Statistics of Financial Markets
Data and Statistical Indicators

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Why do we need data?

⊡ Financial markets are complex systems.

⊡ Understanding their dynamics requires data.

⊡ Understanding the Past

▶ Historical data helps us understand past market behavior.

▶ Identify trends.

▶ Analyze how markets have reacted to different events.

⊡ Predicting the Future

▶ Analyzing historical data can provide insights into potential future market movements

▶ Inform investment decisions.

Where do we need data?

⊡ Risk Management

▶ Data is essential for quantifying and managing financial risks, such as market volatility, credit risk, and liquidity risk.

⊡ Valuation

▶ Data is used to value financial assets, such as stocks, bonds, and derivatives.

⊡ Trading

▶ Traders rely on real-time market data and historical data to make trading decisions and execute trades.

⊡ Regulation

▶ Regulators use market data to monitor market activity, identify potential risks, and ensure market integrity.
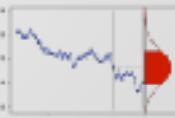
Examples of Data Usage

⊡ Technical Analysis

▶ Traders use historical price and volume data to identify patterns and trends in the market.

⊡ Fundamental Analysis

▶ Investors analyze financial statements and other fundamental data to assess the intrinsic value of a company.

⊡ Quantitative Finance

▶ Quantitative analysts (quants) develop trading strategies and manage risk.

⊡ Algorithmic Trading

▶ Automated trading systems rely on real-time market data to execute trades at high speed.

Data Challenges

⊡ Data Quality

▶ Financial data can be noisy, incomplete, or inaccurate.

⊡ Data Availability

▶ Some data may be difficult or expensive to obtain.

⊡ Data Volume

▶ The sheer volume of financial data can be overwhelming.

⊡ Data Interpretation

▶ Interpreting financial data requires expertise and understanding of market dynamics.

"Data! Data! Data!
I can't make bricks
without clay"

SHERLOCK HOLMES
In The Adventure of The Copper Beeches
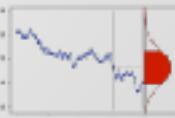
Outline

1. Motivation ✓

2. Price Data

3. Returns

4. Volatility Estimators

5. Conclusions and Homework

6. References

OHLC

⊡ Open Price O_t

▶ The first traded price at the beginning of the time period t.

▶ It reflects the initial market sentiment and can be influenced by news events occurring after the previous close.

⊡ High Price H_t

▶ The highest traded price during period t.

⊡ Low Price L_t

▶ The lowest traded price during period t.

⊡ Close Price P_t

▶ The last traded price at the end of period t.

▶ It is often considered the most significant price because it reflects the final consensus of value for that period.

OHLC

Adjusted Close Price

⊡ P_t^{adj}
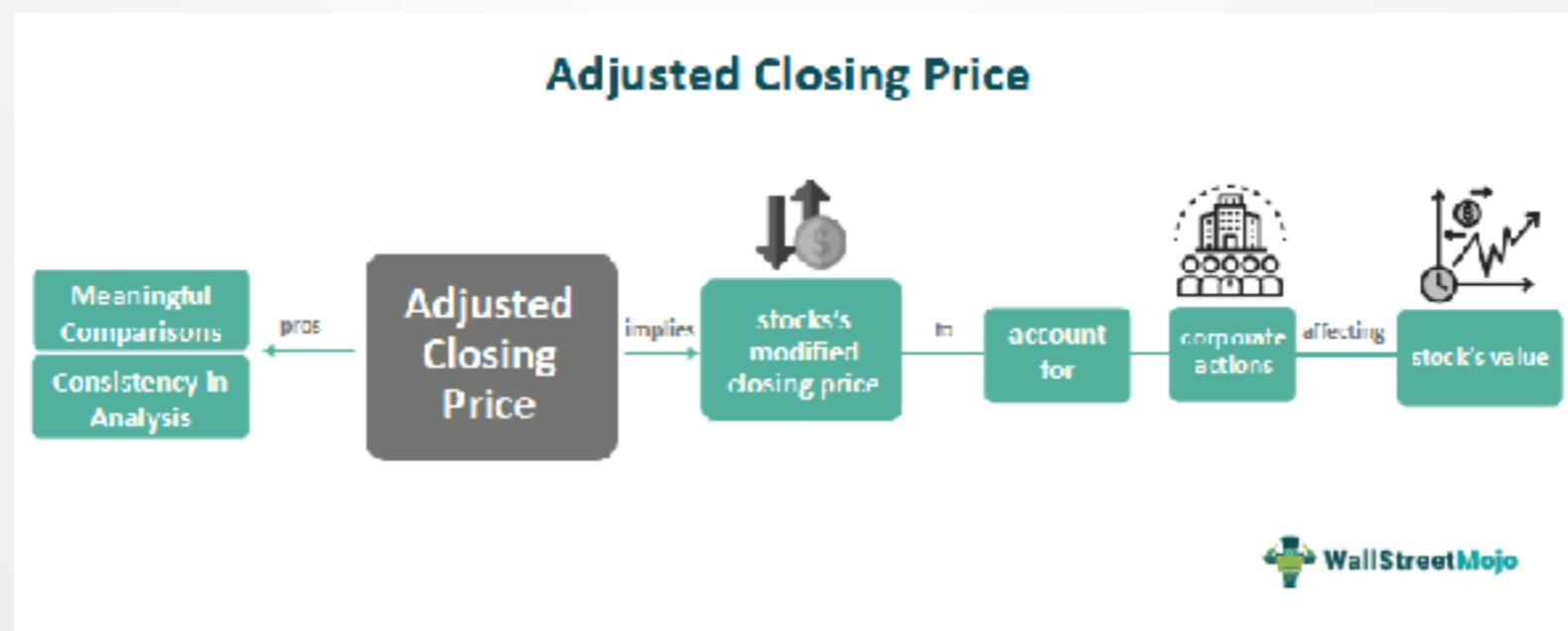
  ▶ Accounts for corporate actions such as dividends, stock splits, and rights offerings that affect a stock's price but not its value.

