Time Series Analysis and Forecasting

Seminar 3: ARIMA Models

Daniel Traian PELE

Bucharest University of Economic Studies
IDA Institute Digital Assets
Blockchain Research Center
AI4EFin – Artificial Intelligence for Energy Finance
Romanian Academy, Institute for Economic Forecasting
MSCA Digital Finance

Seminar Outline

Today's Activities:

1. **Review Quiz** — Checking understanding of ARIMA concepts
2. **True/False Questions** — Conceptual checks
3. **Practice Problems** — Calculations with ARIMA
4. **Worked Examples** — Real-world applications
5. **Discussion Questions** — Practical applications
6. **AI-Assisted Exercises** — Human vs. AI modeling

Quiz 1: Integration Order

Question

A time series Y_t requires two differences to become stationary. What is its order of integration?

A) $I(0)$
B) $I(1)$
C) $I(2)$
D) Cannot be determined

Quiz 1: Integration Order

Question

A time series Y_t requires two differences to become stationary. What is its order of integration?

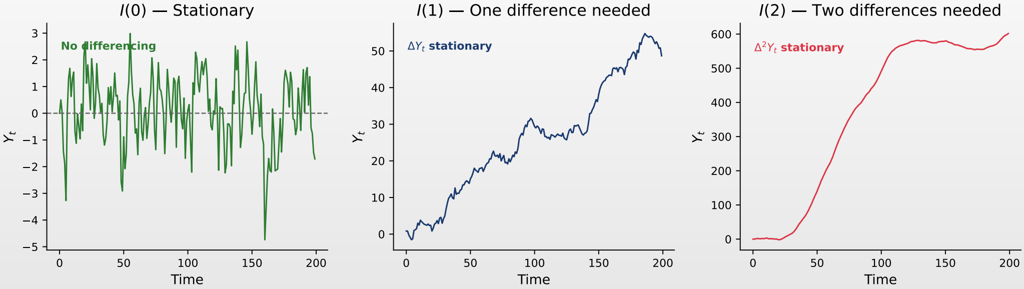
Answer: C – $I(2)$

Definition: $Y_t \sim I(d)$ if $\Delta^d Y_t$ is stationary but $\Delta^{d-1} Y_t$ is not.
Example: If Y_t follows $\Delta^2 Y_t = \varepsilon_t$, then:

- $\Delta Y_t = \Delta Y_{t-1} + \varepsilon_t$ (still has unit root)
- $\Delta^2 Y_t = \varepsilon_t$ (white noise, stationary)

Real-world: Price levels may be $I(2)$ when inflation itself is non-stationary.

Visual: Integrated Processes



$I(0)$: stationary. $I(1)$: one difference needed. $I(2)$: two differences needed to become stationary.

TSA_ch3_def_integrated

Quiz 2: Random Walk Properties

Question

For a random walk $Y_t = Y_{t-1} + \varepsilon_t$ with $\text{Var}(\varepsilon_t) = \sigma^2$, what is $\text{Var}(Y_t)$?

A) σ^2

B) $t \cdot \sigma^2$

C) σ^2/t

D) $\sigma^2/(1 - \phi^2)$

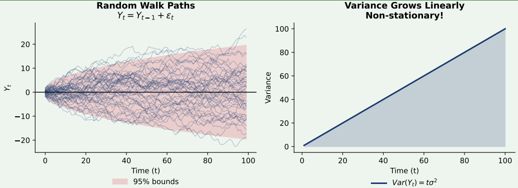
TSA_ch3_rw_variance

Quiz 2: Random Walk Properties

Question

For a random walk $Y_t = Y_{t-1} + \varepsilon_t$ with $\text{Var}(\varepsilon_t) = \sigma^2$, what is $\text{Var}(Y_t)$?

Answer: B – $t \cdot \sigma^2$



Proof: $Y_t = \sum_{i=1}^{t} \varepsilon_i \Rightarrow \text{Var}(Y_t) = t\sigma^2$ (grows linearly $\Rightarrow$ non-stationary!)

TSA_ch3_rw_variance

Quiz 3: ADF Test Hypotheses

Question

In the Augmented Dickey-Fuller test, what is the null hypothesis?

A) The series is stationary

B) The series has a unit root

C) The series has no autocorrelation

D) The series is normally distributed

Quiz 3: ADF Test Hypotheses

Question

In the Augmented Dickey-Fuller test, what is the null hypothesis?

