Time Series Analysis and Forecasting

Chapter 9: Prophet & TBATS

Seminar

Seminar Outline

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Quiz 1: Multiple Seasonality Challenge

Question

Why can't standard SARIMA handle hourly electricity demand data?

A) SARIMA can only handle monthly data

B) SARIMA allows only one seasonal period (m parameter)

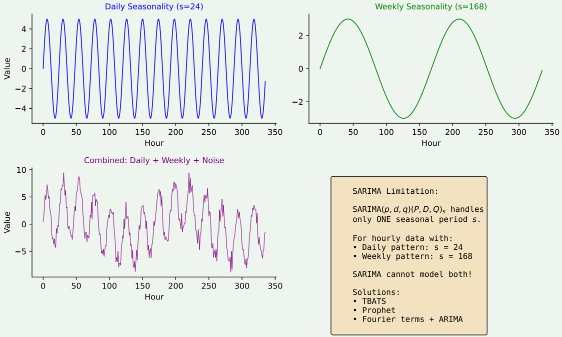
C) SARIMA doesn't support trend components

D) SARIMA requires normally distributed data

Answer on next slide...

Quiz 1: Answer

Answer: B – SARIMA allows only one seasonal period



SARIMA Limitation:

SARIMA$(p, d, q)(P, D, Q)_s$ handles only ONE seasonal period s.

For hourly data with:
- Daily pattern: s = 24
- Weekly pattern: s = 168

SARIMA cannot model both!

Solutions:
- TBATS
- Prophet
- Fourier terms + ARIMA

Key: Hourly data has daily (24h), weekly (168h), and annual (8760h) patterns. SARIMA's single *m* parameter cannot capture all these simultaneously.

Quiz 2: TBATS Acronym

Question

What does TBATS stand for?

A) Trend, Baseline, ARMA, Transform, Seasonal

B) Trigonometric, Box-Cox, ARMA, Trend, Seasonal

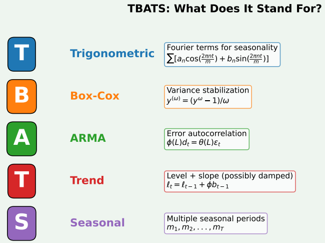
C) Time-Based Automatic Time Series

D) Temporal Bayesian Adaptive Trend System

Answer on next slide...

Quiz 2: Answer

Answer: B – Trigonometric, Box-Cox, ARMA, Trend, Seasonal



TBATS: What Does It Stand For?

T **Trigonometric**: Fourier terms for seasonality $\sum[a_n\cos(\frac{2\pi nt}{m}) + b_n\sin(\frac{2\pi nt}{m})]$

B **Box-Cox**: Variance stabilization $y^{(\omega)} = (y^{\omega} - 1)/\omega$

A **ARMA**: Error autocorrelation $\phi(L)d_t = \theta(L)\varepsilon_t$

T **Trend**: Level + slope (possibly damped) $\ell_t = \ell_{t-1} + \phi b_{t-1}$

S **Seasonal**: Multiple seasonal periods $m_1, m_2, \ldots, m_T$

TBATS: Trigonometric (Fourier for seasonality), Box-Cox (variance stabilization), ARMA (error autocorrelation), Trend (damped local), Seasonal (multiple periods).

Quiz 3: Fourier Terms

Question

In TBATS, increasing the number of Fourier harmonics (K) for a seasonal pattern:

A) Always improves forecast accuracy

B) Allows more flexible (complex) seasonal shapes

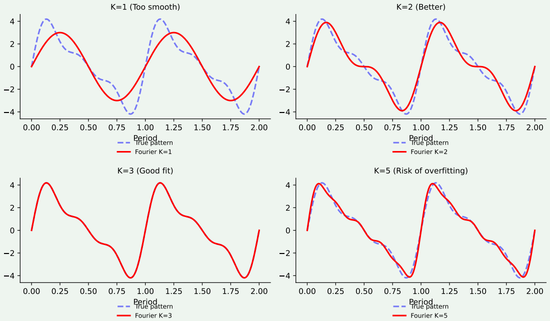
C) Reduces the model complexity

D) Eliminates the need for Box-Cox transformation

Answer on next slide...

