Time Series Analysis and Forecasting

Seminar 8: Modern Extensions

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Seminar Outline

- **Multiple Choice Quiz** – Knowledge check
- **True/False** – Conceptual checks
- **Calculation Exercises** – Applied practice
- **AI-Assisted Exercise** – Critical thinking
- **Summary** – Key takeaways

Quiz 1: Hurst Exponent

Question

A time series has Hurst exponent $H = 0.8$. What does this indicate?

Answer choices

(A) The series is a pure random walk

(B) The series has long memory and is persistent (trend-following)

(C) The series is anti-persistent (mean-reverting)

(D) The series is stationary I(0)

Answer on next slide...

Quiz 1: Answer

Answer: B – Long memory and persistence

Answer choices

(A) The series is a pure random walk ×

(B) **The series has long memory and is persistent (trend-following)** ✓

(C) The series is anti-persistent (mean-reverting) ×

(D) The series is stationary I(0) ×

- $H = 0.5$: Random walk (no memory)
- $0.5 < H < 1$: **Persistence** – trend continues
- $0 < H < 0.5$: Anti-persistence – mean reversion
- With $H = 0.8 > 0.5$: large values tend to be followed by large values

TSA_ch8_hurst_interpretation

Quiz 2: Fractional Differencing Parameter

Question

In the ARFIMA(p, d, q) model, the parameter d can take values:

Answer choices

(A) Only integer values (0, 1, 2, ...)

(B) Only $d = 0$ or $d = 1$

(C) Any real value, including fractional

(D) Only negative values

Answer on next slide...

Quiz 2: Answer

Answer: C – Any real value

Answer choices

(A) Only integer values (0, 1, 2, ...) ×

(B) Only $d = 0$ or $d = 1$ ×

(C) **Any real value, including fractional** ✓

(D) Only negative values ×

- **Fractional differencing**: $(1 - L)^d$ with $d \in \mathbb{R}$
- $d = 0$: Stationary series (ARMA); $0 < d < 0.5$: Long memory, stationary
- $d = 0.5$: Stationary/non-stationary boundary; $d = 1$: Full differencing (classic ARIMA)
- **Relation to Hurst**: $d = H - 0.5$

TSA_ch8_arfima_estimation

Quiz 3: Long Memory in Financial Series

Question

In which financial series is long memory most commonly documented?

Answer choices

(A) Stock prices
(B) Daily returns
(C) Volatility (squared returns)
(D) Trading volume

Answer on next slide...

Quiz 3: Answer

Answer: C – Volatility

Answer choices

- **(A)** Stock prices ×
- **(B)** Daily returns ×
- **(C)** **Volatility (squared returns)** ✓
- **(D)** Trading volume ×

- **Returns**: Approximately memoryless ($H \approx 0.5$)
- **Volatility**: Pronounced long memory ($H \approx 0.7 - 0.9$)
- Volatility clustering: turbulent periods followed by turbulent periods
- This stylized fact is the basis for FIGARCH and HAR-RV models

TSA_ch8_volatility_long_memory

Quiz 4: Feature Engineering

Question

To apply Random Forest to time series, we must create:

Answer choices

(A) Dummy variables for each observation

(B) Lag features and rolling statistics

(C) Fourier transforms of the series

(D) Only the first difference of the series

Answer on next slide...

Quiz 4: Answer

Answer: B – Lag features and rolling statistics

Answer choices

(A) Dummy variables for each observation ×

(B) **Lag features and rolling statistics** ✓

(C) Fourier transforms of the series ×

(D) Only the first difference of the series ×

- **Lag features**: $y_{t-1}, y_{t-2}, \ldots, y_{t-k}$
- **Rolling statistics**: rolling mean, rolling std, min/max over window
- **Calendar features**: day of week, month, etc.
- Transforms the forecasting problem into a **supervised regression** problem

TSA_ch8_model_predictions

Quiz 5: Time Series Cross-Validation

Question

Why can't we use standard k-fold cross-validation for time series?

Answer choices

(A) It's too slow for long series

(B) It violates temporal order and causes data leakage

(C) It only works for classification

(D) It requires too much data

Answer on next slide...

Quiz 5: Answer

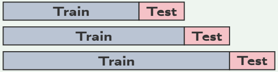
Answer: B – Violates temporal order

Answer choices

(A) It's too slow for long series ×
(B) **It violates temporal order and causes data leakage** ✓
(C) It only works for classification ×
(D) It requires too much data ×

- Shuffles observations temporally $\Rightarrow$ trains on future, tests on past
- **Data leakage** $\Rightarrow$ overestimated performance

Solution: Time Series Split (Walk-Forward)



TSA_ch8_timeseries_cv

Quiz 6: Feature Importance in Random Forest

Question

Feature importance in Random Forest for time series helps us:

Answer choices

(A) Eliminate all low-importance variables

(B) Identify which lags and features are most predictive

(C) Determine Granger causality

(D) Calculate confidence intervals

Answer on next slide...

