



Chapter 3: ARIMA Models

Seminar



Seminar Outline

- 1 Review Quiz
- 2 True/False Questions
- 3 Practice Problems
- 4 Worked Examples
- 5 Real Data Analysis
- 6 Discussion Topics
- 7 Summary

Quiz 1: Integration Order

Question

A time series Y_t requires two differences to become stationary. What is its order of integration?

- ☐ A) $I(0)$
- ☐ B) $I(1)$
- ☐ C) $I(2)$
- ☐ D) Cannot be determined

Quiz 1: Integration Order

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- ☐ D) Cannot be determined

Answer: C – $I(2)$

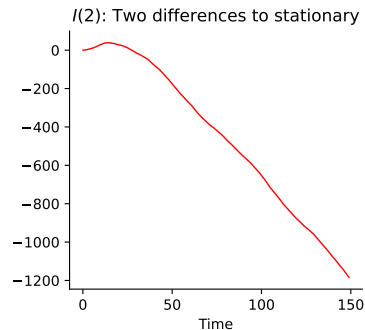
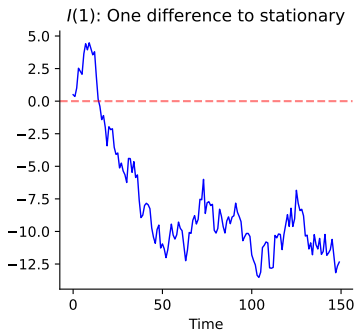
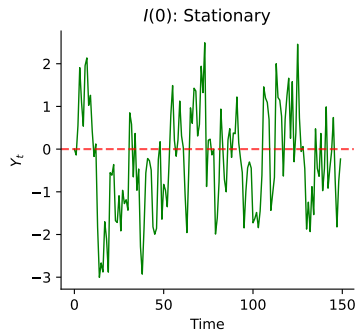
Definition: $Y_t \sim I(d)$ if $\Delta^d Y_t$ is stationary but $\Delta^{d-1} Y_t$ is not.

Example: If Y_t follows $\Delta^2 Y_t = \varepsilon_t$, then:

- $\Delta Y_t = \Delta Y_{t-1} + \varepsilon_t$ (still has unit root)
- $\Delta^2 Y_t = \varepsilon_t$ (white noise, stationary)

Real-world: Price levels may be $I(2)$ when inflation itself is non-stationary.

Visual: Integrated Processes



$I(0)$: stationary. $I(1)$: one difference needed. $I(2)$: two differences needed to become stationary.

Quiz 2: Random Walk Properties

Question

For a random walk $Y_t = Y_{t-1} + \varepsilon_t$ with $\text{Var}(\varepsilon_t) = \sigma^2$, what is $\text{Var}(Y_t)$?

- ☐ A) σ^2
- ☐ B) $t \cdot \sigma^2$
- ☐ C) σ^2/t
- ☐ D) $\sigma^2/(1 - \phi^2)$

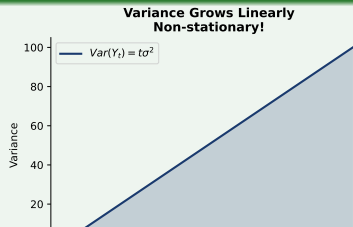
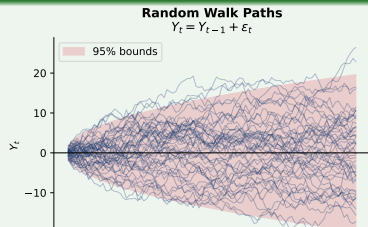
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- ☒ B) $t \cdot \sigma^2$
- ☐ C) σ^2/t
- ☐ D) $\sigma^2/(1 - \phi^2)$

Answer: B – $t \cdot \sigma^2$



Quiz 3: ADF Test Hypotheses

Question

In the Augmented Dickey-Fuller test, what is the null hypothesis?

