



# Time Series Analysis and Forecasting

## Chapter 4: SARIMA Models



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## Learning Objectives

By the end of this chapter, you will be able to:

1. **Identify** seasonal patterns in time series data
2. **Apply** seasonal differencing to remove seasonal unit roots
3. **Build** and estimate SARIMA models with seasonal components
4. **Interpret** seasonal ACF/PACF patterns for model identification
5. **Evaluate** forecasts using rolling window methods for seasonal data
6. **Apply** the complete methodology on real data (airline passengers)

## Outline

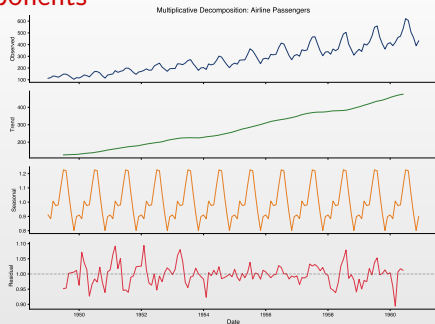
- Motivation
- Seasonality in Time Series
- Seasonal Differencing
- The SARIMA Model
- Seasonal ACF and PACF Patterns
- Estimation and Diagnostics
- Forecasting with SARIMA
- Case Study: Airline Passengers
- Practical Aspects
- AI Use Case
- Summary
- Quiz

## Why SARIMA? Seasonality Is Everywhere



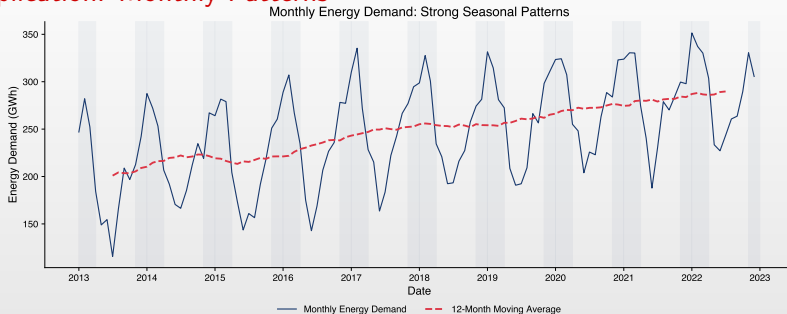
- Retail sales exhibit clear **annual patterns**: December peaks, January troughs
- Standard ARIMA models cannot capture these **repeating seasonal cycles**
- Ignoring seasonality leads to systematic forecast errors

## Understanding Seasonal Components



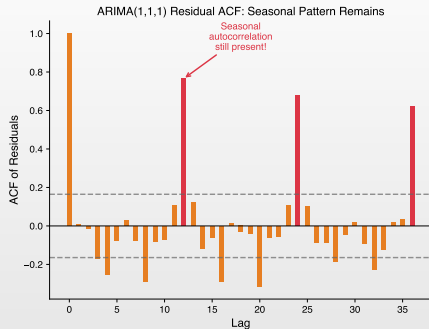
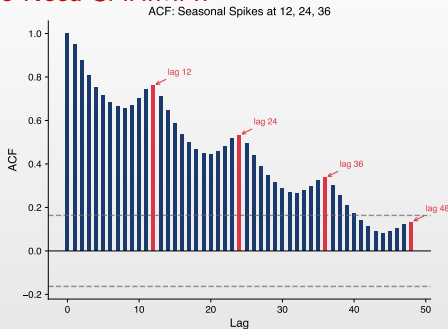
- ▣ Seasonal time series = **Trend** + **Seasonal Pattern** + **Residuals**
- ▣ Decomposition helps visualize each component separately
- ▣ SARIMA models capture both trend dynamics and seasonal behavior

## Real-World Application: Monthly Patterns



- Energy demand shows strong **monthly seasonality** (heating/cooling cycles)
- Pattern repeats predictably each year with slight variations
- Utility companies use SARIMA forecasts for capacity planning

## Why Do We Need SARIMA?



- Left: Seasonal ACF shows spikes at lags 12, 24, 36... (annual pattern)
- Right: ARIMA residuals still show seasonal autocorrelation  $\Rightarrow$  model is incomplete
- SARIMA adds **seasonal AR and MA terms** to capture these patterns

## What is Seasonality?

### Definition 1 (Seasonality)

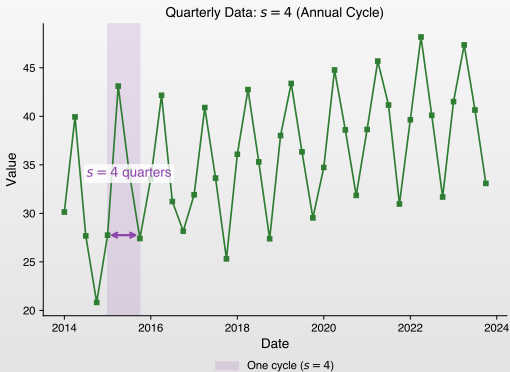
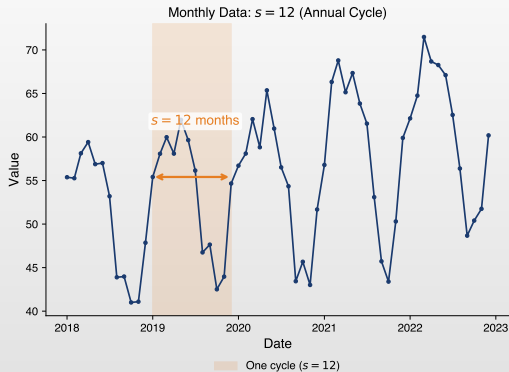
A time series exhibits seasonality when it shows regular, periodic fluctuations that repeat over a fixed period  $s$  (the seasonal period).

### Common Seasonal Periods

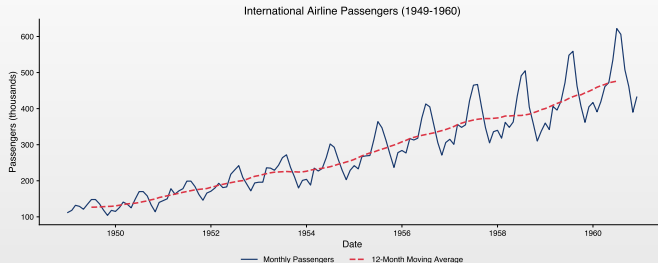
- Monthly data:  $s = 12$  (annual cycle)
- Quarterly data:  $s = 4$  (annual cycle)
- Weekly data:  $s = 52$  (annual) or  $s = 7$  (weekly pattern)
- Daily data:  $s = 7$  (weekly pattern)



## What is Seasonality?



## Example: Airline Passengers Data



- Monthly international airline passengers (1949–1960)
- Clear **upward trend** and **growing seasonal amplitude**
- Summer peaks reflect vacation travel patterns

## Examples of Seasonal Data

### Economic Series

- ▣ Retail sales (holiday peaks)
- ▣ Tourism (summer/winter)
- ▣ Agricultural production
- ▣ Energy consumption
- ▣ Employment (hiring cycles)

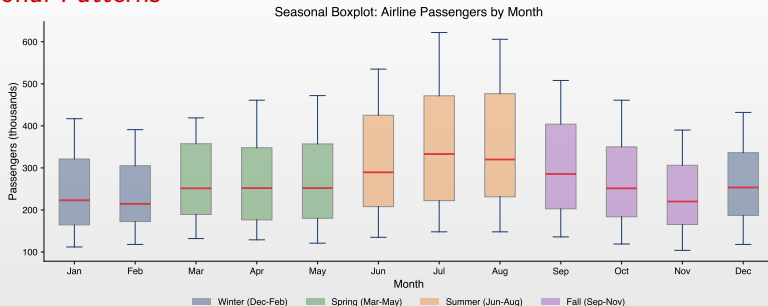
### Other Domains

- ▣ Weather/temperature
- ▣ Website traffic
- ▣ Hospital admissions
- ▣ Transportation usage
- ▣ Electricity demand

### Why It Matters

Ignoring seasonality leads to biased forecasts and invalid inference!

