Time Series Analysis and Forecasting

Chapter 0: Fundamentals

Daniel Traian PELE

Bucharest University of Economic Studies
IDA Institute Digital Assets
Blockchain Research Center
AI4EFin Artificial Intelligence for Energy Finance
Romanian Academy, Institute for Economic Forecasting
MSCA Digital Finance

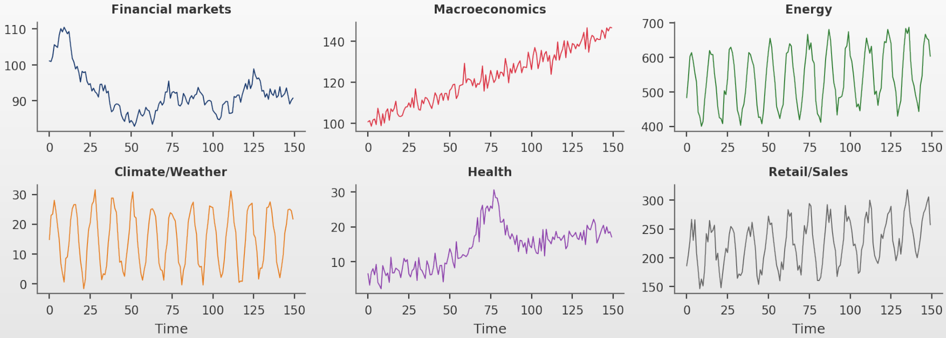
Learning Objectives

By the end of this chapter, you will be able to:

1. **Define** time series and distinguish them from cross-sectional and panel data
2. **Decompose** time series into trend-cycle, seasonality, and residual components
3. **Apply** exponential smoothing methods (SES, Holt, Holt-Winters, ETS)
4. **Evaluate** forecasts using MAE, RMSE, MAPE, sMAPE
5. **Implement** train/validation/test splitting and cross-validation
6. **Model** seasonality using dummy variables or Fourier terms
7. **Remove** trend and seasonality through appropriate methods
8. **Distinguish** between deterministic and stochastic trends

Outline

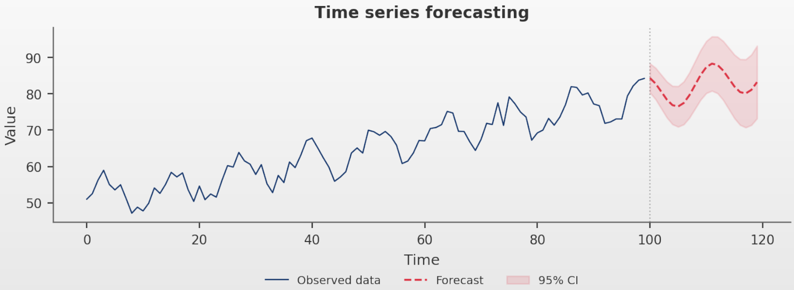
Time Series Are Everywhere



TSA_ch0_real_data

- **Finance**: Stock prices, exchange rates, volumes
- **Economics**: GDP, unemployment, inflation rates
- **Business**: Sales, website traffic, demand
- **Science**: Temperature, pollution, vital signs

Why Do We Study Time Series?

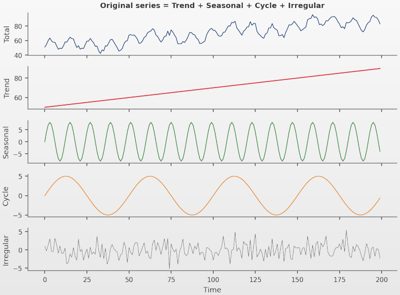


TSA_ch0_real_data

Main objective: forecasting

- We use historical patterns to predict future values $\Rightarrow$ essential for business planning, risk management, and policy decisions

Understanding Time Series Structure

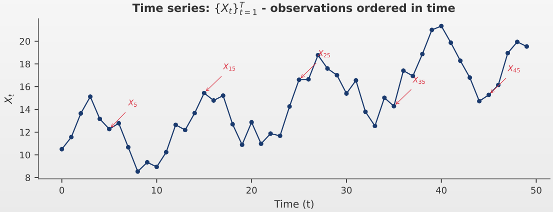


TSA ch0 real data

Decomposition

- Most time series can be decomposed into: **trend-cycle** + **seasonality** + **noise**

Time Series: Conceptual Illustration



TSA_ch0_definition

Fundamental Elements

- **Formal notation**: X_t = value at time t, $t \in \{1, 2, \ldots, T\}$
- **Autocorrelation**: $\rho_k = \text{Corr}(X_t, X_{t-k})$ — measures temporal dependence

Definition of a Time Series

Definition 1 (Time Series)

- **Time series**: a sequence of observations $\{X_t\}$ indexed by time: $\{X_t : t \in \mathcal{T}\}$ where $\mathcal{T}$ is a set of indices representing time points

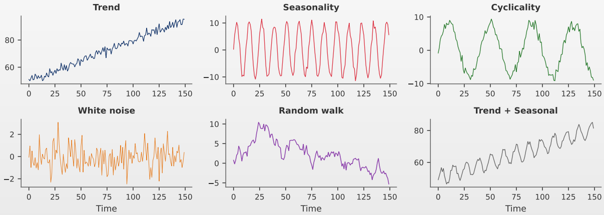
Key Characteristics

- **Ordered**: natural temporal order
- **Dependent**: consecutive observations are correlated
- **Discrete/Continuous**: $t = 1, 2, 3, \ldots$

Notation

- X_t: observation at time t
- $\{X_t\}_{t=1}^{T}$: series with T observations

Common Patterns in Time Series

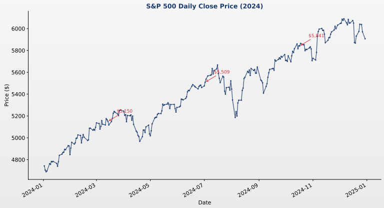


TSA_ch0_definition

Types of Patterns

- **Trend**: long-term increase or decrease
- **Seasonal**: regular periodic patterns
- **Cyclical**: medium-term fluctuations (2–10 years)
- **Random**: unpredictable fluctuations