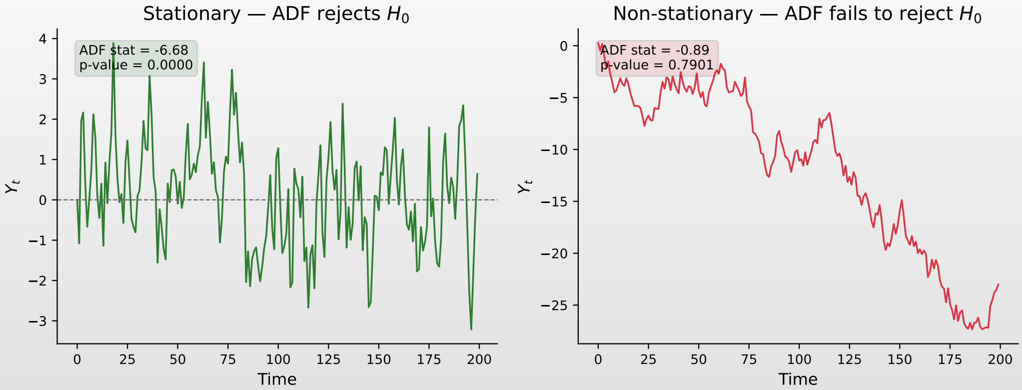
Answer: B – The series has a unit root

ADF regression: $\Delta Y_t = \alpha + \gamma Y_{t-1} + \sum_{j=1}^{p} \delta_j \Delta Y_{t-j} + \varepsilon_t$
Hypotheses:

- $H_0 : \gamma = 0$ (unit root, non-stationary)
- $H_1 : \gamma < 0$ (stationary)

Decision: Reject H_0 if t-statistic $<$ critical value (e.g., -2.86 at 5%)
Note: Uses special Dickey-Fuller distribution, not standard t.

Visual: ADF Test



Left: stationary – ADF rejects unit root. Right: non-stationary – ADF fails to reject.

TSA_ch3_def_adf

Quiz 4: ARIMA Notation

Question

What does ARIMA(2,1,1) mean?

A) AR(2) on differenced data with MA(1) errors

B) AR(1) with 2 differences and MA(1)

C) MA(2) with 1 difference and AR(1)

D) 2 lags, 1 trend, 1 seasonal component

Quiz 4: ARIMA Notation

Question

What does ARIMA(2,1,1) mean?

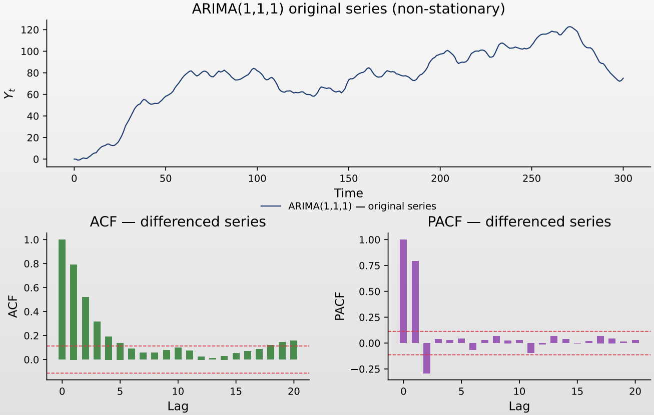
Answer: A – AR(2) on differenced data with MA(1) errors

ARIMA(p, d, q): $\phi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t$
ARIMA(2,1,1) expands to:

$$(1-\phi_1 L - \phi_2 L^2)(1-L)Y_t = (1+\theta_1 L)\varepsilon_t$$

Or equivalently: $(1-\phi_1 L - \phi_2 L^2)\Delta Y_t = (1+\theta_1 L)\varepsilon_t$
Interpretation: First difference the series, then fit ARMA(2,1) to ΔY_t.

Visual: ARIMA Process



Top: original ARIMA series. Bottom: after differencing, use ACF/PACF to identify AR and MA orders.

TSA_ch3_def_arima

Quiz 5: Difference Operator

Question

What is $(1 - L)^2 Y_t$ expanded?

A) $Y_t - Y_{t-1}$

B) $Y_t - 2Y_{t-1} + Y_{t-2}$

C) $Y_t + 2Y_{t-1} + Y_{t-2}$

D) $Y_t - Y_{t-2}$

Quiz 5: Difference Operator

Question

What is $(1 - L)^2 Y_t$ expanded?

Answer: B – $Y_t - 2Y_{t-1} + Y_{t-2}$

Binomial expansion: $(1 - L)^2 = 1 - 2L + L^2$
Applied: $(1 - L)^2 Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$ (the "change in changes")

Quiz 6: KPSS vs ADF

Question

How does the KPSS test differ from the ADF test?

A) KPSS tests for seasonality, ADF tests for trends
B) KPSS has stationarity as null, ADF has unit root as null
C) KPSS is more powerful than ADF
D) There is no difference

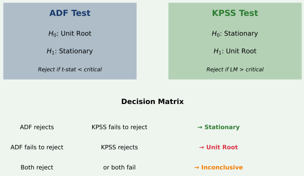
TSA_ch3_adf_kpss

Quiz 6: KPSS vs ADF

Question

How does the KPSS test differ from the ADF test?

