



# Time Series Analysis and Forecasting

## Chapter 10: Comprehensive Review



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## Learning Objectives

By the end of this chapter, you will be able to:

- Apply the complete forecasting workflow from data to evaluation
- Select appropriate models based on data characteristics
- Evaluate forecast accuracy using proper metrics and cross-validation
- Integrate knowledge from all previous chapters in practice



## Outline

Forecasting Methodology

Case Study 1: Bitcoin Volatility (GARCH)

Case Study 2: Sunspot Cycles (Fourier)

Case Study 3: Unemployment (Prophet)

Case Study 4: Multivariate Analysis (VAR)

Synthesis and Guidelines

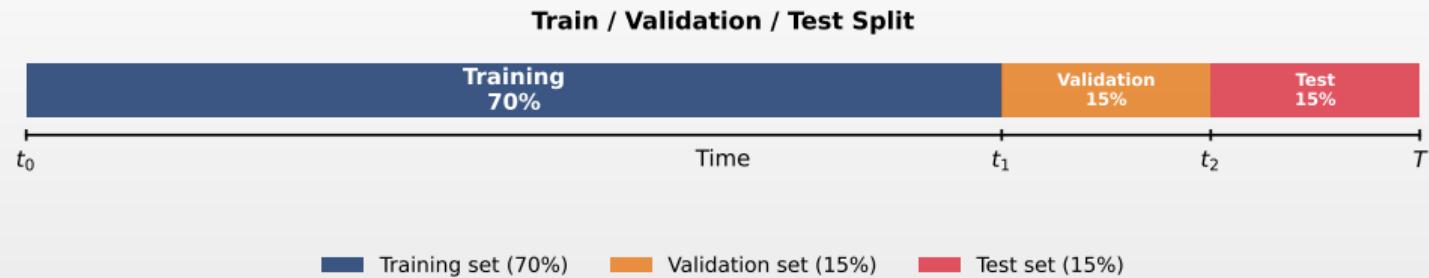
AI Use Case

Quiz

Summary



## Train/Validation/Test Framework



## The Scientific Approach to Forecasting

### Research Question

How do we **rigorously evaluate** forecast performance while avoiding overfitting?

### The Fundamental Problem

- In-sample fit  $\neq$  Out-of-sample performance
- Models can “memorize” training data without learning patterns
- Solution:**
  - ▶ Proper train/validation/test methodology

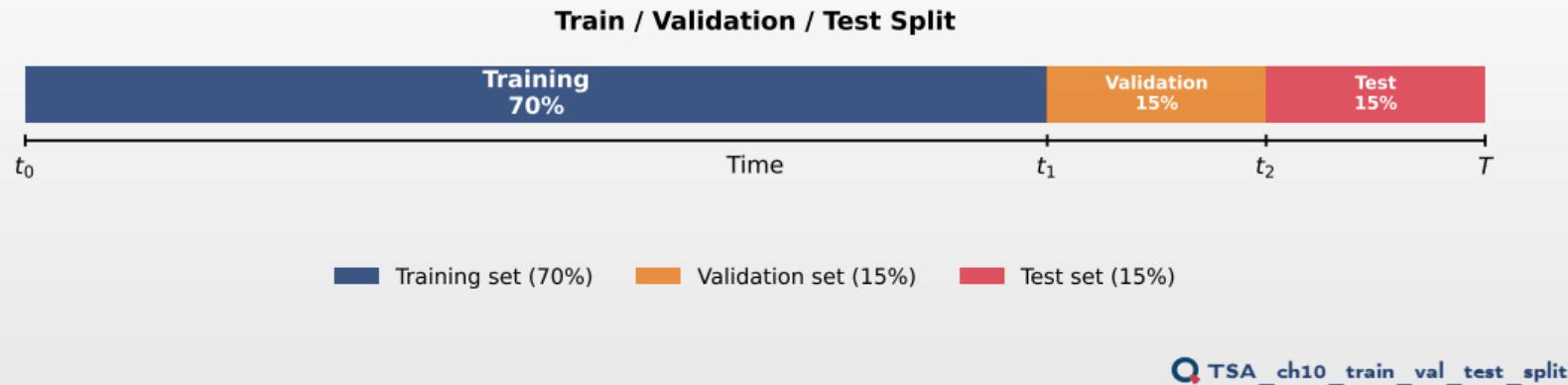
### Key Principle

“The test set must remain **untouched** until final evaluation.”

— Standard practice in machine learning and econometrics



## Train/Validation/Test Framework



Q TSA\_ch10\_train\_val\_test\_split



## Evaluation Metrics

### Definition 1 (Forecast Error Metrics)

Let  $y_t$  be actual,  $\hat{y}_t$  forecast:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_t (y_t - \hat{y}_t)^2}, \quad \text{MAE} = \frac{1}{n} \sum_t |y_t - \hat{y}_t|, \quad \text{MAPE} = \frac{100\%}{n} \sum_t \left| \frac{y_t - \hat{y}_t}{y_t} \right|$$

#### When to Use Each

- RMSE**: Penalizes large errors
- MAE**: Robust to outliers
- MAPE**: Scale-independent (%)

#### Caution

- MAPE undefined when  $y_t = 0$
- Compare on **same** test set
- Report **out-of-sample** metrics



## Forecast Evaluation Beyond RMSE

RMSE is not universally optimal

- The metric must be chosen based on the **economic objective**

Other relevant metrics

- **MASE** (Mean Absolute Scaled Error) — comparison with naïve
- **Directional Accuracy** — correct direction of change
- **Quantile Loss** — for VaR and probabilistic forecasts
- **CRPS** (Continuous Ranked Probability Score) — full distribution

Example: Quantile Loss

$$QL_\alpha(y_t, \hat{q}_t) = \begin{cases} \alpha(y_t - \hat{q}_t), & y_t > \hat{q}_t \\ (1 - \alpha)(\hat{q}_t - y_t), & y_t \leq \hat{q}_t \end{cases}$$



## Formal Forecast Comparison: Diebold–Mariano

### Problem

- ☐ Lower RMSE  $\neq$  statistically significant difference

### Definition 2 (Diebold–Mariano Test)

Define the loss differential:  $d_t = L(e_{1t}) - L(e_{2t})$

Test statistic:

$$DM = \frac{\bar{d}}{\sqrt{\widehat{\text{Var}}(\bar{d})}} \xrightarrow{d} N(0, 1)$$

### Hypotheses

- ☐  $H_0$ : equal predictive performance
- ☐  $H_1$ : different performance

### Key message

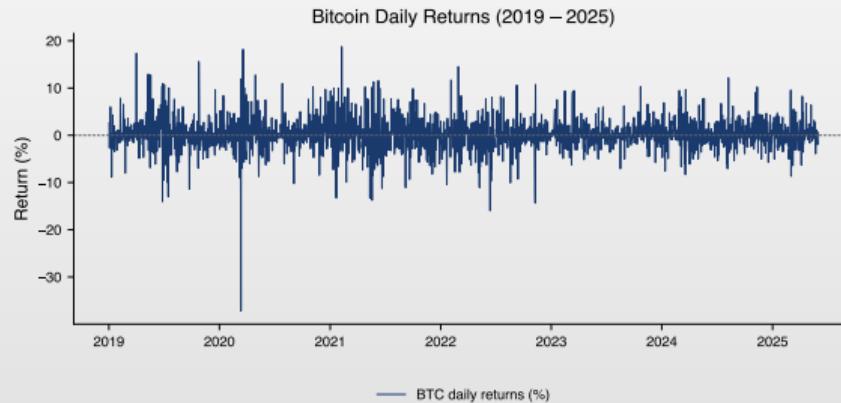
- ☐ Large  $|DM| \Rightarrow$  significant difference
- ☐ Model comparison must be **statistically grounded**



## Bitcoin: Volatility Clustering

### Observation

- Large returns follow large returns, small follow small—**volatility clustering**



Q TSA\_ch10\_btc\_returns

## Bitcoin: Problem Statement

### Research Question

Can we forecast Bitcoin's **volatility** using GARCH models?

### Data Characteristics

- Source: Yahoo Finance (BTC-USD)
- Period: Jan 2019 – Jan 2025
- Frequency: Daily
- Observations:  $\approx 2,200$  days

### Stylized Facts

- Returns: near-zero mean
- Fat tails ( $kurtosis > 3$ )
- Volatility clustering

### Key Insight

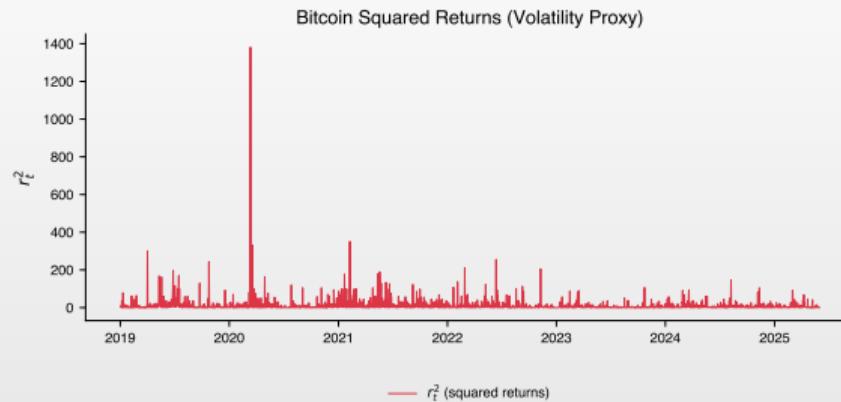
Financial returns are typically:

- **Unpredictable** in mean
- **Predictable** in variance

⇒ Focus on **volatility forecasting**



## Bitcoin: Evidence for GARCH



 TSA\_ch10\_btc\_acf\_squared



## GARCH Model Specification

### Definition 3 (GARCH(p,q) Model)

Let  $r_t$  denote returns. The GARCH(p,q) model:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t \stackrel{iid}{\sim} N(0, 1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2$$

where  $\omega > 0$ ,  $\alpha_i \geq 0$ ,  $\beta_j \geq 0$ , and  $\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1$ .

### Model Variants

- GARCH(1,1)**: Most common
- GJR-GARCH**: Leverage effect
- EGARCH**: Log-variance, asymmetric

### Interpretation

- $\alpha$ : Shock impact (ARCH effect)
- $\beta$ : Volatility persistence
- $\alpha + \beta \approx 1$ : High persistence



## GARCH: Stationarity and Unconditional Variance

### Theorem 1 (Covariance Stationarity of GARCH(1,1))

If  $\alpha_1 + \beta_1 < 1$ , then  $\{\varepsilon_t\}$  is covariance stationary with:

$$\bar{\sigma}^2 = \mathbb{E}[\sigma_t^2] = \frac{\omega}{1 - \alpha_1 - \beta_1}$$

### Derivation

Take expectations of both sides of the variance equation:

$$\begin{aligned}\mathbb{E}[\sigma_t^2] &= \omega + \alpha_1 \mathbb{E}[\varepsilon_{t-1}^2] + \beta_1 \mathbb{E}[\sigma_{t-1}^2] \\ \bar{\sigma}^2 &= \omega + (\alpha_1 + \beta_1) \bar{\sigma}^2 \quad (\text{stationarity}) \\ \bar{\sigma}^2 &= \frac{\omega}{1 - \alpha_1 - \beta_1}\end{aligned}$$

### Multi-Step Forecasts Converge to $\bar{\sigma}^2$

As  $h \rightarrow \infty$ :  $\mathbb{E}_t[\sigma_{t+h}^2] \rightarrow \bar{\sigma}^2$  at rate  $(\alpha_1 + \beta_1)^h$ .