## Visualizing Seasonal Patterns



- Box plot reveals consistent seasonal pattern across years
- July–August: highest passenger counts (summer travel)
- November–February: lowest counts (winter months)

## Deterministic vs Stochastic Seasonality

### Deterministic Seasonality

- ▣ **Fixed pattern:**  $Y_t = \sum_{j=1}^s \gamma_j D_{jt} + \varepsilon_t$ 
  - ▶  $D_{jt}$  are seasonal dummies
- ▣ **Note:** either omit one dummy variable and include a constant, or impose  $\sum \gamma_j = 0$ , to avoid perfect collinearity
- ▣ Pattern constant over time
- ▣ Same amplitude every year
- ▣ Removed by regression on dummies
- ▣ ACF: sharp cutoff at seasonal lags
- ▣ **Example:** University enrollment peaks every September by the same amount

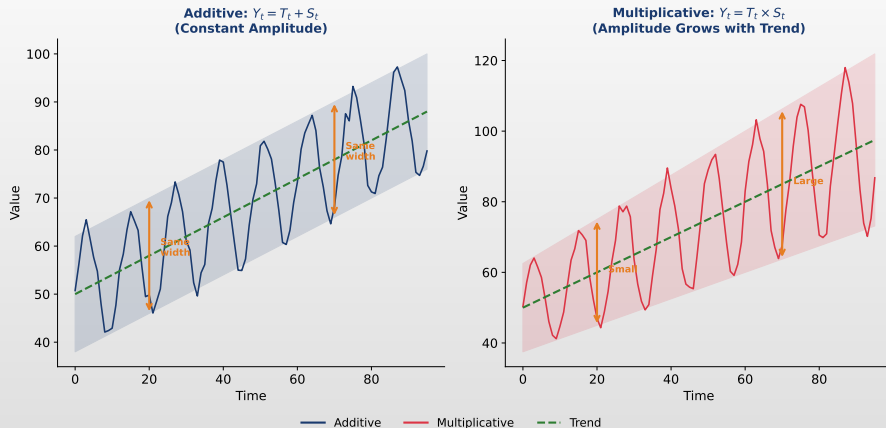
### Stochastic Seasonality

- ▣ **Evolving pattern:**  $\Delta_s Y_t = Y_t - Y_{t-s}$ 
  - ▶ Exhibits dependence structure
- ▣ Pattern evolves over time
- ▣ Amplitude may grow or shrink
- ▣ Requires seasonal differencing
- ▣ ACF: slow decay at seasonal lags
- ▣ **Example:** Retail sales peaks grow larger each December

### How to decide?

- ▣ Slow ACF decay at lags  $s, 2s, 3s, \dots \Rightarrow$  stochastic (use  $\Delta_s$ )
- ▣ Sharp cutoff  $\Rightarrow$  deterministic (use dummies)
- ▣ Use HEGY or Canova-Hansen tests to confirm

## Additive vs Multiplicative Seasonality



## Additive vs Multiplicative Seasonality

Additive:  $Y_t = T_t + S_t + \varepsilon_t$

- ▣ Seasonal amplitude **constant**
- ▣ No transformation needed
- ▣ Ex: temperatures, university enrollment

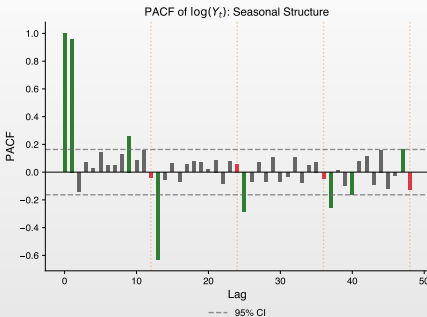
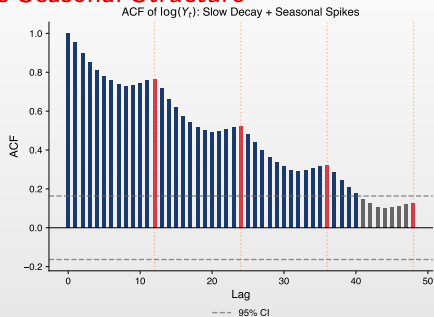
Multiplicative:  $Y_t = T_t \cdot S_t \cdot \varepsilon_t$

- ▣ Amplitude **grows with level**
- ▣ Requires log transform (Box-Cox)
- ▣ Ex: Airline, retail sales, GDP

### First practical decision

- ▣ Amplitude grows with the trend?  $\Rightarrow$  multiplicative  $\Rightarrow$  apply log/Box-Cox *before* differencing

## ACF Reveals Seasonal Structure



- **Slow decay** at all lags indicates non-stationarity (trend)
- **Spikes at lags 12, 24, 36** confirm seasonal pattern ( $s = 12$ )
- ACF at seasonal lags: slow decay  $\Rightarrow$  needs seasonal differencing



## Detecting Seasonality

### Visual Methods

- ▣ Time series plot – look for repeating patterns
- ▣ Seasonal subseries plot – compare same seasons across years
- ▣ ACF plot – spikes at seasonal lags ( $s, 2s, 3s, \dots$ )

### Statistical Tests

- ▣ Seasonal unit root tests (HEGY, Canova-Hansen, OCSB<sup>a</sup>)
- ▣ F-test for seasonal dummy variables
- ▣ Kruskal-Wallis test (non-parametric)

### ACF Signature

- ▣ Strong seasonality: ACF shows significant spikes at lags  $s, 2s, 3s, \dots$

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<sup>a</sup>Osborn-Chui-Smith-Birchenhall — the default test in `auto_arima`

## F-Test for Seasonal Dummy Variables: Intuition

### What does this test do?

- ▣ **Goal:** test whether mean values differ significantly across seasons
- ▣ **Logic:** if the mean in January  $\neq$  February  $\neq \dots \neq$  December  $\Rightarrow$  seasonality
- ▣ **Method:** compare a model WITH seasonal dummy variables vs. a model WITHOUT

### Models compared

- ▣ **Restricted:**  $Y_t = \alpha + \varepsilon_t$     **Unrestricted:**  $Y_t = \alpha + \sum_{j=1}^{s-1} \gamma_j D_{jt} + \varepsilon_t$
- ▣ where  $D_{jt} = 1$  if observation  $t$  is in season  $j$ , 0 otherwise

### Key idea

- ▣ If adding seasonal dummy variables **significantly reduces** prediction errors, then seasonality is present

## F-Test for Seasonal Dummy Variables: Formula and Example

### F-statistic formula

- **Formula:**  $F = \frac{(SSR_R - SSR_U)/(s-1)}{SSR_U/(n-s)} \sim F_{s-1, n-s}$ 
  - ▶  $SSR_R$ : sum of squared residuals from the restricted model (no dummies)
  - ▶  $SSR_U$ : sum of squared residuals from the unrestricted model (with dummies)
  - ▶  $s - 1$ : number of restrictions (monthly: 11, quarterly: 3)

### Numerical example (Monthly data, $n=120$ )

- $SSR_R = 15000, SSR_U = 8500, s = 12$
- $F = \frac{(15000 - 8500)/11}{8500/108} = \frac{590.9}{78.7} = 7.51$
- Critical value  $F_{0.05, 11, 108} \approx 1.87$ . Since  $7.51 > 1.87$ : **Reject  $H_0 \Rightarrow$  Seasonality present!**

## Kruskal-Wallis Test: Intuition

### What does this test do?

- **Non-parametric test:** checks whether observations from different seasons come from the same distribution (no distributional assumptions needed)
- **Mechanism:** ranks all observations 1 to  $N$ , then checks whether ranks are uniformly distributed across seasons
- **Conclusion:** if one season consistently has higher/lower ranks  $\Rightarrow$  seasonality (e.g., summer always has high ranks  $\Rightarrow$  seasonal energy consumption)

### Why use it instead of the F-test?

- **No normality assumption** – works with any distribution; ideal for heavy-tailed or skewed data
- **Robust to outliers** – ranks bound the influence of any single observation

### Limitation

- Less powerful than the F-test when data ARE normally distributed

## Kruskal-Wallis Test: Formula and Example

### Test statistic

$$\square H = \frac{12}{N(N+1)} \sum_{j=1}^s \frac{R_j^2}{n_j} - 3(N+1) \quad \text{where } N = \text{total obs.}, n_j = \text{obs. in season } j, R_j = \text{rank sum}$$

### Example: Quarterly sales (n=20, s=4)

- Data ranked 1–20. Rank sums: Q1:  $R_1=15$ , Q2:  $R_2=35$ , Q3:  $R_3=70$ , Q4:  $R_4=90$
- $H = \frac{12}{20 \times 21} \left( \frac{15^2}{5} + \frac{35^2}{5} + \frac{70^2}{5} + \frac{90^2}{5} \right) - 3(21) = 19.57$
- Critical value  $\chi_{0.05,3}^2 = 7.81$ . Since  $19.57 > 7.81$ : **Reject  $H_0 \Rightarrow$  Seasonality!**

### In Python

- **Implementation:** `scipy.stats.kruskal(q1, q2, q3, q4)`

## HEGY Test: What Problem Does It Solve?

### Key question

- ▣ **Problem:** given a seasonal series, we need to determine the type of differencing (wrong choice  $\Rightarrow$  over- or under-differencing)
- ▣ **Regular differencing**  $(1 - L)^d$ ?  $\Rightarrow$  set  $d = 1$ ; **Seasonal differencing**  $(1 - L^s)^D$ ?  $\Rightarrow$  set  $D = 1$
- ▣ **HEGY** (Hylleberg, Engle, Granger, Yoo, 1990): tests for both types of unit roots simultaneously!

