



Time Series Analysis and Forecasting

Chapter 9: Prophet and TBATS



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Learning Objectives

By the end of this chapter, you will be able to:

- Handle time series with multiple seasonal patterns
- Use Facebook Prophet for flexible forecasting with holidays
- Apply TBATS models for complex seasonality
- Compare and select between modern forecasting methods



Outline



The Problem: Complex Seasonal Patterns

Real-World Examples

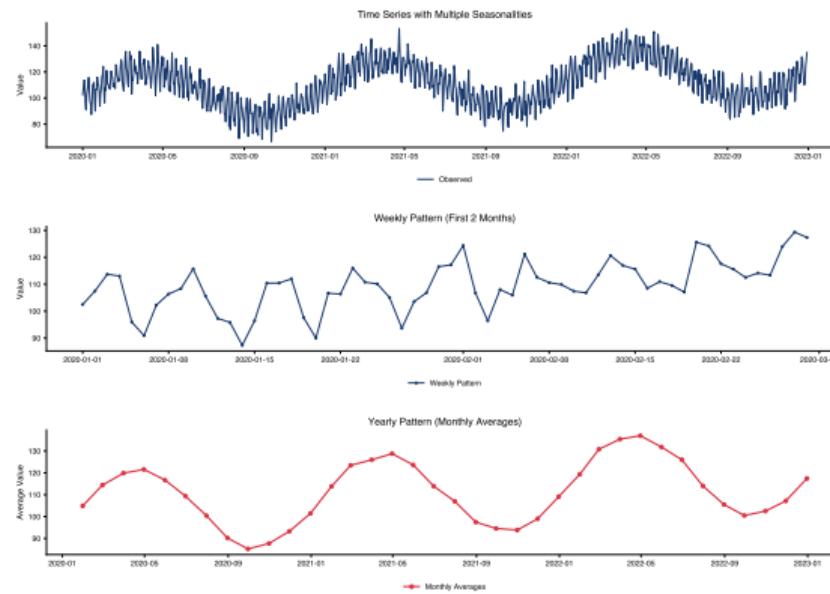
- **Hourly electricity demand:** Daily + Weekly + Annual patterns
- **Website traffic:** Daily + Weekly + Holiday effects
- **Retail sales:** Weekly + Monthly + Annual + Holiday effects
- **Call center volume:**
 - ▶ Hourly + Daily + Weekly patterns

SARIMA Limitation

Standard SARIMA(p, d, q)(P, D, Q) $_s$ handles only **one** seasonal period s .

For hourly data with daily AND weekly patterns, we need $s_1 = 24$ and $s_2 = 168$.

Example: Hourly Data with Multiple Seasonalities



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Solutions for Multiple Seasonalities

Traditional Approaches

- Fourier terms:** Add sin/cos regressors
- Dummy variables:** Many parameters
- Nested models:** Complex specification

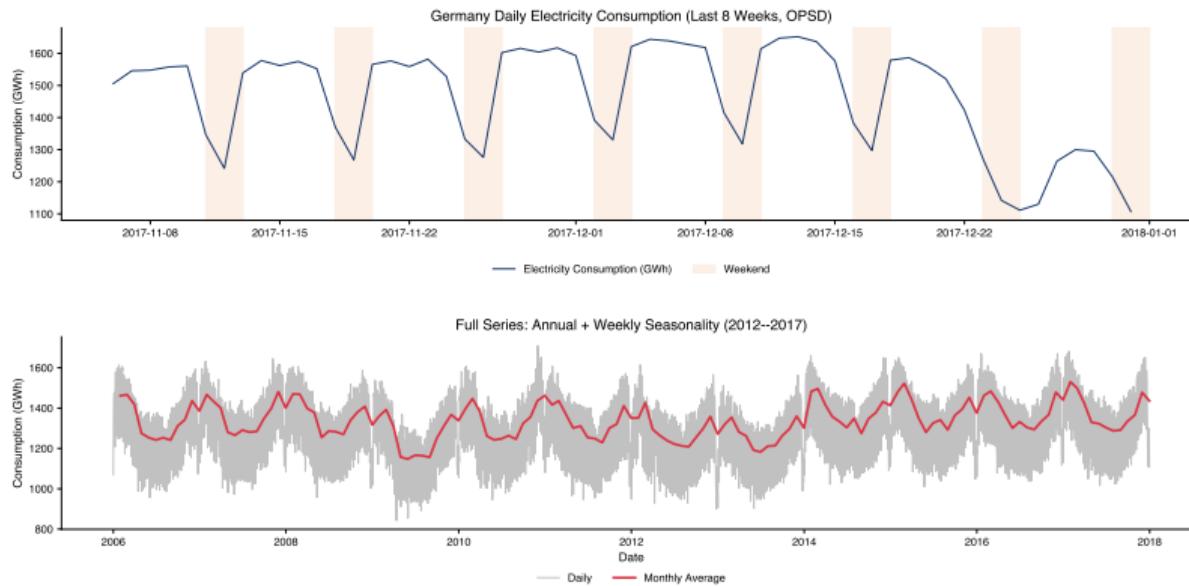
Modern Approaches

- TBATS:** Automatic, handles many periods
- Prophet:** Flexible, interpretable
- Neural methods:**
 - ▶ Deep learning

Method	Max Seasonalities	Interpretable
SARIMA	1	Yes
Fourier + ARIMA	Multiple	Moderate
TBATS	Multiple	Moderate
Prophet	Multiple	Yes