## Bitcoin: Model Selection on Validation Set

### Methodology

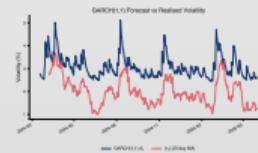
Fit each model on training data, evaluate on validation set.

Model	AIC	BIC	Val MAE	Selection
GARCH(1,1)	6,994.8	7,020.6	2.638	Best
GARCH(2,1)	6,993.7	7,024.6	2.640	
GJR-GARCH(1,1)	6,983.7	7,014.6	2.669	
EGARCH(1,1)	—	—	—	Failed*

\*Analytic forecasts not available for  $h > 1$

### Result

GARCH(1,1) selected based on lowest validation MAE for volatility forecasts.



## Bitcoin: Data Split and Stationarity

### Data Split

Set	Period	N
Training (70%)	2019-01 to 2023-03	1,543
Validation (20%)	2023-03 to 2024-06	441
Test (10%)	2024-06 to 2025-01	221
<b>Total</b>	<b>2,205</b>	

### Stationarity Tests

Series	ADF	Result
Prices	$p = 0.50$	Non-stationary
Returns	$p < 0.01$	Stationary

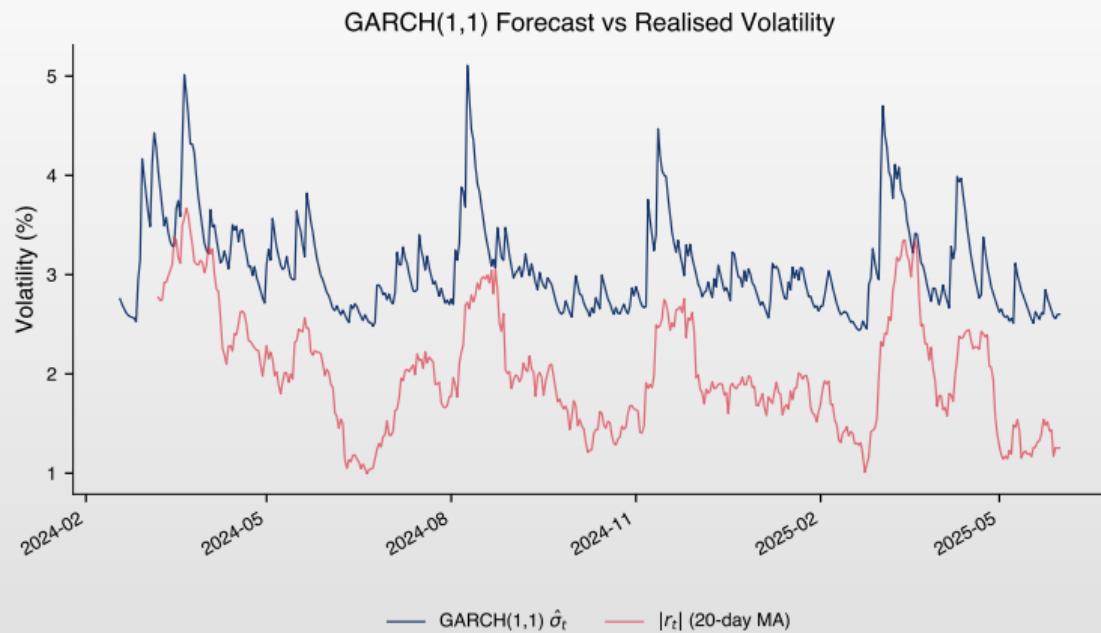
⇒ Model **returns**, not prices

### Why Stationarity Matters

- ◻ GARCH requires weakly stationary input
- ◻ Prices follow random walk; returns are stationary



## Bitcoin: Final Test Set Evaluation



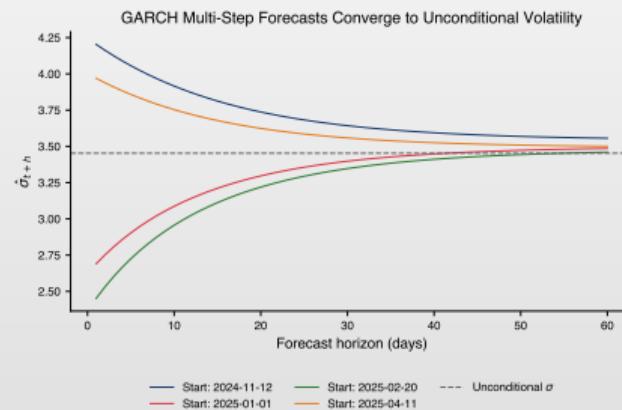
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## GARCH: Multi-Step Forecasts Converge

### Key Insight

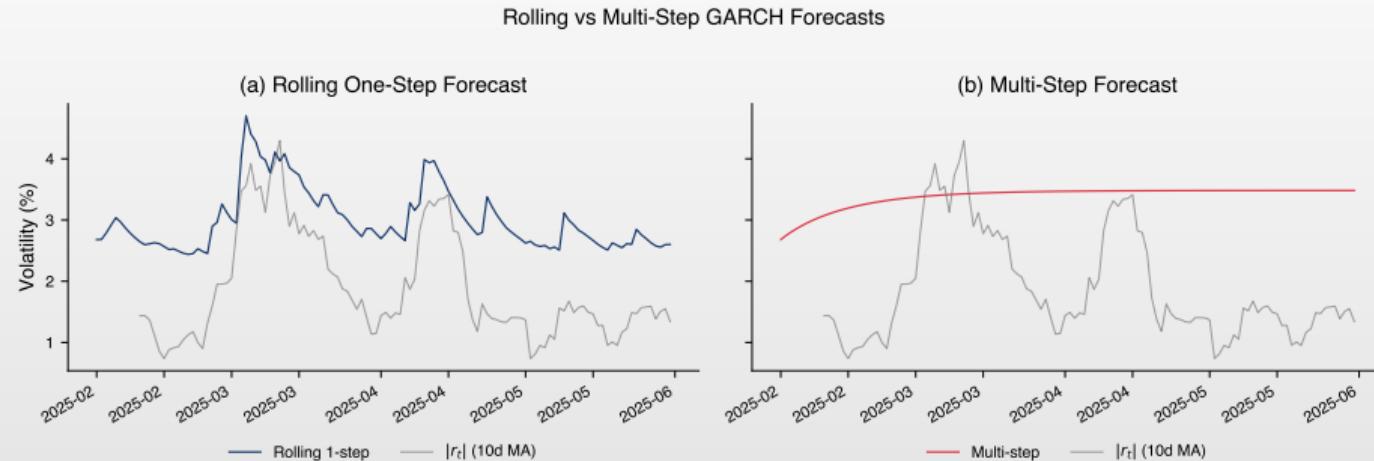
- Multi-step forecasts converge to  $\bar{\sigma}^2 = \frac{\omega}{1-\alpha-\beta}$
- Use rolling forecasts



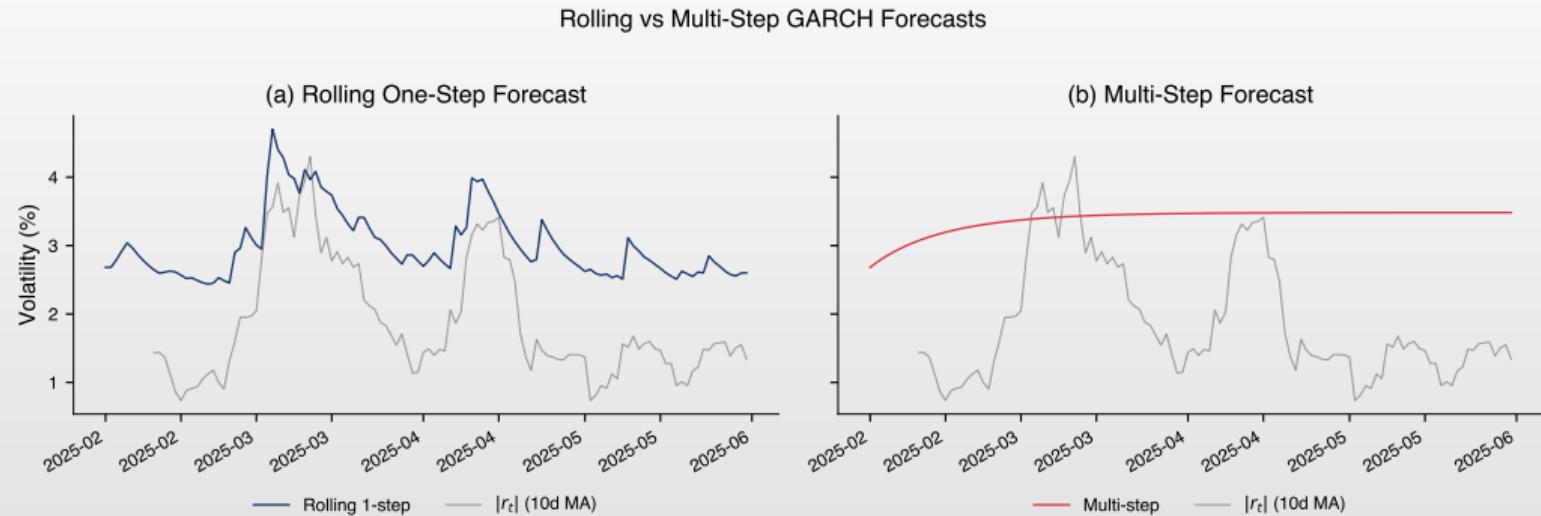
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## GARCH: Rolling One-Step-Ahead Solution



## GARCH: Rolling One-Step-Ahead Solution



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## GARCH: Innovation Distributions

### Model

$$r_t = \mu + \sigma_t z_t$$

- Choices for  $z_t$ :  $\mathcal{N}(0, 1)$  (normal) or  $t_\nu$  (heavy tails)

### Crypto: why it matters

- Kurtosis  $> 3$  (heavy tails)
- Frequent extreme events
- Normality **underestimates** tail risk

### Practical implication

- Student-t distribution produces **more realistic** VaR estimates
- Degrees of freedom  $\nu$  control tail thickness



## Application: Conditional Value-at-Risk

### Definition 4 (Conditional VaR at level $\alpha$ )

$$\text{VaR}_{t+1}^{\alpha} = \mu_{t+1} + \sigma_{t+1} \cdot z_{\alpha}$$

- $\sigma_{t+1}^2$  comes from the GARCH model
- $z_{\alpha}$  = quantile of the chosen distribution (Normal or Student-t)

### Key insight

- Returns are hard to predict
- Volatility is **predictable**

### Conclusion

- GARCH is a **risk model**, not a return model
- VaR = the direct application



## Risk Model Validation: Backtesting

### Kupiec Test (Unconditional Coverage)

$$LR_{uc} = -2 \ln \left( \frac{(1 - \alpha)^{T-x} \alpha^x}{(1 - \hat{\rho})^{T-x} \hat{\rho}^x} \right) \sim \chi^2(1), \quad x = \text{violations}, \quad \hat{\rho} = x/T$$

