Blood Flow in the Human Circulatory System

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November 14, 2025

Report of modern techniques for modeling the motion of blood within a Human's Macrocirculatory System.

Keywords: *computational hemodynamics, 0D blood-flow, 1D blood-flow, 2D-blood-flow, PINN's, finite element methods, discontinuous galerkin, Lax-Wendroff, fluid-structure ineraction (FSI)*

Contents

1 Preliminaries

1.1 Notation

$\mathbb{R}$	set of real numbers
$\mathbb{R}^+$	set of positive real numbers
$\mathbb{R}^-$	set of negative real numbers
$\mathbb{R}^n$	n-dimensional real vector space
$\Omega \subset \mathbb{R}^n$	a connected open subset of $\mathbb{R}^n$
$\overline{\Omega}$	the closure of Ω
$\partial\Omega$	the boundary of Ω
dx	Lebesgue measure on $\mathbb{R}^n$
dS	surface measure on $\partial\Omega$
dV	volume measure on Ω
∇	gradient operator
$\Delta = \nabla^2 = \nabla \cdot \nabla(\cdot)$	Laplace operator
div	divergence of a vector field
div	divergence of a tensor
$\boldsymbol{v}_i$	i-th component of vector $\boldsymbol{v}$
$\langle \cdot, \cdot \rangle_X$	inner product on vector space X
$\langle \boldsymbol{u}, \boldsymbol{v} \rangle$	inner product of vectors $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^n$
$\dfrac{\partial}{\partial \hat{\boldsymbol{n}}} = \langle \nabla, \hat{\boldsymbol{n}} \rangle$	normal derivative on $\partial\Omega$
$\Vert \cdot \Vert$	L^2-norm
$C^k(\Omega)$	space of k times continuously differentiable functions on Ω
$C_0^k(\Omega)$	space of k times continuously differentiable functions with compact support in Ω
$C_0^k(\overline{\Omega})$	space of k times continuously differentiable functions which have bounded and uniformly continuous derivatives up to order k with compact support in Ω
$C_0^\infty(\Omega)$	space of smooth functions with compact support in Ω
$L^p(\Omega)$	Lebesgue space of p-integrable functions on Ω

1.2 Symbols and Abbreviations

$\therefore$	consequently
$\because$	because
$\implies$	implies
$\iff$	if and only if
$:=$	defines
$\equiv$	equivalence
a.e.	almost everywhere
e.g.	”exempli gratia” (for example)
i.e.	”id est” (that means)
s.t.	such that
m.b.s.	m.b.s.
w.r.t.	with respect to
wlog	without loss of generality
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
PDES	System of Partial Differential Equations
IC	Initial Condition
BC	Boundary Condition
0D	Zero dimensional
1D	One dimensional
2D	Two dimensional
3D	Three dimensional
FSI	Fluid-Structure Interaction
WHO	World-Health Organization
SB	Stenotic Blockage
bpm	beats per minute
RBC	Red Blood Cell

1.3 Parameters and Units

ρ	density of blood $\left[\frac{kg}{m^3}\right]$
η	dynamic viscosity $\left[Pa \cdot s\right]$
μ	kinematic viscosity $\left[\frac{m^2}{s}\right]$
τ	shear stress
$\dot{\gamma}$	shear rate
R	radius of vessel with diameter $2R$
$\boldsymbol{u}$	velocity field
p	pressure field
W_0	Womersley number $\left[-\right]$
Re	Reynolds number $\left[-\right]$
Pe	Péclet number $\left[-\right]$
c	concentration of a material element
D	diffusion coefficient $\left[\frac{m^2}{s}\right]$
t	time $\left[s\right]$
T	terminal time $\left[s\right], t > 0$
ω	angular frequency $\left[\frac{rad}{s}\right]$
$\boldsymbol{f_b}$	body force per unit volume $\left[\frac{N}{m^3}\right]$

1.4 Mathematical Foundations

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2 Introduction

Hemodynamics studies the kinematics of blood. Our interest is the kinematic motion of blood within the human macrocirculatory system, i.e. the flow of blood in large vessels such as arteries and veins. Blood is observed as a complex fluid of formed elements suspended in plasma, thus, the rheological behavior of blood is non-trivial. We report techniques and methodologies for modeling blood's motion in large vessels.

Our report is organized as follows.

We start by stating the report's motivation, then in Sec. 2.1 we provide a brief physiological review of the human circulatory system. We follow with Sec. 2.2, which reviews the continuum hypothesis, a necessary postulate in fluid mechanics which treats blood as a continuous medium. This framing reduces the hemodynamic problem to describing blood's motion as a continuum. In Sec. 2.3 we discuss various rheological assumptions and constitutive relations used to model blood as an incompressible fluid. In Sec. 2.4 we derive the incompressible Newtonian Navier–Stokes (NS) equations governing the motion of blood in large vessels.

Motivation Coronary artery stenosis (CAS) is the narrowing of the coronary arteries due to the buildup of plaque. Such narrowing can restrict blood flow to the heart muscle, which may lead to various cardiovascular problems. Current methods for predicting a stenotic blockage (SB) in a coronary artery are rudimentary, and often SB predicition doesn't mean obstruction [[1]]. Additionally, current clinical methods for assessing the severity of a SB rely on imaging techniques such as angiography, intravascular ultrasound (IVUS), and optical coherence tomography (OCT) to visualize the arteries and identify areas of narrowing. Such methods provide valuable information about the anatomy of the arteries, but they do not provide direct information about the functional significance of the CAS. Functional assessment of CAS typically involves measuring the fractional flow reserve (FFR), which is the ratio of the blood pressure downstream of the stenosis to the blood pressure upstream of the stenosis during maximum blood flow. However, measuring FFR requires the use of a pressure wire, which can be invasive and carries some risks. Therefore, there is a need for non-invasive methods to assess the functional significance of CAS.

2.1 Physiology

2.2 Continuum

2.3 Blood Model

2.4 Navier-Stokes

3 Appendix

Bibliography

References

[1] Francois Derimay, Gerard Finet, and Gilles Rioufol. "Coronary Artery Stenosis Prediction Does Not Mean Coronary Artery Stenosis Obstruction". In: *European Heart Journal* (2021). DOI: `10.1093/eurheartj/ehab332`. URL: `https://watermark.silverchair.com/ehab332.pdf`.

Code Listings

Code listings

Code 1: Algorithm 16.5

```
function foo()
    println("Hello World)
end
```