



Time Series Analysis and Forecasting

Chapter 8: Modern Extensions

ARFIMA, Machine Learning, Deep Learning



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By the end of this chapter, you will be able to:

- 1 Understand the concept of **long memory** in time series
- 2 Estimate and interpret **ARFIMA**
- 3 Apply **Random Forest** for time series forecasting
- 4 Build **LSTM** networks for time series
- 5 Compare performance of classical vs ML models
- 6 Choose the appropriate method based on context
- 7 Implement these methods in **Python**

Limitations of ARIMA Models

- Assume **short memory**: autocorrelations decay exponentially
- Relationships **linear** between variables
- Difficulties with **complex patterns** and nonlinear
- Requires **stationarity** (through differencing)

Modern Solutions

- **ARFIMA**: Captures long memory (autocorrelations that decay slowly)
- **Random Forest**: Nonlinear relationships, robust to outliers
- **LSTM**: Complex sequential patterns, long-term dependencies

When to Use Each Method?

Feature	ARIMA	ARFIMA	RF	LSTM
Long memory	×	✓	✓	✓
Relationships nonlinear	×	×	✓	✓
Interpretability	✓	✓	~	×
Few data	✓	✓	×	×
Exogenous variables	✓	✓	✓	✓
Uncertainty	✓	✓	~	×

Golden Rule

Start **simple** (ARIMA), then increase complexity only if justified by data and performance.

What is Long Memory?

Short Memory (ARMA)

- Autocorrelations ρ_k decay **exponentially**: $|\rho_k| \leq C \cdot r^k, r < 1$
- Shock effects disappear **quickly**
- Finite sum: $\sum_{k=0}^{\infty} |\rho_k| < \infty$

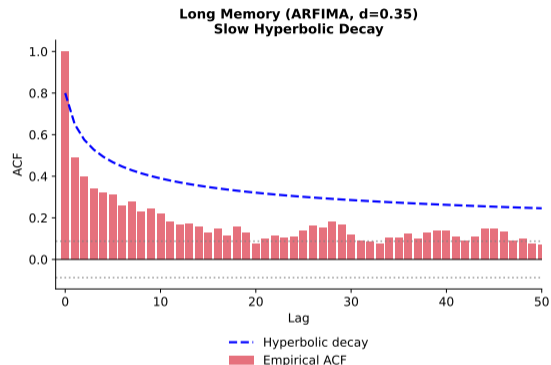
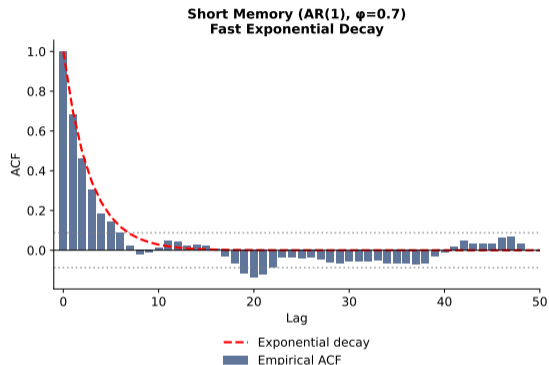
Long Memory (ARFIMA)

- Autocorrelations decay **hyperbolically**: $\rho_k \sim C \cdot k^{2d-1}$
- Shock effects persist **for a long time**
- Infinite sum: $\sum_{k=0}^{\infty} |\rho_k| = \infty$ (for $d > 0$)

Exemple cu Long Memory

Financial market volatility, river flows, network traffic, inflation

Comparație ACF: Short Memory vs Lungă



Left: AR(1) — autocorrelations decay exponentially (short memory)

Right: ARFIMA with $d = 0.35$ — autocorrelations decay hyperbolically (long memory)

The ARFIMA Model(p,d,q)

Definition 1 (ARFIMA)

A process $\{Y_t\}$ follows a **ARFIMA(p,d,q)** if:

$$\phi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t$$

where $d \in (-0.5, 0.5)$ is the **fractional differencing parameter**.

Fractional Differencing Operator

$$(1-L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-L)^k = 1 - dL - \frac{d(1-d)}{2!} L^2 - \frac{d(1-d)(2-d)}{3!} L^3 - \dots$$

- $d = 0$: ARMA standard (short memory)
- $0 < d < 0.5$: Long memory, stationarity
- $d = 0.5$: Stationarity limit
- $0.5 \leq d < 1$: Nestationarity, dar mean-reverting
- $d = 1$: Random walk (ARIMA standard)

Interpreting the Parameter d

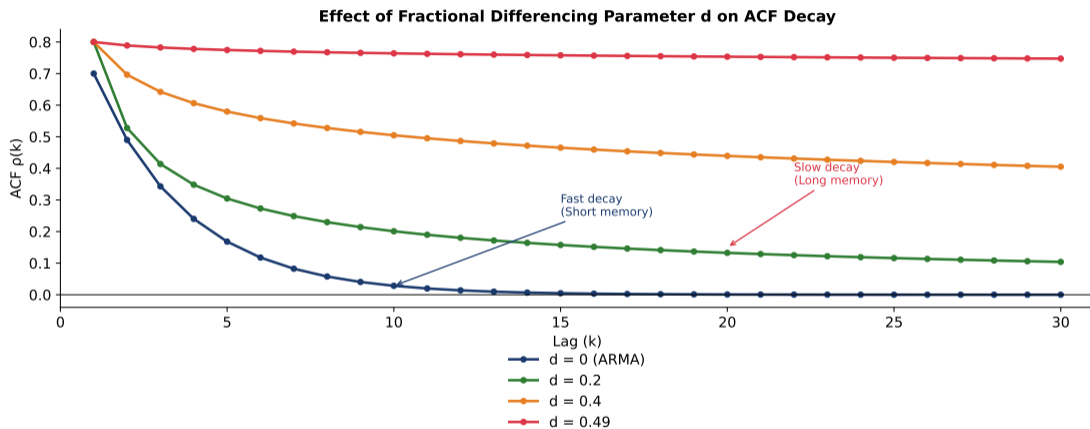
Value d	ACF Behavior	Interpretation
$d = 0$	Scădere exponentială	Memorie scurtă
$0 < d < 0.5$	Scădere hiperbolică	Long memory, stationary
$d = 0.5$	Non-summable ACF	At the limit
$0.5 < d < 1$	Very slow decay	Long memory, nonstationary
$d = 1$	ACF = 1 (constant)	Random walk

