



Time Series Analysis and Forecasting

Chapter 9: Prophet & TBATS

Seminar



Seminar Outline

- 1 Review Quiz
- 2 True/False Questions
- 3 Practice Problems
- 4 Worked Examples
- 5 Discussion Topics
- 6 Exercises for Self-Study

Quiz 1: Multiple Seasonality Challenge

Question

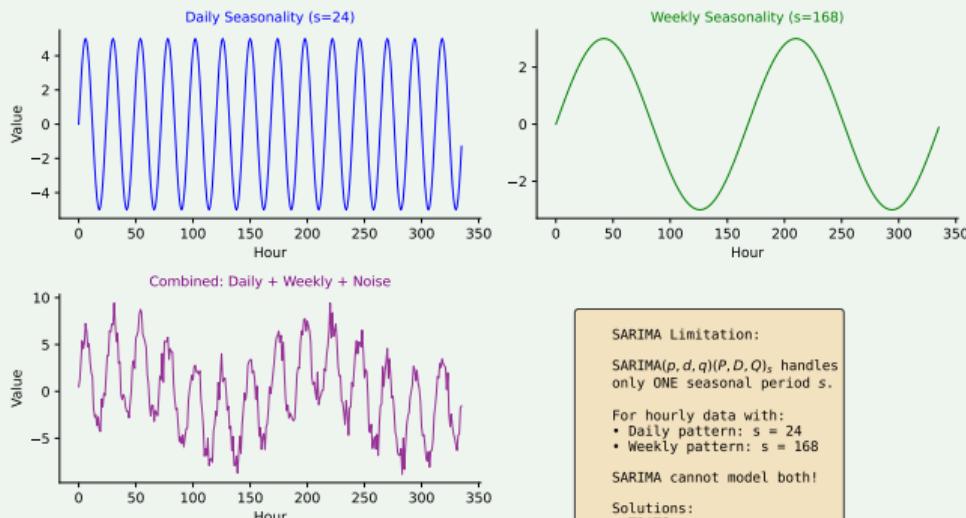
Why can't standard SARIMA handle hourly electricity demand data?

- (A) SARIMA can only handle monthly data
- (B) SARIMA allows only one seasonal period (m parameter)
- (C) SARIMA doesn't support trend components
- (D) SARIMA requires normally distributed data

Answer on next slide...

Quiz 1: Answer

Answer: B – SARIMA allows only one seasonal period



Key: Hourly data has daily (24h), weekly (168h), and annual (8760h) patterns. SARIMA's single m parameter cannot capture all these simultaneously.

Quiz 2: TBATS Acronym

Question

What does TBATS stand for?

- A) Trend, Baseline, ARMA, Transform, Seasonal
- B) Trigonometric, Box-Cox, ARMA, Trend, Seasonal
- C) Time-Based Automatic Time Series
- D) Temporal Bayesian Adaptive Trend System

Answer on next slide...

Quiz 2: Answer

Answer: B – Trigonometric, Box-Cox, ARMA, Trend, Seasonal

TBATS: What Does It Stand For?



Trigonometric

Fourier terms for seasonality
 $\sum [a_n \cos(\frac{2\pi n t}{m}) + b_n \sin(\frac{2\pi n t}{m})]$

Box-Cox

Variance stabilization
 $y^{(\omega)} = (y^\omega - 1)/\omega$

ARMA

Error autocorrelation
 $\phi(L)d_t = \theta(L)\varepsilon_t$

Trend

Level + slope (possibly damped)
 $l_t = l_{t-1} + \phi b_{t-1}$

Seasonal

Multiple seasonal periods
 m_1, m_2, \dots, m_T

TBATS components:

- Trigonometric: Fourier terms for seasonality
- Box-Cox: Variance stabilization
- ARMA: Error autocorrelation