Quiz 3: Answer

Answer: B – Allows more flexible seasonal shapes



Trade-off: More harmonics = more flexibility but also more parameters.

$$s_t^{(i)} = \sum_{j=1}^{K_i} \left[a_j^{(i)} \cos\left(\frac{2\pi jt}{m_i}\right) + b_j^{(i)} \sin\left(\frac{2\pi jt}{m_i}\right) \right]$$

Quiz 4: Prophet Decomposition

Question

Prophet decomposes a time series into which components?

A) AR, MA, and seasonal components

B) Trend, seasonality, holidays, and error

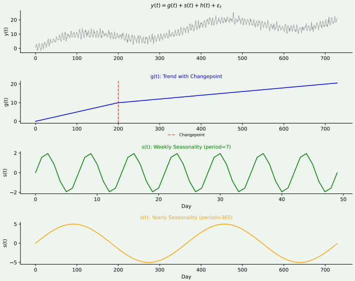
C) Mean, variance, and autocorrelation

D) Level, slope, and curvature

Answer on next slide...

Quiz 4: Answer

Answer: B – Trend, seasonality, holidays, and error



Prophet: $y(t) = g(t) + s(t) + h(t) + \varepsilon_t$ **where** $g(t)$ **= trend,** $s(t)$ **= seasonality (Fourier),** $h(t)$ **= holidays,** ε_t **= error.**

Quiz 5: Prophet vs TBATS

Question

When would you choose Prophet over TBATS?

A) When you need automatic model selection

B) When you have known holidays and changepoints to incorporate

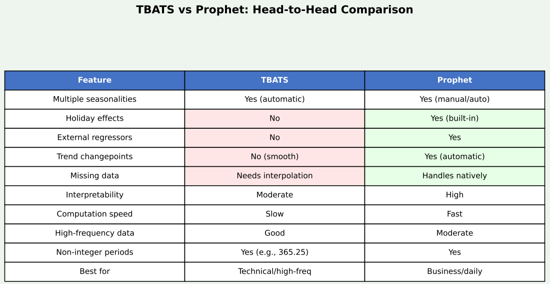
C) When you need the most parsimonious model

D) When your data has no trend

Answer on next slide...

Quiz 5: Answer

Answer: B – Known holidays and changepoints



TBATS vs Prophet: Head-to-Head Comparison

Feature	TBATS	Prophet
Multiple seasonalities	Yes (automatic)	Yes (manual/auto)
Holiday effects	No	Yes (built-in)
External regressors	No	Yes
Trend changepoints	No (smooth)	Yes (automatic)
Missing data	Needs interpolation	Handles natively
Interpretability	Moderate	High
Computation speed	Slow	Fast
High-frequency data	Good	Moderate
Non-integer periods	Yes (e.g., 365.25)	Yes
Best for	Technical/high-freq	Business/daily

Prophet advantages: Easy holiday integration, analyst-in-the-loop, handles missing data, interpretable components.

TBATS advantages: Automatic model selection, handles complex seasonality without domain expertise.

Quiz 6: Seasonality Mode

Question

For retail sales data where December sales are 3x the monthly average, which seasonality mode is more appropriate in Prophet?

A) Additive seasonality

B) Multiplicative seasonality

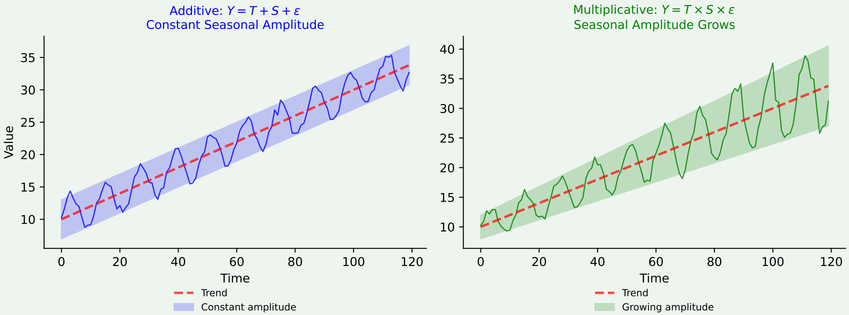
C) Both work equally well

D) Neither—use ARIMA instead

Answer on next slide...