Answer: B – Reversed null hypotheses



Strategy: Use both tests together for robust inference!

TSA_ch3_adf_kpss

Quiz 7: Overdifferencing

Question

If $Y_t \sim I(1)$ and we compute $\Delta^2 Y_t$, what happens?

A) We get a better stationary series
B) We introduce artificial negative autocorrelation
C) The variance decreases
D) Nothing changes

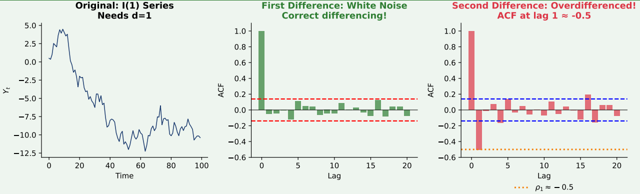
TSA_ch3_overdifferencing

Quiz 7: Overdifferencing

Question

If $Y_t \sim I(1)$ and we compute $\Delta^2 Y_t$, what happens?

Answer: B – Artificial negative autocorrelation



Diagnostic: ACF at lag 1 ≈ -0.5 signals overdifferencing. Reduce d by 1!

TSA_ch3_overdifferencing

Quiz 8: Forecast Variance

Question

For an ARIMA(0,1,0) model (random walk), how does forecast variance behave as horizon h increases?

A) Stays constant

B) Decreases to zero

C) Grows linearly with h

D) Converges to a finite limit

Quiz 8: Forecast Variance

Question

For an ARIMA(0,1,0) model (random walk), how does forecast variance behave as horizon h increases?

Answer: C – Grows linearly with h

Random walk forecast: $\hat{Y}_{T+h|T} = Y_T$ (best forecast is current value)
Forecast error: $Y_{T+h} - \hat{Y}_{T+h|T} = \sum_{i=1}^{h} \varepsilon_{T+i}$
Variance:

$$\text{Var}(Y_{T+h} - \hat{Y}_{T+h|T}) = h\sigma^2$$

95% CI: $Y_T \pm 1.96\sqrt{h}\sigma$ (widens with $\sqrt{h}$)

Quiz 9: Unit Root Test Power

Question

The ADF test has low power when:

A) Sample size is very large

B) The true root is close to but not equal to 1

C) The series has no trend

D) The series is clearly stationary

Quiz 9: Unit Root Test Power

Question

The ADF test has low power when:

Answer: B – Root close to but not equal to 1

Example: AR(1) with $\phi = 0.95$ vs random walk ($\phi = 1$) – similar ACF patterns!
Low power: High probability of Type II error (failing to reject false H_0)
Solutions: Larger samples, Phillips-Perron test, panel unit root tests

Quiz 10: ARIMA Model Selection

Question

After differencing once, the ACF shows a spike at lag 1 only, and PACF decays. The appropriate model is:

A) ARIMA(1,1,0)
B) ARIMA(0,1,1)
C) ARIMA(1,1,1)
D) ARIMA(0,2,1)

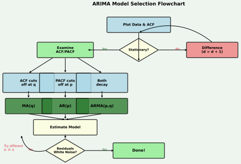
TSA_ch3_arima_flowchart

Quiz 10: ARIMA Model Selection

Question

After differencing once, the ACF shows a spike at lag 1 only, and PACF decays. The appropriate model is:

Answer: B – ARIMA(0,1,1)



Pattern: ACF cuts off at lag 1, PACF decays $\Rightarrow$ MA(1). Full model: ARIMA(0,1,1) = IMA(1,1)

TSA_ch3_arima_flowchart

Quiz 11: Trend Stationarity vs Difference Stationarity

Question

A trend-stationary process is made stationary by:

A) Taking first differences

B) Removing the deterministic trend via regression

C) Taking second differences

D) Applying seasonal adjustment

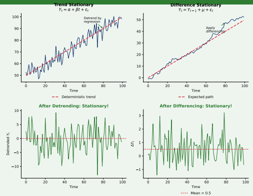
TSA_ch3_trend_vs_diff

Quiz 11: Trend Stationarity vs Difference Stationarity

Question

A trend-stationary process is made stationary by:

Answer: B – Removing deterministic trend via regression



Trend-stationary: Detrend (shocks are temporary). **Difference-stationary**: Difference (shocks are permanent).

