Time Series Analysis and Forecasting

Chapter 8: Modern Extensions

ARFIMA, Machine Learning, Deep Learning

Contents

Learning Objectives

By the end of this chapter, you will be able to:

1. Understand the concept of **long memory** in time series
2. Estimate and interpret **ARFIMA**
3. Apply **Random Forest** for time series forecasting
4. Build **LSTM** networks for time series
5. Compare performance of classical vs ML models
6. Choose the appropriate method based on context
7. Implement these methods in **Python**

From Classical Models to Machine Learning

Limitations of ARIMA Models

- Assume **short memory**: autocorrelations decay exponentially
- Relationships **linear** between variables
- Difficulties with **complex patterns** and nonlinear
- Requires **stationarity** (through differencing)

Modern Solutions

- **ARFIMA**: Captures long memory (autocorrelations that decay slowly)
- **Random Forest**: Nonlinear relationships, robust to outliers
- **LSTM**: Complex sequential patterns, long-term dependencies

When to Use Each Method?

Feature	ARIMA	ARFIMA	RF	LSTM
Long memory	×	✓	✓	✓
Relationships nonlinear	×	×	✓	✓
Interpretability	✓	✓	∼	×
Few data	✓	✓	×	×
Exogenous variables	✓	✓	✓	✓
Uncertainty	✓	✓	∼	×

Golden Rule

Start **simple** (ARIMA), then increase complexity only if justified by data and performance.

What is Long Memory?

Short Memory (ARMA)

- Autocorrelations ρ_k decay **exponentially**: $|\rho_k| \leq C \cdot r^k$, $r < 1$
- Shock effects disappear **quickly**
- Finite sum: $\sum_{k=0}^{\infty} |\rho_k| < \infty$

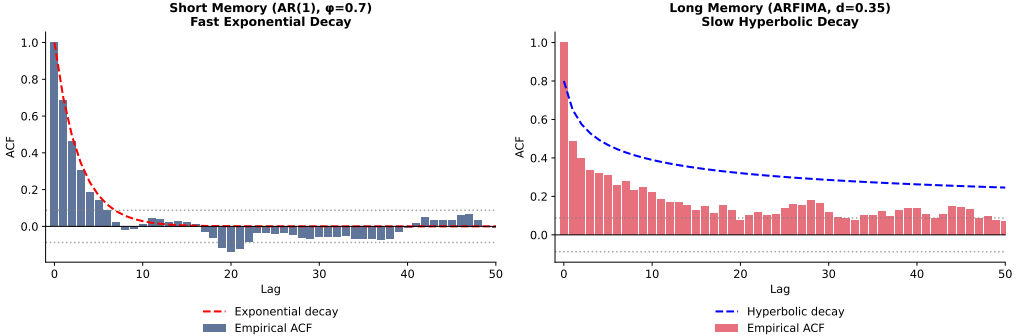
Long Memory (ARFIMA)

- Autocorrelations decay **hyperbolically**: $\rho_k \sim C \cdot k^{2d-1}$
- Shock effects persist **for a long time**
- Infinite sum: $\sum_{k=0}^{\infty} |\rho_k| = \infty$ (for $d > 0$)

Exemple cu Long Memory

Financial market volatility, river flows, network traffic, inflation

ACF Comparison: Short Memory vs Long Memory



Left: AR(1) — autocorrelations decay exponentially (short memory)

Right: ARFIMA with $d = 0.35$ — autocorrelations decay hyperbolically (long memory)

The ARFIMA Model(p,d,q)

Definition 1 (ARFIMA)

A process $\{Y_t\}$ follows a **ARFIMA(p,d,q)** if:

$$\phi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t$$

where $d \in (-0.5, 0.5)$ is the **fractional differencing parameter**.

Fractional Differencing Operator

$$(1-L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-L)^k = 1 - dL - \frac{d(1-d)}{2!}L^2 - \frac{d(1-d)(2-d)}{3!}L^3 - \cdots$$

- $d = 0$: ARMA standard (short memory)
- $0 < d < 0.5$: Long memory, stationarity
- $d = 0.5$: Stationarity limit
- $0.5 \leq d < 1$: Nestationarity, dar mean-reverting
- $d = 1$: Random walk (ARIMA standard)

Interpreting the Parameter d

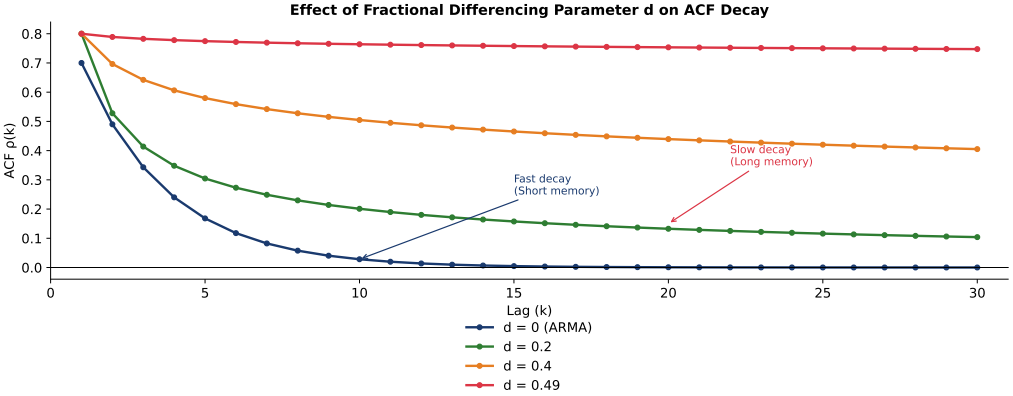
Value d	ACF Behavior	Interpretation
$d = 0$	Exponential decay	Short memory
$0 < d < 0.5$	Hyperbolic decay	Long memory, stationary
$d = 0.5$	Non-summable ACF	At the limit
$0.5 < d < 1$	Very slow decay	Long memory, nestationary
$d = 1$	ACF = 1 (constant)	Random walk