Quiz 3: Fourier Terms

Question

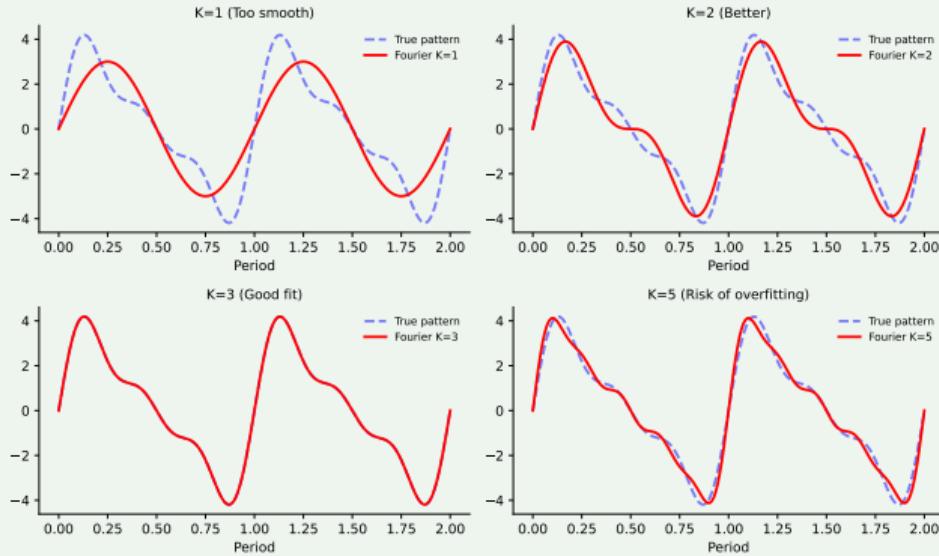
In TBATS, increasing the number of Fourier harmonics (K) for a seasonal pattern:

- A) Always improves forecast accuracy
- B) Allows more flexible (complex) seasonal shapes
- C) Reduces the model complexity
- D) Eliminates the need for Box-Cox transformation

Answer on next slide...

Quiz 3: Answer

Answer: B – Allows more flexible seasonal shapes



Trade-off: More harmonics = more flexibility but also more parameters.

$$s_t^{(i)} = \sum_{j=1}^{K_i} \left[a_j^{(i)} \cos \left(\frac{2\pi j t}{m_i} \right) + b_j^{(i)} \sin \left(\frac{2\pi j t}{m_i} \right) \right]$$

Quiz 4: Prophet Decomposition

Question

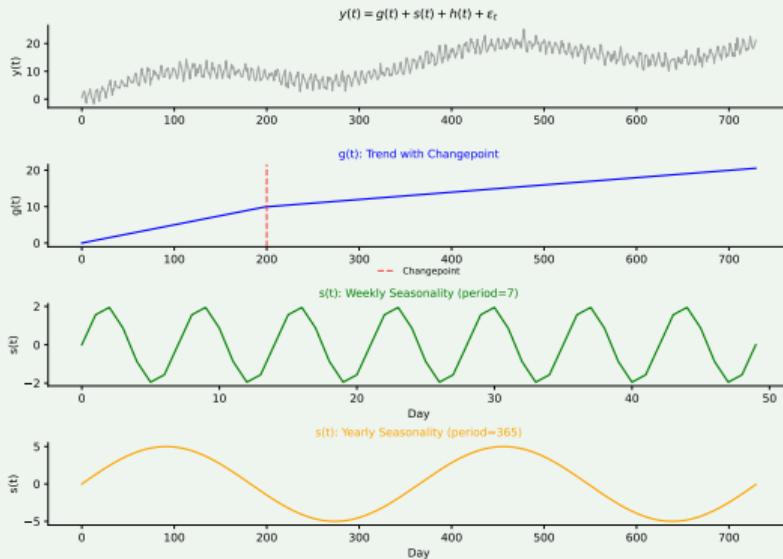
Prophet decomposes a time series into which components?

- (A) AR, MA, and seasonal components
- (B) Trend, seasonality, holidays, and error
- (C) Mean, variance, and autocorrelation
- (D) Level, slope, and curvature

Answer on next slide...

Quiz 4: Answer

Answer: B – Trend, seasonality, holidays, and error



Prophet model: $y(t) = g(t) + s(t) + h(t) + \varepsilon_t$

- $g(t)$: Trend (piecewise linear or logistic growth)
- $s(t)$: Seasonality (Fourier series)
- $h(t)$: Holiday effects

Quiz 5: Prophet vs TBATS

Question

When would you choose Prophet over TBATS?

