Time Series Analysis and Forecasting

Chapter 9: Prophet and TBATS

Modern Models for Multiple Seasonalities

Outline

The Problem: Complex Seasonal Patterns

Real-World Examples

- **Hourly electricity demand**: Daily + Weekly + Annual patterns
- **Website traffic**: Daily + Weekly + Holiday effects
- **Retail sales**: Weekly + Monthly + Annual + Holiday effects
- **Call center volume**: Hourly + Daily + Weekly patterns

SARIMA Limitation

Standard SARIMA$(p, d, q)(P, D, Q)_s$ handles only **one** seasonal period s.

For hourly data with daily AND weekly patterns, we need $s_1 = 24$ and $s_2 = 168$.

Solutions for Multiple Seasonalities

Traditional Approaches

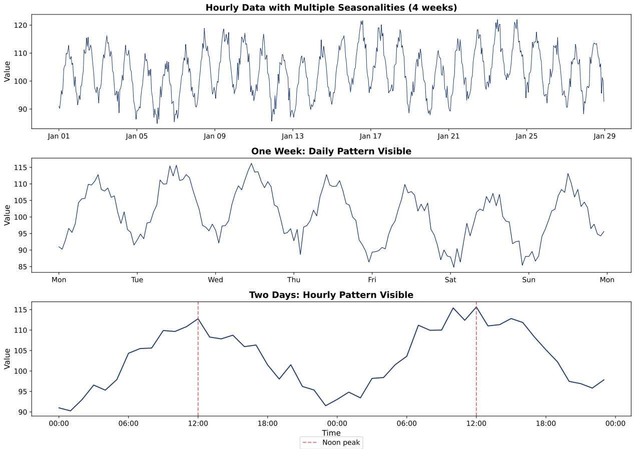
- **Fourier terms**: Add sin/cos regressors
- **Dummy variables**: Many parameters
- **Nested models**: Complex specification

Modern Approaches

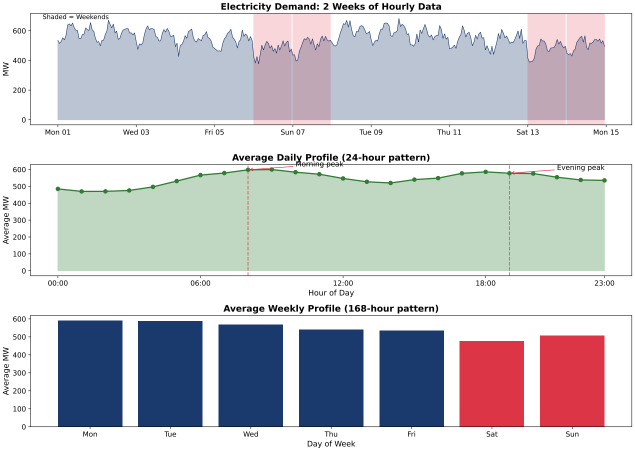
- **TBATS**: Automatic, handles many periods
- **Prophet**: Flexible, interpretable
- **Neural methods**: Deep learning

Method	**Max Seasonalities**	**Interpretable**
SARIMA	1	Yes
Fourier + ARIMA	Multiple	Moderate
TBATS	Multiple	Moderate
Prophet	Multiple	Yes

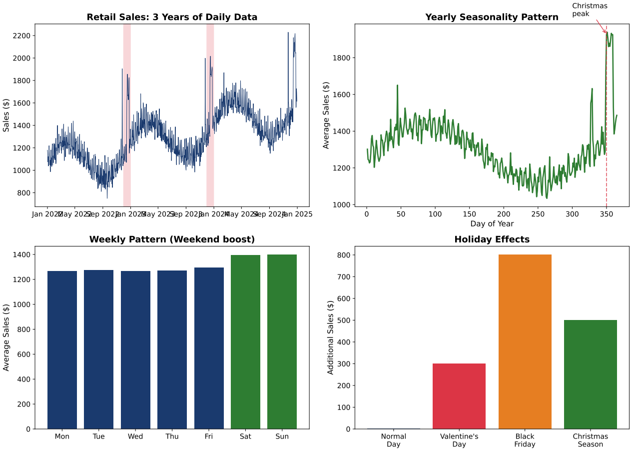
Example: Hourly Data with Multiple Seasonalities



Real Example: Electricity Demand



Real Example: Retail Sales with Holidays



TBATS: What Does It Stand For?