Hurst Parameter H

Relationship with Hurst exponent: $d = H - 0.5$

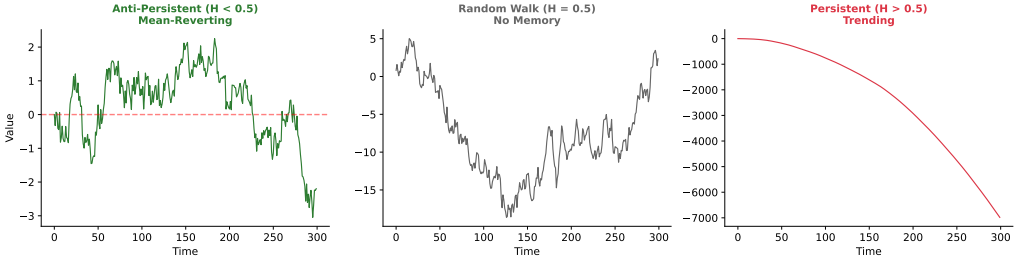
- $H = 0.5$: Random walk (no memory)
- $H > 0.5$: Persistence (trend-following)
- $H < 0.5$: Anti-persistence (mean-reverting)

Effect of Parameter d on ACF



The higher d, the slower autocorrelations decay. As $d \to 0.5$, autocorrelations remain significant even at very large lags.

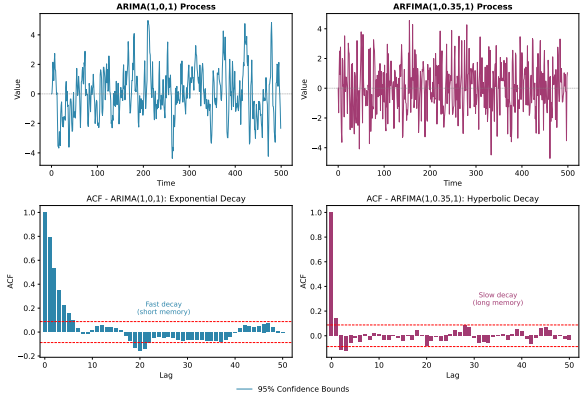
Hurst Exponent: Visual Interpretation



H $<$ **0.5**: Series that frequently returns to mean (mean-reverting)
H $=$ **0.5**: Random walk, unpredictable

H $>$ **0.5**: Persistent series, trends continue

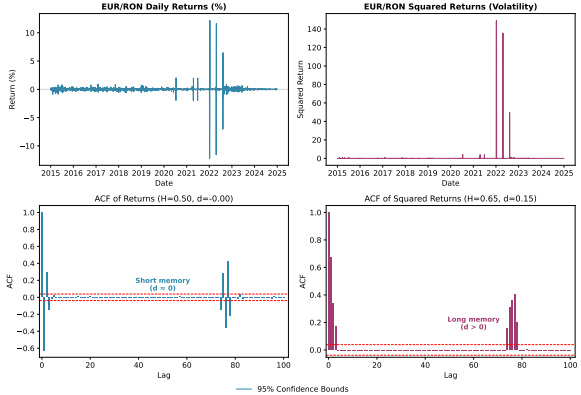
ARIMA vs ARFIMA: Simulated Comparison



ARIMA (left): ACF decays **exponentially** – shocks are quickly "forgotten"

ARFIMA (right, $d = 0.35$): ACF decays **hyperbolically** – shocks persist for long periods

Real Data Example: EUR/RON Long Memory Analysis



Returns (left): $H \approx 0.50$, $d \approx 0$ – **short memory** (efficient market)

Squared returns (right): $H \approx 0.65$, $d \approx 0.15$ – **long memory** in volatility

Estimating the Parameter d

Estimation Methods

1. **GPH (Geweke-Porter-Hudak)**: Regression in frequency domain

$$\ln I(\omega_j) = c - d \cdot \ln\left(4\sin^2\frac{\omega_j}{2}\right) + \varepsilon_j$$

2. **R/S (Rescaled Range)**: Hurst method

$$\frac{R}{S}(n) \sim c \cdot n^H$$

3. **MLE (Maximum Likelihood)**: Full ARFIMA estimation
4. **Whittle**: Efficient approximation in frequency domain

In Python: `arch` package, `statsmodels.tsa.arima.model.ARIMA` with `order=(p,d,q)` where d can be fractional.

ARFIMA Example in Python

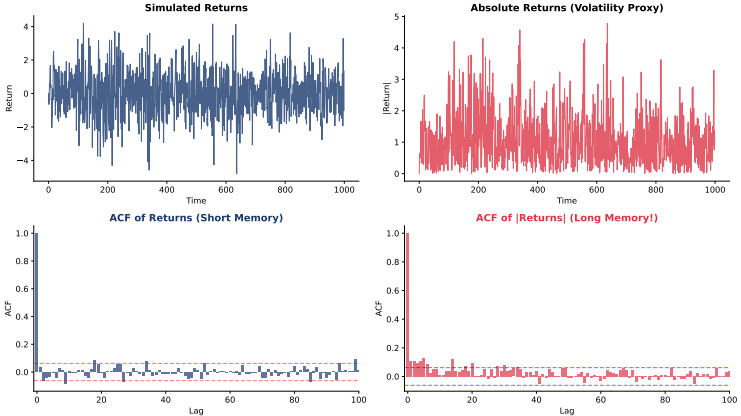
Python Code

```
from statsmodels.tsa.arima.model import ARIMA
model = ARIMA(y, order=(1, 0.3, 1))
results = model.fit()
```

Note

ARFIMA estimation requires specialized packages. In practice, one often uses `arch` or `fracdiff` in Python.