Practical Example: Real Financial Data

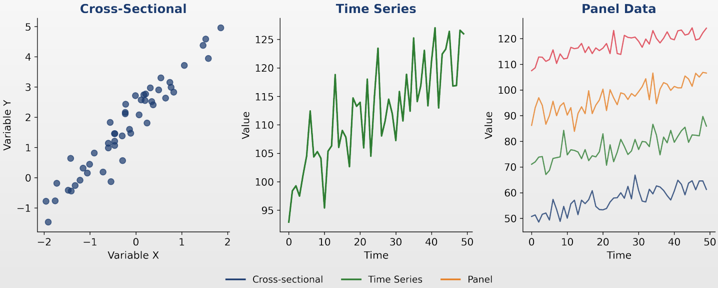


TSA_ch0_definition

S&P 500 (2024)

- **Daily frequency**: $\approx$ 252 trading days/year
- **Observed characteristics**: upward trend, volatility clustering, persistence (momentum)

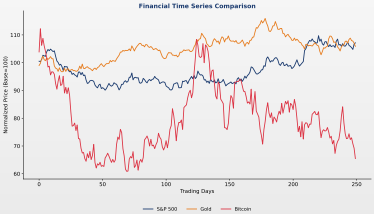
Data Types: Comparison



Data Type	Units (N)	Time (T)	Example
Cross-sectional	Many	1	Survey of 1000 households
Time series	1	Many	Daily S&P 500 prices
Panel	Many	Many	GDP of 50 countries, 20 years

TSA_ch0_definition

Examples of Time Series Data

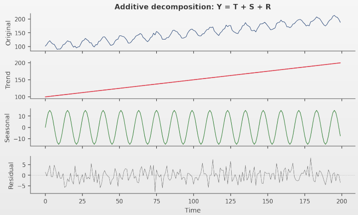


TSA_ch0_real_data

Real financial data

- **Source**: Yahoo Finance (2019–2025), normalized to base 100
- **Bitcoin**: most volatile; **Gold**: most stable

Time Series Decomposition: Visual Example



TSA_ch0_decomposition

Components Explained

- **Original**: observed series
- **Trend-Cycle**: long-term movement
- **Seasonal**: periodic pattern
- **Residual**: random noise

Why Do We Decompose a Time Series?

Objectives

- Understanding underlying patterns
- Removing seasonality for modeling
- Identifying trend direction
- Isolating irregular fluctuations
- Improving forecast accuracy

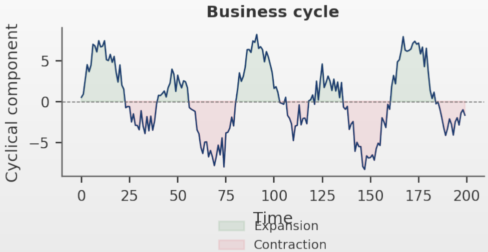
Components

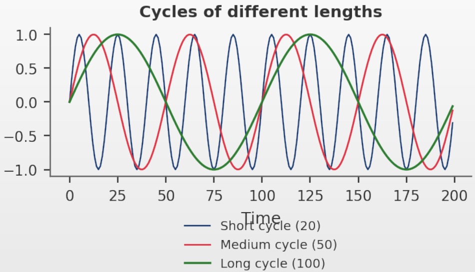
- T_t: Trend-Cycle
  - Long-term movement
- S_t: Seasonal
  - Regular periodic pattern
- ε_t: Residual
  - Random noise

Classical decomposition models

- **Additive**: $X_t = T_t + S_t + \varepsilon_t$
  - Constant seasonal amplitude
- **Multiplicative**: $X_t = T_t \times S_t \times \varepsilon_t$
  - Seasonal amplitude grows with the level

The Cyclical Component





TSA_ch0_decomposition

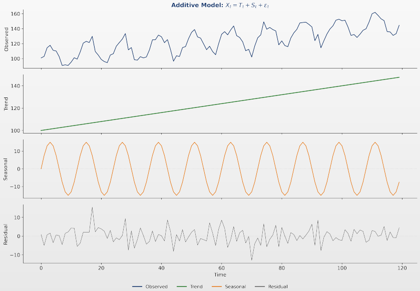
Characteristics

- **Duration**: medium-term fluctuations (2–10 years)
- **Aperiodic**: no fixed period (vs seasonality)
- **Origin**: reflects business cycles

In Practice

- **Combination**: cycle combined with trend
- **Difficulty**: hard to identify in short series
- **Solution**: usually absorbed into trend-cycle

Additive Decomposition: Visualization



TSA_ch0_decomposition

Interpretation

- **Decomposition**: Original = Trend + Seasonal + Residual
- **Property**: constant seasonal amplitude, does not depend on the level

The Additive Decomposition Model

Model

- **Equation**: $X_t = T_t + S_t + \varepsilon_t$
  - Components are added together to form the observed series

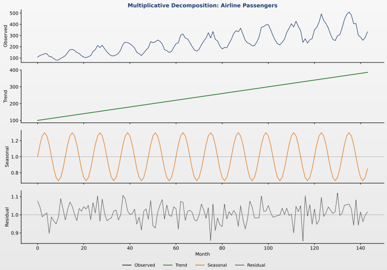
When to Use

- **Constant seasonal fluctuations**
  - Amplitude does not depend on the level
- **Stable series variance**
  - Measures dispersion around the mean
  - Estimator: $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

Properties

- **Error**: $\mathbb{E}[\varepsilon_t] = 0$ (zero mean)
- **Seasonal**: $\sum_{j=1}^{s} S_j = 0$ (seasonal sum is zero)
- **Units**: S_t are the same as X_t

