



# Time Series Analysis and Forecasting

Chapter 9: Prophet and TBATS



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## Learning Objectives

By the end of this chapter, you will be able to:

- Handle time series with multiple seasonal patterns
- Use Facebook Prophet for flexible forecasting with holidays
- Apply TBATS models for complex seasonality
- Compare and select between modern forecasting methods



## Outline

Multiple Seasonalities

TBATS Model

Facebook Prophet

Comparison and Guidelines

Case Study

AI Use Case

Quiz

Summary



## The Problem: Complex Seasonal Patterns

### Real-World Examples

- **Hourly electricity demand:** Daily + Weekly + Annual patterns
- **Website traffic:** Daily + Weekly + Holiday effects
- **Retail sales:** Weekly + Monthly + Annual + Holiday effects
- **Call center volume:**
  - ▶ Hourly + Daily + Weekly patterns

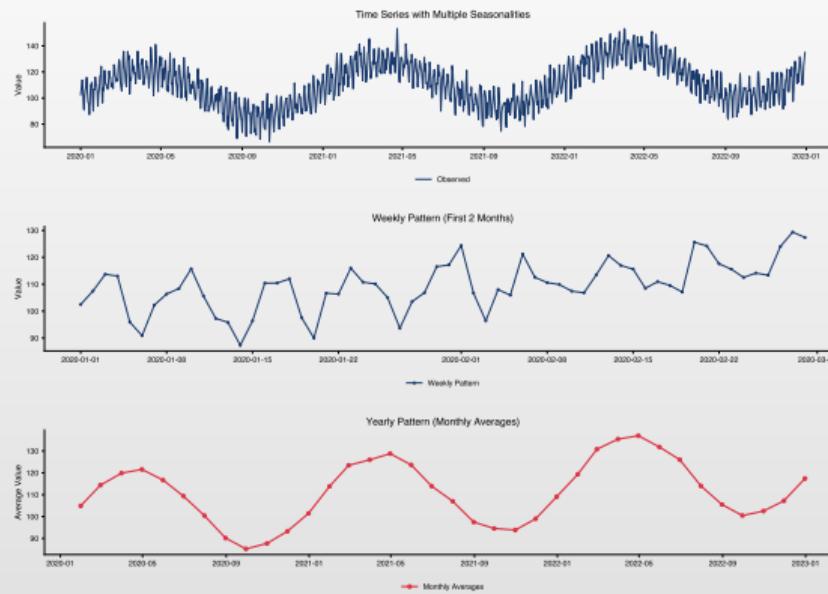
### SARIMA Limitation

Standard SARIMA( $p, d, q)(P, D, Q)_s$  handles only **one** seasonal period  $s$ .

For hourly data with daily AND weekly patterns, we need  $s_1 = 24$  and  $s_2 = 168$ .



## Example: Hourly Data with Multiple Seasonalities



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## Solutions for Multiple Seasonalities

### Traditional Approaches

- Fourier terms:** Add sin/cos regressors
- Dummy variables:** Many parameters
- Nested models:** Complex specification

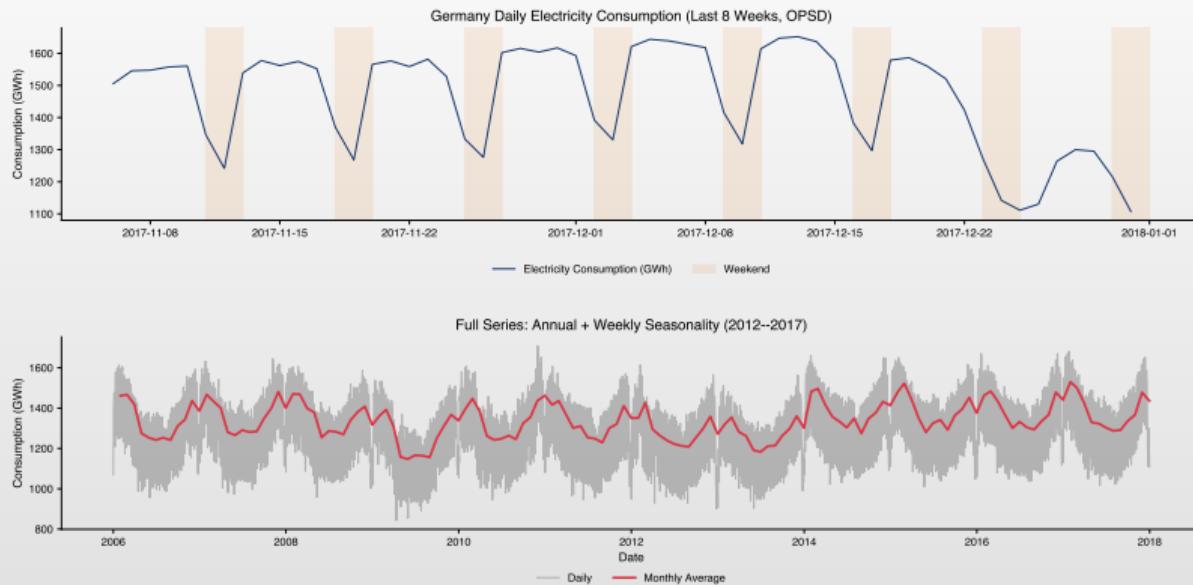
### Modern Approaches

- TBATS:** Automatic, handles many periods
- Prophet:** Flexible, interpretable
- Neural methods:**
  - ▶ Deep learning

Method	Max Seasonalities	Interpretable
SARIMA	1	Yes
Fourier + ARIMA	Multiple	Moderate
TBATS	Multiple	Moderate
Prophet	Multiple	Yes