Quiz 6: Answer

Answer: B – Identifies predictive features

Answer choices

(A) Eliminate all low-importance variables ×

(B) **Identify which lags and features are most predictive** ✓

(C) Determine Granger causality ×

(D) Calculate confidence intervals ×

- Understanding temporal structure and selecting optimal number of lags
- Importance does **NOT** imply causality
- Correlated variables may share importance
- Use for interpretation, not causal inference

TSA_ch8_case_feature_importance

Quiz 7: LSTM Advantage

Question

What is the main advantage of LSTM over simple RNNs?

Answer choices

(A) It's faster to train

(B) It solves the vanishing/exploding gradient problem

(C) It requires less data

(D) It's easier to interpret

Answer on next slide...

Quiz 7: Answer

Answer: B – Solves the gradient problem

Answer choices

(A) It's faster to train ×

(B) **It solves the vanishing/exploding gradient problem** ✓

(C) It requires less data ×

(D) It's easier to interpret ×

- Simple RNN: gradients decay exponentially, cannot learn long-term dependencies
- LSTM gates: **Forget gate** (what to forget), **Input gate** (what to remember), **Output gate** (what to output)
- **Cell state**: highway for information flow – gradients can flow without degradation

TSA_ch8_lstm_architecture

Quiz 8: Data Preparation for LSTM

Question

Before training an LSTM, data should be:

Answer choices

(A) Log-transformed

(B) Normalized/scaled to [0,1] or [-1,1]

(C) Differenced twice

(D) Converted to integers

Answer on next slide...

Quiz 8: Answer

Answer: B – Normalized/scaled

Answer choices

(A) Log-transformed ×

(B) **Normalized/scaled to [0,1] or [-1,1]** ✓

(C) Differenced twice ×

(D) Converted to integers ×

- Activation functions (sigmoid, tanh) work in limited ranges
- **Min-Max**: $x' = \frac{x - x_{min}}{x_{max} - x_{min}} \succ [0,\ 1]$
- **Standard**: $x' = \frac{x - \mu}{\sigma} \succ$ mean 0, std 1
- **Important**: Fit on train, transform on train+test!

TSA_ch8_data_split

Quiz 9: LSTM Hyperparameters

Question

Which is NOT a typical LSTM hyperparameter?

Answer choices

(A) Number of units (neurons) per layer

(B) Input sequence length

(C) Learning rate

(D) Differencing parameter d

Answer on next slide...

Quiz 9: Answer

Answer: D – The d parameter

Answer choices

(A) Number of units (neurons) per layer ×
(B) Input sequence length ×
(C) Learning rate ×
(D) **Differencing parameter d** ✓

- d is specific to ARFIMA models, not LSTM!
- **Architecture**: number of layers, units/layer
- **Sequence**: lookback length
- **Training**: learning rate, batch size, epochs
- **Regularization**: dropout, early stopping

TSA_ch8_case_lstm_training

True or False? — Questions

Statement	**T/F?**
1. ARFIMA models can capture long-term dependence.	?
2. The parameter d in ARFIMA must be an integer.	?
3. LSTM networks are better than ARIMA in all situations.	?
4. Random Forest requires manually created features.	?
5. Standard cross-validation (k-fold) is suitable for time series.	?
6. The Hurst exponent $H > 0.5$ indicates positive long memory.	?

True or False? — Answers

Statement	T/F	Explanation
1. ARFIMA captures long-term dependence.	T	**Fractional d**
2. d in ARFIMA must be an integer.	F	$d \in (0, 0.5)$ **fractional**
3. LSTM better than ARIMA always.	F	**Depends on data and sample**
4. RF requires manual features.	T	**Lags, calendar, etc.**
5. k-fold CV suitable for time series.	F	**Violates temporal ordering**
6. $H > 0.5$ indicates positive long memory.	T	$H = 0.5$: **no memory**

Exercise 1: Hurst Exponent Estimation

Problem

- **Task**: Given daily Bitcoin returns, estimate the Hurst exponent using the R/S method and interpret the result.
- **Formula**: $\log(R/S) = H \cdot \log(n) + c$

Solution

1. Calculate mean over subintervals of different lengths n
2. For each n: calculate Range(R) and Std(S)
3. The ratio R/S grows as n^H
4. Fit regression: $\log(R/S) = H \cdot \log(n) + c$

Python code: `nolds.hurst_rs(returns)`

TSA_ch8_hurst_interpretation

Exercise 1: Solution and Interpretation

Typical Bitcoin Results

- Returns: $H \approx 0.45 - 0.55$ (approximately random walk)
- Volatility ($|$returns$|$): $H \approx 0.75 - 0.85$ (long memory!)

