Time Series Analysis and Forecasting

Seminar 5: GARCH and Volatility Models

Daniel Traian PELE

Bucharest University of Economic Studies
IDA Institute Digital Assets
Blockchain Research Center
AI4EFin Artificial Intelligence for Energy Finance
Romanian Academy, Institute for Economic Forecasting
MSCA Digital Finance

Seminar Outline

- **Multiple Choice Quiz** – Knowledge check
- **True/False** – Conceptual checks
- **Practice Problems** – Applied practice
- **Python Workflow** – Hands-on coding
- **AI-Assisted Exercise** – Critical thinking
- **Summary** – Key takeaways

Quiz 1: Volatility Clustering

Question

What is "volatility clustering"?

Answer choices

(A) Volatility is constant over time

(B) Periods of high volatility tend to be followed by periods of high volatility

(C) Returns are correlated over time

(D) The return distribution is normal

Answer on next slide...

Quiz 1: Answer

Answer: B – Periods of high volatility tend to be followed by periods of high volatility

Answer choices

(A) Volatility is constant over time ×

(B) **Periods of high volatility tend to be followed by periods of high volatility** ✓

(C) Returns are correlated over time ×

(D) The return distribution is normal ×

- **Volatility clustering** is a stylized fact observed in financial time series
- "Turbulent" periods (with large movements) tend to persist
- This implies that conditional variance σ_t^2 is **predictable**
- GARCH models capture exactly this phenomenon!

TSA_ch5_clustering

Quiz 2: GARCH Parameters

Question

In the GARCH(1,1) model: $\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$, what does the parameter α represent?

Answer choices

(A) Volatility persistence

(B) Baseline volatility level

(C) Reaction to recent shocks (news coefficient)

(D) Unconditional variance

Answer on next slide...

Quiz 2: Answer

Answer: C – Reaction to recent shocks (news coefficient)

Answer choices

(A) Volatility persistence ×
(B) Baseline volatility level ×
(C) **Reaction to recent shocks (news coefficient)** ✓
(D) Unconditional variance ×

- ω = baseline (floor) volatility level
- α = **reaction** to squared innovations ("news")
- β = volatility **persistence** (memory)
- $\alpha + \beta$ = total persistence
- A large α means volatility reacts strongly to recent shocks

TSA_ch5_garch

Quiz 3: Stationarity Condition

Question

What is the stationarity condition for GARCH(1,1)?

Answer choices

(A) $\omega > 0$

(B) $\alpha + \beta = 1$

(C) $\alpha + \beta < 1$

(D) $\alpha > \beta$

Answer on next slide...

Quiz 3: Answer

Answer: C – $\alpha + \beta < 1$

Answer choices

(A) $\omega > 0$ ×

(B) $\alpha + \beta = 1$ ×

(C) $\alpha + \beta < 1$ ✓

(D) $\alpha > \beta$ ×

- $\omega > 0$ (ensures positive variance)
- $\alpha \geq 0$, $\beta \geq 0$ (non-negativity)
- $\alpha + \beta < 1$ (**covariance stationarity**)
- If $\alpha + \beta = 1 \Rightarrow$ IGARCH (shocks have permanent effect)

TSA_ch5_garch_sim

Quiz 4: Unconditional Variance

Question

What is the formula for unconditional variance in GARCH(1,1)?

Answer choices

(A) $\bar{\sigma}^2 = \omega$

(B) $\bar{\sigma}^2 = \dfrac{\omega}{1-\alpha}$

(C) $\bar{\sigma}^2 = \dfrac{\omega}{1-\alpha-\beta}$

(D) $\bar{\sigma}^2 = \dfrac{\omega}{\alpha+\beta}$

Answer on next slide...

Quiz 4: Answer

Answer: C – $\bar{\sigma}^2 = \dfrac{\omega}{1-\alpha-\beta}$

Answer choices

(A) $\bar{\sigma}^2 = \omega$ ×

(B) $\bar{\sigma}^2 = \dfrac{\omega}{1-\alpha}$ ×

(C) $\bar{\sigma}^2 = \dfrac{\omega}{1-\alpha-\beta}$ ✓

(D) $\bar{\sigma}^2 = \dfrac{\omega}{\alpha+\beta}$ ×

- Taking unconditional expectation of GARCH(1,1):
- $\mathbb{E}[\sigma_t^2] = \omega + \alpha\mathbb{E}[\varepsilon_{t-1}^2] + \beta\mathbb{E}[\sigma_{t-1}^2]$
- $\bar{\sigma}^2(1-\alpha-\beta) = \omega$
- $\bar{\sigma}^2 = \frac{\omega}{1-\alpha-\beta}$

TSA_ch5_garch_sim

Quiz 5: Leverage Effect

Question

What is the "leverage effect"?

