Time Series Analysis and Forecasting

Seminar 0: Fundamentals

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Seminar Outline

Today's Activities:

1. **Quick Review** — Key concepts summary
2. **Multiple Choice Quiz** — Test your understanding
3. **True/False Questions** — Conceptual checks
4. **Calculation Exercises** — Hands-on practice
5. **Key Formulas** — Reference sheet

Key Formulas to Remember

Decomposition:

- Additive: $X_t = T_t + S_t + \varepsilon_t$
- Multiplicative: $X_t = T_t \times S_t \times \varepsilon_t$

Exponential Smoothing:

- SES: $\hat{X}_{t+1} = \alpha X_t + (1-\alpha)\hat{X}_t$
- Holt: adds trend b_t
- HW: adds seasonality S_t

Error Metrics:

- $\text{MAE} = \frac{1}{n}\sum |e_t|$
- $\text{RMSE} = \sqrt{\frac{1}{n}\sum e_t^2}$
- $\text{MAPE} = \frac{100}{n}\sum \left|\frac{e_t}{X_t}\right|$

Moving Average:

- $\hat{T}_t = \frac{1}{2k+1}\sum_{j=-k}^{k} X_{t+j}$

Quiz 1: Time Series Basics

Question

Which of the following is NOT a characteristic of time series data?

A. Observations are ordered in time
B. Consecutive observations are usually correlated
C. Observations are independent and identically distributed
D. Data has a natural temporal ordering

Quiz 1: Answer

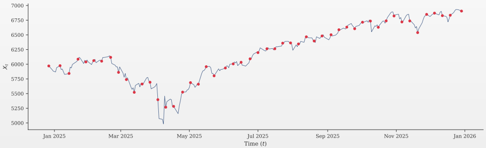
Answer: C

Observations are independent and identically distributed

Explanation:

Time series observations are typically **dependent** (autocorrelated), not independent.

This temporal dependence makes time series analysis unique.



Time series: temporally ordered observations

TSA_ch0_definition

Quiz 2: Decomposition

Question

When should you use multiplicative decomposition instead of additive?

A. When the seasonal pattern has constant amplitude

B. When the variance of the series is stable over time

C. When seasonal fluctuations grow proportionally with the level

D. When the time series has no trend component

Quiz 2: Answer

Answer: C

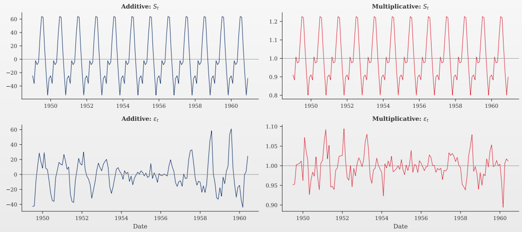
When seasonal fluctuations grow proportionally with the level

Multiplicative:

$$X_t = T_t \times S_t \times \varepsilon_t$$

Use when seasonal amplitude scales with level (fan pattern).

Additive: constant amplitude.



Left: Additive; Right: Multiplicative

TSA_ch0_additive_mult

Quiz 3: Exponential Smoothing

Question

In Simple Exponential Smoothing with $\alpha = 0.9$, what happens?

A. Forecasts are very smooth and stable

B. Recent observations have very little weight

C. Forecasts react quickly to recent changes

D. The forecast is essentially a long-term average

Quiz 3: Answer

Answer: C

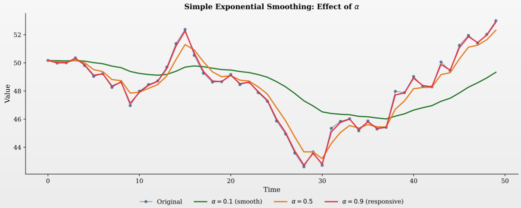
Forecasts react quickly to recent changes

With $\alpha = 0.9$:

$$\hat{X}_{t+1} = 0.9X_t + 0.1\hat{X}_t$$

90% weight on most recent observation!

Small $\alpha \Rightarrow$ smooth; Large $\alpha \Rightarrow$ reactive.



