



Time Series Analysis and Forecasting

Chapter 10: Comprehensive Review



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Learning Objectives

By the end of this chapter, you will be able to:

- Apply the complete forecasting workflow from data to evaluation
- Select appropriate models based on data characteristics
- Evaluate forecast accuracy using proper metrics and cross-validation
- Integrate knowledge from all previous chapters in practice



Outline

Forecasting Methodology

Case Study 1: Bitcoin Volatility (GARCH)

Case Study 2: Sunspot Cycles (Fourier)

Case Study 3: Unemployment (Prophet)

Case Study 4: Multivariate Analysis (VAR)

Synthesis and Guidelines

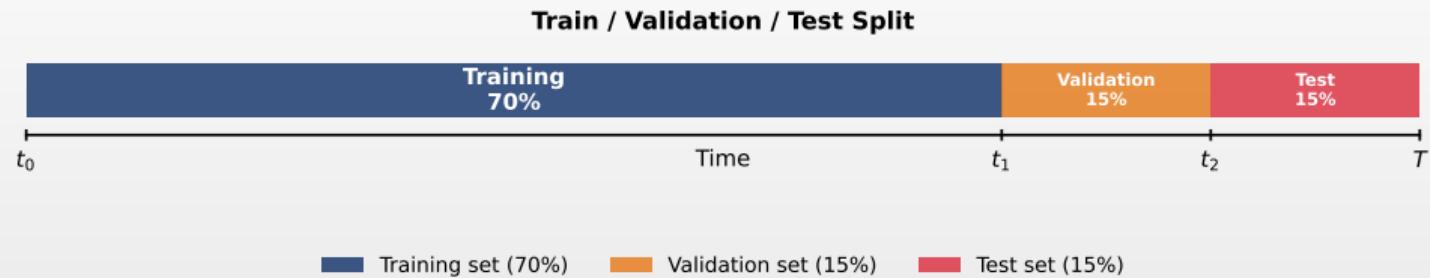
AI Use Case

Quiz

Summary



Train/Validation/Test Framework



The Scientific Approach to Forecasting

Research Question

How do we **rigorously evaluate** forecast performance while avoiding overfitting?

The Fundamental Problem

- In-sample fit \neq Out-of-sample performance
- Models can “memorize” training data without learning patterns
- **Solution:**
 - ▶ Proper train/validation/test methodology

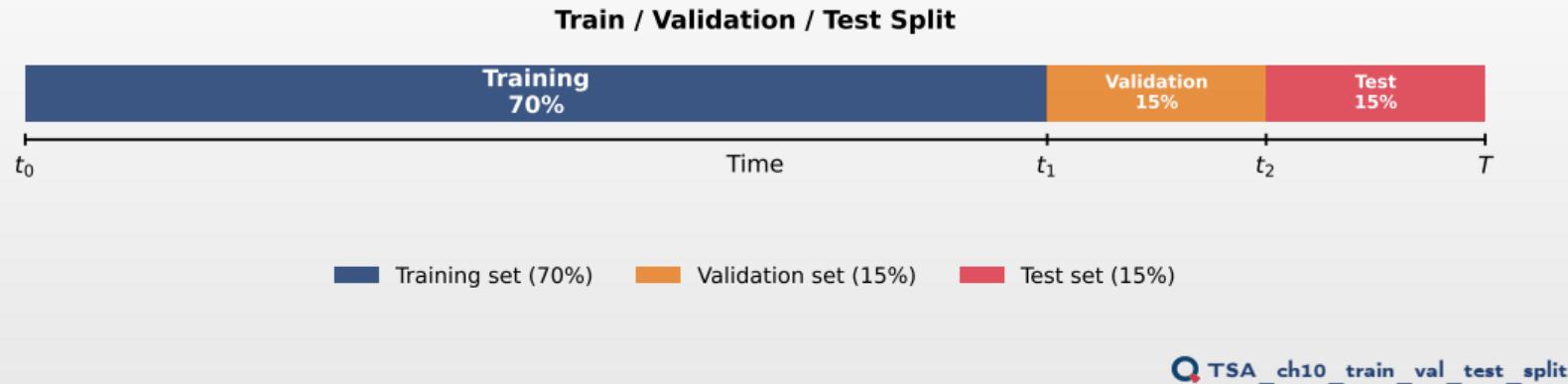
Key Principle

“The test set must remain **untouched** until final evaluation.”

— Standard practice in machine learning and econometrics



Train/Validation/Test Framework



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Evaluation Metrics

Definition 1 (Forecast Error Metrics)

Let y_t be actual, \hat{y}_t forecast:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_t (y_t - \hat{y}_t)^2}, \quad \text{MAE} = \frac{1}{n} \sum_t |y_t - \hat{y}_t|, \quad \text{MAPE} = \frac{100\%}{n} \sum_t \left| \frac{y_t - \hat{y}_t}{y_t} \right|$$

When to Use Each

- RMSE**: Penalizes large errors
- MAE**: Robust to outliers
- MAPE**: Scale-independent (%)

Caution

- MAPE undefined when $y_t = 0$
- Compare on **same** test set
- Report **out-of-sample** metrics



Forecast Evaluation Beyond RMSE

Alternative metrics

- MASE**: comparison with naïve forecast
- Directional Accuracy**: correct direction
- Quantile Loss**: for VaR
- CRPS**: full distribution

Example: Quantile Loss

$$QL_\alpha = \begin{cases} \alpha(y_t - \hat{q}_t), & y_t > \hat{q}_t \\ (1 - \alpha)(\hat{q}_t - y_t), & y_t \leq \hat{q}_t \end{cases}$$

Bitcoin Results (GARCH volatility)

Metric	Value
RMSE	2.21
MAE	1.89
MASE	0.98
Dir. Accuracy	28.7%

- MASE < 1: GARCH beats naïve
- DA 28.7%: volatility direction is hard



Formal Forecast Comparison: Diebold–Mariano

Definition 2 (Diebold–Mariano Test)

Loss differential: $d_t = L(e_{1t}) - L(e_{2t})$, Statistic: $DM = \frac{\bar{d}}{\sqrt{\widehat{\text{Var}}(\bar{d})}} \xrightarrow{d} N(0, 1)$

Hypotheses

- H_0 : equal predictive accuracy
- H_1 : one model is significantly better
- Large $|DM|$ \Rightarrow reject H_0

Bitcoin Result (GARCH volatility)

- Normal vs Student-t: $DM = -0.51$
- $p = 0.612$ — **do not reject H_0**
- Similar accuracy, but Student-t preferred by AIC ($\Delta = 509$)

Key message

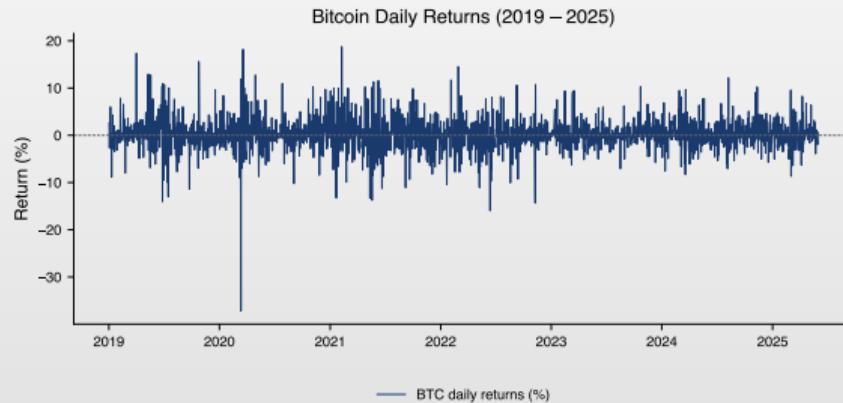
- Lower RMSE \neq significant difference — formal testing is **mandatory**



Bitcoin: Volatility Clustering

Observation

- Large returns follow large returns, small follow small—**volatility clustering**



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Bitcoin: Problem Statement

Research Question

Can we forecast Bitcoin's **volatility** using GARCH models?

Data Characteristics

- Source: Yahoo Finance (BTC-USD)
- Period: Jan 2019 – Jan 2025
- Frequency: Daily
- Observations: $\approx 2,200$ days

Stylized Facts

- Returns: near-zero mean
- Fat tails ($kurtosis > 3$)
- Volatility clustering

Key Insight

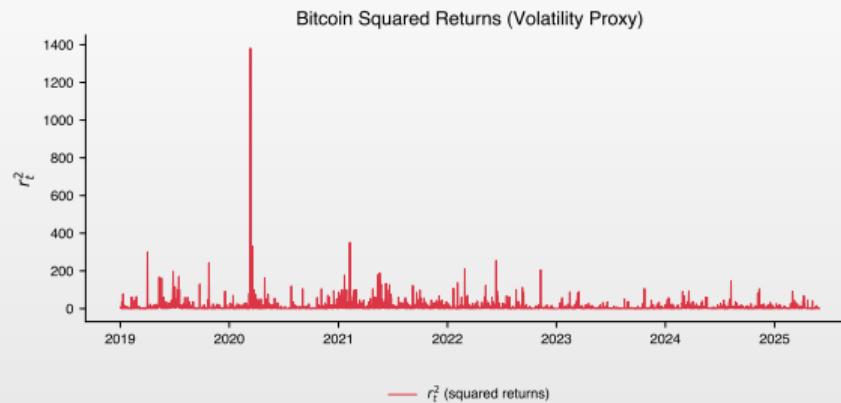
Financial returns are typically:

- **Unpredictable** in mean
- **Predictable** in variance

⇒ Focus on **volatility forecasting**



Bitcoin: Evidence for GARCH



 TSA_ch10_btc_acf_squared



GARCH Model Specification

Definition 3 (GARCH(p,q) Model)

Let r_t denote returns. The GARCH(p,q) model:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t \stackrel{iid}{\sim} N(0, 1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2$$

where $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1$.

Model Variants

- GARCH(1,1)**: Most common
- GJR-GARCH**: Leverage effect
- EGARCH**: Log-variance, asymmetric

Interpretation

- α : Shock impact (ARCH effect)
- β : Volatility persistence
- $\alpha + \beta \approx 1$: High persistence



GARCH: Stationarity and Unconditional Variance

Theorem 1 (Covariance Stationarity of GARCH(1,1))

If $\alpha_1 + \beta_1 < 1$, then $\{\varepsilon_t\}$ is covariance stationary with:

$$\bar{\sigma}^2 = \mathbb{E}[\sigma_t^2] = \frac{\omega}{1 - \alpha_1 - \beta_1}$$

Derivation

Take expectations of both sides of the variance equation:

$$\begin{aligned}\mathbb{E}[\sigma_t^2] &= \omega + \alpha_1 \mathbb{E}[\varepsilon_{t-1}^2] + \beta_1 \mathbb{E}[\sigma_{t-1}^2] \\ \bar{\sigma}^2 &= \omega + (\alpha_1 + \beta_1) \bar{\sigma}^2 \quad (\text{stationarity}) \\ \bar{\sigma}^2 &= \frac{\omega}{1 - \alpha_1 - \beta_1}\end{aligned}$$

Multi-Step Forecasts Converge to $\bar{\sigma}^2$

As $h \rightarrow \infty$: $\mathbb{E}_t[\sigma_{t+h}^2] \rightarrow \bar{\sigma}^2$ at rate $(\alpha_1 + \beta_1)^h$.