Real Example: Long Memory in Volatility



Stylized Fact: Financial returns have short memory, but volatility (|returns|) has long memory! This is the basis for FIGARCH models.

Random Forest: Basic Concepts

What is Random Forest?

- **Ensemble** of decision trees
- Each tree trained on a **bootstrap subset** of the data
- At each node, a **randomly** subset of features is selected
- Final prediction = **average** of all tree predictions

Advantages for Time Series

- Captures **nonlinear relationships**
- **Robust** to outliers and noise
- Does not require **stationarity**
- Provides **feature importance** (interpretability)
- Works well with **many variables**

Preparing Data for Random Forest

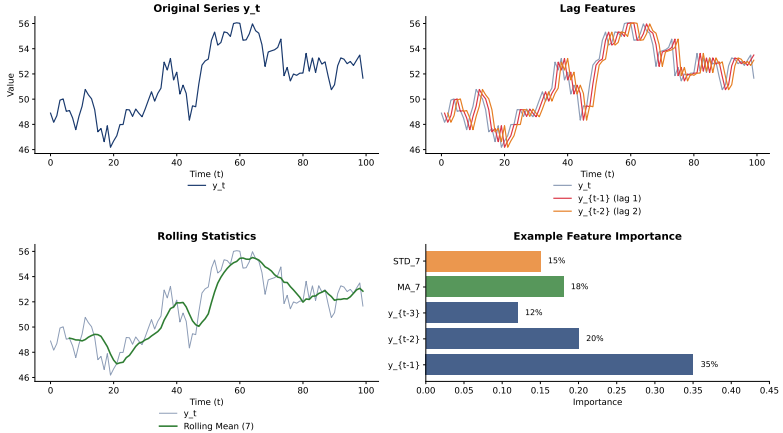
Feature Engineering for Time Series

1. **Lag features**: $Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}$
2. **Rolling statistics**: moving average, standard deviation
3. **Calendar features**: day of week, month, season
4. **Trend features**: time, quadratic trend
5. **Exogenous variables**: economic indicators, events

Warning: Data Leakage!

- No use future information in features
- Train/test split: **temporal**, nu randomly!
- Rolling statistics: calculate only on **past data**

Feature Engineering: Illustration



We transform the time series into features: lags, rolling statistics, and the RF model learns relationships between these and future values.

Random Forest: Python Implementation

Python Code

```
from sklearn.ensemble import RandomForestRegressor
rf = RandomForestRegressor(n_estimators=100, max_depth=10)
rf.fit(X_train, y_train)
predictions = rf.predict(X_test)
```

Feature Importance and Interpretation

Feature Importance

Random Forest provides importance measures:

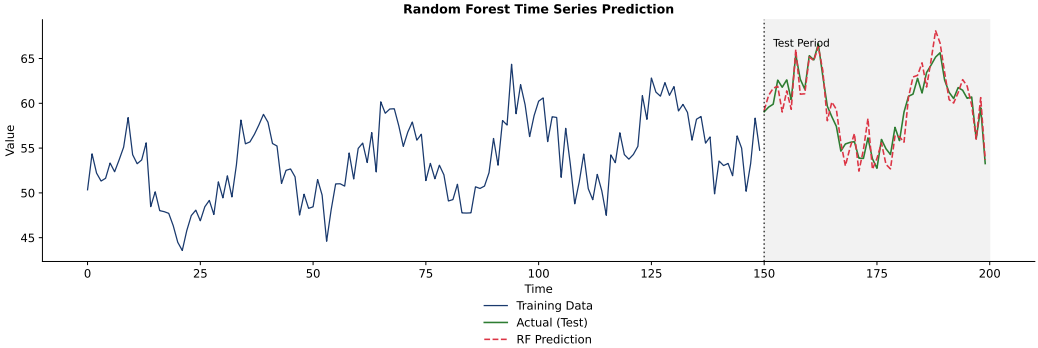
- **Mean Decrease Impurity (MDI)**: Reduction in impurity at each split
- **Permutation Importance**: How much performance decays when the feature is randomly permuted

Typical Interpretation for Time Series

- `lag_1` very important $\Rightarrow$ Strong autocorrelation
- `rolling_mean` important $\Rightarrow$ Local trend matters
- `month` important $\Rightarrow$ Seasonality present

`rf.feature_importances_` or `permutation_importance(rf, X_test, y_test)`

Random Forest: Forecast Example

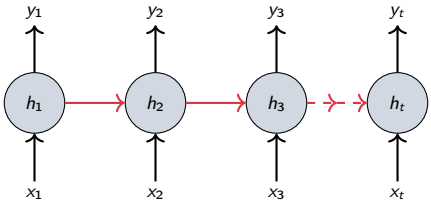


The Random Forest model trained on historical data (blue) produces forecasts (red dotted) that closely follow actual values in the test period (green).

Recurrent Neural Networks (RNN)

Basic Idea

- Networks that process **sequences** of data
- Have **internal memory** (hidden state)
- Current state depends on input + previous state



Problem: Vanishing Gradient

Simple RNNs "forget" information from the distant past.