- (A) When you need automatic model selection
- (B) When you have known holidays and changepoints to incorporate
- (C) When you need the most parsimonious model
- (D) When your data has no trend

Answer on next slide...

Quiz 5: Answer

Answer: B – Known holidays and changepoints

TBATS vs Prophet: Head-to-Head Comparison

Feature	TBATS	Prophet
Multiple seasonalities	Yes (automatic)	Yes (manual/auto)
Holiday effects	No	Yes (built-in)
External regressors	No	Yes
Trend changepoints	No (smooth)	Yes (automatic)
Missing data	Needs interpolation	Handles natively
Interpretability	Moderate	High
Computation speed	Slow	Fast
High-frequency data	Good	Moderate
Non-integer periods	Yes (e.g., 365.25)	Yes
Best for	Technical/high-freq	Business/daily

Prophet advantages: Easy holiday integration, analyst-in-the-loop, handles missing data, interpretable components.

TBATS advantages: Automatic model selection, handles complex seasonality without domain expertise.

Quiz 6: Seasonality Mode

Question

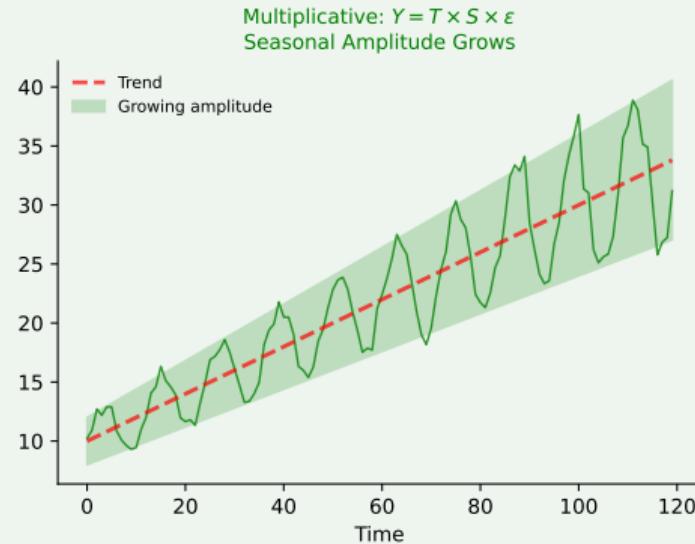
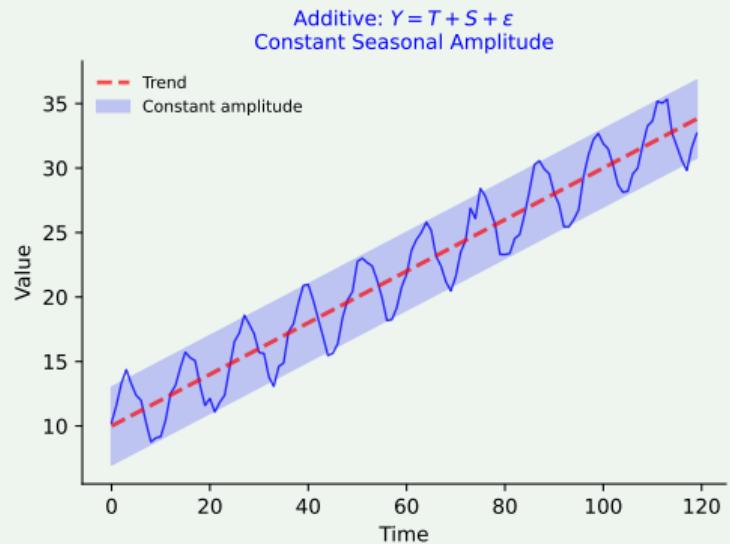
For retail sales data where December sales are 3x the monthly average, which seasonality mode is more appropriate in Prophet?

- (A) Additive seasonality
- (B) Multiplicative seasonality
- (C) Both work equally well
- (D) Neither—use ARIMA instead

Answer on next slide...