Quiz 6: Answer

Answer: B – Multiplicative seasonality



Key: When seasonal amplitude scales with the level, use multiplicative.
Additive: $y = g(t) + s(t)$ (constant seasonal effect)

Multiplicative: $y = g(t) \cdot (1 + s(t))$ (proportional seasonal effect)

Quiz 7: Prophet Changepoints

Question

In Prophet, changepoints allow the model to:

A) Change the seasonal period automatically

B) Adjust the trend slope at specific points in time

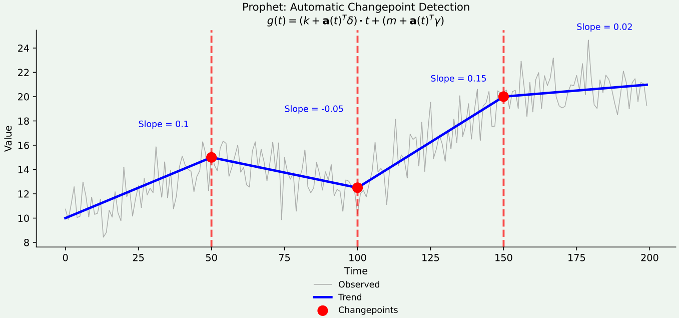
C) Switch between additive and multiplicative modes

D) Detect and remove outliers

Answer on next slide...

Quiz 7: Answer

Answer: B – Adjust trend slope at specific points



Changepoints: Allow piecewise linear trend with different slopes.

$$g(t) = (k + \mathrm{a}(t)^\top \delta) \cdot t + (m + \mathrm{a}(t)^\top \gamma)$$

Prophet automatically detects changepoints or you can specify them manually.

Quiz 8: Model Selection

Question

You have daily call center data with weekly seasonality only. Which model is most appropriate?

A) TBATS (designed for multiple seasonality)

B) Prophet (handles any seasonality well)

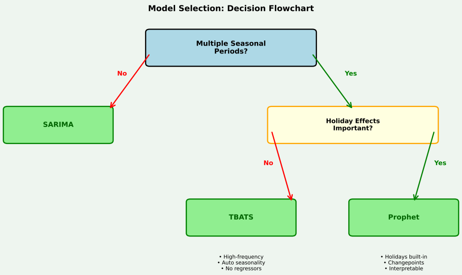
C) Standard SARIMA (simpler and sufficient)

D) LSTM neural network (most flexible)

Answer on next slide...

Quiz 8: Answer

Answer: C – Standard SARIMA is sufficient



Model Selection: Decision Flowchart

Principle of parsimony: Use the simplest model that fits the data.
With only weekly seasonality ($m = 7$), SARIMA works fine.

Use TBATS/Prophet when you *need* multiple seasonalities or special features.

Quiz 9: Prophet Uncertainty

Question

Prophet generates prediction intervals by:

A) Assuming normally distributed residuals

B) Sampling from the posterior distribution of parameters

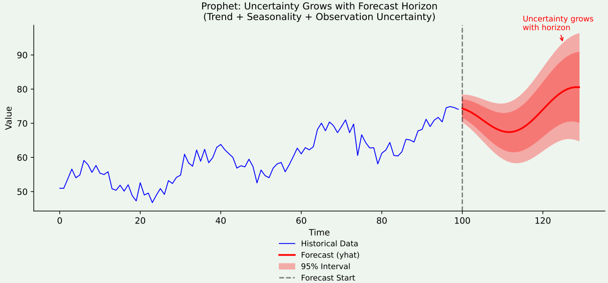
C) Using bootstrap resampling of historical errors

D) Applying a fixed multiplier to point forecasts

Answer on next slide...

Quiz 9: Answer

Answer: B – Sampling from posterior distribution



Prophet uses Bayesian estimation: MAP for point forecasts, MCMC/simulation for intervals. Uncertainty from both trend (changepoints) and observation noise.

Quiz 10: Practical Application

Question

For forecasting hourly energy demand with daily, weekly, and annual patterns plus holiday effects, which approach is best?

A) SARIMA with $m = 24$

B) TBATS with three seasonal periods

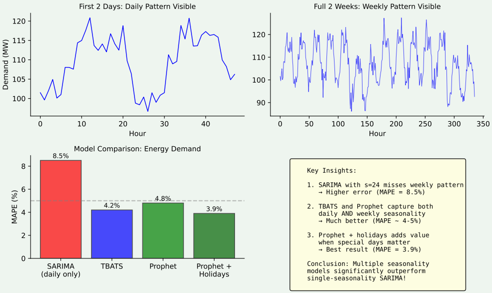
C) Prophet with custom holidays

D) Either TBATS or Prophet, depending on whether holidays are important

Answer on next slide...