### Why not just use ADF?

- ▣ **ADF:** tests only for a regular unit root at frequency zero — completely ignores seasonal components
- ▣ **Limitation:** seasonal data may have unit roots at seasonal frequencies that ADF misses!

### HEGY tests multiple frequencies

- ▣ **Quarterly:** tests at  $0, \pi, \pm\pi/2$  (3 freq.  $\Rightarrow$  one regular + two seasonal roots)
- ▣ **Monthly:** tests at  $0, \pi, \pm\pi/6, \pm\pi/3, \pm\pi/2, \pm2\pi/3, \pm5\pi/6$  (6 freq.  $\Rightarrow$  one regular + five seasonal roots)

## HEGY Test: Auxiliary Regression (Quarterly)

### HEGY auxiliary regression

- ▣ **Quarterly data** ( $s = 4$ ):  $\Delta_4 y_t = \pi_1 z_{1,t-1} + \pi_2 z_{2,t-1} + \pi_3 z_{3,t-2} + \pi_4 z_{4,t-2} + \sum_{j=1}^k \phi_j \Delta_4 y_{t-j} + \varepsilon_t$ 
  - Lags  $k$  chosen by AIC/BIC to eliminate residual autocorrelation

### Transformed variables

- ▣  $z_{1t}$ :  $(1 + L + L^2 + L^3)y_t$  — captures the zero-frequency component (trend)
- ▣  $z_{2t}$ :  $-(1 - L + L^2 - L^3)y_t$  — captures the frequency- $\pi$  component (semi-annual)
- ▣  $z_{3t}, z_{4t}$ : capture the frequency  $\pm\pi/2$  components (annual cycle)

### Hypotheses

- ▣  $H_0 : \pi_1 = 0$ : unit root at frequency 0  $\Rightarrow$  requires  $d = 1$
- ▣  $H_0 : \pi_2 = 0$ : unit root at frequency  $\pi$   $\Rightarrow$  requires  $D = 1$
- ▣  $H_0 : \pi_3 = \pi_4 = 0$ : unit root at frequency  $\pm\pi/2$   $\Rightarrow$  requires  $D = 1$

## HEGY Test: Decision Rules with Examples

### HEGY critical values (5%, $n=100$ , with constant)

Test	Statistic	Critical value	If NOT rejected...
$t_1$ ( $\pi_1 = 0$ )	t-stat	-2.88	Requires $d = 1$
$t_2$ ( $\pi_2 = 0$ )	t-stat	-2.88	Requires $D = 1$
$F_{34}$ ( $\pi_3 = \pi_4 = 0$ )	F-stat	6.57	Requires $D = 1$

### Example: Quarterly GDP

- ▣ **HEGY results:**  $t_1 = -1.52$ ,  $t_2 = -4.21$ ,  $F_{34} = 2.15$
- ▣  $t_1 = -1.52 > -2.88$ : Cannot reject  $\Rightarrow$  **requires**  $d = 1$
- ▣  $t_2 = -4.21 < -2.88$ : Reject  $\Rightarrow$  no unit root at  $\pi$
- ▣  $F_{34} = 2.15 < 6.57$ : Cannot reject  $\Rightarrow$  **requires**  $D = 1$
- ▣ **Conclusion:** Use SARIMA with  $d = 1, D = 1$



## Canova-Hansen Test: The Opposite of HEGY

### HEGY vs Canova-Hansen: Different null hypotheses!

	HEGY	Canova-Hansen
$H_0$	Seasonal unit root	No seasonal unit root
$H_1$	No seasonal unit root	Seasonal unit root
Reject $H_0$	Use seasonal dummies	Use differencing $(1 - L^5)$
Do not reject	Use differencing $(1 - L^5)$	Use seasonal dummies

### Why does it matter?

- HEGY: “Prove there is NO unit root” (conservative towards differencing)
- CH: “Prove there IS a unit root” (conservative towards dummies)
- Use **both** tests for robust conclusions!

## Canova-Hansen Test: Formula

### Testing procedure

- ▣ **Step 1:** Regress  $y_t$  on seasonal dummies:  $y_t = \sum_{j=1}^s \gamma_j D_{jt} + u_t$
- ▣ **Step 2:** Compute partial sums at seasonal frequency  $\lambda_i$ :
  - ▶  $S_{it}^{(c)} = \sum_{j=1}^t \hat{u}_j \cos(\lambda_i j), \quad S_{it}^{(s)} = \sum_{j=1}^t \hat{u}_j \sin(\lambda_i j)$

### LM test statistic

- ▣  $LM_i = \frac{1}{T^2 \hat{\omega}_i} \left[ \sum_{t=1}^T (S_{it}^{(c)})^2 + \sum_{t=1}^T (S_{it}^{(s)})^2 \right]$
- ▣ where  $\hat{\omega}_i$  = consistent estimator of the spectral density at frequency  $\lambda_i$

### Decision

- ▣ **Rule:** reject  $H_0$  (stationarity) if  $LM > \text{critical value} \Rightarrow$  seasonal differencing is needed

## Summary: Choosing the Right Seasonality Test

Test	$H_0$	If rejected	Best for
F-test	No seasonality	Seasonality exists	Normal data
Kruskal-Wallis	No difference across seasons	Seasonality exists	Non-normal, outliers
HEGY	Unit root exists	Use dummies	Determining $d$ , $D$
Canova-Hansen	No unit root	Use $(1 - L^s)$	Confirming stability

### Key idea

- ▣ **F-test / Kruskal-Wallis:** “*Does seasonality exist?*”
- ▣ **HEGY / Canova-Hansen:** “*What type?*” (deterministic vs stochastic)

## Box-Cox Transformation: Variance Stabilization

### Box-Cox Family of Transformations

- ▣ **Formula:**  $Y_t^{(\lambda)} = (Y_t^\lambda - 1)/\lambda$  if  $\lambda \neq 0$ ;  $\ln(Y_t)$  if  $\lambda = 0$
- ▣ **Special cases:**  $\lambda=1$  (none),  $\lambda=0$  (log),  $\lambda=0.5$  ( $\sqrt{\cdot}$ ).  $\lambda < 1$  compresses large values;  $\lambda > 1$  amplifies

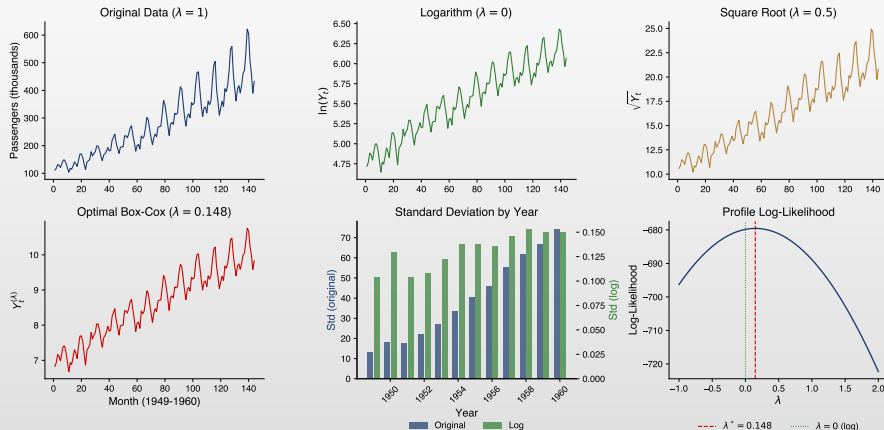
### Automatic Selection of $\lambda$

- ▣ **Profile likelihood:** maximizes log-likelihood as a function of  $\lambda$  (most common; provides CI)
- ▣ **Guerrero (1993):** minimizes CV of seasonal sub-series (preferred for varying amplitude)
- ▣ **Python:** `boxcox(y)` from `scipy.stats` or `BoxCox.lambda_(y)` in R