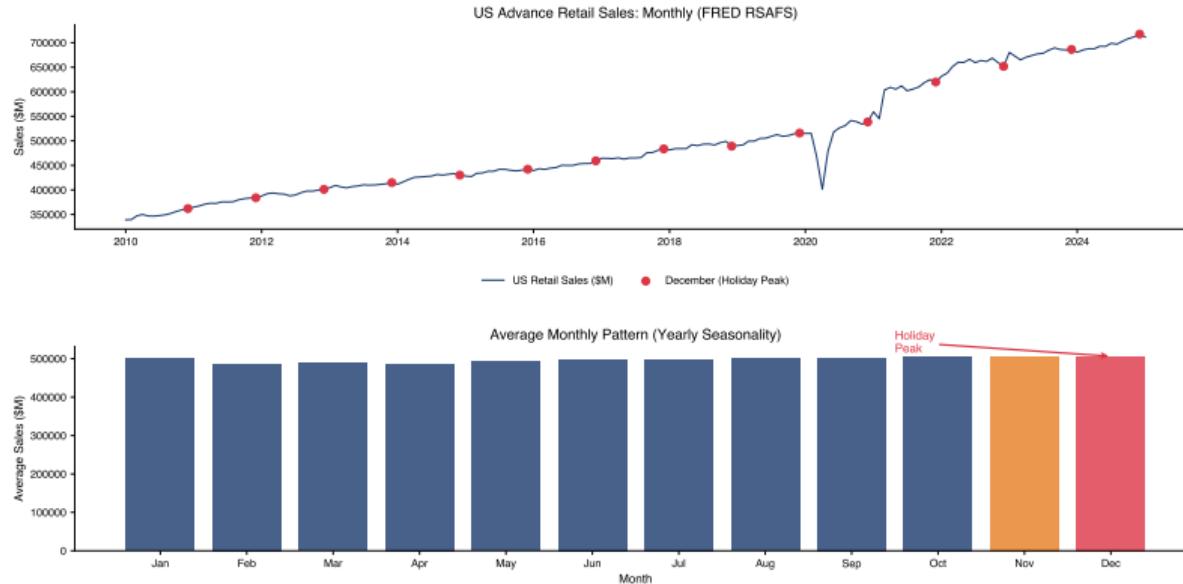
Real Example: Electricity Demand



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Real Example: Retail Sales with Holidays



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Researcher Spotlight: Rob J. Hyndman



*1967
W Wikipedia

Biography

- Australian statistician, Professor at Monash University
- One of the most influential researchers in time series forecasting
- Creator of the widely-used `forecast` package for R
- Editor-in-Chief of the *International Journal of Forecasting* (2005–2018)

Key Contributions

- **TBATS model** (2011) — trigonometric Box-Cox ARMA with multiple seasonal periods
- **ETS framework** — exponential smoothing state space models with automatic selection
- **forecast package** for R — the standard toolkit for time series forecasting
- **Hierarchical forecasting** and forecast reconciliation methods



TBATS: What Does It Stand For?

TBATS Components

- T** Trigonometric seasonality using Fourier terms
- B** Box-Cox transformation for variance stabilization
- A** ARMA errors for remaining autocorrelation
- T** Trend component (possibly damped)
- S** Seasonal components (multiple allowed)

Key Innovation: Trigonometric Seasonality

$$s_t^{(i)} = \sum_{j=1}^{k_i} \left[s_j^{(i)} \cos\left(\frac{2\pi j t}{m_i}\right) + s_j^{*(i)} \sin\left(\frac{2\pi j t}{m_i}\right) \right]$$

m_i = seasonal period, k_i = number of harmonics



Box-Cox Transformation

Definition 1 (Box-Cox Transformation)

The Box-Cox transformation with parameter ω is defined as:

$$y_t^{(\omega)} = \begin{cases} \frac{y_t^\omega - 1}{\omega} & \text{if } \omega \neq 0 \\ \ln(y_t) & \text{if } \omega = 0 \end{cases}$$

Purpose

- Variance stabilization:** Makes variance constant over time
- Normalization:** Reduces skewness in the data
- Common values: $\omega = 0$ (log), $\omega = 0.5$ (square root), $\omega = 1$ (no transform)



TBATS Model Structure

State Space Representation

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^T s_{t-m_i}^{(i)} + d_t \quad (1)$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t, \quad b_t = \phi b_{t-1} + \beta d_t \quad (2)$$

- $y_t^{(\omega)}$: Box-Cox transformed observation
- ℓ_t : local level (smoothed mean)
- b_t : trend with damping $\phi \in (0, 1)$
- $s_t^{(i)}$: i -th seasonal component
- d_t : ARMA(p, q) error process
- α, β : smoothing parameters



TBATS: Trigonometric Seasonality State Evolution

Definition 2 (Trigonometric State-Space Recursion)

For each seasonal component with period m_i and k_i harmonics, define states:

$$\begin{pmatrix} s_{j,t}^{(i)} \\ s_{j,t}^{*(i)} \end{pmatrix} = \begin{pmatrix} \cos(\lambda_j) & \sin(\lambda_j) \\ -\sin(\lambda_j) & \cos(\lambda_j) \end{pmatrix} \begin{pmatrix} s_{j,t-1}^{(i)} \\ s_{j,t-1}^{*(i)} \end{pmatrix} + \begin{pmatrix} \gamma_1^{(i)} \\ \gamma_2^{(i)} \end{pmatrix} d_t$$