  ▶ Ensures that historical price data reflects the true economic value of the stock over time, allowing for accurate performance analysis and comparison.

⊡ $P_t^{adj} = P_t \times A_t$

  ▶ A_t: cumulative adjustment factor at time t.

  ▶ $A_t = \prod_{i=1}^{t} F_i.$

  ▶ F_i is the individual adjustment factor for a corporate action occurring at time i.

Adjusted Closing Price

Individual Adjustment Factors

- ⊡ Stock Splits and Reverse Splits
  - ▶ $F_i = \text{Split Ratio}_i$
- ⊡ Stock Dividends
  - ▶ For a stock dividend of $d_i\%$ at time i (expressed as a decimal):
    $F_i = 1 + d_i.$
  - ▶ For example, a 10% stock dividend has an adjustment factor
    $F_i = 1 + 0.10 = 1.10.$
- ⊡ Cash Dividends
  - ▶ For significant cash dividends (rare for most stocks) $F_i = \dfrac{P_{i-1} - D_i}{P_{i-1}}.$
  - ▶ D_i is the cash dividend per share at time i.
  - ▶ P_{i-1} is the close price before the dividend is paid.
- ⊡ Right Offerings
  - ▶ Shareholders can buy n new shares at a subscription price P_{rights} for
    every m shares held: $F_i = \dfrac{P_{i-1} + \left(\frac{n}{m} \times P_{\text{rights}}\right)}{P_{i-1}}.$

Price chart

▫ Stock Splits and Reverse Splits

▶ $F_i = $ Split Ratio$_i$

▫ Stock Dividends

▶ For a stock dividend of $d_i\%$ at time i (expressed as a decimal):
$F_i = 1 + d_i.$

▶ For example, a 10% stock dividend has an adjustment factor
$F_i = 1 + 0.10 = 1.10.$

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Returns

- Return measures the gain or loss of an investment over a period, expressed as a percentage of the investment's initial cost.

- Simple returns

  - $$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}.$$

  - P_t is the price at time t.

Returns

⊡ Log-Returns

▶ $$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln P_t - \ln P_{t-1}.$$
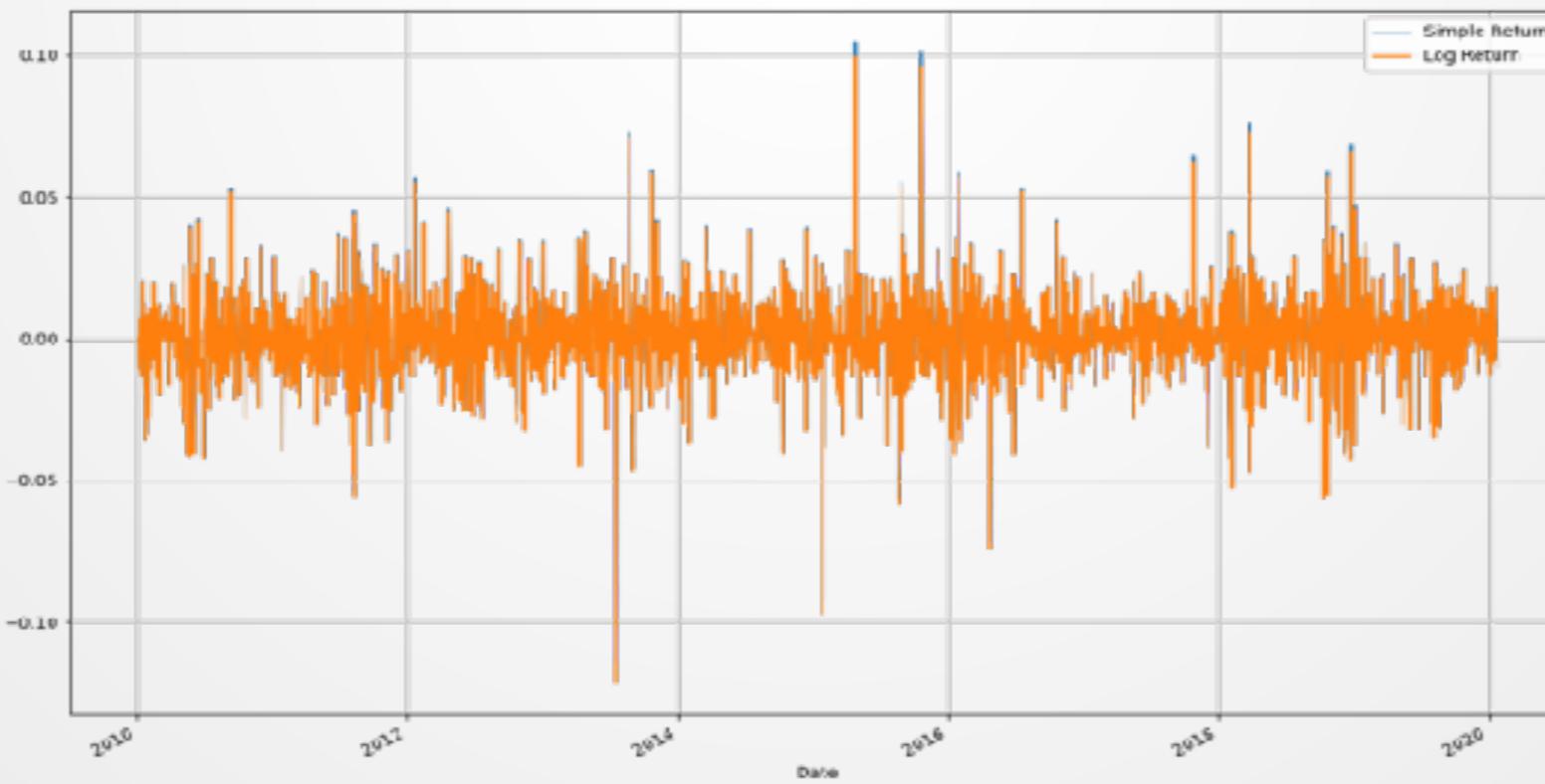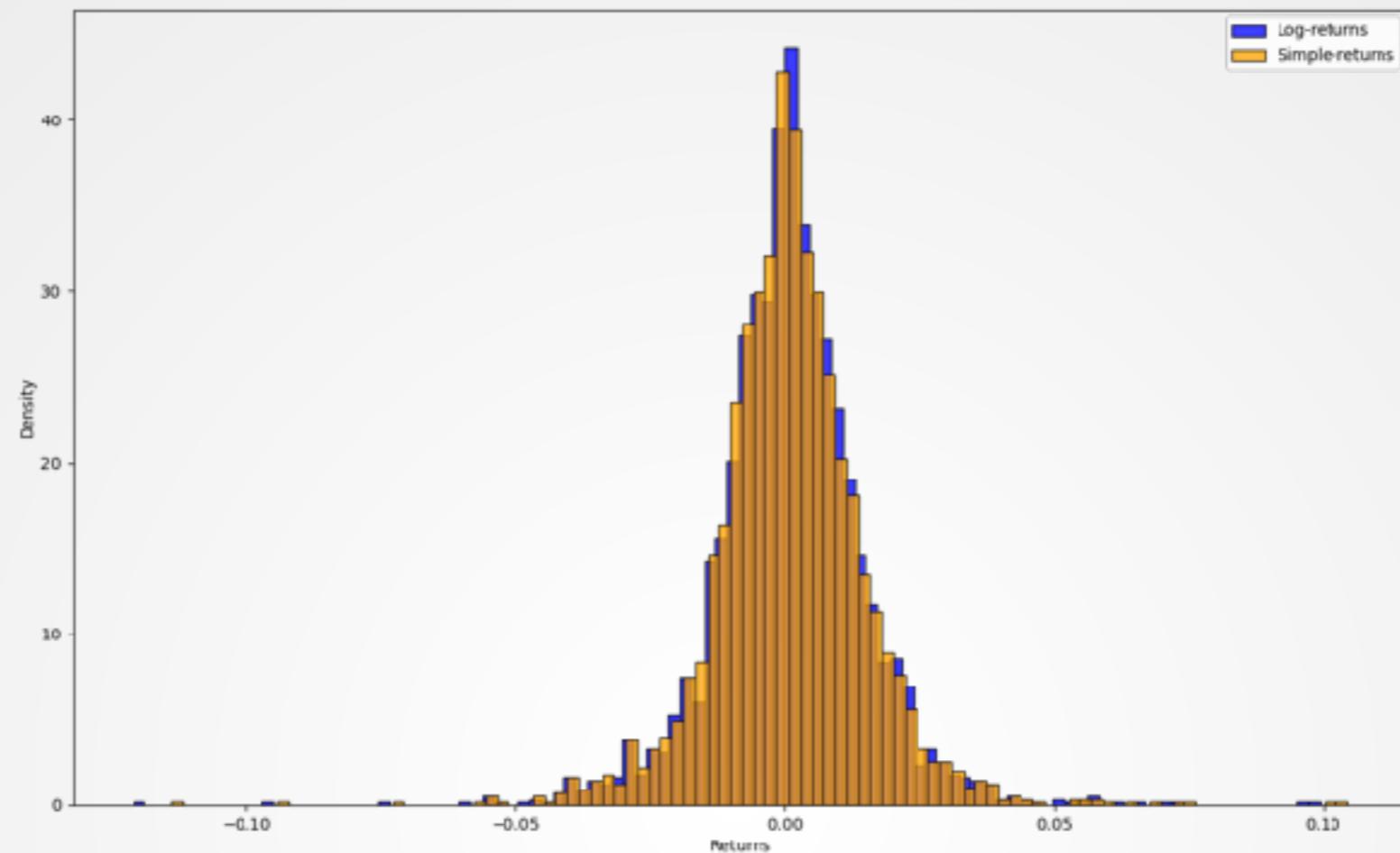
▶ Log-Returns are time-additive:

$$r_{t+2/t} = \ln\left(\frac{P_{t+2}}{P_t}\right) = \ln\left(\frac{P_{t+2}}{P_{t+1}}\right) + \ln\left(\frac{P_{t+1}}{P_t}\right) = r_{t+2} + r_{t+1}.$$

▶ $r_t = \ln\left(1 + R_t\right)$, i.e. for small changes in price: $r_t \approx R_t$.

▶ Under certain conditions, log-returns are more likely to follow a normal distribution.

▶ Log-returns simplify the mathematical handling of compounded returns.

▶ Enable more convenient statistical analysis, especially in the context of portfolio management and risk analysis.

Returns

Cumulative Returns

⊡ Cumulative returns represent the total change in the value of an investment over a period.

⊡ Can be calculated using either simple returns or log-returns.

⊡ Cumulative Simple Return

▶ $\text{CSR} = \prod_{t=1}^{n} \left(1 + R_t - c_t \right) - 1.$

▶ $R_t = \dfrac{P_t - P_{t-1}}{P_{t-1}}.$

▶ c_t % transaction costs.

⊡ Cumulative Log-Return

▶ $\text{CLR} = \sum_{t=1}^{n} \left(r_t - c_t \right).$

▶ $r_t = \log\left(\dfrac{P_t}{P_{t-1}} \right).$

▶ c_t % transaction costs.

Volatility

- Volatility is a statistical measure of the dispersion of returns for a given security or market index.
- It represents the degree of variation of an asset's price over time.
- Several estimators have been developed to capture the complexities of financial market volatility.
  - ▶ Historical Volatility
  - ▶ Parkinson's Estimator
  - ▶ Garman-Klass Estimator
  - ▶ Rogers-Satchell Estimator
  - ▶ Yang-Zhang Estimator
  - ▶ Realized Volatility
  - ▶ GARCH Models
  - ▶ HEAVY Models
  - ▶ Stochastic Volatility Models with Jumps
  - ▶ Implied Volatility