Answer choices

(A) Positive shocks increase volatility more than negative shocks

(B) Negative shocks increase volatility more than positive shocks

(C) Volatility is independent of shock sign

(D) Returns are asymmetric

Answer on next slide...

Quiz 5: Answer

Answer: B – Negative shocks increase volatility more than positive shocks

Answer choices

(A) Positive shocks increase volatility more than negative shocks ×

(B) **Negative shocks increase volatility more than positive shocks** ✓

(C) Volatility is independent of shock sign ×

(D) Returns are asymmetric ×

- Empirically observed in stock markets
- When prices fall, firm leverage increases (debt becomes larger relative to equity)
- This makes the firm riskier $\Rightarrow$ higher volatility
- Standard GARCH **cannot** capture this effect (depends on ε^2)
- Solutions: **EGARCH**, **GJR-GARCH**, **TGARCH**

TSA_ch5_egarch

Quiz 6: EGARCH Leverage

Question

In the EGARCH model, a negative γ parameter indicates:

Answer choices

(A) Absence of leverage effect

(B) Presence of leverage effect

(C) Constant volatility

(D) Non-stationary model

Answer on next slide...

Quiz 6: Answer

Answer: B – Presence of leverage effect

Answer choices

(A) Absence of leverage effect ×
(B) **Presence of leverage effect** ✓
(C) Constant volatility ×
(D) Non-stationary model ×

- EGARCH(1,1): $\ln(\sigma_t^2) = \omega + \alpha(|z_{t-1}| - \mathbb{E}[|z|]) + \gamma z_{t-1} + \beta \ln(\sigma_{t-1}^2)$
- $\gamma < 0$: negative shock $(z < 0) \Rightarrow$ increases $\ln(\sigma_t^2)$
- $\gamma > 0$: inverse effect (less common)
- $\gamma = 0$: symmetric effect (like GARCH)

TSA_ch5_nic

Quiz 7: EGARCH Advantages

Question

What is the main advantage of EGARCH over GARCH?

Answer choices

(A) Faster to estimate

(B) No non-negativity constraints needed

(C) Fewer parameters

(D) Easier to interpret

Answer on next slide...

Quiz 7: Answer

Answer: B – No non-negativity constraints needed

Answer choices

(A) Faster to estimate ×

(B) **No non-negativity constraints needed** ✓

(C) Fewer parameters ×

(D) Easier to interpret ×

- Models $\ln(\sigma_t^2)$, not σ_t^2
- $\sigma_t^2 = e^{\ln(\sigma_t^2)} > 0$ **automatically**, regardless of parameter values
- GARCH requires $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$
- During estimation, these constraints can cause convergence problems

TSA_ch5_egarch_sim

Quiz 8: ARCH Effects Test

Question

Which test do we use to detect ARCH effects in residuals?

Answer choices

(A) Dickey-Fuller test

(B) Ljung-Box test on residuals

(C) Engle's ARCH-LM test

(D) Breusch-Pagan test

Answer on next slide...

Quiz 8: Answer

Answer: C – Engle's ARCH-LM test

Answer choices

(A) Dickey-Fuller test ×

(B) Ljung-Box test on residuals ×

(C) **Engle's ARCH-LM test** ✓

(D) Breusch-Pagan test ×

- Estimate mean model, obtain residuals $\hat{\varepsilon}_t$
- Regress: $\hat{\varepsilon}_t^2 = \beta_0 + \beta_1 \hat{\varepsilon}_{t-1}^2 + \cdots + \beta_q \hat{\varepsilon}_{t-q}^2 + u_t$
- Test statistic: $LM = T \cdot R^2 \sim \chi^2(q)$ under H_0
- H_0: No ARCH effects H_1: ARCH effects present

TSA_ch5_arch_sim

Quiz 9: Volatility Persistence

Question

For S&P 500, typical values of $\alpha + \beta$ in GARCH(1,1) are:

Answer choices

(A) 0.50 – 0.70

(B) 0.70 – 0.85

(C) 0.95 – 0.99

(D) Greater than 1

Answer on next slide...

