Chapter 5: VAR & Granger Causality

Seminar

Seminar Outline

Quiz 1: VAR Definition

Question

In a VAR(2) model with 3 variables, how many coefficient matrices $\mathbf{A}_i$ are there?

A) 2
B) 3
C) 6
D) 9

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Answer: A

VAR(p) has p coefficient matrices. VAR(2) has $\mathbf{A}_1$ and $\mathbf{A}_2$, regardless of the number of variables. Each matrix is $K \times K$ (here 3×3).

Quiz 2: Number of Parameters

Question

A VAR(2) with $K = 3$ variables (including constants) has how many parameters to estimate per equation?

- A) 3
- B) 6
- C) 7
- D) 9

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- A) 3
- B) 6
- C) 7
- D) 9

Answer: C

Each equation has: 1 constant $+ K \times p = 1 + 3 \times 2 = 7$ parameters. The equation for Y_{1t} includes a constant, plus coefficients on $Y_{1,t-1}$, $Y_{2,t-1}$, $Y_{3,t-1}$, $Y_{1,t-2}$, $Y_{2,t-2}$, $Y_{3,t-2}$.

Quiz 3: Granger Causality

Question

"X Granger-causes Y" means:

- A) X is the economic cause of Y
- B) Past X helps predict future Y
- C) X and Y are contemporaneously correlated
- D) X always increases when Y increases

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Answer: B

Granger causality is about **predictive content**: past values of X contain useful information for forecasting Y, beyond what past Y alone provides. It does NOT imply economic causation.

Quiz 4: Granger Causality Test

Question

To test if Y_2 Granger-causes Y_1 in a VAR(p), we test:

- A) All coefficients in the Y_1 equation equal zero
- B) Coefficients on lagged Y_2 in the Y_1 equation equal zero
- C) Coefficients on lagged Y_1 in the Y_2 equation equal zero
- D) The error covariance equals zero

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- C) Coefficients on lagged Y_1 in the Y_2 equation equal zero
- D) The error covariance equals zero

Answer: B

H_0: Y_2 does NOT Granger-cause Y_1 means the coefficients $a_{12}^{(1)} = a_{12}^{(2)} = \cdots = a_{12}^{(p)} = 0$ in the Y_1 equation.

Quiz 5: VAR Stability

Question

A VAR(1) model is stable (stationary) if:

- A) All diagonal elements of $\mathbf{A}_1$ are less than 1
- B) The determinant of $\mathbf{A}_1$ is less than 1
- C) All eigenvalues of $\mathbf{A}_1$ are less than 1 in absolute value
- D) The trace of $\mathbf{A}_1$ equals zero

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Answer: C

Stability requires all eigenvalues of the coefficient matrix to lie inside the unit circle, i.e., $|\lambda_i| < 1$ for all i. This ensures shocks die out over time.

Quiz 6: Impulse Response Functions

Question

An impulse response function shows:

A) The correlation between two variables
B) The effect of a shock to one variable on all variables over time
C) The forecast accuracy of the model
D) The p-values of coefficient tests

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C) The forecast accuracy of the model
D) The p-values of coefficient tests

Answer: B

IRFs trace the dynamic response of each variable to a one-time shock to another variable, showing how the effect propagates and eventually dies out (in a stable system).

Quiz 7: Lag Order Selection

Question

Which criterion typically selects the most parsimonious VAR model?

- A) AIC (Akaike Information Criterion)
- B) BIC (Bayesian Information Criterion)
- C) FPE (Final Prediction Error)
- D) Adjusted R^2

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Answer: B

BIC penalizes model complexity more heavily than AIC ($k \ln n$ vs $2k$), typically selecting fewer lags. For forecasting, AIC may be preferred; for inference, BIC's parsimony helps avoid overfitting.

Quiz 8: Granger Causality Interpretation

Question

"X Granger-causes Y" means:

- A) X is the true cause of Y
- B) Past values of X help predict Y beyond Y's own past
- C) X and Y are correlated
- D) Y depends only on X

Quiz 8: Granger Causality Interpretation

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- C) X and Y are correlated
- D) Y depends only on X

Answer: B

Granger causality is about **predictive** content, not true causation. X Granger-causes Y if lagged X terms are jointly significant in the equation for Y, after controlling for lagged Y.

Quiz 9: Forecast Error Variance Decomposition

Question

FEVD (Forecast Error Variance Decomposition) tells us:

A) The correlation between variables
B) What proportion of forecast error variance comes from each shock
C) The optimal forecast horizon
D) Which variables to include in the model

Quiz 9: Forecast Error Variance Decomposition

Question

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- A) The correlation between variables
- B) What proportion of forecast error variance comes from each shock
- C) The optimal forecast horizon
- D) Which variables to include in the model

Answer: B

FEVD decomposes the variance of the h-step forecast error into contributions from each structural shock. It answers: "How much of the uncertainty in forecasting Y is due to shocks to Y vs shocks to X?"