2011 Tōhoku, Japan, earthquake and tsunami

Historical Volatility
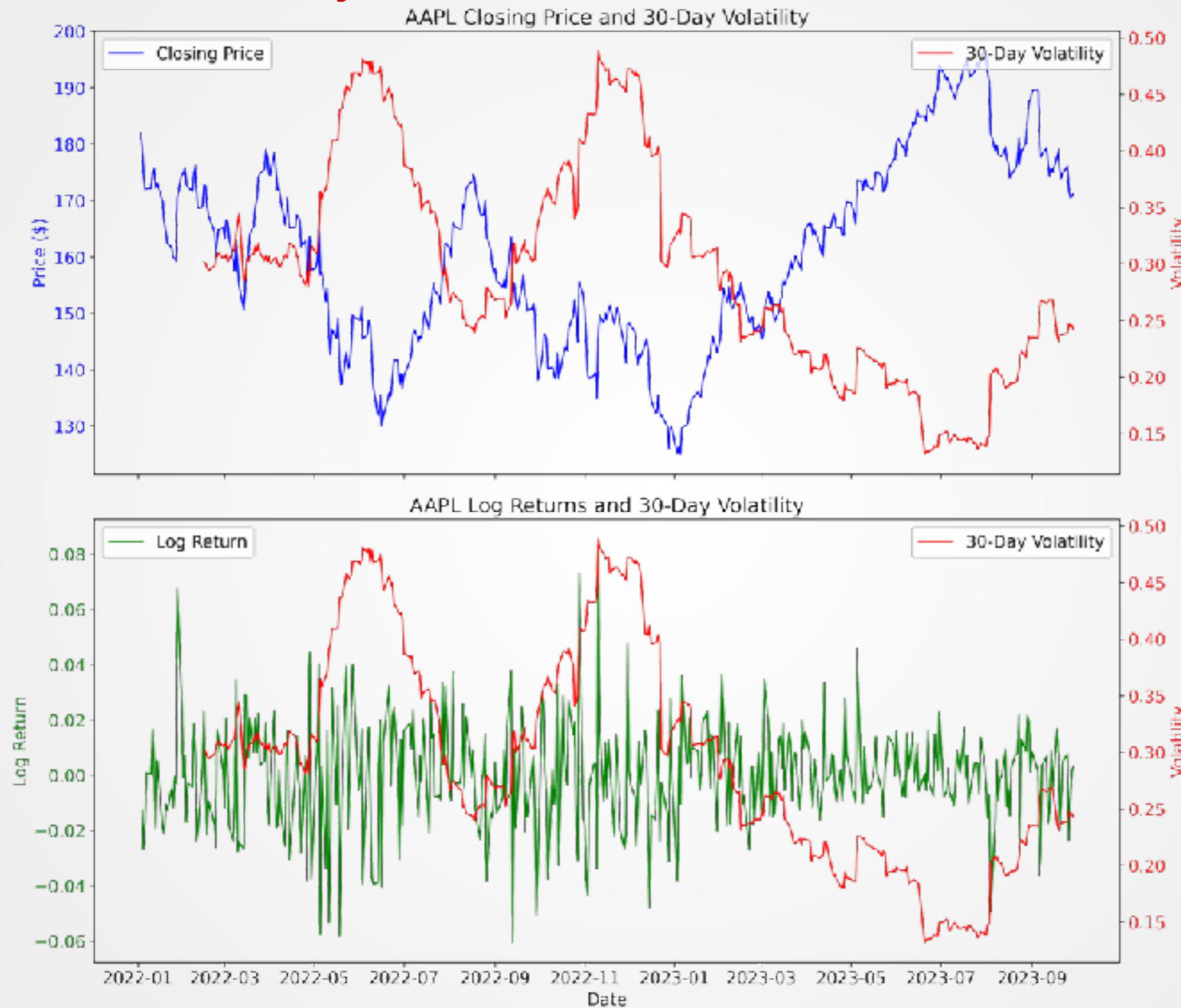
⊡ Measures the dispersion or variability of returns over a specified period in the past.

⊡ Historical volatility is calculated as the standard deviation of past returns:

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (r_t - \bar{r})^2}.$$

⊡ Annualizing Volatility

▶ Since volatility is often expressed on an annual basis, daily volatility can be annualized using the following formula:

$$\sigma_{\text{annual}} = \sigma_{\text{daily}} \times \sqrt{T}.$$

▶ T is the number of trading periods in a year (typically 252 trading days).

▶ σ_{daily} is the standard deviation of the daily returns.

⊡ Limitations of Historical Volatility

▶ Past Performance May Not Predict Future Volatility.

▶ Does Not Account for Shocks or Market Regime Changes.

Historical Volatility

Square Root of Time Rule

⊡ Fundamental concept in financial risk management and quantitative finance.

⊡ It provides a method for scaling volatility (or risk) over different time horizons, assuming that returns are IID and follow a normal distribution.

⊡ This rule allows practitioners to estimate the volatility of returns over longer or shorter periods when the volatility for a specific period is known.

▶ Suppose we have an asset with returns $r_t \sim \mathcal{N}(\mu, \sigma^2)$ i.i.d.

▶ The total return over n periods is:

▶ $R_n = r_1 + r_2 + \ldots + r_n.$

▶ Since the returns are IID, the expected value and variance of R_n are:

▶ $E[R_n] = n\mu$ and $\text{Var}(R_n) = n\sigma^2.$

▶ The standard deviation (volatility) over n periods is therefore:

$\sigma_n = \sqrt{\text{Var}(R_n)} = \sqrt{n}\sigma.$

▶ This shows that volatility scales with the square root of time.

⊡ In practice, this rule is used to scale daily volatility to annual volatility or vice versa.

▶ If the daily volatility of an asset is σ_{daily}, the annualized volatility is:

▶ $\sigma_{\text{annual}} = \sigma_{\text{daily}} \times \sqrt{N}.$

▶ N is the number of trading days in a year (typically 252).

Louis Bachelier
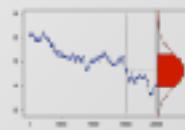
Parkinson's Estimator

⊡ Traditional estimators often rely solely on closing prices, potentially overlooking valuable

information contained within the trading period.

⊡ Parkinson (1980) introduced an estimator that utilizes the high and low prices within a period,

offering a more efficient volatility estimate under specific assumptions.

⊡ Assumptions:

▶ continuous trading.