### Christoffersen Test

- Checks **independence** of violations
- $LR_{cc} = LR_{uc} + LR_{ind} \sim \chi^2(2)$

### Interpretation

- Too many violations  $\Rightarrow$  risk underestimated
- Too few  $\Rightarrow$  model too conservative

### Principle

- A risk model must be **validated**, not just estimated



## GARCH Limitations and Modern Extensions

### Limitations

- Does not capture **jumps**
- Constant parameters over time
- Sensitive to chosen distribution
- Does not model different **regimes**

### Extensions

- GJR-GARCH:** leverage effect
- EGARCH:** asymmetric shocks
- Markov-Switching GARCH:** regimes
- Realized volatility (HAR)
- Hybrid GARCH + ML

### Key message

- GARCH is a **starting point**, not the end of risk modeling



## Bitcoin: Key Findings

### Summary

1. Returns are stationary; prices are not
2. GARCH(1,1) outperforms more complex variants
3. High persistence ( $\alpha + \beta = 0.93$ )
4. Volatility is predictable even when returns are not

### Practical Implications

- ☐ Risk management: VaR, Expected Shortfall
- ☐ Option pricing requires volatility forecasts
- ☐ Portfolio optimization with time-varying risk

### Limitations

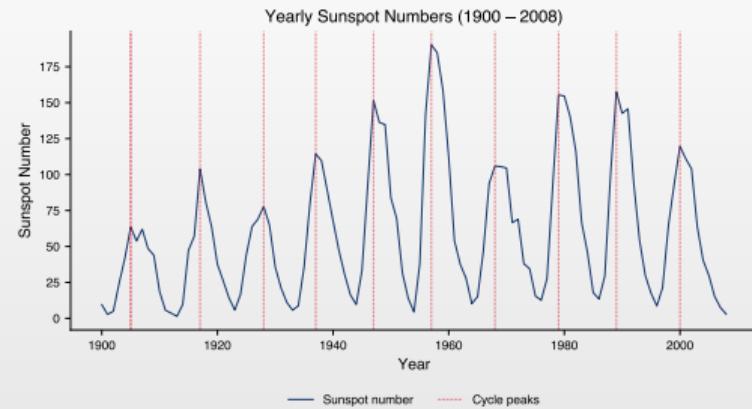
- ☐ GARCH assumes symmetric shocks
- ☐ Does not capture jumps
- ☐ Normal distribution may be restrictive

### Extensions

- ☐ Student-t innovations
- ☐ Realized volatility
- ☐ HAR models

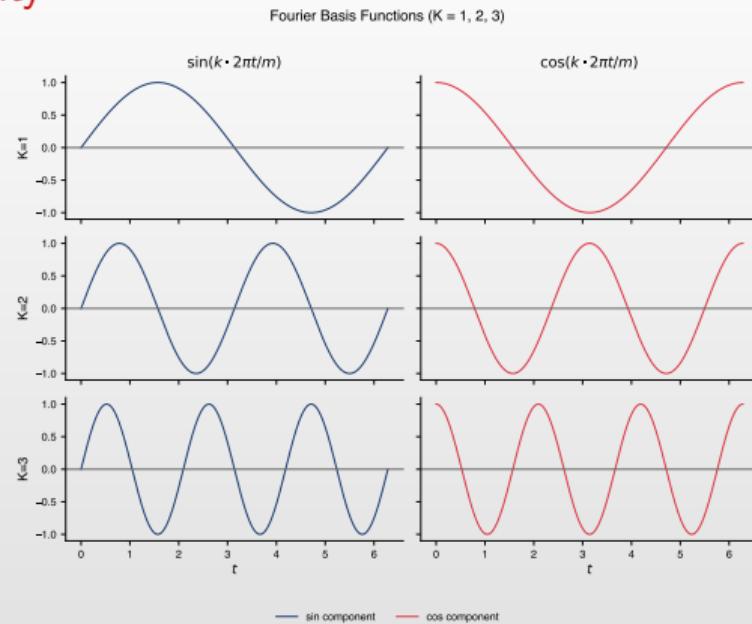


## Sunspots: The 11-Year Solar Cycle

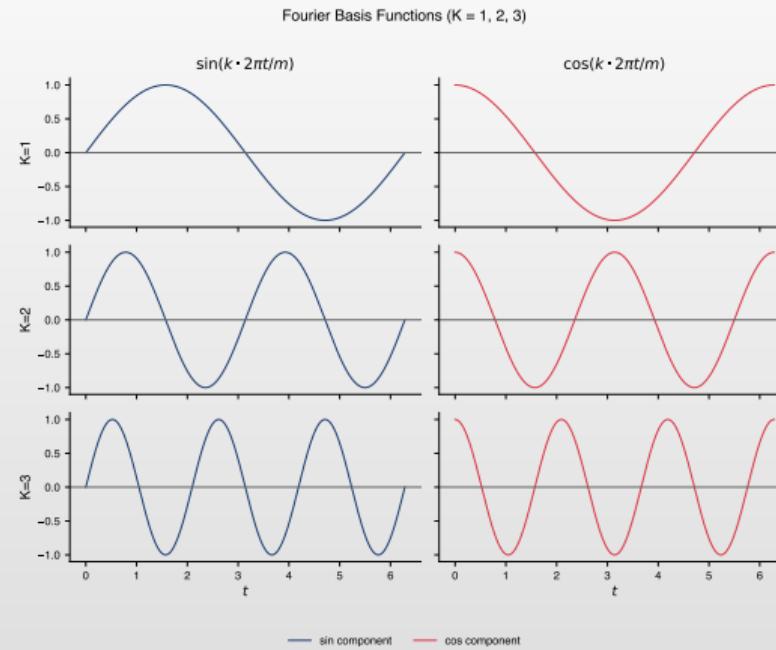


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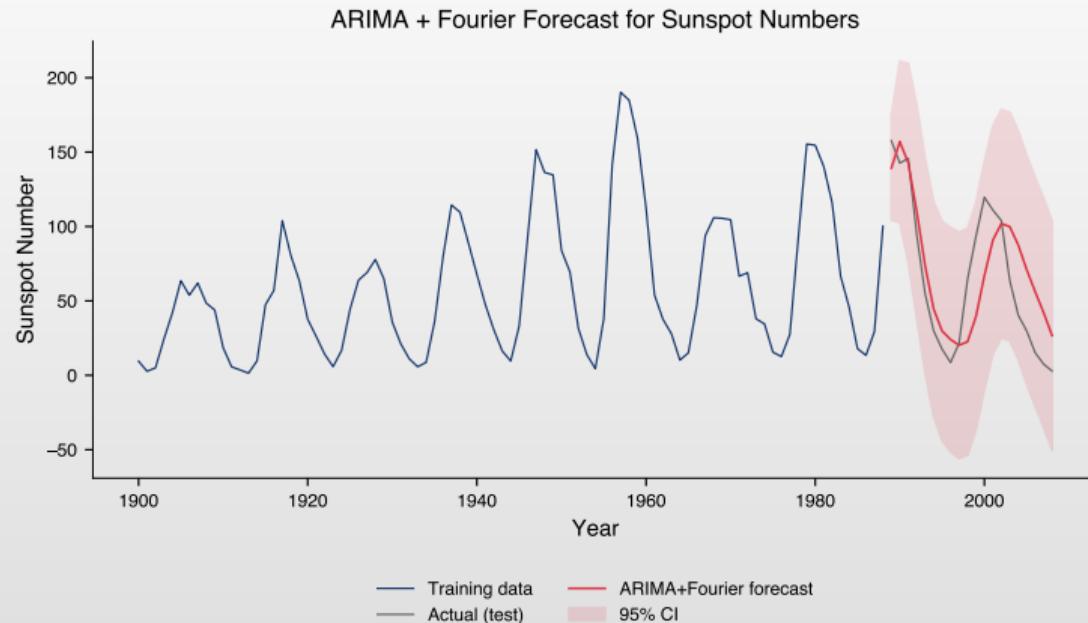
## Fourier Terms for Seasonality



## Fourier Terms for Seasonality



## Sunspots: Forecast Results



## Sunspots: Model Selection

### Methodology

Compare  $K = 1, 2, 3, 4$  Fourier harmonics on validation set.

Data Split	Set	Period	N
	Training (70%)	1900–1975	76
	Validation (20%)	1976–1997	22
	Test (10%)	1998–2008	11
Total			109

Model Comparison			
K	AIC	Val RMSE	
1	665.9	87.15	
2	668.0	86.92	
3	671.8	<b>86.81</b>	Best
4	674.5	87.93	

### Result

$K = 3$  Fourier harmonics selected (6 parameters for 11-year cycle).



## Overfitting in Choosing $K$

### Overfitting risk

- $K$  too large = memorizing historical cycle
- Model fits noise, not signal
- Test performance **degrades**

### Solution: validation

- Select  $K$  on **validation** set
- Evaluate on **test** — untouched
- Trade-off: complexity vs generalization

### Fourier $\approx$ periodic regression

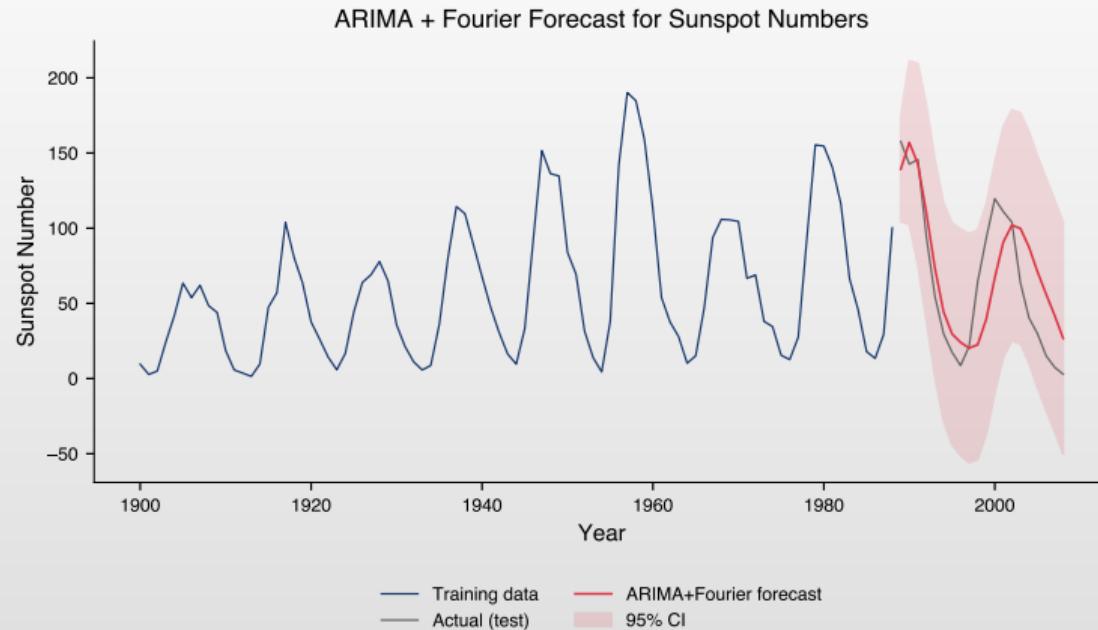
- Each harmonic adds 2 parameters (sin, cos)
- $K = 3$ : 6 extra parameters
- $K = 6$ : 12 parameters — overfitting risk

