



Chapter 1: Introduction to Time Series

Seminar



Today's Activities:

1. **Quick Review** – Key concepts recap
2. **Multiple Choice Quizzes** – Test your understanding
3. **True/False Questions** – Conceptual checks
4. **Calculation Exercises** – Hands-on practice
5. **Python Exercises** – Coding practice
6. **Discussion Questions** – Critical thinking

Key Formulas to Remember

Decomposition:

- Additive: $X_t = T_t + S_t + \varepsilon_t$
- Multiplicative: $X_t = T_t \times S_t \times \varepsilon_t$

Exponential Smoothing:

- SES: $\hat{X}_{t+1} = \alpha X_t + (1 - \alpha)\hat{X}_t$
- Holt: adds trend b_t
- HW: adds seasonal S_t

Stationarity:

- $\mathbb{E}[X_t] = \mu$ (constant)
- $\text{Var}(X_t) = \sigma^2$ (constant)
- $\text{Cov}(X_t, X_{t+h}) = \gamma(h)$

Random Walk:

- $X_t = X_{t-1} + \varepsilon_t$
- $\text{Var}(X_t) = t\sigma^2$ (grows!)

Key Concepts Summary

| Concept | Key Point | When to Use |
|------------------------|-----------------------------------|---------------------------|
| Additive decomp. | Constant seasonal amplitude | Stable variance |
| Multiplicative decomp. | Seasonal grows with level | Increasing variance |
| SES | Level only (α) | No trend, no seasonality |
| Holt | Level + Trend (α, β) | Trend, no seasonality |
| Holt-Winters | Level + Trend + Seasonal | Trend and seasonality |
| ADF Test | H_0 : unit root | Test for non-stationarity |
| KPSS Test | H_0 : stationary | Confirm stationarity |
| Differencing | Remove stochastic trend | Random walk, unit root |
| Regression | Remove deterministic trend | Linear/polynomial trend |

[QUIZ] Quiz 1: Time Series Basics

Question: Which of the following is NOT a characteristic of time series data?

- ☐ A. Observations are ordered in time
- ☐ B. Consecutive observations are typically correlated
- ☐ C. Observations are independent and identically distributed
- ☐ D. The data has a natural temporal ordering

Think about it before moving to the next slide...

[QUIZ] Quiz 1: Answer

Question: Which is NOT a characteristic of time series data?

- ☐ A. Observations are ordered in time ✗
- ☐ B. Consecutive observations are typically correlated ✗
- ☒ C. **Observations are independent and identically distributed** ✓
- ☐ D. The data has a natural temporal ordering ✗

Explanation

Time series observations are typically **dependent** (autocorrelated), not independent. The assumption of i.i.d. observations is fundamental to cross-sectional analysis but violated in time series. This temporal dependence is what makes time series analysis unique and requires specialized methods.

[QUIZ] Quiz 2: Decomposition

Question: When should you use multiplicative decomposition instead of additive?

- ☐ A. When the seasonal pattern has constant amplitude
- ☐ B. When the variance of the series is stable over time
- ☐ C. When the seasonal fluctuations grow proportionally with the level
- ☐ D. When the time series has no trend component

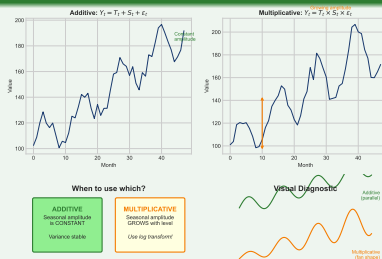
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[QUIZ] Quiz 2: Answer

Question: When should you use multiplicative decomposition?

- A. When the seasonal pattern has constant amplitude ✗
- B. When the variance of the series is stable over time ✗
- C. **When the seasonal fluctuations grow proportionally with the level ✓**
- D. When the time series has no trend component ✗

Explanation



[QUIZ] Quiz 3: Exponential Smoothing

Question: In Simple Exponential Smoothing with $\alpha = 0.9$, what happens?

- ☐ A. Forecasts are very smooth and stable
- ☐ B. Recent observations have very little weight
- ☐ C. Forecasts react quickly to recent changes
- ☐ D. The forecast is essentially a long-term average

Think about it before moving to the next slide...

[QUIZ] Quiz 3: Answer

Question: In SES with $\alpha = 0.9$, what happens?