### Why not just logarithm?

- ▣ Log ( $\lambda=0$ ) assumes variance  $\propto$  level — not always the case (e.g., may need  $\lambda = 0.3$ )
- ▣ Box-Cox chooses the optimal transformation based on data, not assumptions

## Box-Cox on the Airline Data: Complete Example



## Box-Cox on the Airline Data: Complete Example

### Result for Airline Passengers

- $\hat{\lambda} = 0.148 \approx 0 \Rightarrow \log$  is nearly optimal
- Standard deviation per year: from increasing (original) to stable (log)

### Bias Correction in Back-Transformation

- On log scale:  $\hat{y}_{T+h}$  is the **median**, not the mean
- Correction:  $\hat{Y}_{T+h} = \exp\left(\hat{y}_{T+h} + \frac{\sigma_h^2}{2}\right)$
- Without correction: systematically under-estimated forecasts!

## STL Decomposition: Modern Alternatives

### STL: Seasonal-Trend Decomposition using Loess (Cleveland et al., 1990)

- ▣ **Advantages:** time-varying seasonality, robust to outliers, any period  $s$
- ▣ **Algorithm:** iterative locally weighted regression (loess)

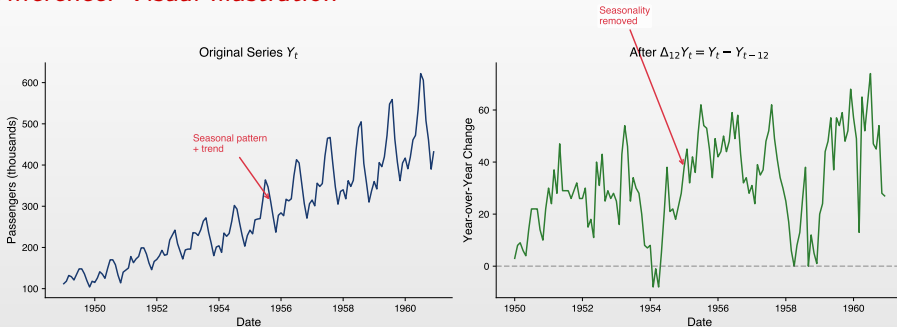
### Key Parameters

- ▣ **Seasonal window** (`seasonal`): controls how quickly seasonality changes
- ▣ **Trend window** (`trend`): smoothing of the trend component
- ▣ **Robustness** (`robust=True`): reduces influence of outliers

### Practical Usage

- ▣ STL for exploration and preprocessing; SARIMA for modeling and forecasting
- ▣ Python: `STL(y, period=12).fit()` from `statsmodels`

## Seasonal Difference: Visual Illustration



- **Left:** Original series with clear seasonal pattern
- **Right:** After  $\Delta_{12} = (1 - L^{12})$ , the seasonal pattern is removed
  - Year-over-year comparison eliminates seasonal effects



## The Seasonal Difference Operator

### Definition 2 (Seasonal Difference)

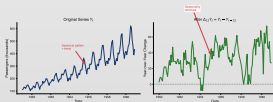
The seasonal difference operator  $\Delta_s$  is defined as:

$$\Delta_s Y_t = (1 - L^s)Y_t = Y_t - Y_{t-s}$$

where  $L^s Y_t = Y_{t-s}$  is the seasonal lag operator.

### Examples

- Monthly data ( $s = 12$ ):  $\Delta_{12} Y_t = Y_t - Y_{t-12}$   
Compares each month to the same month last year
- Quarterly data ( $s = 4$ ):  $\Delta_4 Y_t = Y_t - Y_{t-4}$   
Compares each quarter to the same quarter last year



## Proof: Seasonal Differencing Removes Deterministic Seasonality

**Claim:** If  $Y_t = \mu_t + \varepsilon_t$  where  $\mu_t = \mu_{t-s}$  (periodic mean), then  $\Delta_s Y_t$  removes the seasonal mean.

- The condition  $\mu_t = \mu_{t-s}$  means the mean repeats exactly every  $s$  periods

**Proof:** Let  $Y_t = \mu_t + \varepsilon_t$  where  $\mu_t$  has period  $s$ . Apply seasonal difference:

$$\begin{aligned}\Delta_s Y_t &= Y_t - Y_{t-s} = (\mu_t + \varepsilon_t) - (\mu_{t-s} + \varepsilon_{t-s}) \\ &= \mu_t - \mu_{t-s} + \varepsilon_t - \varepsilon_{t-s} \\ &= 0 + \varepsilon_t - \varepsilon_{t-s} \quad (\text{since } \mu_t = \mu_{t-s})\end{aligned}$$

**Properties of  $\Delta_s Y_t = \varepsilon_t - \varepsilon_{t-s}$ :**

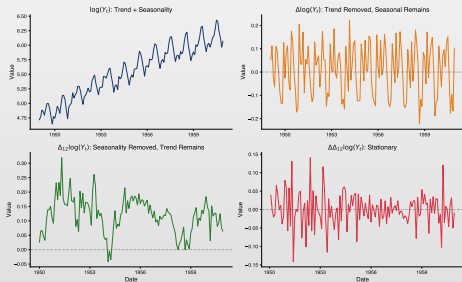
- $\mathbb{E}[\Delta_s Y_t] = 0$  (constant mean);  $\text{Var}(\Delta_s Y_t) = 2\sigma^2$  (constant variance)
  - ▶ Both properties are required for second-order stationarity
- Autocovariance:  $\gamma(s) = -\sigma^2$ ,  $\gamma(k) = 0$  for  $k \neq 0, s$ 
  - ▶ ACF shows a single spike at lag  $s$ , then zero

### Result

Seasonal differencing transforms periodic seasonal pattern into MA(1) at seasonal lag.

## Effect of Differencing Operations

- Regular differencing removes trend but seasonal pattern remains
- Seasonal differencing removes seasonality but trend pattern remains
- Both differences** needed to achieve stationarity



## Combining Regular and Seasonal Differencing

### Full Differencing

For series with both trend and seasonality:

$$\Delta\Delta_s Y_t = (1 - L)(1 - L^s)Y_t$$

### Expansion

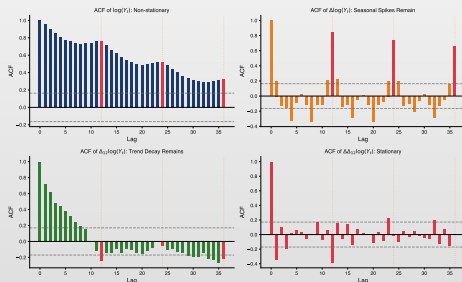
$(1 - L)(1 - L^s)Y_t = Y_t - Y_{t-1} - Y_{t-s} + Y_{t-s-1}$ . For monthly:  $\Delta\Delta_{12} Y_t = Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13}$

### Order of Differencing

$d$ : regular differences (trend removal);  $D$ : seasonal differences (seasonal trend removal)

## ACF Before and After Differencing

- Original ACF: slow decay indicates non-stationarity
- After  $\Delta$ : seasonal spikes remain at lags 12, 24, 36
- After  $\Delta_{12}$ : trend decay remains at early lags
- After  $\Delta\Delta_{12}$ : ACF cuts off  $\Rightarrow$  **stationary**



## Seasonal Integration

### Definition 3 (Seasonally Integrated Process)

A series  $Y_t$  is **seasonally integrated** of order  $(d, D)_s$ , written  $Y_t \sim I(d, D)_s$ , if:

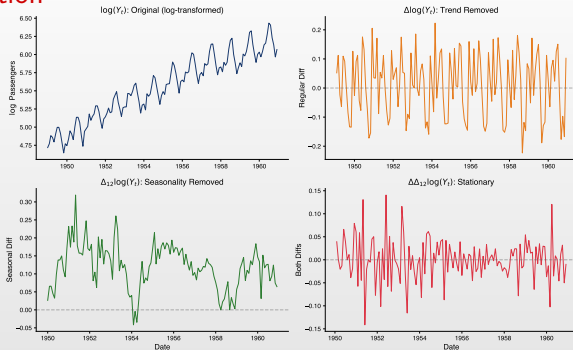
$$(1 - L)^d (1 - L^s)^D Y_t$$

is stationary.

