



Time Series Analysis and Forecasting

Seminar 1: Stochastic Processes and Stationarity



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Seminar Outline

Seminar structure:

- 1. Overview** – Key concepts summary
- 2. Review Quiz** – Knowledge check
- 3. True/False Questions** – Conceptual checks
- 4. Practice Problems** – Applied practice
- 5. Worked Examples** – Coding practice
- 6. Discussion Topics** – Critical thinking
- 7. AI-Assisted Exercises** – Critical thinking



Key Formulas

Decomposition:

- Additive: $X_t = T_t + S_t + \varepsilon_t$
- Multiplicative: $X_t = T_t \times S_t \times \varepsilon_t$

Exponential Smoothing:

- SES: $\hat{X}_{t+1} = \alpha X_t + (1 - \alpha) \hat{X}_t$
- Holt: adds trend b_t
- HW: adds seasonality S_t

Stationarity:

- $\mathbb{E}[X_t] = \mu$ (constant)
- $\text{Var}(X_t) = \sigma^2$ (constant)
- $\text{Cov}(X_t, X_{t+h}) = \gamma(h)$

Random walk:

- $X_t = X_{t-1} + \varepsilon_t$
- $\text{Var}(X_t) = t\sigma^2$ (grows with time)



Summary: Concepts and Methods

| Concept | Key idea | When to use |
|------------------------------|-----------------------------------|---------------------------|
| Additive decomposition | Constant seasonal amplitude | Stable variance |
| Multiplicative decomposition | Seasonality grows with level | Increasing variance |
| SES | Level only (α) | No trend, no seasonality |
| Holt | Level + Trend (α, β) | Trend, no seasonality |
| Holt-Winters | Level + Trend + Seasonality | Trend and seasonality |
| ADF Test | H_0 : unit root | Test for non-stationarity |
| KPSS Test | H_0 : stationary | Confirm stationarity |
| Differencing | Remove stochastic trend | Random walk, unit root |
| Regression | Remove deterministic trend | Linear/polynomial trend |



Quiz 1: Stationarity

Question

A random walk process $X_t = X_{t-1} + \varepsilon_t$ is:

- A. Strictly stationary
- B. Weakly stationary
- C. Non-stationary because variance grows with time
- D. Stationary after adding a constant

Answer on next slide...



Quiz 1: Answer

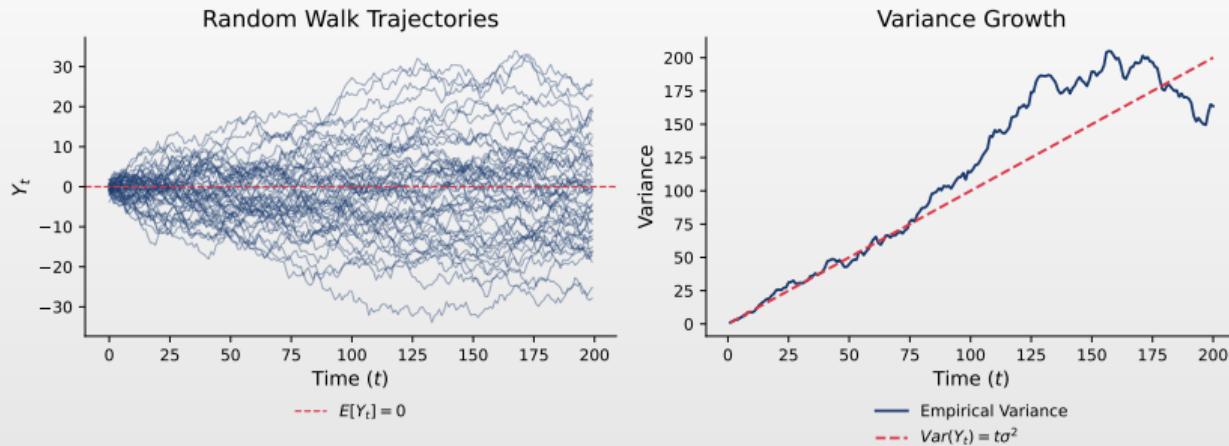
Answer: C – Non-stationary because variance grows with time