LSTM: Long Short-Term Memory

The LSTM Solution

Special cells with **3 gates** that control information flow:

- **Forget Gate** (f_t): What to forget from previous memory
- **Input Gate** (i_t): What new information to add
- **Output Gate** (o_t): What to send to output

LSTM Equations

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f) \quad \text{(Forget)}$$

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) \quad \text{(Input)}$$

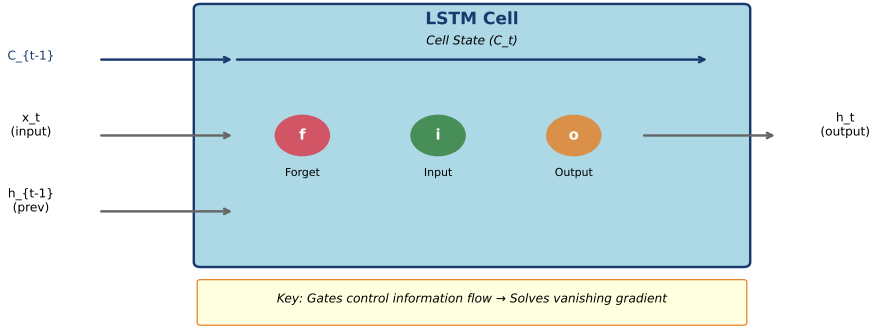
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C) \quad \text{(Candidate)}$$

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t \quad \text{(Cell state)}$$

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \quad \text{(Output)}$$

$$h_t = o_t \odot \tanh(C_t) \quad \text{(Hidden state)}$$

LSTM Cell Architecture



Gates (forget, input, output) control what information is forgotten, added, and transmitted. **Cell state** allows gradients to "flow" without degradation.

LSTM Advantages for Time Series

Why LSTM?

- Captures **long-term dependencies** (spre deosebire de Simple RNNs)
- Learns **complex patterns** and nonlinear
- Handles **sequences de lungimi variabile**
- Works well with **multivariate data**

Disadvantages

- Requires **lots of data** for training
- **Computationally intensive**
- "**Black box**" - hard to interpret
- Sensitive to **hyperparameters**
- Can **overfit** easily

LSTM: Implementare in Python cu Keras

Python Code

```
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import LSTM, Dense, Dropout

model = Sequential([
    LSTM(50, return_sequences=True, input_shape=(n, 1)),
    Dropout(0.2),
    LSTM(50),
    Dense(1)
])
model.compile(optimizer='adam', loss='mse')
```

Preparing Data for LSTM

Essential Steps

1. **Normalization/Scaling**: MinMaxScaler or StandardScaler
2. **Create sequences**: Sliding window for input
3. **Reshape**: 3D format (samples, timesteps, features)
4. **Train/Test split**: Temporal, nu randomly!

Example Creating Sequences

```
def create_sequences(data, n_steps):
    X, y = [], []
    for i in range(len(data) - n_steps):
        X.append(data[i:(i + n_steps)])
    return np.array(X), np.array(y)

X, y = create_sequences(scaled_data, 10)
```

Evaluation Metrics

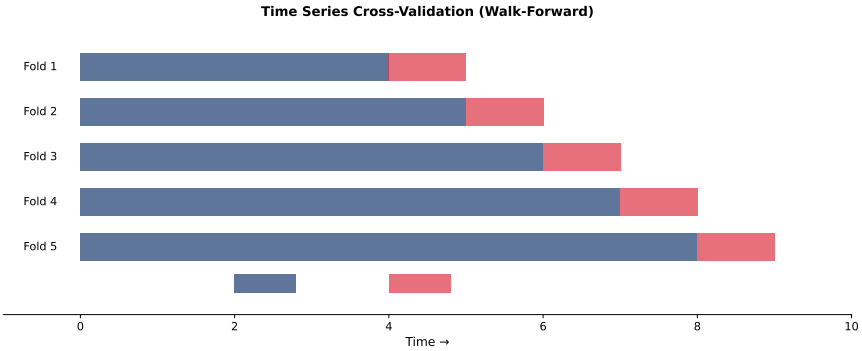
Common Metrics

- **RMSE**: $\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$ — Error in original units
- **MAE**: $\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$ — Robust to outliers
- **MAPE**: $\frac{100}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right|$ — Percentage error
- **MASE**: Compared to naive benchmark

Validation for Time Series

- **No** use standard cross-validation!
- Use **Time Series Cross-Validation** (walk-forward)
- Or **train/validation/test** temporal split