▶ zero drift in the price process.

⊡ By leveraging the range—the difference between the highest and lowest prices—it captures intra-

period price variability more effectively than estimators based on closing prices alone.
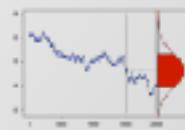
⊡
$$\sigma_P = \frac{1}{\sqrt{4 \ln 2}} \sqrt{\frac{1}{n} \sum_{t=1}^{n} \left(\ln \left(\frac{H_t}{L_t} \right) \right)^2}.$$

▶ H_t is the highest price on day t.

▶ L_t is the lowest price on day t.

▶ n is the number of periods (e.g., days).

Applications

- ⊡ Volatility Estimation in Financial Markets
  - ▶ The Parkinson estimator is widely used when closing prices may not fully capture price movements.
  - ▶ Parkinson estimator provides more accurate volatility estimates compared to traditional methods, such as close-to-close estimators (Fiszeder & Perczak, 2013; Jacob & Vipul, 2008).
- ⊡ Comparative Analysis of Volatility Estimators
  - ▶ The Parkinson estimator is often included in comparative studies alongside other volatility estimators, such as Garman-Klass and Rogers-Satchell (Lapinova & Saichev, 2017; Vasileiou, 2023).
- ⊡ Risk Management and Derivative Pricing
  - ▶ The Parkinson estimator is utilized to improve the pricing of derivatives and to assess the risk associated with various financial instruments.
  - ▶ High volatility markets, such as equity and options markets (Jacob & Vipul, 2008; Vasileiou, 2023).
- ⊡ Forecasting and Trading Strategies
  - ▶ The Parkinson estimator has been integrated into various forecasting models and trading strategies.
  - ▶ Trading strategies based on volatility estimates from the Parkinson method can lead to improved performance and profitability compared to those based on simpler models (Fiszeder & Perczak, 2013; Jacob & Vipul, 2008).
- ⊡ Application in Emerging Markets
  - ▶ The Parkinson estimator is particularly useful in emerging markets where data may be less reliable and market conditions more volatile.
  - ▶ Effective volatility estimation even when closing prices are subject to significant noise (Jacob & Vipul, 2008; Yang & Zhang, 2000).

Garman-Klass Estimator

☑ Incorporates OHLC prices to provide a more accurate volatility estimate under the assumption of zero drift (Garman & Klass, 1980) :

$$\sigma_{\text{GK}} = \sqrt{\frac{1}{n} \sum_{t=1}^{n} \left[0.5 \left(\ln \left(\frac{H_t}{L_t} \right) \right)^2 - (2 \ln 2 - 1) \left(\ln \left(\frac{C_t}{O_t} \right) \right)^2 \right]}.$$

☑ Derivation

▶ Geometric Brownian Motion (GBM) model with zero drift ($\mu = 0$) and continuous trading.

▶ The key idea is to decompose the total variance into components that capture different aspects of price movements within a trading period.

▶ Under GBM, the logarithmic price changes are normally distributed.

▶ The total variance σ^2 over a period can be expressed as the sum of the variances of the individual components:

☑

$$\sigma^2 = \text{Var} \left(\ln \left(\frac{C_t}{O_t} \right) \right) + \text{Var} \left(\ln \left(\frac{H_t}{L_t} \right) \right) - 2\text{Cov} \left(\ln \left(\frac{C_t}{O_t} \right), \ln \left(\frac{H_t}{L_t} \right) \right).$$

▶ Garman and Klass derived the expected values of these components under the GBM framework and combined them to form an estimator that minimizes the variance of the estimate.

Advantages and Limitations

- ⊡ Efficiency and Advantages
  - ▶ The GK estimator is more efficient estimators that use only closing prices or the high-low range.
  - ▶ It incorporates additional information from the opening and closing prices.
  - ▶ This inclusion reduces the estimator's variance, making it a more accurate measure of the true volatility.
  - ▶ Under ideal conditions, the GK estimator is approximately 7.4 times more efficient than the close-to-close estimator and more efficient than Parkinson's estimator.
  - ▶ The increased efficiency is due to the estimator's ability to account for both the intra-day range and the price movements from open to close.
- ⊡ Assumptions and Limitations
  - ▶ GK assumes that the expected return of the asset is zero over the estimation period.
  - ▶ Continuous Trading: trading occurs continuously without interruptions.
  - ▶ No Price Jumps: price changes are continuous and follow a Brownian motion without jumps.
  - ▶ No Opening Price Bias: the opening price reflects the true price at the start of the trading period.
- ⊡ Violations of these assumptions, such as overnight gaps or jumps due to news events, can introduce biases into the volatility estimate.

Rogers - Satchell Estimator

⊡ The RS estimator is defined as:
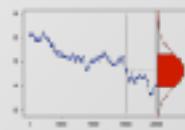
$$\sigma_{RS} = \sqrt{\frac{1}{n} \sum_{t=1}^{n} \left[\ln\left(\frac{H_t}{C_t}\right) \ln\left(\frac{H_t}{O_t}\right) + \ln\left(\frac{L_t}{C_t}\right) \ln\left(\frac{L_t}{O_t}\right) \right]}.$$

⊡ Risk Management
  ▶ Assessing the risk associated with holding or trading an asset.
⊡ Option Pricing
  ▶ Providing accurate volatility estimates crucial for pricing derivatives.
⊡ Portfolio Optimization
  ▶ Informing asset allocation decisions based on volatility estimates.
⊡ Advantages
  ▶ Drift Adjustment: Accounts for the drift component, offering a more accurate measure when the asset price trends.
  ▶ Incorporates Multiple Price Points: OHLC prices, capturing more information about intra-period price movements.
⊡ Limitations
  ▶ Data Requirements: Requires detailed price data OHLC for each period.
⊡ Assumptions
  ▶ Assumes that the asset price follows a geometric Brownian motion with drift, which may not always hold true.

Applications

⊡ Volatility Estimation in Emerging Markets

▶ The RS estimator is frequently employed in emerging markets to provide accurate volatility estimates that account for price extremes.

▶ The RS estimator performs well in environments characterized by high volatility and outliers (Öztürk et al. (2016)).