Multiplicative Decomposition: Real Data



Example

- Box-Jenkins data: monthly passengers (1949–1960). Seasonal amplitude increases with the level

TSA_ch0_decomposition

The Multiplicative Decomposition Model

Model

- **Equation**: $X_t = T_t \times S_t \times \varepsilon_t \Rightarrow$ components are multiplied

When to Use

- **Growing fluctuations**: seasonality increases with the level
- **Heteroscedasticity**: variance increases over time
- **Examples**: economic/financial data

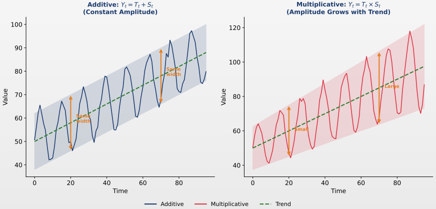
Properties

- **Error**: $\mathbb{E}[\varepsilon_t] = 1$ (centered at 1)
- **Seasonal**: $\frac{1}{s}\sum_{j=1}^{s} S_j = 1$ (mean is 1)
- **Units**: S_t is a dimensionless ratio

Tip

- **Log transformation**: multiplicative $\Rightarrow$ additive: $\log X_t = \log T_t + \log S_t + \log \varepsilon_t$

Additive vs Multiplicative: Comparison

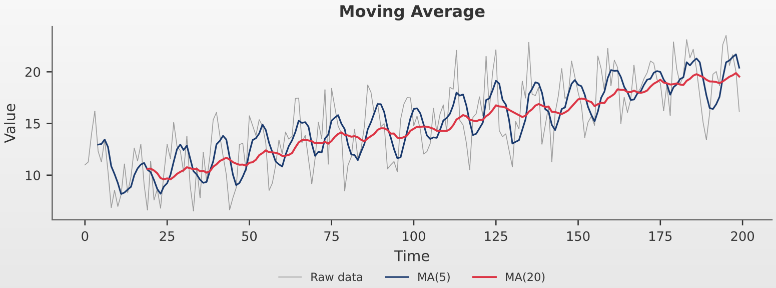


TSA_ch0_decomposition

Key Difference

- **Multiplicative**: seasonal component is a *ratio*, centered at value 1
- **Additive**: seasonal component in *absolute units*, centered at value 0

Centered Moving Average: Visual Illustration



Interpretation

- **Smoothing**: removes short-term fluctuations
- **Result**: reveals the underlying trend

TSA_ch0_definition

Trend Estimation: Moving Average

Definition 2 (Centered Moving Average)

- **Centered moving average** of order $2q + 1$:

$$\hat{T}_t = \frac{1}{2q+1} \sum_{j=-q}^{q} X_{t+j} \tag{1}$$

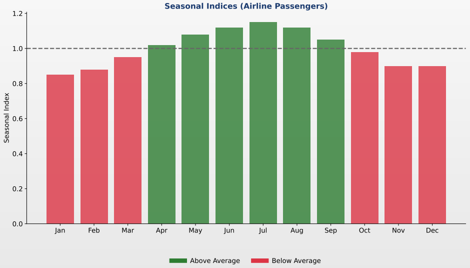
For Seasonal Data

- **Odd period** s
  - Use simple average
- **Even period** s
  - $2 \times s$ MA with half weights

Properties

- **Smoothing**: removes seasonal & random components
- **Large window** $\Rightarrow$ smoother estimate
- **Disadvantage**: data loss at endpoints

Seasonal Indices: Interpretation



Interpretation

- $S_t > 1$: above-average activity; $S_t < 1$: below average. Travel peak in July–August

TSA_ch0_seasonal

Classical Decomposition Algorithm

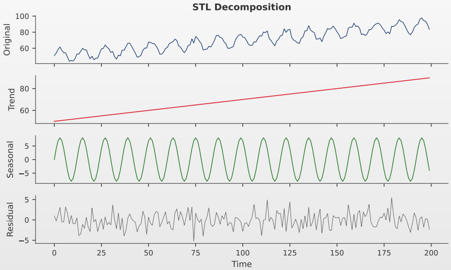
Steps for Multiplicative Decomposition

- **Step 1** $\Rightarrow$ **Estimate Trend**: $\hat{T}_t = MA_s(X_t)$
  - Centered moving average of order equal to the seasonal period
- **Step 2** $\Rightarrow$ **Detrend**: $D_t = X_t / \hat{T}_t$
- **Step 3** $\Rightarrow$ **Estimate Seasonal**: $\hat{S}_j = \text{mean}(D_t \text{ for season } j)$
- **Step 4** $\Rightarrow$ **Normalize**: scale so that $\frac{1}{s}\sum_{j=1}^{s} \hat{S}_j = 1$
- **Step 5** $\Rightarrow$ **Compute Residuals**: $\hat{\varepsilon}_t = X_t / (\hat{T}_t \times \hat{S}_t)$

Note

- **For additive decomposition**: operations change
  - Division $\Rightarrow$ subtraction
  - Multiplication $\Rightarrow$ addition

STL Decomposition: Visual Illustration



Key Idea

- STL (Seasonal-Trend-Loess): separates trend + seasonal + remainder using LOESS regression

TSA_ch0_decomposition

STL Decomposition: A Modern Approach

Definition 3 (STL - Seasonal-Trend Decomposition using LOESS)

