Time Series Analysis and Forecasting

Master in Statistics and Data Science

Academic Year 2025-2026

Outline

1. Introduction and Lag Operator
2. Autoregressive (AR) Models
3. Moving Average (MA) Models
4. ARMA Models
5. Model Identification
6. Parameter Estimation
7. Model Diagnostics
8. Forecasting with ARMA
9. Practical Implementation
10. Summary

Recap: Stationarity

From Chapter 1: A process $\{X_t\}$ is **weakly stationary** if:

1. $\mathbb{E}[X_t] = \mu$ (constant mean)
2. $\text{Var}(X_t) = \sigma^2 < \infty$ (constant, finite variance)
3. $\text{Cov}(X_t, X_{t+h}) = \gamma(h)$ (covariance depends only on lag h)

Why stationarity matters for ARMA:

- ARMA models assume the underlying process is stationary
- Non-stationary data must be differenced first (ARIMA)
- Stationarity ensures stable model parameters

Today: We build models for stationary time series using past values and past errors.

The Lag Operator (Backshift Operator)

Definition 1 (Lag Operator)

The **lag operator** L (or backshift operator B) shifts a time series back by one period:

$$LX_t = X_{t-1}$$

Properties:

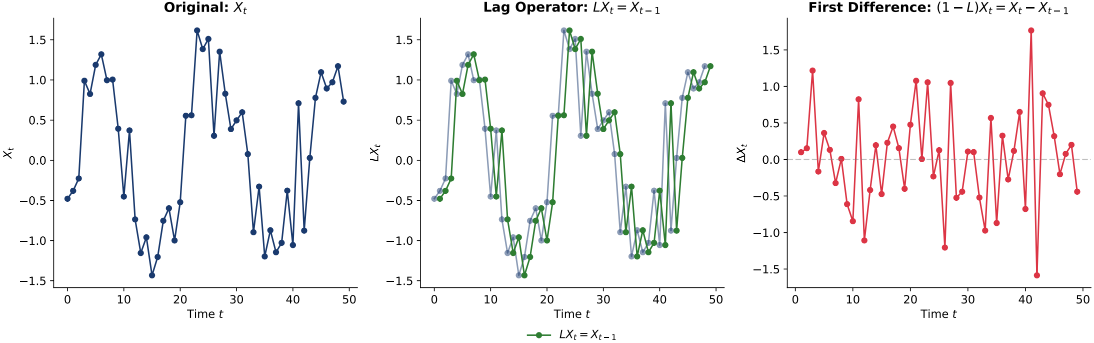
- $L^k X_t = X_{t-k}$ (shift back k periods)
- $L^0 X_t = X_t$ (identity)
- $(1 - L)X_t = X_t - X_{t-1} = \Delta X_t$ (first difference)
- $(1 - L)^d X_t = \Delta^d X_t$ (d-th difference)

Lag Polynomials:

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$$

$$\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q$$

Lag Operator: Visual Illustration



Key insight: The lag operator is the foundation of ARMA model notation

White Noise Process

Definition 2 (White Noise)

A process $\{\varepsilon_t\}$ is **white noise**, denoted $\varepsilon_t \sim WN(0, \sigma^2)$, if:

1. $\mathbb{E}[\varepsilon_t] = 0$ for all t
2. $\mathrm{Var}(\varepsilon_t) = \sigma^2$ for all t
3. $\mathrm{Cov}(\varepsilon_t, \varepsilon_s) = 0$ for all $t \neq s$

Properties:

- White noise is the "building block" of ARMA models
- ACF: $\rho(0) = 1$, $\rho(h) = 0$ for $h \neq 0$
- PACF: same pattern
- **Gaussian white noise:** additionally $\varepsilon_t \sim N(0, \sigma^2)$

Note: White noise is *not* predictable — it's pure randomness.

AR(1) Model: Definition

Definition 3 (AR(1) Process)

An **autoregressive process of order 1** is:

$$X_t = c + \phi X_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim WN(0, \sigma^2)$ and $|\phi| < 1$ for stationarity.

Interpretation:

- c: constant (intercept)
- ϕ: autoregressive coefficient — measures persistence
- ε_t: innovation (unpredictable shock)

Using lag operator:

$$(1 - \phi L)X_t = c + \varepsilon_t$$

$$\phi(L)X_t = c + \varepsilon_t \quad \text{where } \phi(L) = 1 - \phi L$$

AR(1) Stationarity Condition

For AR(1) to be stationary: $|\phi| < 1$

Intuition:

- If $|\phi| < 1$: shocks decay over time → stationary
- If $|\phi| = 1$: random walk → non-stationary (unit root)
- If $|\phi| > 1$: explosive process → non-stationary

Characteristic equation:

$$\phi(z) = 1 - \phi z = 0 \implies z = \frac{1}{\phi}$$

Stationarity requires the root $z = 1/\phi$ to lie **outside the unit circle**, i.e., $|z| > 1$, which means $|\phi| < 1$.

AR(1) Properties

For a stationary AR(1) with $|\phi| < 1$:

Mean:

$$\mu = \mathbb{E}[X_t] = \frac{c}{1-\phi}$$

Variance:

$$\gamma(0) = \text{Var}(X_t) = \frac{\sigma^2}{1-\phi^2}$$

Autocovariance:

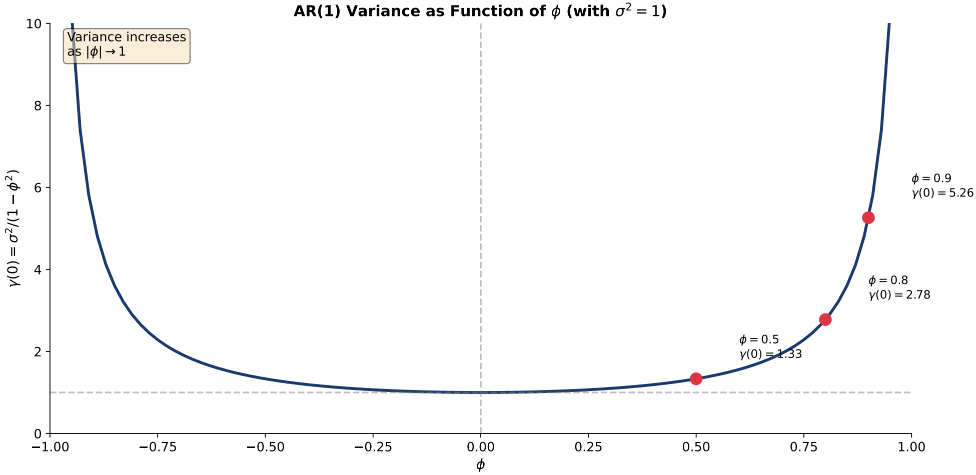
$$\gamma(h) = \phi^h \gamma(0) = \frac{\phi^h \sigma^2}{1-\phi^2}$$