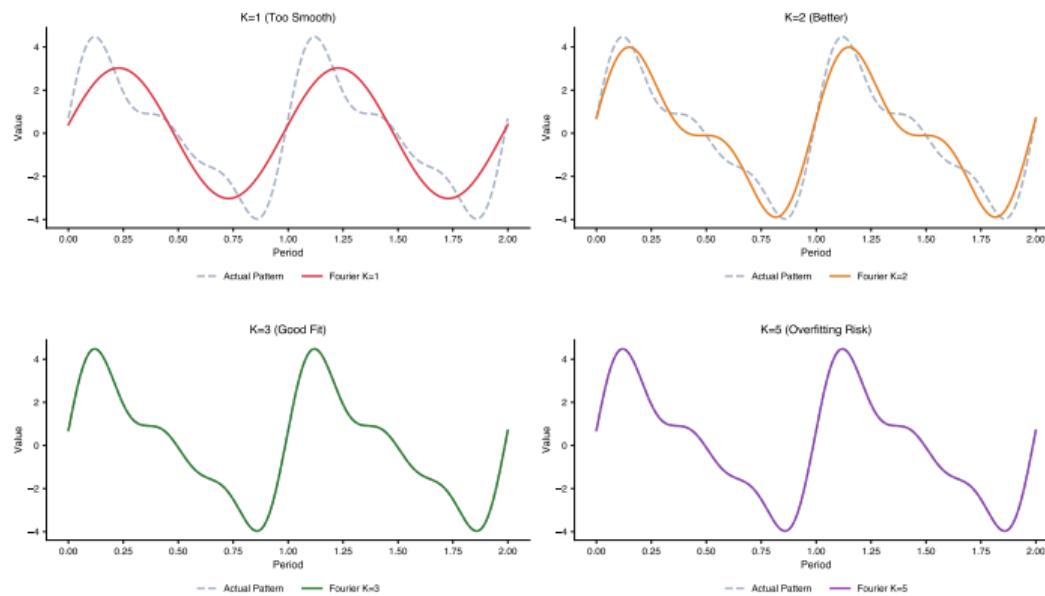
where $\lambda_j = \frac{2\pi j}{m_i}$ is the j -th harmonic frequency.

Interpretation

- The rotation matrix preserves the periodic structure
- Total seasonal: $s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$
- Parameters: $2k_i$ states per seasonal period



Fourier Approximation of Seasonality



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TBATS: Choosing the Number of Harmonics

Why Fourier/Trigonometric Terms?

1. **Parsimonious:** $2k$ parameters vs m dummy variables
2. **Smooth:** Captures smooth seasonal patterns naturally
3. **Flexible:** Number of harmonics k controls complexity
4. **Non-integer periods:** Can handle $s = 365.25$ for daily data

Low k (few harmonics)

- Smooth pattern
- Fewer parameters
- May miss sharp peaks

High k (many harmonics)

- Can capture any pattern
- More parameters ($2k$ total)
- Maximum useful: $k \leq \lfloor m/2 \rfloor$



TBATS: Key Features

Automatic Model Selection

TBATS automatically determines:

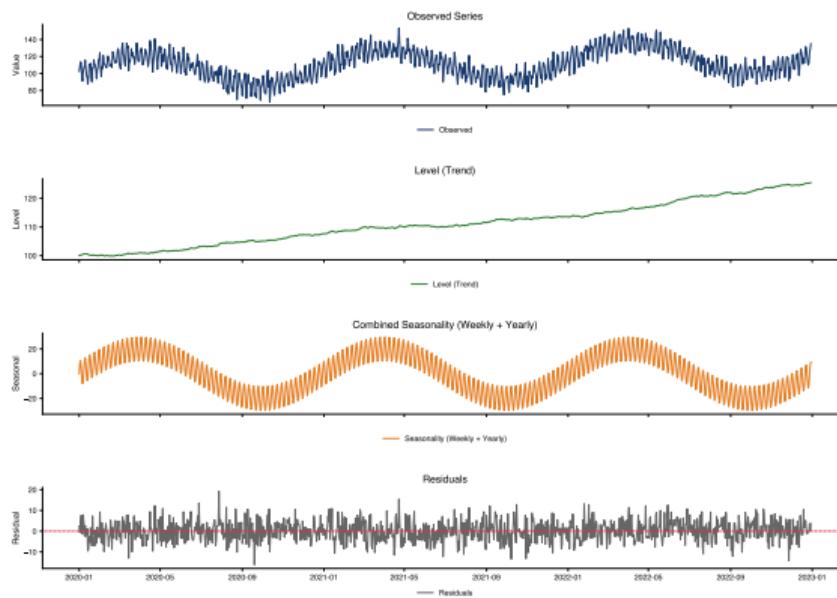
- Box-Cox parameter ω for variance stabilization
- Number of harmonics k_i for each seasonal period
- ARMA orders (p, q) for residual autocorrelation
- Damped vs non-damped trend specification

BATS vs TBATS

- BATS**: Traditional seasonal states (dummy variables)
- TBATS**: Trigonometric (Fourier) seasonal representation
- TBATS more parsimonious for long seasonal periods



TBATS Decomposition Example



 TSA_ch9_tbats_decomposition



TBATS: Advantages and Limitations

Advantages

- Handles **multiple** seasonal periods
- Automatic** model selection
- Handles **non-integer** periods (365.25)
- Box-Cox** for heteroskedasticity
- Good for **high-frequency** data

Limitations

- Computationally intensive**
- No external regressors**
- Less **interpretable** than Prophet
- Can be **slow** for very long series
- Requires **sufficient data per season**



Prophet: Overview

What is Prophet?