Quiz 9: Answer

Answer: C – 0.95 – 0.99

Answer choices

(A) 0.50 – 0.70 ×
(B) 0.70 – 0.85 ×
(C) **0.95 – 0.99** ✓
(D) Greater than 1 ×

- Financial time series exhibit very persistent volatility
- $\alpha + \beta \approx 0.98$ for S&P 500; Half-life $\approx$ 35–60 days

Series	$\alpha + \beta$
S&P 500	0.97–0.99
Bitcoin	0.90–0.98
EUR/USD	0.96–0.99

TSA_ch5_sp500_garch

Quiz 10: Innovation Distributions

Question

Which distribution is most commonly used for GARCH innovations to capture fat tails?

Answer choices

(A) Normal

(B) Uniform

(C) Student-t

(D) Exponential

Answer on next slide...

Quiz 10: Answer

Answer: C – Student-t

Answer choices

(A) Normal ×
(B) Uniform ×
(C) **Student-t** ✓
(D) Exponential ×

- **Normal**: standard, but underestimates extreme risk
- **Student-t**: fat tails, parameter ν (degrees of freedom)
- **GED**: Generalized Error Distribution, flexible
- **Skewed Student-t**: asymmetry + fat tails
- For S&P 500: $\nu \approx 5$–8 (significantly fatter tails than normal)

TSA_ch5_stylized

True or False? — Questions

Statement	T/F?
1. ARIMA models can capture volatility clustering.	?
2. In GARCH(1,1), if $\alpha + \beta = 1$, the model is called IGARCH.	?
3. GJR-GARCH uses an indicator variable for negative shocks.	?
4. GARCH volatility forecasts converge to zero in the long run.	?
5. EGARCH can have negative parameters without generating negative variance.	?
6. Value at Risk (VaR) can be calculated using GARCH volatility forecasts.	?

True or False? — Answers

Statement	T/F	Explanation
1. ARIMA models can capture volatility clustering.	F	**Assumes constant variance**
2. In GARCH(1,1), if $\alpha + \beta = 1$, the model is called IGARCH.	T	**Volatility has unit root**
3. GJR-GARCH uses an indicator variable for negative shocks.	T	$I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$
4. GARCH volatility forecasts converge to zero in the long run.	F	**Converges to** $\bar{\sigma}^2$
5. EGARCH can have negative parameters without generating negative variance.	T	**Models** $\ln(\sigma_t^2)$
6. Value at Risk (VaR) can be calculated using GARCH volatility forecasts.	T	$\text{VaR}_\alpha = z_\alpha \cdot \sigma_{t+1}$

Exercise 1: Calculating Unconditional Variance

Problem

A GARCH(1,1) model has estimated parameters:

- $\omega = 0.000002$, $\alpha = 0.08$, $\beta = 0.90$

Calculate: (a) Daily unconditional variance; (b) Daily unconditional volatility (%); (c) Annualized volatility (252 trading days); (d) Volatility half-life

Solution

- **(a)** $\bar{\sigma}^2 = \dfrac{\omega}{1-\alpha-\beta} = \dfrac{0.000002}{0.02} = 0.0001$
- **(b)** $\bar{\sigma} = \sqrt{0.0001} = 0.01 = 1\%$ per day
- **(c)** $\sigma_{\text{annual}} = 0.01 \times \sqrt{252} = 15.87\%$ per year
- **(d)** $HL = \dfrac{\ln(0.5)}{\ln(0.98)} = \dfrac{-0.693}{-0.0202} \approx 34$ days

TSA_ch5_garch_sim

Exercise 2: Volatility Forecast

Problem

- Using GARCH(1,1): $\omega = 0.000002$, $\alpha = 0.08$, $\beta = 0.90$
- At time T: $\varepsilon_T = -0.03$ (3% drop), $\sigma_T^2 = 0.0004$
- Calculate: (a) σ^2_{T+1}; (b) σ^2_{T+5}; (c) σ^2_{T+100}

Solution

- **(a)** $\sigma^2_{T+1} = 0.000002 + 0.08 \times (0.03)^2 + 0.90 \times 0.0004 = 0.000434$; Vol: 2.08%
- **(b)** $\mathbb{E}_T[\sigma^2_{T+5}] = \bar{\sigma}^2 + (0.98)^4(\sigma^2_{T+1} - \bar{\sigma}^2) = 0.000408$; Vol: 2.02%
- **(c)** $\mathbb{E}_T[\sigma^2_{T+100}] = 0.0001 + (0.98)^{99} \times 0.000334 \approx 0.000145$; Vol: 1.20%

