Time Series Analysis and Forecasting

Chapter 8: Modern Extensions

Seminar

Seminar Outline

Seminar structure:

1. **Review Quiz** – Knowledge check
2. **True/False Questions** – Conceptual checks
3. **Practice Problems** – Applied practice
4. **Summary** – Key takeaways
5. **AI-Assisted Exercises** – Critical thinking

Quiz 1: Hurst Exponent

Question

A time series has Hurst exponent $H = 0.8$. What does this indicate?

Answer choices

(A) The series is a pure random walk

(B) The series has long memory and is persistent (trend-following)

(C) The series is anti-persistent (mean-reverting)

(D) The series is stationary I(0)

Answer on next slide...

Quiz 1: Answer

Answer: B – Long memory and persistence

Hurst Exponent Interpretation:

- $H = 0.5$: Random walk (no memory)
- $0.5 < H < 1$: **Persistence** – trend continues
- $0 < H < 0.5$: Anti-persistence – mean reversion

With $H = 0.8 > 0.5$:

- The series has **long memory**
- Large values tend to be followed by large values
- Autocorrelations decay slowly (hyperbolically, not exponentially)

TSA_ch8_hurst

Quiz 2: Fractional Differencing Parameter

Question

In the ARFIMA(p, d, q) model, the parameter d can take values:

Answer choices

(A) Only integer values (0, 1, 2, ...)

(B) Only $d = 0$ or $d = 1$

(C) Any real value, including fractional

(D) Only negative values

Answer on next slide...

Quiz 2: Answer

Answer: C – Any real value

Fractional differencing: $(1 - L)^d$ with $d \in \mathbb{R}$

Interpretation of d values:

- $d = 0$: Stationary series (ARMA)
- $0 < d < 0.5$: Long memory, stationary
- $d = 0.5$: Stationary/non-stationary boundary
- $0.5 < d < 1$: Long memory, non-stationary
- $d = 1$: Full differencing (classic ARIMA)

Relation to Hurst: $d = H - 0.5$

TSA_ch8_arfima

Quiz 3: Long Memory in Financial Series

Question

In which financial series is long memory most commonly documented?

Answer choices

(A) Stock prices

(B) Daily returns

(C) Volatility (squared returns)

(D) Trading volume

Answer on next slide...

Quiz 3: Answer

Answer: C – Volatility

Stylized facts from finance:

- **Returns**: Approximately memoryless ($H \approx 0.5$)
- **Volatility**: Pronounced long memory ($H \approx 0.7 - 0.9$)

Why?

- Volatility clustering: turbulent periods followed by turbulent periods
- Persistence of shocks in variance
- FIGARCH: explicitly models long memory in volatility

This stylized fact is the basis for FIGARCH and HAR-RV models.

TSA_ch8_long_memory

Quiz 4: Feature Engineering

Question

To apply Random Forest to time series, we must create:

Answer choices

(A) Dummy variables for each observation

(B) Lag features and rolling statistics

(C) Fourier transforms of the series

(D) Only the first difference of the series

Answer on next slide...

Quiz 4: Answer

Answer: B – Lag features and rolling statistics

Feature Engineering for Time Series:

- **Lag features**: $y_{t-1}, y_{t-2}, \ldots, y_{t-k}$
- **Rolling statistics**:
  - Rolling mean: $\bar{y}_{t,w}$
  - Rolling standard deviation: $\sigma_{t,w}$
  - Min/Max over window
- **Calendar features**: day of week, month, etc.

Important: Transforms the forecasting problem into a supervised regression problem!

TSA_ch8_ml_forecast

Quiz 5: Time Series Cross-Validation

Question

Why can't we use standard k-fold cross-validation for time series?

Answer choices

(A) It's too slow for long series

(B) It violates temporal order and causes data leakage

(C) It only works for classification

(D) It requires too much data

Answer on next slide...

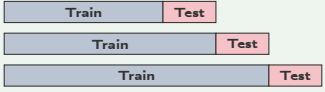
Quiz 5: Answer

Answer: B – Violates temporal order

Problem with standard k-fold:

- Shuffles observations temporally
- Trains on future data, tests on past
- **Data leakage** $\Rightarrow$ overestimated performance

Solution: Time Series Split (Walk-Forward)



TSA_ch8_cross_validation

Quiz 6: Feature Importance in Random Forest

Question

Feature importance in Random Forest for time series helps us:

Answer choices

(A) Eliminate all low-importance variables

(B) Identify which lags and features are most predictive

(C) Determine Granger causality

(D) Calculate confidence intervals

Answer on next slide...