### Common Cases

- ▣  $I(1, 0)_{12}$ : Regular unit root only (monthly)
- ▣  $I(0, 1)_{12}$ : Seasonal unit root only
- ▣  $I(1, 1)_{12}$ :
  - ▶ Both regular and seasonal unit roots

## SARIMA: Visual Illustration



- Original  $\Rightarrow$  regular difference (removes trend)  $\Rightarrow$  seasonal difference (removes seasonality)
- Apply minimum differencing needed to achieve stationarity

## SARIMA Model Definition

### Definition 4 ( $\text{SARIMA}(p, d, q) \times (P, D, Q)_s$ )

The **Seasonal ARIMA** model is:

$$\phi(L)\Phi(L^s)(1-L)^d(1-L^s)^D Y_t = c + \theta(L)\Theta(L^s)\varepsilon_t$$

### Components

- ▣  $\phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p$ : Non-seasonal AR
- ▣  $\Phi(L^s) = 1 - \Phi_1 L^s - \dots - \Phi_P L^{Ps}$ : Seasonal AR
- ▣  $\theta(L) = 1 + \theta_1 L + \dots + \theta_q L^q$ : Non-seasonal MA
- ▣  $\Theta(L^s) = 1 + \Theta_1 L^s + \dots + \Theta_Q L^{Qs}$ : Seasonal MA
- ▣  $(1-L)^d$ :
  - ▶ Regular differencing;  $(1-L^s)^D$ : Seasonal differencing



## Proof: Multiplicative Seasonal Structure

**Why multiplicative?** Consider  $\text{SARIMA}(1, 0, 0) \times (1, 0, 0)_s$ :

$$(1 - \phi L)(1 - \Phi L^s)Y_t = \varepsilon_t$$

**Expand:**  $(1 - \phi L)(1 - \Phi L^s)Y_t = Y_t - \phi Y_{t-1} - \Phi Y_{t-s} + \phi\Phi Y_{t-s-1}$

### Interpretation (Monthly, $s = 12$ )

$Y_t$  depends on:  $Y_{t-1}$  (last month),  $Y_{t-12}$  (same month last year),  $Y_{t-13}$  (interaction).

**Parsimony:** Multiplicative form uses 2 parameters ( $\phi, \Phi$ ); additive would need 3+.

## SARIMA Notation

### Full Specification

$\text{SARIMA}(p, d, q) \times (P, D, Q)_s$  has 7 parameters to specify:

Parameter	Meaning
$p$	Non-seasonal AR order
$d$	Non-seasonal differencing order
$q$	Non-seasonal MA order
$P$	Seasonal AR order
$D$	Seasonal differencing order
$Q$	Seasonal MA order
$s$	Seasonal period

### Example

$\text{SARIMA}(1, 1, 1) \times (1, 1, 1)_{12}$ : Monthly data with AR(1), MA(1), seasonal AR(1), seasonal MA(1), and both regular and seasonal differencing.

## Common SARIMA Models

Airline Model:  $\text{SARIMA}(0, 1, 1) \times (0, 1, 1)_s$

$(1 - L)(1 - L^s)Y_t = (1 + \theta L)(1 + \Theta L^s)\varepsilon_t$  - Classic model (Box & Jenkins, 1970)

$\text{SARIMA}(1, 0, 0) \times (1, 0, 0)_s$

$(1 - \phi L)(1 - \Phi L^s)Y_t = \varepsilon_t$  - Pure seasonal and non-seasonal AR

$\text{SARIMA}(0, 1, 1) \times (0, 1, 0)_s$

$(1 - L)(1 - L^s)Y_t = (1 + \theta L)\varepsilon_t$  - Random walk + seasonal diff + MA(1)

## ACF/PACF for Seasonal Models

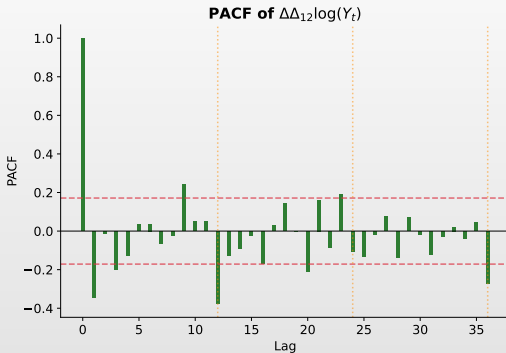
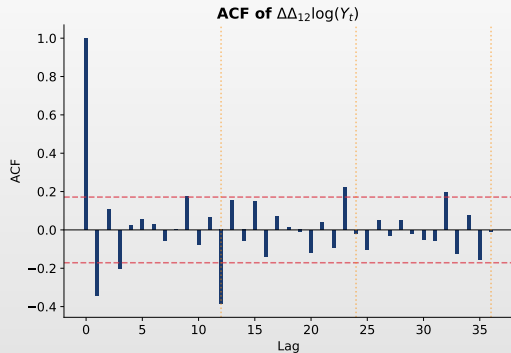
### Key Insight

Seasonal models show patterns at both:

- Non-seasonal lags:  $1, 2, 3, \dots$
- Seasonal lags:  $s, 2s, 3s, \dots$

Model	ACF	PACF
SAR( $P$ )	Decays at $s, 2s, \dots$	Cuts off after $P_s$
SMA( $Q$ )	Cuts off after $Q_s$	Decays at $s, 2s, \dots$
SARMA	Decays at seasonal lags	Decays at seasonal lags

## Example: Airline Model ACF/PACF



## Example: Airline Model ACF/PACF

ACF:  $\Delta\Delta_{12} \log(Y_t)$

- ▣ Spike at lag 1  $\leftarrow$  MA(1),  $\theta$
- ▣ Spike at lag 12  $\leftarrow$  SMA(1),  $\Theta$
- ▣ Rest  $\approx$  zero

PACF: exponential decay

- ▣ Decays at lags 1, 2, 3, ...
- ▣ Decays at lags 12, 24, 36
- ▣  $\Rightarrow$  **MA, not AR**

▣ **Conclusion:** ACF cuts off  $\Rightarrow$  MA; PACF decays  $\Rightarrow$  not AR. Model:  $(0, 1, 1) \times (0, 1, 1)_{12}$

## Model Identification Guidelines

### Step-by-Step Process

1. Examine ACF for slow decay at seasonal lags  $\Rightarrow$  seasonal differencing
2. After differencing, check ACF/PACF patterns
3. Non-seasonal behavior at lags  $1, 2, \dots, s - 1$
4. Seasonal behavior at lags  $s, 2s, 3s, \dots$

### Practical Tips

- Start with  $d \leq 1$  and  $D \leq 1$
- Usually  $P, Q \leq 2$  is sufficient
- Use information criteria (AIC, BIC) for final selection
- Auto-SARIMA algorithms can help

## Estimation Methods

### Maximum Likelihood Estimation

Standard approach for SARIMA:

- ▣ Conditional MLE (conditional on initial values)
- ▣ Exact MLE (via Kalman filter)

### Computational Considerations

- ▣ More parameters than ARIMA  $\Rightarrow$  more data needed
- ▣ Seasonal parameters estimated from lags  $s, 2s, \dots$
- ▣ Need sufficient seasonal cycles (at least 3-4 years of monthly data)



## Exact Likelihood: Prediction Error Decomposition

### Why the Kalman Filter?

- ▣ **SARIMA**: has the structure of a state-space model
- ▣ **Kalman filter**: recursively computes prediction errors  $v_t$  and their variances  $f_t$ , without conditioning on initial values

### Exact Log-Likelihood (Prediction Error Decomposition)

- ▣ **Formula**:  $\ell(\theta) = -\frac{T}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T \left[ \ln(f_t) + \frac{v_t^2}{f_t} \right]$
- ▣  $v_t$ :  $Y_t - \hat{Y}_{t|t-1}$  (innovation);  $f_t$ :  $\text{Var}(v_t)$  (innovation variance)