For random walk: $X_t = \sum_{i=1}^t \varepsilon_i$

- $\mathbb{E}[X_t] = 0$ (constant mean – OK)
- $\text{Var}(X_t) = t\sigma^2$ (variance depends on t – NOT OK!)
- Variance is **not** constant \Rightarrow violates the stationarity condition
 - ▶ **Solution:** differencing gives $\Delta X_t = \varepsilon_t$ — stationary



Visual: Random Walk vs Stationary



- Random walk trajectories wander unpredictably
- Variance grows linearly with time \Rightarrow non-stationary

 TSA_ch1_random_walk



Quiz 2: Unit Root Tests

Question

You run ADF and KPSS tests. ADF fails to reject H_0 , and KPSS rejects H_0 . What do you conclude?

- A. The series is stationary
- B. The series has a unit root (non-stationary)
- C. The results are inconclusive
- D. Additional tests are needed

Answer on next slide...



Quiz 2: Answer

Answer: B – The series has a unit root (non-stationary)

- ADF: H_0 = unit root. Fail to reject \Rightarrow evidence FOR unit root
- KPSS: H_0 = stationary. Reject \Rightarrow evidence AGAINST stationarity
- Both tests agree: the series is **non-stationary**
 - ▶ **Next step:** difference the series before modeling with ARMA



Quiz 3: Trend Types

Question

A deterministic trend can be removed by:

- A. Differencing
- B. Regression on time
- C. Seasonal adjustment
- D. Moving average smoothing

Answer on next slide...



Quiz 3: Answer

Answer: B – Regression on time

- Deterministic trend:** $Y_t = \alpha + \beta t + \varepsilon_t$ (β fixed)
- Removal method:** regress Y_t on t , analyze residuals $\hat{\varepsilon}_t$
- Why not differencing?**
 - ▶ Differencing gives $\Delta Y_t = \beta + \Delta \varepsilon_t$ — removes the trend but leaves a constant
 - ▶ Differencing is correct only for *stochastic* trends (unit roots)



Quiz 4: ACF Interpretation

Question

If the ACF of a time series decays very slowly (remains significant for many lags), this suggests:

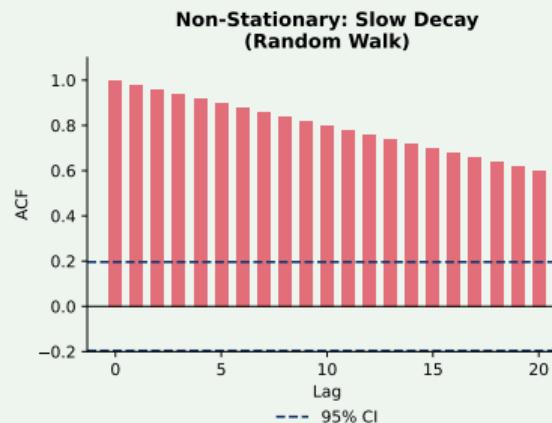
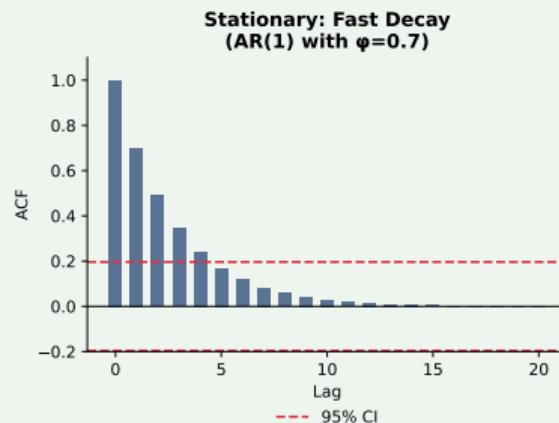
- A. The series is white noise
- B. The series is likely non-stationary
- C. The series has no autocorrelation
- D. The series is perfectly predictable

Answer on next slide...