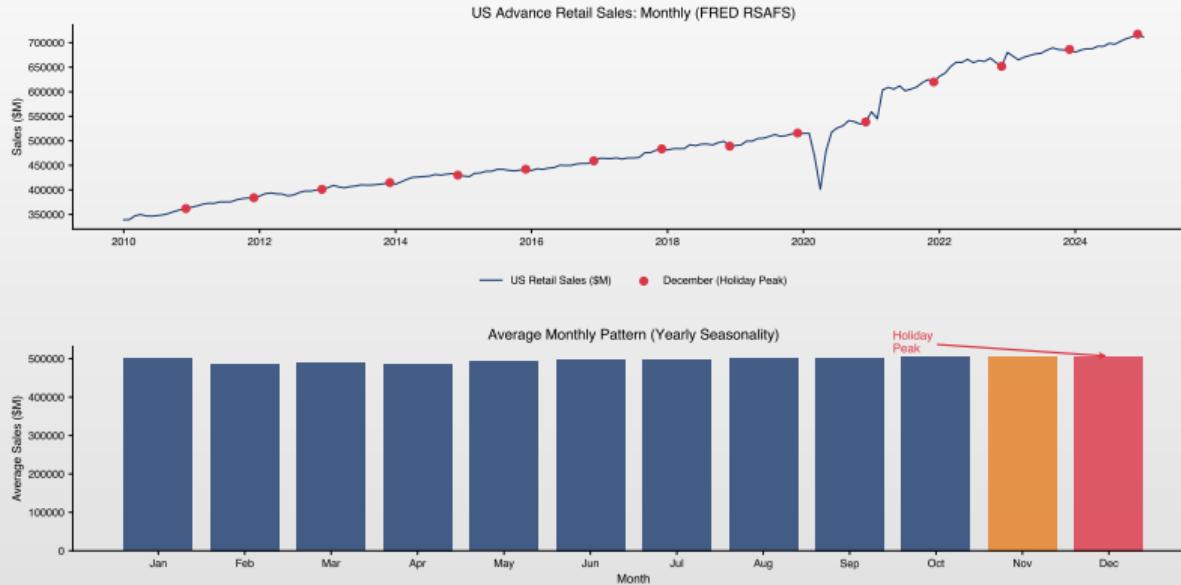
## Real Example: Electricity Demand



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## Real Example: Retail Sales with Holidays



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## Researcher Spotlight: Rob J. Hyndman



\*1967  
W Wikipedia

### Biography

- Australian statistician, Professor at Monash University
- One of the most influential researchers in time series forecasting
- Creator of the widely-used `forecast` package for R
- Editor-in-Chief of the *International Journal of Forecasting* (2005–2018)

### Key Contributions

- **TBATS model** (2011) — trigonometric Box-Cox ARMA with multiple seasonal periods
- **ETS framework** — exponential smoothing state space models with automatic selection
- **forecast package** for R — the standard toolkit for time series forecasting
- **Hierarchical forecasting** and forecast reconciliation methods

## TBATS: What Does It Stand For?

### TBATS Components

- T** Trigonometric seasonality using Fourier terms
- B** Box-Cox transformation for variance stabilization
- A** ARMA errors for remaining autocorrelation
- T** Trend component (possibly damped)
- S** Seasonal components (multiple allowed)

### Key Innovation: Trigonometric Seasonality

$$s_t^{(i)} = \sum_{j=1}^{k_i} \left[ s_j^{(i)} \cos\left(\frac{2\pi j t}{m_i}\right) + s_j^{*(i)} \sin\left(\frac{2\pi j t}{m_i}\right) \right]$$

$m_i$  = seasonal period,  $k_i$  = number of harmonics



## Box-Cox Transformation

### Definition 1 (Box-Cox Transformation)

The Box-Cox transformation with parameter  $\omega$  is defined as:

$$y_t^{(\omega)} = \begin{cases} \frac{y_t^\omega - 1}{\omega} & \text{if } \omega \neq 0 \\ \ln(y_t) & \text{if } \omega = 0 \end{cases}$$

### Purpose

- Variance stabilization:** Makes variance constant over time
- Normalization:** Reduces skewness in the data
- Common values:  $\omega = 0$  (log),  $\omega = 0.5$  (square root),  $\omega = 1$  (no transform)



## TBATS Model Structure

### State Space Representation

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^T s_{t-m_i}^{(i)} + d_t \quad (1)$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t, \quad b_t = \phi b_{t-1} + \beta d_t \quad (2)$$

- $y_t^{(\omega)}$ : Box-Cox transformed observation
- $\ell_t$ : local level (smoothed mean)
- $b_t$ : trend with damping  $\phi \in (0, 1)$
- $s_t^{(i)}$ :  $i$ -th seasonal component
- $d_t$ : ARMA( $p, q$ ) error process
- $\alpha, \beta$ : smoothing parameters



## TBATS: Trigonometric Seasonality State Evolution

### Definition 2 (Trigonometric State-Space Recursion)

For each seasonal component with period  $m_i$  and  $k_i$  harmonics, define states:

$$\begin{pmatrix} s_{j,t}^{(i)} \\ s_{j,t}^{*(i)} \end{pmatrix} = \begin{pmatrix} \cos(\lambda_j) & \sin(\lambda_j) \\ -\sin(\lambda_j) & \cos(\lambda_j) \end{pmatrix} \begin{pmatrix} s_{j,t-1}^{(i)} \\ s_{j,t-1}^{*(i)} \end{pmatrix} + \begin{pmatrix} \gamma_1^{(i)} \\ \gamma_2^{(i)} \end{pmatrix} d_t$$

