



# Time Series Analysis and Forecasting

Seminar 3: ARIMA Models



Daniel Traian PELE

Bucharest University of Economic Studies

IDA Institute Digital Assets

Blockchain Research Center

AI4EFIN – Artificial Intelligence for Energy Finance

Romanian Academy, Institute for Economic Forecasting

MSCA Digital Finance

## Seminar Outline

### Today's Activities:

- 1. Review Quiz** — Checking understanding of ARIMA concepts
- 2. True/False Questions** — Conceptual checks
- 3. Practice Problems** — Calculations with ARIMA
- 4. Worked Examples** — Real-world applications
- 5. Discussion Questions** — Practical applications
- 6. AI-Assisted Exercises** — Human vs. AI modeling



## Quiz 1: Integration Order

### Question

A time series  $Y_t$  requires two differences to become stationary. What is its order of integration?

- A)  $I(0)$
- B)  $I(1)$
- C)  $I(2)$
- D) Cannot be determined



## Quiz 1: Integration Order

### Question

A time series  $Y_t$  requires two differences to become stationary. What is its order of integration?

Answer: C – I(2)

**Definition:**  $Y_t \sim I(d)$  if  $\Delta^d Y_t$  is stationary but  $\Delta^{d-1} Y_t$  is not.

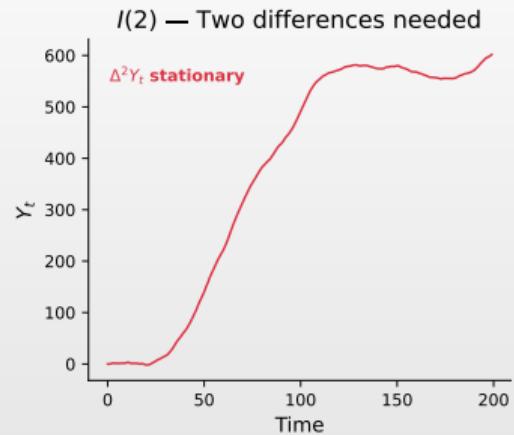
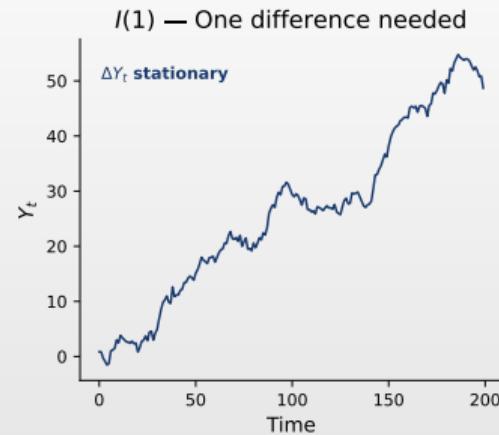
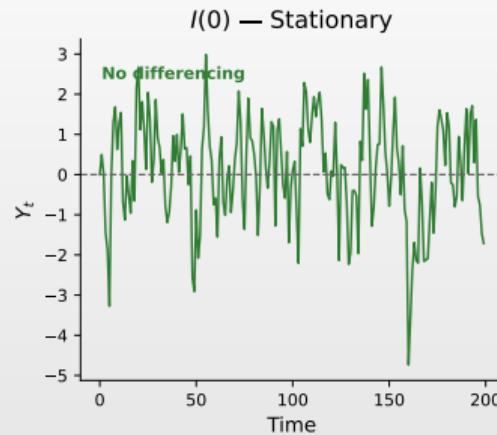
**Example:** If  $Y_t$  follows  $\Delta^2 Y_t = \varepsilon_t$ , then:

- $\Delta Y_t = \Delta Y_{t-1} + \varepsilon_t$  (still has unit root)
- $\Delta^2 Y_t = \varepsilon_t$  (white noise, stationary)

**Real-world:** Price levels may be  $I(2)$  when inflation itself is non-stationary.



## Visual: Integrated Processes



$I(0)$ : stationary.  $I(1)$ : one difference needed.  $I(2)$ : two differences needed to become stationary.

[TSA\\_ch3\\_def\\_integrated](#)



## Quiz 2: Random Walk Properties

### Question

For a random walk  $Y_t = Y_{t-1} + \varepsilon_t$  with  $\text{Var}(\varepsilon_t) = \sigma^2$ , what is  $\text{Var}(Y_t)$ ?

- A)  $\sigma^2$
- B)  $t \cdot \sigma^2$
- C)  $\sigma^2/t$
- D)  $\sigma^2/(1 - \phi^2)$

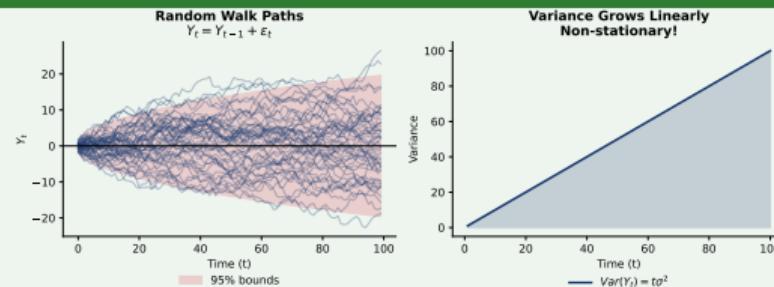
 TSA\_ch3\_rw\_variance

## Quiz 2: Random Walk Properties

### Question

For a random walk  $Y_t = Y_{t-1} + \varepsilon_t$  with  $\text{Var}(\varepsilon_t) = \sigma^2$ , what is  $\text{Var}(Y_t)$ ?

Answer:  $B - t \cdot \sigma^2$



Proof:  $Y_t = \sum_{i=1}^t \varepsilon_i \Rightarrow \text{Var}(Y_t) = t\sigma^2$  (grows linearly  $\Rightarrow$  non-stationary!)

Q TSA\_ch3\_rw\_variance



## Quiz 3: ADF Test Hypotheses

### Question

In the Augmented Dickey-Fuller test, what is the null hypothesis?