Autocorrelation (ACF):

$$\rho(h) = \phi^h$$

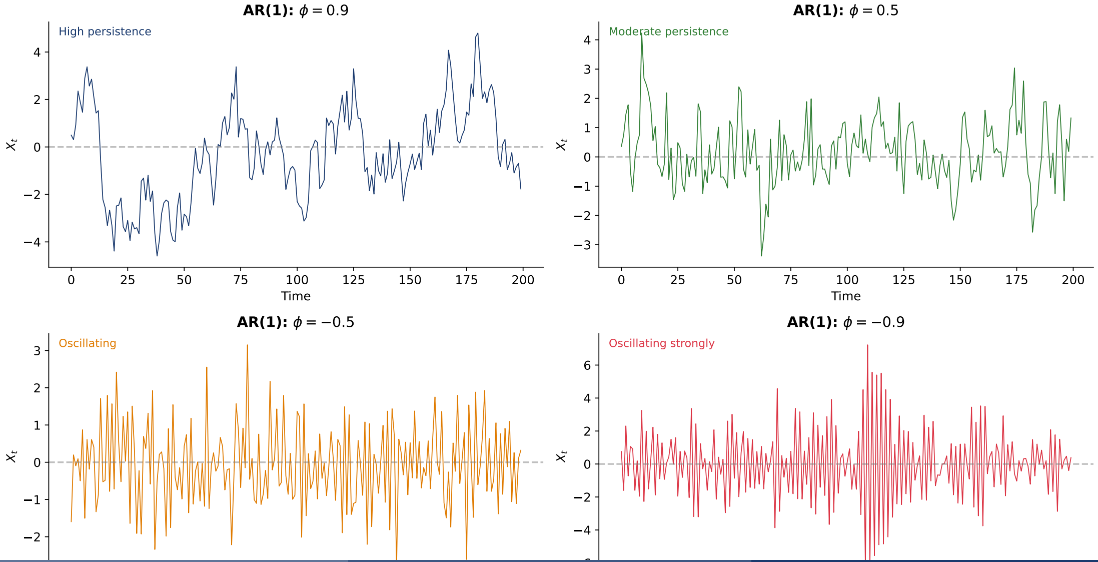
Key insight: ACF decays exponentially at rate ϕ

AR(1) Variance as Function of ϕ

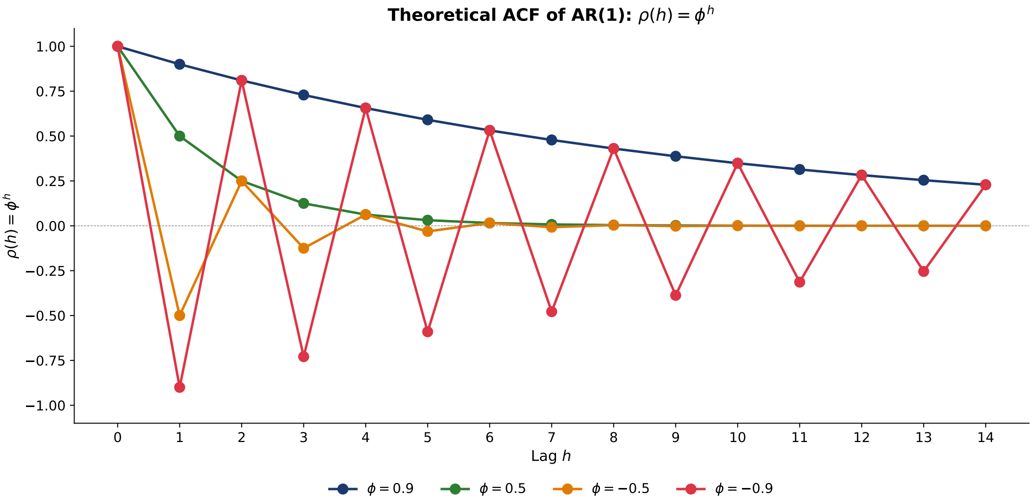


Key insight: As $|\phi| \to 1$, variance explodes → non-stationarity

AR(1) Simulations: Effect of ϕ



AR(1) Theoretical ACF



Pattern: $\rho(h) = \phi^h$ — exponential decay (or alternating for $\phi < 0$)

AR(1) ACF and PACF Patterns

ACF of AR(1):

- Decays exponentially: $\rho(h) = \phi^h$
- If $\phi > 0$: all positive, gradual decay
- If $\phi < 0$: alternating signs, decay in magnitude

PACF of AR(1):

- **Cuts off after lag 1**
- $\pi_1 = \phi$, $\pi_k = 0$ for $k > 1$

	ACF	**PACF**
AR(1)	Exponential decay	Cuts off at lag 1

This is the key identification pattern for AR(1)!

AR(p) Model: General Form

Definition 4 (AR(p) Process)

An **autoregressive process of order p** is:

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + \varepsilon_t$$

Using lag operator:

$$\phi(L)X_t = c + \varepsilon_t$$

where $\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$

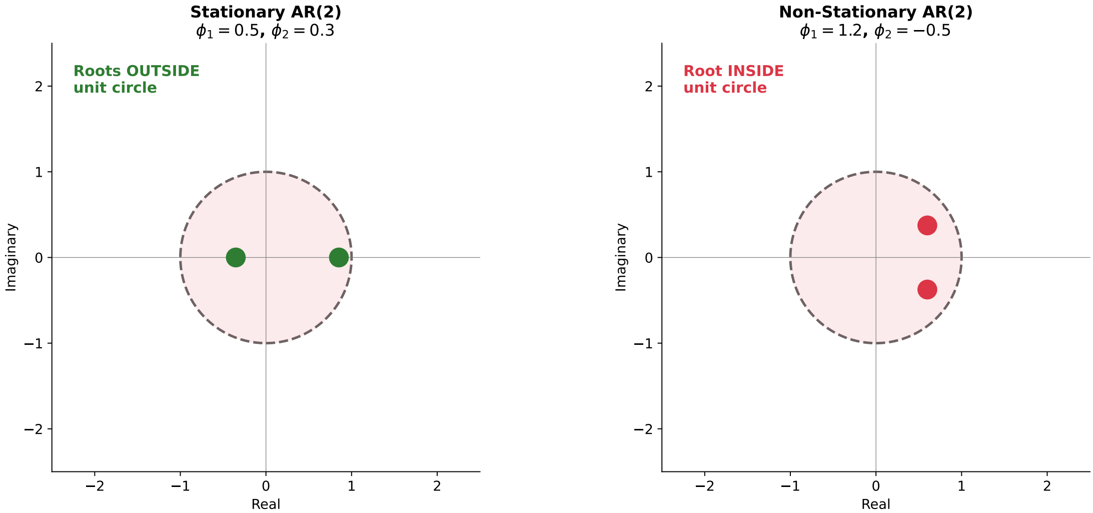
Stationarity condition:

- All roots of $\phi(z) = 0$ must lie **outside** the unit circle
- Equivalently: all roots have modulus > 1

PACF pattern:

- PACF cuts off after lag p
- ACF decays (exponentially or with damped oscillations)

AR(2) Stationarity: Unit Circle Visualization



Rule: All roots of $\phi(z) = 0$ must lie **outside** the shaded unit circle

AR(2) Model

Definition 5 (AR(2) Process)

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t$$

Stationarity conditions for AR(2):

1. $\phi_1 + \phi_2 < 1$
2. $\phi_2 - \phi_1 < 1$
3. $|\phi_2| < 1$

ACF behavior depends on roots:

- **Real roots:** mixture of two exponential decays
- **Complex roots:** damped sinusoidal pattern (pseudo-cycles)

PACF: Cuts off after lag 2 ($\pi_k = 0$ for $k > 2$)

MA(1) Model: Definition

Definition 6 (MA(1) Process)

A **moving average process of order 1** is:

$$X_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1}$$

where $\varepsilon_t \sim WN(0, \sigma^2)$.

Interpretation:

- μ: mean of the process
- θ: MA coefficient — measures impact of past shock
- Current value depends on current and one past shock

Using lag operator:

$$X_t = \mu + \theta(L)\varepsilon_t$$

where $\theta(L) = 1 + \theta L$

Key property: MA processes are **always stationary** for any finite θ

MA(1) Properties

For MA(1): $X_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1}$

Mean:

$$\mathbb{E}[X_t] = \mu$$

Variance:

$$\gamma(0) = \mathrm{Var}(X_t) = \sigma^2(1 + \theta^2)$$

Autocovariance:

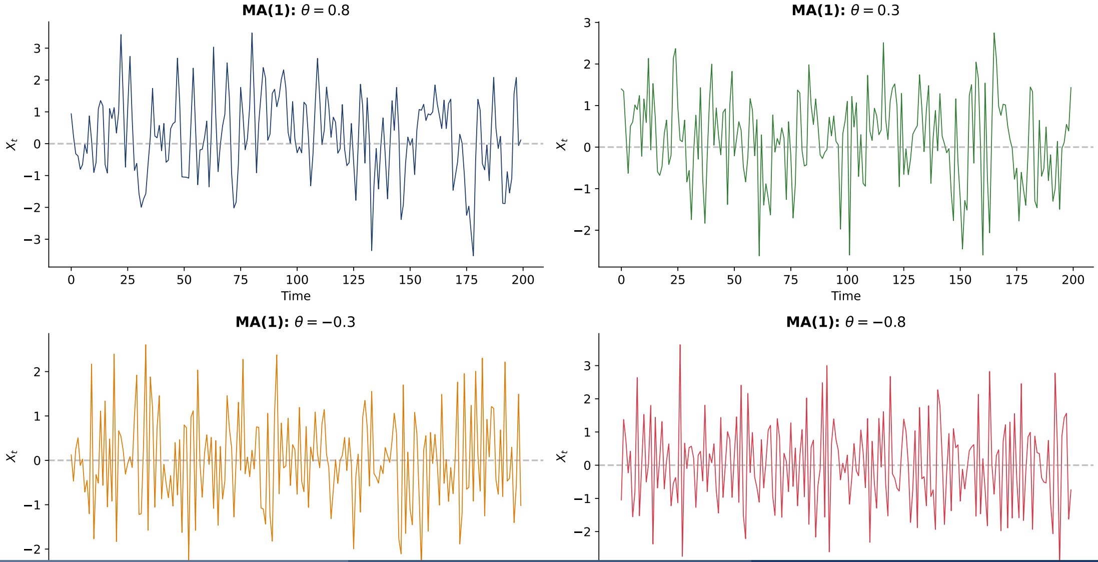
$$\gamma(1) = \theta\sigma^2, \quad \gamma(h) = 0 \text{ for } h > 1$$

Autocorrelation (ACF):

$$\rho(1) = \frac{\theta}{1 + \theta^2}, \quad \rho(h) = 0 \text{ for } h > 1$$

Key insight: ACF **cuts off** after lag 1

MA(1) Simulations: Effect of θ



MA(1) ACF and PACF Patterns

ACF of MA(1):

- **Cuts off after lag 1**
- $\rho(1) = \frac{\theta}{1+\theta^2}$, $\rho(h) = 0$ for $h > 1$
- Note: $|\rho(1)| \leq 0.5$ always (maximum at $\theta = \pm 1$)

PACF of MA(1):

- Decays exponentially (or with alternating signs)
- Does *not* cut off

	ACF	**PACF**
MA(1)	Cuts off at lag 1	Exponential decay

This is the opposite pattern from AR(1)!