TSA_ch3_trend_vs_diff

Quiz 12: ARIMA Invertibility

Question

ARIMA(0,1,1) with $\theta_1 = 1.2$ is:

A) Stationary and invertible
B) Non-stationary but invertible
C) Non-stationary and non-invertible
D) Stationary but non-invertible

Quiz 12: ARIMA Invertibility

Question

ARIMA(0,1,1) with $\theta_1 = 1.2$ is:

Answer: C – Non-stationary and non-invertible

Stationarity: $d = 1$ (unit root) $\Rightarrow$ Non-stationary
Invertibility: MA root $z = -1/1.2 = -0.833$ is inside unit circle; $|\theta_1| > 1 \Rightarrow$ Non-invertible
Fix: Rewrite with $\theta^* = 1/1.2 = 0.833$ and adjust variance.

Quiz 13: Spurious Regression

Question

Regressing one random walk on another independent random walk typically shows:

A) No significant relationship

B) High R^2 and significant t-statistics (spuriously)

C) Negative correlation

D) Perfect multicollinearity

Quiz 13: Spurious Regression

Question

Regressing one random walk on another independent random walk typically shows:

Answer: B – High R^2 and significant t-statistics (spuriously)

Granger & Newbold (1974): Spurious regression phenomenon
Symptoms: High R^2 (> 0.9), significant t-stats, low Durbin-Watson ($\ll 2$), non-stationary residuals
Solutions: Difference both series, or test for cointegration

Quiz 14: Long-Run Forecast

Question

The long-run forecast from ARIMA(1,1,0) with $\phi_1 = 0.7$ converges to:

A) Zero

B) The unconditional mean

C) A linear trend extrapolation

D) The last observed value

Quiz 14: Long-Run Forecast

Question

The long-run forecast from ARIMA(1,1,0) with $\phi_1 = 0.7$ converges to:

Answer: C – A linear trend extrapolation

Model: $(1-\phi_1 L)(1-L)Y_t = c + \varepsilon_t$; **Long-run**: $\hat{Y}_{T+h} \approx Y_T + h \cdot \frac{c}{1-\phi_1}$
Key differences: Stationary ARMA $\to$ mean; I(1) no drift $\to$ last value; I(1) with drift $\to$ linear

True/False Questions

Question

Determine if each statement is True or False:

1. An I(2) process requires two differences to become stationary.
2. The ADF test always includes a constant term.
3. ARIMA(0,1,0) is another name for a random walk.
4. Differencing a stationary series makes it “more stationary.”
5. The KPSS test has stationarity as the null hypothesis.
6. ARIMA models can only capture linear patterns.

Answer on next slide...

True/False: Solutions

Answers

1. An I(2) process requires two differences to become stationary. **TRUE**
   I(d) means d differences needed. I(2) = two unit roots.
2. The ADF test always includes a constant term. **FALSE**
   You choose: no constant, constant only, or constant + trend.
3. ARIMA(0,1,0) is another name for a random walk. **TRUE**
   $(1-L)Y_t = \varepsilon_t \Rightarrow Y_t = Y_{t-1} + \varepsilon_t$.
4. Differencing a stationary series makes it "more stationary." **FALSE**
   Over-differencing creates non-invertible MA; hurts model performance.
5. The KPSS test has stationarity as the null hypothesis. **TRUE**
   KPSS: H_0 = stationary. Opposite of ADF.
6. ARIMA models can only capture linear patterns. **TRUE**
   ARIMA is linear in parameters. Nonlinear patterns need GARCH, neural nets, etc.

Problem 1: Unit Root Testing

Exercise

You have quarterly GDP data for 80 quarters. The ADF test (with constant and trend) gives a test statistic of -2.85. The 5% critical value is -3.41.

1. What is your conclusion about stationarity?
2. What would you do next?

Problem 1: Unit Root Testing

Exercise

You have quarterly GDP data for 80 quarters. The ADF test (with constant and trend) gives a test statistic of -2.85. The 5% critical value is -3.41.

1. What is your conclusion about stationarity?
2. What would you do next?

Solution

1. Since $-2.85 > -3.41$, we **fail to reject** H_0. The data appears to have a unit root (non-stationary).
2. Take the first difference ΔY_t and repeat the ADF test on the differenced series to confirm it is now stationary.

Problem 2: Model Identification

Exercise

After differencing a time series once, the ACF shows:

- Significant spike at lag 1 ($\rho_1 = 0.4$)
- All other lags insignificant

The PACF shows gradual decay.
What ARIMA model is suggested?

Problem 2: Model Identification

Exercise

After differencing a time series once, the ACF shows:

- Significant spike at lag 1 ($\rho_1 = 0.4$)
- All other lags insignificant

The PACF shows gradual decay.
What ARIMA model is suggested?