- A) The series is stationary
- B) The series has a unit root
- C) The series has no autocorrelation
- D) The series is normally distributed



## Quiz 3: ADF Test Hypotheses

### Question

In the Augmented Dickey-Fuller test, what is the null hypothesis?

Answer: B – The series has a unit root

**ADF regression:**  $\Delta Y_t = \alpha + \gamma Y_{t-1} + \sum_{j=1}^p \delta_j \Delta Y_{t-j} + \varepsilon_t$

**Hypotheses:**

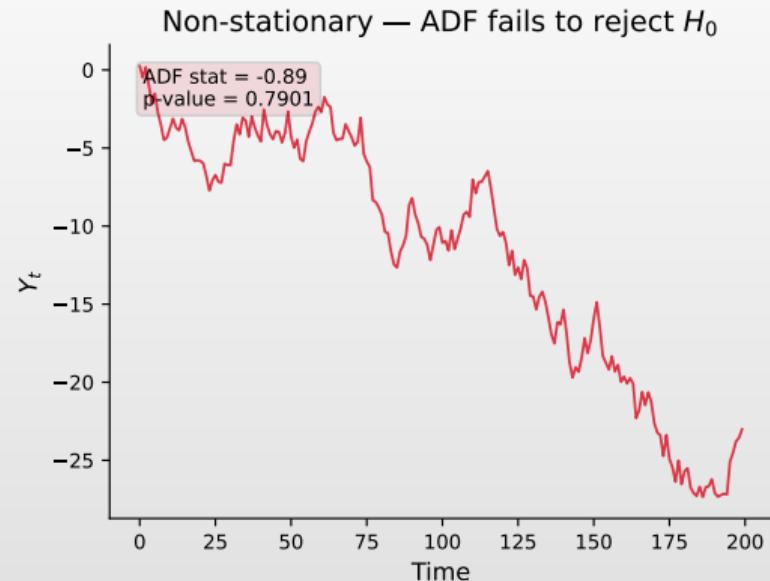
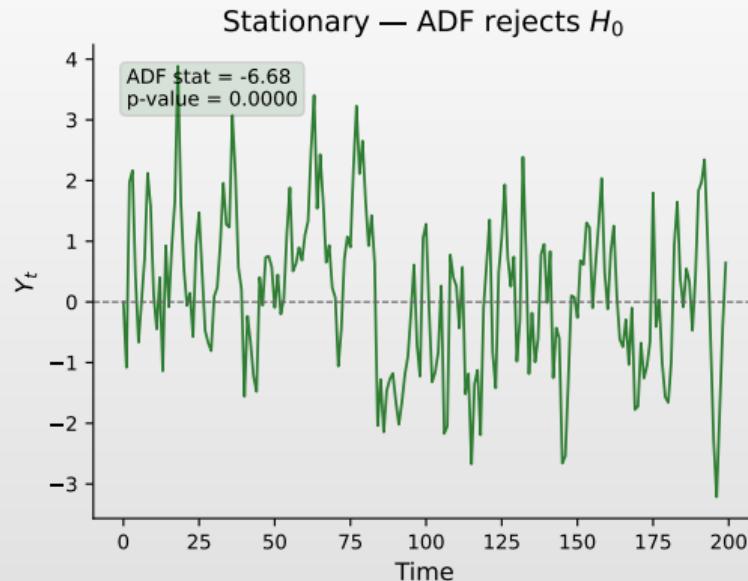
- $H_0 : \gamma = 0$  (unit root, non-stationary)
- $H_1 : \gamma < 0$  (stationary)

**Decision:** Reject  $H_0$  if  $t$ -statistic < critical value (e.g.,  $-2.86$  at 5%)

**Note:** Uses special Dickey-Fuller distribution, not standard  $t$ .



## Visual: ADF Test



Left: stationary – ADF rejects unit root. Right: non-stationary – ADF fails to reject.

Q TSA\_ch3\_def\_adf



## Quiz 4: ARIMA Notation

### Question

What does ARIMA(2,1,1) mean?

- A) AR(2) on differenced data with MA(1) errors
- B) AR(1) with 2 differences and MA(1)
- C) MA(2) with 1 difference and AR(1)
- D) 2 lags, 1 trend, 1 seasonal component



## Quiz 4: ARIMA Notation

### Question

What does ARIMA(2,1,1) mean?

Answer: A – AR(2) on differenced data with MA(1) errors

**ARIMA( $p, d, q$ ):**  $\phi(L)(1 - L)^d Y_t = \theta(L)\varepsilon_t$

**ARIMA(2,1,1) expands to:**

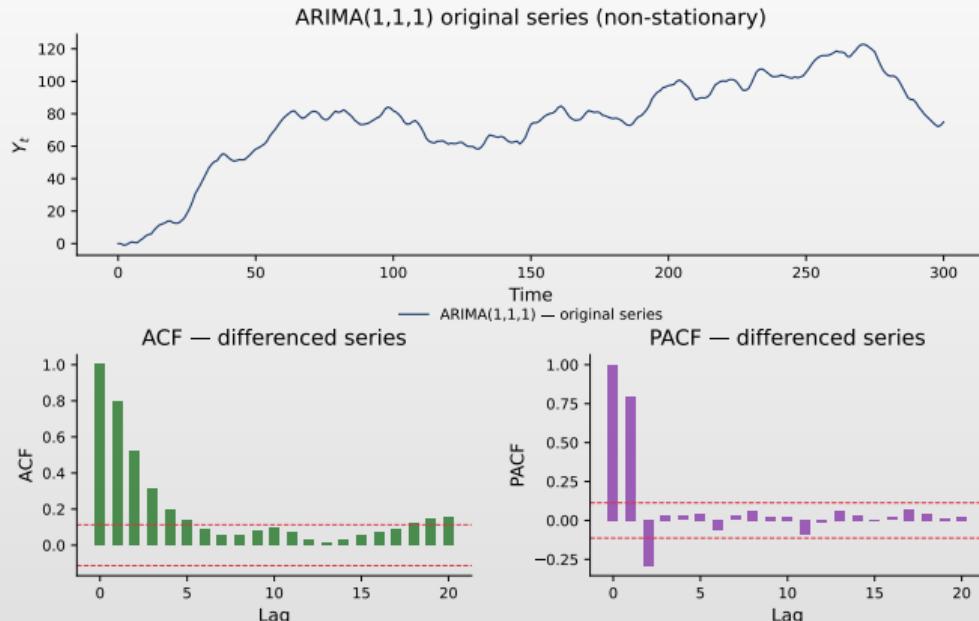
$$(1 - \phi_1 L - \phi_2 L^2)(1 - L)Y_t = (1 + \theta_1 L)\varepsilon_t$$

Or equivalently:  $(1 - \phi_1 L - \phi_2 L^2)\Delta Y_t = (1 + \theta_1 L)\varepsilon_t$

**Interpretation:** First difference the series, then fit ARMA(2,1) to  $\Delta Y_t$ .



## Visual: ARIMA Process



Top: original ARIMA series. Bottom: after differencing, use ACF/PACF to identify AR and MA orders.

