Time Series Analysis and Forecasting

Chapter 10: Comprehensive Review

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Learning Objectives

By the end of this chapter, you will be able to:

- Apply the complete forecasting workflow from data to evaluation
- Select appropriate models based on data characteristics
- Evaluate forecast accuracy using proper metrics and cross-validation
- Integrate knowledge from all previous chapters in practice

Outline

The Scientific Approach to Forecasting

Research Question

How do we **rigorously evaluate** forecast performance while avoiding overfitting?

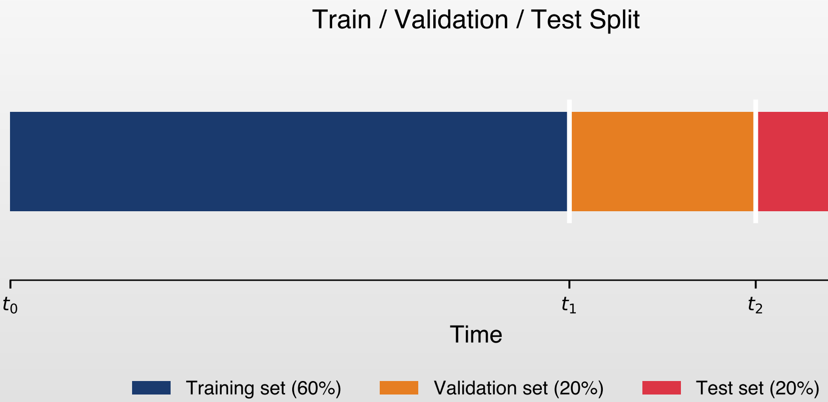
The Fundamental Problem

- In-sample fit $\neq$ Out-of-sample performance
- Models can "memorize" training data without learning patterns
- **Solution**:
  - Proper train/validation/test methodology

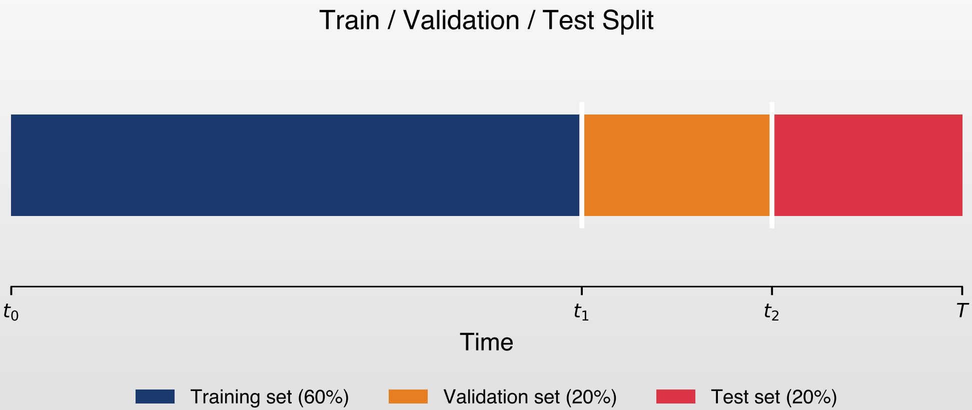
Key Principle

"The test set must remain **untouched** until final evaluation."
— Standard practice in machine learning and econometrics

Train/Validation/Test Framework



Train/Validation/Test Framework



TSA_ch10_train_val_test_split

Evaluation Metrics

Definition 1 (Forecast Error Metrics)

Let y_t be actual, $\hat{y}_t$ forecast:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_t (y_t - \hat{y}_t)^2}, \quad \text{MAE} = \frac{1}{n}\sum_t |y_t - \hat{y}_t|, \quad \text{MAPE} = \frac{100\%}{n}\sum_t \left|\frac{y_t - \hat{y}_t}{y_t}\right|$$

When to Use Each

- **RMSE**: Penalizes large errors
- **MAE**: Robust to outliers
- **MAPE**: Scale-independent (%)

Caution

- MAPE undefined when $y_t = 0$
- Compare on **same** test set
- Report **out-of-sample** metrics

Bitcoin: Problem Statement

Research Question

Can we forecast Bitcoin's **volatility** using GARCH models?

Data Characteristics

- Source: Yahoo Finance (BTC-USD)
- Period: Jan 2019 – Jan 2025
- Frequency: Daily
- Observations: $\approx 2,200$ days

Stylized Facts

- Returns: near-zero mean
- Fat tails (kurtosis > 3)
- Volatility clustering

Key Insight

Financial returns are typically:

- **Unpredictable** in mean
- **Predictable** in variance

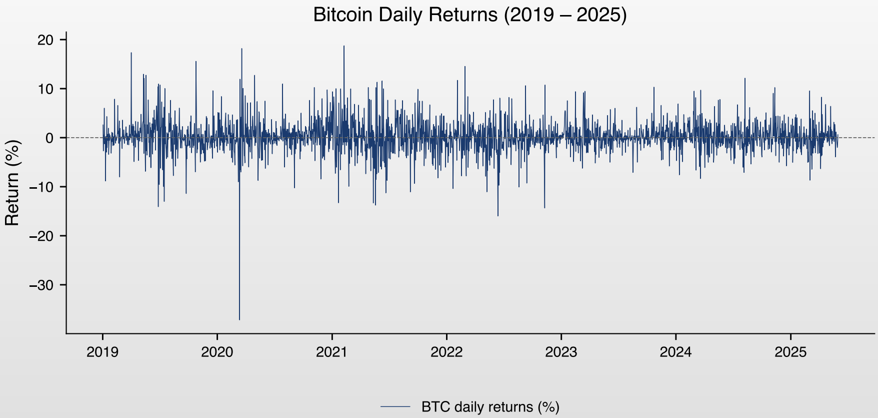
$\Rightarrow$ Focus on **volatility forecasting**

Bitcoin: Volatility Clustering

Observation

- Large returns follow large returns, small follow small—**volatility clustering**

Bitcoin: Volatility Clustering



TSA_ch10_btc_returns

Bitcoin: Evidence for GARCH

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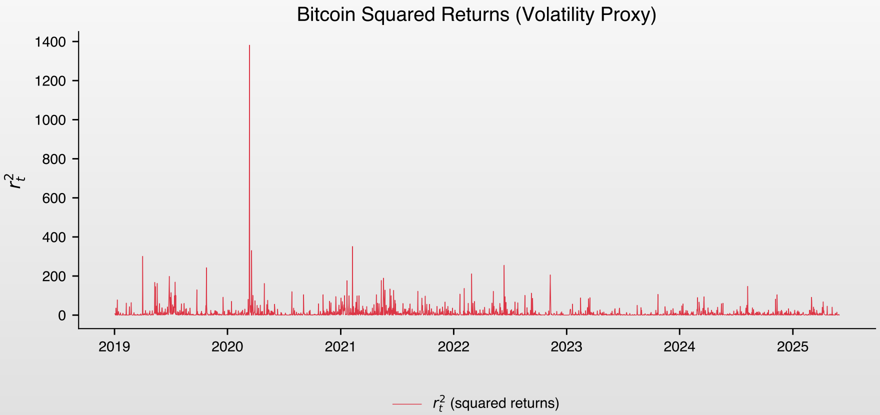


TSA_ch10_btc_acf_squared

GARCH Model Specification

Definition 2 (GARCH(p,q) Model)

Let r_t denote returns. The GARCH(p,q) model:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t \overset{iid}{\sim} N(0,1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

where $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$.