Bitcoin: Model Selection on Validation Set

Methodology

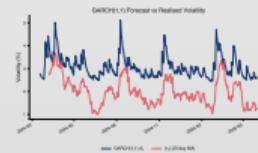
Fit each model on training data, evaluate on validation set.

Model	AIC	BIC	Val MAE	Selection
GARCH(1,1)	6,994.8	7,020.6	2.638	Best
GARCH(2,1)	6,993.7	7,024.6	2.640	
GJR-GARCH(1,1)	6,983.7	7,014.6	2.669	
EGARCH(1,1)	—	—	—	Failed*

*Analytic forecasts not available for $h > 1$

Result

GARCH(1,1) selected based on lowest validation MAE for volatility forecasts.



Bitcoin: Data Split and Stationarity

Data Split

Set	Period	N
Training (70%)	2019-01 to 2023-03	1,543
Validation (20%)	2023-03 to 2024-06	441
Test (10%)	2024-06 to 2025-01	221
Total	2,205	

Stationarity Tests

Series	ADF	Result
Prices	$p = 0.50$	Non-stationary
Returns	$p < 0.01$	Stationary

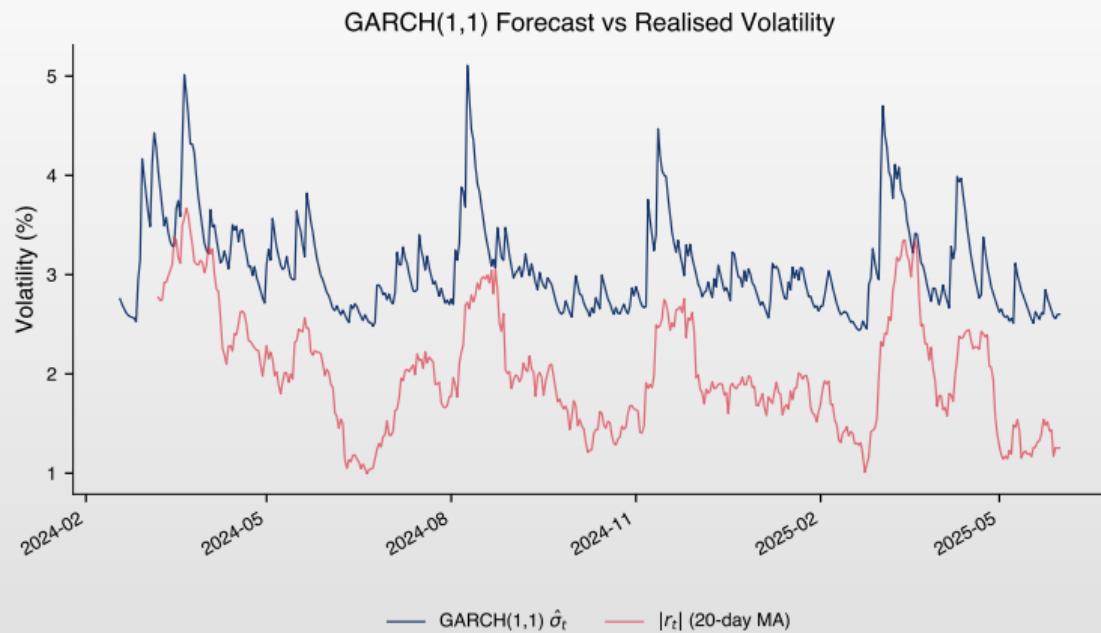
⇒ Model **returns**, not prices

Why Stationarity Matters

- ◻ GARCH requires weakly stationary input
- ◻ Prices follow random walk; returns are stationary



Bitcoin: Final Test Set Evaluation



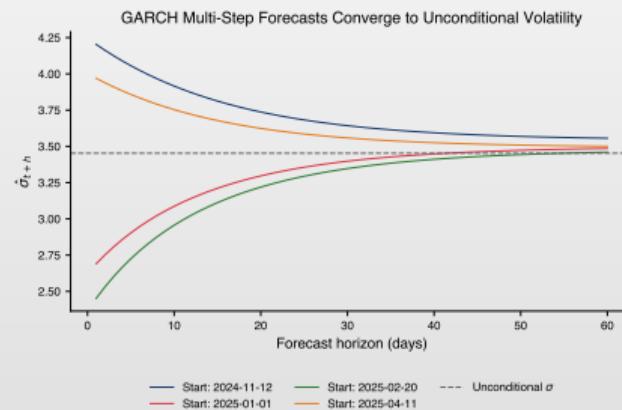
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GARCH: Multi-Step Forecasts Converge

Key Insight

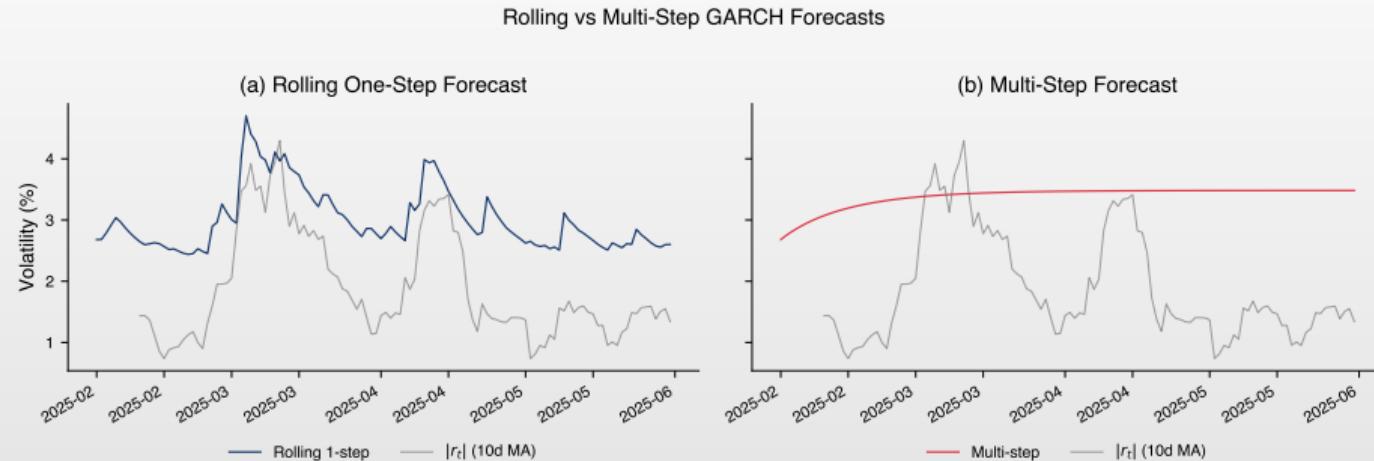
- Multi-step forecasts converge to $\bar{\sigma}^2 = \frac{\omega}{1-\alpha-\beta}$
- Use rolling forecasts



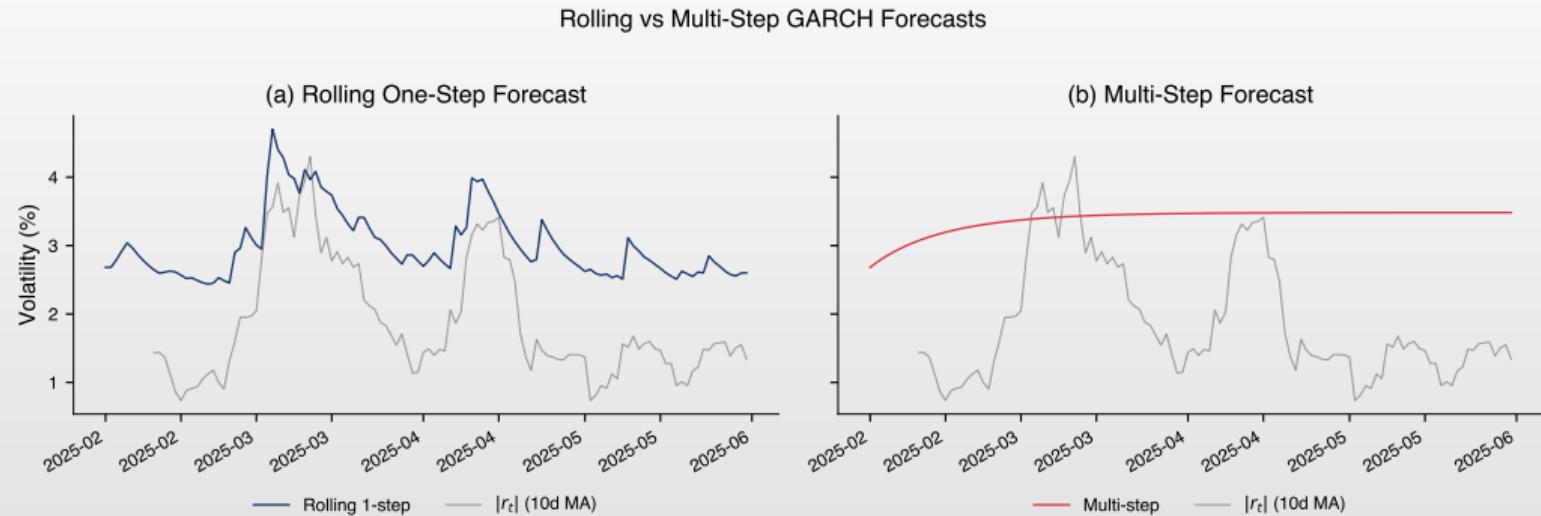
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GARCH: Rolling One-Step-Ahead Solution



GARCH: Rolling One-Step-Ahead Solution



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GARCH: Innovation Distributions

Model

$$r_t = \mu + \sigma_t z_t$$

- Options for z_t : $\mathcal{N}(0, 1)$ (normal) or t_ν (fat tails)

Bitcoin: empirical evidence

- Residual kurtosis: **13.81** (Normal = 3)
- Skewness: -0.29
- Jarque-Bera: 9085, $p < 0.001$
- Normality **underestimates** tail risk

Student-t: the right choice

- $\hat{\nu} = 2.96$ degrees of freedom
- AIC Normal: 9769 vs Student-t: **9260**
- $\Delta\text{AIC} = 509$ — **overwhelming** evidence
- Fat tails = **more realistic** VaR estimates



Application: Conditional Value-at-Risk

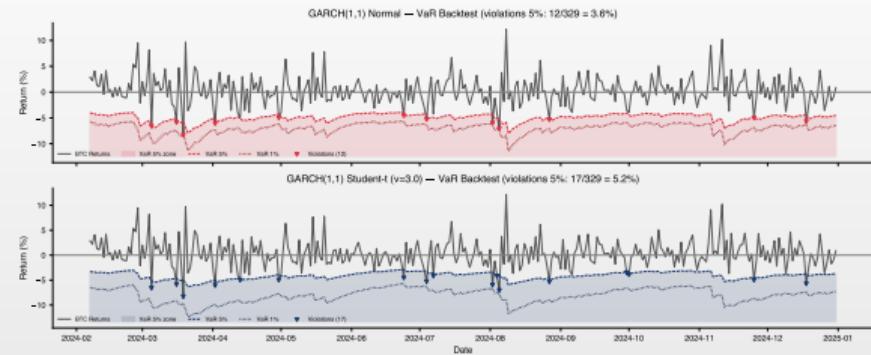
Definition 4 (Conditional VaR at level α)

$$\text{VaR}_{t+1}^{\alpha} = \mu_{t+1} + \sigma_{t+1} \cdot z_{\alpha}$$

Bitcoin Results (test: 329 days)

	Normal	Student-t
Violations 5%	12 (3.6%)	17 (5.2%)
Violations 1%	1 (0.3%)	0 (0.0%)
Kupiec p (5%)	0.238	0.890

Student-t: $5.2\% \approx 5\%$ — perfect coverage. Normal: too conservative (3.6%).