- A. Forecasts are very smooth and stable ✗
- B. Recent observations have very little weight ✗
- C. **Forecasts react quickly to recent changes ✓**
- D. The forecast is essentially a long-term average ✗

Explanation

With $\alpha = 0.9$: $\hat{X}_{t+1} = 0.9X_t + 0.1\hat{X}_t$

This means 90% weight on the most recent observation! High α values make forecasts very responsive to new data. Low α (e.g., 0.1) produces smoother, more stable forecasts that average over more history.

[QUIZ] Quiz 4: Stationarity

Question: A random walk process $X_t = X_{t-1} + \varepsilon_t$ is:

- ☐ A. Strictly stationary
- ☐ B. Weakly stationary
- ☐ C. Non-stationary because variance grows with time
- ☐ D. Stationary after adding a constant

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[QUIZ] Quiz 4: Answer

Question: A random walk is:

- A. Strictly stationary ✗
- B. Weakly stationary ✗
- C. **Non-stationary because variance grows with time ✓**
- D. Stationary after adding a constant ✗

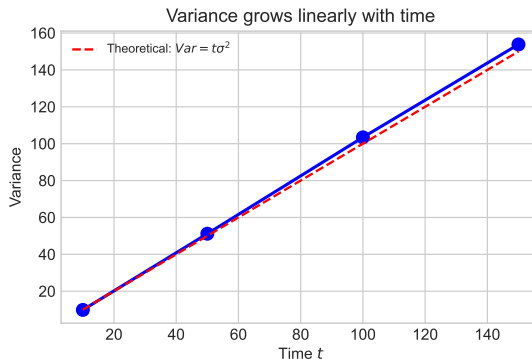
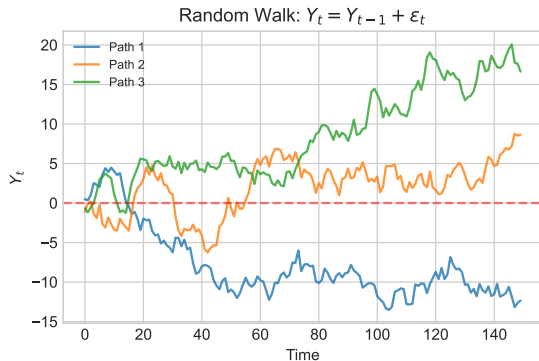
Explanation

For random walk: $X_t = \sum_{i=1}^t \varepsilon_i$

- $\mathbb{E}[X_t] = 0$ (constant mean – OK)
- $\text{Var}(X_t) = t\sigma^2$ (variance depends on t – NOT OK!)

Since variance is not constant, the process violates the stationarity condition. Solution: **differencing** gives $\Delta X_t = \varepsilon_t$ which IS stationary.

Visual: Random Walk vs Stationary



Random walk paths wander unpredictably; variance grows linearly with time \Rightarrow non-stationary.

[QUIZ] Quiz 5: Unit Root Tests

Question: You run ADF and KPSS tests. ADF fails to reject H_0 , and KPSS rejects H_0 . What do you conclude?

- ☐ A. The series is stationary
- ☐ B. The series has a unit root (non-stationary)
- ☐ C. The results are inconclusive
- ☐ D. You need to run more tests

Think about it before moving to the next slide...

[QUIZ] Quiz 5: Answer

Question: ADF fails to reject, KPSS rejects. Conclusion?

- A. The series is stationary ✗
- B. **The series has a unit root (non-stationary) ✓**
- C. The results are inconclusive ✗
- D. You need to run more tests ✗

Explanation

- ADF: $H_0 = \text{unit root}$. Fail to reject \Rightarrow evidence FOR unit root
- KPSS: $H_0 = \text{stationary}$. Reject \Rightarrow evidence AGAINST stationarity

Both tests agree: the series is **non-stationary**. You should difference the series before modeling with ARMA.

[QUIZ] Quiz 6: Forecast Evaluation

Question: Which metric is most appropriate for comparing forecast accuracy across different time series with different scales?

- ☐ A. Mean Absolute Error (MAE)
- ☐ B. Root Mean Squared Error (RMSE)
- ☐ C. Mean Absolute Percentage Error (MAPE)
- ☐ D. Mean Squared Error (MSE)

Think about it before moving to the next slide...

[QUIZ] Quiz 6: Answer

Question: Best metric for comparing across different scales?

- A. Mean Absolute Error (MAE) ✗
- B. Root Mean Squared Error (RMSE) ✗
- C. **Mean Absolute Percentage Error (MAPE) ✓**
- D. Mean Squared Error (MSE) ✗

Explanation

$MAPE = \frac{100}{n} \sum \left| \frac{e_t}{X_t} \right|$ expresses errors as **percentages**.