Quiz 10: Answer

Answer: D – TBATS or Prophet depending on needs



```
Key Insights:

1. SARIMA with s=24 misses weekly pattern
   → Higher error (MAPE = 8.5%)

2. TBATS and Prophet capture both
   daily AND weekly seasonality
   → Much better (MAPE ~ 4-5%)

3. Prophet + holidays adds value
   when special days matter
   → Best result (MAPE = 3.9%)

Conclusion: Multiple seasonality
models significantly outperform
single-seasonality SARIMA!
```

Both handle multiple seasonality: Holiday effects crucial ⇒ Prophet; automatic selection ⇒ TBATS. Often try both and compare via cross-validation.

True/False Questions

Determine if each statement is True or False:

1. Prophet was developed by Facebook (Meta) for business forecasting.
2. TBATS can only handle two seasonal periods at most.
3. In Prophet, the default trend is logistic growth.
4. Fourier terms approximate seasonality using sine and cosine functions.
5. Prophet requires equally spaced time series data.
6. The Box-Cox transformation in TBATS stabilizes variance.

Answers on next slide...

True/False: Solutions

1. Prophet was developed by Facebook (Meta) for business forecasting. **TRUE**

   Released in 2017, designed for “analyst in the loop” forecasting at scale.

2. TBATS can only handle two seasonal periods at most. **FALSE**

   TBATS can handle any number of seasonal periods (e.g., daily, weekly, annual).

3. In Prophet, the default trend is logistic growth. **FALSE**

   Default is piecewise linear. Logistic growth must be explicitly specified.

4. Fourier terms approximate seasonality using sine and cosine functions. **TRUE**

   $s(t) = \sum_{k=1}^{K}[a_k \cos(2\pi kt/m) + b_k \sin(2\pi kt/m)]$

5. Prophet requires equally spaced time series data. **FALSE**

   Prophet handles missing data and irregular timestamps gracefully.

6. The Box-Cox transformation in TBATS stabilizes variance. **TRUE**

   $y^{(\lambda)} = (y^{\lambda} - 1)/\lambda$ for $\lambda \neq 0$; $\log(y)$ for $\lambda = 0$.

Fourier Series Representation

Definition: Fourier Harmonics

A **Fourier series** represents a periodic function $s(t)$ with period m as:

$$s(t) = a_0 + \sum_{k=1}^{K} \left[a_k \cos\left(\frac{2\pi kt}{m}\right) + b_k \sin\left(\frac{2\pi kt}{m}\right) \right]$$

where:

- k: harmonic number (frequency multiplier)
- a_k, b_k: Fourier coefficients (estimated from data)
- K: number of harmonics (controls flexibility)

Why It Works

Fourier's Theorem: Any periodic function can be expressed as a (possibly infinite) sum of sine and cosine waves. With finite K, we get an approximation.

Problem 1: Fourier Terms Calculation

Exercise

For daily data with weekly seasonality ($m = 7$), you want to use Fourier terms with $K = 3$ harmonics.

How many parameters does this add to the model?

Answer on next slide...

Problem 1: Solution

Solution: 6 parameters

Each harmonic requires 2 parameters (sine and cosine coefficients):

$$s(t) = \sum_{k=1}^{K} \left[a_k \cos\left(\frac{2\pi kt}{m}\right) + b_k \sin\left(\frac{2\pi kt}{m}\right) \right]$$

With $K = 3$ harmonics:

- $k = 1$: a_1, b_1 (fundamental frequency $1/7$ cycles per day)
- $k = 2$: a_2, b_2 (first overtone, $2/7$ cycles per day)
- $k = 3$: a_3, b_3 (second overtone, $3/7$ cycles per day)

Total: $2 \times K = 2 \times 3 = 6$ parameters

Nyquist Limit

Maximum useful $K = \lfloor m/2 \rfloor$. With m discrete observations per period, we can identify at most $m/2$ distinct frequencies. For $m = 7$: $K_{\text{max}} = 3$.