Quiz 4: Answer

Answer: B – The series is likely non-stationary



- Stationary:** ACF decays quickly ($\rho_k = \phi^k \rightarrow 0$)
- Non-stationary:** ACF stays near 1 \Rightarrow differencing needed

Quiz 5: Holt's Method

Question

Holt's exponential smoothing differs from SES by adding:

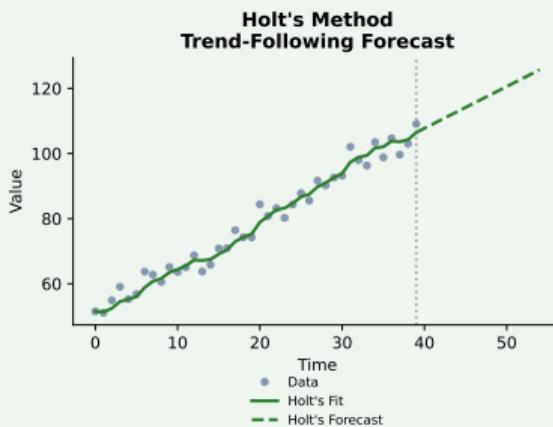
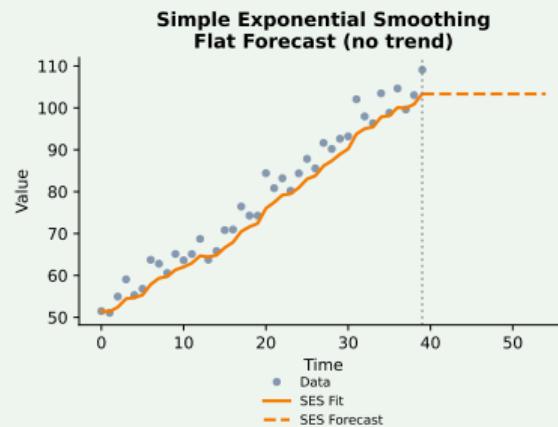
- A. A seasonal component
- B. A trend component
- C. A cyclical component
- D. An irregular component

Answer on next slide...



Quiz 5: Answer

Answer: B – A trend component



- Holt:** $L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + b_{t-1}); \quad b_t = \beta(L_t - L_{t-1}) + (1 - \beta)b_{t-1}$
- Forecast:** $\hat{Y}_{t+h} = L_t + h \cdot b_t$



Quiz 6: White Noise

Question

Which property is NOT required for a process to be white noise?

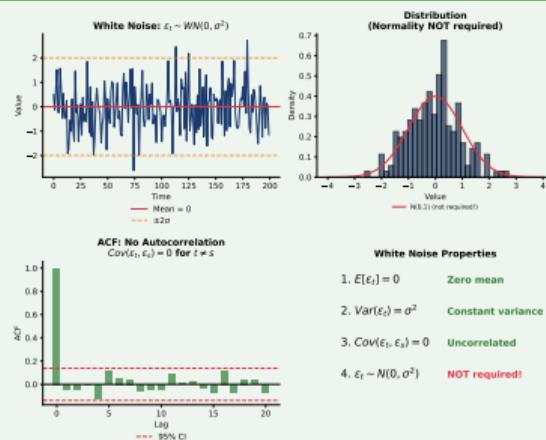
- A. $\mathbb{E}[\varepsilon_t] = 0$
- B. $\text{Var}(\varepsilon_t) = \sigma^2$ (constant)
- C. $\text{Cov}(\varepsilon_t, \varepsilon_s) = 0$ for $t \neq s$
- D. $\varepsilon_t \sim N(0, \sigma^2)$

Answer on next slide...



