Time Series Analysis and Forecasting

Chapter 10: Comprehensive Review

Complete Analysis with Real Data

Outline

Course Overview: Methods Covered

Classical Methods

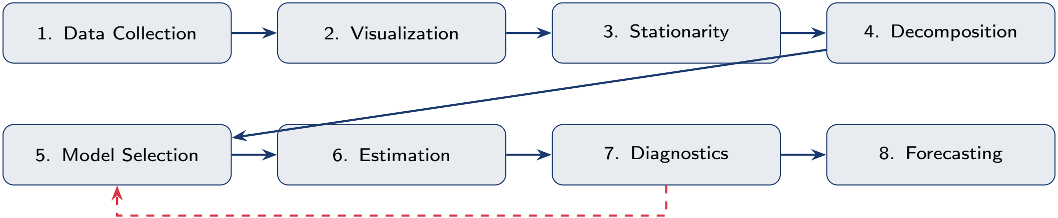
- Ch 1: Time Series Fundamentals
- Ch 2: ARMA Models
- Ch 3: ARIMA Models
- Ch 4: SARIMA Models
- Ch 5: GARCH Models

Advanced Methods

- Ch 6: VAR & Granger Causality
- Ch 7: Cointegration & VECM
- Ch 8: Modern Extensions
- Ch 9: Prophet & TBATS

Today: Apply ALL to Real Data!

The Complete Analysis Workflow



Key Principle

Model diagnostics may require returning to model selection (iterative process)

Real Datasets for This Chapter

Bitcoin

- Daily 2019-2024
- Volatility clustering
- ARIMA + GARCH

Sunspots

- Yearly 1900-2023
- 11-year cycle
- Fourier terms

- Monthly 2010-2023
- COVID-19 shock
- Prophet

Economic VAR

- Quarterly 2000-2023
- GDP, Inflation, etc.
- Multivariate VAR

Bitcoin: Data Overview

- **Data:** Bitcoin daily prices and log returns (2019-2024)
- **Key events:** COVID crash, 2021 bull run, crypto winter 2022

Step 1: Stationarity Testing

Augmented Dickey-Fuller Test

- H_0: Unit root (non-stationary)
- H_1: Stationary

Results on Bitcoin:

Series	ADF Statistic	p-value
Prices	-0.87	0.79
Log Returns	-42.1	< 0.001

$\Rightarrow$ Prices: non-stationary (random walk)
$\Rightarrow$ Returns: stationary

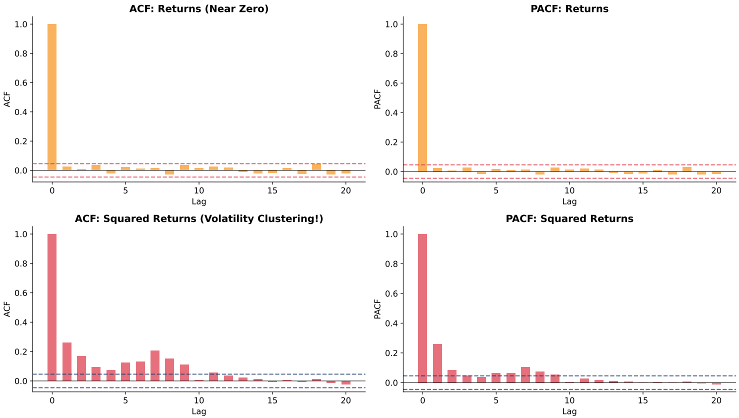
KPSS Test (Confirmation)

- H_0: Stationary
- H_1: Unit root

Prices: KPSS $= 5.83$**
Returns: KPSS $= 0.12$

Both tests confirm: use log returns!

Step 2: ACF/PACF Analysis of Returns



- **Returns:** Near white noise (weak linear dependence)
- **Squared returns:** Strong persistence $\Rightarrow$ volatility clustering
- **Implication:** GARCH model essential for Bitcoin!