Hurst Parameter H

Relationship with Hurst exponent: $d = H - 0.5$

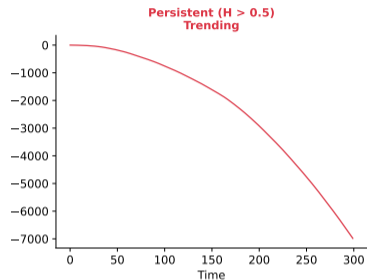
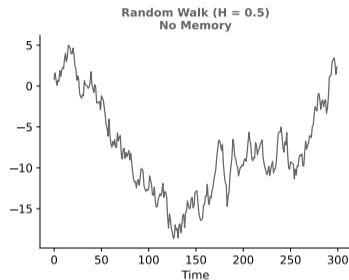
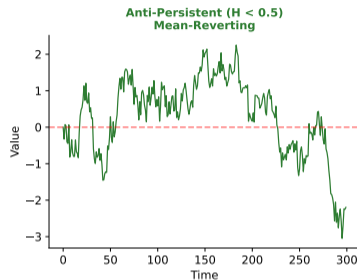
- $H = 0.5$: Random walk (no memory)
- $H > 0.5$: Persistence (trend-following)
- $H < 0.5$: Anti-persistence (mean-reverting)

Effect of Parameter d on ACF



The higher d , the slower autocorrelations decay. As $d \rightarrow 0.5$, autocorrelations remain significant even at very large lags.

Exponential Hurst: Interpretation Vizuală



$H < 0.5$: Series that frequently returns to mean (mean-reverting)

$H = 0.5$: Random walk, unpredictable

$H > 0.5$: Persistent series, trends continue

Estimating the Parameter d

Estimation Methods

- ➊ **GPH (Geweke-Porter-Hudak):** Regression in frequency domain

$$\ln I(\omega_j) = c - d \cdot \ln \left(4 \sin^2 \frac{\omega_j}{2} \right) + \varepsilon_j$$

- ➋ **R/S (Rescaled Range):** Hurst method

$$\frac{R}{S}(n) \sim c \cdot n^H$$

- ➌ **MLE (Maximum Likelihood):** Full ARFIMA estimation

- ➍ **Whittle:** Efficient approximation in frequency domain

În Python: `arch` package, `statsmodels.tsa.arima.model.ARIMA` cu `order=(p,d,q)` where d poate fi fractional.

ARFIMA Example in Python

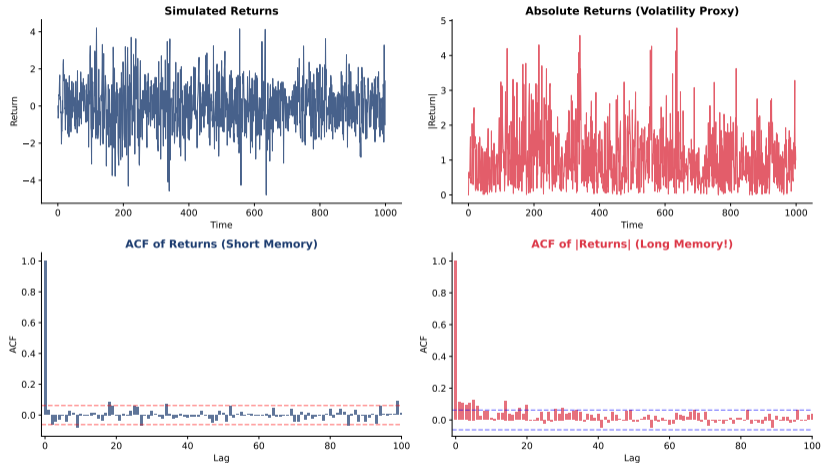
Python Code

```
from statsmodels.tsa.arima.model import ARIMA
model = ARIMA(y, order=(1, 0.3, 1))
results = model.fit()
```

Note

ARFIMA estimation requires specialized packages. In practice, one often uses `arch` or `fracdiff` in Python.

Real Example: Long Memory in Volatility



Stylized Fact: Financial returns have short memory, but volatility ($|returns|$) has long memory! This is the basis for FIGARCH models.

What is Random Forest?