Quiz 6: Answer

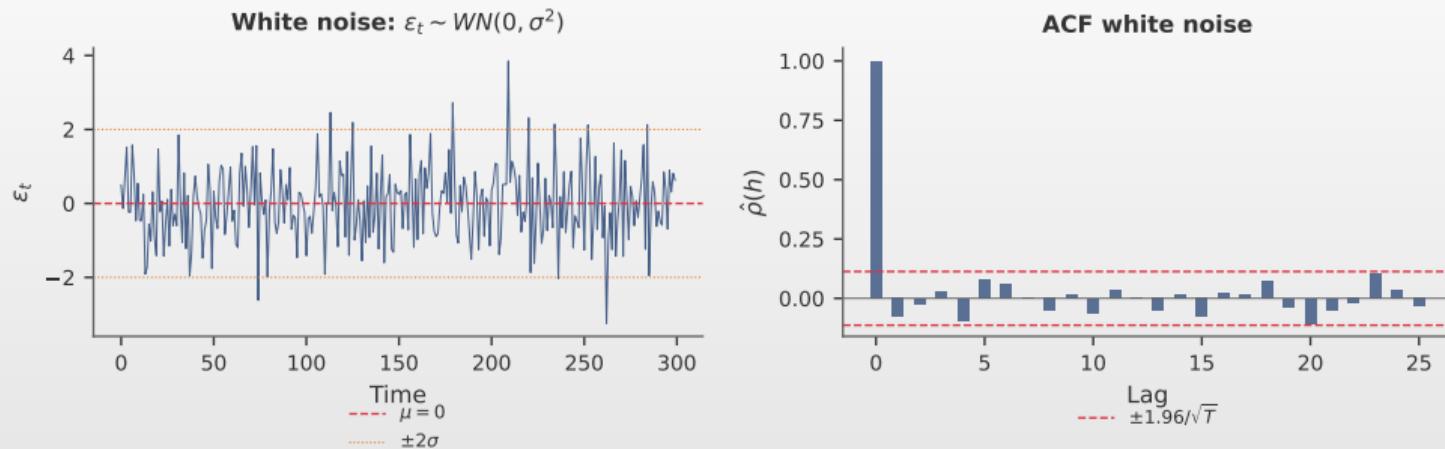
Answer: D – Normality is NOT required



- White noise:** zero mean, constant variance, uncorrelated
- Gaussian white noise:** adds normality \Rightarrow independent (not just uncorrelated)



Visual: White Noise Properties



- Left: white noise fluctuates around zero
- Right: ACF shows no autocorrelation (all values ≈ 0 after lag 0)

Q TSA_ch1_white_noise



Quiz 7: Forecast Horizon

Question

As forecast horizon h increases, what typically happens to forecast intervals?

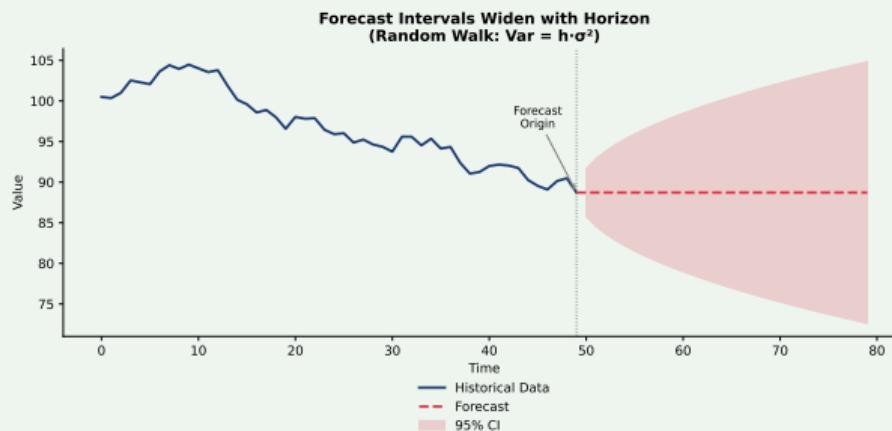
- A. They become narrower
- B. They stay the same width
- C. They become wider
- D. They disappear

Answer on next slide...



Quiz 7: Answer

Answer: C – They become wider



- Random walk: $\text{Var} = h\sigma^2$ (grows linearly)
- 95% CI: $\hat{Y}_{t+h} \pm 1.96\sqrt{h}\sigma$ (widens with \sqrt{h})



Quiz 8: Seasonality Detection

Question

The ACF shows significant spikes at lags 12, 24, and 36 for monthly data. This suggests:

- A. No seasonality
- B. Annual seasonality
- C. Weekly seasonality
- D. Random noise

Answer on next slide...



Quiz 8: Answer

Answer: B – Annual seasonality

- Pattern recognition:**
 - ▶ Lag 12: correlation with the same month last year
 - ▶ Lag 24: same month two years ago
 - ▶ Lag 36: same month three years ago
- Seasonal period:** $s = 12$ (monthly data with annual cycle)
- Common patterns:** retail sales (December), energy consumption (summer/winter), tourism



Quiz 9: MAPE Limitation

Question

MAPE (Mean Absolute Percentage Error) should NOT be used when:

- A. Comparing models on the same dataset
- B. The actual values can be zero or near zero
- C. Forecasting stock prices
- D. The data has a trend

Answer on next slide...