- ☐ A) The series is stationary
- ☐ B) The series has a unit root
- ☐ C) The series has no autocorrelation
- ☐ D) The series is normally distributed

Quiz 3: ADF Test Hypotheses

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- ☐ D) The series is normally distributed

Answer: B – The series has a unit root

ADF regression: $\Delta Y_t = \alpha + \gamma Y_{t-1} + \sum_{j=1}^p \delta_j \Delta Y_{t-j} + \varepsilon_t$

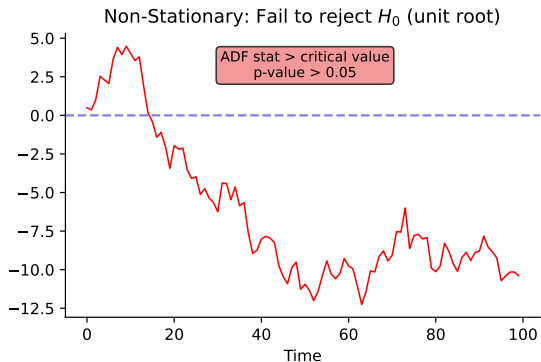
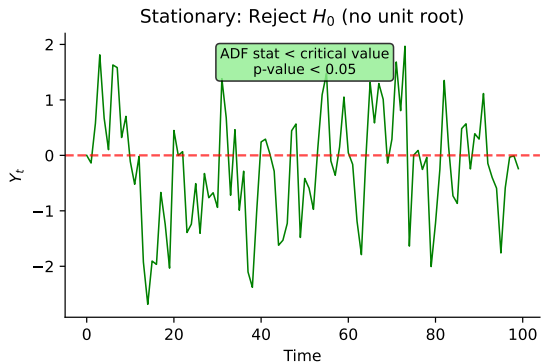
Hypotheses:

- $H_0 : \gamma = 0$ (unit root, non-stationary)
- $H_1 : \gamma < 0$ (stationary)

Decision: Reject H_0 if t -statistic $<$ critical value (e.g., -2.86 at 5%)

Note: Uses special Dickey-Fuller distribution, not standard t .

Visual: ADF Test



Left: stationary – ADF rejects unit root. Right: non-stationary – ADF fails to reject.

Quiz 4: ARIMA Notation

Question

What does ARIMA(2,1,1) mean?

- ☐ A) AR(2) on differenced data with MA(1) errors
- ☐ B) AR(1) with 2 differences and MA(1)
- ☐ C) MA(2) with 1 difference and AR(1)
- ☐ D) 2 lags, 1 trend, 1 seasonal component

Quiz 4: ARIMA Notation

Question

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- ☒ A) AR(2) on differenced data with MA(1) errors
- ☐ B) AR(1) with 2 differences and MA(1)
- ☐ C) MA(2) with 1 difference and AR(1)
- ☐ D) 2 lags, 1 trend, 1 seasonal component

Answer: A – AR(2) on differenced data with MA(1) errors

ARIMA(p, d, q): $\phi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t$

ARIMA(2,1,1) expands to:

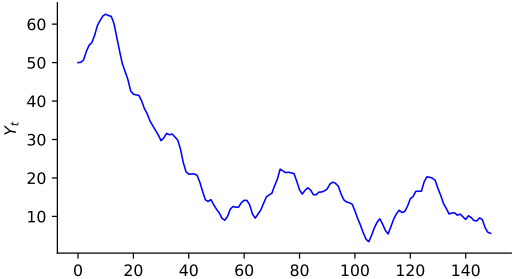
$$(1 - \phi_1 L - \phi_2 L^2)(1 - L)Y_t = (1 + \theta_1 L)\varepsilon_t$$

Or equivalently: $(1 - \phi_1 L - \phi_2 L^2)\Delta Y_t = (1 + \theta_1 L)\varepsilon_t$

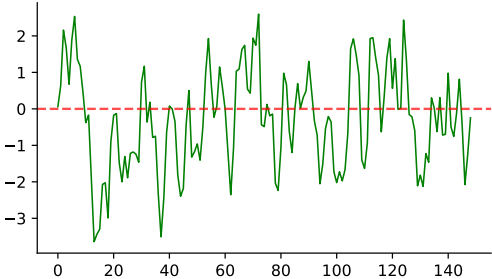
Interpretation: First difference the series, then fit ARMA(2,1) to ΔY_t .

Visual: ARIMA Process

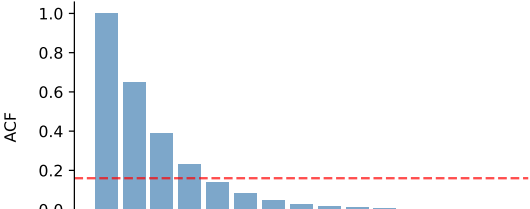
Original: Y_t (ARIMA(1,1,1))



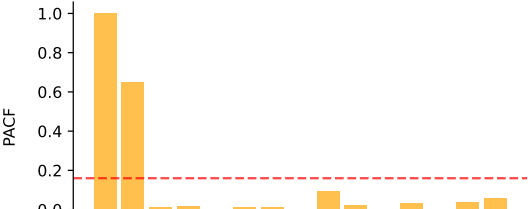
After $d = 1$ difference: ARMA(1,1)



ACF of ΔY_t



PACF of ΔY_t



Quiz 5: Difference Operator

Question

What is $(1 - L)^2 Y_t$ expanded?