- **STL**: uses locally weighted regression (LOESS): $X_t = T_t + S_t + R_t$

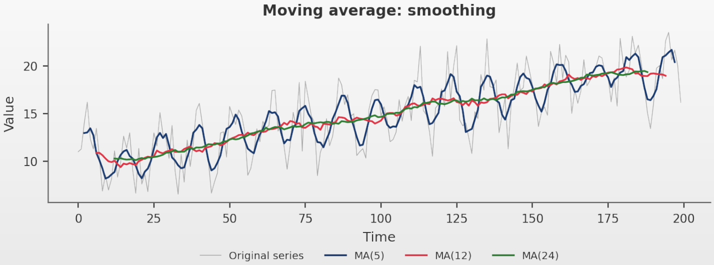
Advantages

- **Flexibility**: any seasonal period
- **Variability**: seasonality can evolve over time
- **Robustness**: resistant to outliers
- **Smoothing**: smooth trend estimates

Key Parameters

- `period`: seasonal period
  - E.g.: 12 for monthly data, 4 for quarterly
- `seasonal`: smoothing window
- `robust`: reduced weight for outliers

Moving Average Smoothing



Window Size Trade-off

- **Small window**: reactive but noisy
  - Captures rapid changes, but amplifies noise
- **Large window**: smooth but lagging
  - Removes noise, but reacts slowly

TSA_ch0_definition

Exponential Smoothing: Overview

Definition

- **Exponential smoothing**: weighted averages of past observations
  - Weights decrease exponentially over time
  - Recent observations receive higher weights

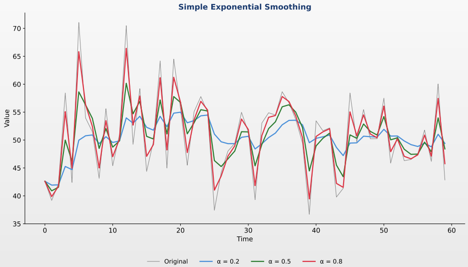
Why Exponential Smoothing?

- **Simple**: easy to implement and understand
  - A single smoothing parameter
- **Adaptive**: higher weights for recent data
- **Versatile**: handles trend and seasonality

Three Main Methods

- **SES** (Simple Exponential Smoothing): level only
  - The simplest exponential method
- **Holt**: level + trend
  - Captures the direction of evolution
- **Holt-Winters**: + seasonality
  - Complete model with all components

Simple Exponential Smoothing: Effect of α



Trade-off

- **Small** $\alpha \Rightarrow$ smooth forecasts
  - More weight on distant history
- **Large** $\alpha \Rightarrow$ tracks the data
  - Fast reaction to recent changes

TSA_ch0_smoothing

Simple Exponential Smoothing (SES)

Model

- **Equation**: $\hat{X}_{t+1|t} = \alpha X_t + (1-\alpha)\hat{X}_{t|t-1}$
  - $\alpha \in (0,1)$ is the smoothing parameter

How It Works

- **Principle**: weights decrease exponentially
- **Large** α
  - Forecast reactive to changes
- **Small** α
  - Smoother, more stable forecast

Level Form

- **Equation**: $\ell_t = \alpha X_t + (1-\alpha)\ell_{t-1}$
  - ℓ_t = estimated level at time t
  - Forecast: $\hat{X}_{t+h|t} = \ell_t$ (constant)

SES: Step-by-Step Numerical Example

Data: Monthly Sales (thousands EUR)

- **Data**: $X_1 = 100,\ X_2 = 110,\ X_3 = 105,\ X_4 = 115,\ X_5 = 120 \quad (\alpha = 0.3,\ \hat{X}_{1|0} = 100)$

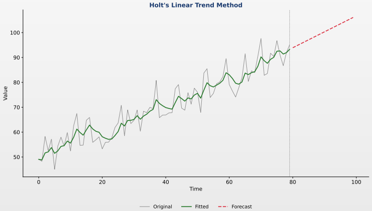
Iterative computation: $\hat{X}_{t+1|t} = \alpha X_t + (1-\alpha)\hat{X}_{t|t-1}$

t	X_t	$\hat{X}_{t\|t-1}$	e_t	Computation $\hat{X}_{t+1\|t}$
1	100	100.00	0.00	$0.3 \times 100 + 0.7 \times 100 = 100.00$
2	110	100.00	10.00	$0.3 \times 110 + 0.7 \times 100 = 103.00$
3	105	103.00	2.00	$0.3 \times 105 + 0.7 \times 103 = 103.60$
4	115	103.60	11.40	$0.3 \times 115 + 0.7 \times 103.6 = 107.02$
5	120	107.02	12.98	$0.3 \times 120 + 0.7 \times 107.02 = 110.91$

Forecast and Evaluation

$\hat{X}_{6|5} = 110.91 \qquad \text{MAE} = 7.28 \qquad \text{RMSE} = 8.97$

Holt's Method: Visualization



TSA_ch0_smoothing

Interpretation

- **Holt's method**: captures level and trend, projects them into the forecast horizon
- α: controls level changes; β^*: controls trend changes

Holt's Linear Trend Method

Equations

- **Level**: $\ell_t = \alpha X_t + (1-\alpha)(\ell_{t-1} + b_{t-1})$
- **Trend**: $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1}$
- **Forecast**: $\hat{X}_{t+h|t} = \ell_t + h \cdot b_t$
  - Extrapolates the linear trend over h steps

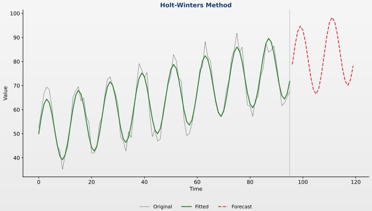
Parameters

- α: level smoothing
  - Controls reactivity to level changes
- β^*: trend smoothing
  - Controls reactivity to slope changes

Components

- ℓ_t: estimated level
  - Local mean of the series
- b_t: estimated trend (slope)
  - Rate of increase/decrease

Holt-Winters: Capturing Seasonality



TSA_ch0_smoothing

Key Feature

- **Complete decomposition**: separates level, trend, and seasonal
- **Seasonal forecasts**: includes both trend and periodic pattern

Holt-Winters Seasonal Method

Equations (Additive Seasonality)