Forecasting procedure developed by Facebook (Meta) in 2017 for **business time series**:

- Strong seasonal effects (daily, weekly, yearly)
- Holiday effects and trend changes (changepoints)
- Handles missing data and outliers

Key Philosophy: “Analyst-in-the-loop”

Designed for analysts with domain knowledge but without time series expertise.



Prophet Model Structure

Decomposition Approach

Prophet uses an **additive decomposition**:

$$y(t) = g(t) + s(t) + h(t) + \varepsilon_t$$

$g(t)$: Trend

- Linear or logistic
- Automatic changepoints
- Growth saturation

$s(t)$: Seasonality

- Fourier series
- Multiple periods
- Custom seasonality

$h(t)$: Holidays

- Country holidays
- Custom events
- Window effects



Prophet: Seasonality Component

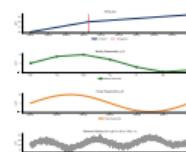
Fourier Series Representation

$$s(t) = \sum_{n=1}^N \left[a_n \cos\left(\frac{2\pi n t}{P}\right) + b_n \sin\left(\frac{2\pi n t}{P}\right) \right]$$

Default Settings

Seasonality	Period	Fourier Order
Yearly	365.25 days	10
Weekly	7 days	3
Daily	1 day	4

Higher N = more flexibility but risk of overfitting



Prophet: Trend Component

Linear Trend with Changepoints

$$g(t) = (k + \mathbf{a}(t)^T \boldsymbol{\delta}) \cdot t + (m + \mathbf{a}(t)^T \boldsymbol{\gamma})$$

- k : base growth rate (slope)
- $\boldsymbol{\delta} = (\delta_1, \dots, \delta_S)$: slope changes at S changepoints
- $\mathbf{a}(t) \in \{0, 1\}^S$: indicator if changepoint s is active at time t

Logistic Growth

For saturating trends:

$$g(t) = \frac{C(t)}{1 + e^{-(k + \mathbf{a}(t)^T \boldsymbol{\delta})(t - m - \mathbf{a}(t)^T \boldsymbol{\gamma})}}$$

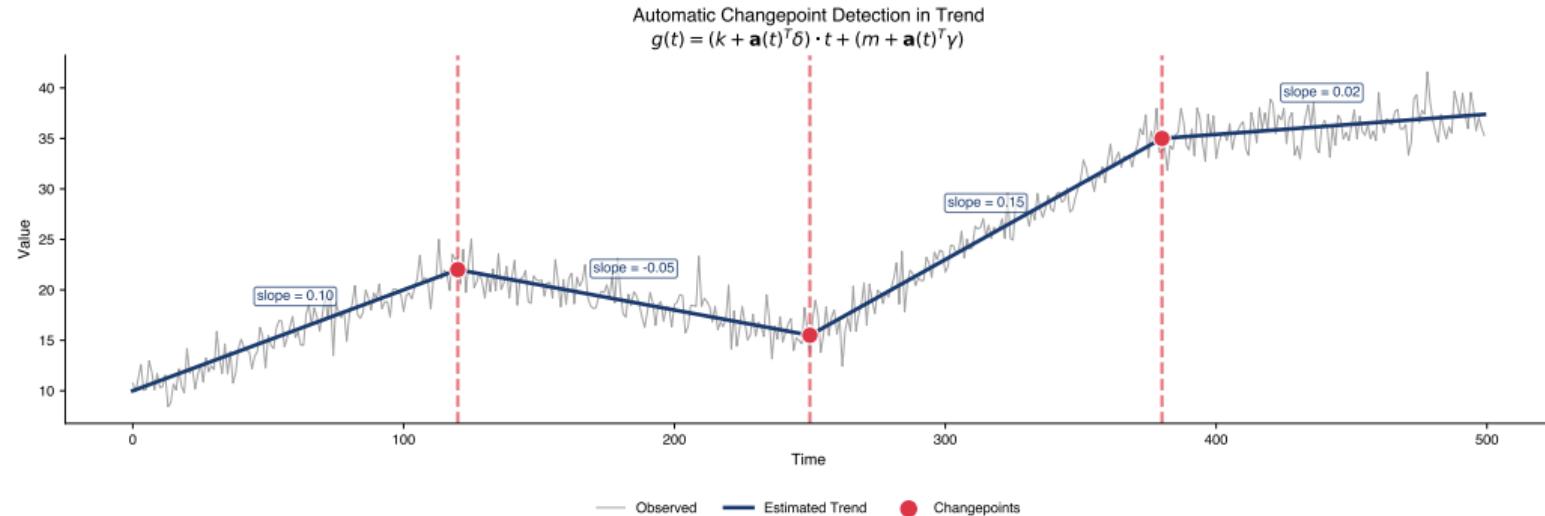
$C(t)$ = time-varying carrying capacity

Continuity Constraint

The offset $\gamma_j = -s_j \cdot \delta_j$ ensures $g(t)$ is continuous at each changepoint s_j .



Trend Changepoint Detection



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Prophet: Holiday Effects

Holiday Model

$$h(t) = Z(t) \cdot \kappa$$

where $Z(t)$ is an indicator matrix for holidays and κ are holiday effects.

Built-in Features

- 60+ countries supported
- Custom holiday definitions
- Window effects (before/after)

Holiday Types

- National holidays
- Religious observances
- Business events



Prophet: Customization Options

Seasonality Customization

- Add custom seasonal periods (monthly, quarterly)
- Control Fourier order for each seasonality
- Enable/disable default seasonalities

External Regressors

Prophet supports adding external variables:

- Weather data, promotions, special events
- Binary or continuous regressors
- Automatic regularization



Prophet: Uncertainty Quantification

Bayesian Framework

Prophet uses a **Laplace prior** on changepoint magnitudes:

$$\delta_j \sim \text{Laplace}(0, \tau), \quad \tau = \text{changepoint_prior_scale}$$

Smaller τ = sparser, smaller changepoints (more regularization).