Model Variants

- **GARCH(1,1)**: Most common
- **GJR-GARCH**: Leverage effect
- **EGARCH**: Log-variance, asymmetric

Interpretation

- α: Shock impact (ARCH effect)
- β: Volatility persistence
- $\alpha + \beta \approx 1$: High persistence

GARCH: Stationarity and Unconditional Variance

Theorem 1 (Covariance Stationarity of GARCH(1,1))

If $\alpha_1 + \beta_1 < 1$, then $\{\varepsilon_t\}$ is covariance stationary with:

$$\bar{\sigma}^2 = \mathbb{E}[\sigma_t^2] = \frac{\omega}{1 - \alpha_1 - \beta_1}$$

Derivation

Take expectations of both sides of the variance equation:

$$\begin{aligned} \mathbb{E}[\sigma_t^2] &= \omega + \alpha_1 \mathbb{E}[\varepsilon_{t-1}^2] + \beta_1 \mathbb{E}[\sigma_{t-1}^2] \\ \bar{\sigma}^2 &= \omega + (\alpha_1 + \beta_1)\bar{\sigma}^2 \quad \text{(stationarity)} \\ \bar{\sigma}^2 &= \frac{\omega}{1 - \alpha_1 - \beta_1} \end{aligned}$$

Multi-Step Forecasts Converge to $\bar{\sigma}^2$

Bitcoin: Data Split and Stationarity

Data Split

Set	Period	N
Training (70%)	2019-01 to 2023-03	1,543
Validation (20%)	2023-03 to 2024-06	441
Test (10%)	2024-06 to 2025-01	221
Total		**2,205**

Stationarity Tests

Series	ADF	Result
Prices	$p = 0.50$	Non-stationary
Returns	$p < 0.01$	Stationary

$\Rightarrow$ Model **returns**, not prices

Why Stationarity Matters

GARCH requires weakly stationary input. Prices follow random walk; returns are stationary.

Bitcoin: Model Selection on Validation Set

Methodology

Fit each model on **training data**, evaluate on **validation set**.

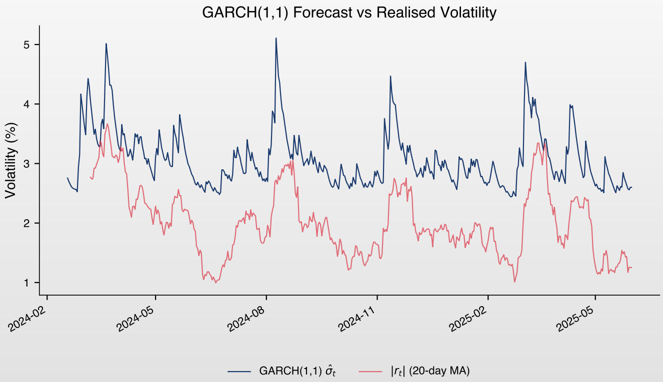
Model	AIC	BIC	Val MAE	Selection
GARCH(1,1)	6,994.8	7,020.6	**2.638**	**Best**
GARCH(2,1)	6,993.7	7,024.6	2.640	
GJR-GARCH(1,1)	6,983.7	7,014.6	2.669	
EGARCH(1,1)	—	—	—	Failed*

*Analytic forecasts not available for $h > 1$

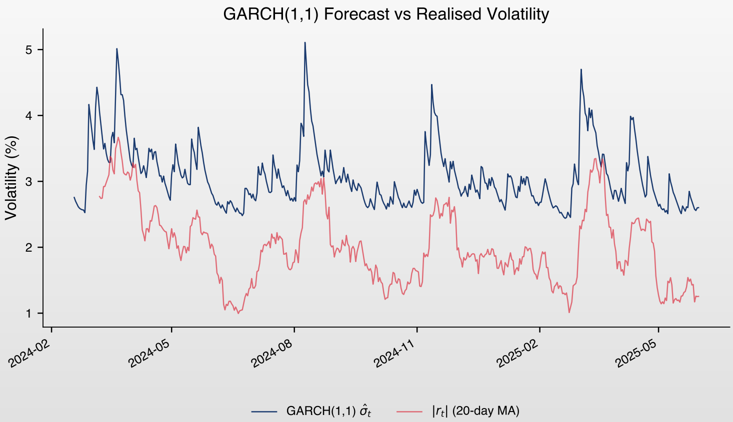
Result

GARCH(1,1) selected based on lowest validation MAE for volatility forecasts.

Bitcoin: Final Test Set Evaluation



Bitcoin: Final Test Set Evaluation



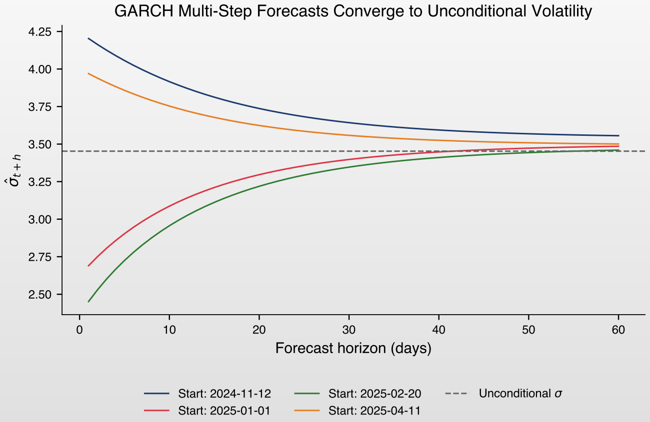
TSA_ch10_garch_forecast

GARCH: Multi-Step Forecasts Converge

Key Insight

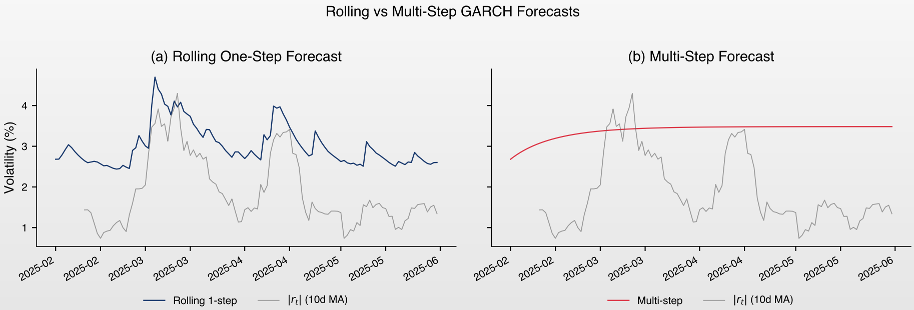
- Multi-step forecasts converge to $\bar{\sigma}^2 = \frac{\omega}{1-\alpha-\beta}$
- Use rolling forecasts

GARCH: Multi-Step Forecasts Converge

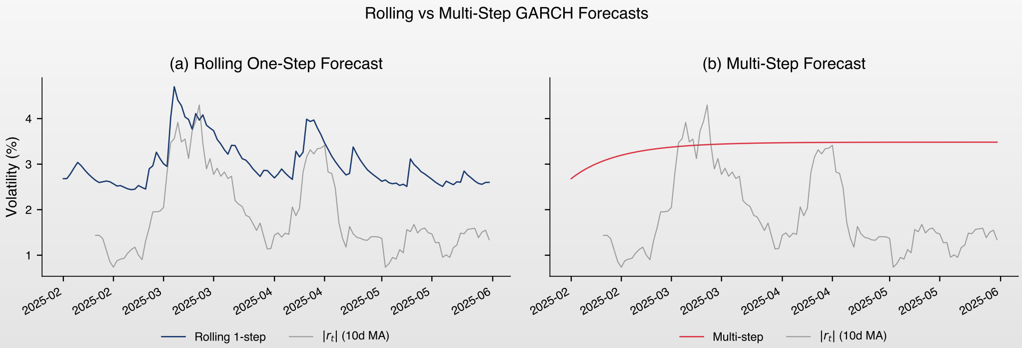


TSA_ch10_garch_convergence

GARCH: Rolling One-Step-Ahead Solution



GARCH: Rolling One-Step-Ahead Solution



TSA_ch10_rolling_vs_multistep

Bitcoin: Key Findings

Summary

1. **Returns are stationary**; prices are not
2. **GARCH(1,1)** outperforms more complex variants
3. **High persistence** ($\alpha + \beta = 0.93$)
4. Volatility is **predictable** even when returns are not

Practical Implications

- Risk management: VaR, Expected Shortfall
- Option pricing requires volatility forecasts
- Portfolio optimization with time-varying risk