### Our results

- $K = 3$  minimizes Val RMSE
- $K = 4$  increases error  $\rightarrow$  overfitting



## Sunspots: Forecast Results



## Sunspots: Key Takeaways

### When to Use Fourier Terms

- Seasonal period  $s$  is **long** (e.g., 11 years, 52 weeks)
- SARIMA would require too many seasonal lags
- Pattern is **smooth and periodic**
- Multiple cycles need to be captured

### Choosing K

- Start with  $K = 1$ , increase until validation error stops improving
- Too high  $K$  = overfitting

### Fourier vs SARIMA

	Fourier	SARIMA
Long seasons	✓	✗
Short seasons	OK	✓
Parameters	$2K$	Many
Flexibility	Fixed	Adaptive

### Applications

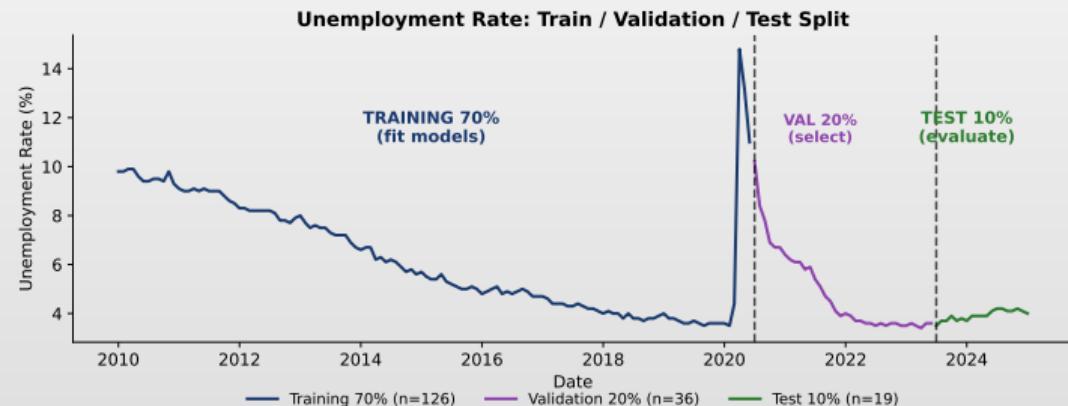
Climate cycles, business cycles, astronomical phenomena



## Unemployment: Train / Validation / Test Split

### Methodology

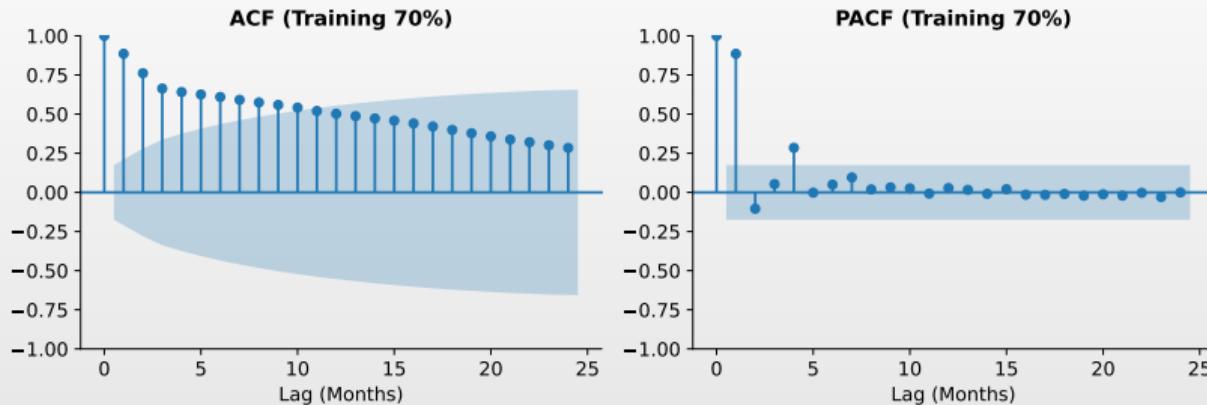
- Training:** Fit models
- Validation:** Select best
- Test:** Final evaluation



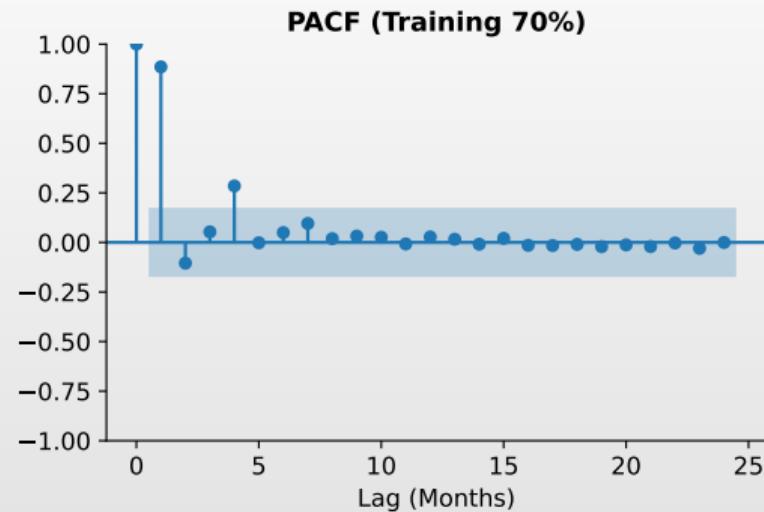
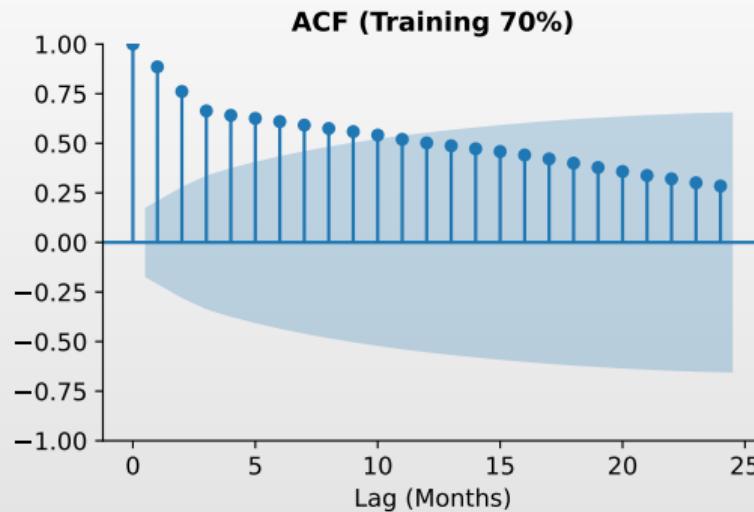
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## Unemployment: Preliminary Analysis

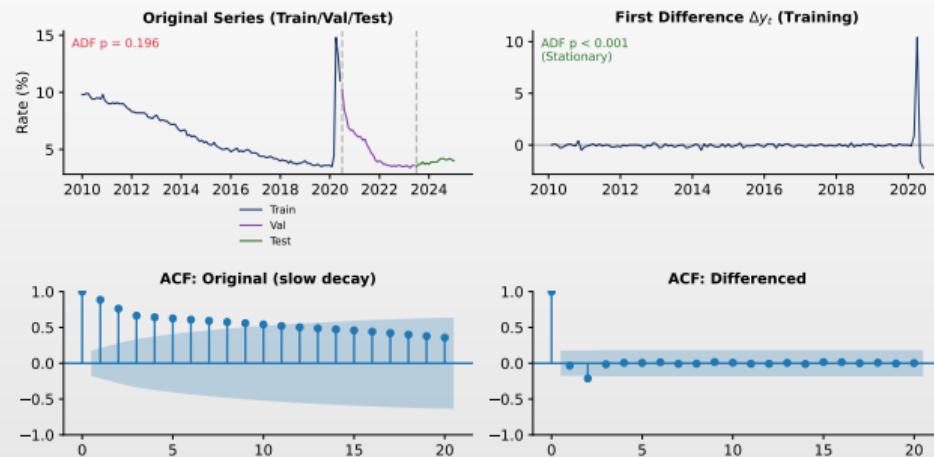


## Unemployment: Preliminary Analysis

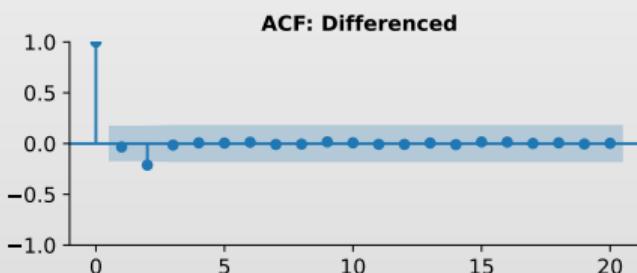
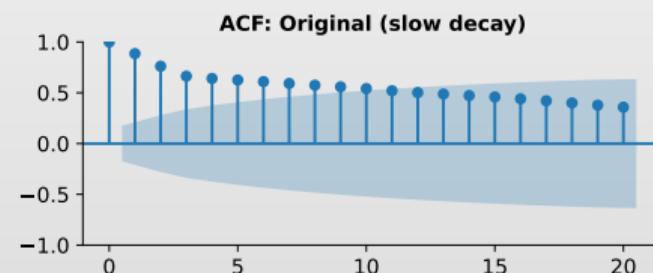
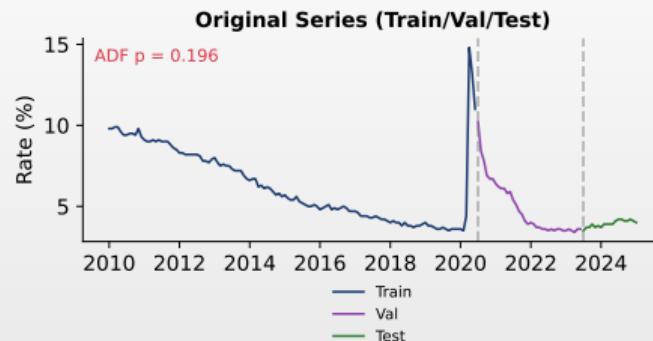


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## Unemployment: Stationarity Tests



## Unemployment: Stationarity Tests



## Structural Breaks: Formal Approach

### Classical methods

- Chow Test:** break at known point
- Bai–Perron:** multiple unknown breaks
- CUSUM:** sequential detection

### Practical trade-off

- SARIMA:** stable, parsimonious, but assumes constant parameters
- Prophet:** flexible, detects changepoints automatically

### Problem

- ADF can confuse **break** with **unit root**
- Zivot–Andrews test: ADF with endogenous break

### Key message

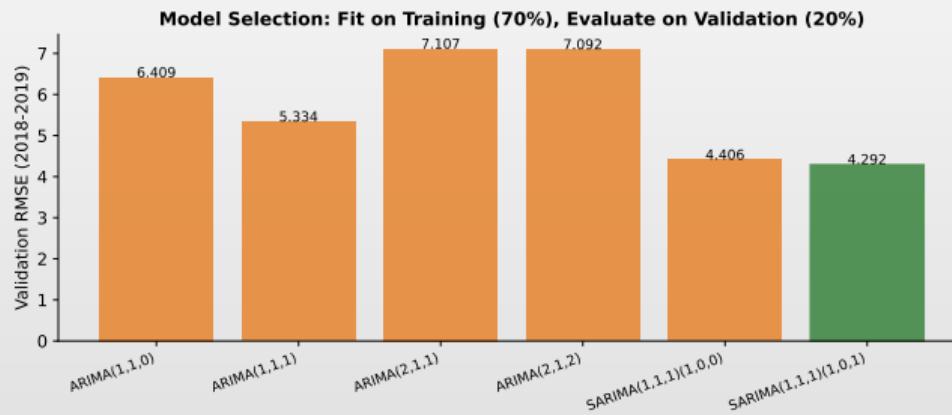
- Model must be adapted to **parameter stability**



## Unemployment: Model Selection (Validation Set)

Best: SARIMA(1,1,1)(1,0,0)<sub>12</sub>

- Selected by lowest validation RMSE



 TSA\_ch10\_sarima\_model\_selection



## Unemployment: SARIMA Parameters

SARIMA(1,1,1)(1,0,0)<sub>12</sub> fitted on Train+Val (2010-2019)

- AR(1):  $\phi_1 = -0.86$
- MA(1):  $\theta_1 = 0.78$
- SAR(12):  $\Phi_1 = -0.08$  (n.s.)