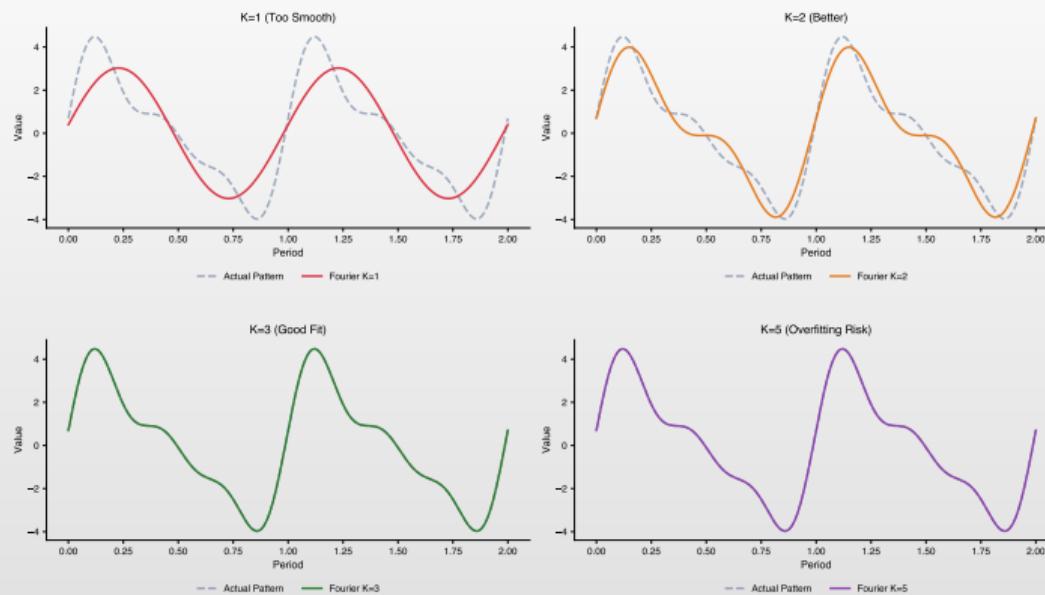
where  $\lambda_j = \frac{2\pi j}{m_i}$  is the  $j$ -th harmonic frequency.

### Interpretation

- The rotation matrix preserves the periodic structure
- Total seasonal:  $s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$
- Parameters:  $2k_i$  states per seasonal period



## Fourier Approximation of Seasonality



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## TBATS: Choosing the Number of Harmonics

### Why Fourier/Trigonometric Terms?

1. **Parsimonious:**  $2k$  parameters vs  $m$  dummy variables
2. **Smooth:** Captures smooth seasonal patterns naturally
3. **Flexible:** Number of harmonics  $k$  controls complexity
4. **Non-integer periods:** Can handle  $s = 365.25$  for daily data

#### Low $k$ (few harmonics)

- Smooth pattern
- Fewer parameters
- May miss sharp peaks

#### High $k$ (many harmonics)

- Can capture any pattern
- More parameters ( $2k$  total)
- Maximum useful:  $k \leq \lfloor m/2 \rfloor$



## TBATS: Key Features

### Automatic Model Selection

TBATS automatically determines:

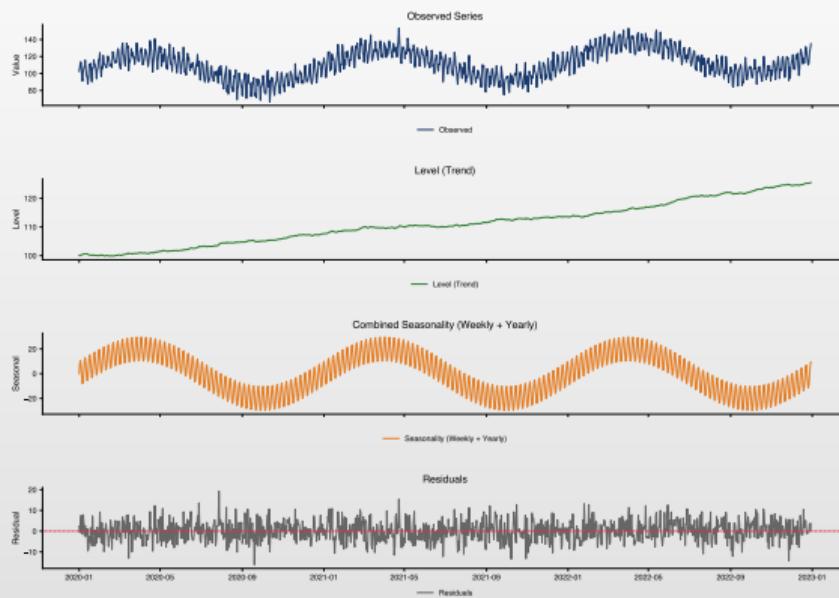
- Box-Cox parameter  $\omega$  for variance stabilization
- Number of harmonics  $k_i$  for each seasonal period
- ARMA orders  $(p, q)$  for residual autocorrelation
- Damped vs non-damped trend specification

### BATS vs TBATS

- BATS**: Traditional seasonal states (dummy variables)
- TBATS**: Trigonometric (Fourier) seasonal representation
- TBATS more parsimonious for long seasonal periods



## TBATS Decomposition Example



 TSA\_ch9\_tbats\_decomposition



## TBATS: Advantages and Limitations

### Advantages

- Handles **multiple** seasonal periods
- Automatic** model selection
- Handles **non-integer** periods (365.25)
- Box-Cox** for heteroskedasticity
- Good for **high-frequency** data

### Limitations

- Computationally intensive**
- No external regressors**
- Less **interpretable** than Prophet
- Can be **slow** for very long series
- Requires **sufficient data per season**



## Prophet: Overview

### What is Prophet?

Forecasting procedure developed by Facebook (Meta) in 2017 for **business time series**:

- Strong seasonal effects (daily, weekly, yearly)
- Holiday effects and trend changes (changepoints)
- Handles missing data and outliers

### Key Philosophy: “Analyst-in-the-loop”

Designed for analysts with domain knowledge but without time series expertise.