⊡ Comparative Analysis of Volatility Estimators

▶ The RS estimator provides robust estimates, particularly in the presence of extreme price movements (Lapinova & Saichev, 2017).

⊡ Application in Cryptocurrency Markets

▶ The RS estimator can capture the unique volatility characteristics of cryptocurrencie (Cadenas-Morales, 2022).

⊡ Risk Management and Derivative Pricing

▶ Accurate measure of volatility in equity and options markets (Fałdziński & Osińska, 2016).

⊡ Forecasting and Trading Strategies

▶ Trading strategies based on RS method can lead to improved performance and profitability compared to those based on simpler models (Banerjee, 2023).
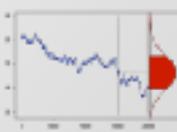
Yang-Zhang Estimator

⊡ The YZ estimator (Yang, 2000) is a combination of the overnight (close-to-open volatility) and a weighted average of the Rogers-Satchell volatility and the day's open-to-close volatility.

⊡ It considered being **14 times** more efficient than the close-to-close estimator:
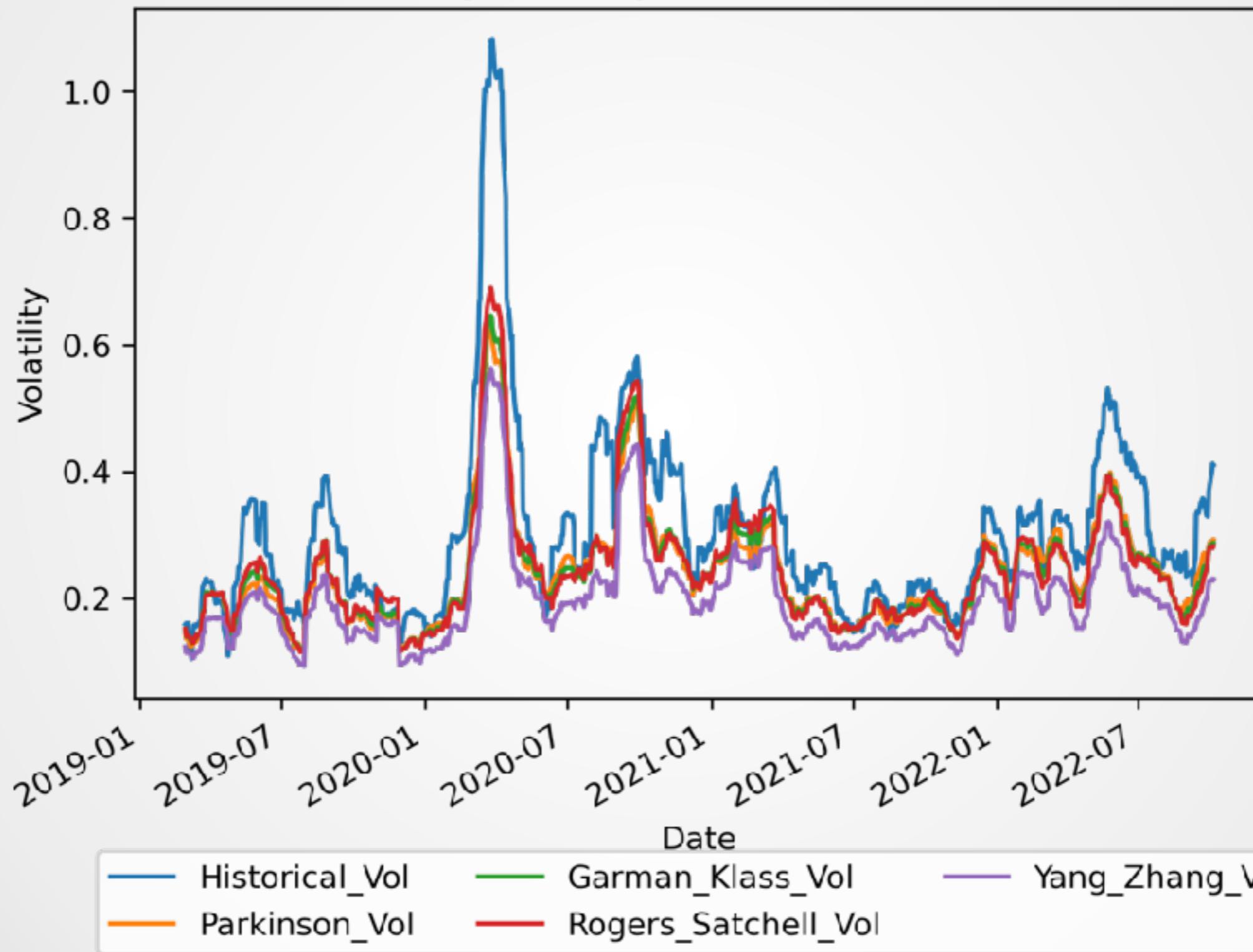
$$\sigma_{YZ} = \sqrt{\sigma_O^2 + k\sigma_C^2 + (1-k)\sigma_{RS}^2}.$$

▶ σ_{YZ}: YZ estimated volatility.

▶ Overnight variance: $\sigma_O^2 = \dfrac{1}{T-1}\sum_{t=1}^{T}\left(\ln\left(\dfrac{O_t}{C_{t-1}}\right) - \text{Avg}\left(\ln\left(\dfrac{O_t}{C_{t-1}}\right)\right)\right)^2.$

▶ Open-to-Close variance: $\sigma_C^2 = \dfrac{1}{T-1}\sum_{t=1}^{T}\left(\ln\left(\dfrac{C_t}{O_t}\right) - \text{Avg}\left(\ln\left(\dfrac{C_t}{O_t}\right)\right)\right)^2.$

▶ RS variance:

▶ $\sigma_{RS}^2 = \dfrac{1}{n}\sum_{t=1}^{n}\left[\ln\left(\dfrac{H_t}{C_t}\right)\ln\left(\dfrac{H_t}{O_t}\right) + \ln\left(\dfrac{L_t}{C_t}\right)\ln\left(\dfrac{L_t}{O_t}\right)\right].$

▶ $k = \dfrac{\alpha - 1}{\alpha + \dfrac{T+1}{T-1}}$: Weighting factor, typically determined empirically (usually $k = 0.34$).