Invertibility of MA Models

Definition 7 (Invertibility)

An MA process is **invertible** if it can be written as an infinite AR process:

$$X_t = \mu + \sum_{j=1}^{\infty} \pi_j (X_{t-j} - \mu) + \varepsilon_t$$

For MA(1): Invertible if $|\theta| < 1$
For MA(q): All roots of $\theta(z) = 0$ must lie outside the unit circle

Why invertibility matters:

- Ensures unique representation
- Required for forecasting and estimation
- Creates correspondence: AR(∞) $\leftrightarrow$ MA(q)

Note: Stationarity is for AR, Invertibility is for MA

MA(q) Model: General Form

Definition 8 (MA(q) Process)

A **moving average process of order q** is:

$$X_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}$$

Using lag operator:

$$X_t = \mu + \theta(L)\varepsilon_t$$

where $\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q$

Properties:

- Always stationary (finite variance)
- ACF cuts off after lag q: $\rho(h) = 0$ for $h > q$
- PACF decays gradually
- Invertible if all roots of $\theta(z) = 0$ lie outside unit circle

ARMA(p,q) Model: Definition

Definition 9 (ARMA(p,q) Process)

An **autoregressive moving average process** of order (p,q) is:

$$X_t = c + \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

Compact form using lag operators:

$$\phi(L)(X_t - \mu) = \theta(L)\varepsilon_t$$

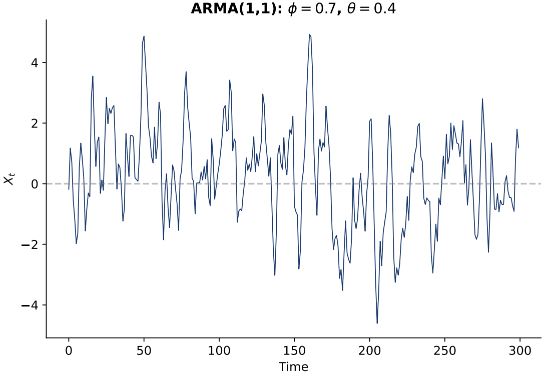
or equivalently:

$$\phi(L)X_t = c + \theta(L)\varepsilon_t$$

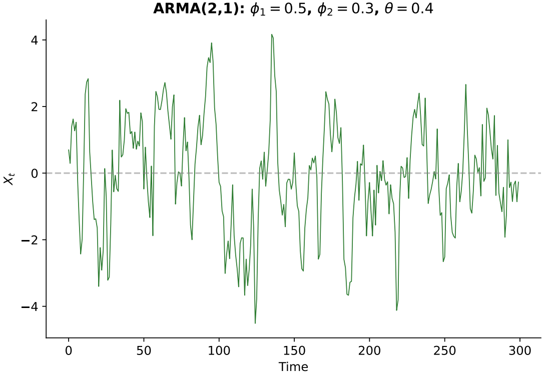
where $\mu = \frac{c}{1-\phi_1-\cdots-\phi_p}$

Key idea: Combines AR and MA components for more flexible modeling

ARMA Examples



ARMA(1,1): $\phi = 0.7$, $\theta = 0.4$



ARMA(2,1): $\phi_1 = 0.5$, $\phi_2 = 0.3$, $\theta = 0.4$

ARMA(1,1) Model

Definition 10 (ARMA(1,1) Process)

$$X_t = c + \phi X_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$$

Properties (assuming stationarity and invertibility):

- Mean: $\mu = \frac{c}{1-\phi}$
- Variance: $\gamma(0) = \frac{(1+2\phi\theta+\theta^2)\sigma^2}{1-\phi^2}$

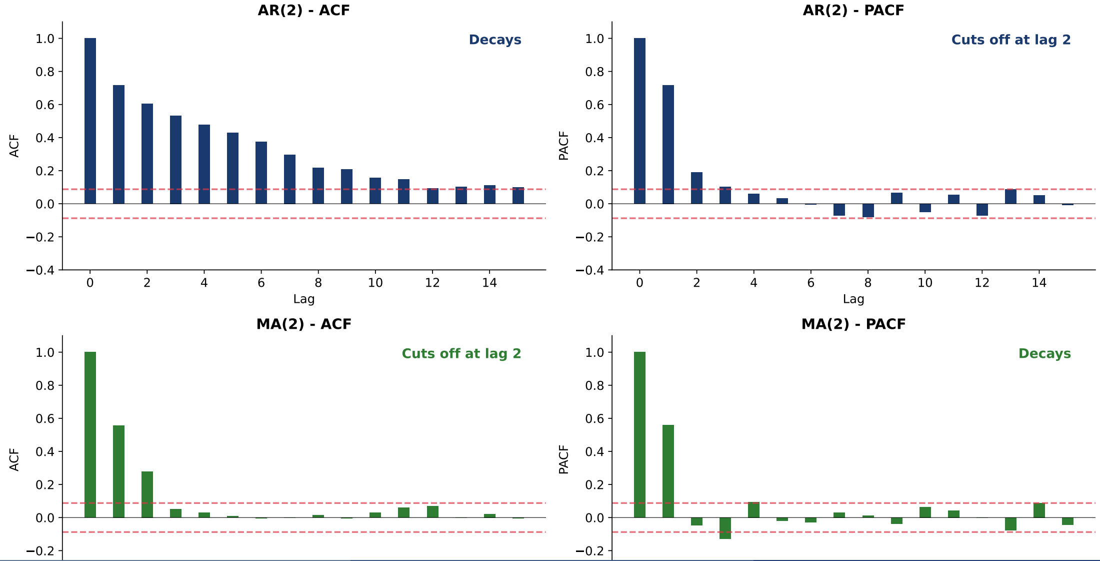
ACF:

$$\rho(1) = \frac{(1+\phi\theta)(\phi+\theta)}{1+2\phi\theta+\theta^2}$$

$$\rho(h) = \phi \cdot \rho(h-1) \quad \text{for } h \geq 2$$

Pattern: ACF decays exponentially after lag 1 (like AR), but starting point depends on both ϕ and θ

ACF/PACF Patterns: AR vs MA vs ARMA



ARMA ACF and PACF Patterns

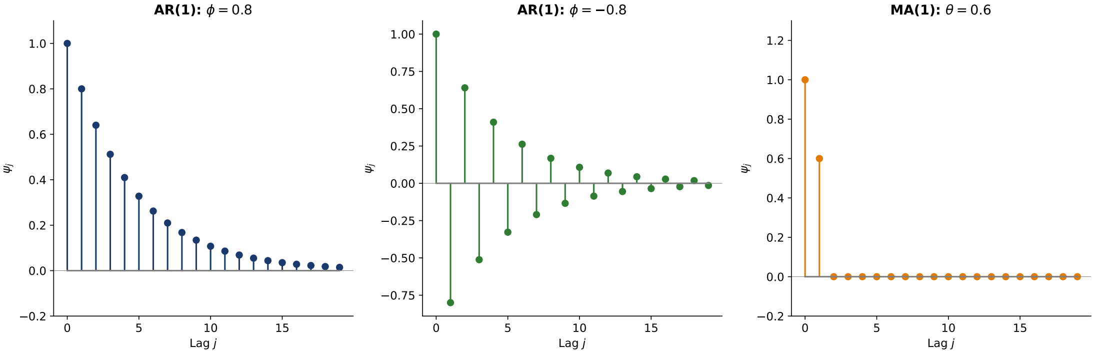
Model	**ACF**	**PACF**
AR(p)	Decays (exp./damped)	Cuts off at lag p
MA(q)	Cuts off at lag q	Decays (exp./damped)
ARMA(p,q)	Decays after lag $q - p$	Decays after lag $p - q$