TSA\_ch3\_def\_arima



## Quiz 5: Difference Operator

### Question

What is  $(1 - L)^2 Y_t$  expanded?

- A)  $Y_t - Y_{t-1}$
- B)  $Y_t - 2Y_{t-1} + Y_{t-2}$
- C)  $Y_t + 2Y_{t-1} + Y_{t-2}$
- D)  $Y_t - Y_{t-2}$



## Quiz 5: Difference Operator

### Question

What is  $(1 - L)^2 Y_t$  expanded?

Answer:  $B - Y_t - 2Y_{t-1} + Y_{t-2}$

**Binomial expansion:**  $(1 - L)^2 = 1 - 2L + L^2$

**Applied:**  $(1 - L)^2 Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$  (the “change in changes”)



## Quiz 6: KPSS vs ADF

### Question

How does the KPSS test differ from the ADF test?

- A) KPSS tests for seasonality, ADF tests for trends
- B) KPSS has stationarity as null, ADF has unit root as null
- C) KPSS is more powerful than ADF
- D) There is no difference

Q TSA\_ch3\_adf\_kpss

## Quiz 6: KPSS vs ADF

### Question

How does the KPSS test differ from the ADF test?

Answer: B – Reversed null hypotheses

ADF Test		KPSS Test	
$H_0$ : Unit Root		$H_0$ : Stationary	
$H_1$ : Stationary		$H_1$ : Unit Root	
Reject if $t\text{-stat} < \text{critical}$		Reject if $LM > \text{critical}$	

Decision Matrix

ADF rejects	KPSS fails to reject	→ Stationary
ADF fails to reject	KPSS rejects	→ Unit Root
Both reject	or both fail	→ Inconclusive

Strategy: Use both tests together for robust inference!

Q TSA\_ch3\_adf\_kpss



## Quiz 7: Overdifferencing

### Question

If  $Y_t \sim I(1)$  and we compute  $\Delta^2 Y_t$ , what happens?

- A) We get a better stationary series
- B) We introduce artificial negative autocorrelation
- C) The variance decreases
- D) Nothing changes

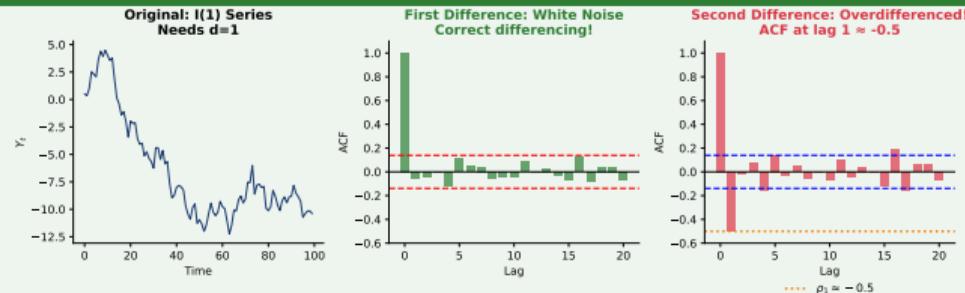
 TSA\_ch3\_overdifferencing

## Quiz 7: Overdifferencing

### Question

If  $Y_t \sim I(1)$  and we compute  $\Delta^2 Y_t$ , what happens?

Answer: B – Artificial negative autocorrelation



Diagnostic: ACF at lag 1  $\approx -0.5$  signals overdifference. Reduce  $d$  by 1!

Q TSA\_ch3\_overdifferencing



## Quiz 8: Forecast Variance

### Question

For an ARIMA(0,1,0) model (random walk), how does forecast variance behave as horizon  $h$  increases?

- A) Stays constant
- B) Decreases to zero
- C) Grows linearly with  $h$
- D) Converges to a finite limit



## Quiz 8: Forecast Variance

### Question

For an ARIMA(0,1,0) model (random walk), how does forecast variance behave as horizon  $h$  increases?

Answer: C – Grows linearly with  $h$

**Random walk forecast:**  $\hat{Y}_{T+h|T} = Y_T$  (best forecast is current value)

**Forecast error:**  $Y_{T+h} - \hat{Y}_{T+h|T} = \sum_{i=1}^h \varepsilon_{T+i}$

**Variance:**

$$\text{Var}(Y_{T+h} - \hat{Y}_{T+h|T}) = h\sigma^2$$

**95% CI:**  $Y_T \pm 1.96\sqrt{h}\sigma$  (widens with  $\sqrt{h}$ )



## Quiz 9: Unit Root Test Power

### Question

The ADF test has low power when:

- A) Sample size is very large
- B) The true root is close to but not equal to 1
- C) The series has no trend
- D) The series is clearly stationary



## Quiz 9: Unit Root Test Power

### Question

The ADF test has low power when:

Answer: B – Root close to but not equal to 1

Example: AR(1) with  $\phi = 0.95$  vs random walk ( $\phi = 1$ ) – similar ACF patterns!