- **Level**: $\ell_t = \alpha(X_t - S_{t-s}) + (1-\alpha)(\ell_{t-1} + b_{t-1})$
- **Trend**: $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1}$
- **Seasonal**: $S_t = \gamma(X_t - \ell_t) + (1-\gamma)S_{t-s}$
- **Forecast**: $\hat{X}_{t+h|t} = \ell_t + h \cdot b_t + S_{t+h-s(k+1)}$
  - Where $k = \lfloor (h-1)/s \rfloor$

Parameters

- α — level
- β^* — trend
- γ — seasonal
- s — seasonal period
  - All in $(0, 1)$; estimated by minimizing error

The ETS Framework: Error-Trend-Seasonality

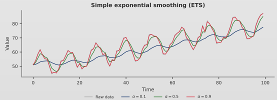
Definition 4 (ETS Models)

- **ETS framework**: generalizes exponential smoothing: ETS(E, T, S)

Component	N	A	M
Error (E)	–	Additive	Multiplicative
Trend (T)	None	Additive	Multiplicative
Seasonal (S)	None	Additive	Multiplicative

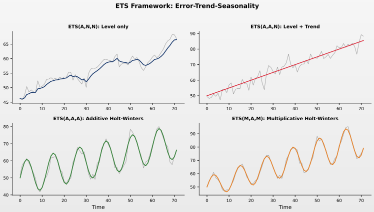
Examples

- **ETS(A,N,N)**: Simple Exponential Smoothing $\Rightarrow$ level only, no trend or seasonality
- **ETS(A,A,N)**: Holt's Linear Method $\Rightarrow$ level + additive trend
- **ETS(A,A,A)**: Additive Holt-Winters $\Rightarrow$ level + trend + additive seasonality



TSA_ch0_smoothing

ETS Model Selection



TSA_ch0_smoothing

Automatic Selection

- **Information criteria**: AIC (Akaike) and BIC (Bayesian)
- **Optimal selection**: balance between fit and complexity

Damped Trend Methods

Damping Parameter

- **Parameter**: $\phi \in (0, 1)$
  - Prevents over-projection of the trend
  - Trend converges to a constant

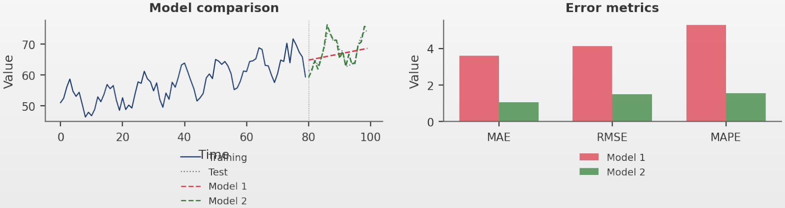
Equations

- **Level**: $\ell_t = \alpha X_t + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1})$
- **Trend**: $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)\phi b_{t-1}$
- **Forecast**: $\hat{X}_{t+h|t} = \ell_t + \phi \frac{1-\phi^h}{1-\phi} b_t$

Key Idea

- **Asymptotic**: as $h \to \infty$, forecast $\to$ constant
  - Prevents unrealistic long-term extrapolation
- **Advantage**: often better for long horizons

Forecast Evaluation: Visual Example



TSA_ch0_forecast_eval

Observations

- **Top**: actual vs. forecast — visual assessment of forecast quality
- **Bottom**: residuals — zero mean, constant variance, no pattern

Forecast Accuracy Metrics

Forecast Error

- **Definition**: $e_t = X_t - \hat{X}_t$ (actual minus predicted)
  - Positive $\Rightarrow$ underestimates; Negative $\Rightarrow$ overestimates

Scale-dependent

- **MAE**: $\frac{1}{n}\sum |e_t|$
- **MSE**: $\frac{1}{n}\sum e_t^2$
- **RMSE**: $\sqrt{\text{MSE}}$

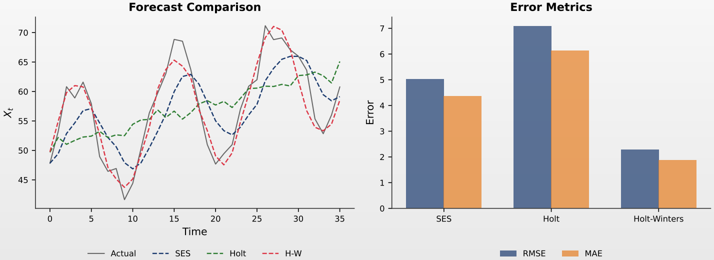
Scale-independent

- **MAPE**: $\frac{100}{n}\sum \left|\frac{e_t}{X_t}\right|$
- **sMAPE**: $\frac{100}{n}\sum \frac{|e_t|}{(|X_t|+|\hat{X}_t|)/2}$

What to Use?

- **Same series**: RMSE, MAE $\Rightarrow$ compare models on the same data
- **Across different series**: MAPE, sMAPE $\Rightarrow$ percentage metrics, scale-independent

Comparing Forecast Methods

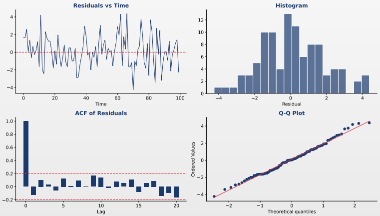


Interpretation

- **Left**: SES, Holt, Holt-Winters forecasts. **Right**: Error metrics. Visual and quantitative comparison

TSA_ch0_forecast_eval

Residual Diagnostics: Visualization



TSA_ch0_forecast_eval

What to Check

- **Time plot**: no systematic patterns
- **Histogram**: normality check
- **ACF**: no significant autocorrelation
- **Q-Q plot**: normality confirmation