**SARIMA(1,1,1)(1,0,1) - Fitted on Train+Val (85%)**

Parameter	Coef	Std Err	P-value	Sig
ar.L1	0.8423	0.2084	0.0001	***
ma.L1	-0.9540	0.1973	0.0000	***
ar.S.L12	0.0326	4.5951	0.9943	
ma.S.L12	-0.0113	4.6087	0.9980	
sigma2	0.8122	0.0608	0.0000	***



## Ljung-Box Test for Residual Autocorrelation

### Definition 5 (Ljung-Box Test)

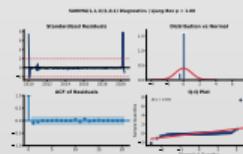
For residuals  $\hat{\varepsilon}_t$  with sample autocorrelations  $\hat{\rho}_k$ , the test statistic:

$$Q(h) = n(n + 2) \sum_{k=1}^h \frac{\hat{\rho}_k^2}{n - k} \stackrel{H_0}{\sim} \chi^2(h - p - q)$$

where  $p, q$  are ARMA orders.  $H_0$ : Residuals are white noise.

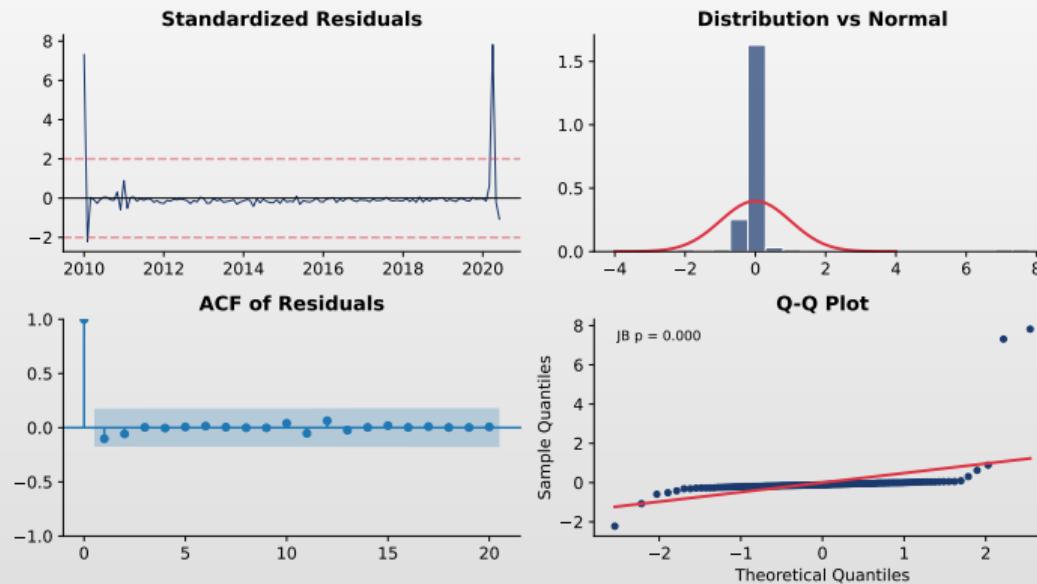
### Interpretation

- Large  $Q$  (small p-value): Reject  $H_0$ , residuals have structure
- Small  $Q$  (large p-value): Fail to reject  $H_0$ , model is adequate
- Rule of thumb: Use  $h = \min(10, n/5)$  for lag order



## Unemployment: SARIMA Diagnostics

SARIMA(1,1,1)(1,0,1) Diagnostics | Ljung-Box p = 1.00



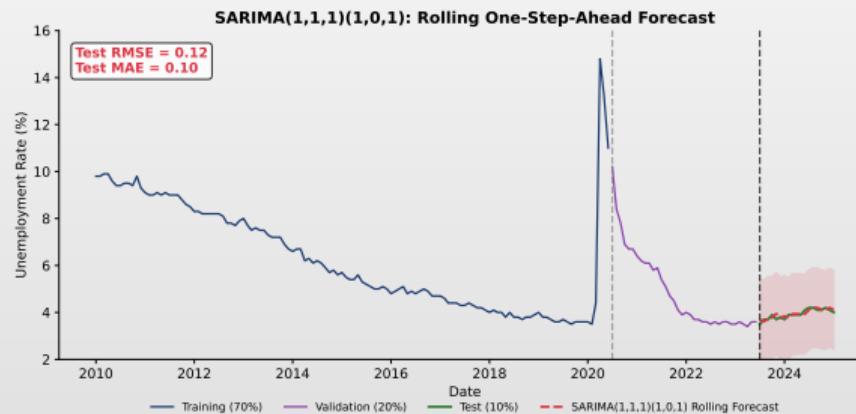
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## Unemployment: SARIMA Rolling Forecast

### Problem: Structural Break

- Rolling one-step-ahead forecast (re-estimate at each  $t$ )
- Test RMSE = 0.12



Q TSA\_ch10\_sarima\_forecast



## Prophet Model

### Definition 6 (Prophet Decomposition)

$$y_t = g(t) + s(t) + h(t) + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2)$$

where  $g(t)$  = trend,  $s(t)$  = seasonality,  $h(t)$  = holidays,  $\sigma^2$  = noise variance (estimated).

### Changepoint Detection

- Automatic location selection
- `changepoint_prior_scale` controls flexibility

### Advantages

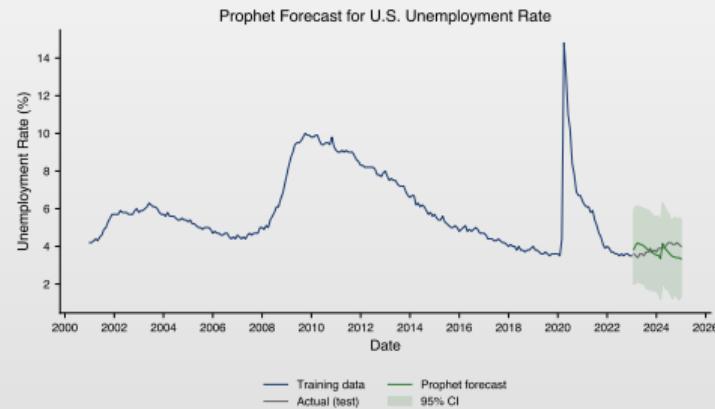
- Handles missing data
- Interpretable components
- Robust to outliers



## Unemployment: Prophet Forecast Results

### Key Finding

- Prophet adapts via changepoint detection
- Test RMSE = 0.58



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## Unemployment: Model Tuning

### Hyperparameter Tuning

Tune `changepoint_prior_scale` on validation set.

Data Split	Set	Period	N
	Training (70%)	2010-01 to 2020-06	126
	Validation (20%)	2020-07 to 2023-06	36
	Test (10%)	2023-07 to 2025-01	19
	<b>Total</b>		<b>181</b>

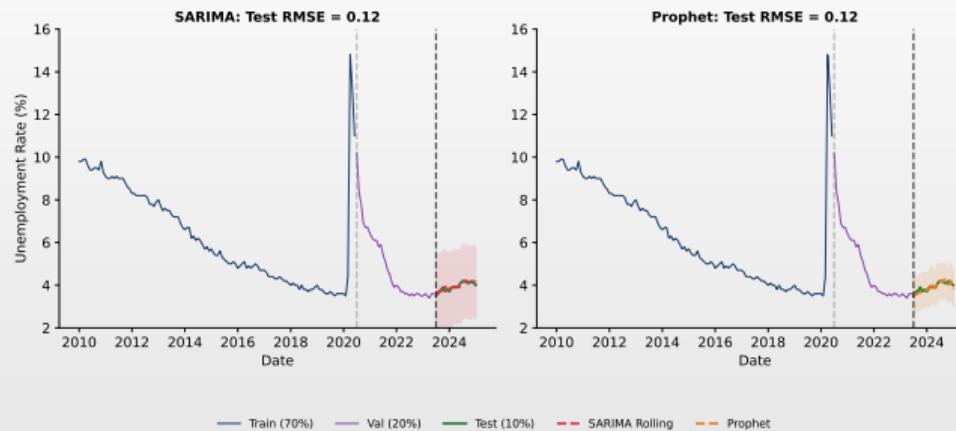
Scale Comparison	Scale	Val RMSE	Best
	0.01	4.21	
	0.05	3.89	
	0.10	<b>3.52</b>	
	0.30	3.67	
	0.50	3.81	