- MAE, RMSE, MSE are **scale-dependent** (units of X_t)
- MAPE is **scale-independent** (always in %)
- Caveat: MAPE fails when X_t is near zero

[QUIZ] Quiz 7: Trend Types

Question: A deterministic trend can be removed by:

- ☐ A. Differencing
- ☐ B. Regression on time
- ☐ C. Seasonal adjustment
- ☐ D. Moving average smoothing

[QUIZ] Quiz 7: Trend Types

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- ☐ B. Regression on time
- ☐ C. Seasonal adjustment
- ☐ D. Moving average smoothing

Answer: B – Regression on time

Deterministic trend: $Y_t = \alpha + \beta t + \varepsilon_t$ where β is fixed.

Removal method: Regress Y_t on t , then analyze residuals $\hat{\varepsilon}_t = Y_t - \hat{\alpha} - \hat{\beta}t$

Why not differencing? Differencing a deterministic trend gives: $\Delta Y_t = \beta + \Delta \varepsilon_t$, which removes the trend but leaves a constant. For *stochastic* trends (unit roots), differencing is correct.

[QUIZ] Quiz 8: ACF Interpretation

Question: If the ACF of a time series decays very slowly (remains significant for many lags), this suggests:

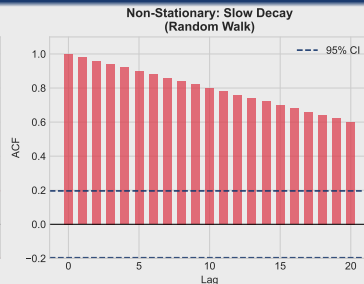
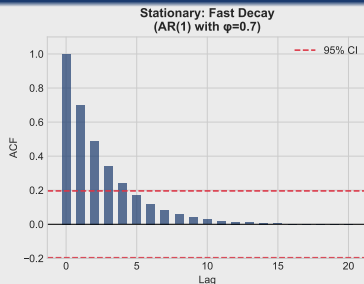
- ☐ A. The series is white noise
- ☐ B. The series is likely non-stationary
- ☐ C. The series has no autocorrelation
- ☐ D. The series is perfectly predictable

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Answer: B – The series is likely non-stationary



[QUIZ] Quiz 9: Holt's Method

Question: Holt's exponential smoothing differs from SES by adding:

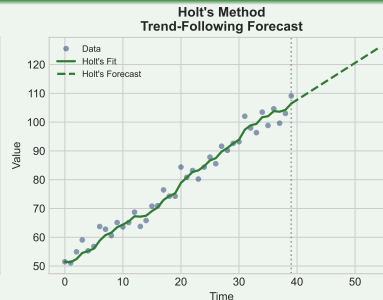
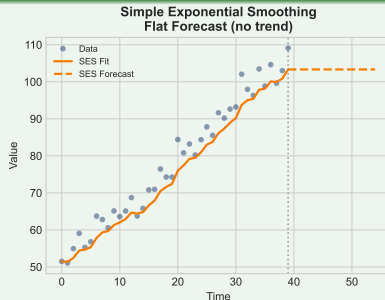
- ☐ A. A seasonal component
- ☐ B. A trend component
- ☐ C. A cyclical component
- ☐ D. An irregular component

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Answer: B – A trend component



[QUIZ] Quiz 10: White Noise

Question: Which property is NOT required for a process to be white noise?

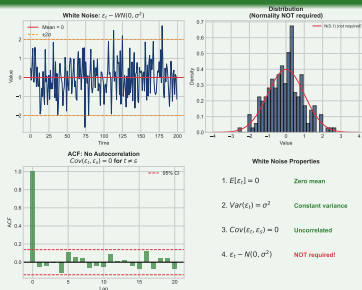
- A. $\mathbb{E}[\varepsilon_t] = 0$
- B. $\text{Var}(\varepsilon_t) = \sigma^2$ (constant)
- C. $\text{Cov}(\varepsilon_t, \varepsilon_s) = 0$ for $t \neq s$
- D. $\varepsilon_t \sim N(0, \sigma^2)$