### Advantages over Conditional MLE

- ▣ Does not require choosing initial values
- ▣ Each term  $\ln(f_t)$  weights observations differently (variable variance at start)
- ▣ Essential for short series where initial values matter
- ▣ Implemented by default in `statsmodels.tsa.SARIMAX()` with `method='mle'`

## Stationarity and Invertibility

### Stationarity Conditions

Both non-seasonal and seasonal AR polynomials must have roots outside the unit circle:

- ▣  $\phi(z) = 0 \Rightarrow |z| > 1$
- ▣  $\Phi(z^s) = 0 \Rightarrow |z| > 1$

### Invertibility Conditions

Both non-seasonal and seasonal MA polynomials must have roots outside the unit circle:

- ▣  $\theta(z) = 0 \Rightarrow |z| > 1$
- ▣  $\Theta(z^s) = 0 \Rightarrow |z| > 1$

## Diagnostic Checking

### Residual Analysis

After fitting SARIMA, check that residuals are white noise:

1. Plot residuals over time (no patterns)
2. ACF of residuals (no significant spikes)
3. Ljung-Box test at multiple lags including seasonal
4. Normality tests (Q-Q plot, Jarque-Bera)

### Important

Check ACF at **both** non-seasonal and seasonal lags!

Significant ACF at lag 12 suggests inadequate seasonal modeling.

## Model Selection Criteria

### Information Criteria

Compare competing SARIMA models using:

- ▣  $AIC = -2 \ln(L) + 2k$
- ▣  $BIC = -2 \ln(L) + k \ln(n)$
- ▣  $AICc = AIC + \frac{2k(k+1)}{n-k-1}$  (corrected for small samples)

where  $k = p + q + P + Q + 1$  (plus 1 for variance).

### Auto-SARIMA

Python's `pmdarima.auto_arima()` with `seasonal=True` automatically searches for optimal  $(p, d, q) \times (P, D, Q)_s$ .

## Hyndman-Khandakar Algorithm (`auto_arima`)

How does automatic selection work? (Hyndman & Khandakar, 2008)

1.  $d$ : successive KPSS tests ( $d = 0, 1, 2$ );  $D$ : OCSB or Canova-Hansen test ( $D = 0, 1$ )
2. **Stepwise search**: starts from initial model, explores neighboring models
3. **Criterion**: AICc (correct for small samples)

### Search Strategy

- ▣ **Initial model**: SARIMA( $2, d, 2$ )( $1, D, 1$ )<sub>s</sub> or SARIMA( $0, d, 0$ )( $0, D, 0$ )<sub>s</sub>
- ▣ **Variations tested**:  $\pm 1$  for each order ( $p, q, P, Q$ ); stops when no neighbor improves AICc
- ▣ **Complexity**:  $O(20-30)$  models evaluated (vs.  $O(k^4)$  for grid search)

Python: `pm.auto_arima(y, seasonal=True, m=12, stepwise=True, trace=True)`

- ▣ Set `stepwise=False` for exhaustive search (slower, sometimes better)

## Point Forecasts

### Forecast Computation

SARIMA forecasts are computed recursively:

- ▣ Replace future  $\varepsilon_{T+h}$  with 0
- ▣ Replace future  $Y_{T+h}$  with forecasts  $\hat{Y}_{T+h|T}$
- ▣ Use known past values  $Y_T, Y_{T-1}, \dots$

### Seasonal Pattern in Forecasts

SARIMA forecasts naturally capture seasonality:

- ▣ Short-term: influenced by recent values
- ▣ Long-term: revert to seasonal pattern

## Forecast Intervals

### Uncertainty Quantification

$(1 - \alpha)\%$  prediction interval:

$$\hat{Y}_{T+h|T} \pm z_{\alpha/2} \sqrt{\text{Var}(e_{T+h})}$$

Variance computed from  $\text{MA}(\infty)$  representation.

### Key Properties

- ▣ Intervals widen with forecast horizon
- ▣ For  $I(1, 1)_s$  series: intervals grow without bound
- ▣ Seasonal pattern visible in point forecasts
- ▣ Uncertainty captures both trend and seasonal variation

## Long-Horizon Forecasts

### Behavior as $h \rightarrow \infty$

- ▣ Point forecasts converge to deterministic seasonal pattern
- ▣ If drift present: linear trend + seasonal pattern
- ▣ Forecast intervals continue to widen

### Practical Implication

- ▣ Short-term: SARIMA captures both short-term dynamics and season
- ▣ Medium-term: Good seasonal forecasts, growing uncertainty
- ▣ Long-term: Mainly reflects seasonal pattern, wide intervals



## The Seasonal Naive Benchmark

### Definition: Seasonal Naive Forecast

- ▣ **Formula:**  $\hat{Y}_{T+h} = Y_{T+h-s}$  (last observed season)
- ▣ **Monthly example:** Forecast for March 2025 = value from March 2024
- ▣ **Interpretation:** “The simplest model that respects seasonality”

### Why is it essential?

- ▣ Any SARIMA model **must** outperform the seasonal naive benchmark
- ▣ If it doesn't  $\Rightarrow$  the model complexity is not justified
- ▣ Surprisingly effective for many series with stable seasonality

### Golden Rule

- ▣ **Always** report SARIMA performance relative to seasonal naive
- ▣ This is the **first thing** a reviewer or manager checks

## The MASE Metric: Proper Evaluation for Seasonal Series

MASE — Mean Absolute Scaled Error (Hyndman & Koehler, 2006)

- **Formula:** 
$$\text{MASE} = \frac{\frac{1}{H} \sum_{h=1}^H |e_{T+h}|}{\frac{1}{T-s} \sum_{t=s+1}^T |Y_t - Y_{t-s}|}$$
- **Numerator:** mean absolute error of the model
- **Denominator:** mean absolute error of seasonal naive (on training data)

### Interpretation

- $\text{MASE} < 1$ : Model is **better** than seasonal naive
- $\text{MASE} = 1$ : Model is **equivalent** to seasonal naive
- $\text{MASE} > 1$ : Model is **worse** — abandon it!

### Why MASE and not MAPE?

- MAPE: undefined for  $Y_t = 0$ ; asymmetric; scale-dependent
- MASE: works with any data; symmetric; comparable across different series

## Forecast Evaluation: Rolling Forecast Origin

### Cross-Validation for Seasonal Time Series

- ▣ **Principle:** re-estimate model  $\rightarrow$  forecast  $h$  steps  $\rightarrow$  advance 1 step  $\rightarrow$  repeat
- ▣ **Fixed window:** training on last  $w$  observations (constant size)
- ▣ **Expanding window:** training from beginning to  $T + i$  (grows)

### Step-by-step procedure

1. Train SARIMA on  $Y_1, \dots, Y_T$ ; forecast  $\hat{Y}_{T+1}, \dots, \hat{Y}_{T+h}$
2. Train SARIMA on  $Y_1, \dots, Y_{T+1}$ ; forecast  $\hat{Y}_{T+2}, \dots, \hat{Y}_{T+h+1}$
3. ... repeat  $N$  times; compute RMSE, MAE, MASE on all  $N$  forecasts

### Important

- ▣ Minimum  $N \geq 2s$  origins (2 complete seasonal cycles) for reliable results
- ▣ Never “look ahead” — test data is strictly after training data

## SARIMA vs Holt-Winters/ETS: When to Use Which?

### Comparison

Criterion	SARIMA	ETS / Holt-Winters
Approach	Box-Jenkins (ACF/PACF)	Exponential smoothing
Seasonality	Stochastic (differencing)	Additive or multiplicative
Interpretation	AR/MA coefficients	Smoothing weights $\alpha, \beta, \gamma$
Flexibility	Very flexible (7 params.)	Less flexible
Automation	<code>auto_arima</code>	<code>ets()</code> / <code>ExponentialSmoothing</code>

### Practical Selection Guide

- ▣ **SARIMA preferred:** series with complex autocorrelation, stochastic seasonality, ARMA components
- ▣ **ETS preferred:** short series, stable seasonality, quick forecasts without diagnostics
- ▣ **Best:** compare both on out-of-sample data and choose the winner

## Case Study: Airline Passengers Data

- Classic Box-Jenkins dataset: monthly airline passengers (1949-1960)
- Clear upward trend and increasing seasonal amplitude
- Multiplicative seasonality suggests log transformation