Volatility Estimators: Example

Rolling Volatility Estimators for AAPL

Sharpe Ratio

⊡ SR measures the excess return of an asset over the risk-free rate, adjusted

for risk (volatility).

⊡ It is a widely used measure of risk-adjusted performance.

⊡
$$SR_t = \frac{\bar{R}_t - R_f}{\sigma_t}.$$
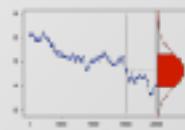
▶ $\bar{R}_t$ is the average return of the asset over time t.

▶ R_f is the risk-free rate, typically based on government bonds.

▶ σ_t is the volatility of returns over time t.

▶ A higher Sharpe ratio indicates better risk-adjusted performance (Sharpe,

1966).

Interpreting the Sharpe Ratio

⊡ Higher values indicate better risk-adjusted returns

⊡ Allows comparison of investments with different risk levels

SR	Interpretation
< 0	Worse than risk-free rate
0 - 0.5	Poor
0.5 - 1	Acceptable
1 - 2	Good
> 2	Excellent

Mathematical framework

⊡ Expected return of a portfolio:

▶ $E[R_p] = \sum\limits_{i=1}^{n} w_i \cdot E[R_i]$

▶ w_i: Weight of asset i in the portfolio, $0 \leq w_i \leq 1, \ \sum w_i = 1.$

▶ $E[R_i]$: Expected return of asset i

▶ n: Number of assets

⊡ In practice, historical mean returns are often used:

▶ $E[R_i] \approx \dfrac{1}{T} \sum\limits_{t=1}^{T} r_{i,t}$

⊡ Annualized from daily returns:

▶ $E[R_i]_{annual} = E[R_i]_{daily} \times 252$
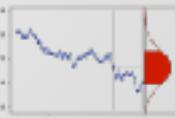
Portfolio Volatility

⊡ Portfolio volatility accounts for correlations between assets
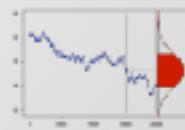
▶ $$\sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_i \sigma_j \rho_{ij}}.$$

▶ In matrix notation: $\sigma_p = \sqrt{\mathbf{w}^T \Sigma \mathbf{w}}$.

▶ $\mathbf{w}$: Vector of asset weights

▶ Σ: Covariance matrix of returns

▶ Annualized from daily volatility: $\sigma_{annual} = \sigma_{daily} \times \sqrt{252}$

Portfolio Optimization
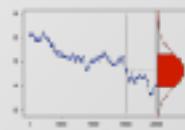
▣ The Efficient Frontier Portfolios

▶ Maximize expected return for a given level of risk

▶ Minimize risk for a given level of expected return

▶ Each point on the frontier is a unique portfolio allocation

▶ Portfolios below the frontier are suboptimal

Efficient Frontier

⊡ Analyzing these assets:

  ▶ SPY: S&P 500 (US Large Cap)

  ▶ QQQ: Nasdaq 100 (Tech)

  ▶ GLD: Gold

  ▶ TLT: Long-Term Treasury Bonds

  ▶ VNQ: Real Estate

⊡ Downloading price data from
2018-01-01 to 2025-03-04.

⊡ Downloaded 1801 days of data.

⊡ Generating 10000 random
portfolios.
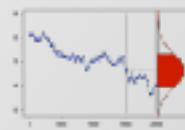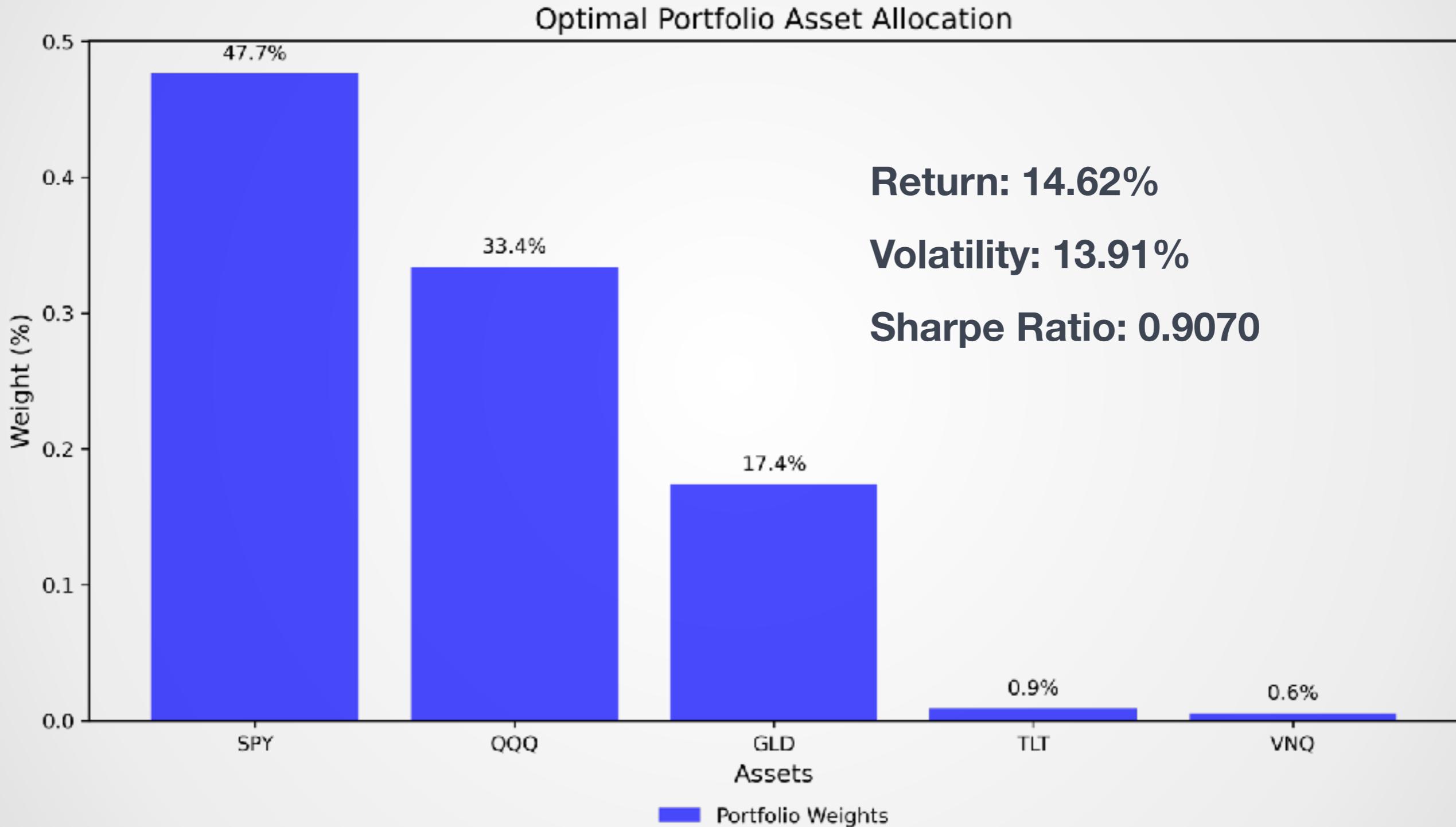
Asset	Return (%)	Volatility (%)	Sharpe Ratio
SPY	14.33	19.35	0.637
QQQ	19.57	24.04	0.7309
GLD	11.62	14.31	0.6725
TLT	-0.37	16.16	-0.1469
VNQ	8.23	22.75	0.2738