Quiz 6: Answer

Answer: B – Identifies predictive features

Uses of feature importance:

- Understanding temporal structure
- Selecting optimal number of lags
- Identifying relevant factors

Caution:

- Importance does NOT imply causality
- Correlated variables may share importance
- Use for interpretation, not causal inference

TSA_ch8_feature_importance

Quiz 7: LSTM Advantage

Question

What is the main advantage of LSTM over simple RNNs?

Answer choices

(A) It's faster to train

(B) It solves the vanishing/exploding gradient problem

(C) It requires less data

(D) It's easier to interpret

Answer on next slide...

Quiz 7: Answer

Answer: B – Solves the gradient problem

Simple RNN Problem:

- Gradients decay exponentially with sequence length
- Cannot learn long-term dependencies

LSTM Solution:

- **Cell state**: Highway for information flow
- **Forget gate**: Decides what to forget
- **Input gate**: Decides what to remember
- **Output gate**: Decides what to output

Gradients can "flow" through the cell state without degradation!

TSA_ch8_lstm

Quiz 8: Data Preparation for LSTM

Question

Before training an LSTM, data should be:

Answer choices

(A) Log-transformed

(B) Normalized/scaled to [0,1] or [-1,1]

(C) Differenced twice

(D) Converted to integers

Answer on next slide...

Quiz 8: Answer

Answer: B – Normalized/scaled

Why normalization?

- Activation functions (sigmoid, tanh) work in limited ranges
- Faster convergence
- Numerical stability

Common methods:

- **Min-Max**: $x' = \frac{x - x_{min}}{x_{max} - x_{min}} \rightarrow [0, 1]$
- **Standard**: $x' = \frac{x - \mu}{\sigma} \rightarrow$ mean 0, std 1

Important: Fit on train, transform on train+test!

TSA_ch8_lstm_preprocessing

Quiz 9: LSTM Hyperparameters

Question

Which is NOT a typical LSTM hyperparameter?

Answer choices

(A) Number of units (neurons) per layer

(B) Input sequence length

(C) Learning rate

(D) Differencing parameter d

Answer on next slide...

Quiz 9: Answer

Answer: D – The d parameter

d is specific to ARFIMA models, not LSTM!

LSTM Hyperparameters:

- **Architecture**: number of layers, units/layer
- **Sequence**: lookback length
- **Training**: learning rate, batch size, epochs
- **Regularization**: dropout, early stopping

Tuning: Grid search or Bayesian optimization with time series CV

TSA_ch8_lstm_tuning

True or False? — Questions

Statement	T/F?
1. ARFIMA models can capture long-term dependence.	?
2. The parameter d in ARFIMA must be an integer.	?
3. LSTM networks are better than ARIMA in all situations.	?
4. Random Forest requires manually created features.	?
5. Standard cross-validation (k-fold) is suitable for time series.	?
6. The Hurst exponent $H > 0.5$ indicates positive long memory.	?

True or False? — Answers

Statement	T/F	Explanation
1. ARFIMA captures long-term dependence.	T	**Fractional d**
2. d in ARFIMA must be an integer.	F	$d \in (0, 0.5)$ **fractional**
3. LSTM better than ARIMA always.	F	**Depends on data and sample**
4. RF requires manual features.	T	**Lags, calendar, etc.**
5. k-fold CV suitable for time series.	F	**Violates temporal ordering**
6. $H > 0.5$ indicates positive long memory.	T	$H = 0.5$**: no memory**

Problem 1: Hurst Exponent Estimation

Exercise

Given daily Bitcoin returns, estimate the Hurst exponent using the R/S method and interpret the result.

Solution Steps:

1. Calculate mean over subintervals of different lengths n
2. For each n: calculate Range(R) and Std(S)
3. The ratio R/S grows as n^H
4. Fit regression: $\log(R/S) = H \cdot \log(n) + c$

Python code: `nolds.hurst_rs(returns)`

Problem 1: Solution and Interpretation

Typical Bitcoin Results

- Returns: $H \approx 0.45 - 0.55$ (approximately random walk)
- Volatility ($|\text{returns}|$): $H \approx 0.75 - 0.85$ (long memory!)