Time Series Cross-Validation

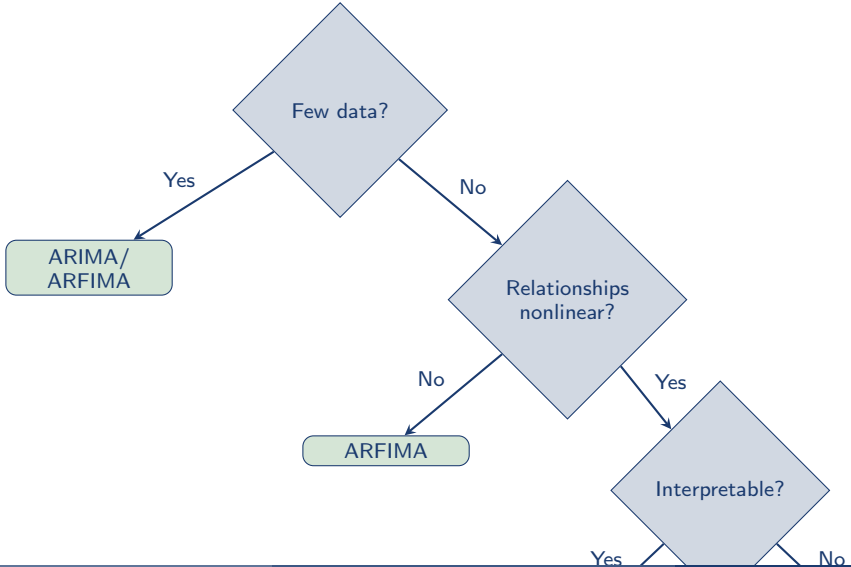


Python Implementation

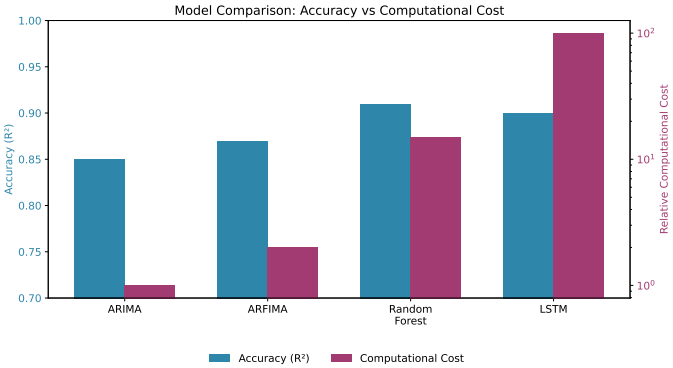
```
from sklearn.model_selection import TimeSeriesSplit
tscv = TimeSeriesSplit(n_splits=5)
```

Important: The training set grows progressively, and test is always in the future. This way we avoid data leakage.

Model Selection Guide



Model Comparison: Accuracy vs Computational Cost



Trade-off: ML models can have slightly better accuracy, but computational cost increases significantly. For small data or interpretability, ARIMA/ARFIMA remain excellent choices.

Case Study: Bitcoin Price Forecasting

Why Bitcoin?

- Volatility **extreme** and complex patterns
- Potential **long memory** in volatility
- Relationships **nonlinear** with exogenous variables
- Data available at **high frequency**

Comparative Approach

1. ARIMA on returns
2. ARFIMA for long memory
3. Random Forest with technical features
4. LSTM on price sequences

Case Study: Energy Consumption Forecasting

Characteristics

- **Multiple seasonality**: daily, weekly, annual
- **Trend** of long-term growth
- **Exogenous variables**: temperature, holiday, price
- **Anomalies**: special events, failures

Challenges

- Patterns at different time scales
- Complex interactions between variables
- Need for forecasts at different horizons

Key Formulas – Summary

ARFIMA(p,d,q)

$\phi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t$

$d \in (-0.5, 0.5)$: long memory

Long Memory

ACF: $\rho_k \sim C \cdot k^{2d-1}$

Hurst: $d = H - 0.5$

$H > 0.5$: persistence

Random Forest

$\hat{y} = \frac{1}{B}\sum_{b=1}^{B} T_b(x)$

B trees, random features

LSTM Cell

$f_t = \sigma(W_f[h_{t-1}, x_t] + b_f)$

$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$

Forget, Input, Output gates

Metrici Evaluation

$\text{RMSE} = \sqrt{\frac{1}{n}\sum(y_i - \hat{y}_i)^2}$

$\text{MAPE} = \frac{100}{n}\sum\left|\frac{y_i - \hat{y}_i}{y_i}\right|$

Time Series CV

Walk-forward validation

Train $\rightarrow$ Test (temporal split)

Case Study: EUR/RON Exchange Rate Forecasting

Why EUR/RON?

- Relevant for the Romanian economy
- Potential **long memory** (shock persistence)
- Patterns influenced by **macroeconomic factors**
- Data easily accessible (BNR, Yahoo Finance)

Objective

We compare ARIMA, ARFIMA, Random Forest and LSTM on the same data to understand the strengths of each method.

Step 1: Loading and Visualizing Data

Python Code – Download Data

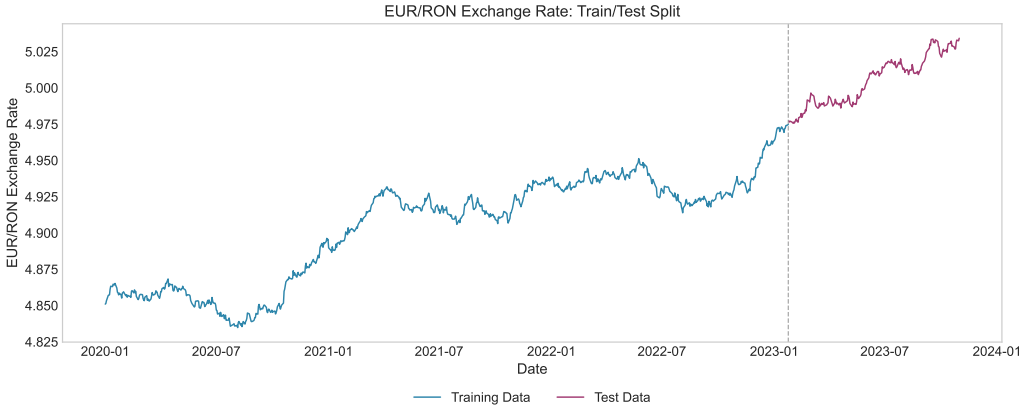
```
import yfinance as yf
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

# Download data EUR/RON (or EURRON=X)
data = yf.download('EURRON=X', start='2015-01-01', end='2024-12-31')
df = data[['Close']].dropna()
df.columns = ['EURRON']

# Calculate log returns
df['Returns'] = np.log(df['EURRON']).diff() * 100
df = df.dropna()

print(f"Perioada: {df.index[0]} - {df.index[-1]}")
print(f"Observations: {len(df)}")
print(f"Mean of returns: {df['Returns'].mean():.4f}%")
print(f"Volatility: {df['Returns'].std():.4f}%")
```