Limitations

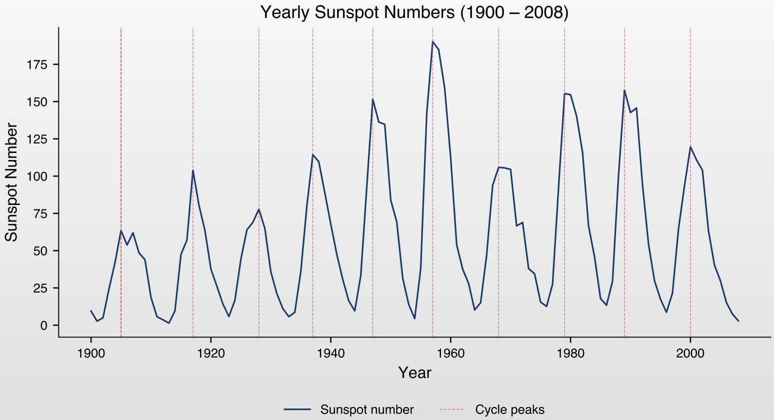
- GARCH assumes **symmetric** shocks
- Does not capture **jumps**
- Normal distribution may be restrictive

Extensions

- Student-t innovations
- Realized volatility
- HAR models

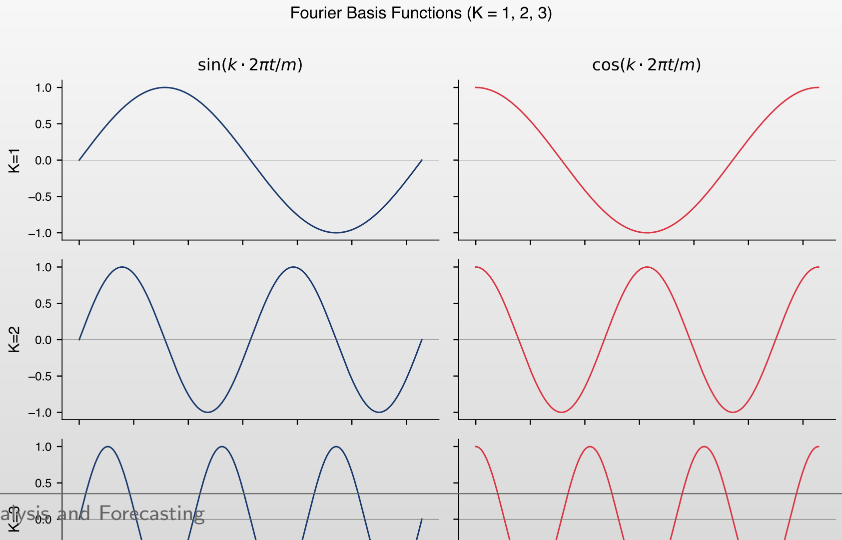
Sunspots: The 11-Year Solar Cycle

Sunspots: The 11-Year Solar Cycle

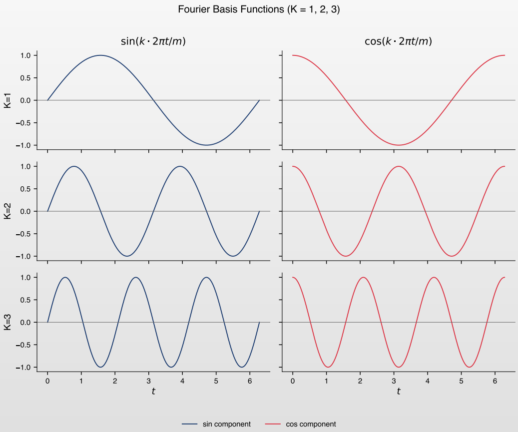


TSA_ch10_sunspots_acf

Fourier Terms for Seasonality



Fourier Terms for Seasonality



TSA_ch10_fourier_terms

Sunspots: Model Selection

Methodology

Compare $K = 1, 2, 3, 4$ Fourier harmonics on validation set.

Data Split

Set	Period	N
Training (70%)	1900–1975	76
Validation (20%)	1976–1997	22
Test (10%)	1998–2008	11
Total		**109**

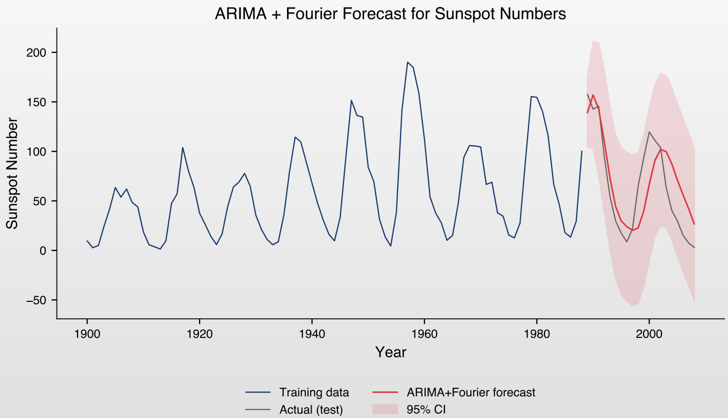
Model Comparison

K	AIC	Val RMSE	
1	665.9	87.15	
2	668.0	86.92	
3	671.8	**86.81**	Best
4	674.5	87.93	

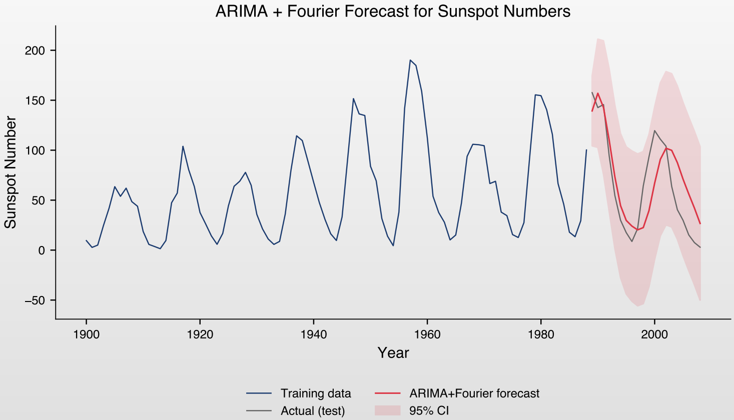
Result

K = 3 Fourier harmonics selected (6 parameters for 11-year cycle).

Sunspots: Forecast Results



Sunspots: Forecast Results



TSA_ch10_sunspot_forecast

Sunspots: Key Takeaways

When to Use Fourier Terms

- Seasonal period s is **long** (e.g., 11 years, 52 weeks)
- SARIMA would require too many seasonal lags
- Pattern is **smooth and periodic**
- Multiple cycles need to be captured

Choosing K

Start with $K = 1$, increase until validation error stops improving. Too high K = overfitting.

Fourier vs SARIMA

	Fourier	**SARIMA**
Long seasons	✓	×
Short seasons	OK	✓
Parameters	$2K$	Many
Flexibility	Fixed	Adaptive

Applications

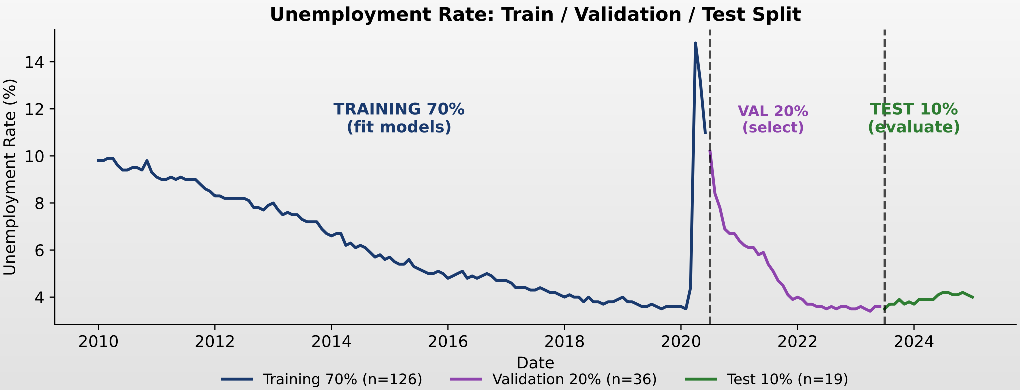
Climate cycles, business cycles, astronomical phenomena