Problem 2: Choosing Seasonality Mode

Exercise

You're forecasting monthly hotel bookings. The data shows:

- July 2020: 1000 bookings (peak season)
- January 2020: 400 bookings (off-season)
- July 2023: 2000 bookings (peak season)
- January 2023: 800 bookings (off-season)

Should you use additive or multiplicative seasonality? Why?

Answer on next slide...

Problem 2: Solution

Solution: Multiplicative seasonality

Analysis: Check if seasonal amplitude is proportional to level.

Year	July	January	Ratio (Jul/Jan)
2020	1000	400	2.5
2023	2000	800	2.5

Key observation: The *ratio* stays constant (2.5), not the difference!

- Additive would mean: July always $+600$ above January
- But 2020: $1000 - 400 = 600$; 2023: $2000 - 800 = 1200$

Conclusion: Use multiplicative: `seasonality_mode='multiplicative'`

Problem 3: TBATS Model Interpretation

Exercise

A TBATS model fitted to hourly electricity data reports:

- Box-Cox $\lambda = 0.5$
- Seasonal periods: $m_1 = 24$, $m_2 = 168$
- Fourier terms: $K_1 = 5$, $K_2 = 3$

What does each component tell you about the data?

Answer on next slide...

Problem 3: Solution

Solution

Box-Cox $\lambda = 0.5$:

- Square root transformation applied
- Data had increasing variance with level
- Transformation: $y^{(0.5)} = \sqrt{y}$

Seasonal periods:

- $m_1 = 24$: Daily pattern (24 hours)
- $m_2 = 168$: Weekly pattern ($7 \times 24 = 168$ hours)

Fourier terms:

- $K_1 = 5$ for daily: Complex intraday pattern (5 harmonics capture peaks, valleys)
- $K_2 = 3$ for weekly: Simpler weekly pattern (weekday vs weekend)

Total seasonal parameters: $2(K_1 + K_2) = 2(5 + 3) = 16$

Problem 4: Prophet Holiday Effects

Exercise

You're forecasting daily restaurant revenue. You want to add these holiday effects to Prophet:

- Valentine's Day (Feb 14) – major boost
- Easter (variable date) – restaurant closed
- Christmas (Dec 25) – restaurant closed

Write the Python code to create the holidays dataframe for 2024-2025.

Answer on next slide...

Problem 4: Solution

Solution

```
import pandas as pd
from prophet import Prophet
# Create holidays dataframe with window effects
holidays = pd.DataFrame({
  ’holiday’:  [’valentines’, ’valentines’, ’easter’, ’easter’,
              ’christmas’, ’christmas’],
  ’ds’:  pd.to_datetime([’2024-02-14’, ’2025-02-14’,
            ’2024-03-31’, ’2025-04-20’,
            ’2024-12-25’, ’2025-12-25’]),
  ’lower_window’:  [-1, -1, 0, 0, -1, -1], # days before
  ’upper_window’:  [0, 0, 0, 0, 0, 0] # days after
})
model = Prophet(holidays=holidays)
model.fit(df) # df must have ’ds’ and ’y’ columns
```

Key Points

- `lower_window=-1`: Effect starts 1 day before
- Easter: closed = zero revenue (handled by model as large negative effect)
- Valentine’s: expect positive effect coefficient

Example: Retail Sales Forecasting with Prophet

Scenario

Monthly retail sales (2018-2023): December peaks, COVID-19 break in 2020, growing trend.

Prophet Configuration

```
model = Prophet(seasonality_mode=’multiplicative’,
  changepoint_prior_scale=0.5, yearly_seasonality=True)
model.add_country_holidays(country_name=’US’)
```

Key Decision

Multiplicative seasonality: December effect proportional to baseline level.

Example: Energy Demand with TBATS

Scenario

Hourly electricity: intraday (24h), weekly (168h), annual (8760h) patterns.

TBATS in R

```
library(forecast)
energy_msts <- msts(energy_data, seasonal.periods = c(24, 168, 8760))
fit <- tbats(energy_msts); fc <- forecast(fit, h = 168)
```

Note

TBATS automatically selects K for each seasonal period via AIC.

Example: Cross-Validation Comparison

Objective

Compare Prophet, TBATS, and SARIMA on 2 years of daily sales data.