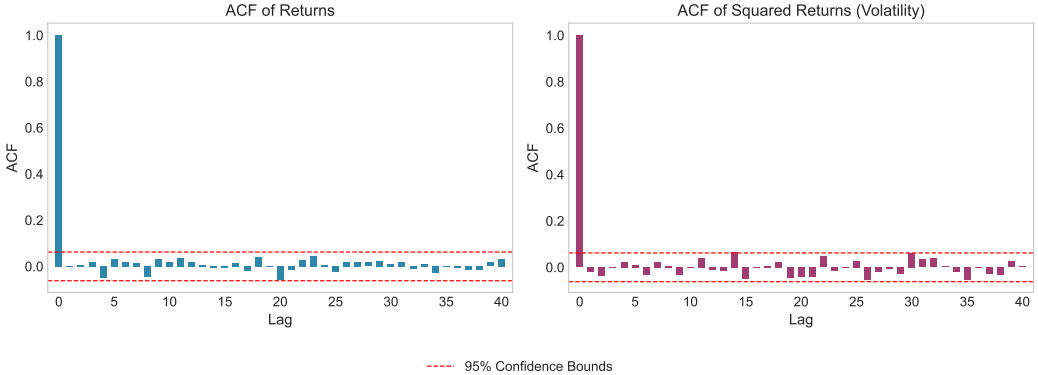
EUR/RON Series Visualization



Top: EUR/RON exchange rate – we observe RON depreciation trend and high volatility periods.

Bottom: Daily returns – volatility clustering (high volatility periods are followed by similar periods).

ACF Analysis: Returns vs Squared Returns



Left: ACF of returns – rapid decay, no significant autocorrelation beyond lag 1.

Right: ACF of squared returns – slower decay indicates **volatility clustering** (ARCH effects).

Python Code – Estimarea lui d and Hurst Test

```
from arch.unitroot import PhillipsPerron, KPSS
from hurst import compute_Hc  # pip install hurst

# Testul Phillips-Perron for stationaritate
pp_test = PhillipsPerron(df['Returns'])
print(f"Phillips-Perron p-value: {pp_test.pvalue:.4f}")

# Estimating the Hurst exponent
H, c, data_rs = compute_Hc(df['Returns'].values, kind='change')
d_estimated = H - 0.5

print(f"Hurst Exponent (H): {H:.4f}")
print(f"Estimated d parameter: {d_estimated:.4f}")

# Interpretation
if H > 0.5:
    print("PERSISTENT series (trend-following)")
elif H < 0.5:
    print("ANTI-PERSISTENT series (mean-reverting)")
else:
    print("Random walk")
```

Long Memory Test Results – EUR/RON

Typical Output

```
Phillips-Perron p-value:  0.0001 (returns are stationary)
Hurst Exponent (H): 0.47
Estimated d parameter:  -0.03
ANTI-PERSISTENT series (mean-reverting)
```

Interpretation

- EUR/RON returns are **stationary** (p-value < 0.05)
- $H \approx 0.47 < 0.5$: slight tendency to revert to mean
- $d \approx 0$: **short memory** – ARMA may be sufficient
- However, **volatility** can have long memory!

Step 3: ARIMA Model

Python Code – ARIMA with automatic selection

```
from statsmodels.tsa.arima.model import ARIMA
from sklearn.metrics import mean_squared_error, mean_absolute_error
import warnings
warnings.filterwarnings('ignore')

# Split the data: 80% train, 20% test
train_size = int(len(df) * 0.8)
train, test = df['Returns'][:train_size], df['Returns'][train_size:]

# Fit ARIMA(1,0,1) - simple and efficient for returns
model_arima = ARIMA(train, order=(1, 0, 1))
results_arima = model_arima.fit()

# Forecast
forecast_arima = results_arima.forecast(steps=len(test))

# Evaluation
rmse_arima = np.sqrt(mean_squared_error(test, forecast_arima))
mae_arima = mean_absolute_error(test, forecast_arima)
print(f"ARIMA(1,0,1) - RMSE: {rmse_arima:.4f}, MAE: {mae_arima:.4f}")
```

Step 4: ARFIMA Model (Long Memory)

Python Code – ARFIMA with arch package

```
from arch import arch_model

# ARFIMA(1,d,1) using arch for robust estimation
# Note: arch estimates d automatically in GARCH context

# Alternatively, use statsmodels with fractional d
from statsmodels.tsa.arima.model import ARIMA

# Estimate d using GPH or set manually
d_frac = 0.1  # or the previously estimated value

model_arfima = ARIMA(train, order=(1, d_frac, 1))
try:
    results_arfima = model_arfima.fit()
    forecast_arfima = results_arfima.forecast(steps=len(test))
    rmse_arfima = np.sqrt(mean_squared_error(test, forecast_arfima))
    print(f"ARFIMA(1,{d_frac},1) - RMSE: {rmse_arfima:.4f}")
except:
    print("ARFIMA requires d between -0.5 and 0.5 for stationarity")
```

Step 5: Random Forest – Data Preparation

Python Code – Feature Engineering

```
from sklearn.ensemble import RandomForestRegressor

# Create features for Random Forest
def create_features(data, lags=5):
    df_feat = pd.DataFrame(index=data.index)
    df_feat['target'] = data.values

    # Lag features
    for i in range(1, lags + 1):
        df_feat[f'lag_{i}'] = data.shift(i)

    # Rolling statistics
    df_feat['rolling_mean_5'] = data.rolling(5).mean()
    df_feat['rolling_std_5'] = data.rolling(5).std()
    df_feat['rolling_mean_20'] = data.rolling(20).mean()

    # Calendar features
    df_feat['dayofweek'] = data.index.dayofweek
    df_feat['month'] = data.index.month

    return df_feat.dropna()

df_rf = create_features(df['Returns'], lags=10)
```