Key identification rule:

- **PACF cuts off** $\rightarrow$ AR process (order = cutoff lag)
- **ACF cuts off** $\rightarrow$ MA process (order = cutoff lag)
- **Both decay** $\rightarrow$ ARMA process

Caution: In practice, sample ACF/PACF are noisy; use confidence bands

Impulse Response Functions



Interpretation: Shows how a unit shock propagates through the system over time

Stationarity and Invertibility Summary

For ARMA(p,q) to be well-behaved:

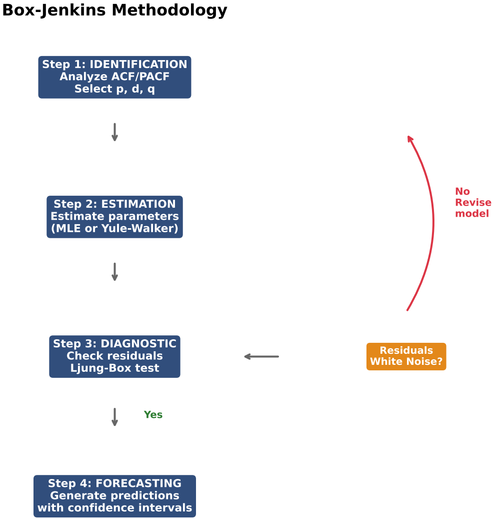
Condition	**Requirement**
Stationarity	Roots of $\phi(z) = 0$ outside unit circle
Invertibility	Roots of $\theta(z) = 0$ outside unit circle

Implications:

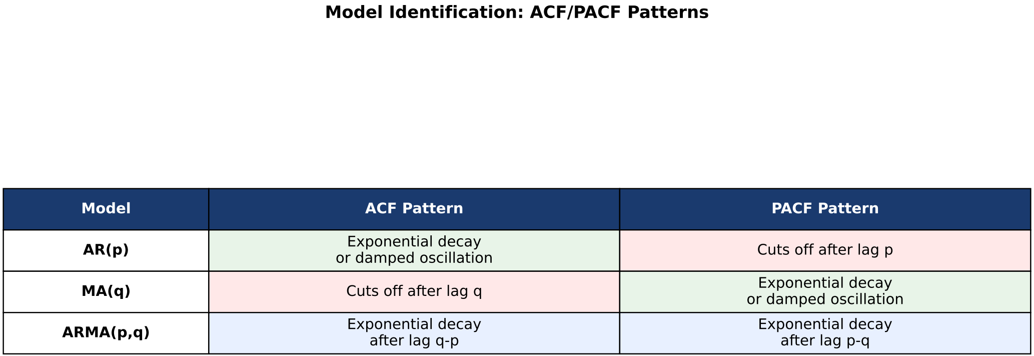
- **Stationarity:** Can write as MA(∞): $X_t = \mu + \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$
- **Invertibility:** Can write as AR(∞): $X_t = \mu + \sum_{j=1}^{\infty} \pi_j (X_{t-j} - \mu) + \varepsilon_t$

Causal representation: X_t depends only on *past* shocks (not future)

The Box-Jenkins Methodology



Model Identification Summary Table



Model Identification: ACF/PACF Patterns

Model	ACF Pattern	PACF Pattern
AR(p)	Exponential decay or damped oscillation	Cuts off after lag p
MA(q)	Cuts off after lag q	Exponential decay or damped oscillation
ARMA(p,q)	Exponential decay after lag q-p	Exponential decay after lag p-q

Practical tip: Start simple (low p, q), increase if diagnostics fail

ACF/PACF Identification Rules

Theoretical patterns for stationary processes:

Model	ACF Pattern	PACF Pattern
AR(1)	Exponential decay	Spike at lag 1, then 0
AR(2)	Damped exponential/sine	Spikes at lags 1-2, then 0
AR(p)	Decays gradually	Cuts off after lag p
MA(1)	Spike at lag 1, then 0	Exponential decay
MA(2)	Spikes at lags 1-2, then 0	Damped exponential/sine
MA(q)	Cuts off after lag q	Decays gradually
ARMA(p,q)	Decays	Decays

Information Criteria

Purpose: Balance goodness-of-fit against model complexity

Akaike Information Criterion (AIC):

$$\text{AIC} = -2\ln(\hat{L}) + 2k$$

Bayesian Information Criterion (BIC/SBC):

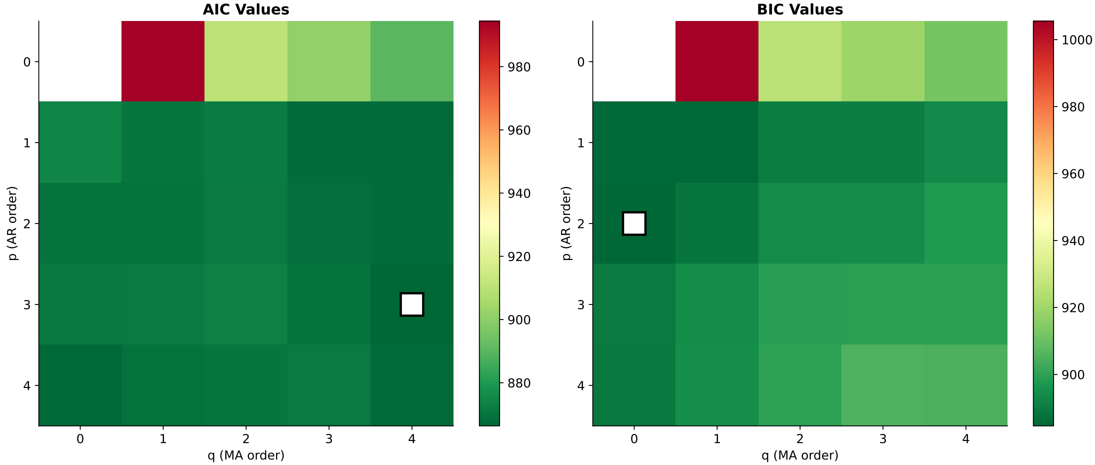
$$\text{BIC} = -2\ln(\hat{L}) + k\ln(n)$$

where $\hat{L}$ = maximized likelihood, k = number of parameters, n = sample size