[QUIZ] Quiz 10: White Noise

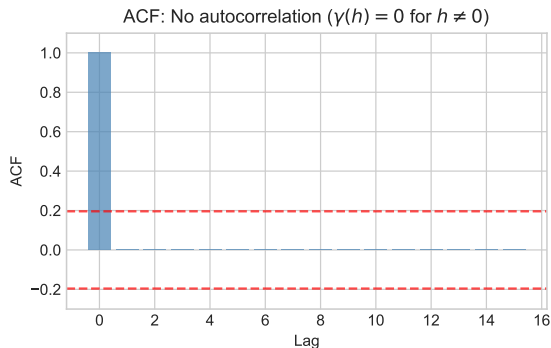
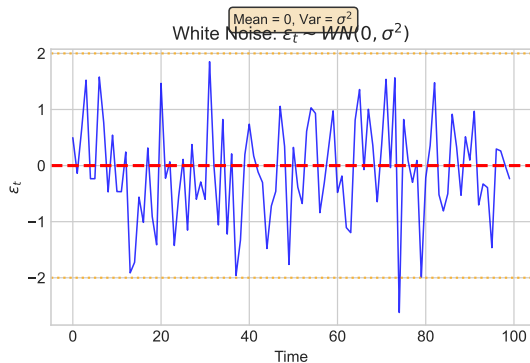
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- C. $\text{Cov}(\varepsilon_t, \varepsilon_s) = 0$ for $t \neq s$
- D. $\varepsilon_t \sim N(0, \sigma^2)$

Answer: D – Normality is NOT required



Visual: White Noise Properties



Left: white noise fluctuates around zero. Right: ACF shows no autocorrelation (all values near zero after lag 0).

[QUIZ] Quiz 11: Forecast Horizon

Question: As forecast horizon h increases, what typically happens to forecast intervals?

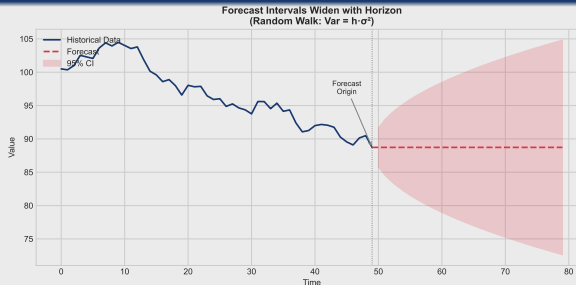
- ☐ A. They become narrower
- ☐ B. They stay the same width
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Answer: C – They become wider



[QUIZ] Quiz 12: Seasonality Detection

Question: The ACF shows significant spikes at lags 12, 24, and 36 for monthly data. This suggests:

- ☐ A. No seasonality
- ☐ B. Annual seasonality
- ☐ C. Weekly seasonality
- ☐ D. Random noise

[QUIZ] Quiz 12: Seasonality Detection

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- ☐ A. No seasonality
- ☒ B. Annual seasonality
- ☐ C. Weekly seasonality
- ☐ D. Random noise

Answer: B – Annual seasonality

Pattern recognition:

- Lag 12: correlation with same month last year
- Lag 24: correlation with same month two years ago
- Lag 36: correlation with same month three years ago

Seasonal period: $s = 12$ (monthly data with annual cycle)

Common patterns: Retail sales (December peaks), energy consumption (summer/winter), tourism data

[QUIZ] Quiz 13: Cross-Validation in Time Series

Question: Why can't we use standard k-fold cross-validation for time series?

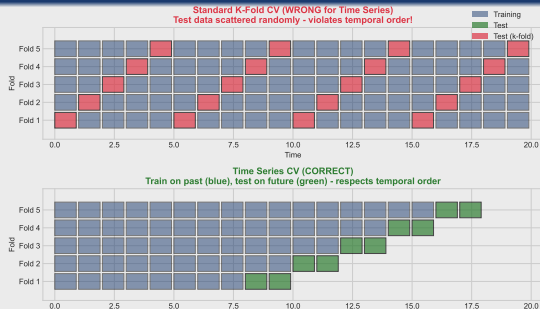
- ☐ A. Time series data is too small
- ☐ B. It would violate temporal ordering (future predicting past)
- ☐ C. Cross-validation is always invalid
- ☐ D. Time series don't need validation

[QUIZ] Quiz 13: Cross-Validation in Time Series

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- ☐ A. Time series data is too small
- ☒ B. It would violate temporal ordering (future predicting past)
- ☐ C. Cross-validation is always invalid
- ☐ D. Time series don't need validation

Answer: B – It would violate temporal ordering



[QUIZ] Quiz 14: MAPE Limitation

Question: MAPE (Mean Absolute Percentage Error) should NOT be used when:

- ☐ A. Comparing models on the same dataset
- ☐ B. The actual values can be zero or near zero
- ☐ C. Forecasting stock prices
- ☐ D. The data has a trend