Effect of α: 0.1 vs 0.9

TSA_ch0_ses

Quiz 4: Moving Averages

Question

A centered moving average of order 5 (MA-5) uses which observations to estimate the trend at time t?

A. $X_t, X_{t+1}, X_{t+2}, X_{t+3}, X_{t+4}$

B. $X_{t-4}, X_{t-3}, X_{t-2}, X_{t-1}, X_t$

C. $X_{t-2}, X_{t-1}, X_t, X_{t+1}, X_{t+2}$

D. X_{t-1}, X_t, X_{t+1}

Quiz 4: Answer

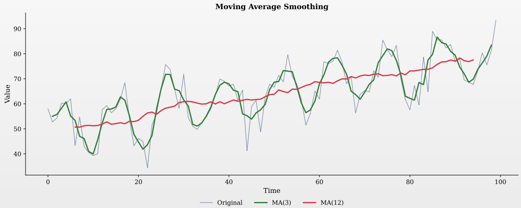
Answer: C

$X_{t-2}, X_{t-1}, X_t, X_{t+1}, X_{t+2}$

Centered MA-k uses $(k-1)/2$ observations on each side of t.

For MA-5: 2 before + t + 2 after.

Larger window $\Rightarrow$ smoother curve.



Moving averages with different windows

TSA_ch0_ma

Quiz 5: Forecast Evaluation

Question

Which metric is most appropriate for comparing forecast accuracy across different time series with different scales?

A. Mean Absolute Error (MAE)
B. Root Mean Squared Error (RMSE)
C. Mean Absolute Percentage Error (MAPE)
D. Mean Squared Error (MSE)

Quiz 5: Answer

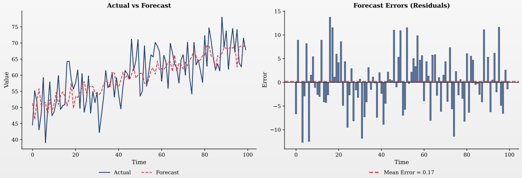
Answer: C

Mean Absolute Percentage Error (MAPE)

$$\text{MAPE} = \frac{100}{n} \sum \left| \frac{e_t}{X_t} \right|$$

MAPE expresses errors as **percentages**, being scale-independent.

Warning: fails when $X_t \approx 0$.



Comparison: MAE, RMSE, MAPE

TSA_ch0_forecast_eval

Quiz 6: Holt-Winters Method

Question

The Holt-Winters method differs from Holt's method by adding:

A. A seasonal component

B. A trend component

C. A cyclical component

D. An irregular component

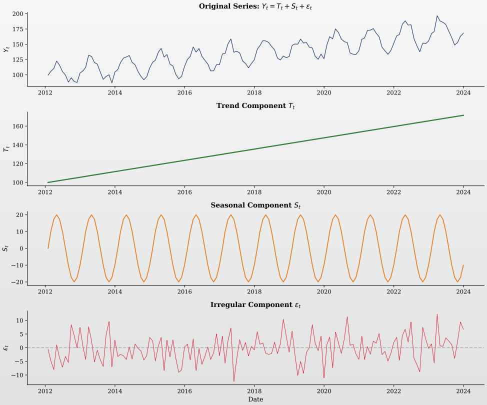
Quiz 6: Answer

Answer: A

A seasonal component

Progression of methods:

- SES: Level (α)
- Holt: Level + Trend (α, β)
- HW: Level + Trend + Season (α, β, γ)



Decomposition: Trend + Season + Residual

TSA_ch0_holt_winters

Quiz 7: Cross-Validation

Question

Why can't we use standard k-fold cross-validation for time series?

A. Time series data is too small
B. It would violate temporal ordering (future predicting past)
C. Cross-validation is always invalid
D. Time series doesn't need validation

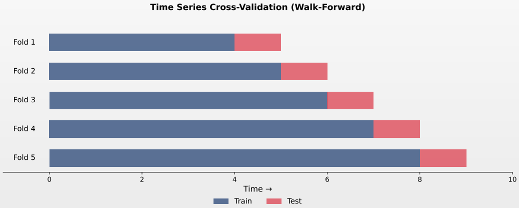
Quiz 7: Answer

Answer: B

It would violate temporal ordering (future predicting past)

Rule: Never use future data to predict the past!