Unemployment: Train / Validation / Test Split

Methodology

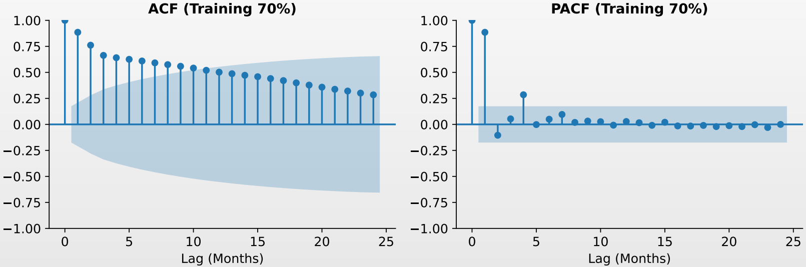
- **Training**: Fit models
- **Validation**: Select best
- **Test**: Final evaluation

Unemployment: Train / Validation / Test Split

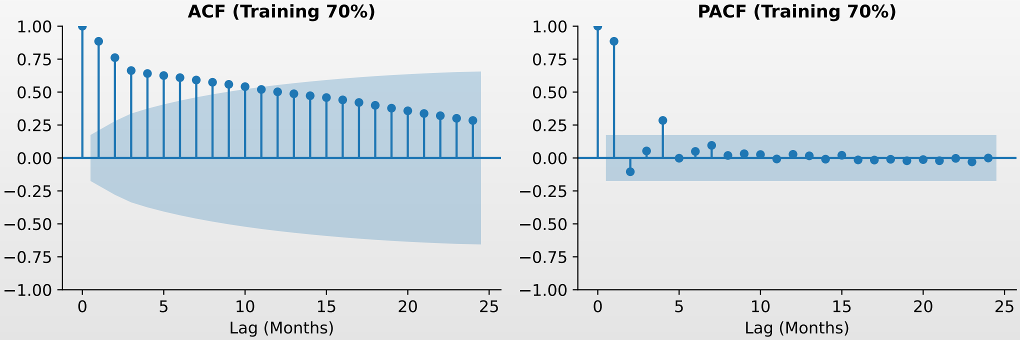


TSA_ch10_unemployment_train_val_test

Unemployment: Preliminary Analysis

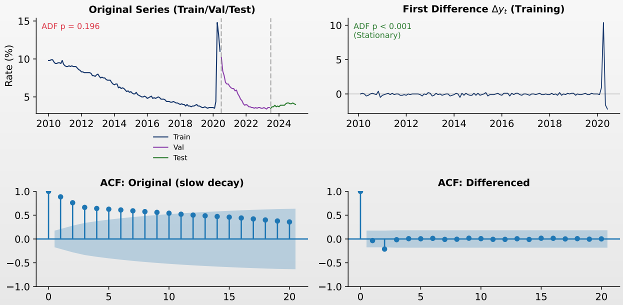


Unemployment: Preliminary Analysis

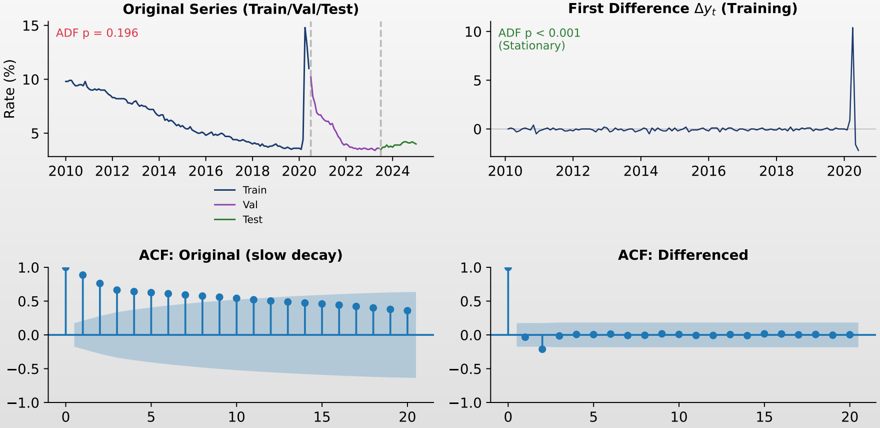


TSA_ch10_unemployment_acf_pacf

Unemployment: Stationarity Tests



Unemployment: Stationarity Tests



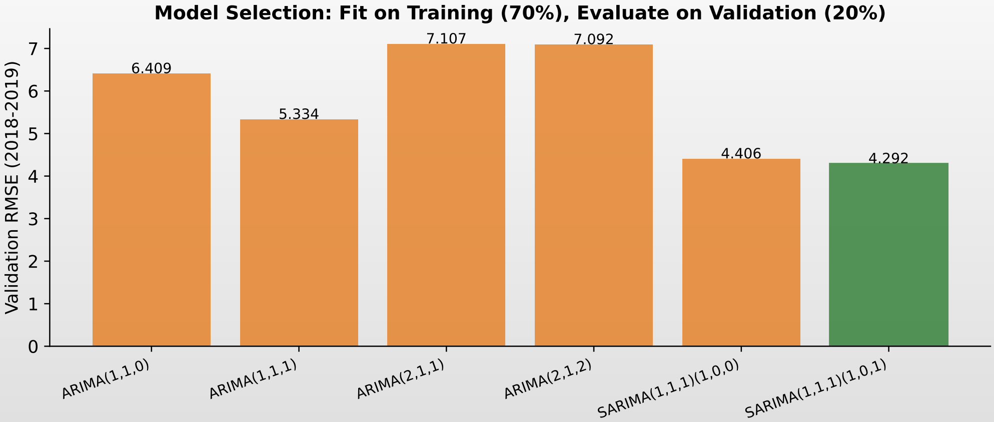
TSA_ch10_unemployment_stationarity

Unemployment: Model Selection (Validation Set)

Best: SARIMA(1,1,1)(1,0,0)$_{12}$

- Selected by lowest validation RMSE

Unemployment: Model Selection (Validation Set)



TSA_ch10_sarima_model_selection

Unemployment: SARIMA Parameters

SARIMA(1,1,1)(1,0,0)$_{12}$ fitted on Train+Val (2010-2019)

- AR(1): $\phi_1 = -0.86$
- MA(1): $\theta_1 = 0.78$
- SAR(12): $\Phi_1 = -0.08$ (n.s.)

Unemployment: SARIMA Parameters

SARIMA(1,1,1)(1,0,1) - Fitted on Train+Val (85%)

Parameter	Coef	Std Err	P-value	Sig
ar.L1	0.8423	0.2084	0.0001	***
ma.L1	-0.9540	0.1973	0.0000	***
ar.S.L12	0.0326	4.5951	0.9943	
ma.S.L12	-0.0113	4.6087	0.9980	
sigma2	0.8122	0.0608	0.0000	***

TSA_ch10_sarima_parameters

Ljung-Box Test for Residual Autocorrelation

Definition 3 (Ljung-Box Test)

For residuals $\hat{\varepsilon}_t$ with sample autocorrelations $\hat{\rho}_k$, the test statistic:

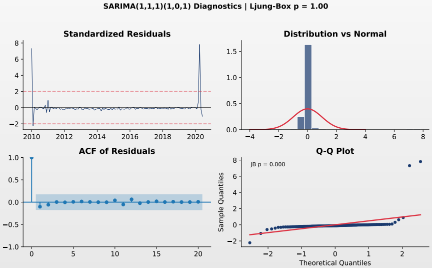
$$Q(h) = n(n+2)\sum_{k=1}^{h}\frac{\hat{\rho}_k^2}{n-k} \overset{H_0}{\sim} \chi^2(h-p-q)$$

where p, q are ARMA orders. H_0: Residuals are white noise.