Quiz 6: Answer

Answer: B – Multiplicative seasonality



Key: When seasonal amplitude scales with the level, use multiplicative.

Additive: $y = g(t) + s(t)$ (constant seasonal effect)

Multiplicative: $y = g(t) \cdot (1 + s(t))$ (proportional seasonal effect)

Quiz 7: Prophet Changepoints

Question

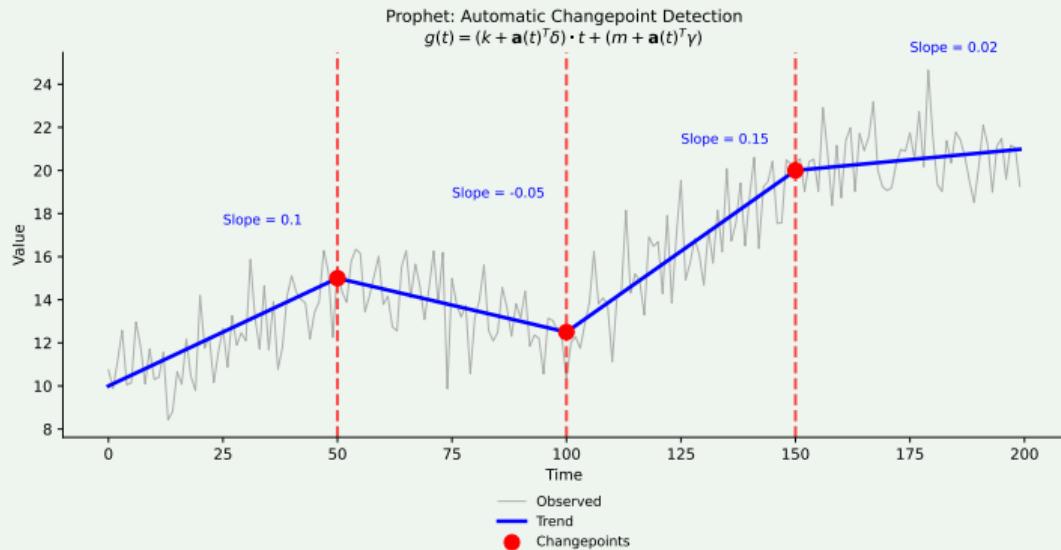
In Prophet, changepoints allow the model to:

- (A) Change the seasonal period automatically
- (B) Adjust the trend slope at specific points in time
- (C) Switch between additive and multiplicative modes
- (D) Detect and remove outliers

Answer on next slide...

Quiz 7: Answer

Answer: B – Adjust trend slope at specific points



Changepoints: Allow piecewise linear trend with different slopes.

$$g(t) = (k + \mathbf{a}(t)^T \delta) \cdot t + (m + \mathbf{a}(t)^T \gamma)$$

Prophet automatically detects changepoints or you can specify them manually.

Quiz 8: Model Selection

Question

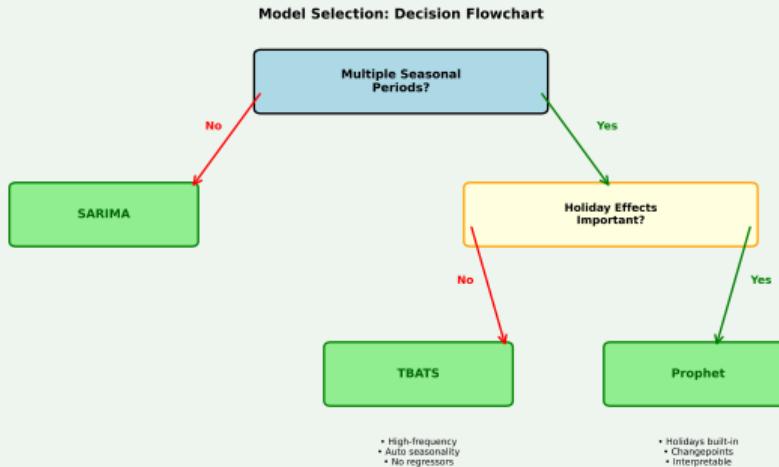
You have daily call center data with weekly seasonality only. Which model is most appropriate?