Quiz 9: Answer

Answer: B – When actual values can be zero or near zero

- **Formula:** $\text{MAPE} = \frac{100\%}{n} \sum_{t=1}^n \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right|$
- **Problem:** $Y_t \approx 0 \Rightarrow \text{MAPE} \rightarrow \infty$
- **Alternatives:**
 - ▶ **SMAPE:** $\frac{200\%}{n} \sum \frac{|Y_t - \hat{Y}_t|}{|Y_t| + |\hat{Y}_t|}$ (bounded 0–200%)
 - ▶ **MASE:** $\frac{1}{n} \sum \frac{|e_t|}{\frac{1}{n-1} \sum |Y_t - Y_{t-1}|}$ (scale-free)



True or False? (Set 1)

Question

Mark each statement as True (T) or False (F):

1. A time series with constant mean is always stationary. _____
2. The variance of a random walk increases linearly with time. _____
3. A stationary process can have time-varying variance. _____
4. ADF and KPSS tests have the same null hypothesis. _____
5. Lower RMSE always means better forecasts. _____
6. Autocorrelation at lag 0 is always equal to 1. _____

Answer on next slide...



True or False: Answers (Set 1)

Answers

1. Constant mean \Rightarrow stationary. **FALSE** — Also need constant variance and covariance depending only on lag.
2. Var of random walk grows linearly with time. **TRUE** — $\text{Var}(X_t) = t\sigma^2$.
3. A stationary process can have time-varying variance. **FALSE** — Weak stationarity requires $\text{Var}(X_t) = \sigma^2$ constant.
4. ADF and KPSS have the same null hypothesis. **FALSE** — ADF: H_0 = unit root. KPSS: H_0 = stationary. Opposite!
5. Lower RMSE \Rightarrow better forecasts. **FALSE** — Scale-dependent; may overfit to extreme values.
6. $\rho(0) = 1$ always. **TRUE** — $\rho(0) = \gamma(0)/\gamma(0) = 1$ by definition.



True or False? (Set 2)

Question

Mark each statement as True (T) or False (F):

1. The ACF of a stationary AR(1) process decays exponentially. _____
2. White noise is always normally distributed. _____
3. Differencing can make a non-stationary series stationary. _____
4. The PACF of a MA(1) process cuts off after lag 1. _____
5. Zero correlation between two variables implies independence. _____
6. Holt-Winters is appropriate for data with no seasonality. _____

Answer on next slide...



True or False: Answers (Set 2)

Answers

1. ACF of a stationary AR(1) decays exponentially. **TRUE** — $\rho(h) = \phi^h$, decays exponentially.
2. White noise is always normally distributed. **FALSE** — Requires only zero mean, constant variance, no autocorrelation.
Gaussian = special case.
3. Differencing can make a non-stationary series stationary. **TRUE** — Removes stochastic trends (unit roots).
4. PACF of a MA(1) cuts off after lag 1. **FALSE** — ACF cuts off for MA. PACF decays exponentially.
5. Zero correlation implies independence. **FALSE** — Zero correlation = no linear relationship. Nonlinear relationships may still exist.
6. Holt-Winters is appropriate for data with no seasonality. **FALSE** — Without seasonality: Holt's method or SES.



Exercise 1: Autocovariance

Problem: For a stationary process with:

- $\mathbb{E}[X_t] = 5$
- $\gamma(0) = 4$ (variance)
- $\gamma(1) = 2$
- $\gamma(2) = 1$

Calculate:

- a) The autocorrelation function $\rho(0), \rho(1), \rho(2)$
- b) $\text{Cov}(X_t, X_{t-1})$
- c) $\text{Corr}(X_5, X_7)$
- d) If $X_t = 6$, what is $\mathbb{E}[X_{t+1}|X_t = 6]$ assuming AR(1)?