Step 3: ARIMA Model for Returns

Model Selection using AIC/BIC:

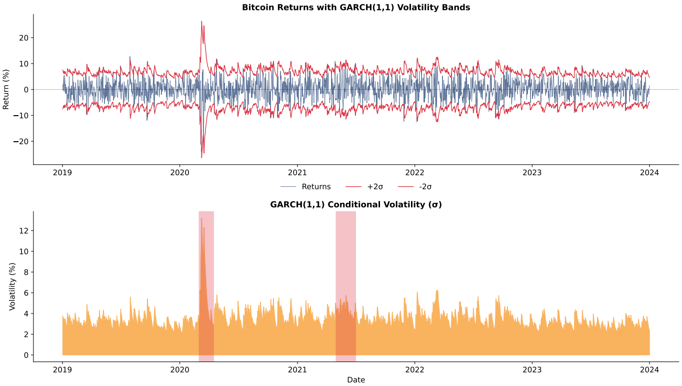
Model	AIC	BIC
ARIMA(0,0,0)	9524	9530
ARIMA(1,0,0)	9522	9534
ARIMA(0,0,1)	9523	9535
ARIMA(1,0,1)	**9520**	**9538**

Best: ARIMA(1,0,1) but marginal improvement

Key Insight

Crypto returns are notoriously unpredictable. The "alpha" is in understanding **volatility dynamics**, not predicting direction!

Step 4: GARCH Model for Volatility



GARCH(1,1) Model:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

- Bitcoin shows $\alpha + \beta \approx 0.95$ (high persistence)
- COVID and May 2021 periods show massive volatility spikes

Bitcoin: Summary of Approach

Raw Prices $\xrightarrow{\text{log diff}}$ Log Returns $\xrightarrow{\text{mean eq.}}$ ARIMA(1,0,1) $\xrightarrow{\text{variance eq.}}$ GARCH(1,1)

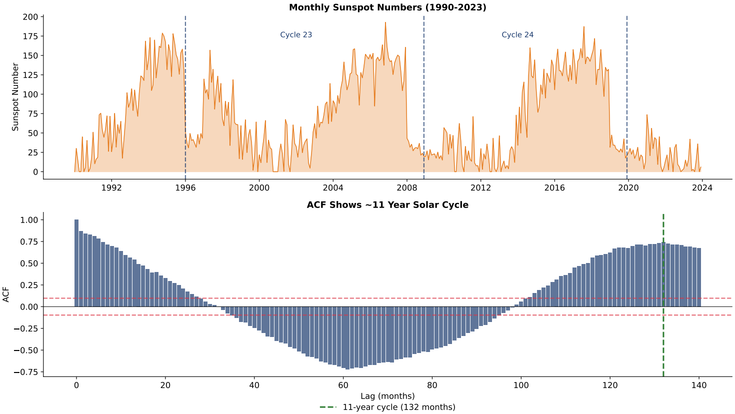
Key Findings:

- Returns are nearly unpredictable
- Extreme volatility clustering
- GARCH captures risk dynamics

Practical Use:

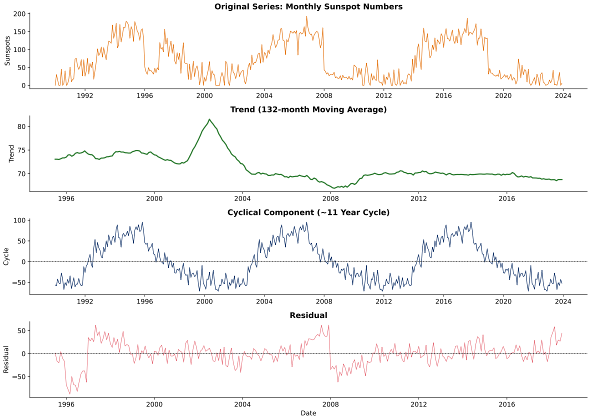
- Risk management (VaR, CVaR)
- Position sizing
- Volatility trading strategies

Sunspots: A Classic Long-Cycle Dataset



- **Data:** Monthly sunspot numbers, 1990-2023
- **Characteristics:** Famous $\sim$11 year solar cycle (132 months)

Step 1: Decomposition Analysis



- **Trend:** Long-term average sunspot activity
- **Cycle:** 11-year solar cycle (Schwabe cycle)
- **Challenge:** Very long seasonal period ($m = 132$)

Step 2: Handling Long Seasonality

The Challenge:

- Standard SARIMA with $m = 132$ requires estimating many parameters
- Seasonal differencing at lag 132 loses 11 years of data!