[QUIZ] Quiz 14: MAPE Limitation

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- ☐ A. Comparing models on the same dataset
- ☐ B. The actual values can be zero or near zero
- ☐ C. Forecasting stock prices
- ☐ D. The data has a trend

Answer: B – When actual values can be zero or near zero

MAPE formula:
$$\text{MAPE} = \frac{100\%}{n} \sum_{t=1}^n \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right|$$

Problem: When $Y_t \approx 0$, division causes $\text{MAPE} \rightarrow \infty$

Alternatives:

- **SMAPE:**
$$\frac{200\%}{n} \sum \frac{|Y_t - \hat{Y}_t|}{|Y_t| + |\hat{Y}_t|} \text{ (bounded 0–200\%)}$$
- **MASE:**
$$\frac{1}{n} \sum \frac{|e_t|}{\frac{1}{n-1} \sum |Y_t - Y_{t-1}|} \text{ (scale-free)}$$

[QUIZ] True or False? (Set 2)

Mark each statement as True (T) or False (F):

- ① A time series with constant mean is always stationary. _____
- ② The variance of a random walk increases linearly with time. _____
- ③ SES forecasts are always flat (constant for all horizons). _____
- ④ ADF and KPSS tests have the same null hypothesis. _____
- ⑤ Lower RMSE always means better forecasts. _____
- ⑥ Autocorrelation at lag 0 is always equal to 1. _____

[QUIZ] True or False: Answers (Set 2)

- ❶ A time series with constant mean is always stationary. **FALSE**
Also need constant variance and covariance depending only on lag.
- ❷ The variance of a random walk increases linearly with time. **TRUE**
 $\text{Var}(X_t) = t\sigma^2$ for random walk starting at X_0 .
- ❸ SES forecasts are always flat (constant for all horizons). **TRUE**
SES has no trend component, so $\hat{X}_{t+h} = L_t$ for all h .
- ❹ ADF and KPSS tests have the same null hypothesis. **FALSE**
ADF: $H_0 = \text{unit root}$. KPSS: $H_0 = \text{stationary}$. Opposite nulls!
- ❺ Lower RMSE always means better forecasts. **FALSE**
Depends on context. RMSE is scale-dependent; may overfit to outliers.
- ❻ Autocorrelation at lag 0 is always equal to 1. **TRUE**
 $\rho(0) = \gamma(0)/\gamma(0) = 1$ by definition.

[QUIZ] True or False?

Mark each statement as True (T) or False (F):

- ① The ACF of a stationary AR(1) process decays exponentially. _____
- ② White noise is always normally distributed. _____
- ③ Differencing can make a non-stationary series stationary. _____
- ④ The PACF of a MA(1) process cuts off after lag 1. _____
- ⑤ You should always use the test set for hyperparameter tuning. _____
- ⑥ Holt-Winters is appropriate for data with no seasonality. _____

Answers on next slide...

[QUIZ] True or False: Answers

- ❶ The ACF of a stationary AR(1) decays exponentially. TRUE
For AR(1): $\rho(h) = \phi^h$, which decays exponentially.
- ❷ White noise is always normally distributed. FALSE
White noise only requires zero mean, constant variance, no autocorrelation. Gaussian white noise is a special case.
- ❸ Differencing can make a non-stationary series stationary. TRUE
Differencing removes stochastic trends (unit roots).
- ❹ The PACF of a MA(1) cuts off after lag 1. FALSE
It's the ACF that cuts off for MA. PACF decays for MA processes.
- ❺ You should always use the test set for hyperparameter tuning. FALSE
Use validation set for tuning. Test set is for final evaluation only!
- ❻ Holt-Winters is appropriate for data with no seasonality. FALSE
Use Holt's method (no seasonal component) or SES for non-seasonal data.

Exercise 1: Simple Exponential Smoothing

Problem: Given the following data and $\alpha = 0.3$:

| t | 1 | 2 | 3 | 4 | 5 |
|-------|----|----|----|----|----|
| X_t | 10 | 12 | 11 | 14 | 13 |

Starting with $\hat{X}_1 = X_1 = 10$, calculate:

- a) The forecasts $\hat{X}_2, \hat{X}_3, \hat{X}_4, \hat{X}_5$
- b) The forecast for $t = 6$: \hat{X}_6
- c) The forecast errors $e_t = X_t - \hat{X}_t$ for $t = 2, 3, 4, 5$
- d) The MAE and RMSE