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Risk Model Validation: Backtesting

Kupiec Test (Unconditional Coverage)

$$LR_{uc} = -2 \ln \left(\frac{(1 - \alpha)^{T-x} \alpha^x}{(1 - \hat{p})^{T-x} \hat{p}^x} \right) \sim \chi^2(1)$$

Christoffersen (Independence)

- Tests **independence** of violations
- $LR_{cc} = LR_{uc} + LR_{ind} \sim \chi^2(2)$

Bitcoin Results (VaR 5%)

	Normal	Student-t
Violations	12/329	17/329
Rate	3.6%	5.2%
Kupiec LR	1.39	0.02
Kupiec p	0.238	0.890
Chr. LR_{ind}	0.00	1.20
Chr. p	1.000	0.272

Conclusion

- Both pass Kupiec ($p > 0.05$)
- Student-t: **more precise coverage**



GARCH Limitations and Modern Extensions

Limitations

- Does not capture **jumps**
- Constant parameters over time
- Sensitive to chosen distribution
- Does not model different **regimes**

Extensions

- GJR-GARCH:** leverage effect
- EGARCH:** asymmetric shocks
- Markov-Switching GARCH:** regimes
- Realized volatility (HAR)
- Hybrid GARCH + ML

Key message

- GARCH is a **starting point**, not the end of risk modeling



Bitcoin: Key Findings

Summary

1. Returns are stationary; prices are not
2. GARCH(1,1) outperforms more complex variants
3. High persistence ($\alpha + \beta = 0.93$)
4. Volatility is predictable even when returns are not

Practical Implications

- ☐ Risk management: VaR, Expected Shortfall
- ☐ Option pricing requires volatility forecasts
- ☐ Portfolio optimization with time-varying risk

Limitations

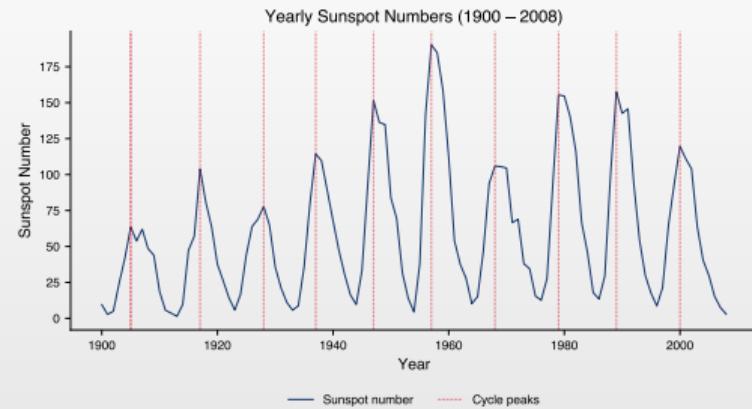
- ☐ GARCH assumes symmetric shocks
- ☐ Does not capture jumps
- ☐ Normal distribution may be restrictive

Extensions

- ☐ Student-t innovations
- ☐ Realized volatility
- ☐ HAR models

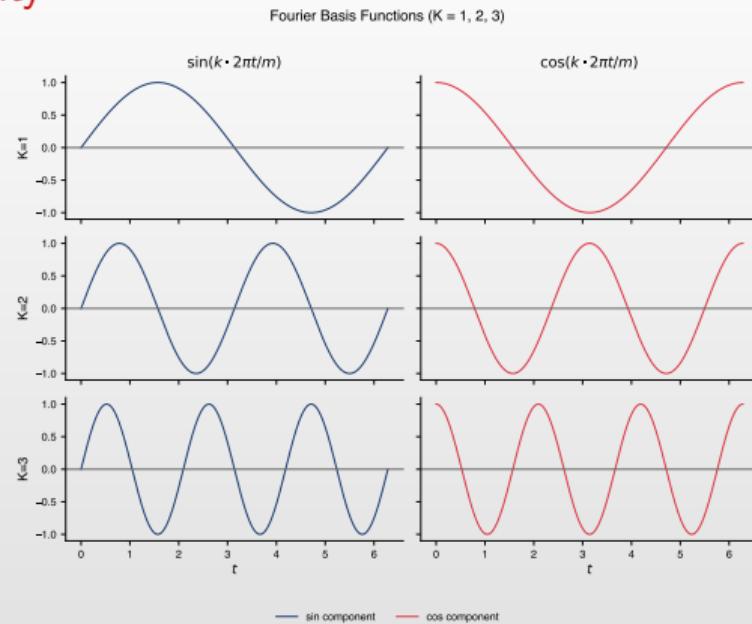


Sunspots: The 11-Year Solar Cycle

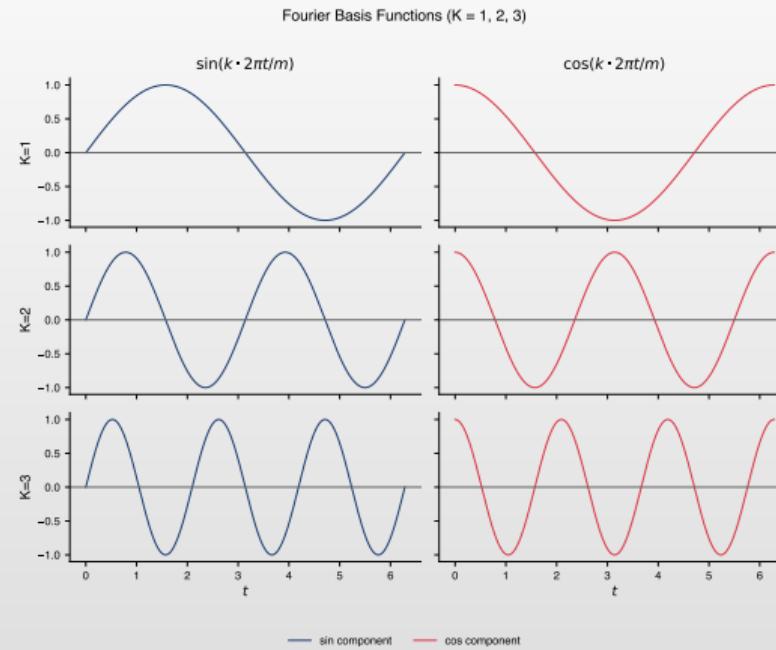


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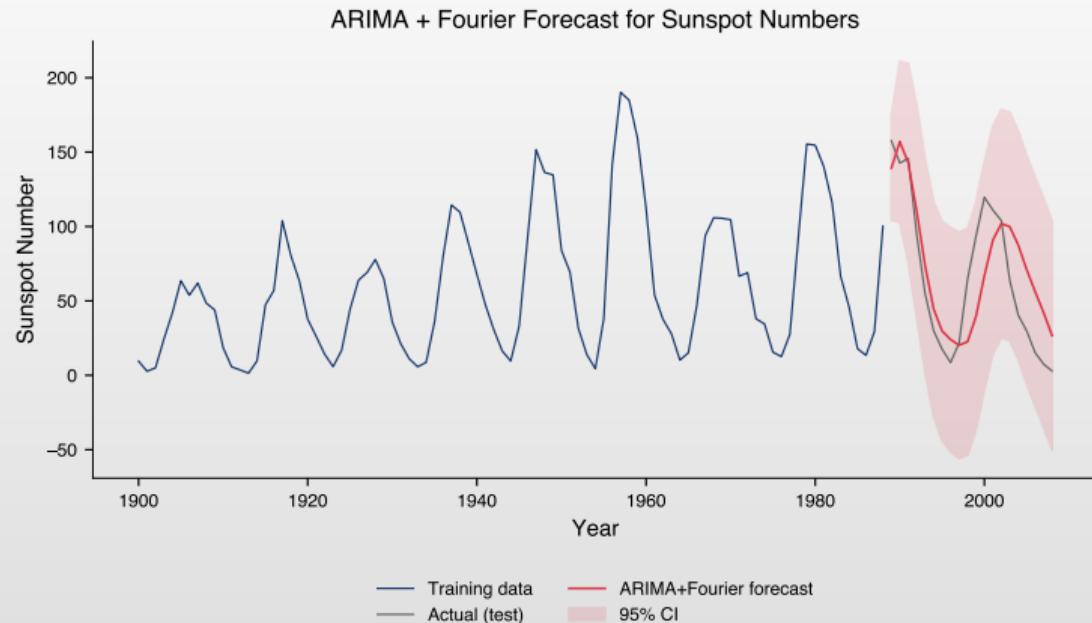
Fourier Terms for Seasonality



Fourier Terms for Seasonality



Sunspots: Forecast Results



Sunspots: Model Selection

Methodology

Compare $K = 1, 2, 3, 4$ Fourier harmonics on validation set.

Data Split	Set	Period	N
	Training (70%)	1900–1975	76
	Validation (20%)	1976–1997	22
	Test (10%)	1998–2008	11
Total			109

Model Comparison			
K	AIC	Val RMSE	
1	665.9	87.15	
2	668.0	86.92	
3	671.8	86.81	Best
4	674.5	87.93	

Result

$K = 3$ Fourier harmonics selected (6 parameters for 11-year cycle).