Residual Diagnostics

Residual Properties

- **Zero mean**: $\mathbb{E}[e_t] = 0$
  - Forecast has no systematic bias
- **Uncorrelated**: $\text{Cov}(e_t, e_{t-k}) = 0$
  - No unexploited information remains
- **Constant variance**: $\text{Var}(e_t) = \sigma^2$
- **Normally distributed**: for confidence intervals

Diagnostic Tests

- **Ljung-Box test** (autocorrelation):
  - $Q = T(T+2)\sum_{k=1}^{h} \frac{\hat{\rho}_k^2}{T-k} \sim \chi_h^2$
- **Jarque-Bera test** (normality):
  - $JB = \frac{T}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) \sim \chi_2^2$
  - S = skewness, K = kurtosis

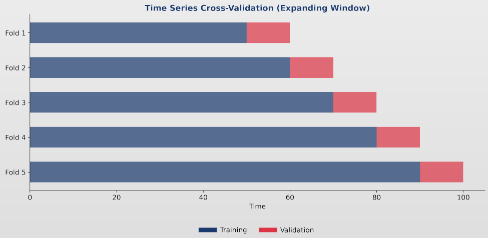
Cross-Validation for Time Series

Why Not Standard CV?

- **Temporal dependence**: observations are correlated
- **Order matters**: chronology must be respected
- **Standard k-fold** $\Rightarrow$ data leakage

CV with Rolling Origin

- **Step 1**: train on $\{X_1, \ldots, X_t\}$
- **Step 2**: forecast $\hat{X}_{t+h}$
- **Step 3**: increment t, repeat



TSA_ch0_forecast_eval

Train / Validation / Test Split

Training Set

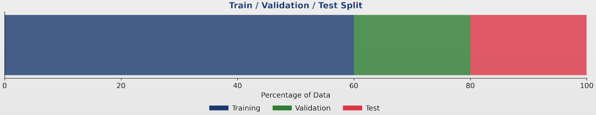
- Fitting model parameters
- Largest portion (60–80%)
- Used for estimation

Validation Set

- Hyperparameter tuning
- Comparing models
- Selecting the best approach

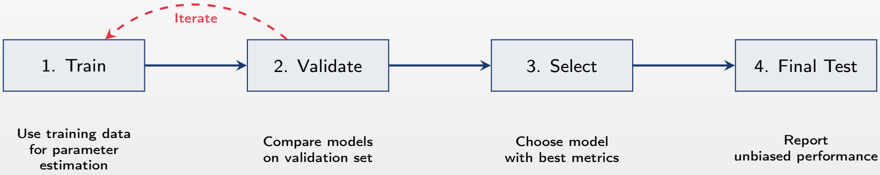
Test Set

- Final evaluation only
- Never used for tuning
- Unbiased performance



TSA_ch0_forecast_eval

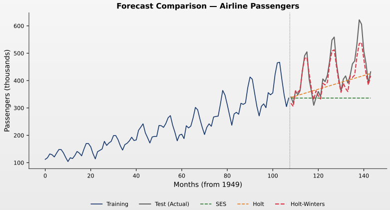
Model Development Workflow



Critical Rule

- **Never use the test set for selection!**
  - Use it only for final evaluation
- **Avoid data leakage**
  - Overly optimistic performance estimates

Real Data: Comparing Forecasts

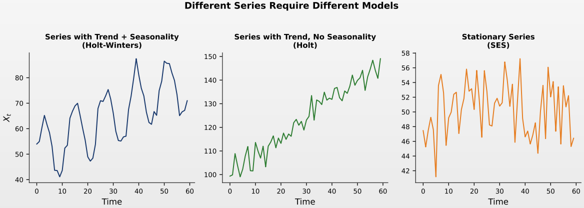


TSA_ch0_forecast_eval

Interpretation

- **Data**: airline passengers
- **Best**: multiplicative Holt-Winters — ideal for data with growing seasonality

Forecast Performance Across Different Datasets

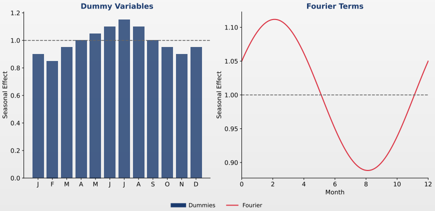


TSA_ch0_forecast_eval

Interpretation

- **Different series**: require different models
- **Seasonal data**: prefer seasonal methods
- **No universal model**: test multiple approaches

Dummy Variables vs Fourier Terms



TSA_ch0_seasonal

Comparison

- **Dummy variables**: capture any shape, require $s-1$ parameters
- **Fourier terms**: only $2K$ parameters, smooth sinusoidal patterns

Modeling Seasonality: Two Approaches

1. Dummy Variables

- **Model**: $X_t = \mu + \sum_{j=1}^{s-1} \gamma_j D_{jt} + \varepsilon_t$
- $D_{jt} = 1$ if t in season j
- $s - 1$ parameters
- Any seasonal pattern

2. Fourier Terms

- **Model**:
  $X_t = \mu + \sum_{k=1}^{K} \left[\alpha_k \sin\left(\frac{2\pi kt}{s}\right) + \beta_k \cos\left(\frac{2\pi kt}{s}\right)\right]$
- Sinusoidal functions
- $2K$ parameters
- Smooth patterns

Trade-off

- **Dummy variables**
  - Any seasonal pattern, but more parameters
- **Fourier terms**
  - Smooth patterns, fewer parameters

Choosing Between Dummy and Fourier

Criterion	Dummy	Fourier
Parameters (monthly)	11	$2K$ (often 4–6)
Seasonal pattern	Any shape	Smooth/sinusoidal
Interpretation	Direct (monthly effects)	Frequency components
High-frequency seasons	Many parameters	Efficient
Multiple seasonality	Complex	Easy (add terms)

Recommendations

- **Use Dummy**
  - Irregular patterns, interpretable coefficients
- **Use Fourier**
  - Smooth patterns, high-frequency seasonality
  - Used in TBATS and Prophet

Why Do We Remove Trend and Seasonality?