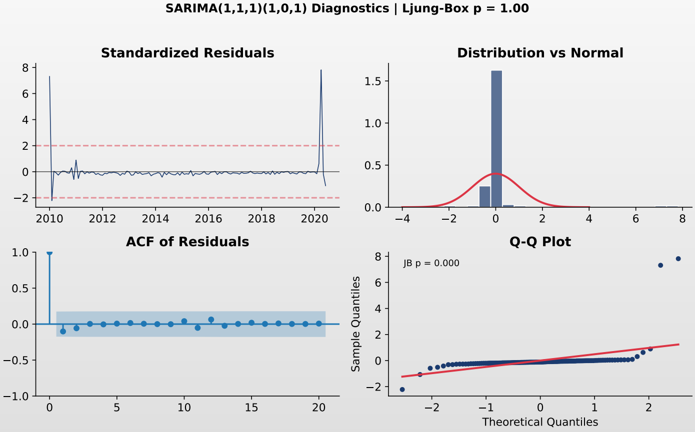
Interpretation

- Large Q (small p-value): Reject H_0, residuals have structure
- Small Q (large p-value): Fail to reject H_0, model is adequate
- Rule of thumb: Use $h = \min(10, n/5)$ for lag order

Unemployment: SARIMA Diagnostics



Unemployment: SARIMA Diagnostics



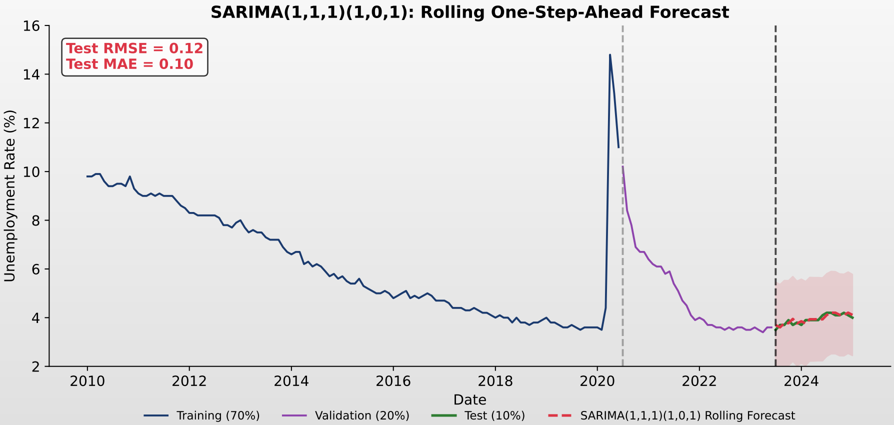
TSA_ch10_sarima_diagnostics

Unemployment: SARIMA Rolling Forecast

Problem: Structural Break

- Rolling one-step-ahead forecast (re-estimate at each t)
- **Test RMSE = 0.12**

Unemployment: SARIMA Rolling Forecast



TSA_ch10_sarima_forecast

Prophet Model

Definition 4 (Prophet Decomposition)

$y_t = g(t) + s(t) + h(t) + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2)$
where $g(t)$ = trend, $s(t)$ = seasonality, $h(t)$ = holidays, σ^2 = noise variance (estimated).

Changepoint Detection

- Automatic location selection
- `changepoint_prior_scale` controls flexibility

Advantages

- Handles missing data
- Interpretable components
- Robust to outliers

Unemployment: Model Tuning

Hyperparameter Tuning

Tune `changepoint_prior_scale` on validation set.

Data Split

Set	Period	N
Training (70%)	2010-01 to 2020-06	126
Validation (20%)	2020-07 to 2023-06	36
Test (10%)	2023-07 to 2025-01	19
Total		**181**

Scale Comparison

Scale	Val RMSE	
0.01	4.21	
0.05	3.89	
0.10	**3.52**	Best
0.30	3.67	
0.50	3.81	

Interpretation

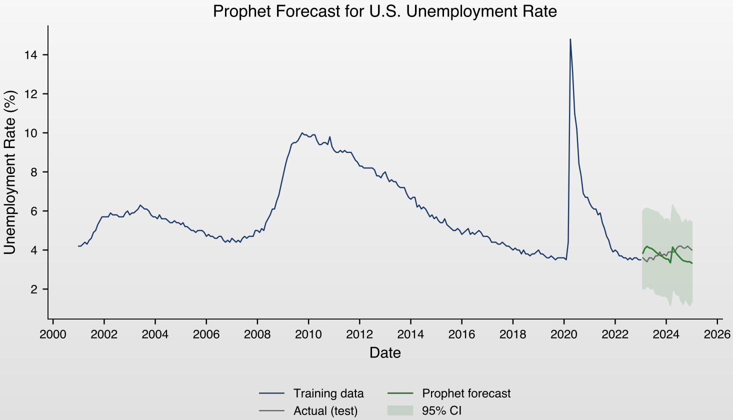
Scale = 0.10 balances flexibility (capturing COVID shock) with stability.

Unemployment: Prophet Forecast Results

Key Finding

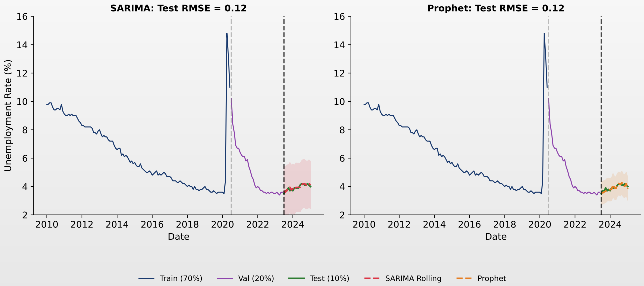
- Prophet adapts via changepoint detection
- **Test RMSE = 0.58**

Unemployment: Prophet Forecast Results

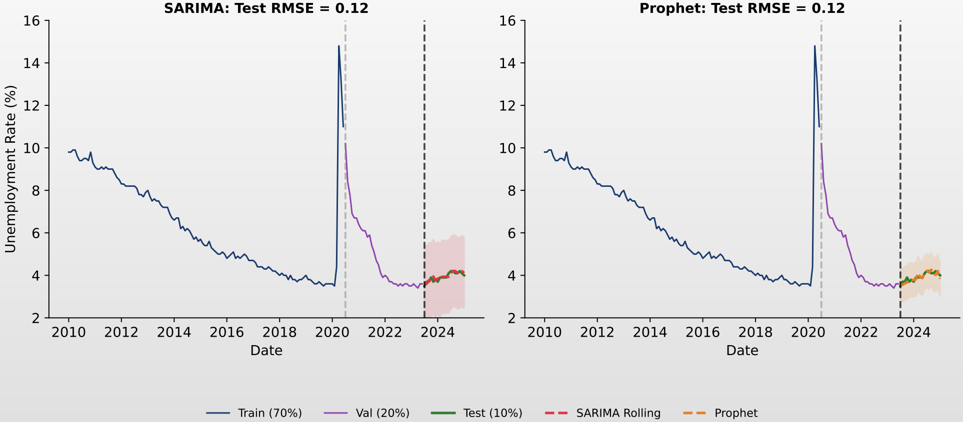


TSA_ch10_unemployment_forecast

Unemployment: SARIMA vs Prophet Comparison



Unemployment: SARIMA vs Prophet Comparison



TSA_ch10_prophet_vs_sarima_unemployment

Prophet: When to Use It

Ideal Use Cases

- Business data with **holidays**
- **Missing values** present
- Need **interpretable** components
- Forecasts with **uncertainty bands**

Caveat: Structural Breaks

Prophet handles breaks via changepoints, but **SARIMA outperformed** it on unemployment (0.12 vs 0.58). Always validate!