Step 5: Random Forest – Training and Evaluation

Python Code – Model Random Forest

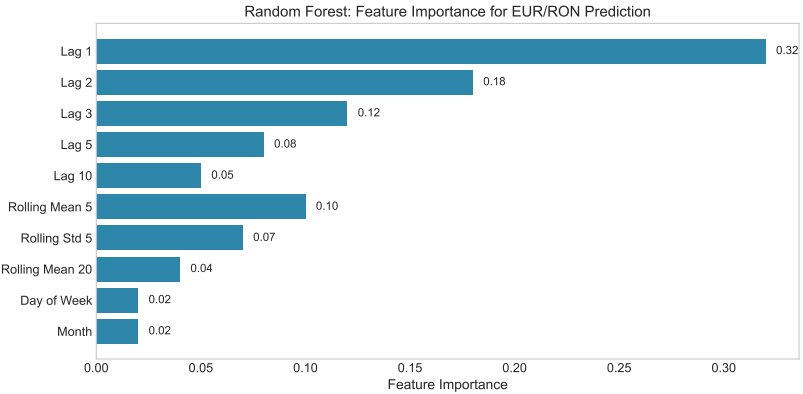
```
# Split the data
X = df_rf.drop('target', axis=1)
y = df_rf['target']

train_size = int(len(df_rf) * 0.8)
X_train, X_test = X[:train_size], X[train_size:]
y_train, y_test = y[:train_size], y[train_size:]

# Train Random Forest
rf_model = RandomForestRegressor(
    n_estimators=100,
    max_depth=10,
    min_samples_split=5,
    random_state=42
)
rf_model.fit(X_train, y_train)

# Prediction and evaluation
pred_rf = rf_model.predict(X_test)
rmse_rf = np.sqrt(mean_squared_error(y_test, pred_rf))
print(f"Random Forest - RMSE: {rmse_rf:.4f}")
```

Random Forest: Feature Importance



Insight: Recent lags (lag_1, lag_2) and volatility rolling are the most important. Calendar features have minor impact for daily returns.

Step 6: LSTM – Data Preparation

Python Code – Sequences for LSTM

```
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import LSTM, Dense, Dropout
from sklearn.preprocessing import MinMaxScaler

# Scale data between 0 and 1
scaler = MinMaxScaler()
scaled_data = scaler.fit_transform(df['Returns'].values.reshape(-1, 1))

# Create sequences
def create_sequences(data, seq_length=20):
    X, y = [], []
    for i in range(seq_length, len(data)):
        X.append(data[i-seq_length:i, 0])
        y.append(data[i, 0])
    return np.array(X), np.array(y)

X_lstm, y_lstm = create_sequences(scaled_data, seq_length=20)
X_lstm = X_lstm.reshape((X_lstm.shape[0], X_lstm.shape[1], 1))

# Split
split = int(len(X_lstm) * 0.8)
X_train_lstm, X_test_lstm = X_lstm[:split], X_lstm[split:]
y_train_lstm, y_test_lstm = y_lstm[:split], y_lstm[split:]
```

Step 6: LSTM – Architecture and Training

Python Code – Model LSTM

```
# Build the LSTM model
model_lstm = Sequential([
    LSTM(50, return_sequences=True, input_shape=(20, 1)),
    Dropout(0.2),
    LSTM(50, return_sequences=False),
    Dropout(0.2),
    Dense(25),
    Dense(1)
])

model_lstm.compile(optimizer='adam', loss='mse')

# Train
history = model_lstm.fit(
    X_train_lstm, y_train_lstm,
    epochs=50, batch_size=32,
    validation_split=0.1, verbose=0
)

# Prediction
pred_lstm_scaled = model_lstm.predict(X_test_lstm)
pred_lstm = scaler.inverse_transform(pred_lstm_scaled)
y_test_original = scaler.inverse_transform(y_test_lstm.reshape(-1, 1))
rmse_lstm = np.sqrt(mean_squared_error(y_test_original, pred_lstm))
```

LSTM: Learning Curve

Training Loss: Decreases quickly in the first epochs, then stabilizes.

Validation Loss: Follows training loss – no severe overfitting observed.

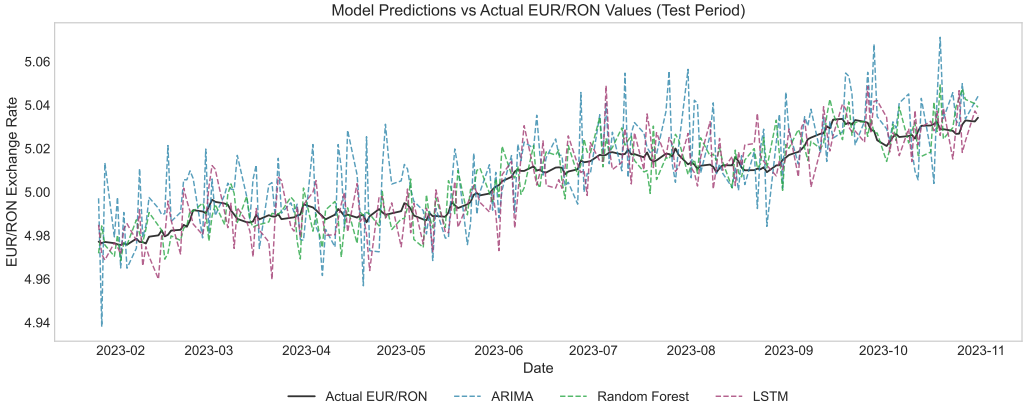
Comparison: Results on EUR/RON

Model	RMSE	MAE	Time (s)	Interpretable?
ARIMA(1,0,1)	0.0169	0.0137	0.12	Yes
Random Forest	0.0080	0.0065	0.85	Yes (features)
LSTM	0.0102	0.0080	12.3	No