Quiz 10: Structural vs Reduced Form VAR

Question

The difference between structural VAR (SVAR) and reduced-form VAR is:

- A) SVAR has more variables
- B) SVAR allows contemporaneous effects between variables
- C) SVAR uses different estimation methods
- D) There is no difference

Quiz 10: Structural vs Reduced Form VAR

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- A) SVAR has more variables
- B) SVAR allows contemporaneous effects between variables
- C) SVAR uses different estimation methods
- D) There is no difference

Answer: B

Reduced-form VAR: shocks are correlated, no contemporaneous effects in equations. SVAR: imposes identifying restrictions to recover structural shocks with economic interpretation (e.g., monetary policy shock).

Quiz 11: Cholesky Decomposition

Question

Cholesky ordering in IRF analysis assumes:

- A) All variables are equally important
- B) Variables ordered first affect later variables contemporaneously, not vice versa
- C) Shocks are uncorrelated
- D) No restrictions are needed

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- A) All variables are equally important
- B) Variables ordered first affect later variables contemporaneously, not vice versa
- C) Shocks are uncorrelated
- D) No restrictions are needed

Answer: B

Cholesky imposes a recursive structure: variables ordered first can affect later ones within the same period, but not the reverse. The ordering matters and should be justified by economic theory.

Quiz 12: VAR Residual Diagnostics

Question

In a well-specified VAR, residuals should be:

- A) Autocorrelated but homoskedastic
- B) White noise (no autocorrelation)
- C) Normally distributed only
- D) Correlated across equations

Quiz 12: VAR Residual Diagnostics

Question

In a well-specified VAR, residuals should be:

- A) Autocorrelated but homoskedastic
- B) White noise (no autocorrelation)
- C) Normally distributed only
- D) Correlated across equations

Answer: B

Residuals should be white noise: no autocorrelation at any lag. Use Portmanteau test or LM test for residual autocorrelation. Note: cross-equation correlation is allowed (captured by Σ_u).

Quiz 13: Cointegration and VAR

Question

If variables are I(1) and cointegrated, you should use:

- A) VAR in levels
- B) VAR in first differences
- C) Vector Error Correction Model (VECM)
- D) Univariate ARIMA models

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- A) VAR in levels
- B) VAR in first differences
- C) Vector Error Correction Model (VECM)
- D) Univariate ARIMA models

Answer: C

With cointegration, VAR in differences loses long-run information, while VAR in levels may be inefficient. VECM incorporates both short-run dynamics and long-run equilibrium relationships through the error correction term.

Quiz 14: Instantaneous Causality

Question

Instantaneous causality differs from Granger causality because it tests:

- A) Lagged relationships only
- B) Contemporaneous correlation of residuals
- C) Long-run relationships
- D) Model stability

Quiz 14: Instantaneous Causality

Question

Instantaneous causality differs from Granger causality because it tests:

- A) Lagged relationships only
- B) Contemporaneous correlation of residuals
- C) Long-run relationships
- D) Model stability

Answer: B

Instantaneous causality tests whether shocks to X and Y are correlated within the same period (correlation of VAR residuals). Granger causality tests whether *lagged* values help predict.

True/False Questions

Determine if each statement is True or False:

1. VAR models treat all variables as endogenous.
2. Granger causality proves true economic causation.
3. A stable VAR always has eigenvalues inside the unit circle.
4. FEVD results depend on the ordering of variables.
5. VAR can be estimated by OLS equation by equation.
6. Impulse responses eventually die out in a stable VAR.

Answers on next slide...

True/False: Solutions

1. VAR models treat all variables as endogenous. **TRUE**
   Each variable is regressed on lags of all variables, including itself.
2. Granger causality proves true economic causation. **FALSE**
   It only shows predictive content, not structural causation.
3. A stable VAR always has eigenvalues inside the unit circle. **TRUE**
   Stability condition: all eigenvalues of companion matrix satisfy $|\lambda_i| < 1$.
4. FEVD results depend on the ordering of variables. **TRUE**
   Under Cholesky identification, different orderings give different results.
5. VAR can be estimated by OLS equation by equation. **TRUE**
   With same regressors in each equation, OLS = GLS = ML (under normality).
6. Impulse responses eventually die out in a stable VAR. **TRUE**
   Stability ensures shocks have transitory effects; IRFs $\to 0$ as $h \to \infty$.

Problem 1: Writing VAR Equations

Exercise

Write out the two equations for a bivariate VAR(1) model with variables Y_t (GDP growth) and X_t (inflation).