Overfitting in Choosing K

Overfitting risk

- K too large = memorizing historical cycle
- Model fits noise, not signal
- Test performance **degrades**

Solution: validation

- Select K on **validation** set
- Evaluate on **test** — untouched
- Trade-off: complexity vs generalization

Fourier \approx periodic regression

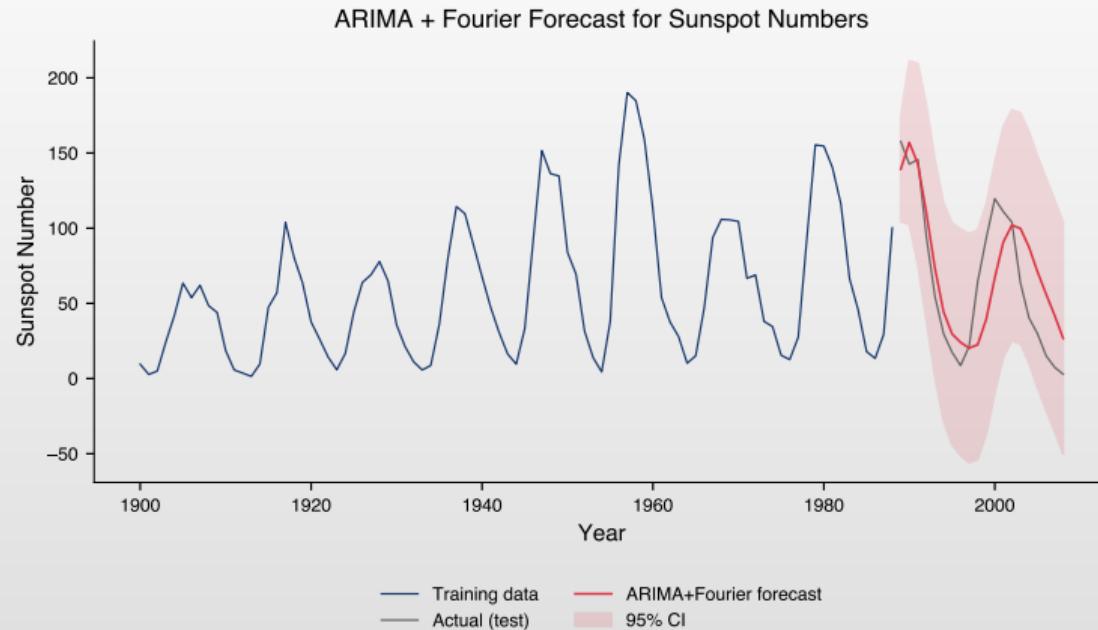
- Each harmonic adds 2 parameters (sin, cos)
- $K = 3$: 6 extra parameters
- $K = 6$: 12 parameters — overfitting risk

Our results

- $K = 3$ minimizes Val RMSE
- $K = 4$ increases error \rightarrow overfitting



Sunspots: Forecast Results



Sunspots: Key Takeaways

When to Use Fourier Terms

- Seasonal period s is **long** (e.g., 11 years, 52 weeks)
- SARIMA would require too many seasonal lags
- Pattern is **smooth and periodic**
- Multiple cycles need to be captured

Choosing K

- Start with $K = 1$, increase until validation error stops improving
- Too high K = overfitting

Fourier vs SARIMA

	Fourier	SARIMA
Long seasons	✓	✗
Short seasons	OK	✓
Parameters	$2K$	Many
Flexibility	Fixed	Adaptive

Applications

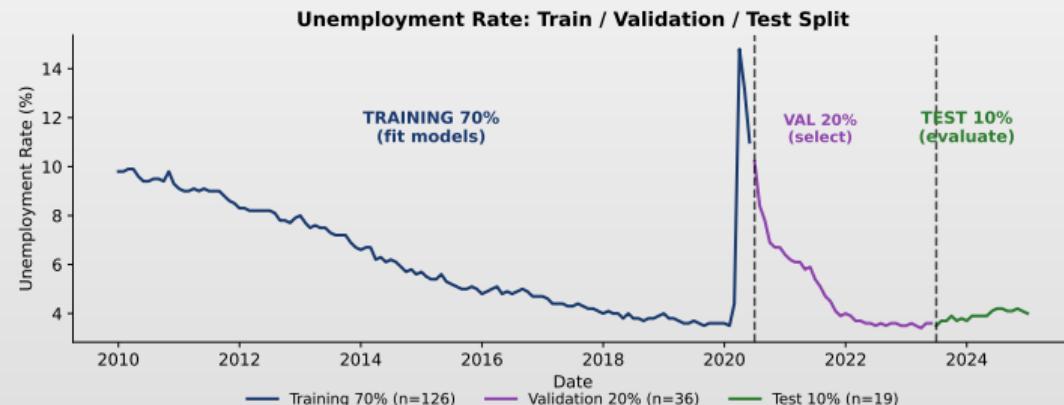
Climate cycles, business cycles, astronomical phenomena



Unemployment: Train / Validation / Test Split

Methodology

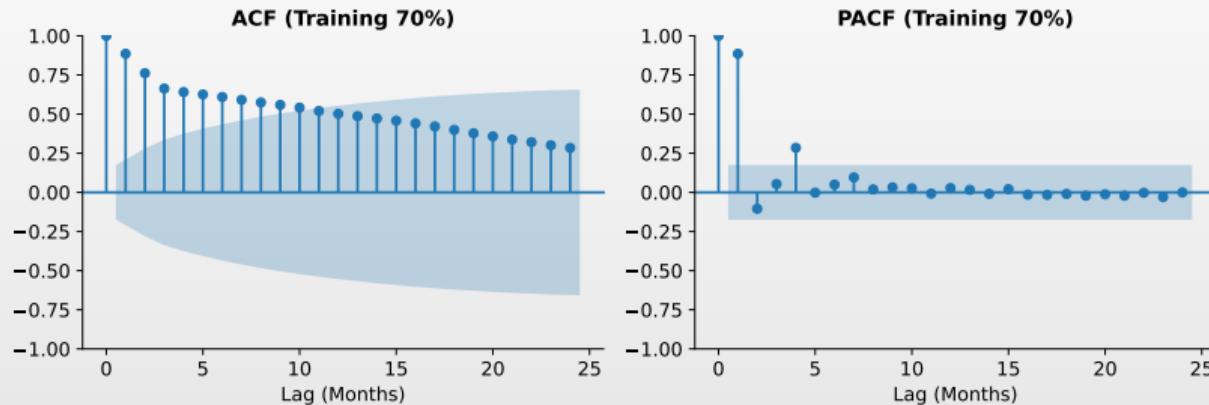
- Training:** Fit models
- Validation:** Select best
- Test:** Final evaluation



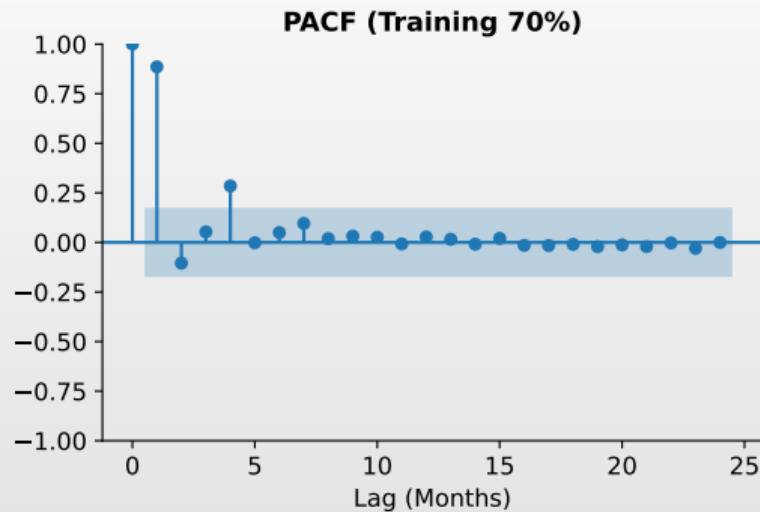
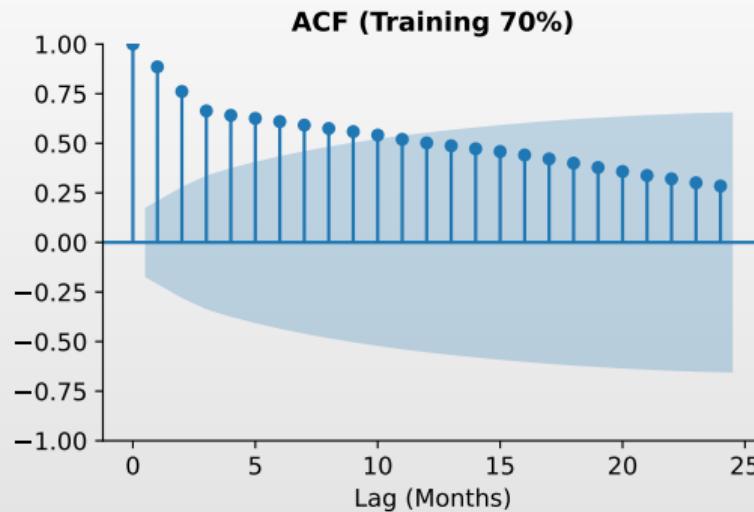
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Unemployment: Preliminary Analysis

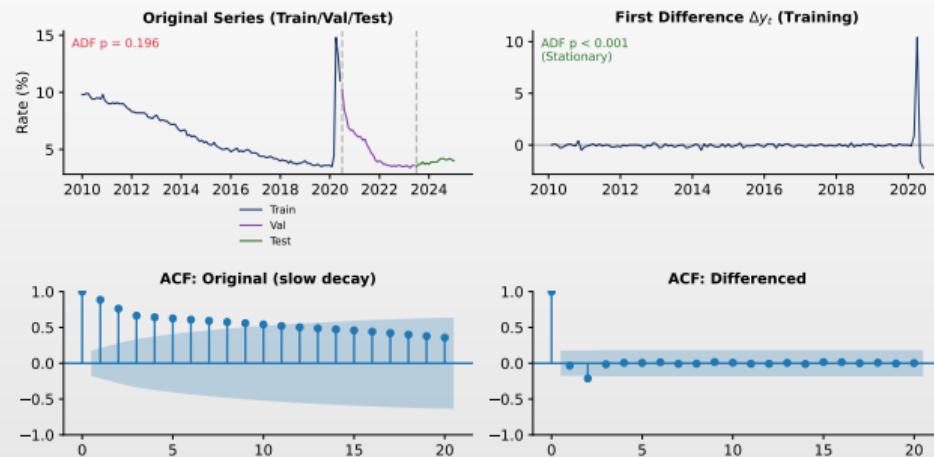


Unemployment: Preliminary Analysis

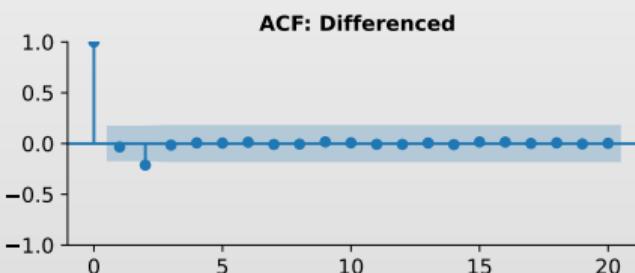
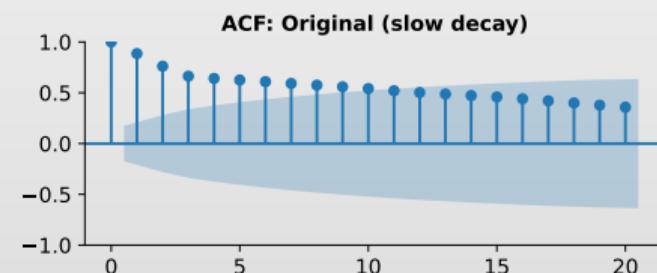


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Unemployment: Stationarity Tests



Unemployment: Stationarity Tests



Structural Breaks: Formal Approach

Classical methods

- Chow Test:** break at known point
- Bai–Perron:** multiple unknown breaks
- CUSUM:** sequential detection

Problem

- ADF can confuse **break** with **unit root**
- Zivot–Andrews test: ADF with endogenous break

Result: Unemployment at COVID (March 2020)

- Chow Test: $F = 21.73, p < 0.001$
- Structural break **confirmed**
- SARIMA: constant parameters — risk
- Prophet: detects changepoints automatically

Key message

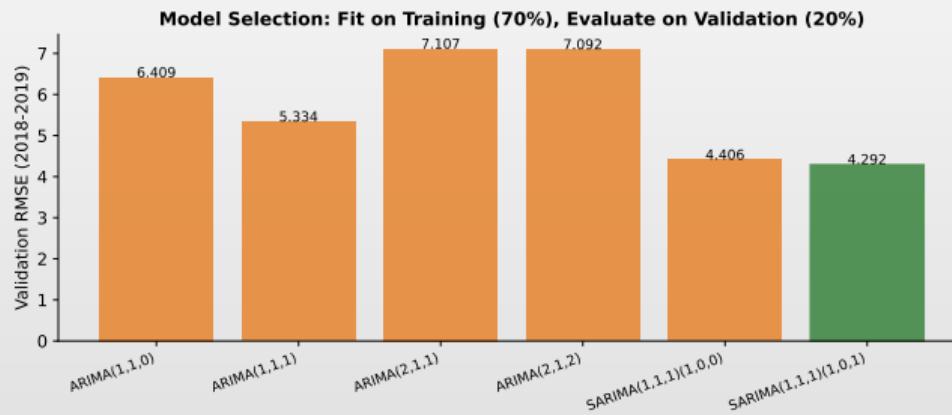
- Model must be adapted to **parameter stability**



Unemployment: Model Selection (Validation Set)

Best: SARIMA(1,1,1)(1,0,0)₁₂

- Selected by lowest validation RMSE



 TSA_ch10_sarima_model_selection



Unemployment: SARIMA Parameters

SARIMA(1,1,1)(1,0,0)₁₂ fitted on Train+Val (2010-2019)

- AR(1): $\phi_1 = -0.86$
- MA(1): $\theta_1 = 0.78$
- SAR(12): $\Phi_1 = -0.08$ (n.s.)