Efficient Frontier

Optimal Portfolio

Optimal Portfolio Asset Allocation

Return: 14.62%

Volatility: 13.91%

Sharpe Ratio: 0.9070
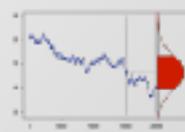
Conclusions

▣ Returns represent the core metric for evaluating asset performance.

▣ Volatility Estimators measure the magnitude of asset price fluctuations, helping to quantify the level of market risk.

▣ Volatility is critical for understanding market uncertainty and is a cornerstone of risk management strategies.

▣ Accurate volatility estimation supports better pricing of derivatives, portfolio management, and strategic decision-making in turbulent markets.

▣ Traders and analysts rely on volatility forecasts to set risk limits and adjust strategies dynamically.

▣ Returns and volatility estimators are fundamental inputs for algorithms in high-frequency trading and automated systems, guiding buy/sell decisions and optimizing portfolios.

Homeworks
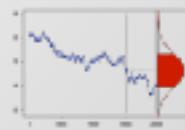
Return Calculation - The Devil's in the Details

⊡ A stock's price at time t is $P_t = \$100$.

    ▶ Over the next three periods, the continuously compounded returns are

$$r_{t+1} = 0.1, \quad r_{t+2} = -0.05, \quad \text{and } r_{t+3} = 0.02.$$

    ▶ Calculate the stock price at times $t + 1, t + 2, t + 3$.

    ▶ Show your calculations.

⊡ Now, assume the simple returns over the same periods are $R_{t+1} = 0.1$, $R_{t+2} = -0.05$ and

$R_{t+3} = 0.02$.

    ▶ Calculate the stock price at times $t + 1, t + 2, t + 3$.

    ▶ Show your calculations.

⊡ Compare the final prices obtained using continuously compounded returns and simple returns.

    ▶ Are they the same?

    ▶ Why or why not?

    ▶ Explain the practical implications of this difference, especially when dealing with longer time horizons.

Return Calculation - The Devil's in the Details

☐ You are given a time series of stock prices $P_0, P_1, \ldots, P_n$.

▶ Derive a general formula for calculating the total continuously compounded return from time 0 to time n.

▶ How does this relate to the individual period returns?

▶ Explain how the concept of time additivity applies to continuously compounded returns.

▶ Why is this property desirable in financial modeling?
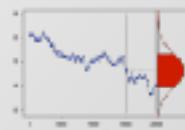
Volatility and Risk - Beyond the Basics

⊡ Define volatility in the context of financial markets.

▶ Relate your definition to the discussion of risk in the provided lecture notes.

⊡ Why is volatility considered a measure of risk?

▶ What kind of risk does it capture?

⊡ Discuss different ways to estimate volatility.

⊡ What are the advantages and disadvantages of using historical data to estimate future volatility?

⊡ What are some other approaches to estimating volatility (mention at least two)?
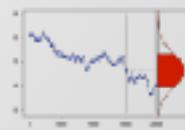
Volatility and Risk - Beyond the Basics

⊡ You are given two assets, A and B. Asset A has a higher expected return than asset B, but also a higher volatility.

▶ How would you decide which asset to invest in? What factors would you consider?

▶ Explain the concept of risk aversion and how it relates to this decision.

⊡ A portfolio consists of two assets, X and Y. The weights of the assets are w_X and w_Y respectively ($w_X + w_Y = 1$).

▶ The volatilities of the individual assets are σ_X and σ_Y, and the correlation between the assets is ρ_{XY}.

▶ Derive the formula for the portfolio volatility σ_P.

▶ Explain how diversification can reduce portfolio volatility.
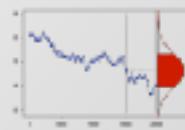
Square Root of Time Rule - When and When Not

⊡ Explain the square root of time rule for scaling volatility.

▶ Write down the formula.

▶ Clearly define all the variables in the formula.

▶ What are the key assumptions underlying the validity of the square root of time rule?

⊡ A stock has a daily volatility of 1.2%.

▶ Using the square root of time rule, estimate its weekly, monthly, and annual volatilities (assume 5 trading days per week, 22 trading days per month, and 252 trading days per year).

▶ Show your calculations.

⊡ Now, imagine that the stock's returns are not independent.

▶ Specifically, assume that the daily returns are positively autocorrelated.

▶ Would you expect the actual annual volatility to be higher or lower than the estimate obtained using the square root of time rule? Explain your reasoning.

▶ How does autocorrelation affect volatility scaling?

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