Solution

- ACF cuts off after lag 1 $\Rightarrow$ MA(1) component
- PACF decays $\Rightarrow$ Confirms MA structure
- Since we differenced once: $d = 1$

Suggested model: ARIMA(0,1,1) or IMA(1,1)

Problem 3: ARIMA Equation

Exercise

Write out the full equation for ARIMA(1,1,1):

$$(1 - \phi_1 L)(1 - L)Y_t = c + (1 + \theta_1 L)\varepsilon_t$$

Expand this completely in terms of Y_t, Y_{t-1}, Y_{t-2}, etc.

Problem 3: ARIMA Equation

Exercise

Write out the full equation for ARIMA(1,1,1):

$$(1 - \phi_1 L)(1 - L)Y_t = c + (1 + \theta_1 L)\varepsilon_t$$

Expand this completely in terms of Y_t, Y_{t-1}, Y_{t-2}, etc.

Solution

Expanding $(1 - \phi_1 L)(1 - L) = 1 - L - \phi_1 L + \phi_1 L^2 = 1 - (1 + \phi_1)L + \phi_1 L^2$:

$$Y_t - (1 + \phi_1)Y_{t-1} + \phi_1 Y_{t-2} = c + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

Or equivalently:

$$Y_t = c + (1 + \phi_1)Y_{t-1} - \phi_1 Y_{t-2} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

Problem 4: Forecast Calculation

Exercise

Given ARIMA(0,1,1): $\Delta Y_t = \varepsilon_t + 0.3\varepsilon_{t-1}$
At time T: $Y_T = 100$, $\hat{\varepsilon}_T = 2$, $\sigma^2 = 4$
Calculate:

1. $\hat{Y}_{T+1|T}$ (one-step forecast)
2. $\hat{Y}_{T+2|T}$ (two-step forecast)

Problem 4: Forecast Calculation

Exercise

Given ARIMA(0,1,1): $\Delta Y_t = \varepsilon_t + 0.3\varepsilon_{t-1}$
At time T: $Y_T = 100$, $\hat{\varepsilon}_T = 2$, $\sigma^2 = 4$
Calculate:

1. $\hat{Y}_{T+1|T}$ (one-step forecast)
2. $\hat{Y}_{T+2|T}$ (two-step forecast)

Solution

1. $\hat{Y}_{T+1|T} = Y_T + 0.3\hat{\varepsilon}_T = 100 + 0.3(2) = 100.6$
2. $\hat{Y}_{T+2|T} = \hat{Y}_{T+1|T} + 0.3 \cdot 0 = 100.6 + 0 = 100.6$
   (Future shocks $\varepsilon_{T+1}, \varepsilon_{T+2}$ are forecast as 0)

Problem 5: Confidence Intervals

Exercise

Continuing from Problem 4, calculate the 95% forecast intervals for $\hat{Y}_{T+1|T}$ and $\hat{Y}_{T+2|T}$.
Recall: $\sigma^2 = 4$, $\theta_1 = 0.3$

Problem 5: Confidence Intervals

Exercise

Continuing from Problem 4, calculate the 95% forecast intervals for $\hat{Y}_{T+1|T}$ and $\hat{Y}_{T+2|T}$.
Recall: $\sigma^2 = 4$, $\theta_1 = 0.3$

Solution

For IMA(1,1), the MA(∞) weights are $\psi_0 = 1$, $\psi_j = 1 + \theta_1$ for $j \geq 1$.
1-step: $\text{Var}(e_{T+1}) = \sigma^2\psi_0^2 = 4$, so $SE = 2$
$100.6 \pm 1.96(2) = [96.68, 104.52]$
2-step: $\text{Var}(e_{T+2}) = \sigma^2(\psi_0^2 + \psi_1^2) = 4(1 + 1.3^2) = 10.76$, $SE = 3.28$
$100.6 \pm 1.96(3.28) = [94.17, 107.03]$

Example: Testing for Unit Root in Stock Prices

Scenario

You have daily closing prices for a stock over 500 days. You want to determine if prices follow a random walk.

Step-by-step Approach

1. **Visual inspection**: Plot prices – likely shows trend
2. **ADF test on prices**: Expect to fail to reject H_0 (unit root)
3. **Take log returns**: $r_t = \ln(P_t/P_{t-1}) = \Delta \ln(P_t)$
4. **ADF test on returns**: Should reject H_0 (stationary)
5. **Conclusion**: Log prices are $I(1)$, returns are $I(0)$

Example: Box-Jenkins for Inflation Data

Scenario

Monthly inflation rates for 10 years. Build an ARIMA model.