- ☐ A) $Y_t - Y_{t-1}$
- ☐ B) $Y_t - 2Y_{t-1} + Y_{t-2}$
- ☐ C) $Y_t + 2Y_{t-1} + Y_{t-2}$
- ☐ D) $Y_t - Y_{t-2}$

Quiz 5: Difference Operator

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- ☒ B) $Y_t - 2Y_{t-1} + Y_{t-2}$
- ☐ C) $Y_t + 2Y_{t-1} + Y_{t-2}$
- ☐ D) $Y_t - Y_{t-2}$

Answer: B - $Y_t - 2Y_{t-1} + Y_{t-2}$

Expansion using binomial theorem:

$$(1 - L)^2 = 1 - 2L + L^2$$

Apply to Y_t :

$$(1 - L)^2 Y_t = Y_t - 2L \cdot Y_t + L^2 \cdot Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$$

Note: This equals $\Delta(\Delta Y_t) = \Delta Y_t - \Delta Y_{t-1}$, the “change in changes”.

Quiz 6: KPSS vs ADF

Question

How does the KPSS test differ from the ADF test?

- ☐ A) KPSS tests for seasonality, ADF tests for trends
- ☐ B) KPSS has stationarity as null, ADF has unit root as null
- ☐ C) KPSS is more powerful than ADF
- ☐ D) There is no difference

Quiz 6: KPSS vs ADF

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- ☐ C) KPSS is more powerful than ADF
- ☐ D) There is no difference

Answer: B – Reversed null hypotheses

ADF Test

H_0 : Unit Root

H_1 : Stationary

Reject if $t\text{-stat} < \text{critical}$

KPSS Test

H_0 : Stationary

H_1 : Unit Root

Reject if $LM > \text{critical}$

Decision Matrix

ADF rejects

KPSS fails to reject

→ **Stationary**

Quiz 7: Overdifferencing

Question

If $Y_t \sim I(1)$ and we compute $\Delta^2 Y_t$, what happens?

- ☒ A) We get a better stationary series
- ☐ B) We introduce artificial negative autocorrelation
- ☐ C) The variance decreases
- ☐ D) Nothing changes

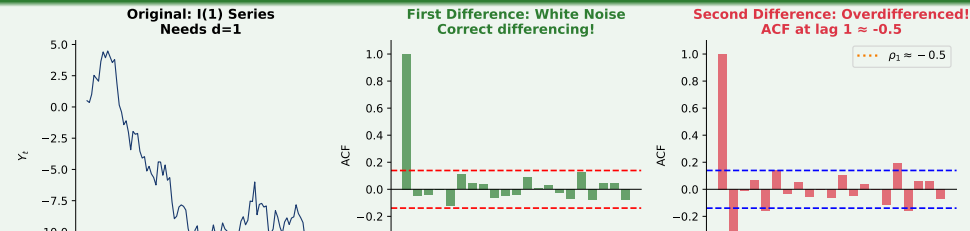
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- ☐ B) We introduce artificial negative autocorrelation
- ☐ C) The variance decreases
- ☐ D) Nothing changes

Answer: B – Artificial negative autocorrelation



Quiz 8: Forecast Variance

Question

For an ARIMA(0,1,0) model (random walk), how does forecast variance behave as horizon h increases?

- ☐ A) Stays constant
- ☐ B) Decreases to zero
- ☐ C) Grows linearly with h
- ☐ D) Converges to a finite limit

Quiz 8: Forecast Variance

Question

For an ARIMA(0,1,0) model (random walk), how does forecast variance behave as horizon h increases?