Formula: $\hat{X}_{t+1} = \alpha X_t + (1 - \alpha)\hat{X}_t$

Exercise 1: Solution

Using $\hat{X}_{t+1} = 0.3X_t + 0.7\hat{X}_t$:

| t | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------|----|----|------|-------|-------|--------------|
| X_t | 10 | 12 | 11 | 14 | 13 | ? |
| \hat{X}_t | 10 | 10 | 10.6 | 10.72 | 11.70 | 12.09 |
| e_t | – | 2 | 0.4 | 3.28 | 1.30 | – |

Calculations:

- $\hat{X}_2 = 0.3(10) + 0.7(10) = 10$
- $\hat{X}_3 = 0.3(12) + 0.7(10) = 10.6$
- $\hat{X}_4 = 0.3(11) + 0.7(10.6) = 10.72$
- $\hat{X}_5 = 0.3(14) + 0.7(10.72) = 11.70$
- $\hat{X}_6 = 0.3(13) + 0.7(11.70) = \mathbf{12.09}$

$$\text{MAE} = \frac{|2| + |0.4| + |3.28| + |1.30|}{4} = 1.745 \quad \text{RMSE} = \sqrt{\frac{4 + 0.16 + 10.76 + 1.69}{4}} = 2.04$$

Exercise 2: Autocovariance

Problem: For a stationary process with:

- $\mathbb{E}[X_t] = 5$
- $\gamma(0) = 4$ (variance)
- $\gamma(1) = 2$
- $\gamma(2) = 1$

Calculate:

- a) The autocorrelation function $\rho(0), \rho(1), \rho(2)$
- b) $\text{Cov}(X_t, X_{t-1})$
- c) $\text{Corr}(X_5, X_7)$
- d) If $X_t = 6$, what is $\mathbb{E}[X_{t+1} | X_t = 6]$ assuming AR(1)?

Exercise 2: Solution

a) Autocorrelations:

$$\rho(h) = \frac{\gamma(h)}{\gamma(0)}$$

- $\rho(0) = \gamma(0)/\gamma(0) = 1$
- $\rho(1) = \gamma(1)/\gamma(0) = 2/4 = 0.5$
- $\rho(2) = \gamma(2)/\gamma(0) = 1/4 = 0.25$

b) $\text{Cov}(X_t, X_{t-1}) = \gamma(1) = 2$ (by stationarity, lag 1 covariance)

c) $\text{Corr}(X_5, X_7) = \rho(|7 - 5|) = \rho(2) = 0.25$

d) For AR(1) with $\phi = \rho(1) = 0.5$:

$$\mathbb{E}[X_{t+1}|X_t] = \mu + \phi(X_t - \mu) = 5 + 0.5(6 - 5) = 5.5$$

Exercise 3: Random Walk Properties

Problem: Consider a random walk $X_t = X_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim WN(0, 4)$ and $X_0 = 100$.

Calculate:

- a) $\mathbb{E}[X_{10}]$
- b) $\text{Var}(X_{10})$
- c) $\text{Cov}(X_5, X_{10})$
- d) The 95% confidence interval for X_{100}
- e) After observing $X_5 = 108$, what is your best forecast for X_6 ?

Exercise 3: Solution

Random walk: $X_t = X_0 + \sum_{i=1}^t \varepsilon_i$ with $\sigma^2 = 4$

a) $\mathbb{E}[X_{10}] = X_0 = 100$ (mean stays at starting value)

b) $\text{Var}(X_{10}) = 10 \times \sigma^2 = 10 \times 4 = 40$

c) $\text{Cov}(X_5, X_{10}) = \min(5, 10) \times \sigma^2 = 5 \times 4 = 20$

d) For X_{100} :

- $\mathbb{E}[X_{100}] = 100$, $\text{Var}(X_{100}) = 400$, $SD = 20$
- 95% CI: $100 \pm 1.96 \times 20 = [60.8, 139.2]$

e) Best forecast: $\hat{X}_6 = X_5 = 108$

(Random walk: best forecast is the last observed value)

Python Exercise 1: Load and Plot

Task: Load S&P 500 data and create a basic time series plot.

Starter Code

```
import yfinance as yf
import matplotlib.pyplot as plt

# Download S&P 500 data
sp500 = yf.download('^GSPC', start='2020-01-01', end='2025-01-01')

# TODO: Plot the closing prices
# TODO: Add title, labels, and grid
# TODO: Calculate and print basic statistics
```

Questions:

- 1 What is the mean and standard deviation of returns?
- 2 Does the series appear stationary? Why or why not?

Python Exercise 2: Decomposition

Task: Perform STL decomposition on airline passengers data.