Workflow

1. **Plot & test**: ADF suggests borderline – try both $d = 0$ and $d = 1$
2. **If** $d = 0$: Fit ARMA models, compare AIC
3. **If** $d = 1$: Examine ACF/PACF of ΔY_t
   - ACF: spike at lag 1, then cuts off
   - PACF: decays
   - $\Rightarrow$ Try ARIMA(0,1,1)
4. **Estimate**: Fit ARIMA(0,1,1), check coefficients
5. **Diagnose**: Ljung-Box on residuals (want $p > 0.05$)
6. **Compare**: AIC of ARIMA(0,1,1) vs ARMA(1,1) on levels

Example: Interpreting Python Output

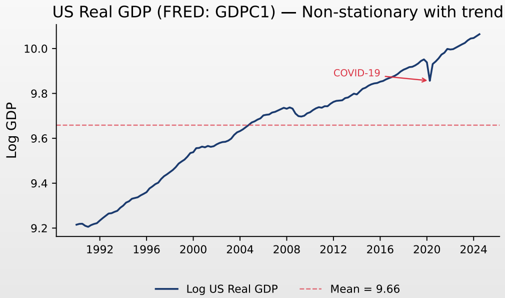
statsmodels ARIMA Output

```
                    ARIMA Model Results
==============================================================
Dep. Variable:              D.y   No. Observations:    99
Model:               ARIMA(1,1,1)    AIC                   285.32
                                  BIC                   295.63
==============================================================
                 coef     std err      z      P>|z|
--------------------------------------------------------------
const          0.0521     0.048    1.085    0.278
ar.L1          0.4532     0.102    4.443    0.000
ma.L1         -0.2891     0.118   -2.450    0.014
sigma2         1.2340     0.176    7.011    0.000
```

Interpretation

- AR coefficient (0.45) is significant, MA coefficient (-0.29) is significant
- Constant (0.052) not significant – could set $c = 0$
- Check: $|\phi_1| < 1$ (stationary), $|\theta_1| < 1$ (invertible) – OK!

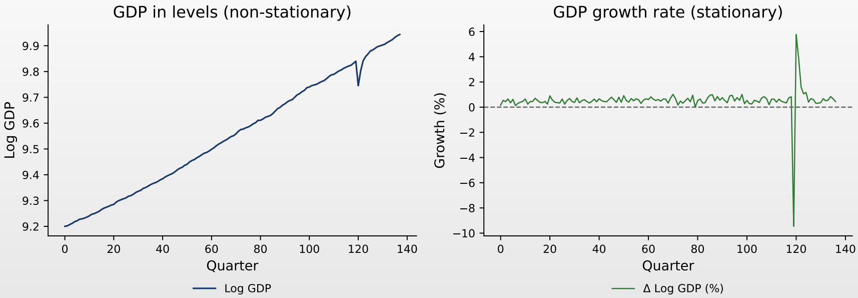
Case Study: US Real GDP (1990–2024)



- US Real GDP in billions of 2017 dollars (quarterly data)
- Clear **upward trend** – typical of macroeconomic series
- Notable drops during recessions (2008-2009, 2020)
- Non-stationary: needs differencing before ARIMA modeling

TSA_ch3_gdp_levels

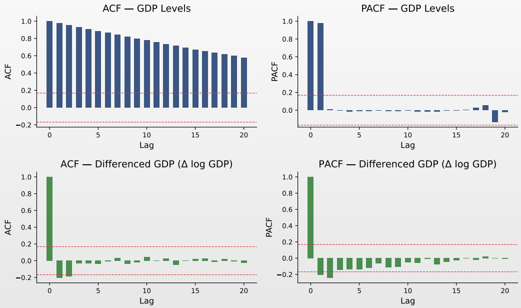
Stationarity Through Differencing



- **Left**: GDP in levels – clear upward trend (non-stationary)
- **Right**: GDP growth rate $= \Delta \log(Y_t) \times 100$ – stationary
- First differencing of log GDP removes the stochastic trend
- Growth rate fluctuates around a constant mean ($\approx 0.6\%$ quarterly)

TSA_ch3_differencing

ACF/PACF: Levels vs Differenced



- **Top row**: ACF/PACF of GDP levels – slow decay indicates non-stationarity
- **Bottom row**: ACF/PACF of GDP growth – mostly within confidence bands
- Pattern suggests low-order ARIMA model is appropriate

TSA_ch3_acf_pacf

ARIMA Estimation Results: US GDP Growth

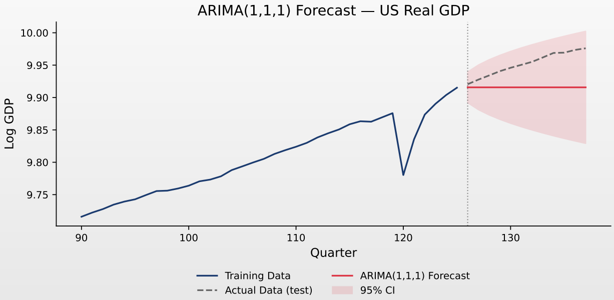
Model: ARIMA$(1, 1, 1)$ on $\log(\text{GDP})$