- ☒ A) Stays constant
- ☐ B) Decreases to zero
- ☐ C) Grows linearly with h
- ☐ D) Converges to a finite limit

Answer: C – Grows linearly with h

Random walk forecast: $\hat{Y}_{T+h|T} = Y_T$ (best forecast is current value)

Forecast error: $Y_{T+h} - \hat{Y}_{T+h|T} = \sum_{i=1}^h \varepsilon_{T+i}$

Variance:

$$\text{Var}(Y_{T+h} - \hat{Y}_{T+h|T}) = h\sigma^2$$

95% CI: $Y_T \pm 1.96\sqrt{h}\sigma$ (widens with \sqrt{h})

Quiz 9: Unit Root Test Power

Question

The ADF test has low power when:

- ☐ A) Sample size is very large
- ☐ B) The true root is close to but not equal to 1
- ☐ C) The series has no trend
- ☐ D) The series is clearly stationary

Quiz 9: Unit Root Test Power

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- ☒ B) The true root is close to but not equal to 1
- ☐ C) The series has no trend
- ☐ D) The series is clearly stationary

Answer: B – Root close to but not equal to 1

Example: AR(1) with $\phi = 0.95$ vs random walk ($\phi = 1$)

Problem: Both have similar ACF patterns (slow decay), but one is stationary!

Low power means: High probability of Type II error (failing to reject false H_0)

Solutions:

- Larger sample sizes
- Phillips-Perron test (robust to heteroskedasticity)
- Panel unit root tests (multiple series)

Quiz 10: ARIMA Model Selection

Question

After differencing once, the ACF shows a spike at lag 1 only, and PACF decays. The appropriate model is:

- ☐ A) ARIMA(1,1,0)
- ☐ B) ARIMA(0,1,1)
- ☐ C) ARIMA(1,1,1)
- ☐ D) ARIMA(0,2,1)

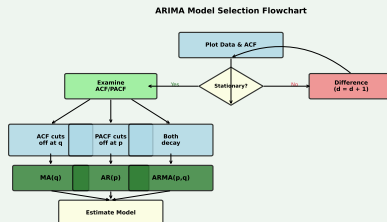
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- B) ARIMA(0,1,1)
- C) ARIMA(1,1,1)
- D) ARIMA(0,2,1)

Answer: B – ARIMA(0,1,1)



Quiz 11: Trend Stationarity vs Difference Stationarity

Question

A trend-stationary process is made stationary by:

- ☐ A) Taking first differences
- ☐ B) Removing the deterministic trend via regression
- ☐ C) Taking second differences
- ☐ D) Applying seasonal adjustment

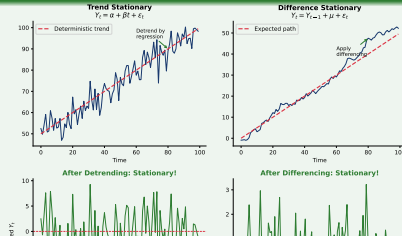
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- ☐ A) Taking first differences
- ☒ B) Removing the deterministic trend via regression
- ☐ C) Taking second differences
- ☐ D) Applying seasonal adjustment

Answer: B – Removing deterministic trend via regression



Quiz 12: ARIMA Invertibility

Question

ARIMA(0,1,1) with $\theta_1 = 1.2$ is:

- ☐ A) Stationary and invertible
- ☐ B) Non-stationary but invertible
- ☐ C) Non-stationary and non-invertible
- ☐ D) Stationary but non-invertible

Quiz 12: ARIMA Invertibility

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- ☐ A) Stationary and invertible
- ☐ B) Non-stationary but invertible
- ☒ C) Non-stationary and non-invertible
- ☐ D) Stationary but non-invertible

Answer: C – Non-stationary and non-invertible

Check stationarity: $d = 1$ means one unit root \Rightarrow **Non-stationary**

Check invertibility: MA polynomial is $\theta(z) = 1 + 1.2z$

- Root: $z = -1/1.2 = -0.833$ (inside unit circle)
- Invertibility requires root outside unit circle
- $|\theta_1| = 1.2 > 1 \Rightarrow$ **Non-invertible**

Fix: Rewrite with $\theta^* = 1/1.2 = 0.833$ and adjust variance.

Quiz 13: Spurious Regression

Question

Regressing one random walk on another independent random walk typically shows:

- ☐ A) No significant relationship
- ☒ B) High R^2 and significant t-statistics (spuriously)
- ☐ C) Negative correlation
- ☐ D) Perfect multicollinearity

Quiz 13: Spurious Regression

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Regressing one random walk on another independent random walk typically shows:

- ☐ A) No significant relationship
- ☒ B) High R^2 and significant t -statistics (spuriously)
- ☐ C) Negative correlation
- ☐ D) Perfect multicollinearity

Answer: B – High R^2 and significant t -statistics (spuriously)

Granger & Newbold (1974): Spurious regression phenomenon

Symptoms:

- High R^2 (often > 0.9) between unrelated series
- Significant t -statistics
- Very low Durbin-Watson statistic ($\ll 2$)
- Non-stationary residuals

Quiz 14: Long-Run Forecast

Question

The long-run forecast from ARIMA(1,1,0) with $\phi_1 = 0.7$ converges to:

- ☐ A) Zero
- ☐ B) The unconditional mean
- ☐ C) A linear trend extrapolation
- ☐ D) The last observed value

Quiz 14: Long-Run Forecast

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- ☐ A) Zero
- ☐ B) The unconditional mean
- ☒ C) A linear trend extrapolation
- ☐ D) The last observed value

Answer: C – A linear trend extrapolation

Model: $(1 - \phi_1 L)(1 - L)Y_t = c + \varepsilon_t$

Long-run forecast: For I(1) models with drift c :

$$\hat{Y}_{T+h} \approx Y_T + h \cdot \frac{c}{1 - \phi_1}$$

Key differences:

- Stationary ARMA: Forecasts \rightarrow unconditional mean

Question

Determine if each statement is True or False:

- ❶ An $I(2)$ process requires two differences to become stationary.
- ❷ The ADF test always includes a constant term.
- ❸ $ARIMA(0,1,0)$ is another name for a random walk.
- ❹ Differencing a stationary series makes it “more stationary.”
- ❺ The KPSS test has stationarity as the null hypothesis.
- ❻ $ARIMA$ models can only capture linear patterns.

Answer on next slide...

Answers

- ① An $I(2)$ process requires two differences to become stationary.
 $I(d)$ means d differences needed. $I(2)$ = two unit roots. TRUE
- ② The ADF test always includes a constant term.
You choose: no constant, constant only, or constant + trend. FALSE
- ③ $ARIMA(0,1,0)$ is another name for a random walk.
 $(1 - L)Y_t = \varepsilon_t \Rightarrow Y_t = Y_{t-1} + \varepsilon_t$. TRUE
- ④ Differencing a stationary series makes it “more stationary.”
Over-differencing creates non-invertible MA; hurts model performance. FALSE
- ⑤ The KPSS test has stationarity as the null hypothesis.
KPSS: H_0 = stationary. Opposite of ADF. TRUE
- ⑥ $ARIMA$ models can only capture linear patterns.
 $ARIMA$ is linear in parameters. Nonlinear patterns need GARCH, neural nets, etc. TRUE

Problem 1: Unit Root Testing

Exercise

You have quarterly GDP data for 80 quarters. The ADF test (with constant and trend) gives a test statistic of -2.85 . The 5% critical value is -3.41 .

- 1 What is your conclusion about stationarity?
- 2 What would you do next?

Problem 1: Unit Root Testing

Exercise

You have quarterly GDP data for 80 quarters. The ADF test (with constant and trend) gives a test statistic of -2.85 . The 5% critical value is -3.41 .

- 1 What is your conclusion about stationarity?
- 2 What would you do next?

Solution

- 1 Since $-2.85 > -3.41$, we **fail to reject** H_0 . The data appears to have a unit root (non-stationary).
- 2 Take the first difference ΔY_t and repeat the ADF test on the differenced series to confirm it is now stationary.

Problem 2: Model Identification

Exercise

After differencing a time series once, the ACF shows:

- Significant spike at lag 1 ($\rho_1 = 0.4$)
- All other lags insignificant

The PACF shows gradual decay.

What ARIMA model is suggested?

Problem 2: Model Identification

Exercise

After differencing a time series once, the ACF shows:

- Significant spike at lag 1 ($\rho_1 = 0.4$)
- All other lags insignificant

The PACF shows gradual decay.

What ARIMA model is suggested?

Solution

- ACF cuts off after lag 1 \Rightarrow MA(1) component
- PACF decays \Rightarrow Confirms MA structure
- Since we differenced once: $d = 1$

Suggested model: ARIMA(0,1,1) or IMA(1,1)

Problem 3: ARIMA Equation

Exercise

Write out the full equation for ARIMA(1,1,1):

$$(1 - \phi_1 L)(1 - L)Y_t = c + (1 + \theta_1 L)\varepsilon_t$$

Expand this completely in terms of Y_t , Y_{t-1} , Y_{t-2} , etc.

Problem 3: ARIMA Equation

Exercise

Write out the full equation for ARIMA(1,1,1):

$$(1 - \phi_1 L)(1 - L)Y_t = c + (1 + \theta_1 L)\varepsilon_t$$

Expand this completely in terms of Y_t , Y_{t-1} , Y_{t-2} , etc.