Solution: Use rolling/expanding window CV.



Time series CV: expanding window

TSA_ch0_cv

Quiz 8: Seasonal Indices

Question

In multiplicative decomposition, if the seasonal index for December is 1.25, this means:

A. December sales are 1.25 units above average
B. December sales are 0.25 units above average
C. December sales are 25% above the trend level
D. December has 125 sales

Quiz 8: Answer

Answer: C

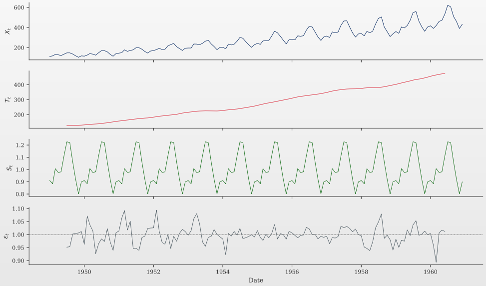
December sales are 25% above the trend level

In multiplicative model:

$$X_t = T_t \times S_t \times \varepsilon_t$$

$S_t = 1.25$ means 125% of trend, i.e., **25% above**.

Indices sum to m (seasons) or average to 1.



Seasonal pattern in airline data

TSA_ch0_seasonal

True or False? — Questions

Statement	T/F?
1. SES forecasts are always flat (constant for all horizons).	?
2. RMSE penalizes large errors more than MAE.	?
3. Multiplicative decomposition requires positive data.	?
4. A larger smoothing parameter α means more smoothing.	?
5. You should always use the test set for hyperparameter tuning.	?
6. Seasonal naive uses the value from one season ago.	?
7. Holt-Winters requires specifying the seasonal period.	?
8. MAPE can be infinite if actual values are zero.	?

True or False? — Answers

Statement	T/F	Explanation
1. SES forecasts are always flat (constant for all horizons).	T	**No trend component**
2. RMSE penalizes large errors more than MAE.	T	**Squared errors**
3. Multiplicative decomposition requires positive data.	T	**Can't multiply by negative**
4. A larger smoothing parameter α means more smoothing.	F	**Larger α = less smooth**
5. You should always use the test set for hyperparameter tuning.	F	**Use validation set!**
6. Seasonal naive uses the value from one season ago.	T	$\hat{x}_{t+h} = x_{t+h-m}$
7. Holt-Winters requires specifying the seasonal period.	T	**Must know m**
8. MAPE can be infinite if actual values are zero.	T	**Division by zero**

Exercise 1: Simple Exponential Smoothing

Problem: Given the following data and $\alpha = 0.3$:

t	1	2	3	4	5
X_t	10	12	11	14	13

Starting with $\hat{X}_1 = X_1 = 10$, calculate:

a) Forecasts $\hat{X}_2, \hat{X}_3, \hat{X}_4, \hat{X}_5$

b) Forecast for $t = 6$: $\hat{X}_6$

c) Forecast errors $e_t = X_t - \hat{X}_t$ for $t = 2, 3, 4, 5$

d) MAE and RMSE

Formula: $\hat{X}_{t+1} = \alpha X_t + (1 - \alpha)\hat{X}_t$

Exercise 1: Solution

Using $\hat{X}_{t+1} = 0.3X_t + 0.7\hat{X}_t$:

t	1	2	3	4	5	6
X_t	10	12	11	14	13	?
$\hat{X}_t$	10	10	10.6	10.72	11.70	**12.09**
e_t	–	2	0.4	3.28	1.30	–

Calculations:

- $\hat{X}_2 = 0.3(10) + 0.7(10) = 10$
- $\hat{X}_3 = 0.3(12) + 0.7(10) = 10.6$
- $\hat{X}_4 = 0.3(11) + 0.7(10.6) = 10.72$
- $\hat{X}_5 = 0.3(14) + 0.7(10.72) = 11.70$
- $\hat{X}_6 = 0.3(13) + 0.7(11.70) = \mathbf{12.09}$