Sources of Uncertainty

1. **Trend:** Future changepoints
2. **Seasonality:** Coefficient variance
3. **Observation:** Residual noise σ^2

Prediction Intervals

- MAP estimation for point forecasts
- Monte Carlo sampling for intervals
- Default: 80% credible interval



Prophet: Tuning Parameters

Key Parameters

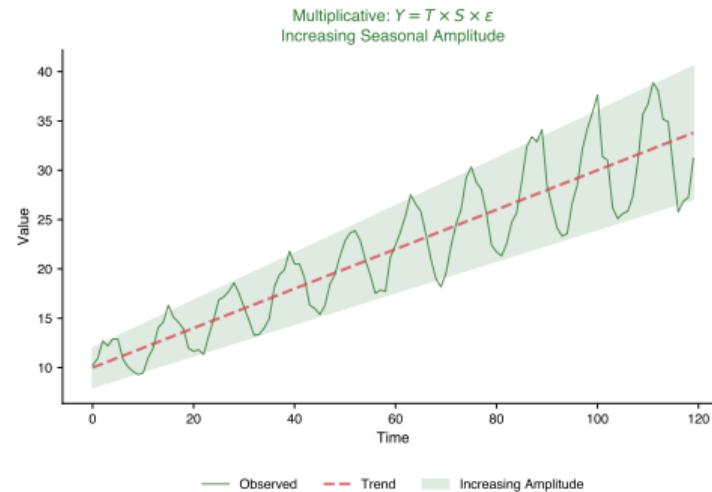
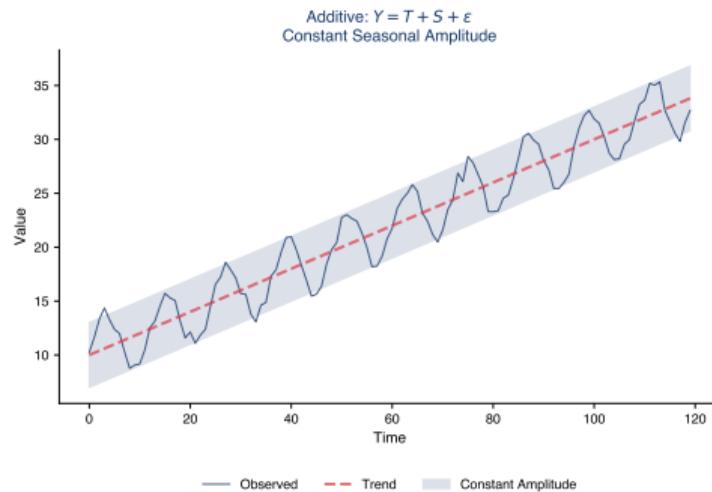
Parameter	Effect
changepoint_prior_scale	Trend flexibility (default: 0.05)
seasonality_prior_scale	Seasonality flexibility (default: 10)
holidays_prior_scale	Holiday effect size (default: 10)
seasonality_mode	'additive' or 'multiplicative'
changepoint_range	Portion of history for changepoints

Practical Tips

- Overfitting trend?** Decrease `changepoint_prior_scale`
- Underfitting seasonality?** Increase `seasonality_prior_scale`
- Seasonal amplitude varies?** Use `seasonality_mode='multiplicative'`



Additive vs Multiplicative Seasonality



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Prophet: Advantages and Limitations

Advantages

- Easy to use:** Minimal tuning needed
- Interpretable:** Clear decomposition
- Handles missing data well**
- Holiday effects built-in**
- Multiple seasonalities**
- External regressors supported**
- Fast fitting**

Limitations

- Not ARIMA-based:** No autocorrelation modeling
- Daily data focus:** Less suited for very high frequency
- Trend assumptions:** Linear/logistic may not fit
- No built-in CV:** Must implement manually
- Overfitting risk with many seasonalities**



TBATS vs Prophet: Head-to-Head

Feature	TBATS	Prophet
Multiple seasonalities	Yes (automatic)	Yes (manual or auto)
Holiday effects	No	Yes (built-in)
External regressors	No	Yes
Trend changepoints	No (smooth)	Yes (automatic)
Missing data	Interpolation needed	Handles natively
Interpretability	Moderate	High
Computation speed	Slow	Fast
High-frequency data	Good	Moderate
Non-integer periods	Yes (e.g., 365.25)	Yes
Uncertainty intervals	Yes	Yes



When to Use Each Model

Use TBATS when:

- High-frequency data
- Multiple seasonal periods
- No external regressors
- Automatic selection preferred

Use Prophet when:

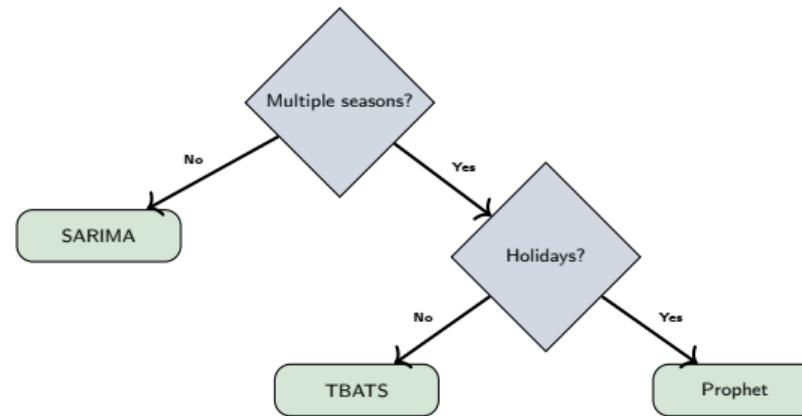
- Business forecasting
- Holiday effects important
- Trend has changepoints
- External regressors available