Starter Code

```
from statsmodels.tsa.seasonal import STL
import pandas as pd

# Load airline passengers
url = 'https://raw.githubusercontent.com/.../airline.csv'
airline = pd.read_csv(url, parse_dates=['Month'],
                      index_col='Month')

# TODO: Apply STL decomposition with period=12
# TODO: Plot all components
# TODO: What percentage of variance is explained by trend?
```

Hint: Use `STL(data, period=12).fit()`

Python Exercise 3: Exponential Smoothing

Task: Compare SES, Holt, and Holt-Winters on real data.

Starter Code

```
from statsmodels.tsa.holtwinters import (SimpleExpSmoothing,
    ExponentialSmoothing)

# Split data: 80% train, 20% test
train = airline[:'1958']
test = airline['1959':]

# TODO: Fit SES, Holt, and Holt-Winters
# TODO: Generate forecasts for test period
# TODO: Calculate RMSE for each method
# TODO: Which method performs best? Why?
```

Python Exercise 4: Stationarity Testing

Task: Test for stationarity using ADF and KPSS tests.

Starter Code

```
from statsmodels.tsa.stattools import adfuller, kpss

# Test S&P 500 prices
prices = sp500['Close']
returns = prices.pct_change().dropna()

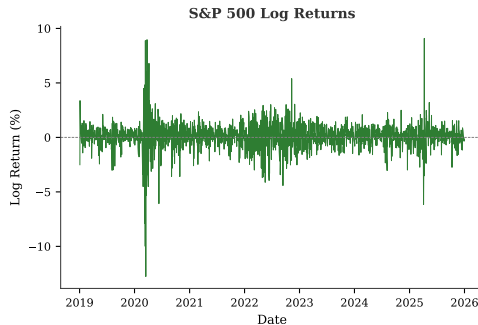
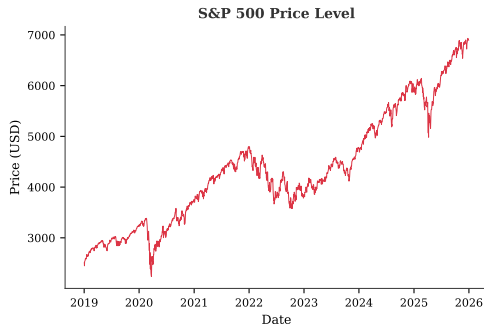
# TODO: Run ADF test on prices and returns
# TODO: Run KPSS test on prices and returns
# TODO: Interpret the results

# ADF: adfuller(series)
# KPSS: kpss(series, regression='c')
```

Questions:

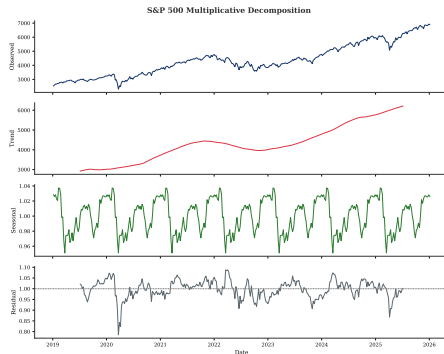
- 1 Are prices stationary? Are returns stationary?
- 2 Do ADF and KPSS agree?

Case Study: S&P 500 Index



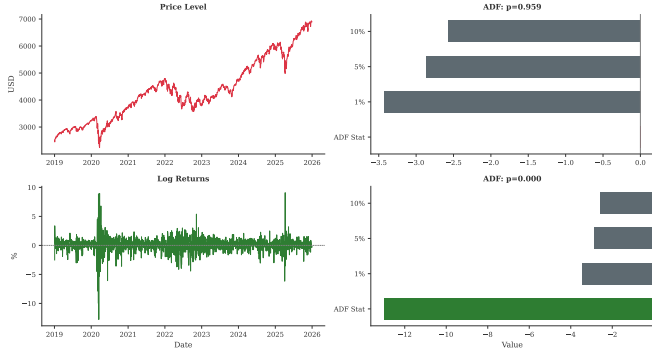
- **Top:** S&P 500 price level – clear upward trend (non-stationary)
- **Bottom:** Daily returns $r_t = \log(P_t/P_{t-1})$ – stationary
- Returns fluctuate around zero mean with no trend
- Volatility clustering visible – periods of high/low volatility

Time Series Decomposition: Real Example



- **Trend:** Long-term direction of the series
- **Seasonal:** Regular periodic patterns
- **Residual:** What remains after removing trend and seasonality
- Decomposition helps understand data structure before modeling