- (A) TBATS (designed for multiple seasonality)
- (B) Prophet (handles any seasonality well)
- (C) Standard SARIMA (simpler and sufficient)
- (D) LSTM neural network (most flexible)

Answer on next slide...

Quiz 8: Answer

Answer: C – Standard SARIMA is sufficient



Principle of parsimony: Use the simplest model that fits the data.

With only weekly seasonality ($m = 7$), SARIMA works fine.

Use TBATS/Prophet when you *need* multiple seasonalities or special features.

Quiz 9: Prophet Uncertainty

Question

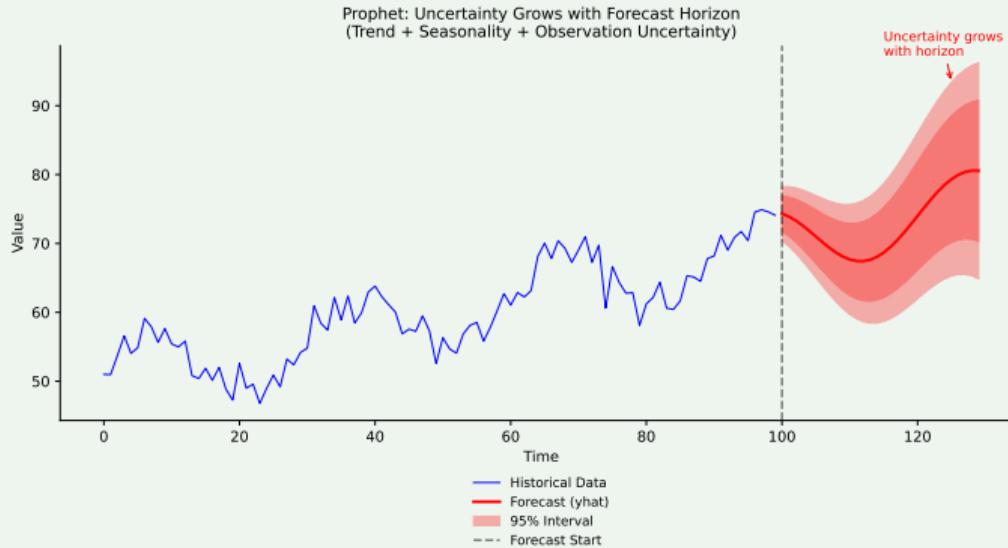
Prophet generates prediction intervals by:

- (A) Assuming normally distributed residuals
- (B) Sampling from the posterior distribution of parameters
- (C) Using bootstrap resampling of historical errors
- (D) Applying a fixed multiplier to point forecasts

Answer on next slide...

Quiz 9: Answer

Answer: B – Sampling from posterior distribution



Prophet uses Bayesian estimation:

- MAP estimation for point forecasts
- MCMC or simulation for uncertainty intervals
- Uncertainty from both trend (changepoints) and observation noise

Quiz 10: Practical Application

Question

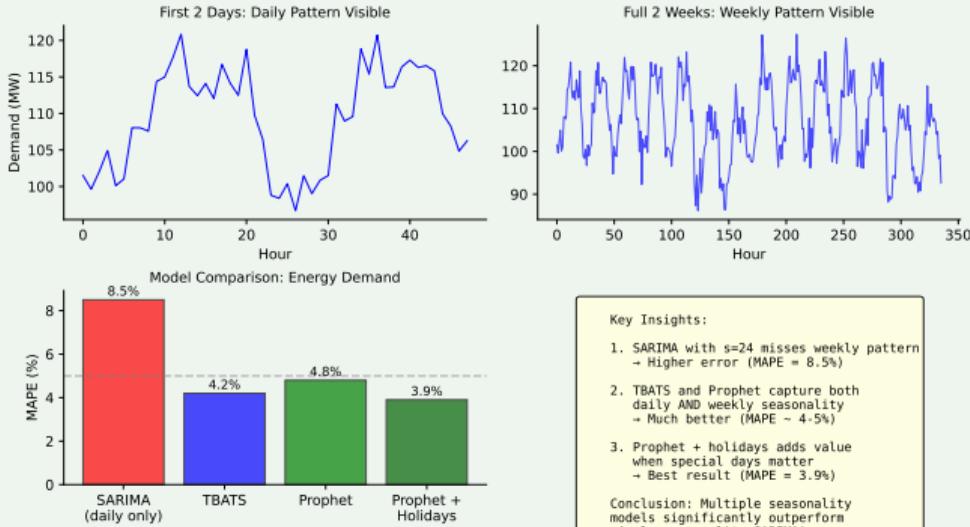
For forecasting hourly energy demand with daily, weekly, and annual patterns plus holiday effects, which approach is best?