TBATS Components

- T **Trigonometric** seasonality using Fourier terms
- B **Box-Cox** transformation for variance stabilization
- A **ARMA** errors for remaining autocorrelation
- T **Trend** component (possibly damped)
- S **Seasonal** components (multiple allowed)

Key Innovation

TBATS uses **trigonometric representation** of seasonality:

$$s_t^{(i)} = \sum_{j=1}^{k_i} \left[s_j^{(i)} \cos\left(\frac{2\pi jt}{m_i}\right) + s_j^{*(i)} \sin\left(\frac{2\pi jt}{m_i}\right) \right]$$

where m_i is the i-th seasonal period and k_i is the number of harmonics.

TBATS Model Structure

Full Model Specification

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{T} s_{t-m_i}^{(i)} + d_t \tag{1}$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t \tag{2}$$

$$b_t = \phi b_{t-1} + \beta d_t \tag{3}$$

$$d_t = \sum_{i=1}^{p} \varphi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \tag{4}$$

Where:

- $y_t^{(\omega)}$ is Box-Cox transformed series (if $\omega \neq 1$)
- ℓ_t is local level, b_t is trend with damping ϕ
- $s_t^{(i)}$ are T seasonal components with periods $m_1, \ldots, m_T$
- d_t is ARMA(p, q) error process

TBATS: Trigonometric Seasonality

Why Fourier/Trigonometric Terms?

1. **Parsimonious**: Fewer parameters than dummy variables
2. **Smooth**: Captures smooth seasonal patterns naturally
3. **Flexible**: Number of harmonics k controls complexity
4. **Non-integer periods**: Can handle $s = 365.25$ for daily data

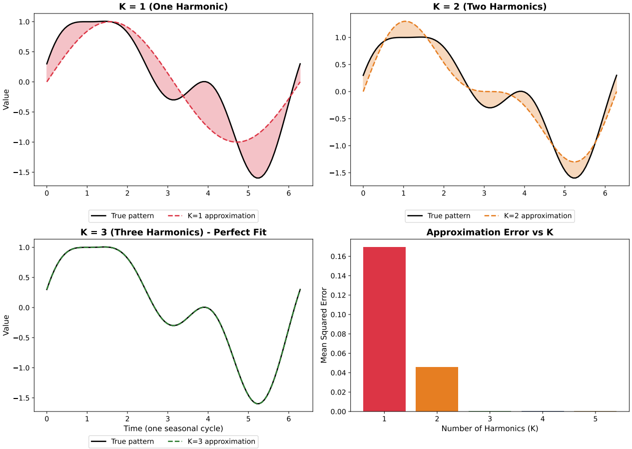
Low k (few harmonics)

- Smooth pattern
- Fewer parameters
- May miss sharp peaks

High k (many harmonics)

- Can capture any pattern
- More parameters
- Risk of overfitting

Fourier Approximation of Seasonality



TBATS in Practice

Python Implementation

`tbats` package provides automatic model selection:

- Automatically selects Box-Cox parameter ω
- Chooses number of harmonics k_i for each seasonal period
- Selects ARMA orders (p, q)
- Tests damped vs non-damped trend

Code Example

```
from tbats import TBATS
estimator = TBATS(seasonal_periods=[7, 365.25])
model = estimator.fit(y)
forecast = model.forecast(steps=30)
```

Note: BATS is the simpler version without trigonometric terms (uses traditional seasonal states).