Parameter	Estimate	Std. Error	z-stat	p-value
ϕ_1 (AR.L1)	0.312	0.185	1.69	0.091
θ_1 (MA.L1)	-0.087	0.203	-0.43	0.668
σ^2	0.00012	–	–	–

Interpretation

- Low-order ARIMA captures GDP dynamics reasonably well
- AR(1) coefficient positive – GDP growth shows persistence
- Alternative: simpler random walk (ARIMA(0,1,0)) often competitive

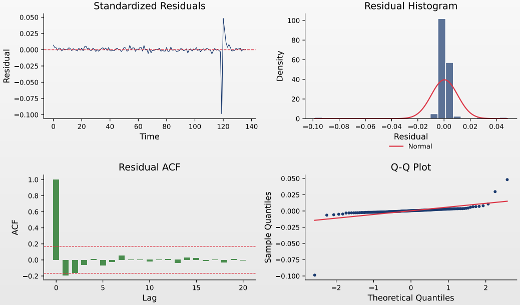
Forecast: ARIMA vs Actual



- Blue: historical training data; Green: actual test data
- Red dashed: ARIMA forecasts with 95% confidence interval
- Forecasts capture the general trend direction
- Confidence intervals widen as forecast horizon increases

TSA_ch3_arima_forecast

Model Diagnostics: Residual Analysis



- Residuals show no systematic patterns over time
- Distribution approximately normal (histogram and Q-Q plot)
- ACF of residuals within bounds – no significant autocorrelation remaining
- Model adequately captures the data generating process

TSA_ch3_diagnostics

Discussion: Deterministic vs Stochastic Trends

Key Question

Why is it important to distinguish between deterministic and stochastic trends?

Discussion Points

- **Wrong treatment consequences**:
  - Detrending a unit root $\Rightarrow$ spurious stationarity
  - Differencing a trend-stationary $\Rightarrow$ overdifferencing
- **Economic interpretation**:
  - Deterministic trend: shocks are temporary
  - Stochastic trend: shocks have permanent effects
- **Policy implications**:
  - Does a recession permanently lower GDP, or does the economy return to trend?

Discussion: Model Selection Criteria

Key Question

When should you use AIC vs BIC for ARIMA model selection?

Considerations

- **AIC**: Minimizes prediction error, may overfit
  - Better for forecasting
  - Tends to select larger models
- **BIC**: Consistent model selection, more parsimonious
  - Better for identifying "true" model
  - Penalizes complexity more heavily
- **Practical advice**: Report both, prefer BIC if they disagree substantially

Discussion: Limitations of ARIMA

Key Question

What are the main limitations of ARIMA models?

Discussion Points

- **Linearity**: Cannot capture nonlinear dynamics
- **Constant variance**: Assumes homoskedasticity (no GARCH effects)
- **No structural breaks**: Parameters assumed constant
- **Univariate**: Ignores relationships with other variables
- **Symmetric**: Treats positive and negative shocks equally
- **Long-horizon forecasts**: Uncertainty grows rapidly

Extensions

These limitations motivate GARCH (volatility), VAR (multivariate), regime-switching models, etc.

Key Points from Today's Seminar

What We Covered

1. **Integration and differencing**: $I(d)$ processes require d differences
2. **Unit root testing**: ADF tests H_0: unit root; KPSS tests H_0: stationary
3. **ARIMA(p,d,q)**: Combines ARMA with differencing
4. **Model identification**: Use ACF/PACF patterns and information criteria
5. **Forecasting**: Point forecasts and growing confidence intervals

Next Seminar

Hands-on Python exercises with real economic data:

- Unit root testing with `statsmodels`
- Auto-ARIMA with `pmdarima`
- Forecasting and model diagnostics

AI in ARIMA Modeling

Context

AI tools can fit ARIMA models and generate forecasts automatically. The critical skill is **evaluating whether the methodology is correct**.

Key questions to ask about any AI-generated ARIMA analysis:

1. Did it test for unit roots **before** choosing the differencing order?
2. Is the differencing order d justified by ADF/KPSS tests?
3. Did it check for overdifferencing (ACF lag $1 \approx -0.5$)?
4. Are residuals white noise (Ljung-Box test)?
5. Do forecast confidence intervals widen appropriately with horizon?

AI Exercise 1: Critique an AI ARIMA Analysis

Scenario

You asked an AI: "Fit the best ARIMA model to this GDP data." It returned:

- Fitted ARIMA(3,2,2) with AIC = 315.7
- No unit root test performed before differencing
- Applied $d = 2$ "to be safe"
- Ljung-Box p-value = 0.04 (reported as "close enough")
- 10-year quarterly forecast with narrow confidence intervals

Your critique:

1. Why is choosing $d = 2$ without testing dangerous? What is overdifferencing?
2. Why is Ljung-Box p = 0.04 **not** acceptable at 5% level?
3. Is ARIMA(3,2,2) likely over-parameterized? What would BIC suggest?
4. What is the correct Box-Jenkins methodology that was skipped?