- (A) SARIMA with $m = 24$
- (B) TBATS with three seasonal periods
- (C) Prophet with custom holidays
- (D) Either TBATS or Prophet, depending on whether holidays are important

Answer on next slide...

Quiz 10: Answer

Answer: D – TBATS or Prophet depending on needs



Both can handle multiple seasonality:

- If holiday effects are crucial ⇒ **Prophet** (explicit holiday modeling)
- If automatic model selection is preferred ⇒ **TBATS**
- Often: Try both and compare via cross-validation

True/False Questions

Determine if each statement is True or False:

- ① Prophet was developed by Facebook (Meta) for business forecasting.
- ② TBATS can only handle two seasonal periods at most.
- ③ In Prophet, the default trend is logistic growth.
- ④ Fourier terms approximate seasonality using sine and cosine functions.
- ⑤ Prophet requires equally spaced time series data.
- ⑥ The Box-Cox transformation in TBATS stabilizes variance.

Answers on next slide...

True/False: Solutions

TRUE

- ① Prophet was developed by Facebook (Meta) for business forecasting.

Released in 2017, designed for “analyst in the loop” forecasting at scale.

FALSE

- ② TBATS can only handle two seasonal periods at most.

TBATS can handle any number of seasonal periods (e.g., daily, weekly, annual).

FALSE

- ③ In Prophet, the default trend is logistic growth.

Default is piecewise linear. Logistic growth must be explicitly specified.

TRUE

- ④ Fourier terms approximate seasonality using sine and cosine functions.

$$s(t) = \sum_{k=1}^K [a_k \cos(2\pi kt/m) + b_k \sin(2\pi kt/m)]$$

FALSE

- ⑤ Prophet requires equally spaced time series data.

Prophet handles missing data and irregular timestamps gracefully.

TRUE

- ⑥ The Box-Cox transformation in TBATS stabilizes variance.

$$y^{(\lambda)} = (y^\lambda - 1)/\lambda \text{ for } \lambda \neq 0; \log(y) \text{ for } \lambda = 0.$$

Problem 1: Fourier Terms Calculation

Exercise

For daily data with weekly seasonality ($m = 7$), you want to use Fourier terms with $K = 3$ harmonics.

How many parameters does this add to the model?

Answer on next slide...

Problem 1: Solution

Solution: 6 parameters

Each harmonic requires 2 parameters (sine and cosine coefficients):

$$s(t) = \sum_{k=1}^K \left[a_k \cos\left(\frac{2\pi k t}{m}\right) + b_k \sin\left(\frac{2\pi k t}{m}\right) \right]$$

With $K = 3$ harmonics:

- $k = 1$: a_1, b_1 (fundamental frequency)
- $k = 2$: a_2, b_2 (first overtone)
- $k = 3$: a_3, b_3 (second overtone)

Total: $2 \times K = 2 \times 3 = 6$ parameters

Note: Maximum useful $K = \lfloor m/2 \rfloor = 3$ for $m = 7$.

Problem 2: Choosing Seasonality Mode

Exercise

You're forecasting monthly hotel bookings. The data shows:

- July 2020: 1000 bookings (peak season)
- January 2020: 400 bookings (off-season)
- July 2023: 2000 bookings (peak season)
- January 2023: 800 bookings (off-season)

Should you use additive or multiplicative seasonality? Why?