TBATS: Advantages and Limitations

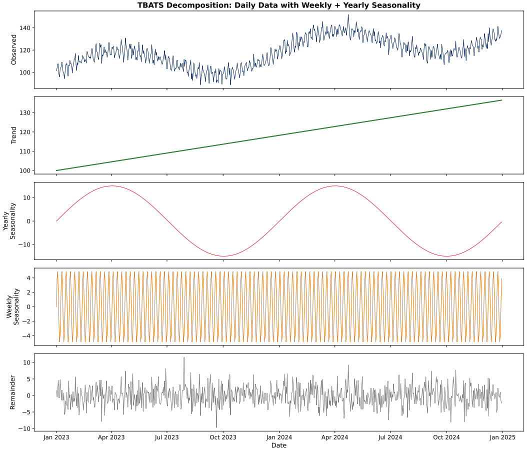
Advantages

- Handles **multiple** seasonal periods
- **Automatic** model selection
- Handles **non-integer** periods (365.25)
- **Box-Cox** for heteroskedasticity
- Good for **high-frequency** data

Limitations

- **Computationally intensive**
- No **external regressors**
- Less **interpretable** than Prophet
- Can be **slow** for very long series
- Requires **sufficient data** per season

TBATS Decomposition Example



Prophet: Overview

What is Prophet?

Prophet is a forecasting procedure developed by Facebook (Meta) in 2017.

Designed for **business time series** with:

- Strong seasonal effects (daily, weekly, yearly)
- Holiday effects
- Trend changes (changepoints)
- Missing data and outliers

Key Philosophy

"Analyst-in-the-loop" forecasting

Prophet is designed to be tuned by analysts who have domain knowledge but may not be time series experts.

Prophet Model Structure

Decomposition Approach

Prophet uses an **additive decomposition**:

$$y(t) = g(t) + s(t) + h(t) + \varepsilon_t$$

$g(t)$: Trend

- Linear or logistic
- Automatic changepoints
- Growth saturation

$s(t)$: Seasonality

- Fourier series
- Multiple periods
- Custom seasonality

$h(t)$: Holidays

- Country holidays
- Custom events
- Window effects

Prophet: Trend Component

Linear Trend with Changepoints

$$g(t) = (k + \mathbf{a}(t)^T \delta) \cdot t + (m + \mathbf{a}(t)^T \gamma)$$

where:

- k is the base growth rate
- δ is a vector of rate adjustments at changepoints
- $\mathbf{a}(t)$ indicates which changepoints are active at time t
- m is the offset, γ ensures continuity

Logistic Growth (for saturating trends)

$$g(t) = \frac{C(t)}{1 + \exp(-(k + \mathbf{a}(t)^T \delta)(t - (m + \mathbf{a}(t)^T \gamma)))}$$

where $C(t)$ is the (possibly time-varying) carrying capacity.

Prophet: Seasonality Component

Fourier Series Representation

For a seasonal period P, Prophet uses:

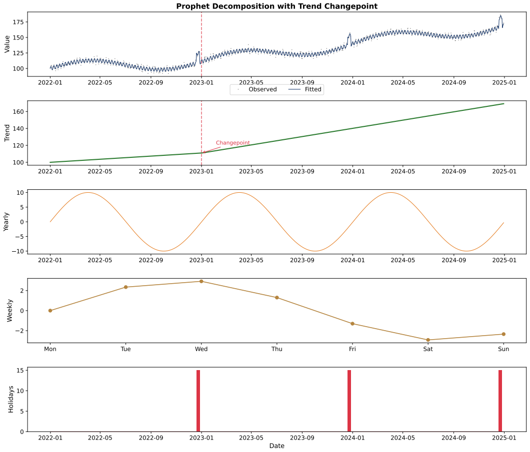
$$s(t) = \sum_{n=1}^{N} \left[a_n \cos\left(\frac{2\pi nt}{P}\right) + b_n \sin\left(\frac{2\pi nt}{P}\right) \right]$$