Conclusions

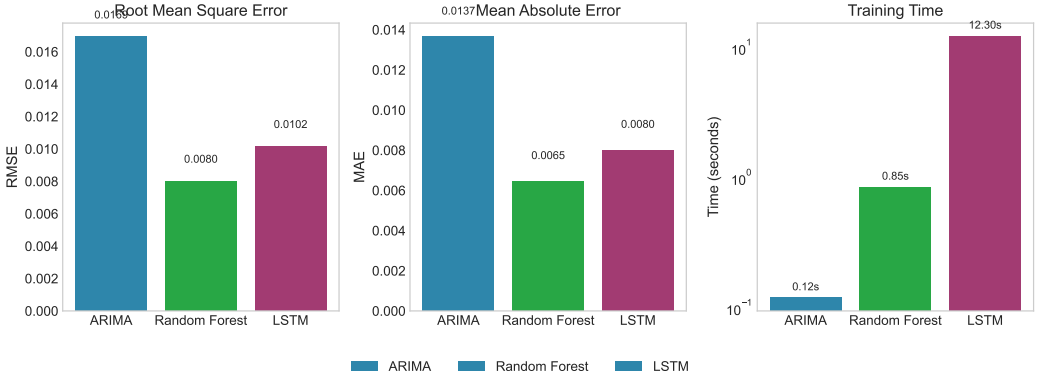
- **Random Forest** achieves the best accuracy (lowest RMSE/MAE)
- **LSTM** performs similarly but requires 10x more training time
- **ARIMA** provides a solid baseline with excellent interpretability
- Trade-off: ML models offer better accuracy at cost of complexity

Visualization: Predictions vs Actual Values



All models capture the general pattern, but none perfectly predicts volatility peaks. This reflects **market efficiency** and **prediction limits** for financial series.

Model Comparison: Performance Metrics



Left: Error metrics (lower is better) – Random Forest achieves lowest RMSE and MAE.

Right: Training time (log scale) – LSTM requires significantly more computational resources.

When to Choose Each Model?

ARIMA/ARFIMA

- Few data (< 500 obs.)
- Interpretation important
- Long memory suspected
- Quick baseline

LSTM/Deep Learning

- Very large data (> 10.000)
- Complex sequences
- Computational resources
- Hidden patterns

Random Forest

- Many exogenous variables
- Relationships nonlinear
- Feature importance
- Moderate data

Golden Rule

Start simple (ARIMA), add complexity only if performance increases significantly!

Example 2: BET Index (Bucharest Stock Exchange)

Characteristics

- **Volatility clustering** strong
- Influenced by international markets
- Lower liquidity than developed markets
- Potential for long memory in volatility

Typical Results (RMSE on returns)

- GARCH(1,1): 1.45 – best for volatility
- ARFIMA for volatility: 1.52
- Random Forest: 1.48
- LSTM: 1.51

Example 3: Inflation Rate in Romania

Characteristics

- Series **monthly** (low frequency)
- **High persistence** – shocks persist
- Influenced by monetary policy
- Strong potential for **long memory**

Typical Results

- ARFIMA cu $d \approx 0.35$ – captures persistence
- ARIMA underestimates shock persistence
- ML does not work well (few data, 300 obs.)

Lesson: For monthly series with few data, classical models (ARFIMA) are superior!

Practical Summary: Model Selection

Criterion	ARIMA	ARFIMA	RF	LSTM
Data needed	Few	Few	Mediumm	Many
Long memory	No	**Yes**	Partial	Partial
Nonlinearity	No	No	**Yes**	**Yes**
Interpretabil	**Yes**	**Yes**	Partial	No
Computation time	Fast	Fast	Medium	Slow
Exog. var.	Limited	Limited	**Yes**	**Yes**

Recommended Workflow

1. Start with **ARIMA** as baseline
2. Test **long memory** $\rightarrow$ ARFIMA if d significant
3. Add **features** $\rightarrow$ Random Forest
4. Only with lots of data and resources $\rightarrow$ LSTM

Summary

What we learned

- **ARFIMA**: Extends ARIMA for long memory (fractional d)
- **Random Forest**: Ensemble of trees, nonlinear relationships, interpretable
- **LSTM**: Deep learning for sequences, complex dependencies
- **Trade-offs**: Complexity vs interpretability vs data requirements

Practical Recommendations

- Start with **simple** models (ARIMA) as baseline
- Use **Time Series CV** for proper evaluation
- ML requires **feature engineering** careful
- LSTM: only with **lots of data** and computational resources

Quiz Fast

1. What does $d = 0.3$ mean in an ARFIMA model?
2. Why use Time Series CV instead of standard k-fold?
3. What is the main advantage of LSTM over Simple RNNs?
4. What type of model would you choose with few data and linear relationships?
5. What does "data leakage" in the context of ML for time series?

Quiz Answers

1. $d = 0.3$: Long memory, series is stationary but autocorrelations decay slowly (hyperbolically). Moderate persistence.
2. **Time Series CV**: To respect temporal order. Standard k-fold would use future data to predict the past (data leakage).
3. **LSTM vs RNN**: LSTM solves the problem "vanishing gradient" through the gating mechanism, allowing learning of long-term dependencies.
4. **Few data, linear relationships**: ARIMA or ARFIMA. ML requires lots of data to generalize well.
5. **Data leakage**: Using future information in features or training. E.g. calculating moving averages using future data, or standard k-fold which mixes temporal order.

What Comes Next?

Extensions and Advanced Topics

- **Transformer** for time series (Temporal Fusion Transformer)
- **Prophet** (Facebook/Meta) for seasonality
- **Neural Prophet**: Prophet + neural networks
- **Ensemble methods**: Combining multiple models
- **Anomaly detection** with ML

Questions?