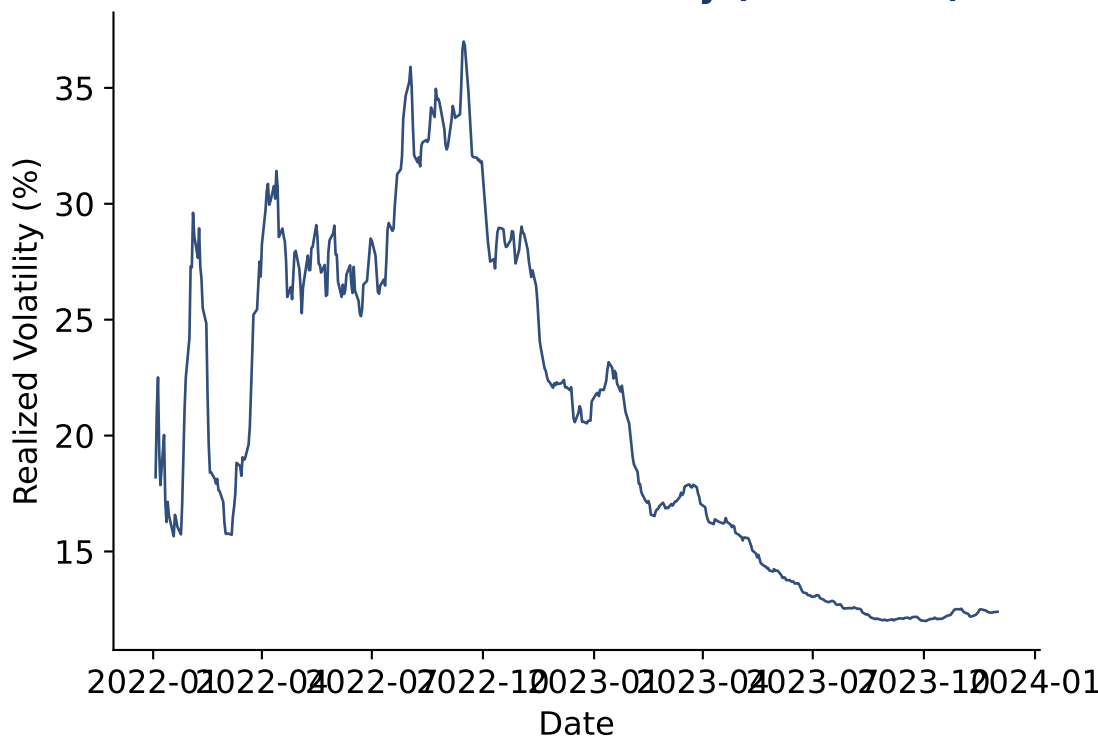
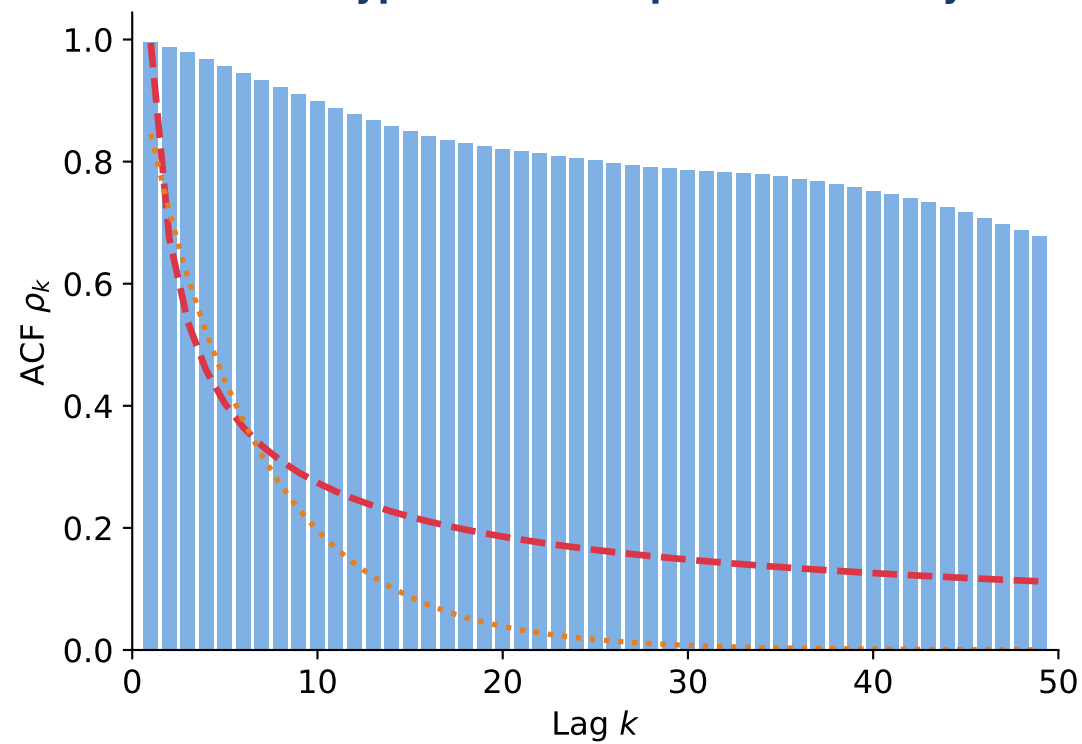