### Interpretation

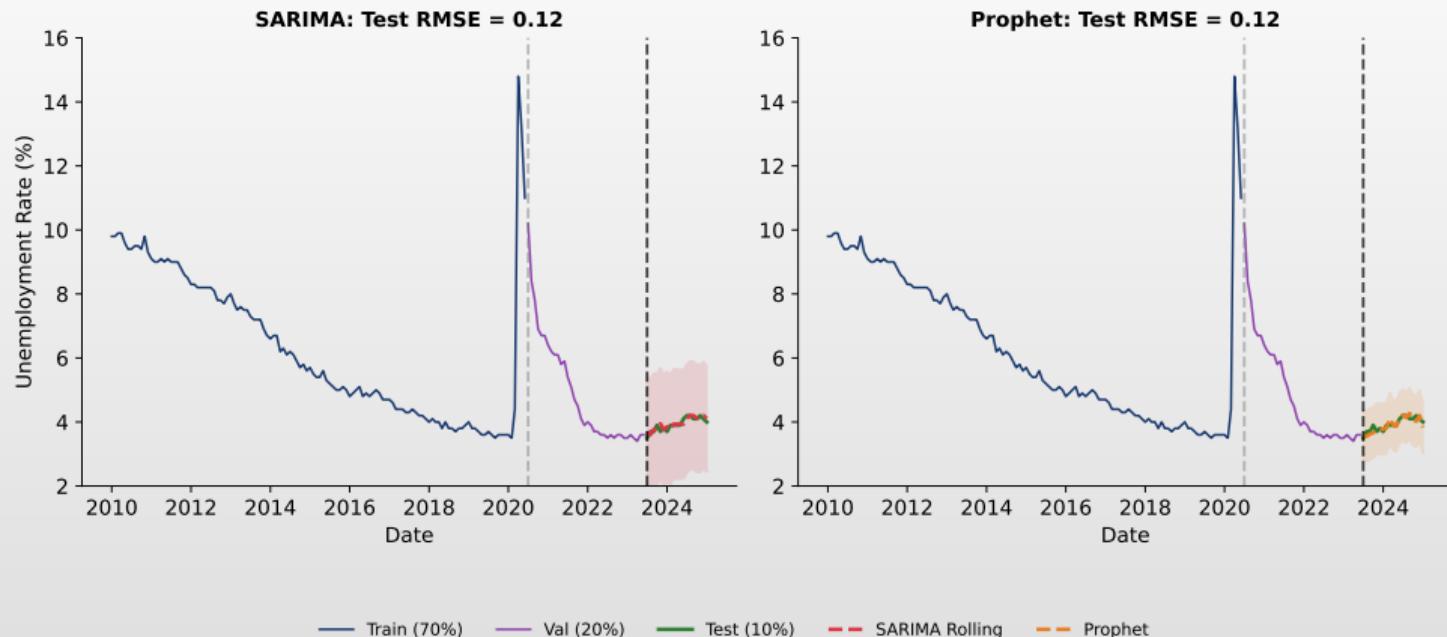
$\text{Scale} = 0.10$  balances flexibility (capturing COVID shock) with stability.



## Unemployment: SARIMA vs Prophet Comparison



## Unemployment: SARIMA vs Prophet Comparison



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## Prophet: When to Use It

### Ideal Use Cases

- Business data with **holidays**
- Missing values** present
- Need **interpretable** components
- Forecasts with **uncertainty bands**

### Caveat: Structural Breaks

Prophet handles breaks via changepoints, but **SARIMA outperformed** it on unemployment (0.12 vs 0.58). Always validate!

### Prophet vs ARIMA

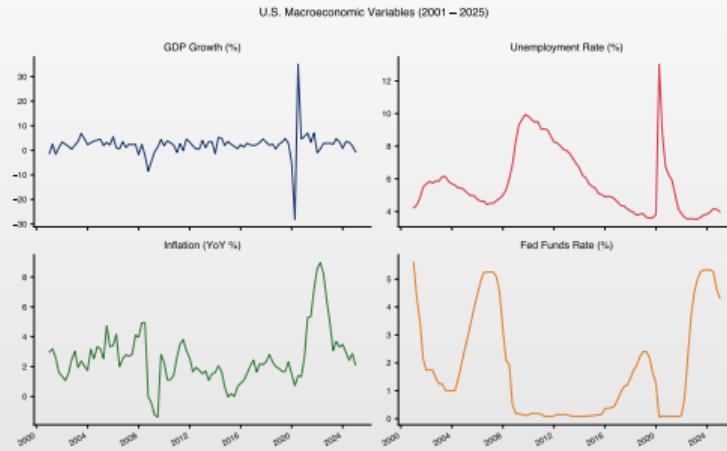
	Prophet	ARIMA
Changepoints	✓	✗
Missing data	✓	✗
Holidays	✓	✗
Speed	Fast	Moderate
Interpretable	✓	✗

### Key Parameters

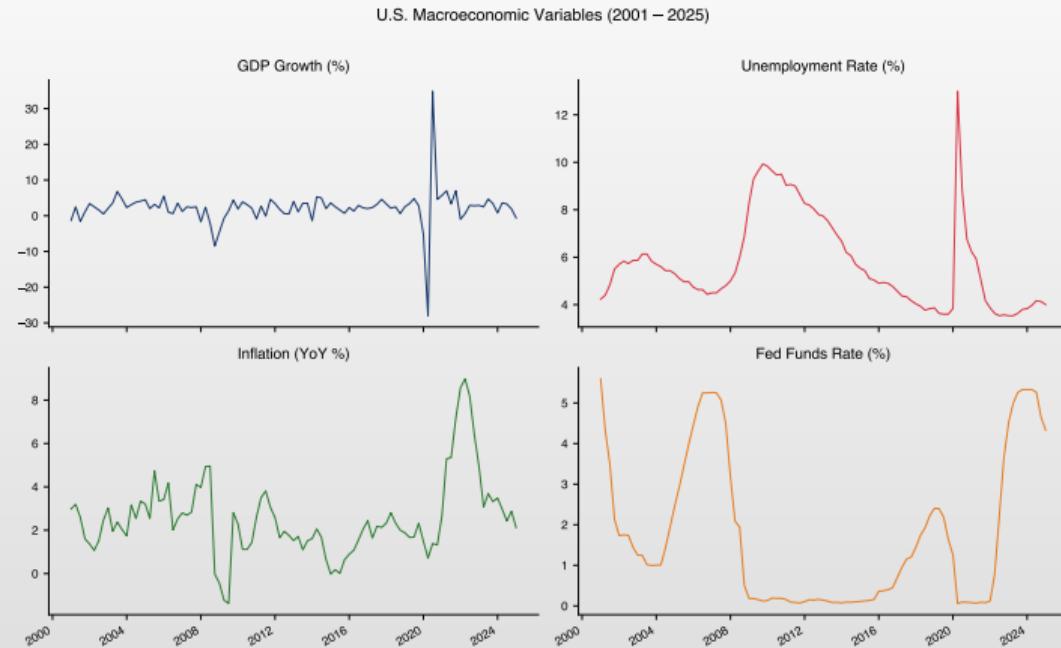
`changepoint_prior_scale`: flexibility  
`seasonality_prior_scale`: smoothness



## VAR: Multivariate Economic Data



## VAR: Multivariate Economic Data



 TSA\_ch10\_economic\_vars



## VAR Model Specification

### Definition 7 (Vector Autoregression VAR(p))

For  $K$  variables  $y_t = (y_{1t}, \dots, y_{Kt})'$ :

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + u_t$$

where  $A_i$  are  $K \times K$  coefficient matrices,  $u_t \sim N(0, \Sigma)$ ,  $\Sigma$  = covariance matrix.

### For Our 4-Variable System

VAR(2) has:

- 4 intercepts
- $2 \times 4 \times 4 = 32$  AR coefficients
- 36 parameters total**

### Lag Selection

Use information criteria:

- AIC**: Tends to overfit
- BIC**: More parsimonious
- Cross-validation on held-out data



## Information Criteria for Model Selection

### Definition 8 (Akaike and Bayesian Information Criteria)

For a model with log-likelihood  $\mathcal{L}$ ,  $k$  parameters, and  $n$  observations:

$$\text{AIC} = -2\mathcal{L} + 2k$$

$$\text{BIC} = -2\mathcal{L} + k \ln(n)$$

#### AIC

- Asymptotically efficient
- May overfit with small  $n$
- Minimizes prediction error

#### BIC

- Consistent (finds true model)
- Heavier penalty:  $\ln(n) > 2$  if  $n > 7$
- More parsimonious



## VAR: Lag Selection and Estimation

### BIC by Lag Order

Lag	BIC
1	-4.810
2	<b>-5.178</b>
3	-4.633
4	-4.614

### Data Split

Set	Period	N
Training (70%)	2001-Q1 to 2017-Q4	67
Validation (20%)	2018-Q1 to 2022-Q4	20
Test (10%)	2023-Q1 to 2025-Q1	10
<b>Total</b>		<b>97</b>

### Validation Check

VAR(2) also achieves lowest validation RMSE.



## VAR Model Stability

### Stability Condition

- ☐ All eigenvalues of the **companion matrix** must lie inside the unit circle:

$$|\lambda_i| < 1, \quad \forall i$$

### If not satisfied

- ☐ Unstable model
- ☐ Explosive IRFs
- ☐ Unsustainable forecasts

### Practical check

- ☐ `results.is_stable()` in statsmodels
- ☐ Stability verification is **mandatory** before IRF



## VAR vs VECM: Cointegration

### Problem

- If variables are  $I(1)$  → VAR on levels produces spurious regressions
- Johansen test: checks for cointegration relationships

### Definition 9 (VECM — Vector Error Correction Model)

$$\Delta y_t = \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + u_t, \quad \Pi = \alpha \beta'$$

### Key message

- VAR is appropriate only for **stationary** series
- Ignoring cointegration produces **spurious regressions**



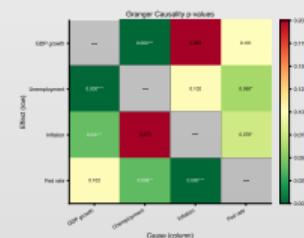
## Granger Causality: Empirical Results

### Interpretation

Each cell shows p-value for testing whether the row variable Granger-causes the column variable. Green:  $p < 0.10$ . Read: row causes column.

### Economic Findings

- Unemp  $\rightarrow$  GDP ( $p = 0.045$ ): Okun's Law
- Fed rate  $\rightarrow$  Inflation ( $p = 0.087$ ): Monetary policy transmission
- GDP  $\rightarrow$  Unemp: Weak evidence



## Granger Causality: Formal Definition

### Definition 10 (Granger Causality)

$X$  Granger-causes  $Y$  if, for some  $h > 0$ :

$$\text{MSE} \left[ \mathbb{E}[Y_{t+h} | \mathcal{F}_t^{X,Y}] \right] < \text{MSE} \left[ \mathbb{E}[Y_{t+h} | \mathcal{F}_t^Y] \right]$$

where  $\mathcal{F}_t^{X,Y}$  includes past values of both  $X$  and  $Y$ , while  $\mathcal{F}_t^Y$  includes only past  $Y$ .

### Important Caveat

Granger causality is **predictive causality**, not true causality. “ $X$  Granger-causes  $Y$ ” means  $X$  contains useful information for forecasting  $Y$ , not that  $X$  causes  $Y$  in a structural sense.

### Test Procedure

Use F-test (or Wald test) to test  $H_0$ : coefficients on lagged  $X$  are jointly zero in the  $Y$  equation.



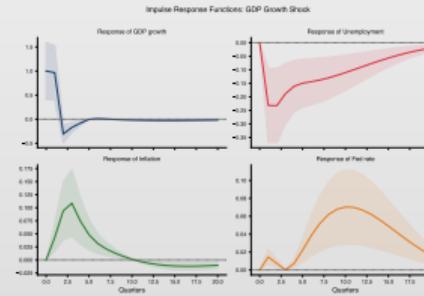
## Impulse Response Functions (IRF)

### What is IRF?

Shows how a 1-unit shock affects others over time.