Exercise 1: Solution

a) Autocorrelations:

$$\rho(h) = \frac{\gamma(h)}{\gamma(0)}$$

- $\rho(0) = \gamma(0)/\gamma(0) = 1$
- $\rho(1) = \gamma(1)/\gamma(0) = 2/4 = 0.5$
- $\rho(2) = \gamma(2)/\gamma(0) = 1/4 = 0.25$

b) $\text{Cov}(X_t, X_{t-1}) = \gamma(1) = 2$ (by stationarity, lag 1 covariance)

c) $\text{Corr}(X_5, X_7) = \rho(|7 - 5|) = \rho(2) = 0.25$

d) For AR(1) with $\phi = \rho(1) = 0.5$:

$$\mathbb{E}[X_{t+1}|X_t] = \mu + \phi(X_t - \mu) = 5 + 0.5(6 - 5) = 5.5$$



Exercise 2: Random Walk Properties

Problem: Consider a random walk $X_t = X_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim WN(0, 4)$ and $X_0 = 100$.

Calculate:

- a) $\mathbb{E}[X_{10}]$
- b) $\text{Var}(X_{10})$
- c) $\text{Cov}(X_5, X_{10})$
- d) The 95% confidence interval for X_{100}
- e) If $X_5 = 108$, what is the optimal forecast for X_6 ?



Exercise 2: Solution

Random walk: $X_t = X_0 + \sum_{i=1}^t \varepsilon_i$ with $\sigma^2 = 4$

a) $\mathbb{E}[X_{10}] = X_0 = 100$ (mean stays at starting value)

b) $\text{Var}(X_{10}) = 10 \times \sigma^2 = 10 \times 4 = 40$

c) $\text{Cov}(X_5, X_{10}) = \min(5, 10) \times \sigma^2 = 5 \times 4 = 20$

d) For X_{100} :

E $[X_{100}] = 100$, Var $(X_{100}) = 400$, SD = 20

95% CI: $100 \pm 1.96 \times 20 = [60.8, 139.2]$

e) Optimal forecast: $\hat{X}_6 = X_5 = 108$

(Random walk property: the optimal forecast is the last observed value)



Python Exercise 1: Import and Visualization

Task: Import S&P 500 data and create a basic time series plot.

Starter Code

```
import yfinance as yf
import matplotlib.pyplot as plt
sp500 = yf.download('^GSPC', start='2020-01-01', end='2025-01-01')
# TODO: Plot the closing prices
# TODO: Add title and labels
# TODO: Calculate and display basic statistics
```

Questions:

1. What is the mean and standard deviation of returns?
2. Does the series appear stationary? Justify your answer.



Python Exercise 2: Decomposition

Task: Apply STL decomposition on airline passengers data.

Starter Code

```
from statsmodels.tsa.seasonal import STL  
import pandas as pd  
  
# Load airline passengers  
url = 'https://raw.githubusercontent.com/..../airline.csv'  
airline = pd.read_csv(url, parse_dates=['Month'],  
                      index_col='Month')  
  
# TODO: Apply STL decomposition with period=12  
# TODO: Plot all components  
# TODO: What percentage of variance is explained by trend?
```

Hint: `STL(data, period=12).fit()`



Python Exercise 3: Exponential Smoothing

Task: Compare SES, Holt, and Holt-Winters methods on real data.

Starter Code

```
from statsmodels.tsa.holtwinters import (SimpleExpSmoothing,  
                                         ExponentialSmoothing)  
  
# Split data: 80% train, 20% test  
train = airline[:'1958']  
test = airline['1959':]  
  
# TODO: Fit SES, Holt, and Holt-Winters  
# TODO: Generate forecasts for the test period  
# TODO: Calculate RMSE for each method  
# TODO: Which method performs best? Why?
```



Python Exercise 4: Stationarity Testing

Task: Test for stationarity using ADF and KPSS tests.

Starter Code

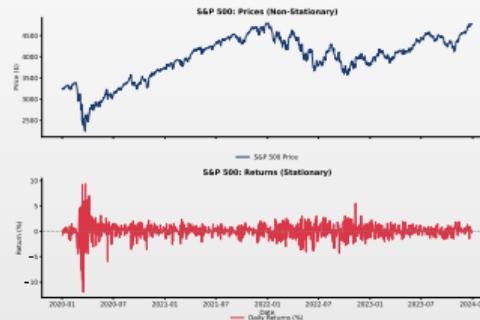
```
from statsmodels.tsa.stattools import adfuller, kpss
prices = sp500['Close']
returns = prices.pct_change().dropna()
# TODO: Run ADF test on prices and returns
# TODO: Run KPSS test on prices and returns
# TODO: Interpret the results
# ADF: adfuller(series) | KPSS: kpss(series, regression='c')
```