TSA_ch5_forecast

Exercise 3: Value at Risk

Problem

- Portfolio: 1,000,000 EUR; $\sigma_{T+1} = 2\%$ daily; Normal distribution, zero mean
- Calculate: (a) VaR 95% (1 day); (b) VaR 99% (1 day); (c) VaR 99% (10 days)
- Quantiles: $z_{0.05} = 1.645$, $z_{0.01} = 2.326$

Solution

- **(a)** $\text{VaR}_{95\%} = 1.645 \times 0.02 \times 1{,}000{,}000 = 32{,}900$ EUR
- **(b)** $\text{VaR}_{99\%} = 2.326 \times 0.02 \times 1{,}000{,}000 = 46{,}520$ EUR
- **(c)** $\text{VaR}_{99\%,10d} = 46{,}520 \times \sqrt{10} = 147{,}100$ EUR

Note: the $\sqrt{T}$ scaling rule assumes i.i.d. returns, which contradicts the GARCH dependence structure.

Caution

In practice, for Student-t distribution, quantiles are larger (fatter tails)!

TSA_ch5_var_es

Exercise 4: Model Identification

Problem

Analyze the following estimation results and identify the model:

Parameter	Estimate	Std. Error
ω	0.0000015	0.0000005
α	0.0550	0.0120
γ	0.0850	0.0180
β	0.9100	0.0150

(a) What model is this? (b) Is leverage effect present? (c) Impact of negative vs positive shocks? (d) Is it stationary?

Solution

- **(a) GJR-GARCH(1,1,1)** — presence of γ parameter (threshold/asymmetry)
- **(b)** Yes: $\gamma = 0.085 > 0$ and significant
- **(c)** Positive shock: $\alpha = 0.055$; Negative shock: $\alpha + \gamma = 0.140$ (2.5x greater!)
- **(d)** $\alpha + \gamma/2 + \beta = 0.055 + 0.0425 + 0.91 = 1.0075$ — **slightly above 1** — technically non-stationary (covariance), very close to IGARCH

Step 1: Load and Prepare Data

```
import pandas as pd
import numpy as np
import yfinance as yf
from arch import arch_model
from arch.unitroot import ADF

# Download S&P 500 data
data = yf.download('^GSPC', start='2010-01-01', end='2024-01-01')
returns = 100 * data['Adj Close'].pct_change().dropna()

# Check stationarity
adf = ADF(returns)
print(f'ADF statistic: {adf.stat:.4f}')
print(f'p-value: {adf.pvalue:.4f}')
```

TSA_ch5_sp500

Step 2: Test for ARCH Effects

```
from statsmodels.stats.diagnostic import het_arch

# ARCH-LM test on residuals
residuals = returns - returns.mean()
lm_stat, lm_pvalue, f_stat, f_pvalue = het_arch(residuals, nlags=10)

print(f'ARCH-LM statistic: {lm_stat:.4f}')
print(f'p-value: {lm_pvalue:.4f}')

if lm_pvalue < 0.05:
    print('=> ARCH effects present! GARCH modeling justified.')
```

TSA_ch5_btc_arch

Step 3: Estimate Models

```
# GARCH(1,1) with Student-t distribution
model_garch = arch_model(returns, vol='Garch', p=1, q=1, dist='t')
res_garch = model_garch.fit(disp='off')
print(res_garch.summary())

# GJR-GARCH(1,1,1)
model_gjr = arch_model(returns, vol='Garch', p=1, o=1, q=1, dist='t')
res_gjr = model_gjr.fit(disp='off')

# EGARCH(1,1)
model_egarch = arch_model(returns, vol='EGARCH', p=1, q=1, dist='t')
res_egarch = model_egarch.fit(disp='off')

# Compare AIC
print(f'GARCH AIC: {res_garch.aic:.2f}')
print(f'GJR AIC: {res_gjr.aic:.2f}')
print(f'EGARCH AIC: {res_egarch.aic:.2f}')
```

TSA_ch5_sp500_comp

Step 4: Diagnostics

```
# Standardized residuals
std_resid = res_gjr.std_resid

# Ljung-Box test on squared residuals
from statsmodels.stats.diagnostic import acorr_ljungbox
lb_test = acorr_ljungbox(std_resid**2, lags=10, return_df=True)
print(lb_test)

# Check for remaining ARCH effects
lm_stat2, lm_pval2, _, _ = het_arch(std_resid, nlags=5)
print(f'ARCH-LM residuals: stat={lm_stat2:.2f}, p={lm_pval2:.4f}')

if lm_pval2 > 0.05:
    print('=> No remaining ARCH effects. Model OK!')
```