- **Ensemble** of decision trees
- Each tree trained on a **bootstrap subset** of the data
- At each node, a **randomly** subset of features is selected
- Final prediction = **average** of all tree predictions

Advantages for Time Series

- Captures **nonlinear relationships**
- **Robust** to outliers and noise
- Does not require **stationarity**
- Provides **feature importance** (interpretability)
- Works well with **many variables**

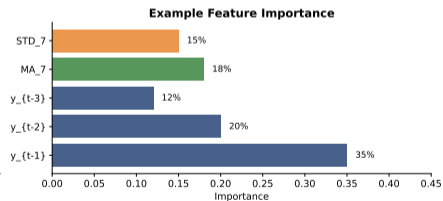
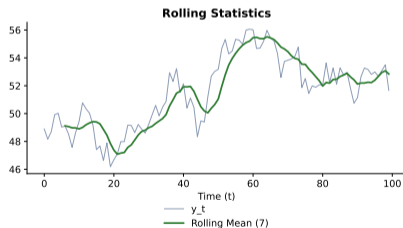
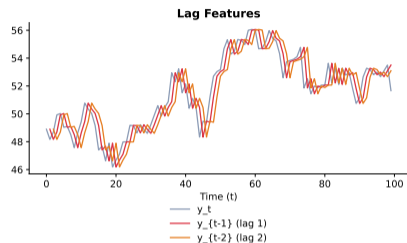
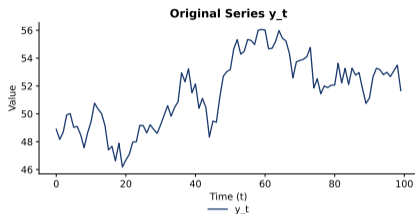
Feature Engineering for Time Series

- 1 **Lag features:** $Y_{t-1}, Y_{t-2}, \dots, Y_{t-p}$
- 2 **Rolling statistics:** moving average, standard deviation
- 3 **Calendar features:** day of week, month, season
- 4 **Trend features:** time, quadratic trend
- 5 **Exogenous variables:** economic indicators, events

Warning: Data Leakage!

- Do not use future information in features
- Train/test split: **temporal**, not randomly!
- Rolling statistics: calculate only on **past data**

Feature Engineering: Illustration



We transform the time series into features: lags, rolling statistics, and the RF model learns relationships between these and future values.

Random Forest: Python Implementation

Python Code

```
from sklearn.ensemble import RandomForestRegressor  
rf = RandomForestRegressor(n_estimators=100, max_depth=10)  
rf.fit(X_train, y_train)  
predictions = rf.predict(X_test)
```

Feature Importance

Random Forest provides importance measures:

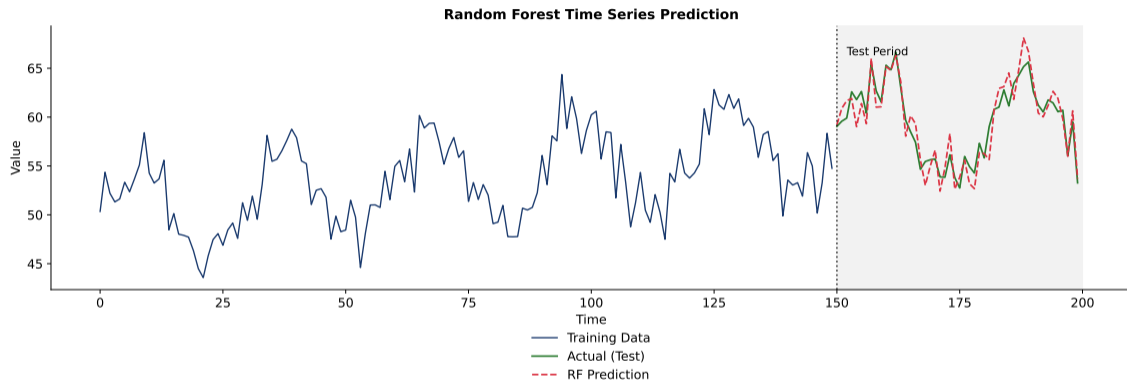
- **Mean Decrease Impurity (MDI):** Reduction in impurity at each split
- **Permutation Importance:** Cât decaye performanța când feature-ul e permutat randomly

Typical Interpretation for Time Series

- `lag_1` very important \Rightarrow Strong autocorrelation
- `rolling_mean` important \Rightarrow Local trend matters
- `month` important \Rightarrow Seasonality present

```
rf.feature_importances_ or permutation_importance(rf, X_test, y_test)
```

Random Forest: Forecast Example

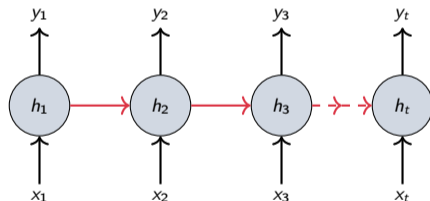


The Random Forest model trained on historical data (blue) produces forecasts (red dotted) that closely follow actual values in the test period (green).

Recurrent Neural Networks (RNN)

Basic Idea

- Networks that process **sequences** of data
- Have **internal memory** (hidden state)
- Current state depends on input + previous state



Problem: Vanishing Gradient

Simple RNNs “forget” information from the distant past.