## Prophet Model Structure

### Decomposition Approach

Prophet uses an **additive decomposition**:

$$y(t) = g(t) + s(t) + h(t) + \varepsilon_t$$

#### $g(t)$ : Trend

- Linear or logistic
- Automatic changepoints
- Growth saturation

#### $s(t)$ : Seasonality

- Fourier series
- Multiple periods
- Custom seasonality

#### $h(t)$ : Holidays

- Country holidays
- Custom events
- Window effects



## Prophet: Seasonality Component

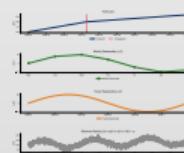
### Fourier Series Representation

$$s(t) = \sum_{n=1}^N \left[ a_n \cos\left(\frac{2\pi n t}{P}\right) + b_n \sin\left(\frac{2\pi n t}{P}\right) \right]$$

### Default Settings

Seasonality	Period	Fourier Order
Yearly	365.25 days	10
Weekly	7 days	3
Daily	1 day	4

Higher  $N$  = more flexibility but risk of overfitting



## Prophet: Trend Component

### Linear Trend with Changepoints

$$g(t) = (k + \mathbf{a}(t)^T \boldsymbol{\delta}) \cdot t + (m + \mathbf{a}(t)^T \boldsymbol{\gamma})$$

- $k$ : base growth rate (slope)
- $\boldsymbol{\delta} = (\delta_1, \dots, \delta_S)$ : slope changes at  $S$  changepoints
- $\mathbf{a}(t) \in \{0, 1\}^S$ : indicator if changepoint  $s$  is active at time  $t$

### Logistic Growth

For saturating trends:

$$g(t) = \frac{C(t)}{1 + e^{-(k + \mathbf{a}(t)^T \boldsymbol{\delta})(t - m - \mathbf{a}(t)^T \boldsymbol{\gamma})}}$$

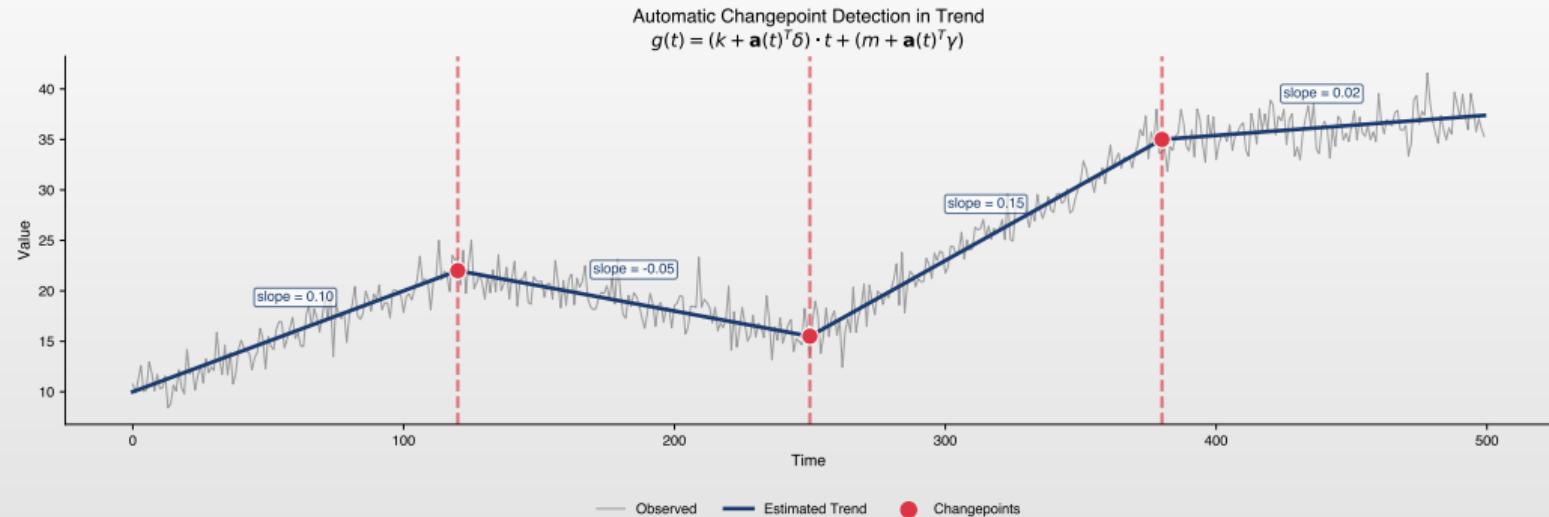
$C(t)$  = time-varying carrying capacity

### Continuity Constraint

The offset  $\gamma_j = -s_j \cdot \delta_j$  ensures  $g(t)$  is continuous at each changepoint  $s_j$ .



## Trend Changepoint Detection



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## Prophet: Holiday Effects

### Holiday Model

$$h(t) = Z(t) \cdot \kappa$$

where  $Z(t)$  is an indicator matrix for holidays and  $\kappa$  are holiday effects.

### Built-in Features

- 60+ countries supported
- Custom holiday definitions
- Window effects (before/after)

### Holiday Types

- National holidays
- Religious observances
- Business events



## Prophet: Customization Options

### Seasonality Customization

- Add custom seasonal periods (monthly, quarterly)
- Control Fourier order for each seasonality
- Enable/disable default seasonalities

### External Regressors

Prophet supports adding external variables:

- Weather data, promotions, special events
- Binary or continuous regressors
- Automatic regularization



## Prophet: Uncertainty Quantification

### Bayesian Framework

Prophet uses a **Laplace prior** on changepoint magnitudes:

$$\delta_j \sim \text{Laplace}(0, \tau), \quad \tau = \text{changepoint\_prior\_scale}$$

Smaller  $\tau$  = sparser, smaller changepoints (more regularization).