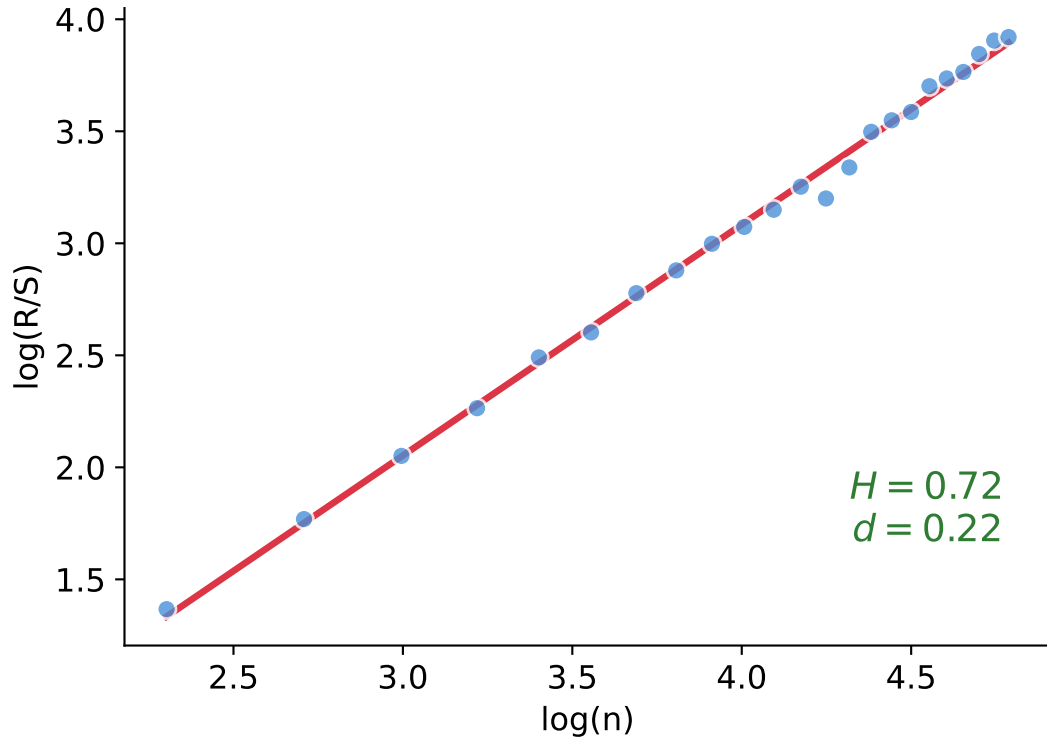
### S&P 500 Realized Volatility (Simulated)



### ACF: Hyperbolic vs Exponential Decay



### R/S Analysis for Hurst Exponent



#### Estimation Results

Sample:  $n = 500$  observations

Hurst Exponent (R/S):  
 $H = 0.72$

Fractional Differencing:  
 $d = H - 0.5 = 0.22$

#### Interpretation:

- $d > 0 \rightarrow$  Long Memory ✓
- ACF decays as  $k^{2d-1}$
- Shocks persist longer than ARMA

Model: ARFIMA(1, 0.22, 0)

#### Real-World Examples:

- Volatility clustering
- Inflation persistence
- Network traffic
- River flow data

Sample ACF    Hyperbolic decay  $k^{2d-1}$     Exponential decay (ARMA)    R/S statistic    Regression line