Reasons for Detrending

- Stationarity requirement
- Focus on fluctuations
- Avoiding spurious regression
- Enabling valid inference

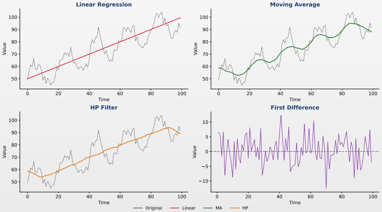
Reasons for Deseasonalizing

- Revealing the underlying trend
- Cross-season comparisons
- Simplifying modeling
- Focus on the irregular component

Important

- **We model the transformed series**
  - With trend and seasonality removed
- **We reverse the transformation**
  - Bring the forecast back to the original scale

Detrending Methods: Comparison



TSA_ch0_detrending

Key Idea

- **Different methods**: produce different residuals
- **Choose by trend type**: consider the analysis objectives

Detrending Methods

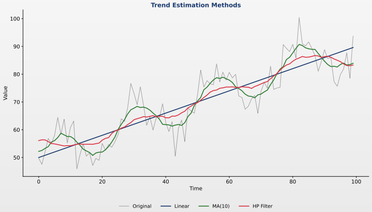
Six Common Detrending Approaches

- **Differencing**: $\Delta X_t = X_t - X_{t-1}$
  - Most commonly used, removes stochastic trend
- **Linear regression**: $\hat{T}_t = \hat{\beta}_0 + \hat{\beta}_1 t$
- **Polynomial**: higher-order polynomial
- **HP filter**: balance between fit and smoothness
- **Moving average**: $\hat{T}_t = MA_q(X_t)$
- **LOESS**: local polynomial regression

The Choice Depends on

- **Nature of the trend**
  - Deterministic vs stochastic
- **Purpose of the analysis**
  - Forecasting vs descriptive analysis

Trend Estimation: Multiple Approaches

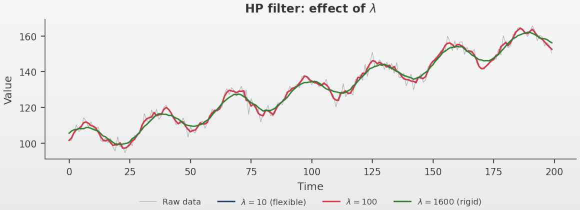


TSA_ch0_detrending

Method Comparison

- **Moving average**: simple but with lag
- **Polynomial regression**: flexible, parametric
- **HP filter**: macroeconomic standard

HP Filter: Effect of λ



TSA_ch0_detrending

Trade-off

- **Small** λ: flexible trend, follows the data closely
- **Large** λ: smooth trend, approaches a linear trend

The Hodrick-Prescott (HP) Filter

Definition 5 (HP Filter)

- **HP filter**: decomposes X_t into trend τ_t and cycle c_t: $X_t = \tau_t + c_t$

$$\min_{\{\tau_t\}} \left\{ \sum_{t=1}^{T} (X_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \right\}$$

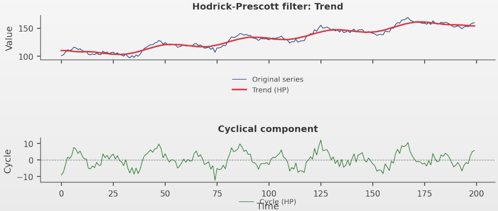
Interpretation

- **First term**
  - Goodness of fit
- **Second term**
  - Smoothness penalty
- λ
  - Controls the balance between fidelity and smoothness

Standard λ Values (Ravn-Uhlig)

- **Annual**
  - $\lambda = 6.25$
- **Quarterly**
  - $\lambda = 1600$ (macroeconomic standard)
- **Monthly**
  - $\lambda = 129600$

HP Filter: Business Cycle Extraction



TSA_ch0_detrending

Application

- **Macroeconomics**: business cycle extraction
- **Common series**: GDP, unemployment, inflation

HP Filter: Limitations

Known Issues

- **Endpoint instability**
  - Trend estimates unreliable at the beginning and end
- **Spurious cycles**
  - Can create artificial dynamics
- **Choice of λ**
  - Results sensitive to the parameter

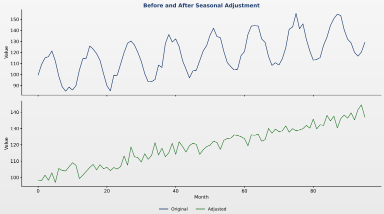
Alternatives

- **Band-pass filters**: Baxter-King, Christiano-Fitzgerald
  - Isolate specific frequencies
- **Hamilton filter**: regression-based
- **Unobserved components**: state-space models

Hamilton's Critique (2018)

- Hamilton (2018) shows that the HP filter introduces **spurious cyclical components**
- Proposes alternative: regression of y_{t+h} on $y_t, y_{t-1}, \ldots, y_{t-p}$ (default $h = 8$, $p = 4$ quarterly)
- Advantage: no λ selection required; no end-of-sample problem

Seasonal Adjustment: Visualization



TSA_ch0_seasonal

Result

- **Seasonally adjusted series**: reveals the underlying trend, removes periodic fluctuations

Seasonal Adjustment Methods

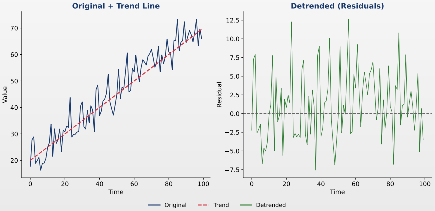
Four Approaches for Seasonal Adjustment

- **Seasonal differencing**: $\Delta_s X_t = X_t - X_{t-s}$
  - Removes periodic pattern, simple to apply
- **Division** (multiplicative): $X_t^{adj} = X_t / \hat{S}_t$
- **Subtraction** (additive): $X_t^{adj} = X_t - \hat{S}_t$
- **X-13ARIMA-SEATS**: official US Census Bureau standard
  - Sophisticated method, used by statistical institutes