### Sources of Uncertainty

1. **Trend:** Future changepoints
2. **Seasonality:** Coefficient variance
3. **Observation:** Residual noise  $\sigma^2$

### Prediction Intervals

- MAP estimation for point forecasts
- Monte Carlo sampling for intervals
- Default: 80% credible interval



## Prophet: Tuning Parameters

### Key Parameters

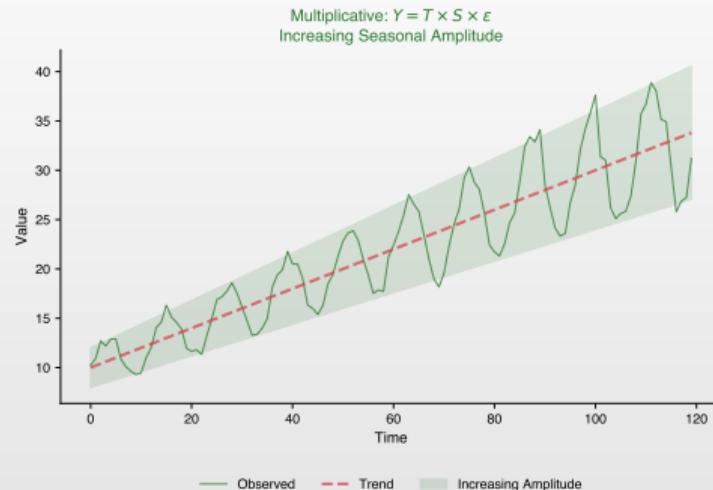
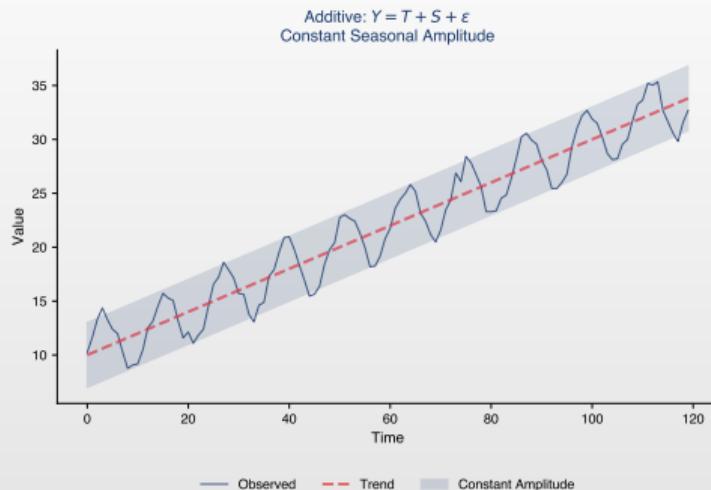
Parameter	Effect
changepoint_prior_scale	Trend flexibility (default: 0.05)
seasonality_prior_scale	Seasonality flexibility (default: 10)
holidays_prior_scale	Holiday effect size (default: 10)
seasonality_mode	'additive' or 'multiplicative'
changepoint_range	Portion of history for changepoints

### Practical Tips

- Overfitting trend?** Decrease `changepoint_prior_scale`
- Underfitting seasonality?** Increase `seasonality_prior_scale`
- Seasonal amplitude varies?** Use `seasonality_mode='multiplicative'`



## Additive vs Multiplicative Seasonality



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## Prophet: Advantages and Limitations

### Advantages

- Easy to use:** Minimal tuning needed
- Interpretable:** Clear decomposition
- Handles missing data well**
- Holiday effects built-in**
- Multiple seasonalities**
- External regressors supported**
- Fast fitting**

### Limitations

- Not ARIMA-based:** No autocorrelation modeling
- Daily data focus:** Less suited for very high frequency
- Trend assumptions:** Linear/logistic may not fit
- No built-in CV:** Must implement manually
- Overfitting risk with many seasonalities**



## TBATS vs Prophet: Head-to-Head

Feature	TBATS	Prophet
Multiple seasonalities	Yes (automatic)	Yes (manual or auto)
Holiday effects	No	Yes (built-in)
External regressors	No	Yes
Trend changepoints	No (smooth)	Yes (automatic)
Missing data	Interpolation needed	Handles natively
Interpretability	Moderate	High
Computation speed	Slow	Fast
High-frequency data	Good	Moderate
Non-integer periods	Yes (e.g., 365.25)	Yes
Uncertainty intervals	Yes	Yes



## When to Use Each Model

### Use TBATS when:

- High-frequency data
- Multiple seasonal periods
- No external regressors
- Automatic selection preferred

### Use Prophet when:

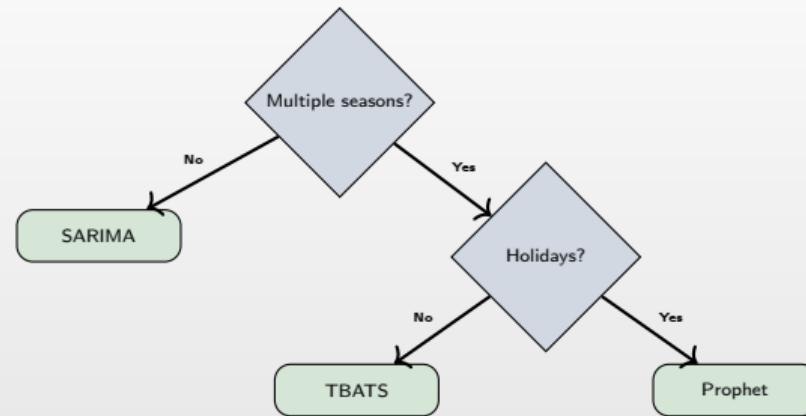
- Business forecasting
- Holiday effects important
- Trend has changepoints
- External regressors available