$\textbf{MAE} = \frac{|2|+|0.4|+|3.28|+|1.30|}{4} = 1.745 \quad \textbf{RMSE} = \sqrt{\frac{4+0.16+10.76+1.69}{4}} = 2.04$

Exercise 2: Error Metrics

Problem: Given actual values and forecasts:

t	1	2	3	4
X_t	100	110	105	120
$\hat{X}_t$	95	108	110	115

Calculate:

a) Forecast errors $e_t = X_t - \hat{X}_t$
b) MAE (Mean Absolute Error)
c) MSE (Mean Squared Error)
d) RMSE (Root Mean Squared Error)
e) MAPE (Mean Absolute Percentage Error)

Exercise 2: Solution

Errors: $e_1 = 5,\ e_2 = 2,\ e_3 = -5,\ e_4 = 5$

a) MAE:

$$\text{MAE} = \frac{1}{4}(|5| + |2| + |-5| + |5|) = \frac{17}{4} = \mathbf{4.25}$$

b) MSE:

$$\text{MSE} = \frac{1}{4}(25 + 4 + 25 + 25) = \frac{79}{4} = \mathbf{19.75}$$

c) RMSE:

$$\text{RMSE} = \sqrt{19.75} = \mathbf{4.44}$$

d) MAPE:

$$\text{MAPE} = \frac{100}{4}\left(\frac{5}{100} + \frac{2}{110} + \frac{5}{105} + \frac{5}{120}\right) = \frac{100}{4}(0.05 + 0.018 + 0.048 + 0.042) = \mathbf{3.95\%}$$

Exercise 3: Seasonal Indices

Problem: Quarterly sales data shows the following seasonal indices (multiplicative model):

Quarter	Q1	Q2	Q3	Q4
Seasonal Index	0.85	1.05	0.90	1.20

The trend forecast for Q4 next year is $T = 1000$ units.

Calculate:

a) Verify that the seasonal indices are properly normalized

b) The seasonally adjusted forecast for Q4

c) If actual Q4 sales were 1150, what is the deseasonalized value?

Exercise 3: Solution

a) Normalization check:

$$\sum S_i = 0.85 + 1.05 + 0.90 + 1.20 = 4.00 \quad \checkmark$$

For 4 quarters, indices should sum to 4 (or average to 1).

b) Seasonally adjusted forecast:
In multiplicative model: $\hat{X}_{Q4} = T \times S_{Q4}$

$$\hat{X}_{Q4} = 1000 \times 1.20 = \textbf{1200 units}$$

c) Deseasonalized value:
To remove seasonality: $X_{deseasonalized} = \frac{X_t}{S_t}$

$$X_{deseasonalized} = \frac{1150}{1.20} = \textbf{958.33 units}$$

This is below the trend (1000), suggesting underlying performance was slightly weak despite the seasonal boost.

Key Formulas Summary

Method	Formula
Additive Decomposition	$X_t = T_t + S_t + \varepsilon_t$
Multiplicative Decomposition	$X_t = T_t \times S_t \times \varepsilon_t$
Simple Moving Average	$\hat{T}_t = \frac{1}{k} \sum_{i=0}^{k-1} X_{t-i}$
SES	$\hat{X}_{t+1} = \alpha X_t + (1-\alpha)\hat{X}_t$
Holt Level	$L_t = \alpha X_t + (1-\alpha)(L_{t-1} + b_{t-1})$
Holt Trend	$b_t = \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1}$
Holt Forecast	$\hat{X}_{t+h} = L_t + h \cdot b_t$
MAE	$\frac{1}{n} \sum_{t=1}^{n} \lvert e_t \rvert$
RMSE	$\sqrt{\frac{1}{n} \sum_{t=1}^{n} e_t^2}$
MAPE	$\frac{100}{n} \sum_{t=1}^{n} \left\lvert \frac{e_t}{X_t} \right\rvert$

Questions?

Good luck with the exercises!