Usage:

- Lower values are better
- BIC penalizes complexity more strongly than AIC
- AIC tends to choose larger models; BIC more parsimonious
- Compare models fit to the *same data*

AIC vs BIC: Model Selection



Note: White square marks the best model; lower values (green) are better

Automatic Model Selection

Grid search approach:

1. Fit ARMA(p,q) for $p = 0, 1, \ldots, p_{max}$ and $q = 0, 1, \ldots, q_{max}$
2. Select model with lowest AIC or BIC
3. Verify with diagnostic checks

In Python (statsmodels):

- `pm.auto_arima()` from `pmdarima` package
- Automatically tests stationarity, searches over orders
- Returns best model by AIC/BIC

Caution:

- Automatic selection is a starting point, not final answer
- Always check diagnostics
- Consider domain knowledge

Estimation Methods Overview

Three main approaches:

1. Method of Moments / Yule-Walker (AR only)

- Match sample autocorrelations to theoretical values
- Simple, closed-form for AR models
- Not efficient for MA components

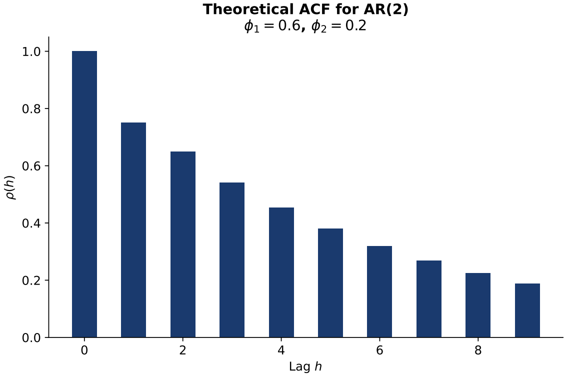
2. Maximum Likelihood Estimation (MLE)

- Most common approach
- Requires distributional assumption (usually Gaussian)
- Efficient and consistent

3. Conditional Least Squares

- Minimize sum of squared residuals
- Conditioning on initial observations
- Computationally simpler than exact MLE

Yule-Walker Equations for AR(p)



Yule-Walker Equations

$$\rho(1) = \phi_1 + \phi_2 \rho(1)$$

$$\rho(2) = \phi_1 \rho(1) + \phi_2$$

Matrix form: $R \cdot \phi = \rho$

R = autocorrelation matrix

Solution: $\hat{\phi} = R^{-1} \rho$

Yule-Walker Equations: Matrix Form

For AR(p): $X_t = \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p} + \varepsilon_t$

Yule-Walker equations:

$$\rho(k) = \phi_1 \rho(k-1) + \phi_2 \rho(k-2) + \cdots + \phi_p \rho(k-p)$$

for $k = 1, 2, \ldots, p$

Matrix form:

$$\begin{pmatrix} \rho(0) & \rho(1) & \cdots & \rho(p-1) \\ \rho(1) & \rho(0) & \cdots & \rho(p-2) \\ \vdots & \vdots & \ddots & \vdots \\ \rho(p-1) & \rho(p-2) & \cdots & \rho(0) \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_p \end{pmatrix} = \begin{pmatrix} \rho(1) \\ \rho(2) \\ \vdots \\ \rho(p) \end{pmatrix}$$

Estimation: Replace $\rho(k)$ with sample autocorrelations $\hat{\rho}(k)$

Maximum Likelihood Estimation

Assuming Gaussian errors: $\varepsilon_t \sim N(0, \sigma^2)$

Log-likelihood for ARMA(p,q):

$$\ell(\boldsymbol{\phi}, \boldsymbol{\theta}, \sigma^2) = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=1}^{n}\varepsilon_t^2$$

where ε_t are the innovations computed recursively.

Estimation procedure:

1. Initialize: use method of moments or OLS for starting values
2. Optimize: numerical methods (e.g., BFGS, Newton-Raphson)
3. Iterate until convergence

In practice: Use `statsmodels.tsa.arima.model.ARIMA`

Standard Errors and Inference

Asymptotic distribution of MLE:

$$\hat{\boldsymbol{\theta}} \xrightarrow{d} N\left(\boldsymbol{\theta}_0, \frac{1}{n}\mathbf{I}(\boldsymbol{\theta}_0)^{-1}\right)$$

where $\mathbf{I}(\boldsymbol{\theta})$ is the Fisher information matrix.

Standard errors: Square root of diagonal of $\frac{1}{n}\hat{\mathbf{I}}^{-1}$

Hypothesis testing:

- $H_0 : \phi_j = 0$ (or $\theta_j = 0$)
- Test statistic: $z = \frac{\hat{\phi}_j}{SE(\hat{\phi}_j)} \sim N(0, 1)$ asymptotically
- Reject if $|z| > 1.96$ at 5% level

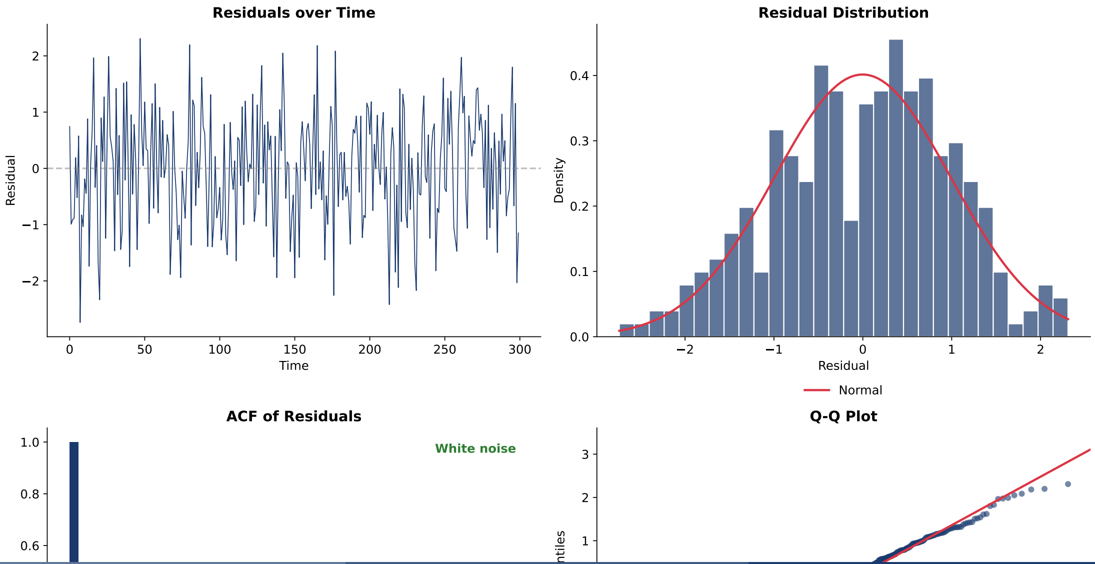
Confidence interval: $\hat{\phi}_j \pm 1.96 \cdot SE(\hat{\phi}_j)$