### General Guideline

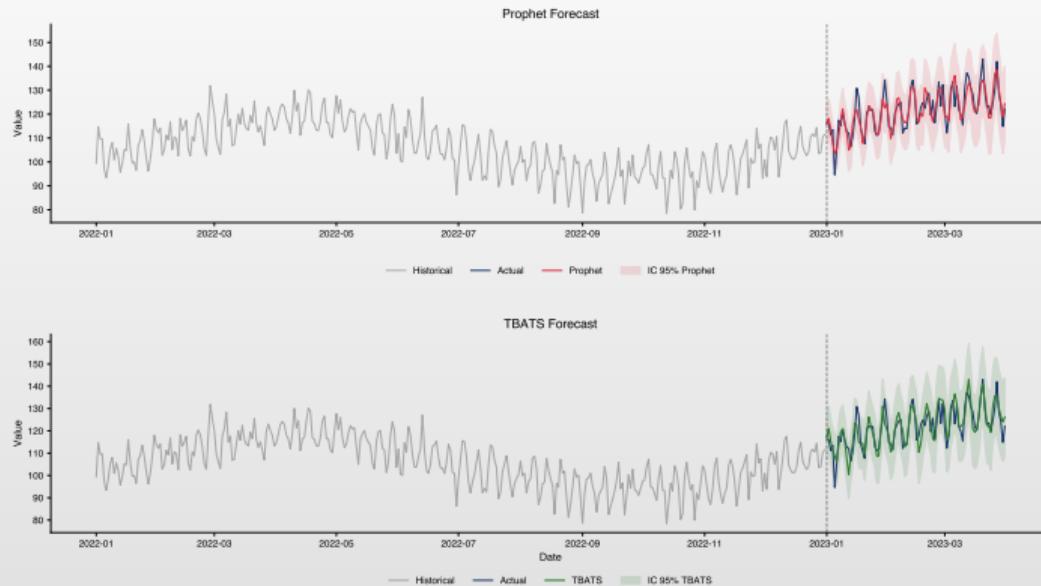
Prophet: business applications with daily data  
TBATS: technical applications with high-frequency data



## Decision Flowchart



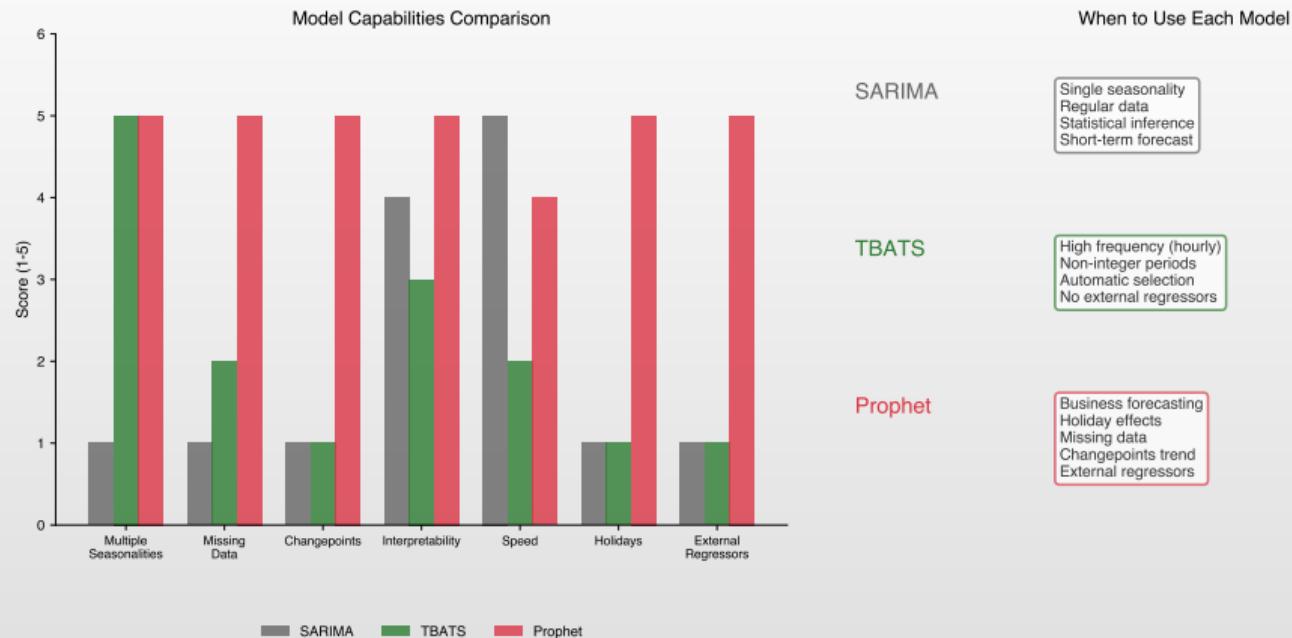
## Prophet vs TBATS: Forecast Comparison



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## Model Selection Guide



## Evaluation Metrics

### Definition 3 (Forecast Accuracy Metrics)

Let  $y_t$  denote actual values,  $\hat{y}_t$  forecasts, and  $n$  the forecast horizon:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{t=1}^n (y_t - \hat{y}_t)^2} \quad (\text{penalizes large errors})$$

$$\text{MAE} = \frac{1}{n} \sum_{t=1}^n |y_t - \hat{y}_t| \quad (\text{robust to outliers})$$

$$\text{MAPE} = \frac{100\%}{n} \sum_{t=1}^n \left| \frac{y_t - \hat{y}_t}{y_t} \right| \quad (\text{scale-free})$$

## Coverage

For prediction intervals  $[\hat{y}_t^L, \hat{y}_t^U]$ , coverage rate is the proportion of actual values falling within the interval. Target: match the nominal level (e.g., 80%).



