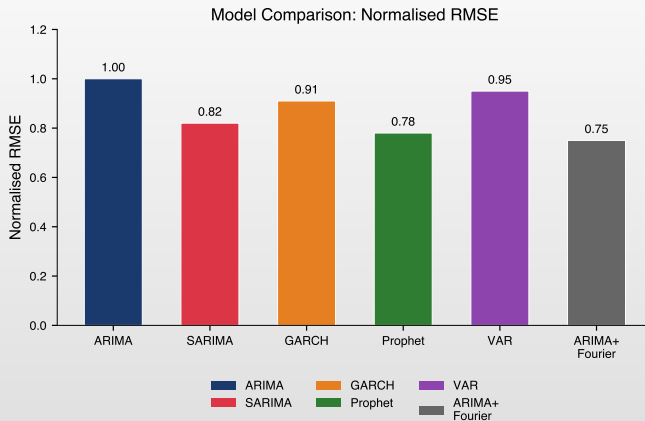
Case	Challenge	Model	RMSE
Bitcoin	Volatility	GARCH	2.15
Sunspots	Seasonality	Fourier	31.10
Unemp	Break	SARIMA	0.12
Economic	Multi-var	VAR	1.53

### Key Principle

**Match model to data characteristics**—no single model dominates.



## Summary: Model Comparison



## Comprehensive Model Comparison

Feature	GARCH	Fourier	Prophet	VAR
Target	Volatility	Level	Level	Multiple
Seasonality	No	Yes (long)	Yes (multi)	No
Structural breaks	No	No	Yes	No
Multiple series	No	No	No	Yes
Interpretable	Medium	High	High	High
Parameters	Few	2K	Auto	Many
Missing data	No	No	Yes	No
Best for	Finance	Cycles	Business	Macro

### Our Results

- GARCH: MAE=1.82 (volatility)
- Fourier: RMSE=31.10 (cycles)
- SARIMA: RMSE=0.12 (breaks)
- VAR: Avg RMSE=1.53 (multi)

### Key Insight

Each model excels in its domain. The art is matching the model to the data characteristics.

## Best Practices for Applied Forecasting

### Methodology

1. **Explore** data
2. **Test** stationarity
3. **Split** train/val/test
4. **Compare** on validation
5. **Report** test metrics

### Common Mistakes

- Peeking at test data
- Over-fitting
- Ignoring assumptions

### Practical Tips

- Start simple (naive)
- Add complexity if needed
- Check residuals
- Report CIs

### Remember

"All models are wrong, but some are useful." — Box

## Key Takeaways

### 1. Rigorous Methodology

- ▶ Train/validation/test split prevents overfitting
- ▶ Test set must remain untouched until final evaluation

### 2. Match Model to Data

- ▶ Financial volatility → GARCH
- ▶ Long seasonality → Fourier terms
- ▶ Structural breaks → Prophet
- ▶ Multiple series → VAR

### 3. Interpret Results Carefully

- ▶ Granger causality  $\neq$  true causality
- ▶ Out-of-sample performance matters most
- ▶ Simpler models often work better

## AI Exercise: Critical Thinking

Prompt to test in ChatGPT / Claude / Copilot

"I have a new dataset of monthly retail sales. Do a complete time series analysis: decomposition, stationarity tests, model selection, forecasting, and evaluation. Make it publication-quality."

**Exercise:**

1. Run the prompt in an LLM of your choice and critically analyze the response.
2. Does it follow the correct workflow? (plot → decompose → test → model → diagnose → forecast)
3. Does it compare multiple models (ETS, ARIMA, SARIMA) with proper benchmarks?
4. Is the train/test split done properly? Is there any data leakage?
5. Does it discuss limitations and assumptions of the chosen model?

**Warning:** AI-generated code may run without errors and look professional. *That does not mean it is correct.*

## Question 1

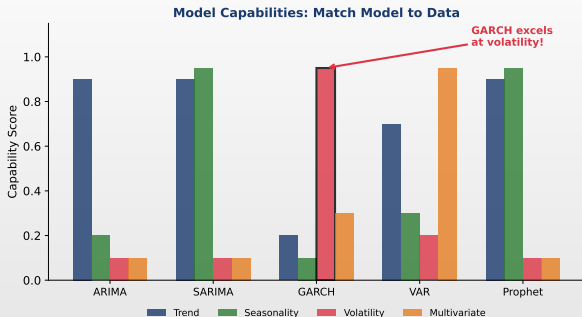
### Question

☐ Which model would you choose to forecast the volatility of financial returns?