Questions:

1. Are prices stationary? Are returns stationary?
2. Do ADF and KPSS results agree?



Case Study: S&P 500 Index

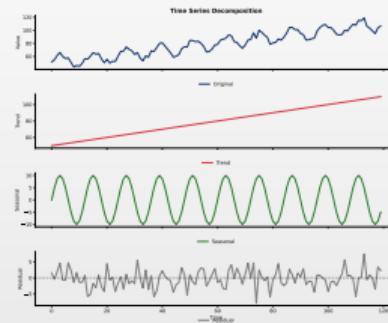


Observations

- **Top:** S&P 500 prices — clear upward trend (non-stationary)
- **Bottom:** Returns $r_t = \log(P_t/P_{t-1})$ — stationary, fluctuations around zero mean
- Volatility clustering visible



Time Series Decomposition: Real Example

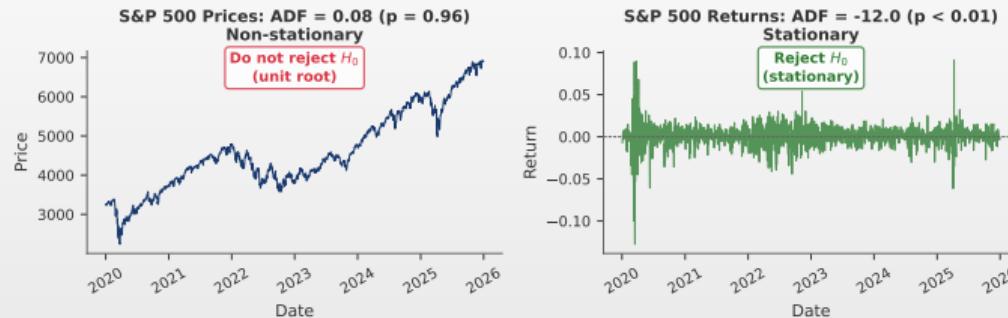


Observations

- **Trend:** Long-term direction
- **Seasonality:** Regular periodic patterns
- **Residual:** What remains after removing trend and seasonality
- Decomposition \Rightarrow understanding structure before modeling



Stationarity Testing: ADF Results



Observations

- ◻ ADF compares test statistic to critical values
- ◻ Test stat. $<$ crit. value \Rightarrow reject H_0 (stationary)
- ◻ **Prices:** ADF $> -2.86 \Rightarrow$ non-stationary
- ◻ **Returns:** ADF $< -2.86 \Rightarrow$ stationary



Stationarity Comparison: Prices vs Returns

ADF Test Results

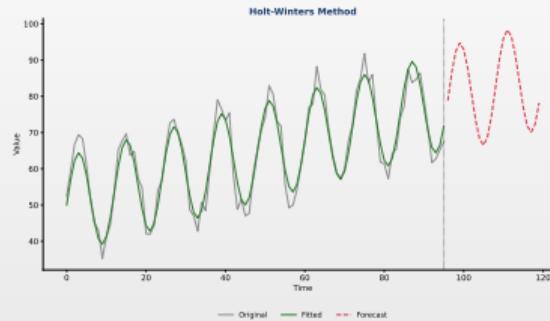
| Series | ADF Statistic | p-value | Conclusion |
|-----------------|---------------|---------|----------------|
| S&P 500 Prices | -0.82 | 0.812 | Non-stationary |
| S&P 500 Returns | -45.3 | < 0.001 | Stationary |

Key Insight

- Financial prices are typically $I(1)$ – integrated of order 1
- Taking first differences (returns) achieves stationarity
- This is why we model **returns**, not prices!



Exponential Smoothing Forecast



Observations

- Holt-Winters: data with trend + seasonality
- α, β, γ control adaptiveness
- Captures trend continuation + seasonal pattern
- Simple yet effective for business applications



Discussion Question 1

Scenario

- Monthly sales data for a retail company
- Clear seasonality (high sales in December) + upward trend
- Seasonal peaks have been getting larger over time

Discuss:

1. Additive or multiplicative decomposition? Justify your answer.
2. Which exponential smoothing method would you recommend?
3. How would you evaluate forecast performance?
4. What are the risks of choosing the wrong decomposition?