General Guideline

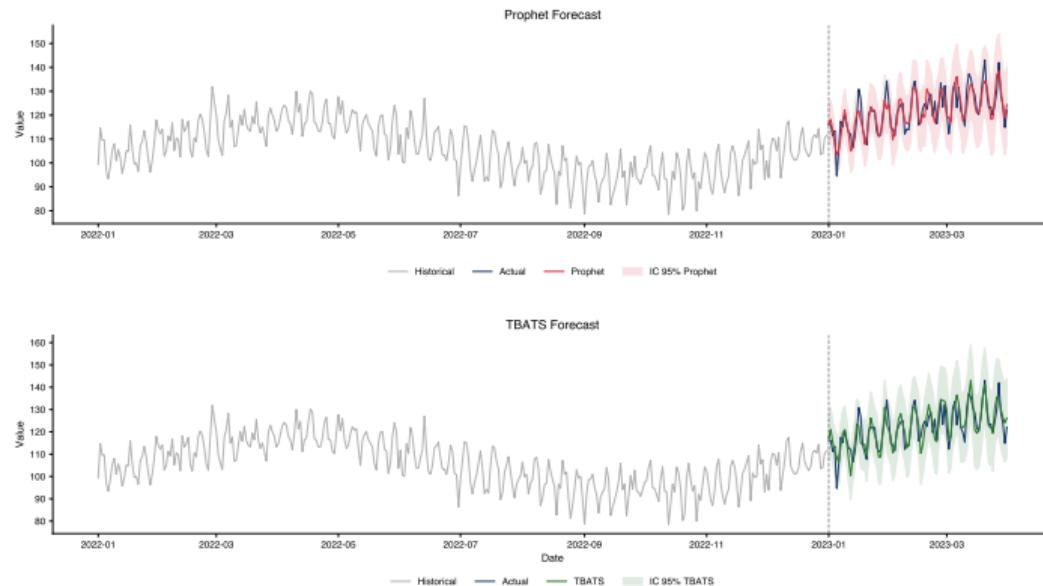
Prophet: business applications with daily data
TBATS: technical applications with high-frequency data



Decision Flowchart



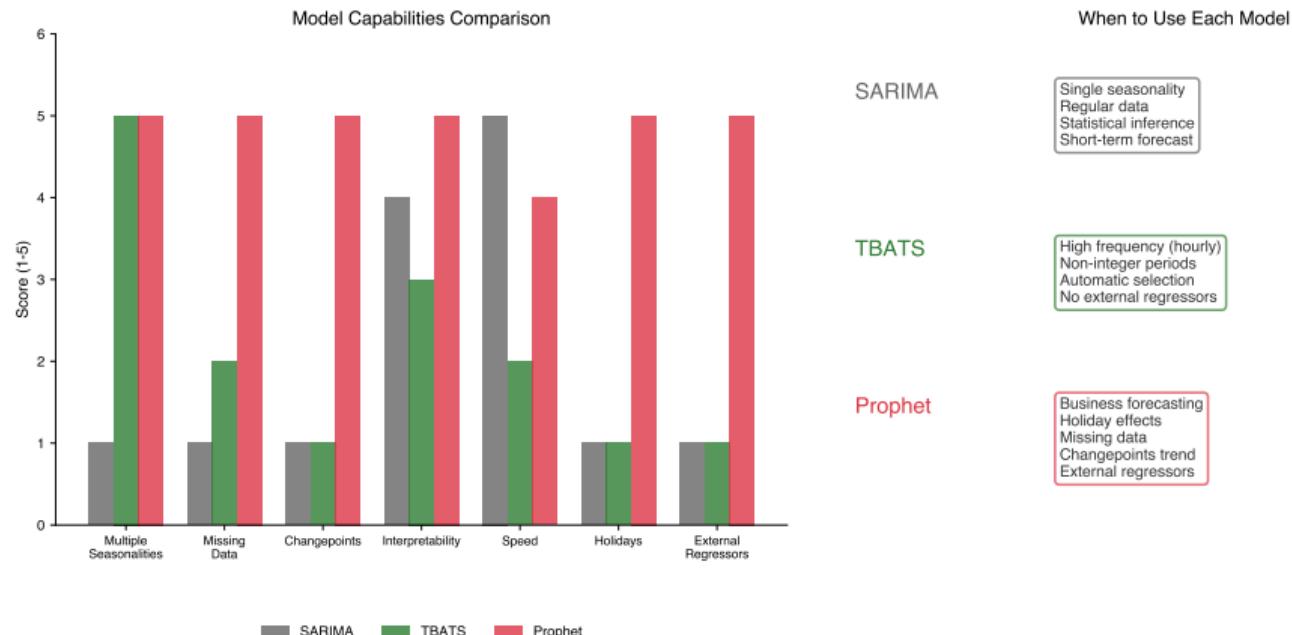
Prophet vs TBATS: Forecast Comparison



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Model Selection Guide



Evaluation Metrics

Definition 3 (Forecast Accuracy Metrics)

Let y_t denote actual values, \hat{y}_t forecasts, and n the forecast horizon:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{t=1}^n (y_t - \hat{y}_t)^2} \quad (\text{penalizes large errors})$$

$$\text{MAE} = \frac{1}{n} \sum_{t=1}^n |y_t - \hat{y}_t| \quad (\text{robust to outliers})$$

$$\text{MAPE} = \frac{100\%}{n} \sum_{t=1}^n \left| \frac{y_t - \hat{y}_t}{y_t} \right| \quad (\text{scale-free})$$

Coverage

For prediction intervals $[\hat{y}_t^L, \hat{y}_t^U]$, coverage rate is the proportion of actual values falling within the interval. Target: match the nominal level (e.g., 80%).