### Answer Choices

- (A) ARIMA — captures trends and autocorrelations
- (B) GARCH — models conditional variance
- (C) Prophet — detects changepoints and seasonality
- (D) VAR — multivariate model for interdependencies

## Question 1: Answer



Answer: (B)

- GARCH captures volatility clustering and time-varying risk. ARIMA models the level, Prophet handles seasonality, VAR captures cross-series dynamics — none model variance directly.

## Question 2

### Question

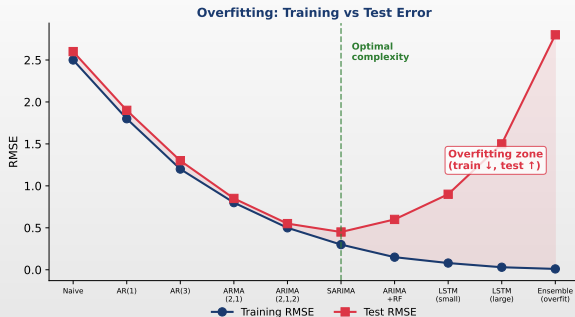
- ☐ A SARIMA model achieves  $\text{RMSE} = 0.05$  on training but  $\text{RMSE} = 2.30$  on test. What does this indicate?

### Answer Choices

- (A) The model is excellent — low training error confirms quality
- (B) The model suffers from overfitting — it memorizes noise
- (C) The test set is faulty and should be replaced
- (D) The difference is normal — all models have higher test error



## Question 2: Answer



Answer: (B)

- A  $46\times$  ratio between test and training RMSE signals severe overfitting. The model fits noise in the training data and fails to generalize. Solution: simpler model, proper validation.

### Question 3

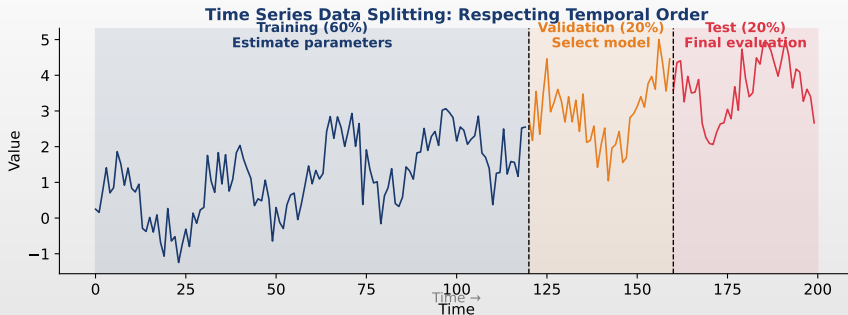
#### Question

☐ Why is it important to separate data into train/validation/test sets?

#### Answer Choices

- (A) To have more training data
- (B) To prevent overfitting and evaluate correctly
- (C) It is just a convention with no real importance
- (D) To reduce computation time

### Question 3: Answer



Answer: (B)

- Train: estimate parameters. Validation: select model/hyperparameters. Test: final unbiased evaluation. Mixing these roles leads to optimistic performance estimates.

## Question 4

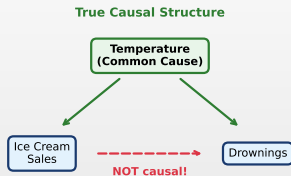
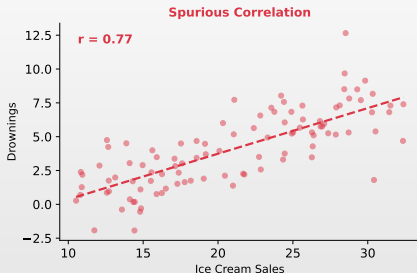
### Question

☐ Is Granger causality equivalent to true (structural) causality?

### Answer Choices

- (A) Yes — if  $X$  predicts  $Y$ , then  $X$  causes  $Y$
- (B) No — it only tests predictive content, not causation
- (C) It depends on the number of lags selected
- (D) Yes, if the p-value is below 0.05

## Question 4: Answer

Granger Causality  $\neq$  True Causality

Answer: (B)

- Granger causality tests whether past  $X$  improves forecasts of  $Y$ . Spurious correlations (e.g., ice cream sales and drownings) can pass the test due to common causes.