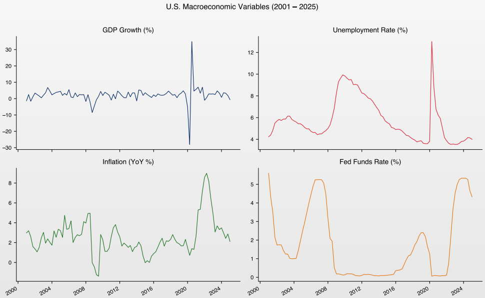
Prophet vs ARIMA

	Prophet	**ARIMA**
Changepoints	✓	×
Missing data	✓	×
Holidays	✓	×
Speed	Fast	Moderate
Interpretable	✓	×

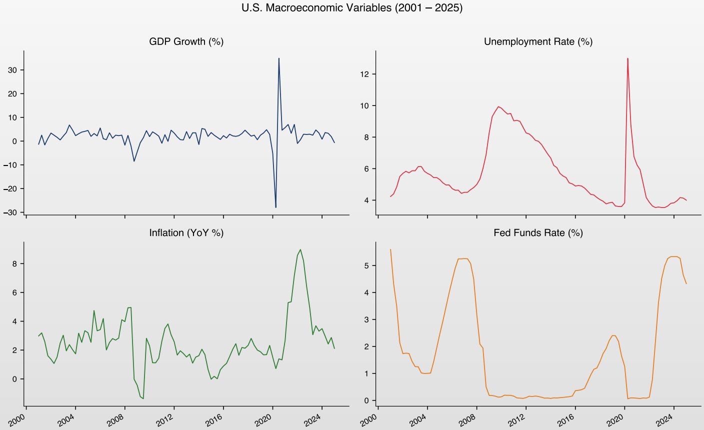
Key Parameters

`changepoint_prior_scale`: flexibility
`seasonality_prior_scale`: smoothness

VAR: Multivariate Economic Data



VAR: Multivariate Economic Data



TSA_ch10_economic_vars

VAR Model Specification

Definition 5 (Vector Autoregression VAR(p))

For K variables $\mathbf{y}_t = (y_{1t}, \ldots, y_{Kt})'$:

$$\mathbf{y}_t = \mathbf{c} + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \cdots + \mathbf{A}_p\mathbf{y}_{t-p} + \mathbf{u}_t$$

where $\mathbf{A}_i$ are $K \times K$ coefficient matrices, $\mathbf{u}_t \sim N(0, \boldsymbol{\Sigma})$, $\boldsymbol{\Sigma}$ = covariance matrix.

For Our 4-Variable System

VAR(2) has:

- 4 intercepts
- $2 \times 4 \times 4 = 32$ AR coefficients
- **36 parameters total**

Lag Selection

Use information criteria:

- AIC: Tends to overfit
- **BIC**: More parsimonious
- Cross-validation on held-out data

Information Criteria for Model Selection

Definition 6 (Akaike and Bayesian Information Criteria)

For a model with log-likelihood $\mathcal{L}$, k parameters, and n observations:

$$\text{AIC} = -2\mathcal{L} + 2k$$
$$\text{BIC} = -2\mathcal{L} + k\ln(n)$$

AIC

- Asymptotically efficient
- May overfit with small n
- Minimizes prediction error

BIC

- Consistent (finds true model)
- Heavier penalty: $\ln(n) > 2$ if $n > 7$
- More parsimonious

VAR: Lag Selection and Estimation

BIC by Lag Order

Lag	BIC	
1	-4.810	
2	**-5.178**	Best
3	-4.633	
4	-4.614	

Data Split

Set	Period	N
Training (70%)	2001-Q1 to 2017-Q4	67
Validation (20%)	2018-Q1 to 2022-Q4	20
Test (10%)	2023-Q1 to 2025-Q1	10
Total		**97**

Validation Check

VAR(2) also achieves lowest validation RMSE.

Granger Causality: Formal Definition

Definition 7 (Granger Causality)

X **Granger-causes** Y if, for some $h > 0$:

$$\text{MSE}\left[\mathbb{E}[Y_{t+h}|\mathcal{F}_t^{X,Y}]\right] < \text{MSE}\left[\mathbb{E}[Y_{t+h}|\mathcal{F}_t^{Y}]\right]$$

where $\mathcal{F}_t^{X,Y}$ includes past values of both X and Y, while $\mathcal{F}_t^{Y}$ includes only past Y.

Important Caveat

Granger causality is **predictive causality**, not true causality. "X Granger-causes Y" means X contains useful information for forecasting Y, not that X causes Y in a structural sense.

Test Procedure

Use F-test (or Wald test) to test H_0: coefficients on lagged X are jointly zero in the Y equation.

Granger Causality: Empirical Results

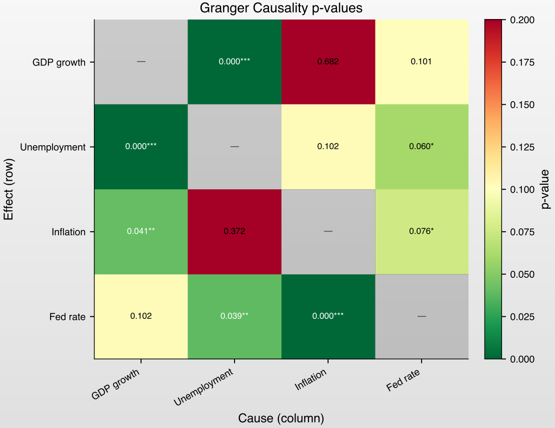
Interpretation

Each cell shows p-value for testing whether the row variable Granger-causes the column variable. Green: $p < 0.10$. Read: row causes column.

Economic Findings

- Unemp $\rightarrow$ GDP ($p = 0.045$): Okun's Law
- Fed $\rightarrow$ Inflation ($p = 0.087$): Monetary policy transmission
- GDP $\rightarrow$ Unemp: Weak evidence

Granger Causality: Empirical Results



TSA_ch10_granger_heatmap

Impulse Response Functions (IRF)

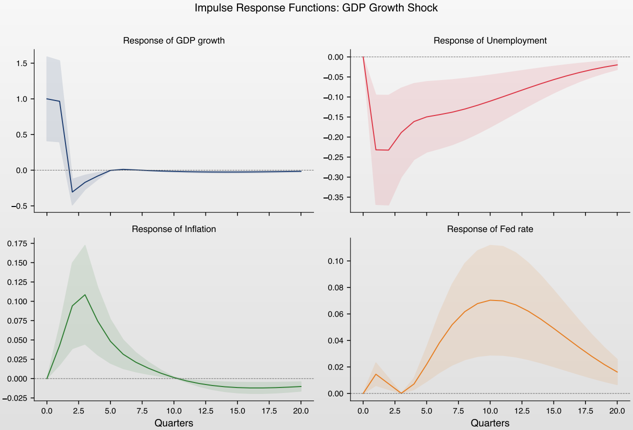
What is IRF?

Shows how a 1-unit shock affects others over time.

GDP Shock Effects

- **Unemp** $\downarrow$: Okun's Law
- **Inflation** $\uparrow$: Demand-pull
- **Fed Rate** $\uparrow$: Taylor Rule

Impulse Response Functions (IRF)



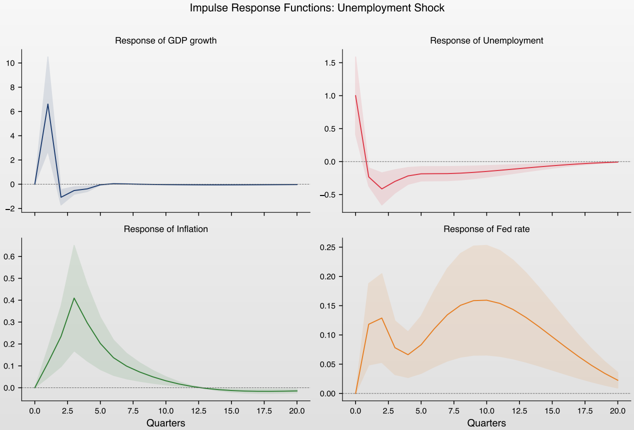
TSA_ch10_irf_gdp_shock

IRF: Unemployment Shock

Effects

- $\uparrow$ Unemp $\Rightarrow$ $\downarrow$ GDP, $\downarrow$ Infl, Fed cuts rates