Option 1: Fourier Terms

- Add sine/cosine regressors
- Period $=$ 132 months
- Fewer parameters than full SARIMA

$$\sum_{k=1}^{K}\left[a_k \sin\left(\frac{2\pi kt}{132}\right) + b_k \cos\left(\frac{2\pi kt}{132}\right)\right]$$

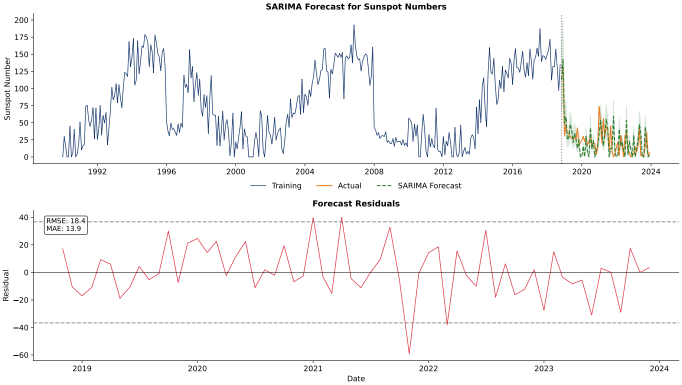
Option 2: AR Model

- High-order AR captures cycle
- AR(12) or AR(24) often sufficient
- Simple and effective

Classic Result

Sunspots are well-modeled by AR(9) or AR(12) models (Yule, 1927)

Step 3: SARIMA Forecasting



Model: AR(12) with Fourier terms for 11-year cycle

- Captures the quasi-periodic behavior
- Forecast uncertainty grows significantly with horizon

Sunspots: Model Comparison

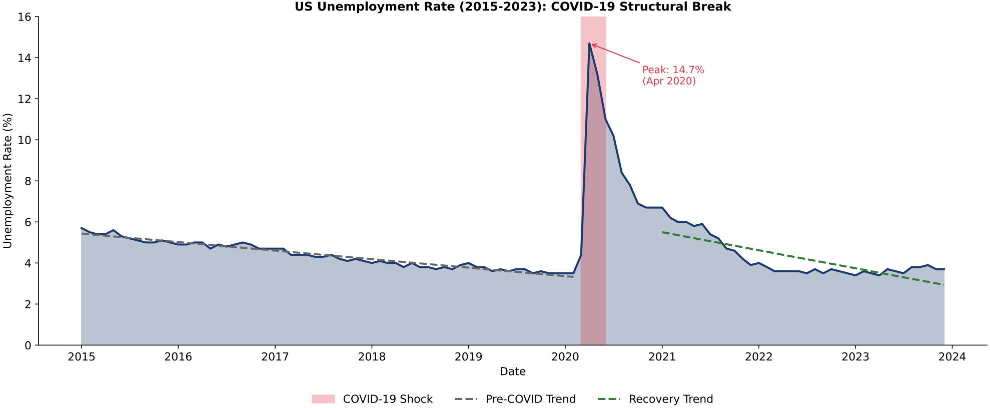
Model	RMSE	MAE	Notes
AR(12)	28.4	22.1	Simple, interpretable
ARIMA(2,0,2) + Fourier	26.8	20.5	Good cycle capture
TBATS	**25.2**	**19.8**	Automatic cycle detection
Prophet	29.1	23.4	Less suited for long cycles

Key Lesson

For very long seasonal periods, consider:

- Fourier regression terms
- TBATS (automatic cycle selection)
- High-order AR models

US Unemployment: COVID-19 Shock



- **Data:** US Unemployment Rate, monthly, 2015-2023 (BLS)
- **Shock:** From 3.5% to 14.7% in one month (April 2020)!

Handling Structural Breaks

Option 1: Truncate Data

- Use only post-COVID data
- Pro: Clean, no breaks
- Con: Lose historical patterns

Option 2: Dummy Variables

- Add COVID indicator
- Pro: Uses all data
- Con: Complex in ARIMA

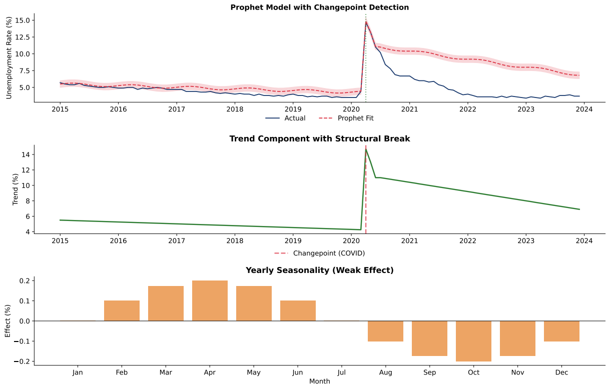
Option 3: Prophet with Changepoints

- Automatic detection
- Pro: Handles breaks naturally
- Con: May need tuning

Recommendation

For COVID-impacted data, Prophet's changepoint detection or regime-switching models work best.