Case Study: Energy Demand Forecasting

Problem

Forecast hourly electricity demand with:

- Daily pattern:** Peak at noon and evening
- Weekly pattern:** Lower on weekends
- Annual pattern:** Higher in summer (AC) and winter (heating)
- Holiday effects:** Lower demand on holidays

Approach

1. Try TBATS with periods [24, 168, 8766]
2. Try Prophet with daily, weekly, yearly seasonality + holidays
3. Compare using cross-validation



Case Study: Results

Model Comparison

Model	MAPE	RMSE	Coverage
SARIMA (daily only)	8.5%	450 MW	75%
TBATS	4.2%	220 MW	82%
Prophet	4.8%	250 MW	85%
Prophet + holidays	3.9%	200 MW	88%

Key Finding

Multiple seasonality models significantly outperform single-seasonality SARIMA.



AI Exercise: Critical Thinking

Prompt to test in ChatGPT / Claude / Copilot

"I have 3 years of hourly energy consumption data. Use Facebook Prophet to forecast the next week. Include holidays and special events. Give me complete Python code."

Exercise:

1. Run the prompt in an LLM of your choice and critically analyze the response.
2. Does Prophet automatically detect multiple seasonalities (daily, weekly)?
3. How are holidays specified? Country-specific or custom events?
4. Does it use cross-validation with cutoffs (performance metrics)?
5. Would TBATS be more appropriate for this frequency? Why or why not?

Warning: AI-generated code may run without errors and look professional. *That does not mean it is correct.*



Question 1

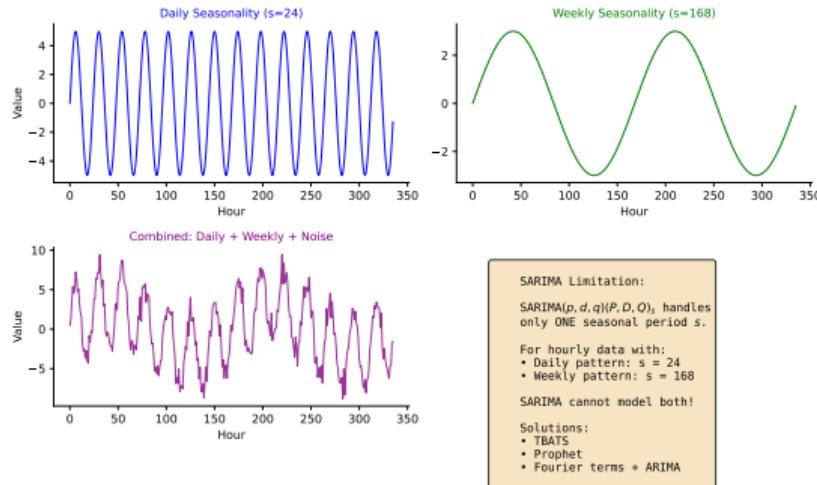
Question

- Why can't standard SARIMA(p, d, q)(P, D, Q) $_s$ model hourly electricity data with both daily and weekly patterns?

Answer Choices

- (A) SARIMA can only handle one seasonal period s at a time
- (B) SARIMA requires normally distributed errors for multiple seasonalities
- (C) SARIMA can handle multiple seasonalities but requires more data
- (D) SARIMA only works with monthly or quarterly data

Question 1: Answer



Answer: (A)

- SARIMA handles only **one** seasonal period s . You cannot set $s = 24$ (daily) and $s = 168$ (weekly) simultaneously in a single SARIMA model.



Question 2

Question

- What does each letter in TBATS represent?

Answer Choices

- (A)** Trend, Bayes, Autoregressive, Time, Stationarity
- (B)** Trigonometric seasonality, Box-Cox, ARMA errors, Trend, Seasonal components
- (C)** Taylor, Box-Cox, ARIMA, Transformation, Smoothing
- (D)** Trigonometric, Bayesian, ARMA, Trend, Spectral analysis



Question 2: Answer

TBATS: What Does It Stand For?

T	Trigonometric	Fourier terms for seasonality $\sum [a_n \cos(\frac{2\pi n t}{m}) + b_n \sin(\frac{2\pi n t}{m})]$
B	Box-Cox	Variance stabilization $y^{(w)} = (y^w - 1)/w$
A	ARMA	Error autocorrelation $\phi(L)d_t = \theta(L)e_t$
T	Trend	Level + slope (possibly damped) $t_t = t_{t-1} + \phi b_{t-1}$
S	Seasonal	Multiple seasonal periods m_1, m_2, \dots, m_T

Answer: (B)

- Trigonometric seasonality, Box-Cox transformation, ARMA errors, Trend, Seasonal components.