Low power: High probability of Type II error (failing to reject false  $H_0$ )

Solutions: Larger samples, Phillips-Perron test, panel unit root tests



## Quiz 10: ARIMA Model Selection

### Question

After differencing once, the ACF shows a spike at lag 1 only, and PACF decays. The appropriate model is:

- A) ARIMA(1,1,0)
- B) ARIMA(0,1,1)
- C) ARIMA(1,1,1)
- D) ARIMA(0,2,1)

Q TSA\_ch3\_arima\_flowchart

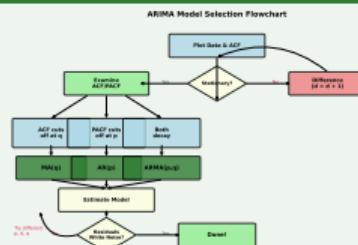


## Quiz 10: ARIMA Model Selection

### Question

After differencing once, the ACF shows a spike at lag 1 only, and PACF decays. The appropriate model is:

Answer: B – ARIMA(0,1,1)



Pattern: ACF cuts off at lag 1, PACF decays  $\Rightarrow$  MA(1). Full model: ARIMA(0,1,1) = IMA(1,1)

Q TSA\_ch3\_arima\_flowchart



## Quiz 11: Trend Stationarity vs Difference Stationarity

### Question

A trend-stationary process is made stationary by:

- A) Taking first differences
- B) Removing the deterministic trend via regression
- C) Taking second differences
- D) Applying seasonal adjustment

Q TSA\_ch3\_trend\_vs\_diff

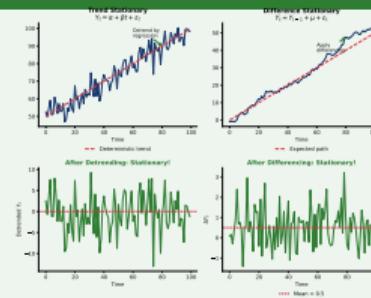


## Quiz 11: Trend Stationarity vs Difference Stationarity

### Question

A trend-stationary process is made stationary by:

Answer: B – Removing deterministic trend via regression



Trend-stationary: Detrend (shocks are temporary). Difference-stationary: Difference (shocks are permanent).

Q TSA\_ch3\_trend\_vs\_diff



## Quiz 12: ARIMA Invertibility

### Question

ARIMA(0,1,1) with  $\theta_1 = 1.2$  is:

- A) Stationary and invertible
- B) Non-stationary but invertible
- C) Non-stationary and non-invertible
- D) Stationary but non-invertible



## Quiz 12: ARIMA Invertibility

### Question

ARIMA(0,1,1) with  $\theta_1 = 1.2$  is:

Answer: C – Non-stationary and non-invertible

Stationarity:  $d = 1$  (unit root)  $\Rightarrow$  Non-stationary

Invertibility: MA root  $z = -1/1.2 = -0.833$  is inside unit circle;  $|\theta_1| > 1 \Rightarrow$  Non-invertible

Fix: Rewrite with  $\theta^* = 1/1.2 = 0.833$  and adjust variance.



## Quiz 13: Spurious Regression

### Question

Regressing one random walk on another independent random walk typically shows:

- A) No significant relationship
- B) High  $R^2$  and significant t-statistics (spuriously)
- C) Negative correlation
- D) Perfect multicollinearity



## Quiz 13: Spurious Regression

### Question

Regressing one random walk on another independent random walk typically shows:

Answer: B – High  $R^2$  and significant t-statistics (spuriously)

**Granger & Newbold (1974):** Spurious regression phenomenon

**Symptoms:** High  $R^2$  ( $> 0.9$ ), significant  $t$ -stats, low Durbin-Watson ( $\ll 2$ ), non-stationary residuals

**Solutions:** Difference both series, or test for cointegration



## Quiz 14: Long-Run Forecast

### Question

The long-run forecast from ARIMA(1,1,0) with  $\phi_1 = 0.7$  converges to:

- A) Zero
- B) The unconditional mean
- C) A linear trend extrapolation
- D) The last observed value



## Quiz 14: Long-Run Forecast

### Question

The long-run forecast from ARIMA(1,1,0) with  $\phi_1 = 0.7$  converges to:

Answer: C – A linear trend extrapolation

Model:  $(1 - \phi_1 L)(1 - L)Y_t = c + \varepsilon_t$ ; Long-run:  $\hat{Y}_{T+h} \approx Y_T + h \cdot \frac{c}{1-\phi_1}$

Key differences: Stationary ARMA  $\rightarrow$  mean; I(1) no drift  $\rightarrow$  last value; I(1) with drift  $\rightarrow$  linear



## True/False Questions

### Question

Determine if each statement is True or False:

1. An I(2) process requires two differences to become stationary.
2. The ADF test always includes a constant term.
3. ARIMA(0,1,0) is another name for a random walk.
4. Differencing a stationary series makes it “more stationary.”
5. The KPSS test has stationarity as the null hypothesis.
6. ARIMA models can only capture linear patterns.

*Answer on next slide...*



## True/False: Solutions

### Answers

- |   |              |
|---|--------------|
| 1. An $I(2)$ process requires two differences to become stationary.                                   | <b>TRUE</b>  |
| <p><math>I(d)</math> means <math>d</math> differences needed. <math>I(2) =</math> two unit roots.</p> |              |
| 2. The ADF test always includes a constant term.  | <b>FALSE</b> |
| <p>You choose: no constant, constant only, or constant + trend.</p>                                   |              |
| 3. ARIMA(0,1,0) is another name for a random walk.  | <b>TRUE</b>  |
| <p><math>(1 - L)Y_t = \varepsilon_t \Rightarrow Y_t = Y_{t-1} + \varepsilon_t</math>.</p>             |              |
| 4. Differencing a stationary series makes it “more stationary.”                                       | <b>FALSE</b> |
| <p>Over-differencing creates non-invertible MA; hurts model performance.</p>                          |              |
| 5. The KPSS test has stationarity as the null hypothesis.   | <b>TRUE</b>  |
| <p>KPSS: <math>H_0</math> = stationary. Opposite of ADF.</p>  |              |
| 6. ARIMA models can only capture linear patterns.   | <b>TRUE</b>  |
| <p>ARIMA is linear in parameters. Nonlinear patterns need GARCH, neural nets, etc.</p>                |              |



## Problem 1: Unit Root Testing

### Exercise

You have quarterly GDP data for 80 quarters. The ADF test (with constant and trend) gives a test statistic of  $-2.85$ . The 5% critical value is  $-3.41$ .