Residual Analysis

If model is correctly specified, residuals should be white noise:

1. **Plot residuals over time**
   - Should fluctuate around zero
   - No obvious patterns or trends
   - Constant variance (no heteroskedasticity)
2. **Check ACF of residuals**
   - All correlations should be within confidence bands
   - No significant spikes $\rightarrow$ white noise
3. **Check histogram / Q-Q plot**
   - Should be approximately normal (if assuming Gaussian)
   - Heavy tails suggest non-normal errors

Residual Diagnostics: Example



Ljung-Box Test

Definition 11 (Ljung-Box Test)

Tests whether residuals are independently distributed (no autocorrelation).

Test statistic:

$$Q(m) = n(n+2)\sum_{k=1}^{m}\frac{\hat{\rho}_k^2}{n-k}$$

Hypotheses:

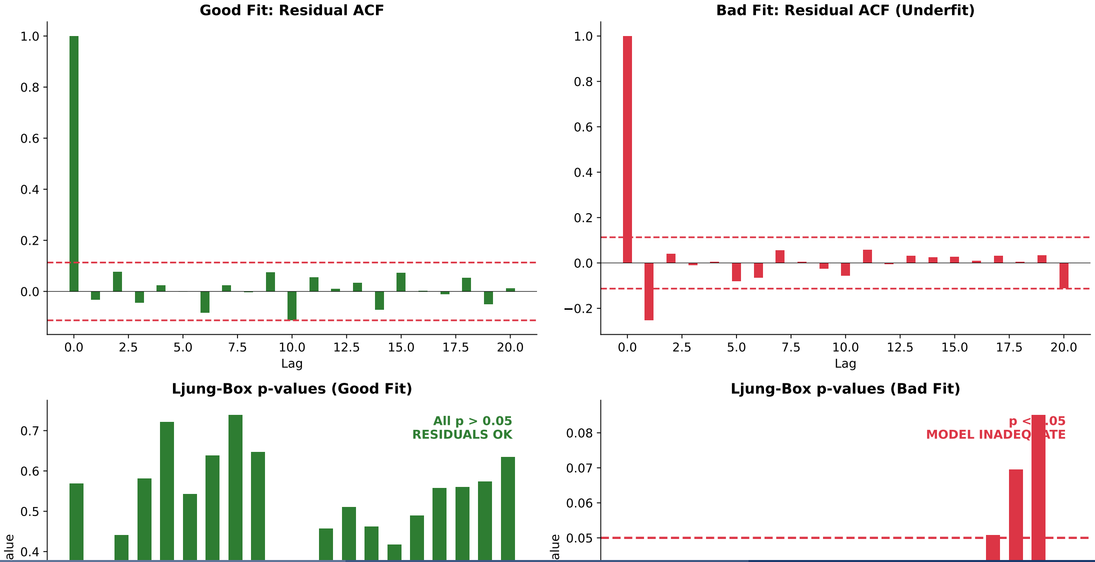
- H_0: Residuals are white noise (no autocorrelation up to lag m)
- H_1: Residuals are autocorrelated

Distribution: Under H_0, $Q(m) \sim \chi^2(m-p-q)$ approximately

Decision:

- p-value $> 0.05 \rightarrow$ fail to reject $H_0 \rightarrow$ residuals look like white noise (good!)
- p-value $< 0.05 \rightarrow$ significant autocorrelation remains $\rightarrow$ model inadequate

Ljung-Box Test: Good vs Bad Model Fit



Good Fit: Residual ACF

Bad Fit: Residual ACF (Underfit)

Ljung-Box p-values (Good Fit)

All p > 0.05
RESIDUALS OK

Ljung-Box p-values (Bad Fit)

p < 0.05
MODEL INADEQUATE

Diagnostic Checklist

A good ARMA model should satisfy:

1. **Stationarity:** AR roots outside unit circle
   ✓ Check with `arroots`
2. **Invertibility:** MA roots outside unit circle
   ✓ Check with `maroots`
3. **White noise residuals:** No significant ACF
   ✓ ACF plot, Ljung-Box test
4. **Normal residuals:** (if assumed)
   ✓ Q-Q plot, Jarque-Bera test
5. **No heteroskedasticity:** Constant variance
   ✓ Plot residuals, ARCH test
6. **Parsimonious:** Lowest AIC/BIC among adequate models

If diagnostics fail: Return to identification, try different orders

Point Forecasts

Optimal forecast: Conditional expectation minimizes MSE

$$\hat{X}_{n+h|n} = \mathbb{E}[X_{n+h}|X_n, X_{n-1}, \dots]$$

For AR(1): $X_t = c + \phi X_{t-1} + \varepsilon_t$

$$\begin{aligned}\hat{X}_{n+1|n} &= c + \phi X_n \\ \hat{X}_{n+2|n} &= c + \phi \hat{X}_{n+1|n} = c(1+\phi) + \phi^2 X_n \\ \hat{X}_{n+h|n} &= \mu + \phi^h (X_n - \mu)\end{aligned}$$

Key property: Forecasts converge to mean μ as $h \to \infty$

For MA(1): $X_t = \mu + \varepsilon_t + \theta \varepsilon_{t-1}$

$$\begin{aligned}\hat{X}_{n+1|n} &= \mu + \theta \varepsilon_n \\ \hat{X}_{n+h|n} &= \mu \quad \text{for } h > 1\end{aligned}$$