Prophet Cross-Validation

```
from prophet.diagnostics import cross_validation, performance_metrics
df_cv = cross_validation(model, initial=’365 days’,
  period=’90 days’, horizon=’30 days’)
metrics = performance_metrics(df_cv)
print(f"MAPE: {metrics[’mape’].mean():.2%}")
```

Typical Results

Model	MAPE	Computation Time
SARIMA (weekly only)	8.5%	Fast
TBATS (weekly + yearly)	6.2%	Moderate
Prophet (weekly + yearly + holidays)	5.8%	Fast

Discussion: When to Use Which Model?

Key Question

You have a new forecasting task. How do you choose between SARIMA, TBATS, and Prophet?

Decision Framework

1. **How many seasonal periods?**
   - One ⇒ SARIMA may suffice
   - Multiple ⇒ TBATS or Prophet
2. **Do you have domain knowledge to encode?**
   - Holidays, events, changepoints ⇒ Prophet
   - Let the data speak ⇒ TBATS
3. **Interpretability requirements?**
   - Need to explain components ⇒ Prophet
   - Just need forecasts ⇒ Either

Discussion: Overfitting with Fourier Terms

Key Question

Can you have too many Fourier terms? What are the symptoms?

Answer: Yes!

Symptoms of overfitting:

- In-sample fit is excellent, but out-of-sample is poor
- Seasonality looks "jagged" or unrealistic
- Forecasts oscillate wildly

Guidelines

- Maximum $K \leq m/2$ (Nyquist limit)
- Start with $K = 3$–5 for most applications
- Use cross-validation to select K
- Prophet default: $K = 10$ for yearly, $K = 3$ for weekly

Discussion: Handling Structural Breaks

Scenario

Your historical data includes COVID-19 period (2020-2021). How do you handle this when forecasting 2024?

Options

1. **Exclude COVID period**: Train only on pre-COVID and post-COVID data
2. **Use changepoints**: Let Prophet detect/specify breaks
3. **Add regressors**: Include COVID indicator variable
4. **Adjustment**: Manually adjust 2020-2021 values to “normal”

Prophet Approach

```
model = Prophet(changepoints=[’2020-03-15’, ’2021-06-01’])
df[’covid’] = (df[’ds’] >= ’2020-03-15’) & (df[’ds’] < ’2021-06-01’)
model.add_regressor(’covid’)
```

Take-Home Exercises

1. **Theoretical**: Prove that $K = m/2$ Fourier terms can represent any periodic function with period m (for even m).
2. **Computation**: For the seasonal pattern below (daily data, weekly cycle), determine the minimum number of Fourier harmonics needed:

   Mon: 100, Tue: 110, Wed: 115, Thu: 110, Fri: 120, Sat: 80, Sun: 65

3. **Applied**: Download hourly electricity demand data from a public source:
   - Fit both TBATS (in R) and Prophet (in Python)
   - Compare forecast accuracy using RMSE and MAPE
   - Visualize the component decompositions
4. **Critical Thinking**: Why might Prophet perform poorly on high-frequency financial data (e.g., minute-by-minute stock prices)?

Exercise Solutions Hints

Hints

1. By Fourier theorem, any periodic function can be represented as sum of sines and cosines. With period m, frequencies are k/m for $k = 1, \ldots, m/2$.
2. The pattern has:
   - One peak (Friday) and one trough (Sunday)
   - Fairly smooth transitions
   - $K = 2$ or $K = 3$ likely sufficient (try and compare)
3. For electricity data:
   - Include daily (24h) and weekly (168h) patterns
   - Add holidays for your region in Prophet
   - Expect MAPE around 3-5% for hourly forecasts
4. Financial data issues:
   - No clear seasonality (market efficiency)
   - High noise-to-signal ratio
   - Prophet designed for "business" data with trends and seasons

Key Takeaways from This Seminar

Multiple Seasonality Models

1. **TBATS**: Automatic, Fourier-based, handles any number of seasonal periods
2. **Prophet**: Analyst-friendly, explicit holiday/event handling, interpretable
3. **Use SARIMA** when only one seasonal period exists

Key Decisions

- **Seasonality mode**: Additive (constant amplitude) vs Multiplicative (proportional)
- **Fourier terms**: More = flexible but risk overfitting; use CV to select
- **Changepoints**: Allow trend to adapt to structural breaks

Remember

Prophet: Great when you have domain knowledge to encode
TBATS: Great for automatic modeling of complex seasonality