SARIMA(1,1,1)(1,0,1) - Fitted on Train+Val (85%)

Parameter	Coef	Std Err	P-value	Sig
ar.L1	0.8423	0.2084	0.0001	***
ma.L1	-0.9540	0.1973	0.0000	***
ar.S.L12	0.0326	4.5951	0.9943	
ma.S.L12	-0.0113	4.6087	0.9980	
sigma2	0.8122	0.0608	0.0000	***



Ljung-Box Test for Residual Autocorrelation

Definition 5 (Ljung-Box Test)

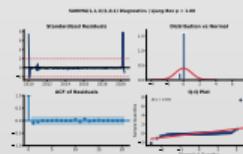
For residuals $\hat{\varepsilon}_t$ with sample autocorrelations $\hat{\rho}_k$, the test statistic:

$$Q(h) = n(n + 2) \sum_{k=1}^h \frac{\hat{\rho}_k^2}{n - k} \stackrel{H_0}{\sim} \chi^2(h - p - q)$$

where p, q are ARMA orders. H_0 : Residuals are white noise.

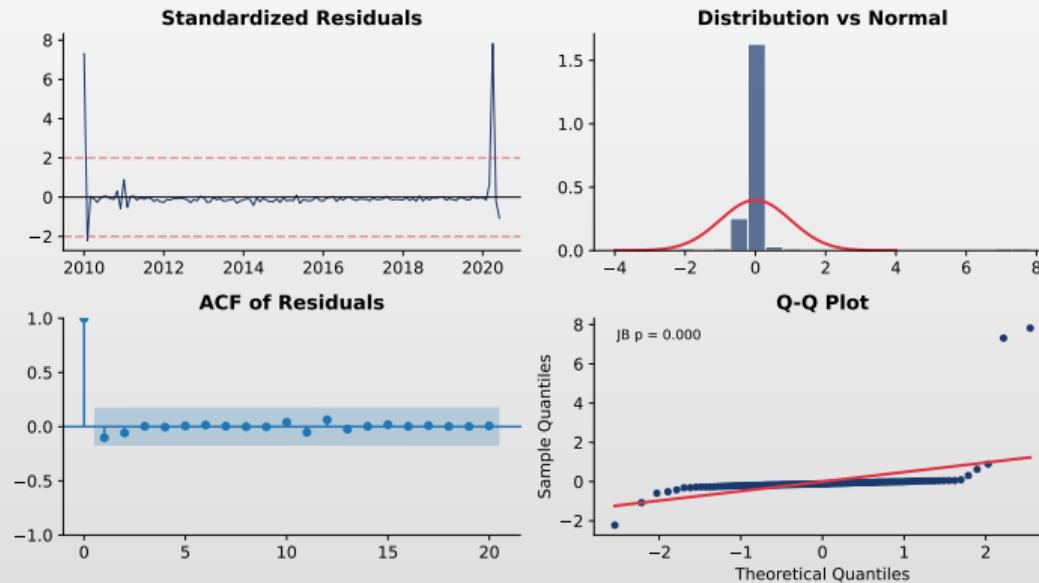
Interpretation

- Large Q (small p-value): Reject H_0 , residuals have structure
- Small Q (large p-value): Fail to reject H_0 , model is adequate
- Rule of thumb: Use $h = \min(10, n/5)$ for lag order



Unemployment: SARIMA Diagnostics

SARIMA(1,1,1)(1,0,1) Diagnostics | Ljung-Box p = 1.00



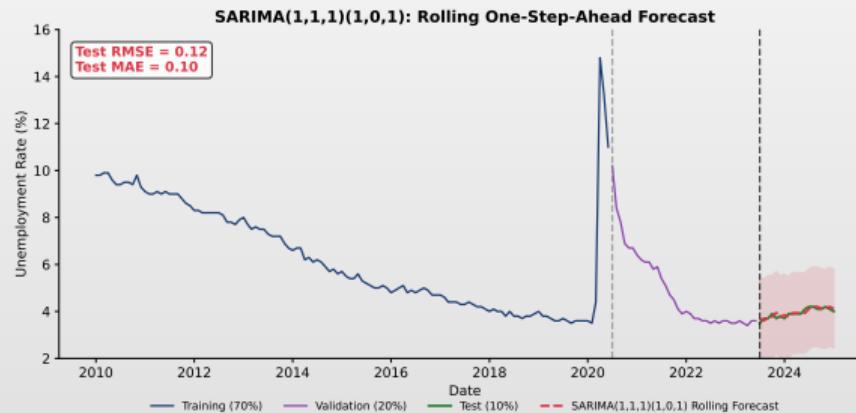
TSA_ch10_sarima_diagnostics



Unemployment: SARIMA Rolling Forecast

Problem: Structural Break

- Rolling one-step-ahead forecast (re-estimate at each t)
- Test RMSE = 0.12



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Prophet Model

Definition 6 (Prophet Decomposition)

$$y_t = g(t) + s(t) + h(t) + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2)$$

where $g(t)$ = trend, $s(t)$ = seasonality, $h(t)$ = holidays, σ^2 = noise variance (estimated).

Changepoint Detection

- Automatic location selection
- `changepoint_prior_scale` controls flexibility

Advantages

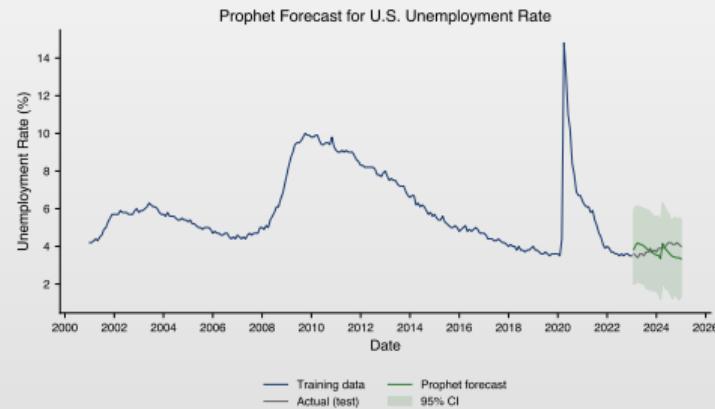
- Handles missing data
- Interpretable components
- Robust to outliers



Unemployment: Prophet Forecast Results

Key Finding

- Prophet adapts via changepoint detection
- Test RMSE = 0.58



 TSA_ch10_unemployment_forecast



Unemployment: Model Tuning

Hyperparameter Tuning

Tune `changepoint_prior_scale` on validation set.

Data Split	Set	Period	N
	Training (70%)	2010-01 to 2020-06	126
	Validation (20%)	2020-07 to 2023-06	36
	Test (10%)	2023-07 to 2025-01	19
	Total		181

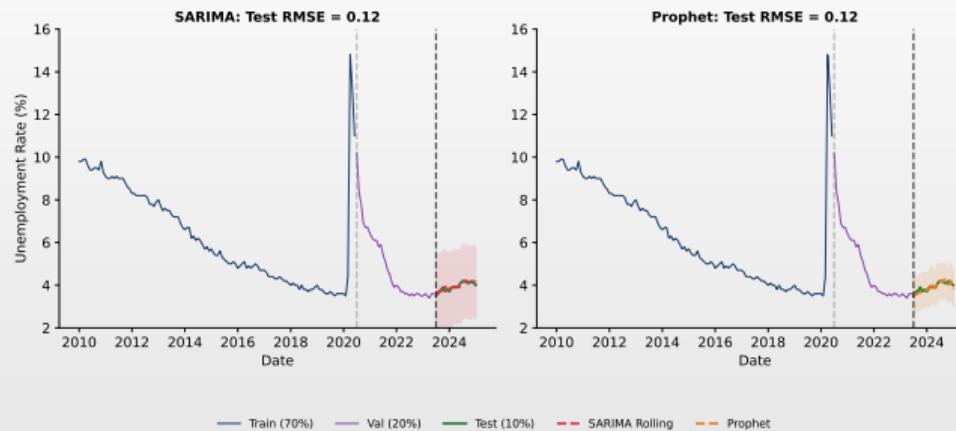
Scale Comparison	Scale	Val RMSE	Best
	0.01	4.21	
	0.05	3.89	
	0.10	3.52	
	0.30	3.67	
	0.50	3.81	

Interpretation

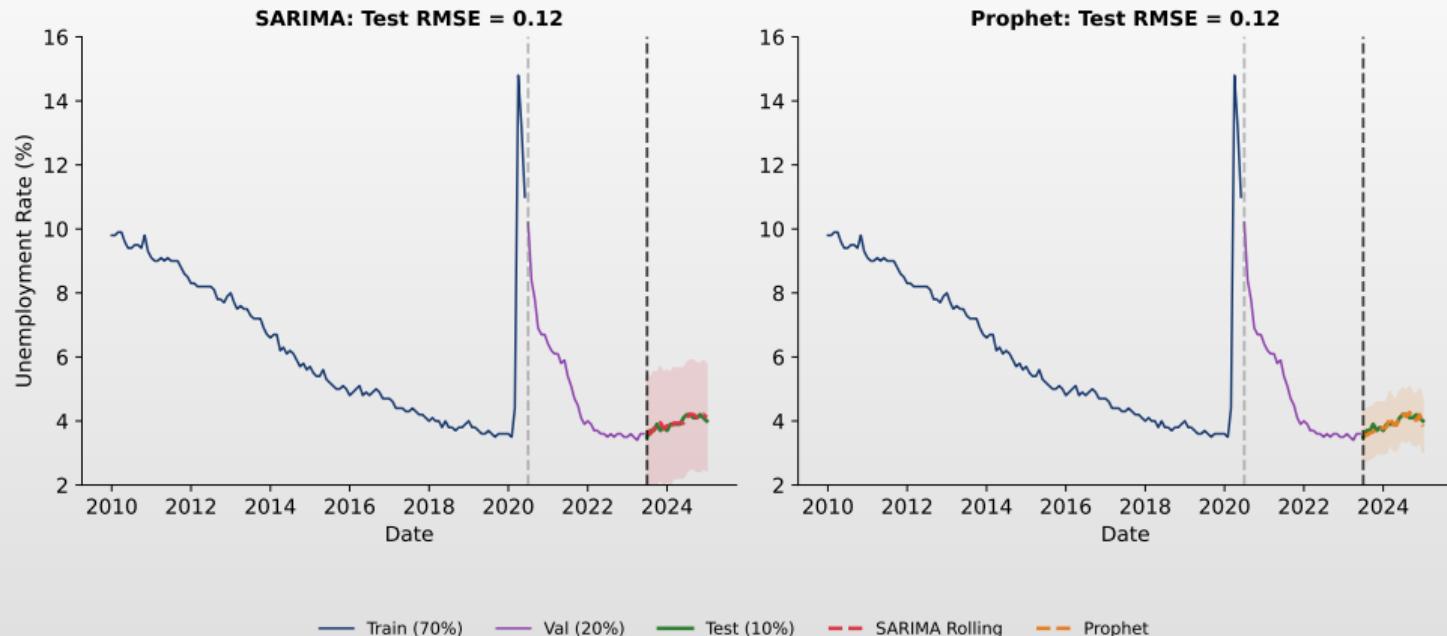
$\text{Scale} = 0.10$ balances flexibility (capturing COVID shock) with stability.