Default Settings

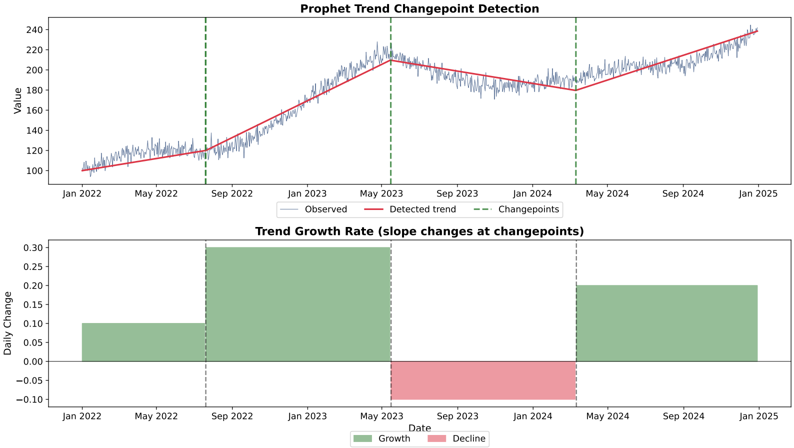
Seasonality	Period	Fourier Order
Yearly	365.25 days	10
Weekly	7 days	3
Daily	1 day	4

Higher Fourier order N = more flexibility (can fit more complex patterns) but higher risk of overfitting.

Prophet Component Decomposition



Trend Changepoint Detection



Prophet: Holiday Effects

Holiday Model

$$h(t) = Z(t) \cdot \boldsymbol{\kappa}$$

where $Z(t)$ is an indicator matrix for holidays and $\boldsymbol{\kappa}$ are holiday effects.

Features

- **Built-in holidays**: 60+ countries supported
- **Custom holidays**: Add your own events (Black Friday, company events)
- **Window effects**: Holidays can affect days before/after
- **Prior scale**: Control regularization of holiday effects

Code Example

```
holidays = pd.DataFrame({'holiday': 'black_friday', ....})
model = Prophet(holidays=holidays)
```

Prophet in Practice

Basic Usage

```
from prophet import Prophet
import pandas as pd

# Data must have columns 'ds' (date) and 'y' (value)
df = pd.DataFrame({'ds': dates, 'y': values})

model = Prophet()
model.fit(df)

future = model.make_future_dataframe(periods=365)
forecast = model.predict(future)
```

Adding Custom Seasonality

```
model = Prophet(weekly_seasonality=False)
model.add_seasonality(name='monthly', period=30.5, fourier_order=5)
model.add_seasonality(name='quarterly', period=91.25, fourier_order=3)
```

Prophet: Uncertainty Quantification

Three Sources of Uncertainty

1. **Trend uncertainty**: Future changepoints are uncertain
2. **Seasonality uncertainty**: Parameter estimation uncertainty
3. **Observation noise**: Inherent randomness

Prediction Intervals

Prophet provides:

- Point forecast: `yhat`
- Lower bound: `yhat_lower`
- Upper bound: `yhat_upper`

Default is 80% interval.
Change with `interval_width=0.95`

Note

Uncertainty grows with forecast horizon, especially for trend uncertainty.