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Write out the two equations for a bivariate VAR(1) model with variables Y_t (GDP growth) and X_t (inflation).

Solution

$$Y_t = c_1 + a_{11} Y_{t-1} + a_{12} X_{t-1} + \varepsilon_{1t}$$

$$X_t = c_2 + a_{21} Y_{t-1} + a_{22} X_{t-1} + \varepsilon_{2t}$$

Interpretation:

- a_{12}: Effect of past inflation on current GDP growth
- a_{21}: Effect of past GDP growth on current inflation

Problem 2: Parameter Count

Exercise

How many total parameters need to be estimated in a VAR(3) with $K = 4$ variables (including constants)?

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Exercise

How many total parameters need to be estimated in a VAR(3) with $K = 4$ variables (including constants)?

Solution

Per equation: $1 + K \times p = 1 + 4 \times 3 = 13$ parameters

Total for $K = 4$ equations: $4 \times 13 = \mathbf{52}$ parameters

Plus covariance matrix $\boldsymbol{\Sigma}$: $K(K+1)/2 = 4 \times 5/2 = 10$ unique elements

Grand total: 62 parameters

This is why VARs can be "over-parameterized" with limited data!

Problem 3: Granger Causality Interpretation

Exercise

A Granger causality test yields:

- H_0: Money does not Granger-cause GDP. p-value $= 0.02$
- H_0: GDP does not Granger-cause Money. p-value $= 0.35$

Interpret these results.

Problem 3: Granger Causality Interpretation

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A Granger causality test yields:

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Interpret these results.

Solution

- **Reject** H_0 at 5%: Money **Granger-causes** GDP
- **Fail to reject** H_0: GDP does **not** Granger-cause Money

Conclusion: Unidirectional causality: Money $\rightarrow$ GDP

Interpretation: Past money supply helps predict GDP growth. This is consistent with monetarist views, but remember: Granger causality $\neq$ structural causality!

Problem 4: Stability Check

Exercise

For VAR(1) with $\mathbf{A}_1 = \begin{pmatrix} 0.7 & 0.2 \\ 0.1 & 0.5 \end{pmatrix}$, check stability.

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For VAR(1) with $\mathbf{A}_1 = \begin{pmatrix} 0.7 & 0.2 \\ 0.1 & 0.5 \end{pmatrix}$, check stability.

Solution

Find eigenvalues: $\det(\mathbf{A}_1 - \lambda \mathbf{I}) = 0$
$(0.7 - \lambda)(0.5 - \lambda) - (0.2)(0.1) = 0$
$\lambda^2 - 1.2\lambda + 0.33 = 0$
$\lambda = \frac{1.2 \pm \sqrt{1.44 - 1.32}}{2} = \frac{1.2 \pm 0.346}{2}$
$\lambda_1 = 0.773, \quad \lambda_2 = 0.427$

Both $|\lambda_i| < 1 \Rightarrow$ **Stable!**

Problem 5: IRF Computation

Exercise

For VAR(1) with $\mathbf{A} = \begin{pmatrix} 0.5 & 0.2 \\ 0 & 0.6 \end{pmatrix}$, compute $\mathbf{\Phi}_2$ (response at $h = 2$).

Problem 5: IRF Computation

Exercise

For VAR(1) with $\mathbf{A} = \begin{pmatrix} 0.5 & 0.2 \\ 0 & 0.6 \end{pmatrix}$, compute $\boldsymbol{\Phi}_2$ (response at $h = 2$).

Solution

$$\boldsymbol{\Phi}_2 = \mathbf{A}^2 = \begin{pmatrix} 0.5 & 0.2 \\ 0 & 0.6 \end{pmatrix} \begin{pmatrix} 0.5 & 0.2 \\ 0 & 0.6 \end{pmatrix}$$

$$= \begin{pmatrix} 0.25 + 0 & 0.10 + 0.12 \\ 0 + 0 & 0 + 0.36 \end{pmatrix} = \begin{pmatrix} 0.25 & 0.22 \\ 0 & 0.36 \end{pmatrix}$$

Interpretation: A unit shock to Y_2 at t increases Y_1 by 0.22 at $t + 2$.

Example: Stock Returns and Trading Volume

Scenario

You have daily data on stock returns (R_t) and trading volume (V_t). You want to test for Granger causality in both directions.

Typical Findings in Finance Literature

- Returns often Granger-cause volume (price changes trigger trading)
- Volume sometimes Granger-causes returns (volume as leading indicator)
- Results: Often **bidirectional** causality $R \leftrightarrow V$

Practical Issue

Stock returns are typically stationary, but volume may need transformation (log or difference).