Solution

Expanding $(1 - \phi_1 L)(1 - L) = 1 - L - \phi_1 L + \phi_1 L^2 = 1 - (1 + \phi_1)L + \phi_1 L^2$:

$$Y_t - (1 + \phi_1)Y_{t-1} + \phi_1 Y_{t-2} = c + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

Or equivalently:

$$Y_t = c + (1 + \phi_1)Y_{t-1} - \phi_1 Y_{t-2} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

Problem 4: Forecast Calculation

Exercise

Given ARIMA(0,1,1): $\Delta Y_t = \varepsilon_t + 0.3\varepsilon_{t-1}$

At time T : $Y_T = 100$, $\hat{\varepsilon}_T = 2$, $\sigma^2 = 4$

Calculate:

- ① $\hat{Y}_{T+1|T}$ (one-step forecast)
- ② $\hat{Y}_{T+2|T}$ (two-step forecast)

Problem 4: Forecast Calculation

Exercise

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At time T : $Y_T = 100$, $\hat{\varepsilon}_T = 2$, $\sigma^2 = 4$

Calculate:

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- ② $\hat{Y}_{T+2|T}$ (two-step forecast)

Solution

- ① $\hat{Y}_{T+1|T} = Y_T + 0.3\hat{\varepsilon}_T = 100 + 0.3(2) = \mathbf{100.6}$
- ② $\hat{Y}_{T+2|T} = \hat{Y}_{T+1|T} + 0.3 \cdot 0 = 100.6 + 0 = \mathbf{100.6}$
(Future shocks $\varepsilon_{T+1}, \varepsilon_{T+2}$ are forecast as 0)

Problem 5: Confidence Intervals

Exercise

Continuing from Problem 4, calculate the 95% forecast intervals for $\hat{Y}_{T+1|T}$ and $\hat{Y}_{T+2|T}$.

Recall: $\sigma^2 = 4$, $\theta_1 = 0.3$

Problem 5: Confidence Intervals

Exercise

Continuing from Problem 4, calculate the 95% forecast intervals for $\hat{Y}_{T+1|T}$ and $\hat{Y}_{T+2|T}$.

Recall: $\sigma^2 = 4$, $\theta_1 = 0.3$

Solution

For IMA(1,1), the MA(∞) weights are $\psi_0 = 1$, $\psi_j = 1 + \theta_1$ for $j \geq 1$.

1-step: $\text{Var}(e_{T+1}) = \sigma^2 \psi_0^2 = 4$, so $SE = 2$

$$100.6 \pm 1.96(2) = [96.68, 104.52]$$

2-step: $\text{Var}(e_{T+2}) = \sigma^2(\psi_0^2 + \psi_1^2) = 4(1 + 1.3^2) = 10.76$, $SE = 3.28$

$$100.6 \pm 1.96(3.28) = [94.17, 107.03]$$

Example: Testing for Unit Root in Stock Prices

Scenario

You have daily closing prices for a stock over 500 days. You want to determine if prices follow a random walk.

Step-by-step Approach

- ➊ **Visual inspection:** Plot prices – likely shows trend
- ➋ **ADF test on prices:** Expect to fail to reject H_0 (unit root)
- ➌ **Take log returns:** $r_t = \ln(P_t/P_{t-1}) = \Delta \ln(P_t)$
- ➍ **ADF test on returns:** Should reject H_0 (stationary)
- ➎ **Conclusion:** Log prices are $I(1)$, returns are $I(0)$

Example: Box-Jenkins for Inflation Data

Scenario

Monthly inflation rates for 10 years. Build an ARIMA model.

Workflow

- ① **Plot & test:** ADF suggests borderline – try both $d = 0$ and $d = 1$
- ② **If $d = 0$:** Fit ARMA models, compare AIC
- ③ **If $d = 1$:** Examine ACF/PACF of ΔY_t
 - ACF: spike at lag 1, then cuts off
 - PACF: decays
 - \Rightarrow Try ARIMA(0,1,1)
- ④ **Estimate:** Fit ARIMA(0,1,1), check coefficients
- ⑤ **Diagnose:** Ljung-Box on residuals (want $p > 0.05$)
- ⑥ **Compare:** AIC of ARIMA(0,1,1) vs ARMA(1,1) on levels