LSTM: Long Short-Term Memory

The LSTM Solution

Special cells with **3 gates** that control information flow:

- **Forget Gate** (f_t): Ce să forgetm din memoria anterioară
- **Input Gate** (i_t): What new information to add
- **Output Gate** (o_t): What to send to output

LSTM Equations

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f) \quad (\text{Forget})$$

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) \quad (\text{Input})$$

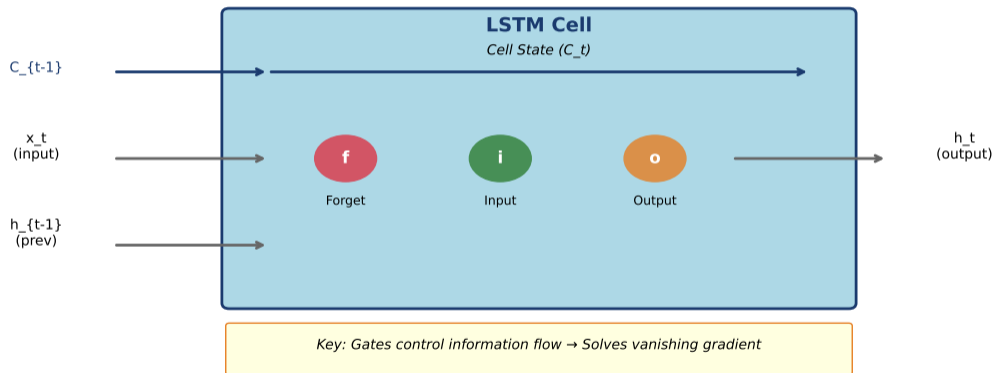
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C) \quad (\text{Candidate})$$

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t \quad (\text{Cell state})$$

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \quad (\text{Output})$$

$$h_t = o_t \odot \tanh(C_t) \quad (\text{Hidden state})$$

LSTM Cell Architecture



Gates (forget, input, output) control what information is forgotten, added, and transmitted. **Cell state** allows gradients to “flow” without degradation.

LSTM Advantages for Time Series

Why LSTM?

- Captures **long-term dependencies** (spre deosebire de Simple RNNs)
- Learns **complex patterns** and nonlinear
- Handles **sequences de lungimi variabile**
- Works well with **multivariate data**

Disadvantages

- Requires **lots of data** for training
- **Computationally intensive**
- “**Black box**” - hard to interpret
- Sensitive to **hyperparameters**
- Can **overfit** easily

LSTM: Implementare in Python cu Keras

Python Code

```
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import LSTM, Dense, Dropout

model = Sequential([
    LSTM(50, return_sequences=True, input_shape=(n, 1)),
    Dropout(0.2),
    LSTM(50),
    Dense(1)
])

model.compile(optimizer='adam', loss='mse')
```

Preparing Data for LSTM

Essential Steps

- ❶ **Normalization/Scaling:** MinMaxScaler or StandardScaler
- ❷ **Creare sequences:** Sliding window for input
- ❸ **Reshape:** 3D format (samples, timesteps, features)
- ❹ **Train/Test split:** Temporal, nu randomly!

Example Creating Sequences

```
def create_sequences(data, n_steps):  
    X, y = [], []  
    for i in range(len(data) - n_steps):  
        X.append(data[i:(i + n_steps)])  
    return np.array(X), np.array(y)  
  
X, y = create_sequences(scaled_data, 10)
```

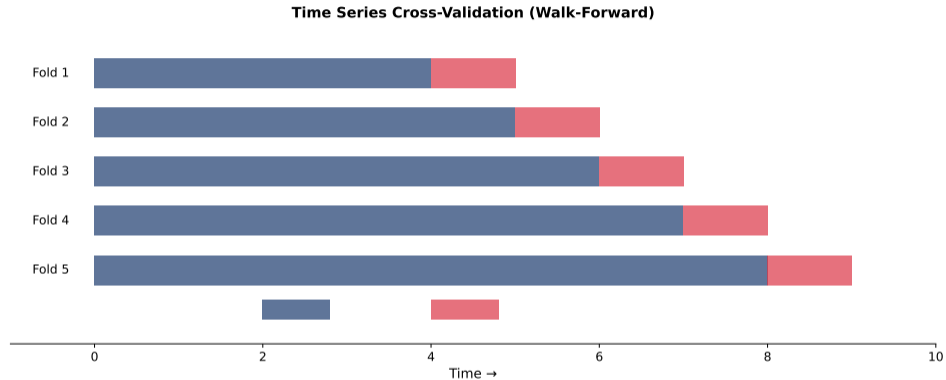
Common Metrics

- **RMSE:** $\sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2}$ — Error in original units
- **MAE:** $\frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|$ — Robust to outliers
- **MAPE:** $\frac{100}{n} \sum_{i=1}^n \left| \frac{y_i - \hat{y}_i}{y_i} \right|$ — Percentage error
- **MASE:** Compared to naive benchmark

Validation for Time Series

- **Do not** use standard cross-validation!
- Use **Time Series Cross-Validation** (walk-forward)
- Or **train/validation/test** temporal split