- Returns are hard to predict (EMH approximately valid)
- Volatility is predictable over long horizons
- Implications for risk management and VaR

Application: FIGARCH models may outperform standard GARCH

TSA_ch8_hurst_interpretation

Exercise 2: Random Forest for Forecasting

Problem

- **Task**: Build a Random Forest model for 1-day ahead Bitcoin price forecasting. Evaluate using TimeSeriesSplit.

Solution Pipeline

1. **Feature engineering**:
   - Lags: $y_{t-1}, y_{t-2}, \ldots, y_{t-7}$
   - Rolling mean/std: 7, 14, 30 days
2. **Train/Test split**: TimeSeriesSplit(n_splits=5)
3. **Model**: RandomForestRegressor(n_estimators=100)
4. **Evaluation**: RMSE, MAE, Direction Accuracy

TSA_ch8_feature_engineering

Exercise 2: Code and Results

Python Code

```
from sklearn.ensemble import RandomForestRegressor
from sklearn.model_selection import TimeSeriesSplit

tscv = TimeSeriesSplit(n_splits=5)
rf = RandomForestRegressor(n_estimators=100)

for train_idx, test_idx in tscv.split(X):
rf.fit(X[train_idx], y[train_idx])
pred = rf.predict(X[test_idx])
```

- Direction accuracy: 52-55% (slightly above random)
- Feature importance: lag-1 and rolling_std dominate

TSA_ch8_rf_prediction

Exercise 3: LSTM for Time Series

Problem

- **Task**: Implement a simple LSTM model for Bitcoin forecasting. Compare with Random Forest.

Simple LSTM Architecture

1. Input: 30-day sequences
2. LSTM layer: 50 units
3. Dense output: 1 neuron (forecast)
4. Loss: MSE, Optimizer: Adam

Important steps:

- MinMaxScaler normalization
- Reshape to [samples, timesteps, features]
- Early stopping to prevent overfitting

TSA_ch8_lstm_architecture

Exercise 3: LSTM Code

Keras/TensorFlow Code

```
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import LSTM, Dense

model = Sequential([
LSTM(50, input_shape=(30, 1)),
Dense(1)
])

model.compile(optimizer='adam', loss='mse')
model.fit(X_train, y_train, epochs=50,
validation_split=0.1, verbose=0)
```

- RMSE similar (LSTM slightly better on smooth data)
- RF: faster, more interpretable
- LSTM: captures complex patterns better

TSA_ch8_lstm_architecture

AI Exercise: Critical Thinking

Prompt to test in ChatGPT / Claude / Copilot

"Download hourly electricity consumption data. Compare three approaches: (1) ARIMA, (2) Random Forest with lag features, (3) LSTM. Use time series cross-validation, report RMSE and directional accuracy for each."

Exercise:

1. Did the AI use proper time series cross-validation (no future data leakage)?
2. Are the lag features for Random Forest correctly constructed?
3. Is the LSTM data properly scaled and sequenced?
4. Does the comparison use the same test set for all three models?
5. Which model performs best and why? Is this consistent with theory?

Warning: AI-generated code may run without errors and look professional. *That does not mean it is correct.*

TSA_ch8_case_comparison

Sum

ARFIMA

- Series with long memory (volatility, hydrology)
- When $0 < d < 0.5$ is theoretically justified
- Statistical interpretability is important

Random Forest

- Nonlinear relationships between features
- Feature importance for understanding
- Structured data, not too long series

LSTM

- Long sequences with complex dependencies
- Sufficient data for deep learning
- Patterns difficult to capture with classical methods

TSA_ch8_case_comparison

Key

ARFIMA and Long Memory

- Fractional differencing: $(1-L)^d y_t = \varepsilon_t$
- Hurst exponent: $d = H - 0.5$
- ACF for long memory: $\rho(k) \sim k^{2d-1}$ (slow decay)

Machine Learning

- Lag feature: $X_t = [y_{t-1}, y_{t-2}, \ldots, y_{t-k}]$
- RMSE: $\sqrt{\frac{1}{n}\sum(y_i - \hat{y}_i)^2}$
- Direction Accuracy: $\frac{1}{n}\sum 1[\text{sign}(\Delta y) = \text{sign}(\Delta \hat{y})]$

LSTM

- Forget gate: $f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$
- Cell update: $C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$

TSA_ch8_arfima_d_effect

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Online Resources and Code

- **Quantlet**: `https://quantlet.com` — Code repository for statistics
- **Quantinar**: `https://quantinar.com` — Quantitative methods learning platform
- **GitHub TSA**: `https://github.com/QuantLet/TSA/tree/main/TSA_ch8` — Python code for this seminar

Thank You!

Questions?

Seminar materials are available at: https://danpele.github.io/Time-Series-Analysis/

Quantlet Quantinar