Example: Interpreting Python Output

statsmodels ARIMA Output

```

                        ARIMA Model Results
=====
Dep. Variable:          D.y    No. Observations:   99
Model:                 ARIMA(1,1,1)    AIC          285.32
                                   BIC          295.63
=====

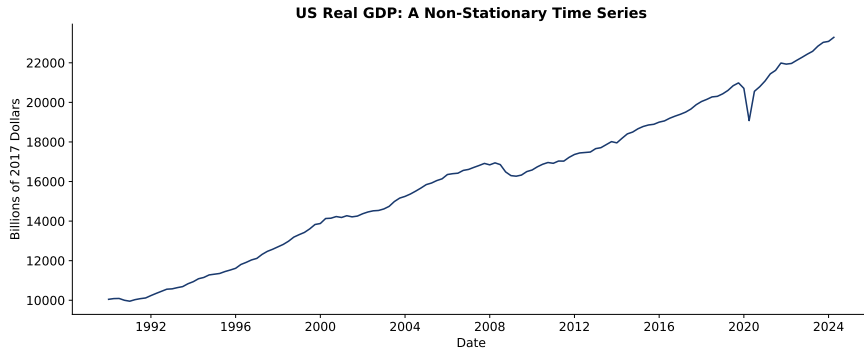
```

	coef	std err	z	P> z
const	0.0521	0.048	1.085	0.278
ar.L1	0.4532	0.102	4.443	0.000
ma.L1	-0.2891	0.118	-2.450	0.014
sigma2	1.2340	0.176	7.011	0.000

Interpretation

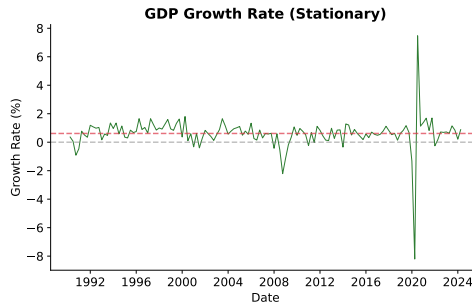
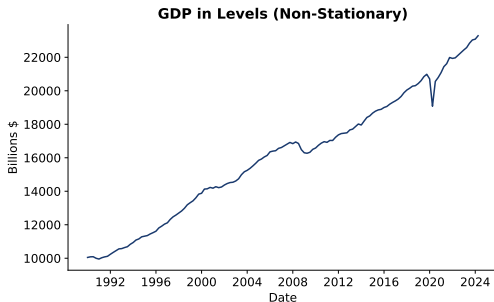
- AR coefficient (0.45) is significant, MA coefficient (-0.29) is significant
- Constant (0.052) not significant – could set $c = 0$
- Check: $|\phi_1| < 1$ (stationary), $|\theta_1| < 1$ (invertible) – OK!

Case Study: US Real GDP (1990–2024)



- US Real GDP in billions of 2017 dollars (quarterly data)
- Clear **upward trend** – typical of macroeconomic series
- Notable drops during recessions (2008-2009, 2020)
- Non-stationary: needs differencing before ARIMA modeling

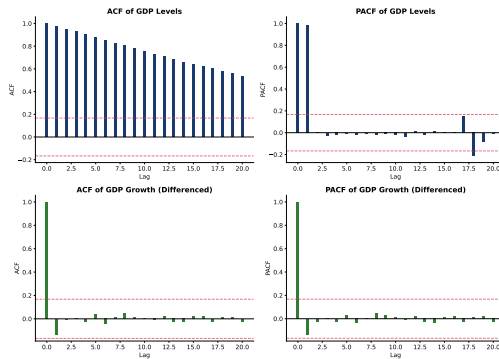
Stationarity Through Differencing



--- Mean = 0.61%

- **Left:** GDP in levels – clear upward trend (non-stationary)
- **Right:** GDP growth rate = $\Delta \log(Y_t) \times 100$ – stationary
- First differencing of log GDP removes the stochastic trend
- Growth rate fluctuates around a constant mean ($\approx 0.6\%$ quarterly)

ACF/PACF: Levels vs Differenced



- **Top row:** ACF/PACF of GDP levels – slow decay indicates non-stationarity
- **Bottom row:** ACF/PACF of GDP growth – mostly within confidence bands
- Pattern suggests low-order ARIMA model is appropriate