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Question 3

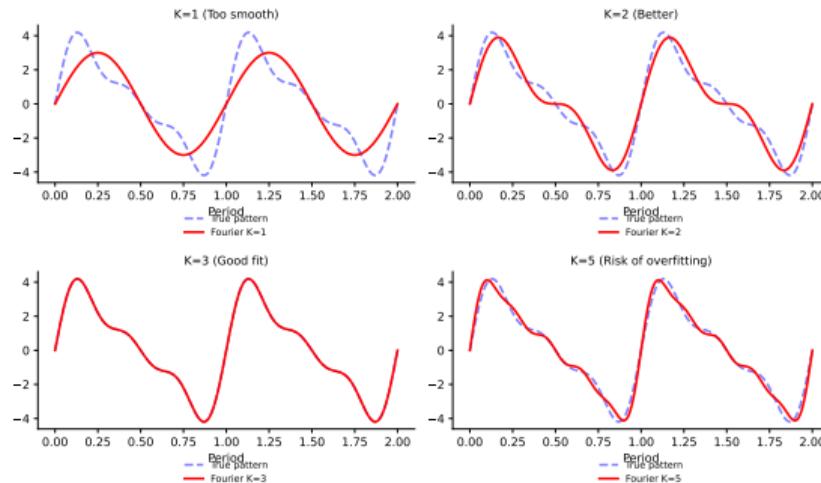
Question

- What happens when we increase the number of Fourier harmonics K ?

Answer Choices

- (A) The model becomes simpler and more robust
- (B) The model captures more complex seasonal patterns but risks overfitting
- (C) The forecast horizon increases proportionally
- (D) The seasonal period s changes automatically

Question 3: Answer



Answer: (B)

- Higher K captures more complex seasonal patterns but increases the risk of overfitting. The maximum is $K \leq s/2$.



Question 4

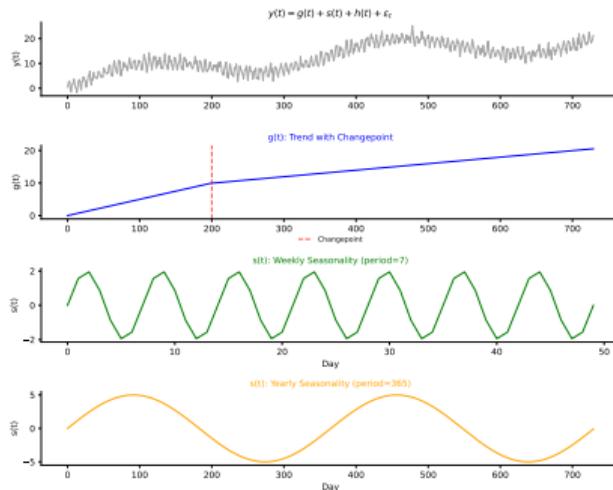
Question

- What are the main components in Prophet's model $y(t) = g(t) + s(t) + h(t) + \varepsilon_t$?

Answer Choices

- (A)** $g(t)$ = GARCH volatility, $s(t)$ = stationarity test, $h(t)$ = heteroskedasticity
- (B)** $g(t)$ = growth (trend with changepoints), $s(t)$ = seasonality, $h(t)$ = holiday effects
- (C)** $g(t)$ = Gaussian noise, $s(t)$ = smoothing, $h(t)$ = harmonic terms
- (D)** $g(t)$ = gradient, $s(t)$ = spectral density, $h(t)$ = Hurst exponent

Question 4: Answer



Answer: (B)

- $g(t)$ = trend with changepoints, $s(t)$ = seasonality (Fourier terms), $h(t)$ = holiday effects, ε_t = error term.



Question 5

Question

- What key features does Prophet have that TBATS lacks?

Answer Choices

- (A)** Trigonometric seasonality and Box-Cox transformation
- (B)** Automatic parameter selection and exponential smoothing
- (C)** Holiday effects, external regressors, trend changepoints, and native missing data handling
- (D)** State-space formulation and ARMA error modeling

Question 5: Answer

TBATS vs Prophet: Head-to-Head Comparison

Feature	TBATS	Prophet
Multiple seasonalities	Yes (automatic)	Yes (manual/auto)
Holiday effects	No	Yes (built-in)
External regressors	No	Yes
Trend changepoints	No (smooth)	Yes (automatic)
Missing data	Needs interpolation	Handles natively
Interpretability	Moderate	High
Computation speed	Slow	Fast
High-frequency data	Good	Moderate
Non-integer periods	Yes (e.g., 365.25)	Yes
Best for	Technical/high-freq	Business/daily

Answer: (C)

- Prophet offers holiday effects, external regressors, trend changepoints, and native missing data handling—features not available in TBATS.

Key Takeaways

What We Learned

- ☐ TBATS handles multiple seasonalities with Fourier terms and Box-Cox transformation
- ☐ Prophet provides interpretable decomposition with trend changepoints and holiday effects
- ☐ Both methods scale better than SARIMA for high-frequency and complex seasonal data

Important

Choose Prophet for business forecasting with holidays and interpretability needs. Use TBATS for automatic modeling of high-frequency data. Always validate with time series cross-validation—never standard k-fold!



Questions?

Questions?

Next Steps:

- Practice with the Jupyter notebook
- Try Prophet on your own data
- Explore NeuralProphet for deep learning extension



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Prophet

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Online Resources and Code

- **Quantlet:** <https://quantlet.com> → Code repository for statistics
- **Quantinar:** <https://quantinar.com> → Learning platform for quantitative methods
- **GitHub TSA:** <https://github.com/QuantLet/TSA> → Python code for this course



Thank You!

Questions?

Course materials available at: <https://danpele.github.io/Time-Series-Analysis/>

 Quantlet

 Quantinar