1. What is your conclusion about stationarity?
2. What would you do next?



## Problem 1: Unit Root Testing

### Exercise

You have quarterly GDP data for 80 quarters. The ADF test (with constant and trend) gives a test statistic of  $-2.85$ . The 5% critical value is  $-3.41$ .

1. What is your conclusion about stationarity?
2. What would you do next?

### Solution

1. Since  $-2.85 > -3.41$ , we **fail to reject  $H_0$** . The data appears to have a unit root (non-stationary).
2. Take the first difference  $\Delta Y_t$  and repeat the ADF test on the differenced series to confirm it is now stationary.



## Problem 2: Model Identification

### Exercise

After differencing a time series once, the ACF shows:

- Significant spike at lag 1 ( $\rho_1 = 0.4$ )
- All other lags insignificant

The PACF shows gradual decay.

What ARIMA model is suggested?



## Problem 2: Model Identification

### Exercise

After differencing a time series once, the ACF shows:

- Significant spike at lag 1 ( $\rho_1 = 0.4$ )
- All other lags insignificant

The PACF shows gradual decay.

What ARIMA model is suggested?

### Solution

- ACF cuts off after lag 1  $\Rightarrow$  MA(1) component
- PACF decays  $\Rightarrow$  Confirms MA structure
- Since we differenced once:  $d = 1$

**Suggested model: ARIMA(0,1,1) or IMA(1,1)**



## Problem 3: ARIMA Equation

### Exercise

Write out the full equation for ARIMA(1,1,1):

$$(1 - \phi_1 L)(1 - L)Y_t = c + (1 + \theta_1 L)\varepsilon_t$$

Expand this completely in terms of  $Y_t$ ,  $Y_{t-1}$ ,  $Y_{t-2}$ , etc.



## Problem 3: ARIMA Equation

### Exercise

Write out the full equation for ARIMA(1,1,1):

$$(1 - \phi_1 L)(1 - L)Y_t = c + (1 + \theta_1 L)\varepsilon_t$$

Expand this completely in terms of  $Y_t$ ,  $Y_{t-1}$ ,  $Y_{t-2}$ , etc.

### Solution

Expanding  $(1 - \phi_1 L)(1 - L) = 1 - L - \phi_1 L + \phi_1 L^2 = 1 - (1 + \phi_1)L + \phi_1 L^2$ :

$$Y_t - (1 + \phi_1)Y_{t-1} + \phi_1 Y_{t-2} = c + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

Or equivalently:

$$Y_t = c + (1 + \phi_1)Y_{t-1} - \phi_1 Y_{t-2} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$



## Problem 4: Forecast Calculation

### Exercise

Given ARIMA(0,1,1):  $\Delta Y_t = \varepsilon_t + 0.3\varepsilon_{t-1}$

At time  $T$ :  $Y_T = 100$ ,  $\hat{\varepsilon}_T = 2$ ,  $\sigma^2 = 4$

Calculate:

1.  $\hat{Y}_{T+1|T}$  (one-step forecast)
2.  $\hat{Y}_{T+2|T}$  (two-step forecast)



## Problem 4: Forecast Calculation

### Exercise

Given ARIMA(0,1,1):  $\Delta Y_t = \varepsilon_t + 0.3\varepsilon_{t-1}$

At time  $T$ :  $Y_T = 100$ ,  $\hat{\varepsilon}_T = 2$ ,  $\sigma^2 = 4$

Calculate:

1.  $\hat{Y}_{T+1|T}$  (one-step forecast)
2.  $\hat{Y}_{T+2|T}$  (two-step forecast)

### Solution

$$1. \hat{Y}_{T+1|T} = Y_T + 0.3\hat{\varepsilon}_T = 100 + 0.3(2) = 100.6$$

$$2. \hat{Y}_{T+2|T} = \hat{Y}_{T+1|T} + 0.3 \cdot 0 = 100.6 + 0 = 100.6$$

(Future shocks  $\varepsilon_{T+1}, \varepsilon_{T+2}$  are forecast as 0)



## Problem 5: Confidence Intervals

### Exercise

Continuing from Problem 4, calculate the 95% forecast intervals for  $\hat{Y}_{T+1|T}$  and  $\hat{Y}_{T+2|T}$ .

Recall:  $\sigma^2 = 4$ ,  $\theta_1 = 0.3$



## Problem 5: Confidence Intervals

### Exercise

Continuing from Problem 4, calculate the 95% forecast intervals for  $\hat{Y}_{T+1|T}$  and  $\hat{Y}_{T+2|T}$ .

Recall:  $\sigma^2 = 4$ ,  $\theta_1 = 0.3$

### Solution

For IMA(1,1), the MA( $\infty$ ) weights are  $\psi_0 = 1$ ,  $\psi_j = 1 + \theta_1$  for  $j \geq 1$ .

**1-step:**  $\text{Var}(e_{T+1}) = \sigma^2 \psi_0^2 = 4$ , so  $SE = 2$

$$100.6 \pm 1.96(2) = [96.68, 104.52]$$

**2-step:**  $\text{Var}(e_{T+2}) = \sigma^2(\psi_0^2 + \psi_1^2) = 4(1 + 1.3^2) = 10.76$ ,  $SE = 3.28$

$$100.6 \pm 1.96(3.28) = [94.17, 107.03]$$



## Example: Testing for Unit Root in Stock Prices

### Scenario

You have daily closing prices for a stock over 500 days. You want to determine if prices follow a random walk.

### Step-by-step Approach

1. **Visual inspection:** Plot prices – likely shows trend
2. **ADF test on prices:** Expect to fail to reject  $H_0$  (unit root)
3. **Take log returns:**  $r_t = \ln(P_t/P_{t-1}) = \Delta \ln(P_t)$
4. **ADF test on returns:** Should reject  $H_0$  (stationary)
5. **Conclusion:** Log prices are  $I(1)$ , returns are  $I(0)$



## Example: Box-Jenkins for Inflation Data

### Scenario

Monthly inflation rates for 10 years. Build an ARIMA model.