### GDP Shock Effects

- Unemp** ↓: Okun's Law
- Inflation** ↑: Demand-pull
- Fed Rate** ↑: Taylor Rule



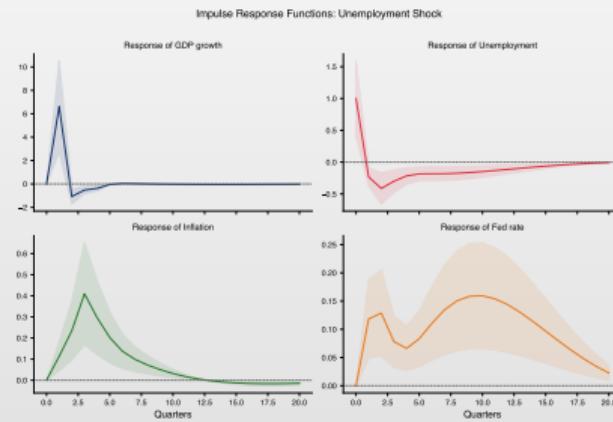
Q TSA\_ch10\_irf\_gdp\_shock



## IRF: Unemployment Shock

### Effects

- ☐  $\uparrow$  Unemp  $\Rightarrow$   $\downarrow$  GDP,  $\downarrow$  Infl, Fed cuts rates



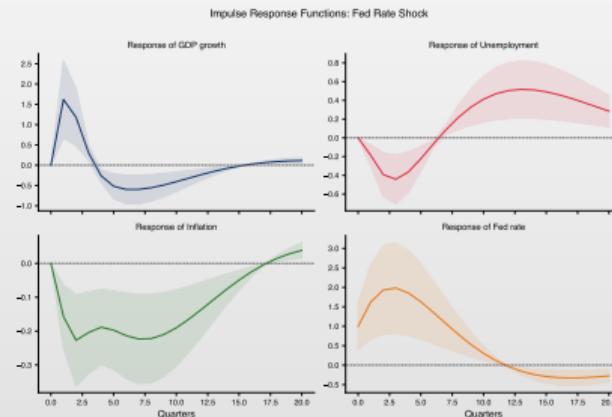
Q TSA\_ch10\_irf\_unemp\_shock



## IRF: Fed Rate Shock

### Monetary Policy

- Rate hike  $\Rightarrow$  GDP  $\downarrow$ , Unemp  $\uparrow$ , Infl  $\downarrow$



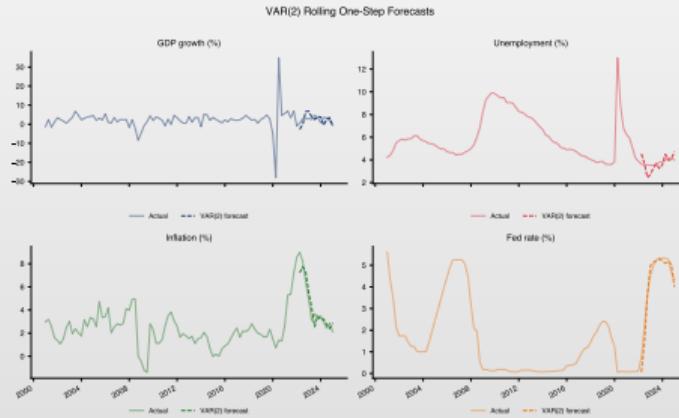
Q TSA\_ch10\_irf\_fed\_shock



## VAR: Forecast (Train/Val/Test)

### Rolling One-Step-Ahead Forecast

- VAR captures GDP-Unemployment dynamics
- COVID shock visible in test period



Q TSA\_ch10\_var\_forecast



## VAR: Test Set Results

### Test Set Performance by Variable

Variable	RMSE	MAE	Dir. Acc.
GDP Growth	1.33	0.99	50%
Unemployment	0.64	0.52	50%
Inflation	1.56	1.12	60%
Fed Rate	2.59	2.45	80%
Average	1.53	1.27	60%

### Strengths

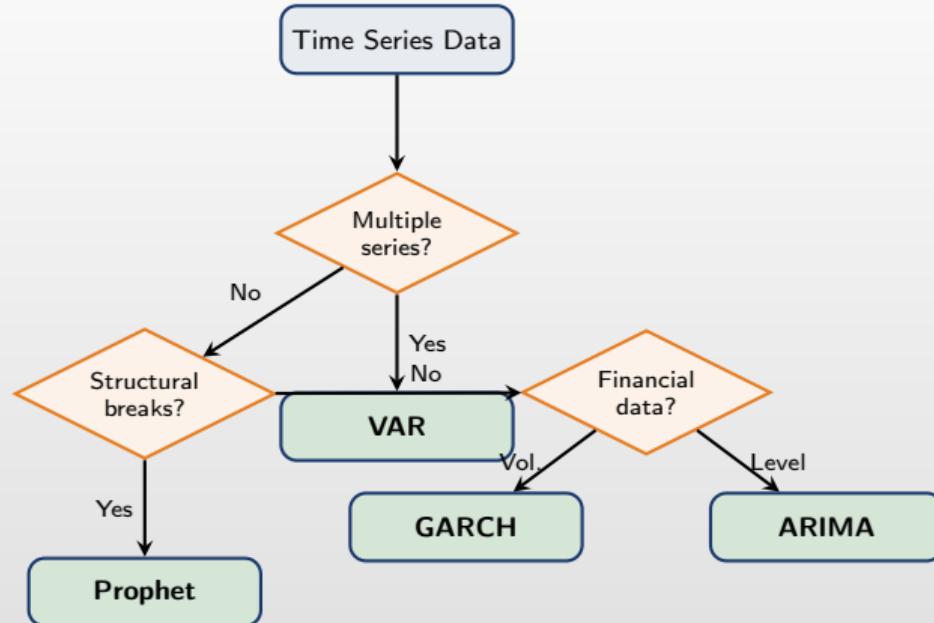
- Cross-variable dynamics
- Good directional accuracy

### Limitations

- Many parameters
- Sensitive to lag selection



## Model Selection Framework



## Summary: Model Comparison

### Results

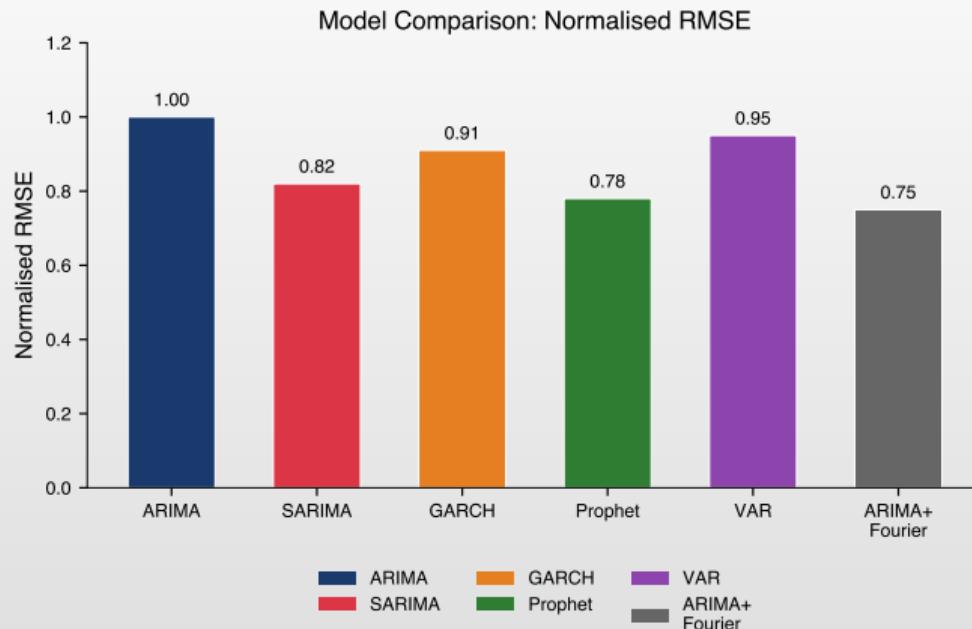
Case	Challenge	Model	RMSE
Bitcoin	Volatility	GARCH	2.15
Sunspots	Seasonality	Fourier	31.10
Unemp	Break	SARIMA	0.12
Economic	Multi-var	VAR	1.53

### Key Principle

**Match model to data characteristics—no single model dominates.**



## Summary: Model Comparison



## Comprehensive Model Comparison

Feature	GARCH	Fourier	Prophet	VAR
<b>Target</b>	Volatility	Level	Level	Multiple
<b>Seasonality</b>	No	Yes (long)	Yes (multi)	No
<b>Structural breaks</b>	No	No	Yes	No
<b>Multiple series</b>	No	No	No	Yes
<b>Interpretable</b>	Medium	High	High	High
<b>Parameters</b>	Few	2K	Auto	Many
<b>Missing data</b>	No	No	Yes	No
<b>Best for</b>	Finance	Cycles	Business	Macro

### Our Results

- GARCH: MAE=1.82 (volatility)
- Fourier: RMSE=31.10 (cycles)
- SARIMA: RMSE=0.12 (breaks)
- VAR: Avg RMSE=1.53 (multi)

### Key Insight

Each model excels in its domain. The art is matching the model to the data characteristics.



## Best Practices for Applied Forecasting

### Methodology

1. **Explore** data
2. **Test** stationarity
3. **Split** train/val/test
4. **Compare** on validation
5. **Report** test metrics

### Common Mistakes

- Peeking at test data
- Over-fitting
- Ignoring assumptions

### Practical Tips

- Start simple (naive)
- Add complexity if needed
- Check residuals
- Report CIs

### Remember

"All models are wrong, but some are useful." — Box



## Forecasting vs Causality vs Decision

Objective	Model	Focus
Pure prediction	ARIMA / ML	Out-of-sample accuracy
Financial risk	GARCH	Volatility, VaR
Macro dynamics	VAR	Multivariate interactions
Structural relations	SVAR / VECM	Causal identification
Regimes	Markov Switching	Regime changes

### Key Message

- There is no universal model
- There is **fit between model and problem**



## Key Takeaways

### 1. Rigorous Methodology

- ▶ Train/validation/test split prevents overfitting
- ▶ Test set must remain untouched until final evaluation

### 2. Match Model to Data

- ▶ Financial volatility → GARCH
- ▶ Long seasonality → Fourier terms
- ▶ Structural breaks → Prophet
- ▶ Multiple series → VAR

### 3. Interpret Results Carefully

- ▶ Granger causality  $\neq$  true causality
- ▶ Out-of-sample performance matters most
- ▶ Simpler models often work better



## The Role of AI in Time Series Modeling

### AI can

- Generate code for estimation and forecasting
- Select models (AutoML, grid search)
- Combine forecasts (ensemble)
- Detect anomalies and patterns

### But cannot

- Replace statistical validation
- Automatically detect **data leakage**
- Guarantee correct economic interpretation
- Verify model assumptions

### Principle

- AI is a **tool**, not an authority
- Statistical validation remains the researcher's responsibility



## AI Exercise: Critical Thinking

### Prompt to test in ChatGPT / Claude / Copilot

"Download monthly US Retail Sales from FRED (series RSXFS) for 2010-01 to 2024-12 (180 observations). Perform a complete time series analysis: decomposition, stationarity tests, model selection (compare ETS, SARIMA, and Prophet), 12-month forecast, and evaluation using RMSE/MAE/MASE on a 70/15/15 temporal split. Give me publication-quality Python code."

### Exercise:

1. Run the prompt in an LLM of your choice and critically analyze the response.
2. Does it follow the correct workflow? (plot → decompose → test → model → diagnose → forecast)
3. Does it compare multiple models (ETS, ARIMA, SARIMA) with proper benchmarks?
4. Is the train/test split done properly? Is there any data leakage?
5. Does it discuss limitations and assumptions of the chosen model?