Unemployment: SARIMA vs Prophet Comparison



Unemployment: SARIMA vs Prophet Comparison



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Prophet: When to Use It

Ideal Use Cases

- Business data with **holidays**
- Missing values** present
- Need **interpretable** components
- Forecasts with **uncertainty bands**

Caveat: Structural Breaks

Prophet handles breaks via changepoints, but **SARIMA outperformed** it on unemployment (0.12 vs 0.58). Always validate!

Prophet vs ARIMA

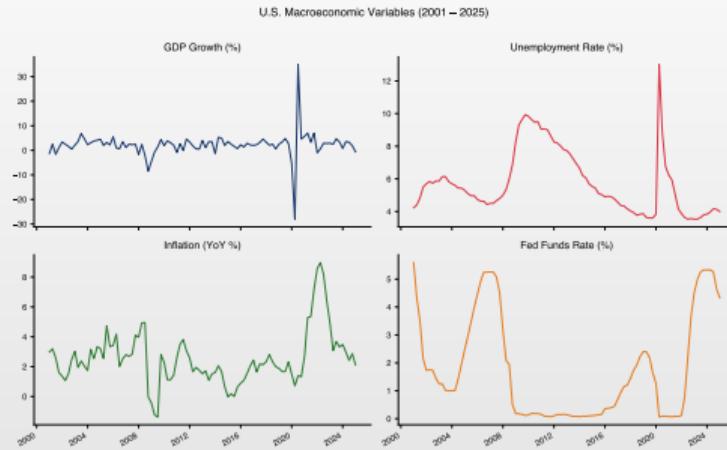
	Prophet	ARIMA
Changepoints	✓	✗
Missing data	✓	✗
Holidays	✓	✗
Speed	Fast	Moderate
Interpretable	✓	✗

Key Parameters

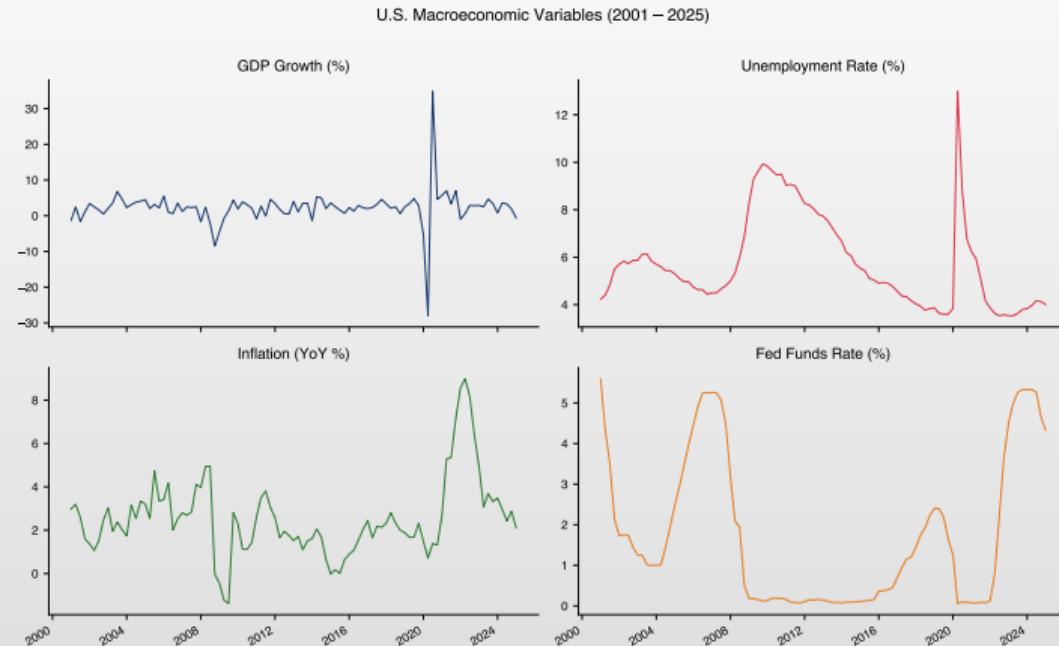
`changepoint_prior_scale`: flexibility
`seasonality_prior_scale`: smoothness



VAR: Multivariate Economic Data



VAR: Multivariate Economic Data



 TSA_ch10_economic_vars



VAR Model Specification

Definition 7 (Vector Autoregression VAR(p))

For K variables $y_t = (y_{1t}, \dots, y_{Kt})'$:

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + u_t$$

where A_i are $K \times K$ coefficient matrices, $u_t \sim N(0, \Sigma)$, Σ = covariance matrix.

For Our 4-Variable System

VAR(2) has:

- 4 intercepts
- $2 \times 4 \times 4 = 32$ AR coefficients
- 36 parameters total**

Lag Selection

Use information criteria:

- AIC**: Tends to overfit
- BIC**: More parsimonious
- Cross-validation on held-out data



Information Criteria for Model Selection

Definition 8 (Akaike and Bayesian Information Criteria)

For a model with log-likelihood \mathcal{L} , k parameters, and n observations:

$$\text{AIC} = -2\mathcal{L} + 2k$$

$$\text{BIC} = -2\mathcal{L} + k \ln(n)$$

AIC

- Asymptotically efficient
- May overfit with small n
- Minimizes prediction error

BIC

- Consistent (finds true model)
- Heavier penalty: $\ln(n) > 2$ if $n > 7$
- More parsimonious



VAR: Lag Selection and Estimation

BIC by Lag Order

Lag	BIC
1	-4.810
2	-5.178
3	-4.633
4	-4.614

Data Split

Set	Period	N
Training (70%)	2001-Q1 to 2017-Q4	67
Validation (20%)	2018-Q1 to 2022-Q4	20
Test (10%)	2023-Q1 to 2025-Q1	10
Total		97

Validation Check

VAR(2) also achieves lowest validation RMSE.



VAR Model Stability

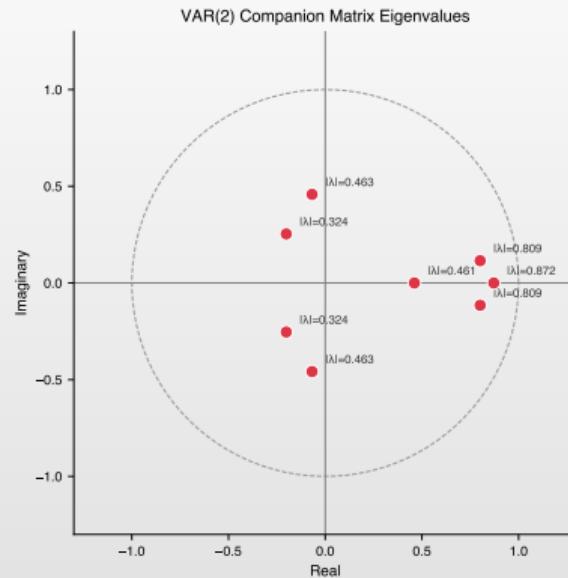
Stability condition

- All eigenvalues of the companion matrix: $|\lambda_i| < 1, \forall i$

VAR(2) Results — economic data

$ \lambda_1 , \lambda_2 $	0.324
$ \lambda_3 , \lambda_4 $	0.463
$ \lambda_5 $	0.461
$ \lambda_6 $	0.872
$ \lambda_7 , \lambda_8 $	0.810

- Max $|\lambda| = 0.872 < 1$ — **stable**



VAR vs VECM: Cointegration

Problem

- ◻ If variables are $I(1)$ > VAR on levels produces spurious regressions

Definition 9 (VECM)

$$\Delta y_t = \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + u_t, \quad \Pi = \alpha \beta'$$

Johansen Test — economic data

r	Trace	CV 5%	Reject?
0	64.09	47.85	Yes
1	24.03	29.80	No
2	11.89	15.49	No
3	1.28	3.84	No

- ◻ 1 cointegrating relation found
- ◻ VECM more appropriate than VAR on levels

Key message

- ◻ VAR on differences: loses long-run relationship; VECM: preserves it through $\Pi = \alpha \beta'$



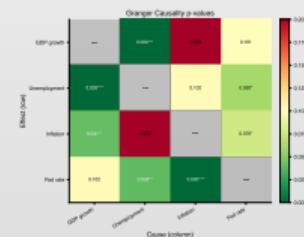
Granger Causality: Empirical Results

Interpretation

Each cell shows p-value for testing whether the row variable Granger-causes the column variable. Green: $p < 0.10$. Read: row causes column.

Economic Findings

- Unemp \rightarrow GDP ($p = 0.045$): Okun's Law
- Fed rate \rightarrow Inflation ($p = 0.087$): Monetary policy transmission
- GDP \rightarrow Unemp: Weak evidence



Granger Causality: Formal Definition

Definition 10 (Granger Causality)

X Granger-causes Y if, for some $h > 0$:

$$\text{MSE} \left[\mathbb{E}[Y_{t+h} | \mathcal{F}_t^{X,Y}] \right] < \text{MSE} \left[\mathbb{E}[Y_{t+h} | \mathcal{F}_t^Y] \right]$$

where $\mathcal{F}_t^{X,Y}$ includes past values of both X and Y , while \mathcal{F}_t^Y includes only past Y .

Important Caveat

Granger causality is **predictive causality**, not true causality. “ X Granger-causes Y ” means X contains useful information for forecasting Y , not that X causes Y in a structural sense.

Test Procedure

Use F-test (or Wald test) to test H_0 : coefficients on lagged X are jointly zero in the Y equation.



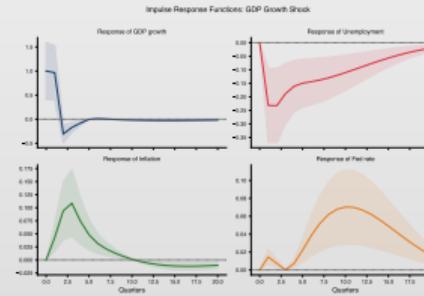
Impulse Response Functions (IRF)

What is IRF?