Prophet: Tuning Parameters

Key Parameters

Parameter	Effect
`changepoint_prior_scale`	Trend flexibility (default: 0.05)
`seasonality_prior_scale`	Seasonality flexibility (default: 10)
`holidays_prior_scale`	Holiday effect size (default: 10)
`seasonality_mode`	'additive' or 'multiplicative'
`changepoint_range`	Portion of history for changepoints

Practical Tips

- **Overfitting trend?** Decrease `changepoint_prior_scale`
- **Underfitting seasonality?** Increase `seasonality_prior_scale`
- **Seasonal amplitude varies?** Use `seasonality_mode='multiplicative'`

Prophet: Advantages and Limitations

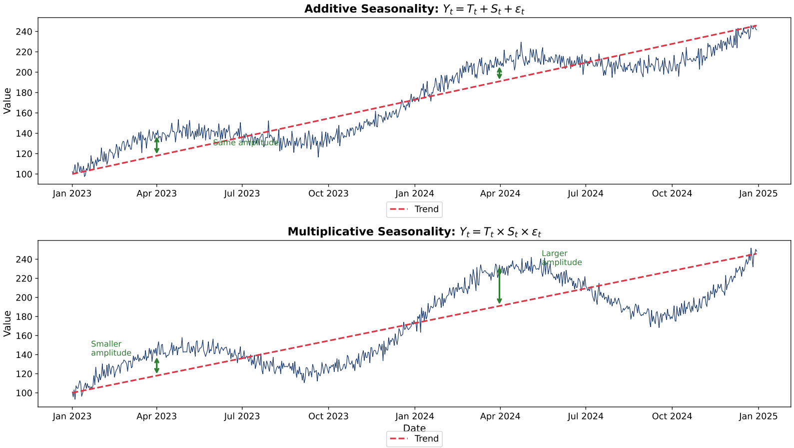
Advantages

- **Easy to use**: Minimal tuning needed
- **Interpretable**: Clear decomposition
- **Handles missing data** well
- **Holiday effects** built-in
- **Multiple seasonalities**
- **External regressors** supported
- **Fast** fitting

Limitations

- **Not ARIMA-based**: No autocorrelation modeling
- **Daily data focus**: Less suited for very high frequency
- **Trend assumptions**: Linear/logistic may not fit
- **No built-in CV**: Must implement manually
- **Overfitting risk** with many seasonalities

Additive vs Multiplicative Seasonality



TBATS vs Prophet: Head-to-Head

Feature	TBATS	Prophet
Multiple seasonalities	Yes (automatic)	Yes (manual or auto)
Holiday effects	No	Yes (built-in)
External regressors	No	Yes
Trend changepoints	No (smooth)	Yes (automatic)
Missing data	Interpolation needed	Handles natively
Interpretability	Moderate	High
Computation speed	Slow	Fast
High-frequency data	Good	Moderate
Non-integer periods	Yes (e.g., 365.25)	Yes
Uncertainty intervals	Yes	Yes

When to Use Each Model

Use TBATS when:

- High-frequency data (hourly, sub-daily)
- Multiple complex seasonal periods
- No external regressors needed
- Automatic model selection preferred
- Traditional state-space framework desired

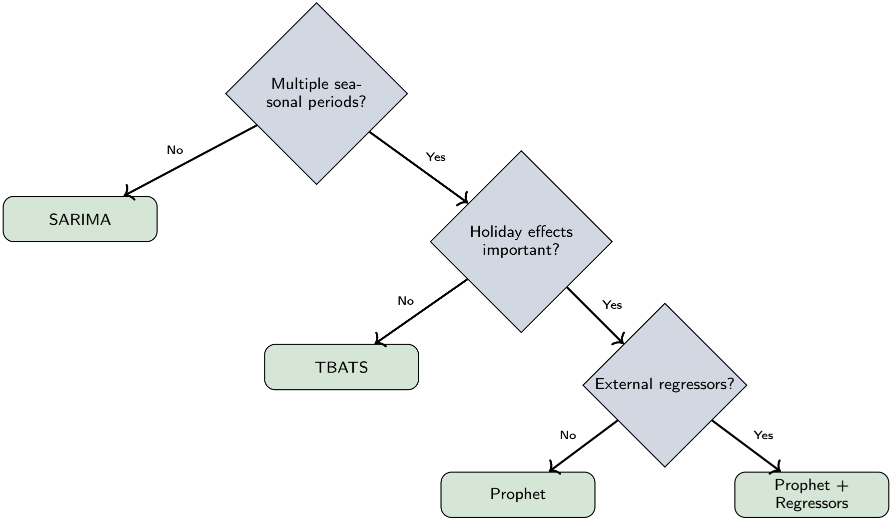
Use Prophet when:

- Business forecasting (daily/weekly)
- Holiday effects are important
- Trend has structural breaks
- Missing data present
- Interpretability is key
- External regressors available

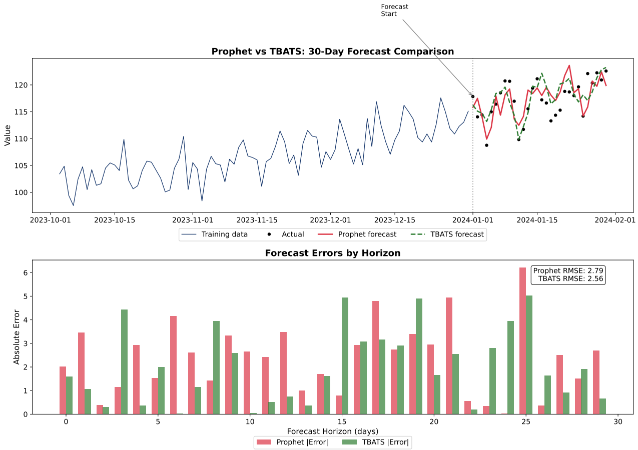
General Guideline

Prophet for business applications with daily data
TBATS for technical applications with high-frequency data

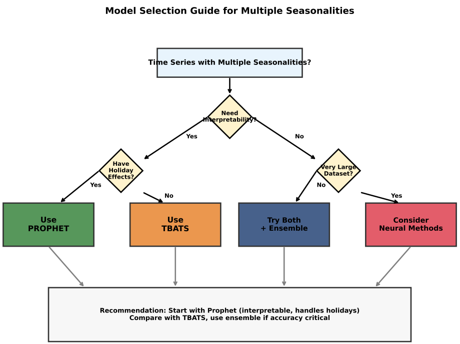
Decision Flowchart



Prophet vs TBATS: Forecast Comparison



Model Selection Guide



Case Study: Energy Demand Forecasting

Problem

Forecast hourly electricity demand with:

- **Daily pattern**: Peak at noon and evening
- **Weekly pattern**: Lower on weekends
- **Annual pattern**: Higher in summer (AC) and winter (heating)
- **Holiday effects**: Lower demand on holidays

Approach

1. Try TBATS with periods [24, 168, 8766]
2. Try Prophet with daily, weekly, yearly seasonality + holidays
3. Compare using cross-validation

Case Study: Results Interpretation

Evaluation Metrics

- **MAPE**: Mean Absolute Percentage Error
- **RMSE**: Root Mean Square Error
- **Coverage**: % of actuals within prediction interval

Typical Findings

Model	MAPE	RMSE	Coverage
SARIMA (daily only)	8.5%	450 MW	75%
TBATS	4.2%	220 MW	82%
Prophet	4.8%	250 MW	85%
Prophet + holidays	3.9%	200 MW	88%

Multiple seasonality models significantly outperform single-seasonality SARIMA.

Key Takeaways

Multiple Seasonalities

- Real-world data often has multiple seasonal patterns
- Standard SARIMA handles only one seasonal period
- TBATS and Prophet are designed for this challenge

Model Selection

- **TBATS**: Automatic, handles high-frequency, no external regressors
- **Prophet**: Interpretable, holiday effects, external regressors
- Both use Fourier terms for efficient seasonality representation

Remember

Always validate with proper time series cross-validation!

Questions?

Next Steps:

- Practice with the Jupyter notebook
- Try Prophet on your own data
- Explore NeuralProphet for deep learning extension