AI Exercise 2: Prompt Refinement for ARIMA

Task

Iteratively improve prompts for fitting an ARIMA model to US GDP data.

Round 1 (vague): *"Fit a time series model to quarterly GDP"*

- What did the AI produce? Did it test stationarity? What's missing?

Round 2 (better): *"Test stationarity with ADF, determine differencing order, fit ARIMA using BIC, check residuals with Ljung-Box"*

- Did the AI follow the Box-Jenkins methodology correctly?

Round 3 (expert): *"Follow Box-Jenkins: (1) plot series & test ADF/KPSS, (2) difference if needed & re-test, (3) identify p,q from ACF/PACF, (4) estimate ARIMA(p,1,q), (5) Ljung-Box on residuals, (6) rolling 1-step forecast on last 20 obs with RMSE"*

- Compare results across all three rounds

AI Exercise 3: Model Selection Competition

Task

Download US Real GDP data (quarterly) using `pandas_datareader` or FRED.

Your approach (manual):

- ADF/KPSS tests → determine d
- ACF/PACF of differenced series → candidate models
- Compare AIC/BIC across ARIMA(1,1,0), ARIMA(0,1,1), ARIMA(1,1,1)
- Residual diagnostics for selected model
- Rolling 1-step forecast on last 20 observations

AI approach:

- Ask AI to "find the best ARIMA model and forecast GDP"

Compare:

- Which differencing order and model did each select?
- Compare out-of-sample RMSE; did the AI check for overdifferencing?
- **Submit:** 1-page reflection on AI strengths and weaknesses

Key Formulas Summary

Concept	Formula
Random walk	$Y_t = Y_{t-1} + \varepsilon_t$
Random walk variance	$\text{Var}(Y_t) = t\sigma^2$
ARIMA(p, d, q)	$\phi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t$
First difference	$\Delta Y_t = Y_t - Y_{t-1} = (1-L)Y_t$
Second difference	$\Delta^2 Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$
ADF regression	$\Delta Y_t = \alpha + \gamma Y_{t-1} + \sum \delta_j \Delta Y_{t-j} + \varepsilon_t$
ADF null	$H_0 : \gamma = 0$ (unit root)
RW forecast	$\hat{Y}_{T+h\|T} = Y_T$
RW forecast CI	$Y_T \pm z_{\alpha/2}\sqrt{h}\,\sigma$
AIC	$-2\ln(\hat{L}) + 2k$
BIC	$-2\ln(\hat{L}) + k\ln(n)$

Notation: $\hat{L}$ = maximum of the likelihood function, k = no. of parameters, n = sample size, σ^2 = white noise variance

What's Next?

Seminar 4: SARIMA Models for Seasonal Data

- **Seasonality**: repetitive patterns at regular intervals
- **Seasonal differencing**: the $(1 - L^s)$ operator
- **SARIMA$(p, d, q)(P, D, Q)_s$**: the seasonal extension of ARIMA
- **Case study**: Airline passenger forecasting with Python

Questions?

Bibliography I

Fundamental textbooks

- Hyndman, R.J., & Athanasopoulos, G. (2021). *Forecasting: Principles and Practice*, 3rd ed., OTexts.
- Shumway, R.H., & Stoffer, D.S. (2017). *Time Series Analysis and Its Applications*, 4th ed., Springer.
- Brockwell, P.J., & Davis, R.A. (2016). *Introduction to Time Series and Forecasting*, 3rd ed., Springer.

Financial time series

- Tsay, R.S. (2010). *Analysis of Financial Time Series*, 3rd ed., Wiley.
- Franke, J., Härdle, W.K., & Hafner, C.M. (2019). *Statistics of Financial Markets*, 4th ed., Springer.

Bibliography II

Modern approaches and Machine Learning

- Nielsen, A. (2019). *Practical Time Series Analysis*, O'Reilly Media.
- Petropoulos, F., et al. (2022). *Forecasting: Theory and Practice*, International Journal of Forecasting.
- Makridakis, S., Spiliotis, E., & Assimakopoulos, V. (2020). The M4 Competition, International Journal of Forecasting.

Online resources and code

- **Quantlet**: `https://quantlet.com` — Code repository for statistics
- **Quantinar**: `https://quantinar.com` — Learning platform for quantitative methods
- **GitHub TSA**: `https://github.com/QuantLet/TSA/tree/main/TSA_ch3` — Python code for this seminar