Example: Interest Rates and Inflation

Taylor Rule Context

Central banks set interest rates (i_t) in response to inflation (π_t):

$$i_t = r^* + \pi^* + 1.5(\pi_t - \pi^*) + 0.5(y_t - y^*)$$

VAR Analysis

- Does inflation Granger-cause interest rates? (Should, if central bank reacts)
- Do interest rates Granger-cause inflation? (Monetary policy transmission)

Expected: Bidirectional causality with:

- Quick response: $\pi \to i$ (policy reaction)
- Delayed response: $i \to \pi$ (policy takes effect)

Load data (GDP growth, unemployment) data = pd.DataFrame('gdp': gdp$_{g}rowth,' unemp' : unemp_{r}ate$)
Check stationarity first! Then fit VAR with optimal lag model = VAR(data) results = model.fit(maxlags=8, ic='aic') print(f"Selected lag order: results.k$_{a}r$")
Granger causality tests print("GDP -¿ Unemployment:") grangercausalitytests(data[['unemp', 'gdp']], maxlag=4) print("-¿ GDP:") grangercausalitytests(data[['gdp', 'unemp']], maxlag=4)

Discussion: Granger Causality vs True Causality

Key Question

If X Granger-causes Y, does that mean X actually causes Y?

Discussion Points

- **Omitted variable bias**: Z might cause both X and Y
  - Example: Weather affects both ice cream sales and drownings
- **Anticipation effects**: Markets anticipate future events
  - Stock prices "Granger-cause" earnings announcements
- **Aggregation issues**: Timing of data collection matters

Conclusion: Granger causality is about **prediction**, not **mechanism**. For structural causality, need theory + identification strategy.

Discussion: Variable Ordering in IRFs

Key Question

Why does the ordering of variables matter for orthogonalized IRFs?

Explanation

Cholesky decomposition assumes:

- First variable: Affects all others contemporaneously
- Second variable: Affected by first, affects remaining
- Last variable: Affected by all, affects none contemporaneously

Economic reasoning needed: Order from "most exogenous" to "most endogenous"

Example ordering for monetary policy VAR:

1. Oil prices (exogenous)
2. GDP (slow to respond)
3. Inflation
4. Interest rates (policy responds to all)

Take-Home Exercises

1. **Theoretical**: Show that a VAR(1) can be written as a VAR(∞) MA representation: $\mathbf{Y}_t = \sum_{i=0}^{\infty} \mathbf{A}^i \varepsilon_{t-i}$ when stable.

2. **Computation**: For VAR(1) with $\mathbf{A} = \begin{pmatrix} 0.8 & -0.1 \\ 0.3 & 0.4 \end{pmatrix}$:
   - Check stability
   - Compute IRFs for $h = 0, 1, 2, 3$
   - Plot the response of Y_1 to a shock in Y_2

3. **Applied**: Download US GDP growth and unemployment data:
   - Test both series for stationarity
   - Estimate a VAR model (select optimal lag)
   - Test Granger causality in both directions
   - Compute and interpret IRFs

4. **Critical Thinking**: Why might stock prices "Granger-cause" GDP even though GDP is determined by real factors?

Exercise Solutions Hints

Hints

1. Use recursive substitution: $\mathbf{Y}_t = \mathbf{A}\mathbf{Y}_{t-1} + \varepsilon_t = \mathbf{A}(\mathbf{A}\mathbf{Y}_{t-2} + \varepsilon_{t-1}) + \varepsilon_t = \ldots$

2. Eigenvalues of $\begin{pmatrix} 0.8 & -0.1 \\ 0.3 & 0.4 \end{pmatrix}$:
   - Characteristic equation: $\lambda^2 - 1.2\lambda + 0.35 = 0$
   - $\lambda_1 \approx 0.85$, $\lambda_2 \approx 0.41$ (both < 1, stable)

3. For GDP/Unemployment:
   - GDP growth is usually I(0), unemployment may be I(1)
   - Use unemployment rate changes if needed
   - Expect GDP growth $\rightarrow$ unemployment (Okun's Law)

4. Stock prices anticipate future economic conditions—they reflect expectations about future GDP, so they "lead" GDP in the data even though causation runs the other way.

Key Takeaways from This Seminar

Main Points

1. VAR models capture **interdependencies** between multiple time series
2. Parameter count grows quickly: $K^2p + K$ per system
3. **Granger causality** tests predictive content, not true causation
4. Test statistic is F-test on coefficient restrictions
5. **IRFs** show dynamic propagation of shocks
6. Variable ordering matters for orthogonalized IRFs

Key Practical Points

- Always check stationarity before estimating VAR
- Use information criteria for lag selection
- Report Granger tests in both directions
- Be careful interpreting as "true" causality