**Warning:** AI-generated code may run without errors and look professional. *That does not mean it is correct.*



## Question 1

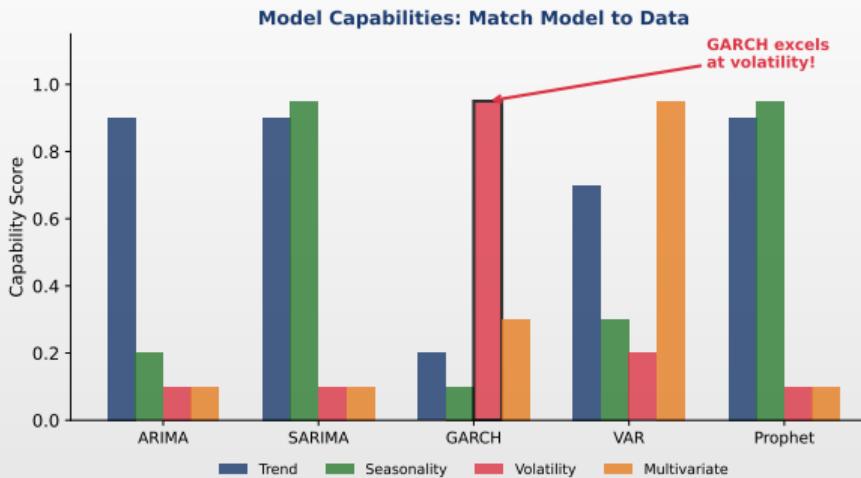
### Question

- Which model would you choose to forecast the volatility of financial returns?

### Answer Choices

- (A)** ARIMA — captures trends and autocorrelations
- (B)** GARCH — models conditional variance
- (C)** Prophet — detects changepoints and seasonality
- (D)** VAR — multivariate model for interdependencies

## Question 1: Answer



Answer: (B)

- GARCH captures volatility clustering and time-varying risk. ARIMA models the level, Prophet handles seasonality, VAR captures cross-series dynamics — none model variance directly.



## Question 2

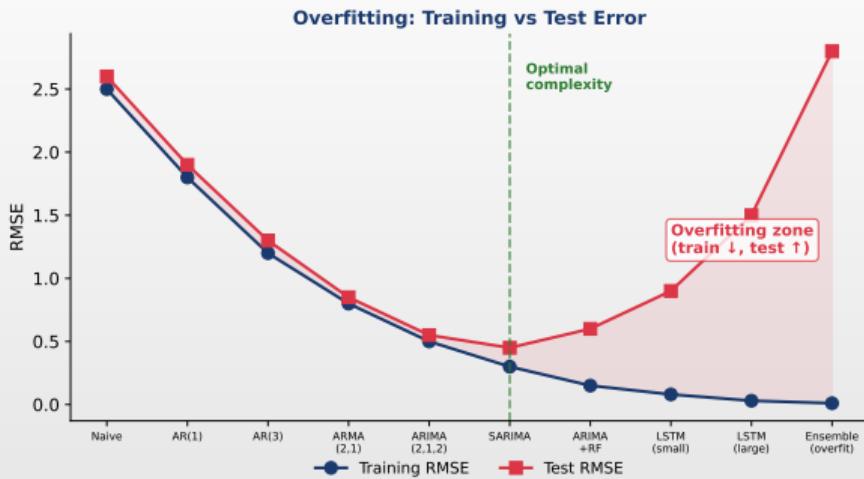
### Question

- A SARIMA model achieves RMSE = 0.05 on training but RMSE = 2.30 on test. What does this indicate?

### Answer Choices

- (A)** The model is excellent — low training error confirms quality
- (B)** The model suffers from overfitting — it memorizes noise
- (C)** The test set is faulty and should be replaced
- (D)** The difference is normal — all models have higher test error

## Question 2: Answer



Answer: (B)

- A  $46\times$  ratio between test and training RMSE signals severe overfitting. The model fits noise in the training data and fails to generalize. Solution: simpler model, proper validation.

Q TSA\_ch10\_quiz2\_overfitting



## Question 3

### Question

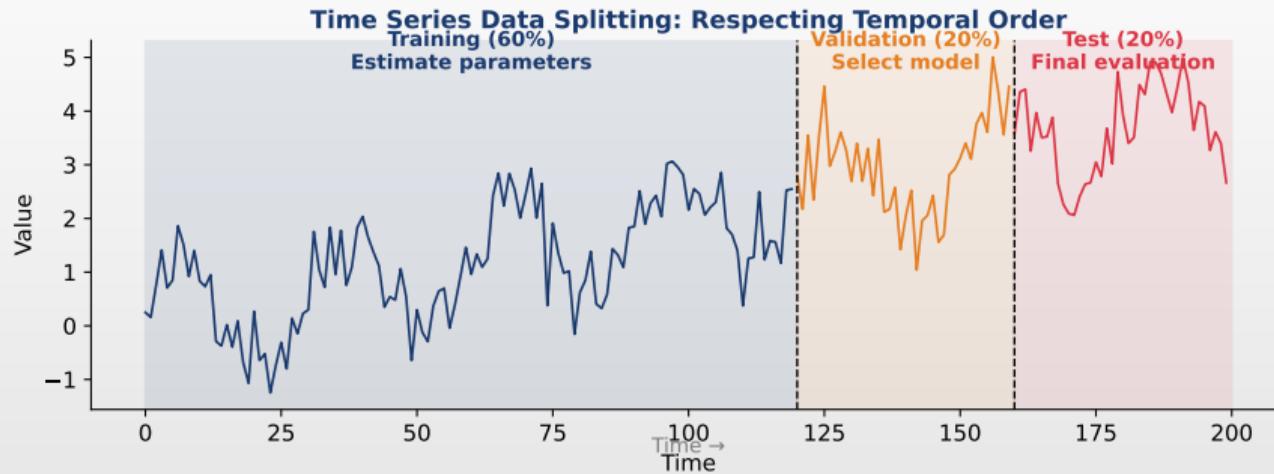
- Why is it important to separate data into train/validation/test sets?

### Answer Choices

- (A)** To have more training data
- (B)** To prevent overfitting and evaluate correctly
- (C)** It is just a convention with no real importance
- (D)** To reduce computation time



## Question 3: Answer



Answer: (B)

- Train: estimate parameters. Validation: select model/hyperparameters. Test: final unbiased evaluation. Mixing these roles leads to optimistic performance estimates.



## Question 4

### Question

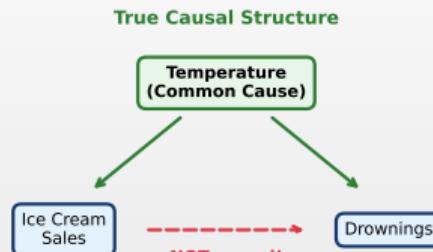
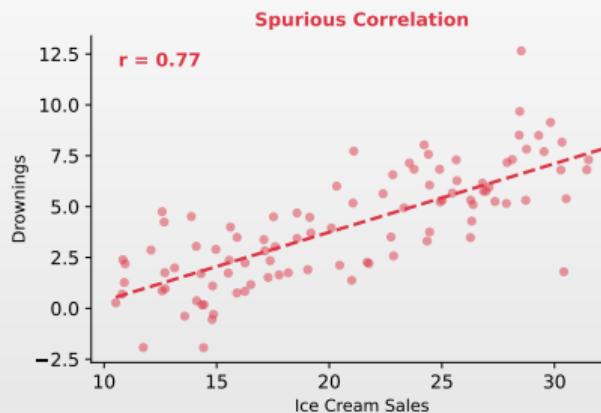
- Is Granger causality equivalent to true (structural) causality?

### Answer Choices

- (A)** Yes — if  $X$  predicts  $Y$ , then  $X$  causes  $Y$
- (B)** No — it only tests predictive content, not causation
- (C)** It depends on the number of lags selected
- (D)** Yes, if the p-value is below 0.05

## Question 4: Answer

Granger Causality ≠ True Causality



Answer: (B)

- Granger causality tests whether past  $X$  improves forecasts of  $Y$ . Spurious correlations (e.g., ice cream sales and drownings) can pass the test due to common causes.



## Question 5

### Question

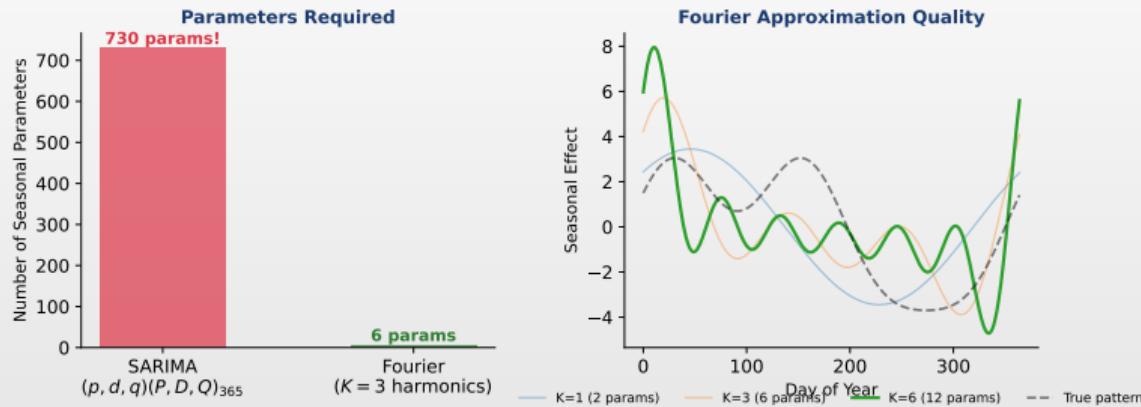
- What model do you use for a series with long seasonality (e.g.,  $s = 365$  days)?

### Answer Choices

- (A)** SARIMA( $p, d, q$ )( $P, D, Q$ )<sub>365</sub>
- (B)** GARCH — models variation
- (C)** ARIMA + Fourier terms or Prophet/TBATS
- (D)** VAR with 365 lags

## Question 5: Answer

Long Seasonality ( $s = 365$ ): Fourier Terms vs SARIMA



Answer: (C)

- SARIMA<sub>365</sub> needs  $\sim 730$  seasonal parameters — infeasible. Fourier terms with  $K = 3$  use only 6 parameters. Prophet and TBATS handle multiple seasonalities automatically.



## Bibliography I

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- Makridakis, S., Spiliotis, E., & Assimakopoulos, V. (2020). The M4 Competition, *International Journal of Forecasting*, 36(1), 54–74.
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## Key Takeaways

### What We Learned

- Model selection depends on data characteristics: stationarity, seasonality, volatility
- The Box-Jenkins methodology provides a systematic framework for time series modeling
- Proper evaluation requires out-of-sample testing and time series cross-validation

### Important

No single model wins everywhere. Match the model to the data: ARIMA for trends, SARIMA for seasonality, GARCH for volatility, VAR/VECM for multivariate dynamics, Prophet/TBATS for complex patterns. Always validate out-of-sample!



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# Thank You!

Questions?

Course materials available at: <https://danpele.github.io/Time-Series-Analysis/>