Time Series Cross-Validation

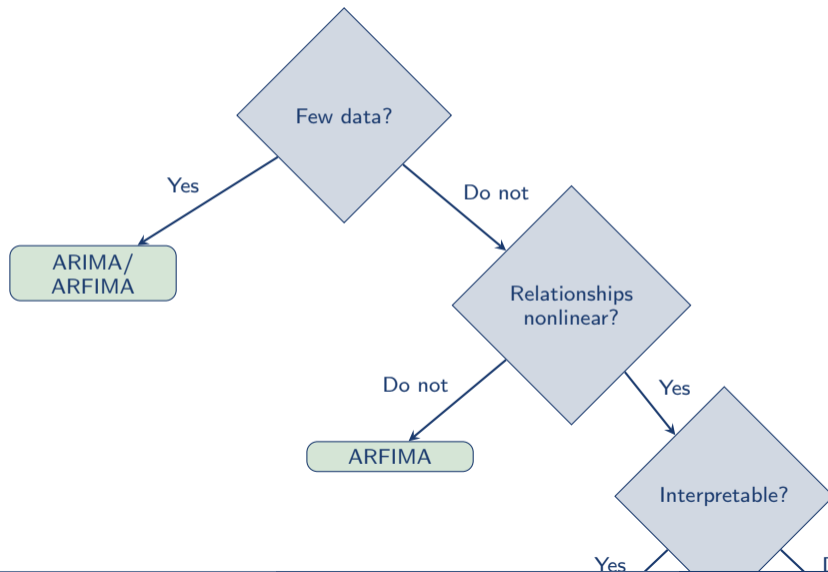


Python Implementation

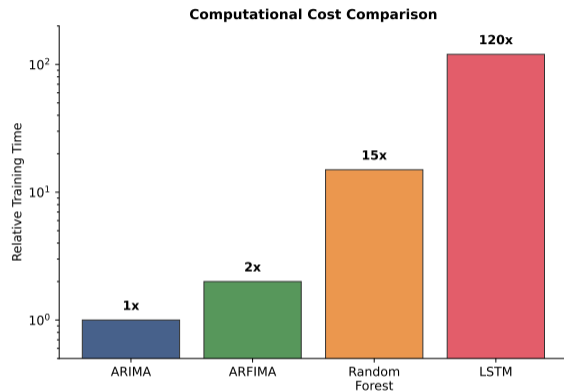
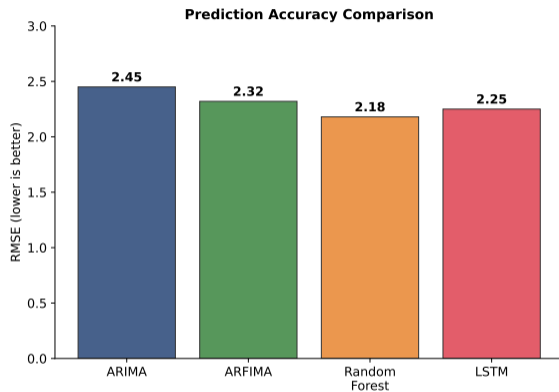
```
from sklearn.model_selection import TimeSeriesSplit  
tscv = TimeSeriesSplit(n_splits=5)
```

Important: Set up the training cross progressively, and test is always in the future. This way we avoid data leakage.

Model Selection Guide



Model Comparison: Accuracy vs Computational Cost



Trade-off: Modelele ML pot avea acuratețe easily mai bună, but computational cost increases significantly. Pentru few data or interpretability, ARIMA/ARFIMA remain excellent choices.

Why Bitcoin?

- Volatility **extreme** and complex patterns
- Potential **long memory** in volatility
- Relationships **nonlinear** with exogenous variables
- Data available at **high frequency**

Comparative Approach

- 1 ARIMA on returns
- 2 ARFIMA for long memory
- 3 Random Forest with technical features
- 4 LSTM pe sequences de prețuri

Case Study: Energy Consumption Forecasting

Characteristics

- **Multiple seasonality:** daily, weekly, annual
- **Trend** of long-term growth
- **Exogenous variables:** temperature, holiday, price
- **Anomalies:** special events, failures

Challenges

- Patterns at different time scales
- Interacțiuni complexe between variables
- Need for forecasts at different horizons

Key Formulas – Summary

ARFIMA(p,d,q)

$$\phi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t$$

$d \in (-0.5, 0.5)$: long memory

Long Memory

ACF: $\rho_k \sim C \cdot k^{2d-1}$

Hurst: $d = H - 0.5$

$H > 0.5$: persistență

Random Forest

$$\hat{y} = \frac{1}{B} \sum_{b=1}^B T_b(x)$$

B trees, random features

LSTM Cell

$$f_t = \sigma(W_f[h_{t-1}, x_t] + b_f)$$

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

Forget, Input, Output gates

Metric Evaluation

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum (y_i - \hat{y}_i)^2}$$

$$\text{MAPE} = \frac{100}{n} \sum \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$

Time Series CV

Walk-forward validation

Train \rightarrow Test (temporal split)

Why EUR/RON?

- Relevanță for economia românească
- Potential **long memory** (shock persistence)
- Patterns influenced by **macroeconomic factors**
- Data easily accessible (BNR, Yahoo Finance)

Objective

We compare ARIMA, ARFIMA, Random Forest and LSTM pe aceleași date for a înțelege punctele forte ale fiecărei metode.