Shows how a 1-unit shock affects others over time.

GDP Shock Effects

- Unemp** ↓: Okun's Law
- Inflation** ↑: Demand-pull
- Fed Rate** ↑: Taylor Rule



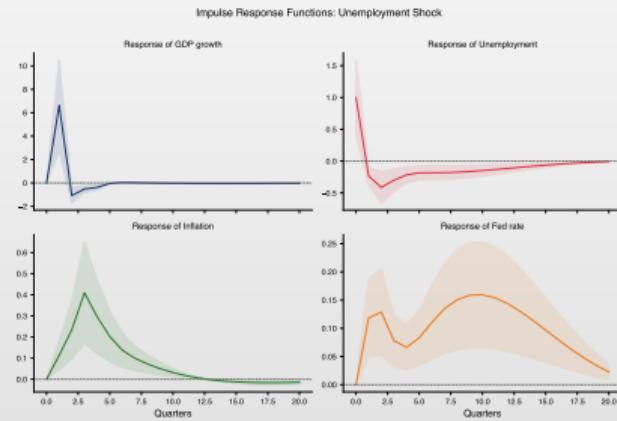
Q TSA_ch10_irf_gdp_shock



IRF: Unemployment Shock

Effects

- ↑ Unemp \Rightarrow ↓ GDP, ↓ Infl, Fed cuts rates

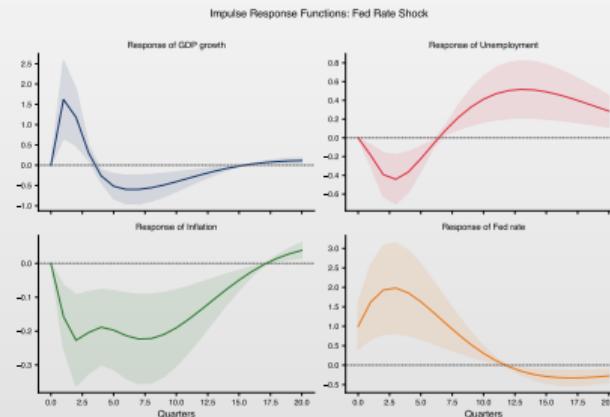


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IRF: Fed Rate Shock

Monetary Policy

- Rate hike \Rightarrow GDP \downarrow , Unemp \uparrow , Infl \downarrow



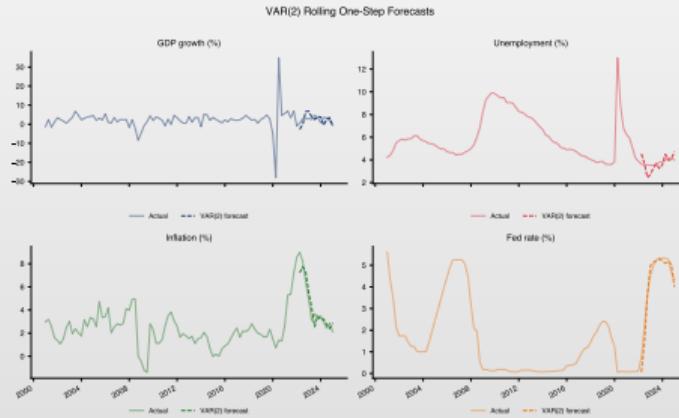
Q TSA_ch10_irf_fed_shock



VAR: Forecast (Train/Val/Test)

Rolling One-Step-Ahead Forecast

- VAR captures GDP-Unemployment dynamics
- COVID shock visible in test period



Q TSA_ch10_var_forecast



VAR: Test Set Results

Test Set Performance by Variable

Variable	RMSE	MAE	Dir. Acc.
GDP Growth	1.33	0.99	50%
Unemployment	0.64	0.52	50%
Inflation	1.56	1.12	60%
Fed Rate	2.59	2.45	80%
Average	1.53	1.27	60%

Strengths

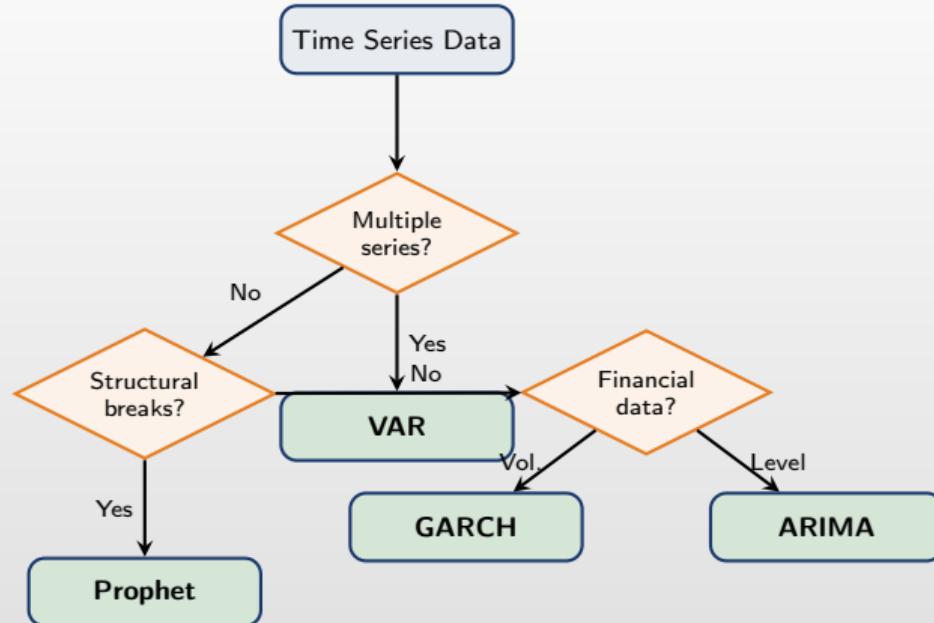
- Cross-variable dynamics
- Good directional accuracy

Limitations

- Many parameters
- Sensitive to lag selection



Model Selection Framework



Summary: Model Comparison

Results

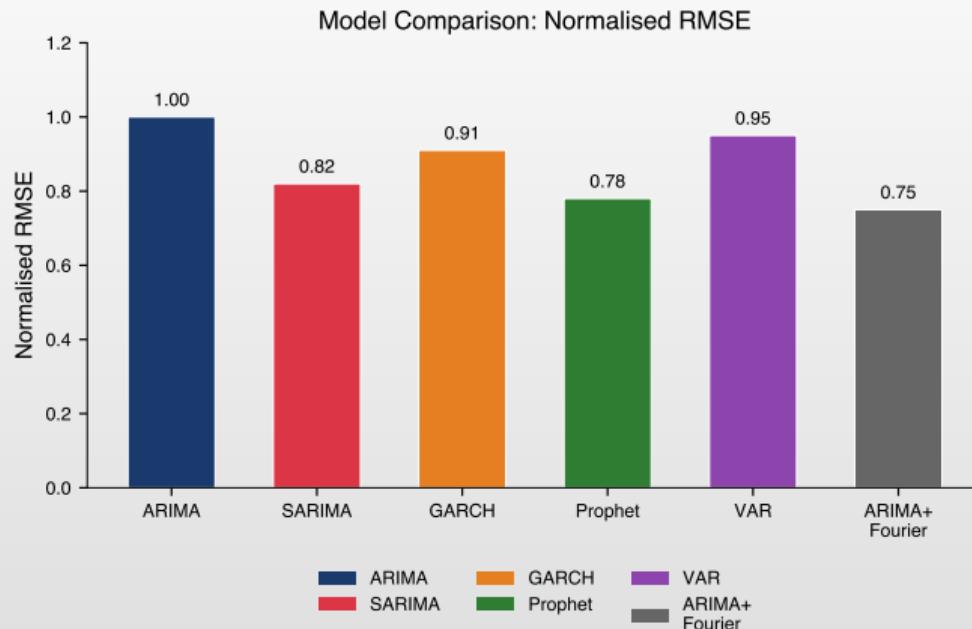
Case	Challenge	Model	RMSE
Bitcoin	Volatility	GARCH	2.15
Sunspots	Seasonality	Fourier	31.10
Unemp	Break	SARIMA	0.12
Economic	Multi-var	VAR	1.53

Key Principle

Match model to data characteristics—no single model dominates.



Summary: Model Comparison



Comprehensive Model Comparison

Feature	GARCH	Fourier	Prophet	VAR
Target	Volatility	Level	Level	Multiple
Seasonality	No	Yes (long)	Yes (multi)	No
Structural breaks	No	No	Yes	No
Multiple series	No	No	No	Yes
Interpretable	Medium	High	High	High
Parameters	Few	2K	Auto	Many
Missing data	No	No	Yes	No
Best for	Finance	Cycles	Business	Macro

Our Results

- GARCH: MAE=1.82 (volatility)
- Fourier: RMSE=31.10 (cycles)
- SARIMA: RMSE=0.12 (breaks)
- VAR: Avg RMSE=1.53 (multi)

Key Insight

Each model excels in its domain. The art is matching the model to the data characteristics.



Best Practices for Applied Forecasting

Methodology

1. **Explore** data
2. **Test** stationarity
3. **Split** train/val/test
4. **Compare** on validation
5. **Report** test metrics

Common Mistakes

- Peeking at test data
- Over-fitting
- Ignoring assumptions

Practical Tips

- Start simple (naive)
- Add complexity if needed
- Check residuals
- Report CIs

Remember

"All models are wrong, but some are useful." — Box



Forecasting vs Causality vs Decision

Objective	Model	Focus
Pure prediction	ARIMA / ML	Out-of-sample accuracy
Financial risk	GARCH	Volatility, VaR
Macro dynamics	VAR	Multivariate interactions
Structural relations	SVAR / VECM	Causal identification
Regimes	Markov Switching	Regime changes

Key Message

- There is no universal model
- There is **fit between model and problem**



Key Takeaways

1. Rigorous Methodology

- ▶ Train/validation/test split prevents overfitting
- ▶ Test set must remain untouched until final evaluation

2. Match Model to Data

- ▶ Financial volatility → GARCH
- ▶ Long seasonality → Fourier terms
- ▶ Structural breaks → Prophet
- ▶ Multiple series → VAR

3. Interpret Results Carefully

- ▶ Granger causality \neq true causality
- ▶ Out-of-sample performance matters most
- ▶ Simpler models often work better



The Role of AI in Time Series Modeling

AI can

- Generate code for estimation and forecasting
- Select models (AutoML, grid search)
- Combine forecasts (ensemble)
- Detect anomalies and patterns

But cannot

- Replace statistical validation
- Automatically detect **data leakage**
- Guarantee correct economic interpretation
- Verify model assumptions

Principle

- AI is a **tool**, not an authority
- Statistical validation remains the researcher's responsibility



AI Exercise: Critical Thinking

Prompt to test in ChatGPT / Claude / Copilot

"Download monthly US Retail Sales from FRED (series RSXFS) for 2010-01 to 2024-12 (180 observations). Perform a complete time series analysis: decomposition, stationarity tests, model selection (compare ETS, SARIMA, and Prophet), 12-month forecast, and evaluation using RMSE/MAE/MASE on a 70/15/15 temporal split. Give me publication-quality Python code."