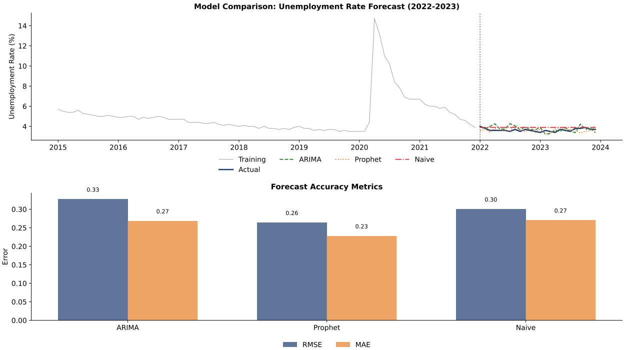
Prophet for Unemployment



Prophet Configuration:

- `changepoint_prior_scale = 0.5` (flexible for COVID shock)
- Automatic changepoint at April 2020
- Captures V-shaped recovery pattern

Model Comparison on Unemployment



Key Lesson

When data has extreme structural breaks:

- Traditional ARIMA may fail or require intervention analysis
- Prophet's flexibility with changepoints captures regime changes
- Consider regime-switching models (Markov-switching)

Why Multivariate Analysis?

Single Series (Univariate):

- ARIMA, GARCH, Prophet
- One variable at a time
- Cannot capture cross-series dynamics

Multiple Series (Multivariate):

- VAR: Vector Autoregression
- VECM: With cointegration
- Captures interdependencies

When to Use VAR?

- Economic indicators (GDP, inflation, unemployment)
- Financial markets (stocks, bonds, FX)
- Supply chain variables
- Any system with feedback loops

VAR Model: The Basics

Vector Autoregression VAR(p):

For k variables, VAR(p) is:

$$\mathrm{y}_t = \mathrm{c} + \mathrm{A}_1\mathrm{y}_{t-1} + \mathrm{A}_2\mathrm{y}_{t-2} + \cdots + \mathrm{A}_p\mathrm{y}_{t-p} + \boldsymbol{\varepsilon}_t$$

Example: VAR(1) with 2 variables

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

Key: Each variable depends on lags of ALL variables

Key Features:

- Captures dynamic feedback
- All variables are endogenous
- Enables Granger causality tests
- Impulse response analysis

Case Study: US Economic Variables

Variables (Quarterly, 2000-2023):

- GDP Growth (YoY %)
- Unemployment Rate (%)
- Inflation (CPI YoY %)
- Federal Funds Rate (%)

Expected Relationships:

- GDP $\uparrow \Rightarrow$ Unemployment $\downarrow$ (Okun's Law)
- GDP $\uparrow \Rightarrow$ Inflation $\uparrow$ (demand-pull)
- Inflation $\uparrow \Rightarrow$ Fed Rate $\uparrow$ (Taylor Rule)

Data Sources (FRED):

- GDPC1: Real GDP
- UNRATE: Unemployment
- CPIAUCSL: Consumer Price Index
- FEDFUNDS: Fed Funds Rate

Granger Causality

Definition:
Variable X *Granger-causes* Y if past values of X help predict Y beyond what past values of Y alone provide.

Test:

- H_0: X does NOT Granger-cause Y
- H_1: X Granger-causes Y
- F-test on coefficient restrictions

Warning

Granger causality $\neq$ True causality!
It measures *predictive* causality, not structural causality.

Example Results:

Cause $\rightarrow$ Effect	p-value
GDP $\rightarrow$ Unemp	0.02*
Unemp $\rightarrow$ GDP	0.15
Inflation $\rightarrow$ Fed	0.01**
Fed $\rightarrow$ Inflation	0.08

GDP leads unemployment
Inflation leads Fed Rate

Impulse Response Functions (IRF)

What is an IRF?
Shows how a one-unit shock to variable X affects all variables over time.