## Case Study: Energy Demand Forecasting

### Problem

Forecast hourly electricity demand with:

- Daily pattern:** Peak at noon and evening
- Weekly pattern:** Lower on weekends
- Annual pattern:** Higher in summer (AC) and winter (heating)
- Holiday effects:** Lower demand on holidays

### Approach

1. Try TBATS with periods [24, 168, 8766]
2. Try Prophet with daily, weekly, yearly seasonality + holidays
3. Compare using cross-validation



## Case Study: Results

### Model Comparison

Model	MAPE	RMSE	Coverage
SARIMA (daily only)	8.5%	450 MW	75%
TBATS	4.2%	220 MW	82%
Prophet	4.8%	250 MW	85%
Prophet + holidays	3.9%	200 MW	88%

### Key Finding

Multiple seasonality models significantly outperform single-seasonality SARIMA.



## AI Exercise: Critical Thinking

Prompt to test in ChatGPT / Claude / Copilot

"I have 3 years of hourly energy consumption data. Use Facebook Prophet to forecast the next week. Include holidays and special events. Give me complete Python code."

**Exercise:**

1. Run the prompt in an LLM of your choice and critically analyze the response.
2. Does Prophet automatically detect multiple seasonalities (daily, weekly)?
3. How are holidays specified? Country-specific or custom events?
4. Does it use cross-validation with cutoffs (performance metrics)?
5. Would TBATS be more appropriate for this frequency? Why or why not?

**Warning:** AI-generated code may run without errors and look professional. *That does not mean it is correct.*



## Question 1

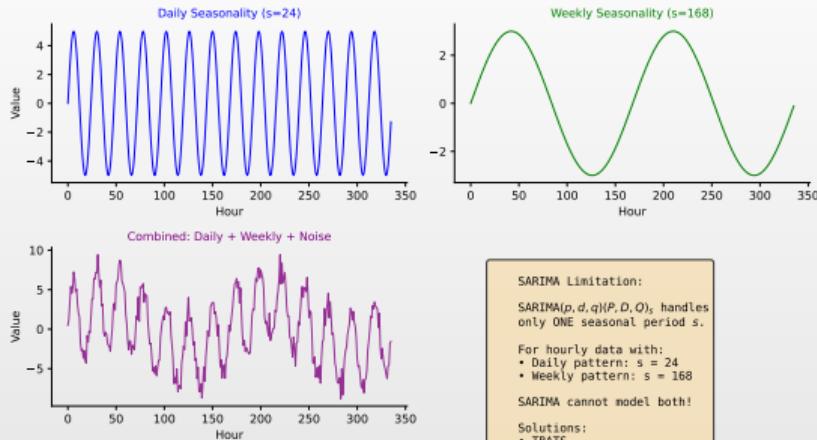
### Question

- Why can't standard SARIMA( $p, d, q$ )( $P, D, Q$ ) $_s$  model hourly electricity data with both daily and weekly patterns?

### Answer Choices

- (A) SARIMA can only handle one seasonal period  $s$  at a time
- (B) SARIMA requires normally distributed errors for multiple seasonalities
- (C) SARIMA can handle multiple seasonalities but requires more data
- (D) SARIMA only works with monthly or quarterly data

## Question 1: Answer



SARIMA Limitation:  
SARIMA( $p, d, q|P, D, Q$ )<sub>s</sub> handles  
only **ONE** seasonal period  $s$ .  
  
For hourly data with:  
• Daily pattern:  $s = 24$   
• Weekly pattern:  $s = 168$   
  
SARIMA cannot model both!  
  
Solutions:  
• TBATS  
• Prophet  
• Fourier terms + ARIMA

Answer: (A)

- SARIMA handles only **one** seasonal period  $s$ . You cannot set  $s = 24$  (daily) and  $s = 168$  (weekly) simultaneously in a single SARIMA model.



## Question 2

### Question

- What does each letter in TBATS represent?

### Answer Choices

- (A)** Trend, Bayes, Autoregressive, Time, Stationarity
- (B)** Trigonometric seasonality, Box-Cox, ARMA errors, Trend, Seasonal components
- (C)** Taylor, Box-Cox, ARIMA, Transformation, Smoothing
- (D)** Trigonometric, Bayesian, ARMA, Trend, Spectral analysis



## Question 2: Answer

### TBATS: What Does It Stand For?

<b>T</b>	<b>Trigonometric</b>	Fourier terms for seasonality $\sum [a_n \cos(\frac{2\pi nt}{m}) + b_n \sin(\frac{2\pi nt}{m})]$
<b>B</b>	<b>Box-Cox</b>	Variance stabilization $y^{(w)} = (y^w - 1)/w$
<b>A</b>	<b>ARMA</b>	Error autocorrelation $\phi(L)d_t = \theta(L)e_t$
<b>T</b>	<b>Trend</b>	Level + slope (possibly damped) $t_t = t_{t-1} + \phi b_{t-1}$
<b>S</b>	<b>Seasonal</b>	Multiple seasonal periods $m_1, m_2, \dots, m_T$

Answer: (B)

- Trigonometric seasonality, Box-Cox transformation, ARMA errors, Trend, Seasonal components.