### Workflow

1. **Plot & test:** ADF suggests borderline – try both  $d = 0$  and  $d = 1$
2. **If  $d = 0$ :** Fit ARMA models, compare AIC
3. **If  $d = 1$ :** Examine ACF/PACF of  $\Delta Y_t$ 
  - ▶ ACF: spike at lag 1, then cuts off
  - ▶ PACF: decays
  - ▶  $\Rightarrow$  Try ARIMA(0,1,1)
4. **Estimate:** Fit ARIMA(0,1,1), check coefficients
5. **Diagnose:** Ljung-Box on residuals (want  $p > 0.05$ )
6. **Compare:** AIC of ARIMA(0,1,1) vs ARMA(1,1) on levels



## Example: Interpreting Python Output

### statsmodels ARIMA Output

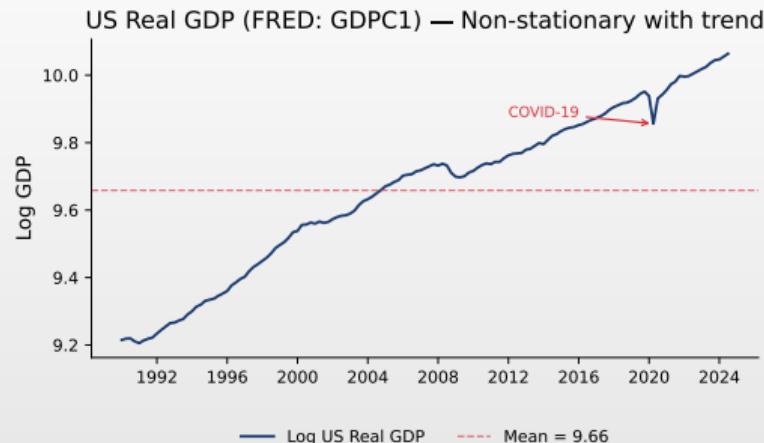
```
ARIMA Model Results
=====
Dep. Variable:      D.y    No. Observations:   99
Model:             ARIMA(1,1,1)    AIC:            285.32
                  BIC:            295.63
=====
                           coef    std err      z    P>|z|
-----
const            0.0521    0.048    1.085    0.278
ar.L1            0.4532    0.102    4.443    0.000
ma.L1           -0.2891    0.118   -2.450    0.014
sigma2          1.2340    0.176    7.011    0.000
```

### Interpretation

- AR coefficient (0.45) is significant, MA coefficient (-0.29) is significant
- Constant (0.052) not significant – could set  $c = 0$
- Check:  $|\phi_1| < 1$  (stationary),  $|\theta_1| < 1$  (invertible) – OK!



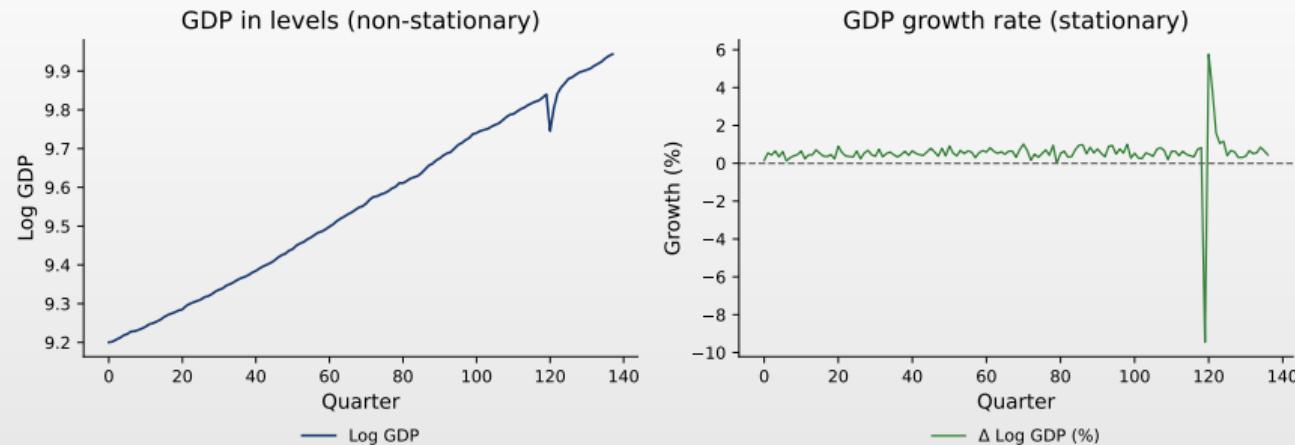
## Case Study: US Real GDP (1990–2024)



- US Real GDP in billions of 2017 dollars (quarterly data)
- Clear **upward trend** – typical of macroeconomic series
- Notable drops during recessions (2008-2009, 2020)
- Non-stationary: needs differencing before ARIMA modeling



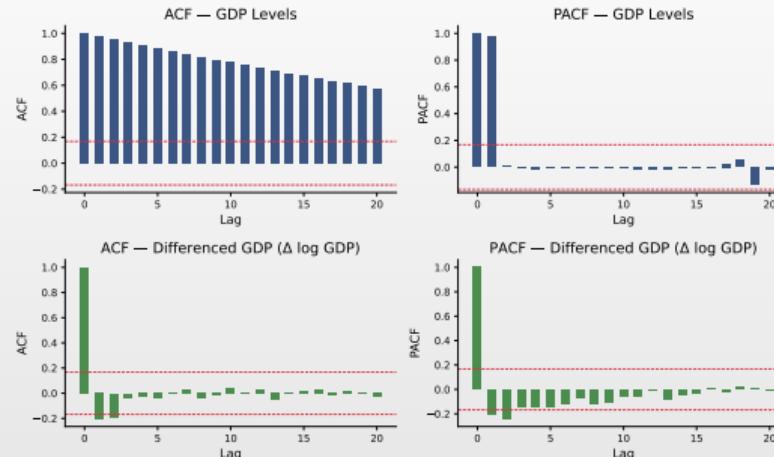
## Stationarity Through Differencing



- Left: GDP in levels – clear upward trend (non-stationary)
- Right:  $\text{GDP growth rate} = \Delta \log(Y_t) \times 100$  – stationary
- First differencing of log GDP removes the stochastic trend
- Growth rate fluctuates around a constant mean ( $\approx 0.6\%$  quarterly)