## Data Splitting Strategy

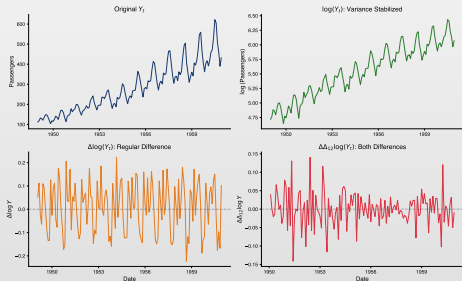
### Time Series Train/Validation/Test Split



- ▣ **Training set (70%)** — Fit model parameters; estimate SARIMA coefficients ( $\phi$ ,  $\theta$ ,  $\Phi$ ,  $\Theta$ )
- ▣ **Validation set (15%)** — Select best model; compare candidates, choose lowest validation error
- ▣ **Test set (15%)** — Final evaluation; unbiased out-of-sample performance, never used during development

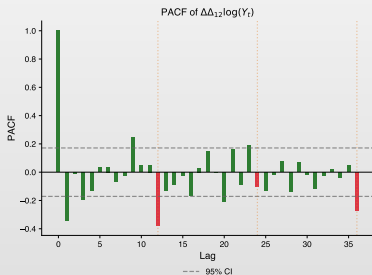
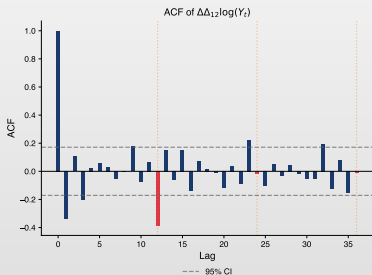
## Step 1: Transformations

- Log transform stabilizes variance (multiplicative  $\Rightarrow$  additive)
- First difference removes trend; seasonal difference removes seasonality
- Double-differenced series appears stationary



## Step 2: ACF/PACF Analysis

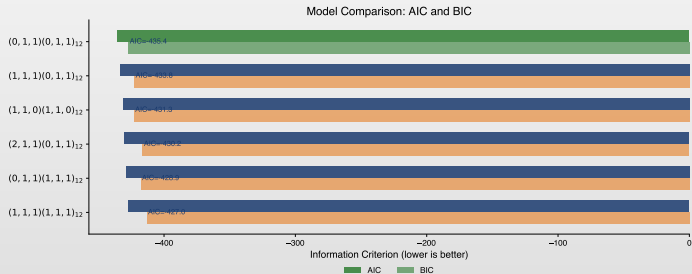
- ACF: Significant spike at lag 1 and lag 12  $\Rightarrow$  MA(1), SMA(1)
- PACF: Exponential decay pattern confirms MA structure
- Suggests SARIMA(0, 1, 1)  $\times$  (0, 1, 1)<sub>12</sub> (airline model)





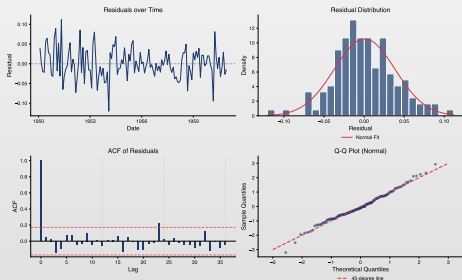
## Step 3: Model Comparison

- Compare candidate SARIMA models using AIC criterion
- SARIMA(0, 1, 1)  $\times$  (0, 1, 1)<sub>12</sub> provides best fit (lowest AIC)
- This is the famous “airline model” identified by Box & Jenkins



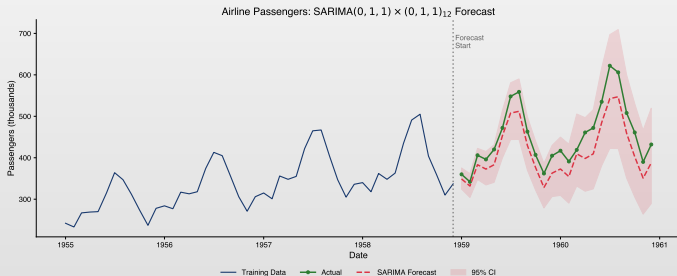
## Step 4: Residual Diagnostics

- Residuals appear random with no remaining autocorrelation
- Q-Q plot shows approximate normality
- Model adequately captures both trend and seasonal structure



## Step 5: Forecasting

- 24-month forecast with 95% confidence interval
- Model captures seasonal pattern and upward trend
- Prediction intervals widen appropriately with forecast horizon



## Practical Pitfalls in SARIMA Modeling

### 1. Over-differencing

- ▣ **Symptom:** ACF at lag 1  $\approx -0.5$  (regular) or at lag  $s \approx -0.5$  (seasonal)
- ▣ **Cause:** applying  $(1 - L)$  or  $(1 - L^s)$  too many times
- ▣ **Solution:** reduce  $d$  or  $D$  by 1 and re-examine ACF/PACF

### 2. Insufficient Data

- ▣ **Minimum:** 3–4 complete seasonal cycles (36–48 monthly obs.); **recommended:** 5+ cycles
- ▣ Seasonal parameters  $\Phi, \Theta$  are estimated from lags  $s, 2s, 3s, \dots$

### 3. Other Common Pitfalls

- ▣ **Root cancellation:**  $\phi \approx \theta$  suggests over-parameterization
- ▣ **Parameters at invertibility boundary:**  $|\theta| \approx 1$  or  $|\Theta| \approx 1$  indicates problems
- ▣ **Forgetting inverse transformation:** forecasts on log scale must be back-transformed!

## X-13ARIMA-SEATS: Official Seasonal Adjustment

### What is seasonal adjustment?

- ▣ **Goal:** remove the seasonal component to reveal the true trend
- ▣ **Users:** Eurostat, US Census Bureau, central banks, national statistical offices
- ▣ **Example:** “GDP grew 0.3% compared to previous quarter” (seasonally adjusted data)

### X-13ARIMA-SEATS (US Census Bureau)

- ▣ **Step 1:** Identify and estimate a regARIMA model (SARIMA + calendar effects)
- ▣ **Step 2:** Extract the seasonal component via SEATS or X-11 filters
- ▣ **Step 3:**  $Y_t^{adjusted} = Y_t - \hat{S}_t$  (additive) or  $Y_t^{adjusted} = Y_t / \hat{S}_t$  (multiplicative)

### Why does it matter for economists?

- ▣ Published macroeconomic data is almost always seasonally adjusted
- ▣ Misinterpreting unadjusted data can lead to erroneous conclusions

## AI Exercise: Critical Thinking

Prompt to test in ChatGPT / Claude / Copilot

"I have the AirPassengers dataset from statsmodels (monthly data, international airline passengers, 1949–1960, 144 obs.). Identify seasonality, apply Box-Cox transform if needed, estimate a SARIMA model, and forecast 12 months. Give me complete Python code with plots."

### Exercise:

1. Run the prompt in an LLM of your choice and critically analyze the response.
2. Does it check seasonality with ACF at lags  $s, 2s, 3s$ ? Does it use STL decomposition?
3. Does it apply Box-Cox *before* differencing? Does it justify the choice of  $\lambda$ ?
4. How does it choose orders  $(p, d, q) \times (P, D, Q)_s$ ? Only `auto_arima` or also ACF/PACF?
5. Does it evaluate with MASE relative to seasonal naïve? Does it use rolling forecast?

**Warning:** AI-generated code may run without errors and look professional. *That does not mean it is correct.*

## Summary

### What We Learned in This Chapter

- Seasonality in time series
  - ▶ Repetitive patterns at regular intervals; additive vs multiplicative
- Seasonal differencing and Box-Cox transformation
  - ▶  $(1 - L^s)$  removes stochastic seasonality; Box-Cox stabilizes variance
- SARIMA( $p, d, q$ )  $\times$  ( $P, D, Q$ )<sub>s</sub> models
  - ▶ Extend ARIMA with seasonal components; automatic selection via `auto_arima`
- Forecasting and evaluation
  - ▶ Benchmark: MASE relative to seasonal naive; rolling forecast out-of-sample

### Key Idea

- **Parsimony principle:** The Airline Model  $(0, 1, 1) \times (0, 1, 1)_{12}$  with only 2 parameters is remarkably effective for many seasonal economic series.