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## Question 3

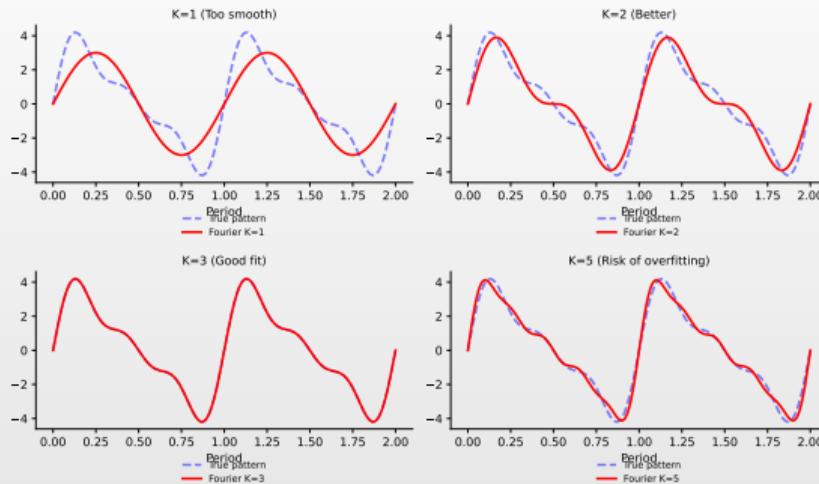
### Question

- What happens when we increase the number of Fourier harmonics  $K$ ?

### Answer Choices

- (A) The model becomes simpler and more robust
- (B) The model captures more complex seasonal patterns but risks overfitting
- (C) The forecast horizon increases proportionally
- (D) The seasonal period  $s$  changes automatically

## Question 3: Answer



Answer: (B)

- Higher  $K$  captures more complex seasonal patterns but increases the risk of overfitting. The maximum is  $K \leq s/2$ .



## Question 4

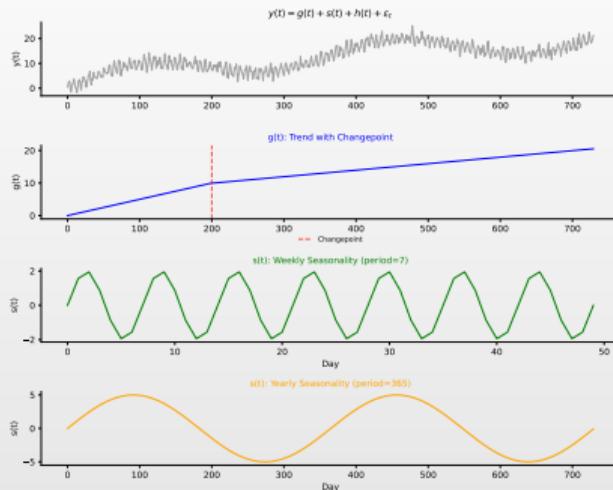
### Question

- What are the main components in Prophet's model  $y(t) = g(t) + s(t) + h(t) + \varepsilon_t$ ?

### Answer Choices

- (A)**  $g(t)$  = GARCH volatility,  $s(t)$  = stationarity test,  $h(t)$  = heteroskedasticity
- (B)**  $g(t)$  = growth (trend with changepoints),  $s(t)$  = seasonality,  $h(t)$  = holiday effects
- (C)**  $g(t)$  = Gaussian noise,  $s(t)$  = smoothing,  $h(t)$  = harmonic terms
- (D)**  $g(t)$  = gradient,  $s(t)$  = spectral density,  $h(t)$  = Hurst exponent

## Question 4: Answer



Answer: (B)

- $g(t)$  = trend with changepoints,  $s(t)$  = seasonality (Fourier terms),  $h(t)$  = holiday effects,  $\varepsilon_t$  = error term.



## Question 5

### Question

- What key features does Prophet have that TBATS lacks?

### Answer Choices

- (A)** Trigonometric seasonality and Box-Cox transformation
- (B)** Automatic parameter selection and exponential smoothing
- (C)** Holiday effects, external regressors, trend changepoints, and native missing data handling
- (D)** State-space formulation and ARMA error modeling

## Question 5: Answer

**TBATS vs Prophet: Head-to-Head Comparison**

Feature	TBATS	Prophet
Multiple seasonalities	Yes (automatic)	Yes (manual/auto)
Holiday effects	No	Yes (built-in)
External regressors	No	Yes
Trend changepoints	No (smooth)	Yes (automatic)
Missing data	Needs interpolation	Handles natively
Interpretability	Moderate	High
Computation speed	Slow	Fast
High-frequency data	Good	Moderate
Non-integer periods	Yes (e.g., 365.25)	Yes
Best for	Technical/high-freq	Business/daily

Answer: (C)

- Prophet offers holiday effects, external regressors, trend changepoints, and native missing data handling—features not available in TBATS.



## Key Takeaways

### What We Learned

- TBATS handles multiple seasonalities with Fourier terms and Box-Cox transformation
- Prophet provides interpretable decomposition with trend changepoints and holiday effects
- Both methods scale better than SARIMA for high-frequency and complex seasonal data

### Important

Choose Prophet for business forecasting with holidays and interpretability needs. Use TBATS for automatic modeling of high-frequency data. Always validate with time series cross-validation—never standard k-fold!



## Questions?

Questions?

### Next Steps:

- Practice with the Jupyter notebook
- Try Prophet on your own data
- Explore NeuralProphet for deep learning extension



## Bibliography I

### Prophet

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- Taylor, J.W. (2003). Short-term Electricity Demand Forecasting Using Double Seasonal Exponential Smoothing, *Journal of the Operational Research Society*, 54(8), 799–805.



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- Hyndman, R.J., & Athanasopoulos, G. (2021). *Forecasting: Principles and Practice*, 3rd ed., OTexts.
- Petropoulos, F., et al. (2022). Forecasting: Theory and Practice, *International Journal of Forecasting*, 38(3), 845–1054.

### Online Resources and Code

- Quantlet: <https://quantlet.com> ↗ Code repository for statistics
- Quantinar: <https://quantinar.com> ↗ Learning platform for quantitative methods
- GitHub TSA: [https://github.com/QuantLet/TSA/tree/main/TSA\\_ch9](https://github.com/QuantLet/TSA/tree/main/TSA_ch9) ↗ Python code for this chapter



# Thank You!

Questions?

Course materials available at: <https://danpele.github.io/Time-Series-Analysis/>