TSA_ch5_diagnostic

Step 5: Forecast and VaR

```
# Forecast 10 days ahead
forecasts = res_gjr.forecast(horizon=10)
vol_forecast = np.sqrt(forecasts.variance.values[-1, :])

print('Volatility forecast (%):', vol_forecast)

# Value at Risk 99%
portfolio_value = 1_000_000
VaR_99 = 2.326 * vol_forecast[0] / 100 * portfolio_value
print(f'VaR 99% (1 day): {VaR_99:,.0f} EUR')

# 10-day VaR
VaR_99_10d = VaR_99 * np.sqrt(10)
print(f'VaR 99% (10 days): {VaR_99_10d:,.0f} EUR')
```

TSA_ch5_btc_garch

AI Exercise: Critical Thinking

Prompt to test in ChatGPT / Claude / Copilot

"Download BTC-USD daily data from Yahoo Finance (last 5 years). Fit a GARCH(1,1) model to log returns. Show the news impact curve, forecast volatility for the next 30 days, and calculate 1-day 99% VaR."

Exercise:

1. Did the AI check for ARCH effects before fitting GARCH?
2. Is the model adequate? (Check Ljung-Box on standardized residuals)
3. Does the news impact curve show asymmetry? Should EGARCH/GJR be used instead?
4. Is the VaR calculation based on the conditional or unconditional distribution?
5. Try changing to EGARCH — does the AI explain why results differ?

Warning: AI-generated code may run without errors and look professional. *That does not mean it is correct.*

Summary: Chapter 5

Key Concepts

1. **ARCH**: conditional variance depends on past shocks
2. **GARCH**: adds persistence through lagged variance
3. **EGARCH/GJR**: capture leverage effect (asymmetry)
4. **Stationarity**: $\alpha + \beta < 1$
5. **Key formulas**: $\bar{\sigma}^2 = \frac{\omega}{1-\alpha-\beta}$, $HL = \frac{\ln(0.5)}{\ln(\alpha+\beta)}$, $\text{VaR}_\alpha = z_\alpha \cdot \sigma \cdot V$

Practical Tip

Use Student-t distribution to capture fat tails. Verify absence of ARCH effects in residuals!

Questions?

Homework Exercises

Exercise 1

Download daily returns for BET (BVB index) and estimate a GARCH(1,1) model. Compare persistence $(\alpha + \beta)$ with S&P 500.

Exercise 2

For Bitcoin, estimate GARCH, EGARCH, and GJR-GARCH. Is leverage effect present for cryptocurrencies?

Exercise 3

Calculate daily VaR for a portfolio of 100,000 EUR invested in EUR/USD, using GARCH-forecasted volatility.

Exercise 4

Compare GARCH(1,1) volatility forecast with realized volatility (sum of squared returns) for a 20-day period.

TSA_ch5_rolling_forecast

Bibliography I

Time Series Fundamentals

- Hyndman, R.J., & Athanasopoulos, G. (2021). *Forecasting: Principles and Practice*, 3rd ed., OTexts.
- Shumway, R.H., & Stoffer, D.S. (2017). *Time Series Analysis and Its Applications*, 4th ed., Springer.
- Brockwell, P.J., & Davis, R.A. (2016). *Introduction to Time Series and Forecasting*, 3rd ed., Springer.

Financial Time Series

- Tsay, R.S. (2010). *Analysis of Financial Time Series*, 3rd ed., Wiley.
- Franke, J., Härdle, W.K., & Hafner, C.M. (2019). *Statistics of Financial Markets*, 4th ed., Springer.

Bibliography II

Modern Approaches and Machine Learning

- Nielsen, A. (2019). *Practical Time Series Analysis*, O'Reilly Media.
- Petropoulos, F., et al. (2022). *Forecasting: Theory and Practice*, International Journal of Forecasting.
- Makridakis, S., Spiliotis, E., & Assimakopoulos, V. (2020). The M4 Competition, International Journal of Forecasting.

Online Resources and Code

- **Quantlet**: https://quantlet.com — Code repository for statistics
- **Quantinar**: https://quantinar.com — Quantitative methods learning platform
- **GitHub TSA**: https://github.com/QuantLet/TSA/tree/main/TSA_ch5 — Python code for this seminar

Thank You!

Questions?

Seminar materials are available at: https://danpele.github.io/Time-Series-Analysis/

Quantlet Quantinar