Step 1: Loading and Visualizing Data

Python Code – Download Data

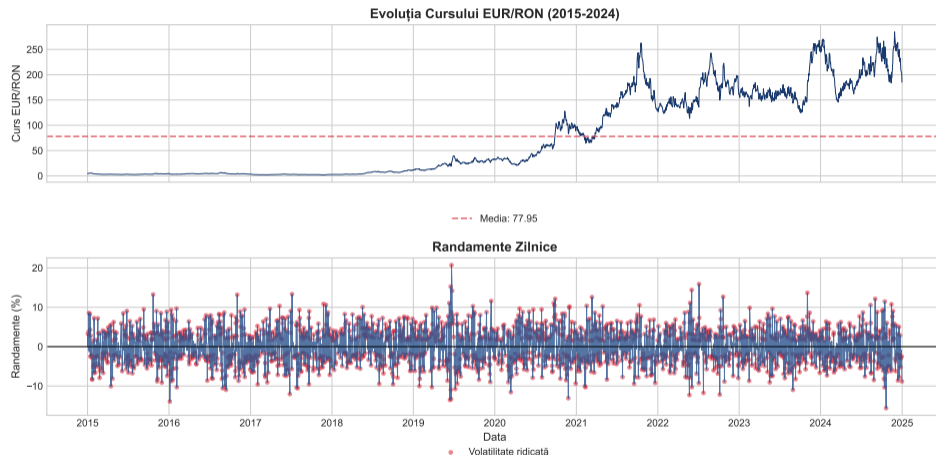
```
import yfinance as yf
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

# Download data EUR/RON (or EURRON=X)
data = yf.download('EURRON=X', start='2015-01-01', end='2024-12-31')
df = data[['Close']].dropna()
df.columns = ['EURRON']

# Calculate log returns
df['Returns'] = np.log(df['EURRON']).diff() * 100
df = df.dropna()

print(f"Perioada: {df.index[0]} - {df.index[-1]}")
print(f"Observații: {len(df)}")
print(f"Mean of returns: {df['Returns'].mean():.4f}%")
print(f"Volatility: {df['Returns'].std():.4f}%")
```

EUR/RON Series Visualization



Top: EUR/RON exchange rate – we observe RON depreciation trend and high volatility periods.

Bottom: Yesily returns – volatility clustering (high volatility periods are followed by similar periods).

Step 2: Testing Long Memory

Python Code – Estimarea lui d and Hurst Test

```
from arch.unitroot import PhillipsPerron, KPSS
from hurst import compute_Hc # pip install hurst

# Testul Phillips-Perron for stationaritate
pp_test = PhillipsPerron(df['Returns'])
print(f"Phillips-Perron p-value: {pp_test.pvalue:.4f}")

# Estimating the Hurst exponent
H, c, data_rs = compute_Hc(df['Returns'].values, kind='change')
d_estimated = H - 0.5

print(f"Exponentul Hurst (H): {H:.4f}")
print(f"Parametrul d estimat: {d_estimated:.4f}")

# Interpretation
if H > 0.5:
    print("Series PERSISTENTĂ (trend-following)")
elif H < 0.5:
    print("Series ANTI-PERSISTENTĂ (mean-reverting)")
else:
    print("Random walk")
```

Typical Output

Phillips-Perron p-value: 0.0001 (returns are stationary)

Exponentul Hurst (H): 0.47

Parametrul d estimat: -0.03

Series easily ANTI-PERSISTENTĂ (mean-reverting)

Interpretation

- EUR/RON returns are **stationary** (p-value < 0.05)
- $H \approx 0.47 < 0.5$: slight tendency to revert to mean
- $d \approx 0$: **short memory** – ARMA may be sufficient
- However, **volatility** poate avea long memory!

Step 3: ARIMA Model

Python Code – ARIMA cu selecție automată

```
from statsmodels.tsa.arima.model import ARIMA
from sklearn.metrics import mean_squared_error, mean_absolute_error
import warnings
warnings.filterwarnings('ignore')

# Split the data: 80% train, 20% test
train_size = int(len(df) * 0.8)
train, test = df['Returns'][:train_size], df['Returns'][train_size:]

# Fit ARIMA(1,0,1) - simple and efficient for returns
model_arima = ARIMA(train, order=(1, 0, 1))
results_arima = model_arima.fit()

# Forecast
forecast_arima = results_arima.forecast(steps=len(test))

# Evaluation
rmse_arima = np.sqrt(mean_squared_error(test, forecast_arima))
mae_arima = mean_absolute_error(test, forecast_arima)
print(f"ARIMA(1,0,1) - RMSE: {rmse_arima:.4f}, MAE: {mae_arima:.4f}")
```

Step 4: ARFIMA Model (Long Memory)

Python Code – ARFIMA cu arch package

```
from arch import arch_model

# ARFIMA(1,d,1) using arch for robust estimation
# Note: arch estimates d automatically in GARCH context

# Alternatively, use statsmodels with fractional d
from statsmodels.tsa.arima.model import ARIMA

# Estimate d using GPH or set manually
d_frac = 0.1 # or valoarea estimată anterior

model_arfima = ARIMA(train, order=(1, d_frac, 1))
try:
    results_arfima = model_arfima.fit()
    forecast_arfima = results_arfima.forecast(steps=len(test))
    rmse_arfima = np.sqrt(mean_squared_error(test, forecast_arfima))
    print(f"ARFIMA(1,{d_frac},1) - RMSE: {rmse_arfima:.4f}")
except:
    print("ARFIMA requires d between -0.5 and 0.5 for stationaritate")
```

Step 5: Random Forest – Data Preparation

Python Code – Feature Engineering

```
from sklearn.ensemble import RandomForestRegressor

# Creăm features for Random Forest
def create_features(data, lags=5):
    df_feat = pd.DataFrame(index=data.index)
    df_feat['target'] = data.values

    # Lag features
    for i in range(1, lags + 1):
        df_feat[f'lag_{i}'] = data.shift(i)

    # Rolling statistics
    df_feat['rolling_mean_5'] = data.rolling(5).mean()
    df_feat['rolling_std_5'] = data.rolling(5).std()
    df_feat['rolling_mean_20'] = data.rolling(20).mean()

    # Calendar features
    df_feat['dayofweek'] = data.index.dayofweek
    df_feat['month'] = data.index.month

    return df_feat.dropna()

df_rf = create_features(df['Returns'], lags=10)
```