Forecast Uncertainty

Forecast error:

$$e_{n+h|n} = X_{n+h} - \hat{X}_{n+h|n}$$

Mean squared forecast error (MSFE):

$$\text{MSFE}(h) = \mathbb{E}[e_{n+h|n}^2] = \sigma^2 \sum_{j=0}^{h-1} \psi_j^2$$

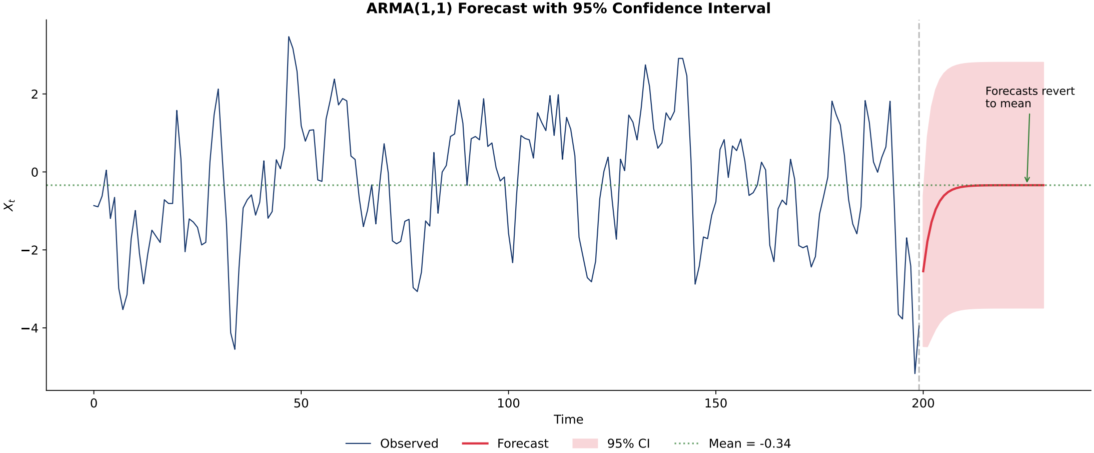
where ψ_j are the MA(∞) coefficients.

For AR(1): $\psi_j = \phi^j$

$$\text{MSFE}(h) = \sigma^2 \frac{1-\phi^{2h}}{1-\phi^2} \to \frac{\sigma^2}{1-\phi^2} = \text{Var}(X_t)$$

Key insight: Forecast uncertainty increases with horizon, eventually reaching unconditional variance

ARMA Forecasting with Confidence Intervals



Confidence Intervals for Forecasts

Assuming Gaussian errors:

$$X_{n+h}|X_n, \ldots \sim N\left(\hat{X}_{n+h|n}, \mathrm{MSFE}(h)\right)$$

$(1-\alpha)$ **confidence interval:**

$$\hat{X}_{n+h|n} \pm z_{\alpha/2} \cdot \sqrt{\mathrm{MSFE}(h)}$$

where $z_{\alpha/2} = 1.96$ for 95% CI.

Properties:

- Intervals widen as horizon increases
- Eventually converge to unconditional interval: $\mu \pm z_{\alpha/2}\sigma_X$
- Width depends on model parameters (AR coefficients, etc.)

In Python: `model.get_forecast(h).conf_int()`

Forecast Evaluation

Out-of-sample testing:

1. Split data: training set (fit model) and test set (evaluate)
2. Generate forecasts for test period
3. Compare forecasts to actual values

Metrics (from Chapter 1):

- $\text{MAE} = \frac{1}{n} \sum |e_t|$
- $\text{RMSE} = \sqrt{\frac{1}{n} \sum e_t^2}$
- $\text{MAPE} = \frac{100}{n} \sum \left| \frac{e_t}{X_t} \right|$

Rolling/expanding window:

- Re-estimate model as new data arrives
- More realistic assessment of forecast performance

Python Implementation: Fitting ARMA

Using statsmodels:

```
from statsmodels.tsa.arima.model import ARIMA

# Fit ARMA(2,1) --- note:  ARIMA(p,d,q) with d=0
model = ARIMA(data, order=(2, 0, 1))
results = model.fit()

# Summary
print(results.summary())

# Forecasting
forecast = results.get_forecast(steps=10)
print(forecast.predicted_mean)
print(forecast.conf_int())
```

Note: ARIMA with $d = 0$ is equivalent to ARMA

Python: Model Selection with pmdarima

Automatic ARIMA selection:

```
import pmdarima as pm

# Auto ARIMA with AIC criterion
model = pm.auto_arima(data,
              start_p=0, max_p=5,
              start_q=0, max_q=5,
              d=0, # No differencing for stationary data
              seasonal=False,
              information_criterion=’aic’,
              trace=True)

print(model.summary())
```

Output: Best model order and fitted parameters

Workflow Summary

1. **Data preparation**
   - Check for missing values, outliers
   - Transform if necessary (log, differencing)
2. **Stationarity check**
   - Visual inspection: time plot, ACF
   - Formal tests: ADF, KPSS
   - Difference if non-stationary
3. **Model identification**
   - ACF/PACF patterns
   - Information criteria grid search
4. **Estimation and diagnostics**
   - Fit model, check significance
   - Residual analysis, Ljung-Box test
5. **Forecasting**
   - Point forecasts with confidence intervals
   - Out-of-sample validation

Key Takeaways

1. **AR(p) models:** Current value depends on p past values
   - Stationarity: roots of $\phi(z)$ outside unit circle
   - PACF cuts off at lag p
2. **MA(q) models:** Current value depends on q past shocks
   - Always stationary; invertibility: roots of $\theta(z)$ outside unit circle
   - ACF cuts off at lag q
3. **ARMA(p,q):** Combines AR and MA for flexible modeling
   - Both ACF and PACF decay
4. **Box-Jenkins:** Identify → Estimate → Diagnose → Forecast
5. **Diagnostics:** Residuals must be white noise
6. **Forecasts:** Converge to mean; uncertainty increases with horizon

Next Chapter Preview

Chapter 3: ARIMA and Seasonal Models

- ARIMA(p,d,q): Integrated models for non-stationary data
- Seasonal ARIMA: SARIMA(p,d,q)(P,D,Q)$_s$
- Seasonal differencing
- Real-world applications with seasonal patterns

Reading:

- Hyndman & Athanasopoulos, *Forecasting: Principles and Practice*, Ch. 9
- Box, Jenkins, Reinsel & Ljung, *Time Series Analysis*, Ch. 3-4