Answer on next slide...

Problem 2: Solution

Solution: Multiplicative seasonality

Analysis: Check if seasonal amplitude is proportional to level.

Year	July	January	Ratio (Jul/Jan)
2020	1000	400	2.5
2023	2000	800	2.5

Key observation: The *ratio* stays constant (2.5), not the difference!

- Additive would mean: July always +600 above January
- But 2020: $1000 - 400 = 600$; 2023: $2000 - 800 = 1200$

Conclusion: Use multiplicative: `seasonality_mode='multiplicative'`

Problem 3: TBATS Model Interpretation

Exercise

A TBATS model fitted to hourly electricity data reports:

- Box-Cox $\lambda = 0.5$
- Seasonal periods: $m_1 = 24, m_2 = 168$
- Fourier terms: $K_1 = 5, K_2 = 3$

What does each component tell you about the data?

Answer on next slide...

Problem 3: Solution

Solution

Box-Cox $\lambda = 0.5$:

- Square root transformation applied
- Data had increasing variance with level
- Transformation: $y^{(0.5)} = \sqrt{y}$

Seasonal periods:

- $m_1 = 24$: Daily pattern (24 hours)
- $m_2 = 168$: Weekly pattern ($7 \times 24 = 168$ hours)

Fourier terms:

- $K_1 = 5$ for daily: Complex intraday pattern (5 harmonics capture peaks, valleys)
- $K_2 = 3$ for weekly: Simpler weekly pattern (weekday vs weekend)

Total seasonal parameters: $2(K_1 + K_2) = 2(5 + 3) = 16$

Problem 4: Prophet Holiday Effects

Exercise

You're forecasting daily restaurant revenue. You want to add these holiday effects to Prophet:

- Valentine's Day (Feb 14) – major boost
- Easter (variable date) – restaurant closed
- Christmas (Dec 25) – restaurant closed

Write the Python code to create the holidays dataframe for 2024-2025.

Answer on next slide...

Problem 4: Solution

Solution

```
import pandas as pd
from prophet import Prophet
holidays = pd.DataFrame({
    'holiday': ['valentines', 'valentines', 'easter', 'easter', 'christmas', 'christmas'],
    'ds': pd.to_datetime(['2024-02-14', '2025-02-14', '2024-03-31', '2025-04-20', ...]),
    'lower_window': [0, 0, 0, 0, 0, 0],
    'upper_window': [0, 0, 0, 0, 0, 0]
})
model = Prophet(holidays=holidays)
model.fit(df)
```

Note: Use `lower_window=-1`, `upper_window=1` to capture effects on adjacent days.

Example: Retail Sales Forecasting with Prophet

Scenario

Monthly retail sales data (2018-2023) with:

- Strong December peaks (Christmas shopping)
- COVID-19 impact in 2020 (structural break)
- Growing trend over time

Prophet Configuration

```
model = Prophet(  
    seasonality_mode='multiplicative',  
    changepoint_prior_scale=0.5,  
    yearly_seasonality=True,  
    weekly_seasonality=False  
)  
  
model.add_country_holidays(country_name='US')
```

Key Decision

Used multiplicative seasonality because December effect is proportional to baseline sales level.

Example: Energy Demand with TBATS

Scenario

Hourly electricity demand with:

- Intraday pattern (24 hours)
- Weekly pattern (168 hours)
- Annual pattern (8760 hours)

TBATS in R

```
library(forecast)
energy_mssts <- msts(energy_data,
  seasonal.periods = c(24, 168, 8760))
fit <- tbats(energy_mssts)
fc <- forecast(fit, h = 168)
```

Note

TBATS automatically selects K for each seasonal period via AIC.

Example: Cross-Validation Comparison

Objective

Compare Prophet, TBATS, and SARIMA on 2 years of daily sales data.

Prophet Cross-Validation

```
from prophet.diagnostics import cross_validation, performance_metrics
df_cv = cross_validation(model, initial='365 days',
    period='90 days', horizon='30 days')
metrics = performance_metrics(df_cv)
print(f"MAPE: {metrics['mape'].mean():.2%}")
```

Typical Results

Model	MAPE	Computation Time
SARIMA (weekly only)	8.5%	Fast
TBATS (weekly + yearly)	6.2%	Moderate
Prophet (weekly + yearly + holidays)	5.8%	Fast

Discussion: When to Use Which Model?