Seasonal Period s

- Monthly: $s = 12$ | Quarterly: $s = 4$

Example: Deterministic Trend



TSA_ch0_detrending

Key

- **Method**: regression
- **Result**: stationary residuals, ACF decays rapidly

Deterministic vs Stochastic Trend

Deterministic Trend

- **Model**: $X_t = \beta_0 + \beta_1 t + \varepsilon_t$
- **Characteristics**:
  - Trend is a function of time
  - ε_t is stationary
- **Method**: detrend by regression

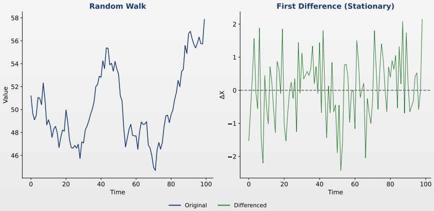
Stochastic Trend

- **Model**: $X_t = X_{t-1} + \varepsilon_t$
- **Characteristics**:
  - Random walk component
  - ΔX_t is stationary
- **Method**: detrend by differencing

Wrong Method = Problems

- **Differencing a deterministic trend** $\Rightarrow$ over-differencing
  - Introduces artificial dependence in the series
- **Regression on a stochastic trend** $\Rightarrow$ spurious regression
  - Invalid statistical results

Example: Stochastic Trend (Random Walk)

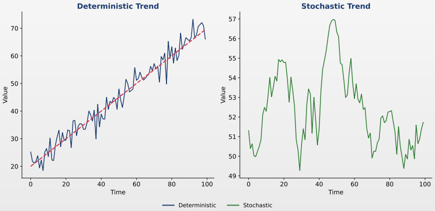


TSA_ch0_detrending

Key

- **Method**: differencing
- **Result**: differences are stationary (white noise)

Side-by-Side Comparison



TSA_ch0_detrending

Remember

- **Deterministic trend**: use regression — trend is a predictable function of time
- **Stochastic trend**: use differencing — trend contains a random component

AI Exercise: Critical Thinking

Prompt to test in ChatGPT / Claude / Copilot

"Using yfinance, download monthly closing prices for Apple (AAPL) from 2015-01 to 2024-12 (120 observations). Decompose the series into trend, seasonality, and residuals. Determine if additive or multiplicative decomposition is more appropriate and forecast the price for the next 12 months. Give me complete Python code with professional charts."

Exercise:

1. Run the prompt in an LLM of your choice and critically analyze the response.
2. What type of decomposition does the model choose? Is it correct? Justify.
3. How does it evaluate forecast quality? Is the metric computed correctly?
4. Check the residuals — do they show unexplained structure?
5. Rewrite the analysis correctly and compare with a seasonal naïve benchmark.

Warning: AI-generated code may run without errors and look professional. *That does not mean it is correct.*

Summary

What We Learned in This Chapter

- Time Series Definition and Characteristics
  - Sequence of temporally ordered observations with dependence
- Decomposition (Additive vs Multiplicative)
  - Components: Trend-Cycle + Seasonal + Residual
- Exponential Smoothing Methods
  - SES (level), Holt (+ trend), Holt-Winters (+ seasonality), ETS
- Forecast Evaluation and Validation
  - Metrics: MAE, RMSE, MAPE; Cross-Validation with rolling origin

Key Idea

- **Understand Before Modeling**:
  - Visualize and decompose the data first
  - Choose additive vs multiplicative based on variance behavior

What's Next?

Chapter 1: Stochastic Processes and Stationarity

- **Stochastic Processes**: mathematical foundation, random variables indexed by time
- **Stationarity**: strict (invariant distribution) vs weak (invariant moments)
- **Fundamental Processes**: white noise and random walk $\Rightarrow$ building blocks for ARIMA
- **ACF and PACF**: tools for model identification

Questions?

Bibliography I

Time Series Fundamentals

- Wold, H. (1938). *A Study in the Analysis of Stationary Time Series*, Almqvist & Wiksell.
- Hamilton, J.D. (1994). *Time Series Analysis*, Princeton University Press.
- Brockwell, P.J., & Davis, R.A. (2016). *Introduction to Time Series and Forecasting*, 3rd ed., Springer.

Decomposition and Exploratory Analysis

- Cleveland, R.B., Cleveland, W.S., McRae, J.E., & Terpenning, I. (1990). STL: A Seasonal-Trend Decomposition Procedure Based on Loess, *Journal of Official Statistics*, 6(1), 3–33.
- Hyndman, R.J., & Athanasopoulos, G. (2021). *Forecasting: Principles and Practice*, 3rd ed., OTexts.

Bibliography II

Exponential Smoothing and ETS Fundamentals

- Holt, C.C. (1957/2004). Forecasting Seasonals and Trends by Exponentially Weighted Moving Averages, *International Journal of Forecasting*, 20(1), 5–10.
- Winters, P.R. (1960). Forecasting Sales by Exponentially Weighted Moving Averages, *Management Science*, 6(3), 324–342.
- Hyndman, R.J., Koehler, A.B., Ord, J.K., & Snyder, R.D. (2008). *Forecasting with Exponential Smoothing: The State Space Approach*, Springer.

Online Resources and Code

- **Quantlet**: `https://quantlet.com` – Code platform for quantitative methods
- **Quantinar**: `https://quantinar.com` – Learning platform for quantitative methods
- **GitHub TSA**: `https://github.com/QuantLet/TSA/tree/main/TSA_ch0` – Python code for this chapter

Thank You!

Questions?

Course materials available at: https://danpele.github.io/Time-Series-Analysis/

Quantlet Quantinar