Interpretation:

- Returns are hard to predict (EMH approximately valid)
- Volatility is predictable over long horizons
- Implications for risk management and VaR

Application: FIGARCH models may outperform standard GARCH

TSA_ch8_hurst

Problem 2: Random Forest for Forecasting

Exercise

Build a Random Forest model for 1-day ahead Bitcoin price forecasting. Evaluate using TimeSeriesSplit.

Pipeline:

1. **Feature engineering**:
   - Lags: $y_{t-1}, y_{t-2}, \ldots, y_{t-7}$
   - Rolling mean/std: 7, 14, 30 days
2. **Train/Test split**: TimeSeriesSplit(n_splits=5)
3. **Model**: RandomForestRegressor(n_estimators=100)
4. **Evaluation**: RMSE, MAE, Direction Accuracy

Problem 2: Code and Results

Python Code

```
from sklearn.ensemble import RandomForestRegressor
from sklearn.model_selection import TimeSeriesSplit

tscv = TimeSeriesSplit(n_splits=5)
rf = RandomForestRegressor(n_estimators=100)

for train_idx, test_idx in tscv.split(X):
    rf.fit(X[train_idx], y[train_idx])
    pred = rf.predict(X[test_idx])
```

Typical results:

- Direction accuracy: 52-55% (slightly above random)
- Feature importance: lag-1 and rolling_std dominate

TSA_ch8_random_forest

Problem 3: LSTM for Time Series

Exercise

Implement a simple LSTM model for Bitcoin forecasting. Compare with Random Forest.

Simple LSTM Architecture:

1. Input: 30-day sequences
2. LSTM layer: 50 units
3. Dense output: 1 neuron (forecast)
4. Loss: MSE, Optimizer: Adam

Important steps:

- ⊡ MinMaxScaler normalization
- ⊡ Reshape to [samples, timesteps, features]
- ⊡ Early stopping to prevent overfitting

Problem 3: LSTM Code

Keras/TensorFlow Code

```
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import LSTM, Dense

model = Sequential([
    LSTM(50, input_shape=(30, 1)),
    Dense(1)
])

model.compile(optimizer='adam', loss='mse')
model.fit(X_train, y_train, epochs=50,
          validation_split=0.1, verbose=0)
```

Typical RF vs LSTM comparison:

- RMSE similar (LSTM slightly better on smooth data)
- RF: faster, more interpretable
- LSTM: captures complex patterns better

TSA_ch8_lstm

Su

ARFIMA

- Series with long memory (volatility, hydrology)
- When $0 < d < 0.5$ is theoretically justified
- Statistical interpretability is important

Random Forest

- Nonlinear relationships between features
- Feature importance for understanding
- Structured data, not too long series

LSTM

- Long sequences with complex dependencies
- Sufficient data for deep learning
- Patterns difficult to capture with classical methods

Ke

ARFIMA and Long Memory

- Fractional differencing: $(1 - L)^d y_t = \varepsilon_t$
- Hurst exponent: $d = H - 0.5$
- ACF for long memory: $\rho(k) \sim k^{2d-1}$ (slow decay)

Machine Learning

- Lag feature: $X_t = [y_{t-1}, y_{t-2}, \ldots, y_{t-k}]$
- RMSE: $\sqrt{\frac{1}{n}\sum(y_i - \hat{y}_i)^2}$
- Direction Accuracy: $\frac{1}{n}\sum 1[\text{sign}(\Delta y) = \text{sign}(\Delta \hat{y})]$

LSTM

- Forget gate: $f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$
- Cell update: $C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$

AI Exercise: Critical Thinking

Test this prompt in ChatGPT/Claude/Copilot

"Download hourly electricity consumption data. Compare three approaches: (1) ARIMA, (2) Random Forest with lag features, (3) LSTM. Use time series cross-validation, report RMSE and directional accuracy for each."

1. Did the AI use proper time series cross-validation (no future data leakage)?
2. Are the lag features for Random Forest correctly constructed?
3. Is the LSTM data properly scaled and sequenced?
4. Does the comparison use the same test set for all three models?
5. Which model performs best and why? Is this consistent with theory?

Warning: AI-generated code may run without errors and look professional. That does not mean it is correct.

Thank You!

Questions?

Seminar materials are available at: https://danpele.github.io/Time-Series-Analysis/

Quantlet Quantinar

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Online Resources and Code

- **Quantlet**: `https://quantlet.com` — Code repository for statistics
- **Quantinar**: `https://quantinar.com` — Quantitative methods learning platform
- **GitHub TSA**: `https://github.com/QuantLet/TSA` — Python code for this seminar