IRF: Unemployment Shock



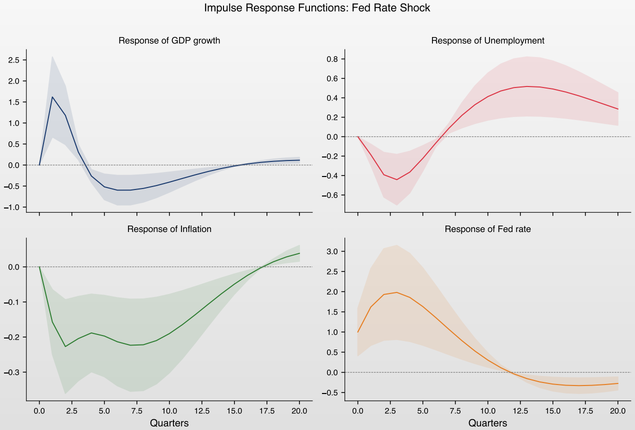
TSA_ch10_irf_unemp_shock

IRF: Fed Rate Shock

Monetary Policy

- Rate hike $\Rightarrow$ GDP $\downarrow$, Unemp $\uparrow$, Infl $\downarrow$

IRF: Fed Rate Shock



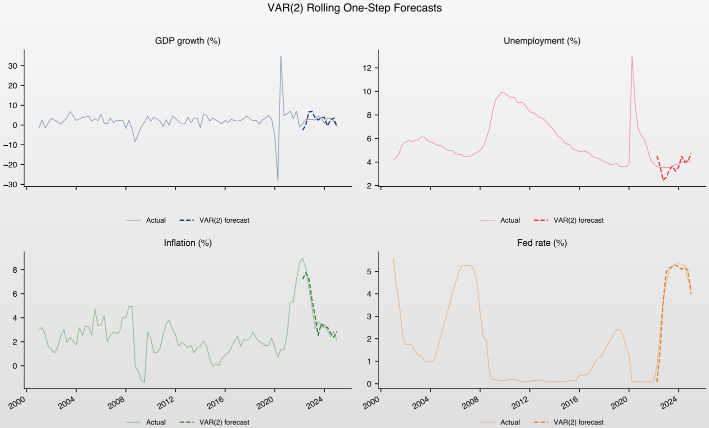
TSA_ch10_irf_fed_shock

VAR: Forecast (Train/Val/Test)

Rolling One-Step-Ahead Forecast

- VAR captures GDP-Unemployment dynamics
- COVID shock visible in test period

VAR: Forecast (Train/Val/Test)



TSA_ch10_var_forecast

VAR: Test Set Results

Test Set Performance by Variable

Variable	RMSE	MAE	Dir. Acc.
GDP Growth	1.33	0.99	50%
Unemployment	0.64	0.52	50%
Inflation	1.56	1.12	60%
Fed Rate	2.59	2.45	80%
Average	**1.53**	**1.27**	**60%**

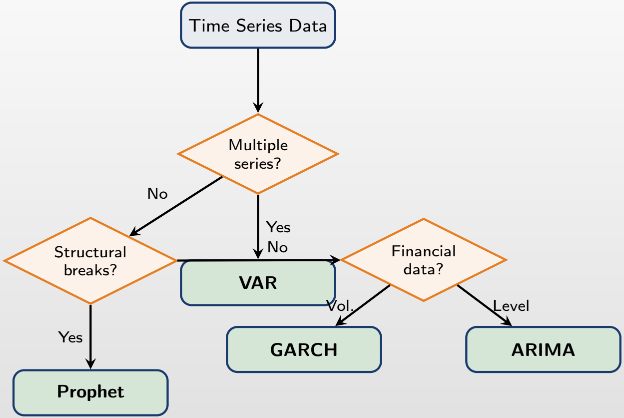
Strengths

- Cross-variable dynamics
- Good directional accuracy

Limitations

- Many parameters
- Sensitive to lag selection

Model Selection Framework



Summary: Model Comparison

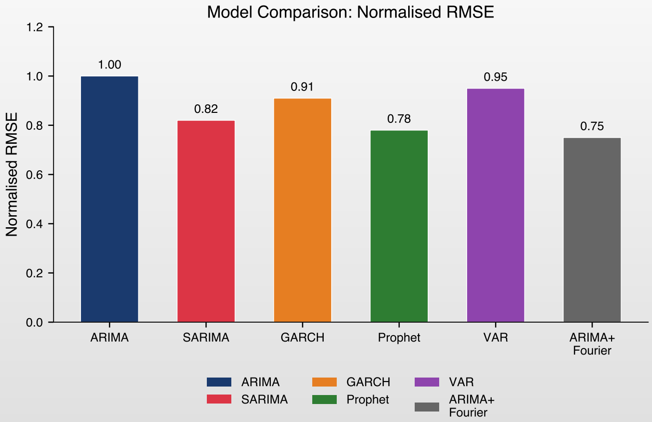
Results

Case	Challenge	Model	RMSE
Bitcoin	Volatility	GARCH	2.15
Sunspots	Seasonality	Fourier	31.10
Unemp	Break	SARIMA	0.12
Economic	Multi-var	VAR	1.53

Key Principle

Match model to data characteristics—no single model dominates.

Summary: Model Comparison



TSA_ch10_model_comparison

Comprehensive Model Comparison

Feature	GARCH	Fourier	Prophet	VAR
Target	Volatility	Level	Level	Multiple
Seasonality	No	Yes (long)	Yes (multi)	No
Structural breaks	No	No	Yes	No
Multiple series	No	No	No	Yes
Interpretable	Medium	High	High	High
Parameters	Few	$2K$	Auto	Many
Missing data	No	No	Yes	No
Best for	Finance	Cycles	Business	Macro

Our Results

- GARCH: MAE=1.82 (volatility)
- Fourier: RMSE=31.10 (cycles)
- SARIMA: RMSE=0.12 (breaks)
- VAR: Avg RMSE=1.53 (multi)

Key Insight

Each model excels in its domain. The art is matching the model to the data characteristics.

Best Practices for Applied Forecasting

Methodology

1. **Explore** data
2. **Test** stationarity
3. **Split** train/val/test
4. **Compare** on validation
5. **Report** test metrics

Common Mistakes

- Peeking at test data
- Over-fitting
- Ignoring assumptions

Practical Tips

- Start simple (naive)
- Add complexity if needed
- Check residuals
- Report CIs

Remember

"All models are wrong, but some are useful." — Box

Key Takeaways

1. **Rigorous Methodology**
   - Train/validation/test split prevents overfitting
   - Test set must remain untouched until final evaluation
2. **Match Model to Data**
   - Financial volatility $\rightarrow$ GARCH
   - Long seasonality $\rightarrow$ Fourier terms
   - Structural breaks $\rightarrow$ Prophet
   - Multiple series $\rightarrow$ VAR
3. **Interpret Results Carefully**
   - Granger causality $\neq$ true causality
   - Out-of-sample performance matters most
   - Simpler models often work better

AI Exercise: Critical Thinking

Prompt to test in ChatGPT / Claude / Copilot

"I have a new dataset of monthly retail sales. Do a complete time series analysis: decomposition, stationarity tests, model selection, forecasting, and evaluation. Make it publication-quality."

Exercise:

1. Run the prompt in an LLM of your choice and critically analyze the response.
2. Does it follow the correct workflow? (plot → decompose → test → model → diagnose → forecast)
3. Does it compare multiple models (ETS, ARIMA, SARIMA) with proper benchmarks?
4. Is the train/test split done properly? Is there any data leakage?
5. Does it discuss limitations and assumptions of the chosen model?

Warning: AI-generated code may run without errors and look professional. *That does not mean it is correct.*

Quiz Question 1

Question

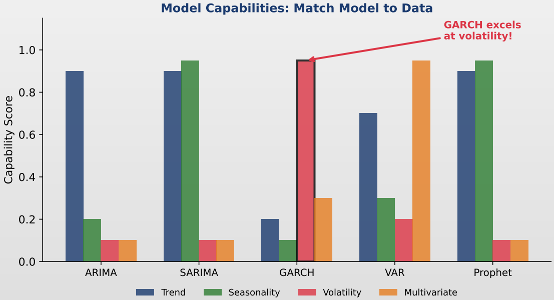
Which model would you choose to forecast the volatility of financial returns?