Key Question

You have a new forecasting task. How do you choose between SARIMA, TBATS, and Prophet?

Decision Framework

① How many seasonal periods?

- One ⇒ SARIMA may suffice
- Multiple ⇒ TBATS or Prophet

② Do you have domain knowledge to encode?

- Holidays, events, changepoints ⇒ Prophet
- Let the data speak ⇒ TBATS

③ Interpretability requirements?

- Need to explain components ⇒ Prophet
- Just need forecasts ⇒ Either

Discussion: Overfitting with Fourier Terms

Key Question

Can you have too many Fourier terms? What are the symptoms?

Answer: Yes!

Symptoms of overfitting:

- In-sample fit is excellent, but out-of-sample is poor
- Seasonality looks “jagged” or unrealistic
- Forecasts oscillate wildly

Guidelines

- Maximum $K \leq m/2$ (Nyquist limit)
- Start with $K = 3-5$ for most applications
- Use cross-validation to select K
- Prophet default: $K = 10$ for yearly, $K = 3$ for weekly

Discussion: Handling Structural Breaks

Scenario

Your historical data includes COVID-19 period (2020-2021). How do you handle this when forecasting 2024?

Options

- ① **Exclude COVID period:** Train only on pre-COVID and post-COVID data
- ② **Use changepoints:** Let Prophet detect/specify breaks
- ③ **Add regressors:** Include COVID indicator variable
- ④ **Adjustment:** Manually adjust 2020-2021 values to “normal”

Prophet Approach

```
model = Prophet(changepoints=['2020-03-15', '2021-06-01'])
df['covid'] = (df['ds'] >= '2020-03-15') & (df['ds'] < '2021-06-01')
model.add_regressor('covid')
```

Take-Home Exercises

- ① **Theoretical:** Prove that $K = m/2$ Fourier terms can represent any periodic function with period m (for even m).
- ② **Computation:** For the seasonal pattern below (daily data, weekly cycle), determine the minimum number of Fourier harmonics needed:

Mon: 100, Tue: 110, Wed: 115, Thu: 110, Fri: 120, Sat: 80, Sun: 65
- ③ **Applied:** Download hourly electricity demand data from a public source:
 - Fit both TBATS (in R) and Prophet (in Python)
 - Compare forecast accuracy using RMSE and MAPE
 - Visualize the component decompositions
- ④ **Critical Thinking:** Why might Prophet perform poorly on high-frequency financial data (e.g., minute-by-minute stock prices)?

Hints

- ① By Fourier theorem, any periodic function can be represented as sum of sines and cosines. With period m , frequencies are k/m for $k = 1, \dots, m/2$.
- ② The pattern has:
 - One peak (Friday) and one trough (Sunday)
 - Fairly smooth transitions
 - $K = 2$ or $K = 3$ likely sufficient (try and compare)
- ③ For electricity data:
 - Include daily (24h) and weekly (168h) patterns
 - Add holidays for your region in Prophet
 - Expect MAPE around 3-5% for hourly forecasts
- ④ Financial data issues:
 - No clear seasonality (market efficiency)
 - High noise-to-signal ratio
 - Prophet designed for “business” data with trends and seasons

Key Takeaways from This Seminar

Multiple Seasonality Models

- ① **TBATS**: Automatic, Fourier-based, handles any number of seasonal periods
- ② **Prophet**: Analyst-friendly, explicit holiday/event handling, interpretable
- ③ Use **SARIMA** when only one seasonal period exists

Key Decisions

- **Seasonality mode**: Additive (constant amplitude) vs Multiplicative (proportional)
- **Fourier terms**: More = flexible but risk overfitting; use CV to select
- **Changepoints**: Allow trend to adapt to structural breaks

Remember

Prophet: Great when you have domain knowledge to encode

TBATS: Great for automatic modeling of complex seasonality