## ACF/PACF: Levels vs Differenced



- Top row: ACF/PACF of GDP levels – slow decay indicates non-stationarity
- Bottom row: ACF/PACF of GDP growth – mostly within confidence bands
- Pattern suggests low-order ARIMA model is appropriate

Q TSA\_ch3\_acf\_pacf



## ARIMA Estimation Results: US GDP Growth

Model: ARIMA(1, 1, 1) on log(GDP)

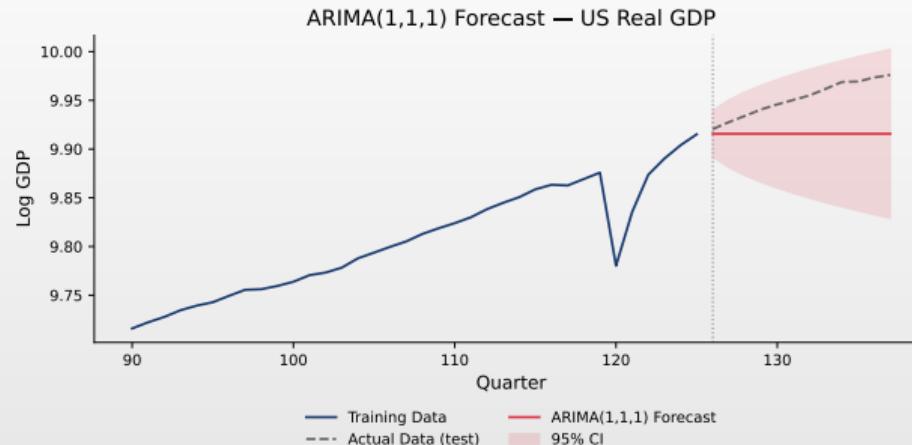
Parameter	Estimate	Std. Error	z-stat	p-value
$\phi_1$ (AR.L1)	0.312	0.185	1.69	0.091
$\theta_1$ (MA.L1)	-0.087	0.203	-0.43	0.668
$\sigma^2$	0.00012	-	-	-

## Interpretation

- Low-order ARIMA captures GDP dynamics reasonably well
- AR(1) coefficient positive – GDP growth shows persistence
- Alternative: simpler random walk (ARIMA(0,1,0)) often competitive



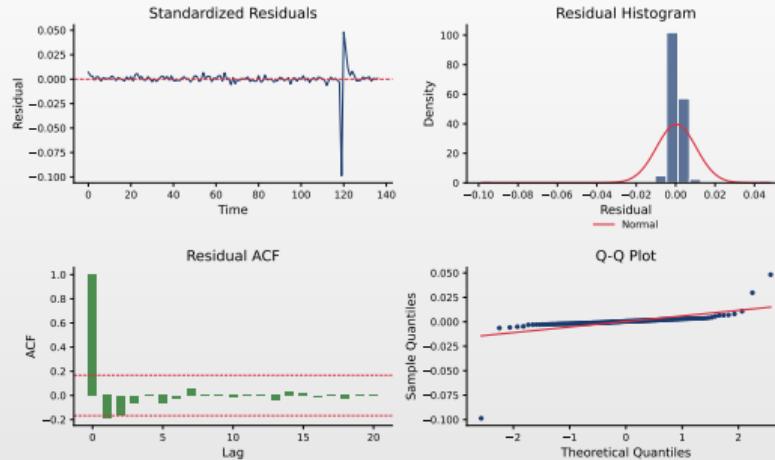
## Forecast: ARIMA vs Actual



- Blue: historical training data; Green: actual test data
- Red dashed: ARIMA forecasts with 95% confidence interval
- Forecasts capture the general trend direction
- Confidence intervals widen as forecast horizon increases



## Model Diagnostics: Residual Analysis



- Residuals show no systematic patterns over time
- Distribution approximately normal (histogram and Q-Q plot)
- ACF of residuals within bounds – no significant autocorrelation remaining
- Model adequately captures the data generating process



## Discussion: Deterministic vs Stochastic Trends

### Key Question

Why is it important to distinguish between deterministic and stochastic trends?

### Discussion Points

- **Wrong treatment consequences:**
  - ▶ Detrending a unit root  $\Rightarrow$  spurious stationarity
  - ▶ Differencing a trend-stationary  $\Rightarrow$  overdifferencing
- **Economic interpretation:**
  - ▶ Deterministic trend: shocks are temporary
  - ▶ Stochastic trend: shocks have permanent effects
- **Policy implications:**
  - ▶ Does a recession permanently lower GDP, or does the economy return to trend?



## Discussion: Model Selection Criteria

### Key Question

When should you use AIC vs BIC for ARIMA model selection?

### Considerations

- **AIC:** Minimizes prediction error, may overfit
  - ▶ Better for forecasting
  - ▶ Tends to select larger models
- **BIC:** Consistent model selection, more parsimonious
  - ▶ Better for identifying “true” model
  - ▶ Penalizes complexity more heavily
- **Practical advice:** Report both, prefer BIC if they disagree substantially



## Discussion: Limitations of ARIMA

### Key Question

What are the main limitations of ARIMA models?

### Discussion Points

- Linearity:** Cannot capture nonlinear dynamics
- Constant variance:** Assumes homoskedasticity (no GARCH effects)
- No structural breaks:** Parameters assumed constant
- Univariate:** Ignores relationships with other variables
- Symmetric:** Treats positive and negative shocks equally
- Long-horizon forecasts:** Uncertainty grows rapidly

### Extensions

These limitations motivate GARCH (volatility), VAR (multivariate), regime-switching models, etc.



## Key Points from Today's Seminar

### What We Covered

1. **Integration and differencing:**  $I(d)$  processes require  $d$  differences
2. **Unit root testing:** ADF tests  $H_0$ : unit root; KPSS tests  $H_0$ : stationary
3. **ARIMA(p,d,q):** Combines ARMA with differencing
4. **Model identification:** Use ACF/PACF patterns and information criteria
5. **Forecasting:** Point forecasts and growing confidence intervals

### Next Seminar

Hands-on Python exercises with real economic data:

- Unit root testing with statsmodels
- Auto-ARIMA with pmdarima
- Forecasting and model diagnostics



## AI in ARIMA Modeling

### Context

AI tools can fit ARIMA models and generate forecasts automatically. The critical skill is **evaluating whether the methodology is correct**.