Discussion Question 2

Scenario

A colleague claims:

- "I ran the ADF test on my stock price data"
- "I got a p-value of 0.65"
- "So my data is stationary and I can fit an ARMA model directly"

Discuss:

1. Where is the reasoning flawed?
2. What are the hypotheses of the ADF test?
3. What steps should be taken before estimating an ARMA model?
4. What role does the KPSS test play in clarifying the situation?



Discussion Question 3

Scenario

- You build a forecasting model: MAPE of 2%
- The manager is impressed and wants immediate deployment

Discuss:

1. What checks are needed before deployment?
2. Is the train/validation/test split correct?
3. Is there a risk of data leakage?
4. What additional checks are needed?
5. How would you monitor model performance in production?



Discussion Question 4

Scenario

Daily electricity demand forecast for the next week:

- Strong daily patterns (peaks at 6pm)
- Weekly patterns (lower on weekends)
- Annual patterns (higher in summer/winter)

Discuss:

1. How would you handle multiple seasonality?
2. Is Holt-Winters appropriate? Justify your answer.
3. What is the advantage of Fourier terms in this case?
4. How would you set up train/validation/test samples?



AI Exercise: Critical Thinking

Prompt to test in ChatGPT / Claude / Copilot

"Using yfinance, download the BTC-USD price series. Is the price series stationary? If not, transform it so that it becomes stationary. Can I use the returns series to predict future prices? Show me."

Exercise:

1. Run the prompt in an LLM of your choice and critically analyze the response.
2. Does the AI correctly interpret the ADF and KPSS null hypotheses?
3. Does it stop at first differencing or over-difference? Why does this matter?
4. Check the conclusion about returns — does "white noise" mean "no structure"?
5. Does it account for volatility clustering (ARCH effects) in the returns?

Warning: AI-generated code may run without errors and look professional. *That does not mean it is correct.*



Conclusions

- **Time series are dependent**
 - ▶ Not i.i.d. like cross-sectional data — autocorrelation is key
- **Choose decomposition wisely**
 - ▶ Multiplicative when seasonal amplitude grows with the level
- **Understand smoothing parameters**
 - ▶ High α = reactive, low α = smooth
- **Test for stationarity**
 - ▶ Use both ADF and KPSS together
- **Proper evaluation**
 - ▶ Never tune on the test set!
- **Random walk is non-stationary**
 - ▶ Variance grows with time: $\text{Var}(X_t) = t\sigma^2$

Next Seminar

- ARMA/ARIMA model identification, estimation, and forecasting



Data Sources and Software

Software Tools:

- statsmodels – Statistical models for Python
- pandas – Data manipulation and time series
- matplotlib, seaborn – Visualization
- scipy – Statistical functions

Data and Examples:

- Simulated time series for illustrations
- Examples based on Hyndman & Athanasopoulos (2021)



References I

Fundamental textbooks

- Hyndman, R.J., & Athanasopoulos, G. (2021). *Forecasting: Principles and Practice*, 3rd ed., OTexts.
- Shumway, R.H., & Stoffer, D.S. (2017). *Time Series Analysis and Its Applications*, 4th ed., Springer.
- Brockwell, P.J., & Davis, R.A. (2016). *Introduction to Time Series and Forecasting*, 3rd ed., Springer.

Financial time series

- Tsay, R.S. (2010). *Analysis of Financial Time Series*, 3rd ed., Wiley.
- Franke, J., Härdle, W.K., & Hafner, C.M. (2019). *Statistics of Financial Markets*, 4th ed., Springer.



References II

Modern approaches and Machine Learning

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- Petropoulos, F., et al. (2022). *Forecasting: Theory and Practice*, International Journal of Forecasting.
- Makridakis, S., Spiliotis, E., & Assimakopoulos, V. (2020). The M4 Competition, International Journal of Forecasting.

Online resources and code

- **Quantlet:** <https://quantlet.com> — Code repository for statistics
- **Quantinar:** <https://quantinar.com> — Quantitative methods learning platform
- **GitHub TSA:** https://github.com/QuantLet/TSA/tree/main/TSA_ch1 — Python code for this seminar



Thank You!

Questions?

Seminar materials are available at: <https://danpele.github.io/Time-Series-Analysis/>