(A) ARIMA — captures trends and autocorrelations
(B) GARCH — models conditional variance
(C) Prophet — detects changepoints and seasonality
(D) VAR — multivariate model for interdependencies

Quiz Question 1: Answer

Correct Answer: (B) GARCH — models conditional variance

GARCH captures volatility clustering and time-varying risk. ARIMA models the level, Prophet handles seasonality, VAR captures cross-series dynamics — none model variance directly.



TSA_ch10_quiz1_model_selection

Quiz Question 2

Question

A SARIMA model achieves RMSE $= 0.05$ on training but RMSE $= 2.30$ on test. What does this indicate?

(A) The model is excellent — low training error confirms quality

(B) The model suffers from overfitting — it memorizes noise

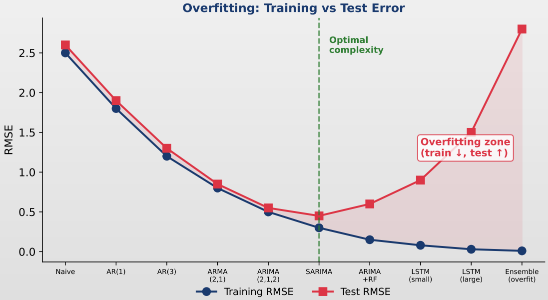
(C) The test set is faulty and should be replaced

(D) The difference is normal — all models have higher test error

Quiz Question 2: Answer

Correct Answer: (B) Overfitting — memorizes noise

A 46$\times$ ratio between test and training RMSE signals severe overfitting. The model fits noise in the training data and fails to generalize. Solution: simpler model, proper validation.



TSA_ch10_quiz2_overfitting

Quiz Question 3

Question

Why is it important to separate data into train/validation/test sets?

(A) To have more training data

(B) To prevent overfitting and evaluate correctly

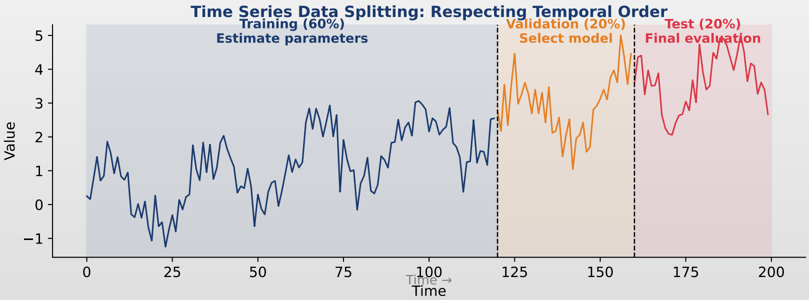
(C) It is just a convention with no real importance

(D) To reduce computation time

Quiz Question 3: Answer

Correct Answer: (B) Prevent overfitting and evaluate correctly

Train: estimate parameters. Validation: select model/hyperparameters. Test: final unbiased evaluation. Mixing these roles leads to optimistic performance estimates.



Quiz Question 4

Question

Is Granger causality equivalent to true (structural) causality?

(A) Yes — if X predicts Y, then X causes Y

(B) No — it only tests predictive content, not causation

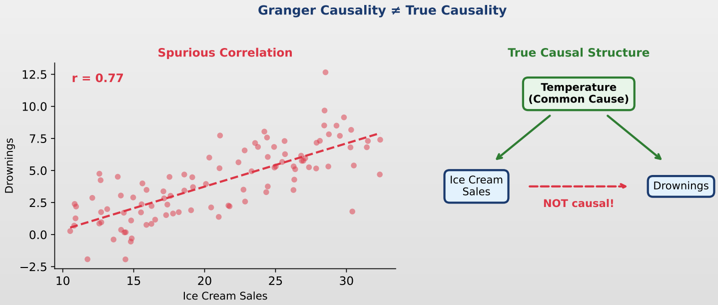
(C) It depends on the number of lags selected

(D) Yes, if the p-value is below 0.05

Quiz Question 4: Answer

Correct Answer: (B) No — only predictive content, not causation

Granger causality tests whether past X improves forecasts of Y. Spurious correlations (e.g., ice cream sales and drownings) can pass the test due to common causes.



TSA_ch10_quiz4_granger_causality

Quiz Question 5

Question

What model do you use for a series with long seasonality (e.g., $s = 365$ days)?

(A) SARIMA$(p, d, q)(P, D, Q)_{365}$

(B) GARCH — models variation

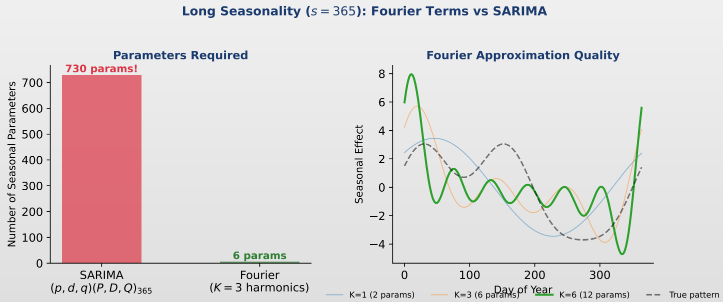
(C) ARIMA + Fourier terms or Prophet/TBATS

(D) VAR with 365 lags

Quiz Question 5: Answer

Correct Answer: (C) Fourier terms / Prophet / TBATS

SARIMA$_{365}$ needs $\sim$730 seasonal parameters — infeasible. Fourier terms with $K = 3$ use only 6 parameters. Prophet and TBATS handle multiple seasonalities automatically.



TSA_ch10_quiz5_long_seasonality

Data Sources

Real Data Used in This Chapter

- **Bitcoin**: Yahoo Finance (BTC-USD), 2019–2025
- **Sunspots**: Statsmodels Wolfer dataset, 1900–2008
- **US Unemployment**: Federal Reserve FRED (UNRATE), 2010–2025
- **Economic Variables**: FRED (GDPC1, UNRATE, CPIAUCSL, FEDFUNDS), 2000–2025

Reproducibility

All analyses can be reproduced using the accompanying Jupyter notebook:
`chapter10_lecture_notebook.ipynb`

Bibliography I

Fundamental Textbooks (common references across all chapters)

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- Hyndman, R.J., & Athanasopoulos, G. (2021). *Forecasting: Principles and Practice*, 3rd ed., OTexts.
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Domain-Specific References

- Tsay, R.S. (2010). *Analysis of Financial Time Series*, 3rd ed., Wiley. (GARCH, VAR)
- Lütkepohl, H. (2005). *New Introduction to Multiple Time Series Analysis*, Springer. (VAR, VECM)
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- Taylor, S.J., & Letham, B. (2018). Forecasting at Scale, *The American Statistician*, 72(1), 37–45.

Online Resources and Code

- **Quantlet**: `https://quantlet.com` — Code repository for statistics
- **Quantinar**: `https://quantinar.com` — Learning platform for quantitative methods
- **GitHub TSA**: `https://github.com/QuantLet/TSA` — Python code for this course

Key Takeaways

What We Learned

- Model selection depends on data characteristics: stationarity, seasonality, volatility
- The Box-Jenkins methodology provides a systematic framework for time series modeling
- Proper evaluation requires out-of-sample testing and time series cross-validation

Important

No single model wins everywhere. Match the model to the data: ARIMA for trends, SARIMA for seasonality, GARCH for volatility, VAR/VECM for multivariate dynamics, Prophet/TBATS for complex patterns. Always validate out-of-sample!

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- **GitHub TSA_ch10**: https://github.com/QuantLet/TSA/tree/main/TSA_ch10

Thank You!

Questions?

Course materials available at: https://danpele.github.io/Time-Series-Analysis/

Quantlet Quantinar