GDP Shock Analysis:

- Positive GDP shock
- $\Rightarrow$ Unemployment decreases
- $\Rightarrow$ Inflation increases
- $\Rightarrow$ Fed raises rates
- Effects persist for several quarters

Interpretation:

- Shows dynamic multiplier effects
- Confidence bands show uncertainty
- Can identify policy transmission

Policy Use

Central banks use IRFs to understand how monetary policy shocks propagate through the economy.

VAR: Model Selection and Diagnostics

Step 1: Lag Order Selection

- Use information criteria (AIC, BIC)
- BIC tends to select simpler models
- Cross-validate on held-out data

Step 2: Stationarity Check

- All variables should be stationary
- Or use VECM if cointegrated
- Test each variable with ADF

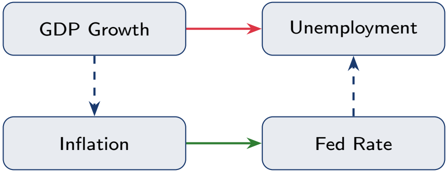
Step 3: Diagnostics

- Residual autocorrelation (Portmanteau)
- Normality (Jarque-Bera)
- Stability (eigenvalue check)

Step 4: Forecast Evaluation

- RMSE for each variable
- Compare to univariate benchmarks
- VAR often wins for interdependent systems

VAR Case Study: Summary



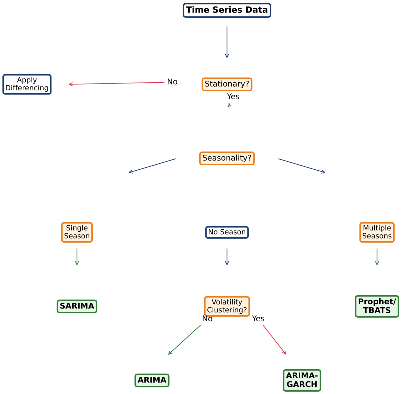
Key Findings:

- GDP Granger-causes unemployment
- Inflation Granger-causes Fed policy
- VAR captures these dynamics

Practical Applications:

- Economic forecasting
- Policy analysis
- Risk management in portfolios

Decision Framework



Model Selection Summary

Data Type	Characteristics	Recommended Model	Alternatives
Financial returns	No trend, volatility clustering	ARIMA-GARCH	EGARCH, GJR
Single seasonality	Trend + one seasonal period	SARIMA	ETS, Prophet
Long cycles	Sunspots, business cycles	AR + Fourier, TBATS	Spectral methods
Structural breaks	COVID, regime changes	Prophet	Intervention ARIMA
Multiple series	Interdependencies	VAR, VECM	Factor models

Forecast Evaluation Metrics

Point Forecast Metrics:

RMSE (Root Mean Square Error):

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

MAE (Mean Absolute Error):

$$\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

MAPE (Mean Absolute % Error):

$$\frac{100}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right|$$

When to Use Each:

- **RMSE**: Penalizes large errors more
- **MAE**: Robust to outliers
- **MAPE**: Scale-independent

Cross-Validation

Always use time series CV:

- Rolling window
- Expanding window
- Never shuffle!

Course Summary: Complete Toolkit

Understanding the Data

- Visualization first!
- Test for stationarity (ADF, KPSS)
- Identify seasonality patterns
- Check for structural breaks

Classical Models

- ARIMA: Non-seasonal data
- SARIMA: Single seasonality
- GARCH: Volatility modeling

Modern Approaches

- Prophet: Interpretable, handles breaks
- TBATS: Multiple/long seasonalities
- VAR/VECM: Multiple time series

Best Practices

- Always check diagnostics
- Use cross-validation
- Compare multiple models
- Domain knowledge matters!

Final Recommendations

1. **Start Simple**: Begin with visualization and basic statistics
2. **Test Assumptions**: Stationarity, normality, independence
3. **Iterate**: Model $\rightarrow$ Diagnose $\rightarrow$ Improve
4. **Compare**: Never rely on a single model
5. **Validate**: Out-of-sample testing is essential
6. **Communicate**: Clear visualizations and interpretations

Remember

"All models are wrong, but some are useful." — George Box

The goal is not perfect prediction, but useful insights and reasonable forecasts.

Questions?

Questions?

Next Steps:

- Practice with the Jupyter notebook
- Apply these methods to your own data
- Compare different models on the same dataset

Course Materials: `github.com/danpele/Time-Series-Analysis`