## What's Next?

### Chapter 5: Volatility Modeling — GARCH

- ▣ **Volatility:** conditional variation of financial returns
- ▣ **ARCH/GARCH:** models for conditional variance
- ▣ **Asymmetric extensions:** GJR-GARCH, EGARCH (leverage effect)
- ▣ **VaR:** Value-at-Risk based on GARCH models
- ▣ **Case study:** S&P 500 returns volatility

Questions?



## Question 1

### Question

□ For monthly data with annual seasonality, what is the seasonal period  $s$ ?

### Answer Choices

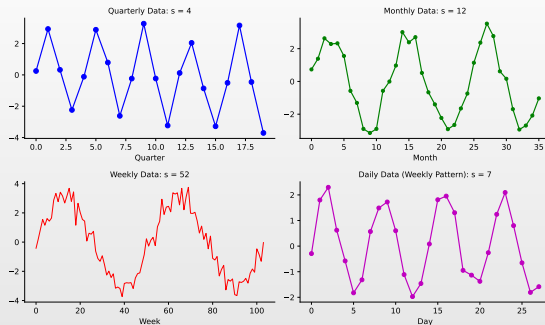
(A)  $s = 4$

(B)  $s = 7$

(C)  $s = 12$

(D)  $s = 52$

## Question 1: Answer



Answer: (C)

- ☐  $s = 12$  (12 months per year). Common periods: Quarterly=4, Monthly=12, Weekly=52, Daily=7, Hourly=24

## Question 2

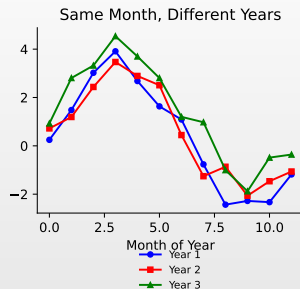
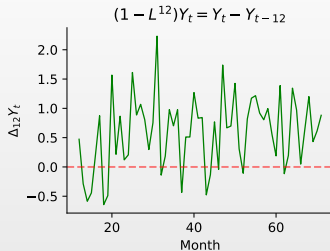
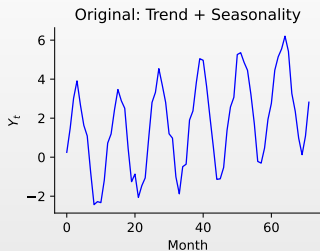
### Question

□ What does the seasonal difference operator  $(1 - L^{12})$  do to a monthly series?

### Answer Choices

- (A) Computes  $Y_t - Y_{t-1}$  (month-to-month change)
- (B) Computes  $Y_t - Y_{t-12}$  (year-over-year change)
- (C) Computes the 12-month moving average
- (D) Removes the trend component only

## Question 2: Answer



Answer: (B)

□  $(1 - L^{12})Y_t = Y_t - Y_{t-12}$  removes the seasonal pattern by comparing same months.

### Question 3

#### Question

□ In  $SARIMA(1, 1, 1) \times (1, 1, 1)_{12}$  notation, what does the  $(1, 1, 1)_{12}$  part represent?

#### Answer Choices

- (A) AR(1), differencing once, MA(1) at the regular level
- (B) Seasonal AR(1), seasonal differencing once, seasonal MA(1)
- (C) 12 AR terms, 12 differences, 12 MA terms
- (D) The model has 12 parameters in total

### Question 3: Answer

Answer: (B)

- Seasonal AR(1), seasonal differencing once, seasonal MA(1)

#### SARIMA Notation Breakdown

$\text{SARIMA}(p, d, q) \times (P, D, Q)_s$ :

$(p, d, q)$  Non-seasonal: AR( $p$ ),  $d$  differences, MA( $q$ )

$(P, D, Q)_s$  Seasonal: SAR( $P$ ),  $D$  seasonal diffs, SMA( $Q$ )

For  $(1, 1, 1) \times (1, 1, 1)_{12}$ :

- Non-seasonal: AR(1), one regular difference, MA(1)
- Seasonal: SAR(1) at lag 12, one  $\Delta_{12}$ , SMA(1) at lag 12

## Question 4

## Question

- ☐ The “Airline Model” is  $\text{SARIMA}(0, 1, 1) \times (0, 1, 1)_{12}$ . How many parameters need to be estimated (excluding variance)?

## Answer Choices

- (A) 1
- (B) 2
- (C) 4
- (D) 12

## Question 4: Answer

Answer: (B)

- ▣ 2 parameters: SARIMA(0, 1, 1)  $\times$  (0, 1, 1)<sub>12</sub>:  $(1 - L)(1 - L^{12})Y_t = (1 + \theta_1 L)(1 + \Theta_1 L^{12})\varepsilon_t$
- ▣ Parameters:  $\theta_1$  (non-seasonal MA) and  $\Theta_1$  (seasonal MA), plus  $\sigma^2$ .

### Why “Airline Model”?

Box & Jenkins (1970) used this model to forecast international airline passengers. Remarkably effective for many seasonal economic series!



## Question 5

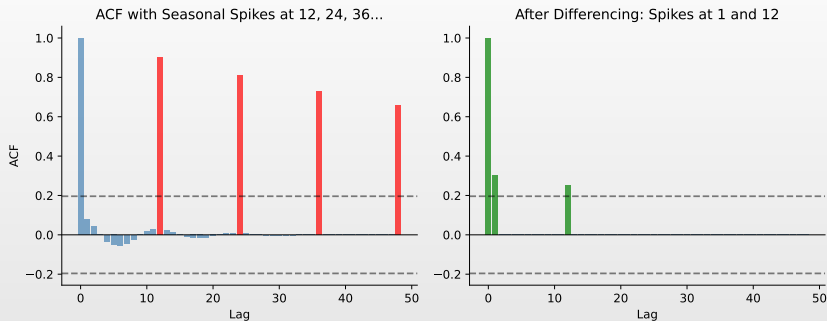
### Question

- ☐ You observe significant ACF spikes at lags 12, 24, and 36 in a monthly series. What does this suggest?

### Answer Choices

- (A) The series has a unit root
- (B) The series has annual seasonality that needs seasonal differencing
- (C) The series follows an AR(36) process
- (D) The series is already stationary

## Question 5: Answer



Answer: (B)

☐ ACF spikes at 12, 24, 36 = stochastic seasonality. Apply  $(1 - L^{12})$  to remove it.

## Question 6

### Question

- ☐ After applying  $(1 - L)(1 - L^{12})$  to a monthly series, the ACF shows a significant spike only at lag 1 and lag 12. What SARIMA model is suggested?

### Answer Choices

- (A)  $\text{SARIMA}(1, 1, 0) \times (1, 1, 0)_{12}$
- (B)  $\text{SARIMA}(0, 1, 1) \times (0, 1, 1)_{12}$
- (C)  $\text{SARIMA}(1, 1, 1) \times (1, 1, 1)_{12}$
- (D)  $\text{SARIMA}(0, 1, 0) \times (0, 1, 0)_{12}$

## Question 6: Answer

Answer: (B)

- ▣ **Model:**  $\text{SARIMA}(0, 1, 1) \times (0, 1, 1)_{12}$  (The Airline Model)

### ACF/PACF Identification Rules

- ▣ **Rule:** for MA processes, ACF cuts off after lag  $q$
- ▣ **ACF spike at lag 1:** MA(1) for non-seasonal part
- ▣ **ACF spike at lag 12:** SMA(1) for seasonal part
- ▣ **Combined:**  $\text{MA}(1) \times \text{SMA}(1) = (0, d, 1) \times (0, D, 1)_{12}$
- ▣ **With**  $d = 1, D = 1$ :  $(0, 1, 1) \times (0, 1, 1)_{12}$

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### Seasonal Decomposition and Diagnostics

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### Online Resources and Code

- ▣ **Quantlet**: <https://quantlet.com> – Code platform for quantitative methods
- ▣ **Quantinar**: <https://quantinar.com> – Learning platform for quantitative methods
- ▣ **GitHub TSA**: [https://github.com/QuantLet/TSA/tree/main/TSA\\_ch4](https://github.com/QuantLet/TSA/tree/main/TSA_ch4) – Python code for this chapter

# Thank You!

## Questions?

Course materials available at: <https://danpele.github.io/Time-Series-Analysis/>



Quantlet



Quantinar