Stationarity Testing: ADF Results



- ADF test compares test statistic to critical values
- If test statistic $<$ critical value \Rightarrow reject unit root (series is stationary)
- Prices: ADF statistic $> -2.86 \Rightarrow$ non-stationary
- Returns: ADF statistic $< -2.86 \Rightarrow$ stationary

Stationarity Comparison: Prices vs Returns

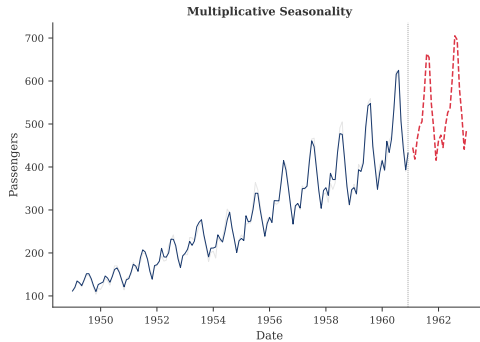
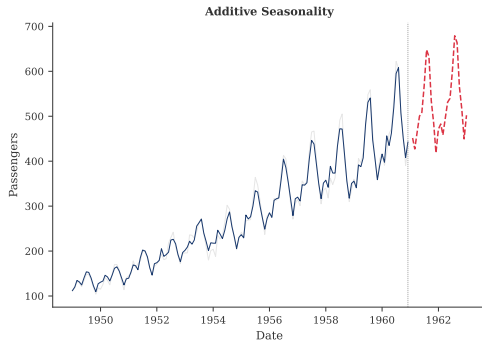
ADF Test Results

| Series | ADF Statistic | p-value | Conclusion |
|-----------------|---------------|---------|----------------|
| S&P 500 Prices | −0.82 | 0.812 | Non-stationary |
| S&P 500 Returns | −45.3 | < 0.001 | Stationary |

Key Insight

Financial prices are typically $I(1)$ – integrated of order 1.
Taking first differences (returns) achieves stationarity.
This is why we model **returns**, not prices!

Exponential Smoothing Forecast



- Holt-Winters method for data with trend and seasonality
- Smoothing parameters α , β , γ control adaptiveness
- Forecasts capture both trend continuation and seasonal pattern
- Simple yet effective for many business applications

Discussion Question 1

Scenario

You are analyzing monthly sales data for a retail company. The data shows clear seasonality (high sales in December) and an upward trend. The seasonal peaks have been getting larger over time.

Discuss:

- 1 Should you use additive or multiplicative decomposition? Why?
- 2 Which exponential smoothing method would you recommend?
- 3 How would you evaluate your forecast model?
- 4 What could go wrong if you used the wrong decomposition?

Discussion Question 2

Scenario

A colleague claims: "I ran the ADF test on my stock price data and got a p-value of 0.65, so my data is stationary and I can fit an ARMA model directly."

Discuss:

- 1 What is wrong with this interpretation?
- 2 What do the ADF hypotheses actually test?
- 3 What should the colleague do before fitting an ARMA model?
- 4 How could the KPSS test help clarify the situation?

Discussion Question 3

Scenario

You're building a forecasting model and achieve excellent results: MAPE of 2% on your dataset. Your manager is impressed and wants to deploy the model immediately.

Discuss:

- 1 What questions should you ask before deployment?
- 2 Did you use proper train/validation/test splits?
- 3 Could there be data leakage in your evaluation?
- 4 What additional diagnostics would you run?
- 5 How would you monitor the model in production?

Discussion Question 4

Scenario

You need to forecast daily electricity demand for the next week. The data shows: (1) strong daily patterns (peaks at 6pm), (2) weekly patterns (lower on weekends), and (3) annual patterns (higher in summer/winter).

Discuss:

- ① How would you handle multiple seasonal patterns?
- ② Would Holt-Winters work here? Why or why not?
- ③ What's the advantage of Fourier terms in this case?
- ④ How would you set up your train/validation/test split?

Key Takeaways from Today

- ① **Time series are dependent** – not i.i.d. like cross-sectional data
- ② **Choose decomposition wisely** – multiplicative when seasonal amplitude grows
- ③ **Understand smoothing parameters** – high α = reactive, low α = smooth
- ④ **Test for stationarity** – use both ADF and KPSS together
- ⑤ **Proper evaluation** – never tune on test set!
- ⑥ **Random walk is non-stationary** – variance grows with time

Next Seminar

ARMA/ARIMA model identification, estimation, and forecasting

Questions?

Good luck with the exercises!

Practice makes perfect.