## Question 5

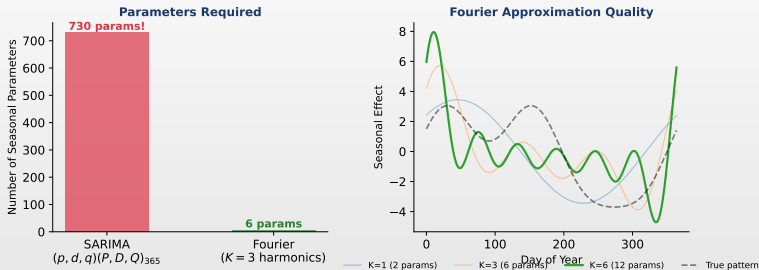
### Question

□ What model do you use for a series with long seasonality (e.g.,  $s = 365$  days)?

### Answer Choices

- (A)  $\text{SARIMA}(p, d, q)(P, D, Q)_{365}$
- (B) GARCH — models variation
- (C) ARIMA + Fourier terms or Prophet/TBATS
- (D) VAR with 365 lags

## Question 5: Answer

Long Seasonality ( $s = 365$ ): Fourier Terms vs SARIMA

Answer: (C)

- SARIMA<sub>365</sub> needs  $\sim 730$  seasonal parameters — infeasible. Fourier terms with  $K = 3$  use only 6 parameters. Prophet and TBATS handle multiple seasonalities automatically.

## Data Sources

### Real Data Used in This Chapter

- ▣ **Bitcoin:** Yahoo Finance (BTC-USD), 2019–2025
- ▣ **Sunspots:** Statsmodels Wolfer dataset, 1900–2008
- ▣ **US Unemployment:** Federal Reserve FRED (UNRATE), 2010–2025
- ▣ **Economic Variables:** FRED (GDPC1, UNRATE, CPIAUCSL, FEDFUNDS), 2000–2025

### Reproducibility

All analyses can be reproduced using the accompanying Jupyter notebook:  
`chapter10_lecture_notebook.ipynb`



## Bibliography I

### Fundamental Textbooks (common references across all chapters)

- Hamilton, J.D. (1994). *Time Series Analysis*, Princeton University Press.
- Hyndman, R.J., & Athanasopoulos, G. (2021). *Forecasting: Principles and Practice*, 3rd ed., OTexts.
- Shumway, R.H., & Stoffer, D.S. (2017). *Time Series Analysis and Its Applications*, 4th ed., Springer.

### Domain-Specific References

- Tsay, R.S. (2010). *Analysis of Financial Time Series*, 3rd ed., Wiley. (GARCH, VAR)
- Lütkepohl, H. (2005). *New Introduction to Multiple Time Series Analysis*, Springer. (VAR, VECM)
- Francq, C., & Zakoïan, J.-M. (2019). *GARCH Models*, 2nd ed., Wiley. (Volatility)

## Bibliography II

### Modern Approaches and Forecasting Competitions

- ▣ Petropoulos, F., et al. (2022). Forecasting: Theory and Practice, *International Journal of Forecasting*, 38(3), 845–1054.
- ▣ Makridakis, S., Spiliotis, E., & Assimakopoulos, V. (2020). The M4 Competition, *International Journal of Forecasting*, 36(1), 54–74.
- ▣ Taylor, S.J., & Letham, B. (2018). Forecasting at Scale, *The American Statistician*, 72(1), 37–45.

### Online Resources and Code

- ▣ **Quantlet**: <https://quantlet.com> ∽ Code repository for statistics
- ▣ **Quantinar**: <https://quantinar.com> ∽ Learning platform for quantitative methods
- ▣ **GitHub TSA**: [https://github.com/QuantLet/TSA/tree/main/TSA\\_ch10](https://github.com/QuantLet/TSA/tree/main/TSA_ch10) ∽ Python code for this chapter

## Key Takeaways

### What We Learned

- Model selection depends on data characteristics: stationarity, seasonality, volatility
- The Box-Jenkins methodology provides a systematic framework for time series modeling
- Proper evaluation requires out-of-sample testing and time series cross-validation

### Important

No single model wins everywhere. Match the model to the data: ARIMA for trends, SARIMA for seasonality, GARCH for volatility, VAR/VECM for multivariate dynamics, Prophet/TBATS for complex patterns. Always validate out-of-sample!

## References



Box, G.E.P., Jenkins, G.M., Reinsel, G.C., & Ljung, G.M. (2015). *Time Series Analysis: Forecasting and Control*. 5th ed., Wiley.



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Sims, C.A. (1980). Macroeconomics and Reality. *Econometrica*, 48(1), 1-48.

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# Thank You!

## Questions?

Course materials available at: <https://danpele.github.io/Time-Series-Analysis/>



Quantlet



Quantinar