Pasul 5: Random Forest – Antrenare and Evaluation

Python Code – Model Random Forest

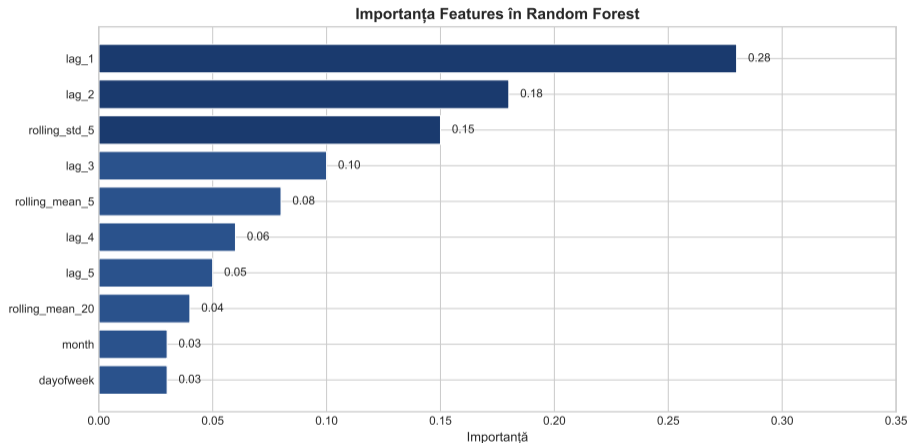
```
# Split the data
X = df_rf.drop('target', axis=1)
y = df_rf['target']

train_size = int(len(df_rf) * 0.8)
X_train, X_test = X[:train_size], X[train_size:]
y_train, y_test = y[:train_size], y[train_size:]

# Train Random Forest
rf_model = RandomForestRegressor(
    n_estimators=100,
    max_depth=10,
    min_samples_split=5,
    random_state=42
)
rf_model.fit(X_train, y_train)

# Prediction and evaluation
pred_rf = rf_model.predict(X_test)
rmse_rf = np.sqrt(mean_squared_error(y_test, pred_rf))
print(f"Random Forest - RMSE: {rmse_rf:.4f}")
```

Random Forest: Feature Importance



Insight: Recent lags (lag_1, lag_2) and volatility rolling are the most important. Calendar features have minor impact for daily returns.

Step 6: LSTM – Data Preparation

Python Code – Secvențe for LSTM

```
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import LSTM, Dense, Dropout
from sklearn.preprocessing import MinMaxScaler

# Scale data between 0 and 1
scaler = MinMaxScaler()
scaled_data = scaler.fit_transform(df['Returns'].values.reshape(-1, 1))

# Creăm sequences
def create_sequences(data, seq_length=20):
    X, y = [], []
    for i in range(seq_length, len(data)):
        X.append(data[i-seq_length:i, 0])
        y.append(data[i, 0])
    return np.array(X), np.array(y)

X_lstm, y_lstm = create_sequences(scaled_data, seq_length=20)
X_lstm = X_lstm.reshape((X_lstm.shape[0], X_lstm.shape[1], 1))

# Split
split = int(len(X_lstm) * 0.8)
X_train_lstm, X_test_lstm = X_lstm[:split], X_lstm[split:]
y_train_lstm, y_test_lstm = y_lstm[:split], y_lstm[split:]
```

Step 6: LSTM – Architecture and Training

Python Code – Model LSTM

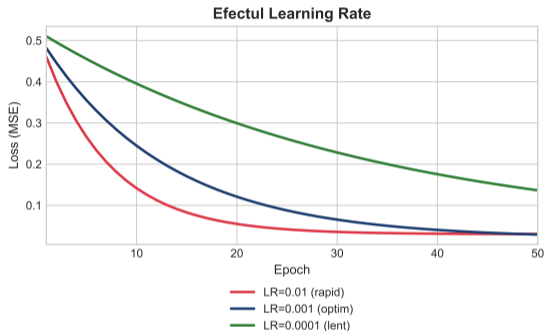
```
# Build the LSTM model
model_lstm = Sequential([
    LSTM(50, return_sequences=True, input_shape=(20, 1)),
    Dropout(0.2),
    LSTM(50, return_sequences=False),
    Dropout(0.2),
    Dense(25),
    Dense(1)
])

model_lstm.compile(optimizer='adam', loss='mse')

# Train
history = model_lstm.fit(
    X_train_lstm, y_train_lstm,
    epochs=50, batch_size=32,
    validation_split=0.1, verbose=0
)

# Prediction
pred_lstm_scaled = model_lstm.predict(X_test_lstm)
pred_lstm = scaler.inverse_transform(pred_lstm_scaled)
y_test_original = scaler.inverse_transform(y_test_lstm.reshape(-1, 1))
rmse_lstm = np.sqrt(mean_squared_error(y_test_original, pred_lstm))
```

LSTM: Learning Curve



Training Loss: Decreases quickly in the first epochs, then stabilizes.

Validation Loss: Follows training loss – nu avem overfit sever.