Exercise:

1. Run the prompt in an LLM of your choice and critically analyze the response.
2. Does it follow the correct workflow? (plot → decompose → test → model → diagnose → forecast)
3. Does it compare multiple models (ETS, ARIMA, SARIMA) with proper benchmarks?
4. Is the train/test split done properly? Is there any data leakage?
5. Does it discuss limitations and assumptions of the chosen model?

Warning: AI-generated code may run without errors and look professional. *That does not mean it is correct.*



Question 1

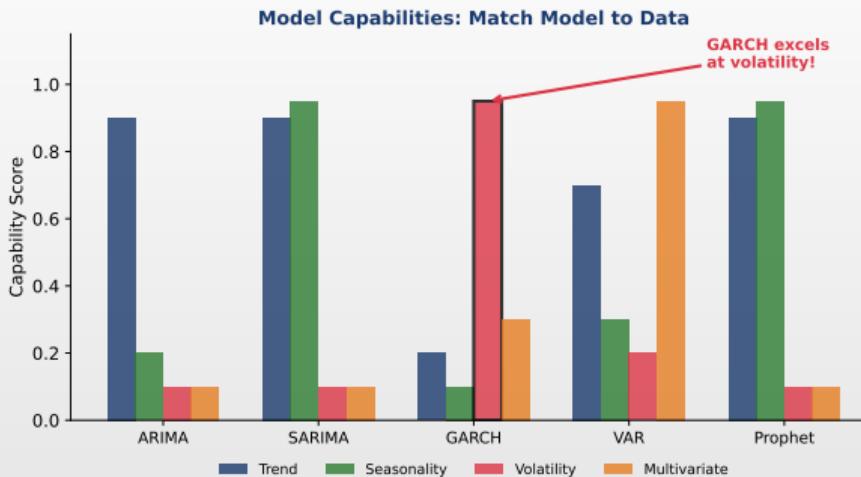
Question

- Which model would you choose to forecast the volatility of financial returns?

Answer Choices

- (A)** ARIMA — captures trends and autocorrelations
- (B)** GARCH — models conditional variance
- (C)** Prophet — detects changepoints and seasonality
- (D)** VAR — multivariate model for interdependencies

Question 1: Answer



Answer: (B)

- GARCH captures volatility clustering and time-varying risk. ARIMA models the level, Prophet handles seasonality, VAR captures cross-series dynamics — none model variance directly.



Question 2

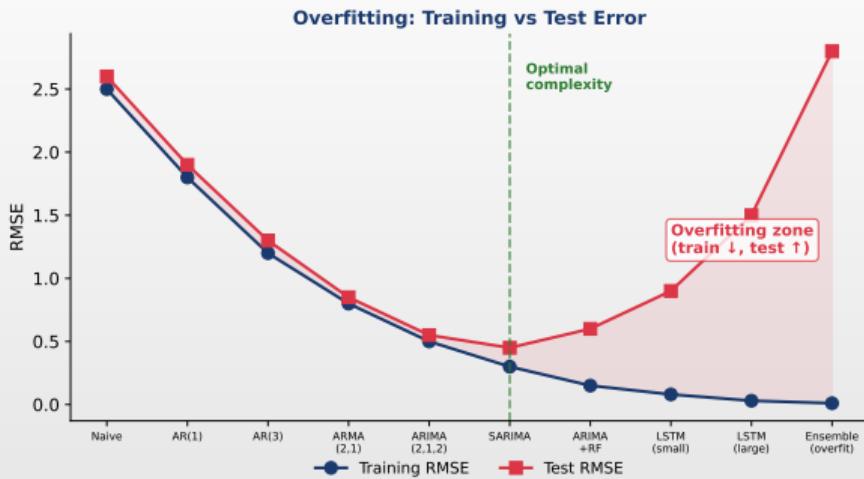
Question

- A SARIMA model achieves RMSE = 0.05 on training but RMSE = 2.30 on test. What does this indicate?

Answer Choices

- (A)** The model is excellent — low training error confirms quality
- (B)** The model suffers from overfitting — it memorizes noise
- (C)** The test set is faulty and should be replaced
- (D)** The difference is normal — all models have higher test error

Question 2: Answer



Answer: (B)

- A $46\times$ ratio between test and training RMSE signals severe overfitting. The model fits noise in the training data and fails to generalize. Solution: simpler model, proper validation.

Q TSA_ch10_quiz2_overfitting



Question 3

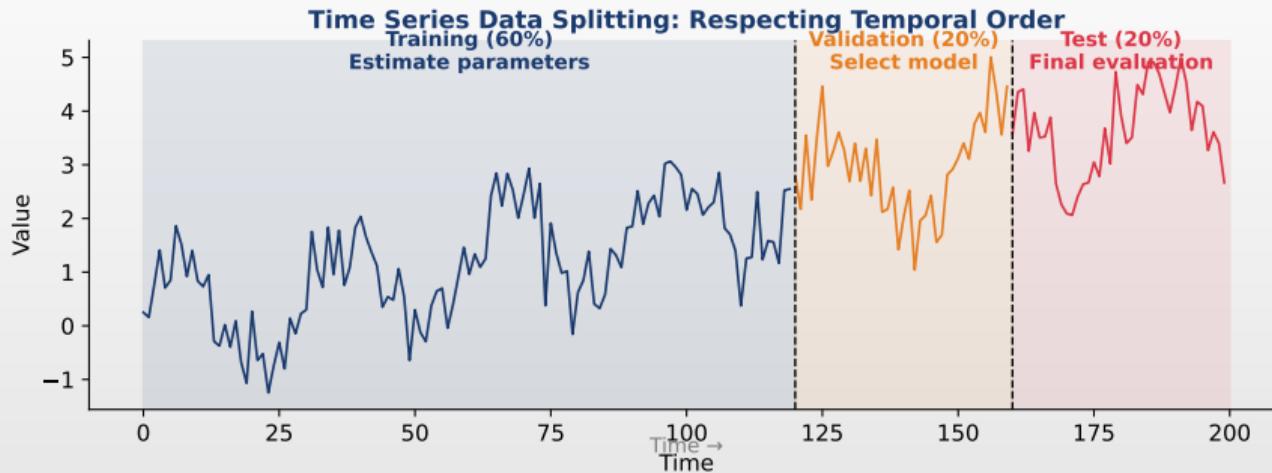
Question

- Why is it important to separate data into train/validation/test sets?

Answer Choices

- (A)** To have more training data
- (B)** To prevent overfitting and evaluate correctly
- (C)** It is just a convention with no real importance
- (D)** To reduce computation time

Question 3: Answer



Answer: (B)

- Train: estimate parameters. Validation: select model/hyperparameters. Test: final unbiased evaluation. Mixing these roles leads to optimistic performance estimates.



Question 4

Question

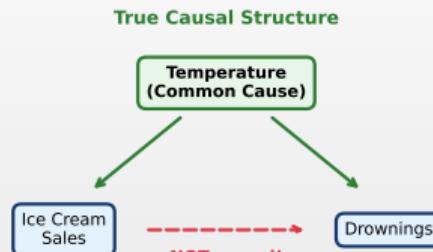
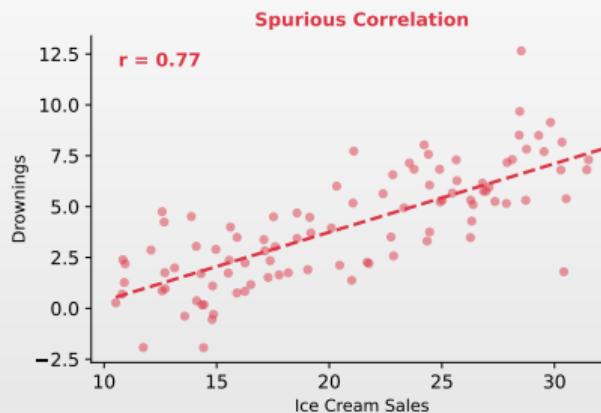
- Is Granger causality equivalent to true (structural) causality?

Answer Choices

- (A)** Yes — if X predicts Y , then X causes Y
- (B)** No — it only tests predictive content, not causation
- (C)** It depends on the number of lags selected
- (D)** Yes, if the p-value is below 0.05

Question 4: Answer

Granger Causality ≠ True Causality



Answer: (B)

- Granger causality tests whether past X improves forecasts of Y . Spurious correlations (e.g., ice cream sales and drownings) can pass the test due to common causes.



Question 5

Question

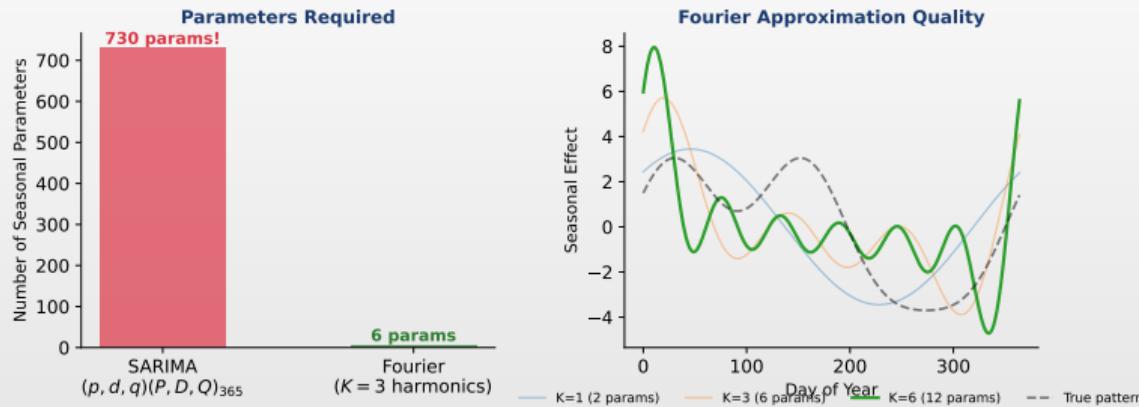
- What model do you use for a series with long seasonality (e.g., $s = 365$ days)?

Answer Choices

- (A)** SARIMA(p, d, q)(P, D, Q)₃₆₅
- (B)** GARCH — models variation
- (C)** ARIMA + Fourier terms or Prophet/TBATS
- (D)** VAR with 365 lags

Question 5: Answer

Long Seasonality ($s = 365$): Fourier Terms vs SARIMA



Answer: (C)

- SARIMA₃₆₅ needs ~ 730 seasonal parameters — infeasible. Fourier terms with $K = 3$ use only 6 parameters. Prophet and TBATS handle multiple seasonalities automatically.



Bibliography I

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Key Takeaways

What We Learned

- Model selection depends on data characteristics: stationarity, seasonality, volatility
- The Box-Jenkins methodology provides a systematic framework for time series modeling
- Proper evaluation requires out-of-sample testing and time series cross-validation

Important

No single model wins everywhere. Match the model to the data: ARIMA for trends, SARIMA for seasonality, GARCH for volatility, VAR/VECM for multivariate dynamics, Prophet/TBATS for complex patterns. Always validate out-of-sample!



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Thank You!

Questions?

Course materials available at: <https://danpele.github.io/Time-Series-Analysis/>