ARIMA Estimation Results: US GDP Growth

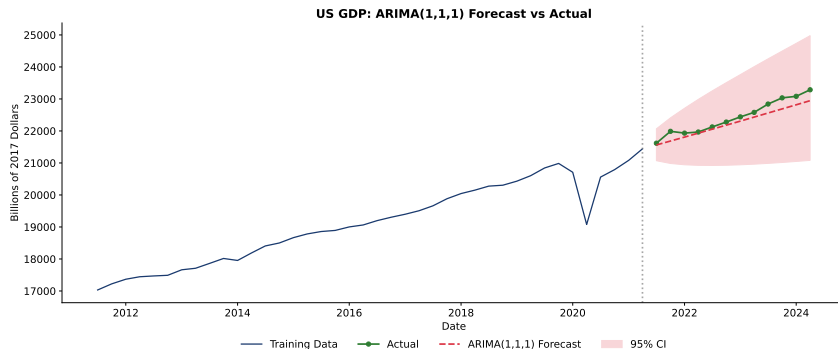
Model: ARIMA(1,1,1) on log(GDP)

Parameter	Estimate	Std. Error	z-stat	p-value
ϕ_1 (AR.L1)	0.312	0.185	1.69	0.091
θ_1 (MA.L1)	-0.087	0.203	-0.43	0.668
σ^2	0.00012	—	—	—

Interpretation

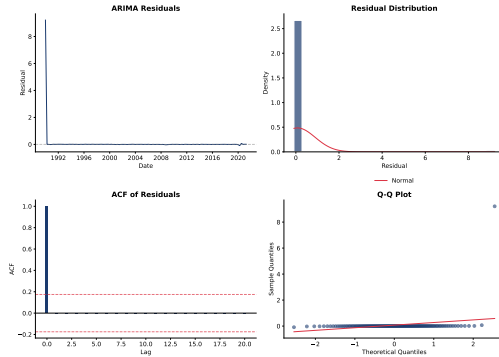
- Low-order ARIMA captures GDP dynamics reasonably well
- AR(1) coefficient positive – GDP growth shows persistence
- Alternative: simpler random walk (ARIMA(0,1,0)) often competitive

Forecast: ARIMA vs Actual



- Blue: historical training data; Green: actual test data
- Red dashed: ARIMA forecasts with 95% confidence interval
- Forecasts capture the general trend direction
- Confidence intervals widen as forecast horizon increases

Model Diagnostics: Residual Analysis



- Residuals show no systematic patterns over time
- Distribution approximately normal (histogram and Q-Q plot)
- ACF of residuals within bounds – no significant autocorrelation remaining
- Model adequately captures the data generating process

Discussion: Deterministic vs Stochastic Trends

Key Question

Why is it important to distinguish between deterministic and stochastic trends?

Discussion Points

- **Wrong treatment consequences:**
 - Detrending a unit root \Rightarrow spurious stationarity
 - Differencing a trend-stationary \Rightarrow over differencing
- **Economic interpretation:**
 - Deterministic trend: shocks are temporary
 - Stochastic trend: shocks have permanent effects
- **Policy implications:**
 - Does a recession permanently lower GDP, or does the economy return to trend?

Key Question

When should you use AIC vs BIC for ARIMA model selection?

Considerations

- **AIC:** Minimizes prediction error, may overfit
 - Better for forecasting
 - Tends to select larger models
- **BIC:** Consistent model selection, more parsimonious
 - Better for identifying “true” model
 - Penalizes complexity more heavily
- **Practical advice:** Report both, prefer BIC if they disagree substantially

Discussion: Limitations of ARIMA

Key Question

What are the main limitations of ARIMA models?

Discussion Points

- **Linearity:** Cannot capture nonlinear dynamics
- **Constant variance:** Assumes homoskedasticity (no GARCH effects)
- **No structural breaks:** Parameters assumed constant
- **Univariate:** Ignores relationships with other variables
- **Symmetric:** Treats positive and negative shocks equally
- **Long-horizon forecasts:** Uncertainty grows rapidly

Extensions

These limitations motivate GARCH (volatility), VAR (multivariate), regime-switching models, etc.

Key Points from Today's Seminar

What We Covered

- 1 **Integration and differencing:** $I(d)$ processes require d differences
- 2 **Unit root testing:** ADF tests H_0 : unit root; KPSS tests H_0 : stationary
- 3 **ARIMA(p,d,q):** Combines ARMA with differencing
- 4 **Model identification:** Use ACF/PACF patterns and information criteria
- 5 **Forecasting:** Point forecasts and growing confidence intervals

Next Seminar

Hands-on Python exercises with real economic data:

- Unit root testing with `statsmodels`
- Auto-ARIMA with `pmdarima`
- Forecasting and model diagnostics