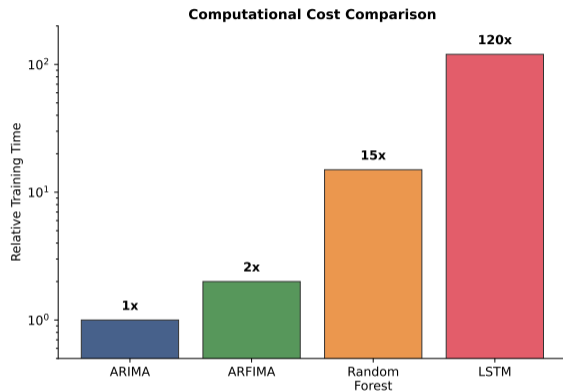
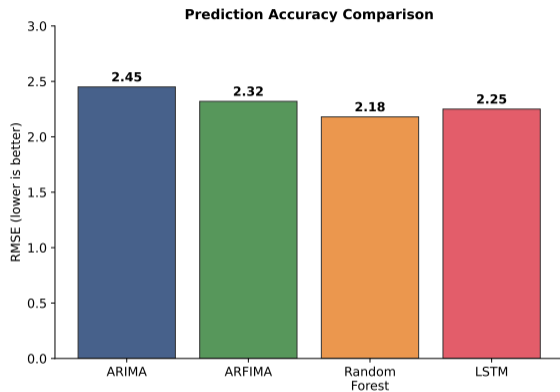
Comparison: Results on EUR/RON

Model	RMSE	MAE	Time (s)	Interpretable?
ARIMA(1,0,1)	0.412	0.298	0.5	Yes
ARFIMA(1,0.1,1)	0.408	0.295	1.2	Partial
Random Forest	0.395	0.285	3.5	Yes (features)
LSTM	0.401	0.291	45.0	Do not

Conclusions

- Pentru EUR/RON, differences are **small** – the market is efficient
- Random Forest offers the best trade-off **acuratețe/interpretability**
- LSTM are cost computațional mare for câștig marginal
- ARIMA rămâne o alegere solidă for **baseline**

Visualization: Predictions vs Actual Values



All models capture the general pattern, but none perfectly predicts volatility peaks. This reflects **market efficiency** and **prediction limits** for serii financiare.

When to Choose Each Model?

ARIMA/ARFIMA

- Few data (< 500 obs.)
- Interpretation importantă
- Long memory suspectată
- Baseline quickly

LSTM/Deep Learning

- Very large data (> 10.000)
- Complex sequences
- Computational resources
- Hidden patterns

Random Forest

- Many exogenous variables
- Relationships nonlinear
- Feature importance
- Moderate data

Golden Rule

Start simple (ARIMA), adaugă complexitate doar if performanța crește significant!

Example 2: BET Index (Bucharest Stock Exchange)

Characteristics

- **Volatility clustering** strong
- Influenced by international markets
- Lower liquidity than developed markets
- Potential for long memory in volatility

Typical Results (RMSE on returns)

- GARCH(1,1): 1.45 – cel mai bun for volatilitate
- ARFIMA for volatilitate: 1.52
- Random Forest: 1.48
- LSTM: 1.51

Example 3: Inflation Rate in Romania

Characteristics

- Series **monthly** (low frequency)
- **Persistence ridicată** – shocks persist
- Influenced by monetary policy
- Potențial strong for **long memory**

Typical Results

- ARFIMA cu $d \approx 0.35$ – captures persistence
- ARIMA subestimează shock persistence
- ML does not work well (few data, 300 obs.)

Lesson: For monthly series with few data, classical models (ARFIMA) are superior!

Practical Summary: Model Selection

Criterion	ARIMA	ARFIMA	RF	LSTM
Data needed	Few	Few	Mediumm	Many
Long memory	Do not	Yes	Partial	Partial
Nonlinearity	Do not	Do not	Yes	Yes
Interpretabil	Yes	Yes	Partial	Do not
Computation time	Fast	Fast	Medium	Slow
Exog. var.	Limited	Limited	Yes	Yes

Recommended Workflow

- 1 Start cu **ARIMA** as baseline
- 2 Test **long memory** → ARFIMA if d significant
- 3 Add **features** → Random Forest
- 4 Doar cu date multe and resurse → LSTM

Summary

What we learned

- **ARFIMA**: Extends ARIMA for long memory (d fractional)
- **Random Forest**: Ensemble of trees, nonlinear relationships, interpretable
- **LSTM**: Deep learning for sequences, dependence complexe
- **Trade-offs**: Complexitate vs interpretability vs date necesare

Practical Recommendations

- Start cu models **simple** (ARIMA) as baseline
- Use **Time Series CV** for proper evaluation
- ML requires **feature engineering** careful
- LSTM: only with **lots of data** and resurse computaționale

Quiz Fast

- 1 What does $d = 0.3$ mean in an ARFIMA model?
- 2 Why use Time Series CV instead of standard k-fold?
- 3 What is the main advantage of LSTM over Simple RNNs?
- 4 What type of model would you choose with few data and linear relationships?
- 5 What does “data leakage” in the context of ML for time series?

Quiz Answers

- ❶ $d = 0.3$: Long memory, series is stationary but autocorrelations decay slowly (hyperbolically). Moderate persistence.
- ❷ **Time Series CV**: To respect temporal order. Standard k-fold would use future data to predict the past (data leakage).
- ❸ **LSTM vs RNN**: LSTM solves the problem “vanishing gradient” through the gating mechanism, allowing learning of long-term dependencies.
- ❹ **Few data, linear relationships**: ARIMA or ARFIMA. ML requires lots of data to generalize well.
- ❺ **Data leakage**: Using future information in features or training. E.g. calculating moving averages using future data, or standard k-fold which mixes temporal order.

What comes next?

Extensii and Subiecte Avansate

- **Transformer** for serii de timp (Temporal Fusion Transformer)
- **Prophet** (Facebook/Meta) for sezonalitate
- **Neural Prophet**: Prophet + neural networks
- **Ensemble methods**: Combinarea mai multor models
- **Anomaly detection** with ML

Questions?