**Key questions to ask about any AI-generated ARIMA analysis:**

1. Did it test for unit roots **before** choosing the differencing order?
2. Is the differencing order  $d$  justified by ADF/KPSS tests?
3. Did it check for overdifferencing ( $ACF$  lag 1  $\approx -0.5$ )?
4. Are residuals white noise (Ljung-Box test)?
5. Do forecast confidence intervals widen appropriately with horizon?



## AI Exercise 1: Critique an AI ARIMA Analysis

### Scenario

You asked an AI: "Fit the best ARIMA model to this GDP data." It returned:

- Fitted ARIMA(3,2,2) with AIC = 315.7
- No unit root test performed before differencing
- Applied  $d = 2$  "to be safe"
- Ljung-Box p-value = 0.04 (reported as "close enough")
- 10-year quarterly forecast with narrow confidence intervals

### Your critique:

1. Why is choosing  $d = 2$  without testing dangerous? What is overdifferencing?
2. Why is Ljung-Box p = 0.04 **not** acceptable at 5% level?
3. Is ARIMA(3,2,2) likely over-parameterized? What would BIC suggest?
4. What is the correct Box-Jenkins methodology that was skipped?



## AI Exercise 2: Prompt Refinement for ARIMA

### Task

Iteratively improve prompts for fitting an ARIMA model to US GDP data.

**Round 1** (vague): “*Fit a time series model to quarterly GDP*”

- What did the AI produce? Did it test stationarity? What's missing?

**Round 2** (better): “*Test stationarity with ADF, determine differencing order, fit ARIMA using BIC, check residuals with Ljung-Box*”

- Did the AI follow the Box-Jenkins methodology correctly?

**Round 3** (expert): “*Follow Box-Jenkins: (1) plot series & test ADF/KPSS, (2) difference if needed & re-test, (3) identify p,q from ACF/PACF, (4) estimate ARIMA(p,1,q), (5) Ljung-Box on residuals, (6) rolling 1-step forecast on last 20 obs with RMSE*”

- Compare results across all three rounds



## AI Exercise 3: Model Selection Competition

### Task

Download US Real GDP data (quarterly) using pandas\_datareader or FRED.

#### Your approach (manual):

- ADF/KPSS tests → determine  $d$
- ACF/PACF of differenced series → candidate models
- Compare AIC/BIC across ARIMA(1,1,0), ARIMA(0,1,1), ARIMA(1,1,1)
- Residual diagnostics for selected model
- Rolling 1-step forecast on last 20 observations

#### AI approach:

- Ask AI to “find the best ARIMA model and forecast GDP”

#### Compare:

- Which differencing order and model did each select?
- Compare out-of-sample RMSE; did the AI check for overdifferencing?
- Submit:** 1-page reflection on AI strengths and weaknesses



## Key Formulas Summary

Concept	Formula
Random walk	$Y_t = Y_{t-1} + \varepsilon_t$
Random walk variance	$\text{Var}(Y_t) = t\sigma^2$
ARIMA( $p, d, q$ )	$\phi(L)(1 - L)^d Y_t = \theta(L)\varepsilon_t$
First difference	$\Delta Y_t = Y_t - Y_{t-1} = (1 - L)Y_t$
Second difference	$\Delta^2 Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$
ADF regression	$\Delta Y_t = \alpha + \gamma Y_{t-1} + \sum \delta_j \Delta Y_{t-j} + \varepsilon_t$
ADF null	$H_0 : \gamma = 0$ (unit root)
RW forecast	$\hat{Y}_{T+h T} = Y_T$
RW forecast CI	$Y_T \pm z_{\alpha/2} \sqrt{h} \sigma$
AIC	$-2 \ln(\hat{L}) + 2k$
BIC	$-2 \ln(\hat{L}) + k \ln(n)$

Notation:  $\hat{L}$  = maximum of the likelihood function,  $k$  = no. of parameters,  $n$  = sample size,  $\sigma^2$  = white noise variance



## What's Next?

### Seminar 4: SARIMA Models for Seasonal Data

- Seasonality:** repetitive patterns at regular intervals
- Seasonal differencing:** the  $(1 - L^s)$  operator
- SARIMA** $(p, d, q)(P, D, Q)_s$ : the seasonal extension of ARIMA
- Case study:** Airline passenger forecasting with Python

Questions?



## Bibliography I

### Fundamental textbooks

- Hyndman, R.J., & Athanasopoulos, G. (2021). *Forecasting: Principles and Practice*, 3rd ed., OTexts.
- Shumway, R.H., & Stoffer, D.S. (2017). *Time Series Analysis and Its Applications*, 4th ed., Springer.
- Brockwell, P.J., & Davis, R.A. (2016). *Introduction to Time Series and Forecasting*, 3rd ed., Springer.

### Financial time series

- Tsay, R.S. (2010). *Analysis of Financial Time Series*, 3rd ed., Wiley.
- Franke, J., Härdle, W.K., & Hafner, C.M. (2019). *Statistics of Financial Markets*, 4th ed., Springer.



## Bibliography II

### Modern approaches and Machine Learning

- Nielsen, A. (2019). *Practical Time Series Analysis*, O'Reilly Media.
- Petropoulos, F., et al. (2022). *Forecasting: Theory and Practice*, International Journal of Forecasting.
- Makridakis, S., Spiliotis, E., & Assimakopoulos, V. (2020). The M4 Competition, International Journal of Forecasting.

### Online resources and code

- **Quantlet:** <https://quantlet.com> — Code repository for statistics
- **Quantinar:** <https://quantinar.com> — Learning platform for quantitative methods
- **GitHub TSA:** [https://github.com/QuantLet/TSA/tree/main/TSA\\_ch3](https://github.com/QuantLet/TSA/tree/main/TSA_ch3) — Python code for this seminar

