Blood Flow in the Human Circulatory System

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Report of modern techniques for modeling the motion of blood within a Human's Macrocirculatory System.

Keywords: *computational hemodynamics, 0D blood-flow, 1D blood-flow, 2D-blood-flow, PINN's, finite element methods, discontinous galerkin, Lax-Wendroff, fluid-structure ineraction (FSI)*

Contents

1 Preliminaries

Notation

$\mathbb{R}$	set of real numbers
$\mathbb{R}^+$	set of positive real numbers
$\mathbb{R}^-$	set of negative real numbers
$\mathbb{R}^n$	n-dimensional real vector space
$\Omega \subset \mathbb{R}^n$	a connected open subset of $\mathbb{R}^n$
$\overline{\Omega}$	the closure of Ω
$\partial\Omega$	the boundary of Ω
dx	Lebesgue measure on $\mathbb{R}^n$
dS	surface measure on $\partial\Omega$
dV	volume measure on Ω
∇	gradient operator
$\Delta = \nabla^2 = \nabla \cdot \nabla(\cdot)$	Laplace operator
div	divergence of a vector field
div	divergence of a tensor
$\boldsymbol{v}_i$	i-th component of vector $\boldsymbol{v}$
$\langle \cdot, \cdot \rangle_X$	inner product on vector space X
$\langle \boldsymbol{u}, \boldsymbol{v} \rangle$	inner product of vectors $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^n$
$\dfrac{\partial}{\partial \hat{\boldsymbol{n}}} = \langle \nabla, \hat{\boldsymbol{n}} \rangle$	normal derivative on $\partial\Omega$
$\| \cdot \|$	L^2-norm
$C^k(\Omega)$	space of k times continuously differentiable functions on Ω
$C_0^k(\Omega)$	space of k times continuously differentiable functions with compact support in Ω
$C_0^k(\overline{\Omega})$	space of k times continuously differentiable functions which have bounded and uniformly continuous derivatives up to order k with compact support in Ω
$C_0^\infty(\Omega)$	space of smooth functions with compact support in Ω
$L^p(\Omega)$	Lebesgue space of p-integrable functions on Ω

Symbols and Abbreviations

$\therefore$	consequently
$\because$	because
$\Longrightarrow$	implies
$\Longleftrightarrow$	if and only if
$:=$	defines
$\equiv$	equivalence
a.e.	almost everywhere
e.g.	"exempli gratia" (for example)
i.e.	"id est" (that means)
s.t.	such that
m.b.s.	m.b.s.
w.r.t.	with respect to
wlog	without loss of generality
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
PDES	System of Partial Differential Equations
IC	Initial Condition
BC	Boundary Condition
0D	Zero dimensional
1D	One dimensional
2D	Two dimensional
3D	Three dimensional
FSI	Fluid-Structure Interaction
WHO	World-Health Organization
SB	Stenotic Blockage
bpm	beats per minute
RBC	Red Blood Cell

Parameters and Units

ρ	density of blood $\quad \left[\frac{kg}{m^3}\right]$
η	dynamic viscosity $\quad \left[Pa \cdot s\right]$
μ	kinematic viscosity $\quad \left[\frac{m^2}{s}\right]$
τ	shear stress
$\dot{\gamma}$	shear rate
R	radius of vessel with diameter $2R$
$\boldsymbol{u}$	velocity field
p	pressure field
W_0	Womersley number $\left[-\right]$
Re	Reynolds number $\left[-\right]$
Pe	Péclet number $\left[-\right]$
c	concentration of a material element
D	diffusion coefficient $\quad \left[\frac{m^2}{s}\right]$
t	time $\quad \left[s\right]$
T	terminal time $\quad \left[s\right], t > 0$
ω	angular frequency $\quad \left[\frac{rad}{s}\right]$
$\boldsymbol{f_b}$	body force per unit volume $\quad \left[\frac{N}{m^3}\right]$

Mathematical Foundations

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2 Introduction

Hemodynamics studies the kinematics of blood. Our interest is the kinematic motion of blood within the Human macrocirculatory system, i.e. the flow of blood in large vessels such as arteries and veins. Blood is observed as a complex fluid of formed elements suspended in plasma, thus, the rheological behavior of blood is non-trivial. We report techniques for modeling bloods' motion in a Human's macrocirculatory system.

After stating our motivation, we begin with physiological review of a Human's circulatory system in Sec. **??**. Then Sec. **??** states the continuum hypothesis, a necessary posulate of fluid mechanics. We adopt the continuum hypothesis and treat blood as a continuous medium, then our hemodynamics problem simplifies describing the motion of a continous media. In Sec. **??**, we discuss assumptions one may impose on the material-rheological properties of blood. By adopting a particular rheological model, we obtain the necessary constitutive relations for a mathematical description of bloods motion. Treating blood as a simple fluid (single-contitute, homogenous, and isotropic mixture) yeilds a Newtonian rheological model of blood (i.m.b. justified such models are valid in large vessels, e.g. arteries and veins, where the shear rates are sufficiently high [[**fung1997biomechanics**], Ch. 2]). Assuming the density of a material element of blood remains constant as it flows within the vessel, is r.t. the incompressibility condition. Together, these assumptions result in an incompressible-newtonian rheological model of blood. Finally, in Sec. **??** the Navier-Stokes (NS) system of Partial Differential Equations (PDEs) governing the motion of an incompressible-newtonian continuum of blood are derived. The NS equations serve as the foundation for all subsequent modeling techniques reported herein.

Motivation Coronary artery stenosis (CAS) is the narrowing of the coronary arteries due to the buildup of plaque. Such narrowing can restrict blood flow to the heart muscle, which may lead to various cardiovascular problems. Current methods for predicting a stenotic blockage (SB) in a coronary artery are rudimentary, and often SB predicition doesn't mean obstruction [[1]]. Additionally, current clinical methods for assessing the severity of a SB rely on imaging techniques such as angiography, intravascular ultrasound (IVUS), and optical coherence tomography (OCT) to visualize the arteries and identify areas of narrowing. These methods provide valuable information about the anatomy of the arteries, but they do not provide direct information about the functional significance of the CAS. Functional assessment of CAS typically involves measuring the fractional flow reserve (FFR), which is the ratio of the blood pressure downstream of the stenosis to the blood pressure upstream of the stenosis during maximum blood flow. However, measuring FFR requires the use of a pressure wire, which can be invasive and carries some risks. Therefore, there is a need for non-invasive methods to assess the functional significance of CAS.

3 Appendix

Bibliography

References

[1] Francois Derimay, Gerard Finet, and Gilles Rioufol. "Coronary Artery Stenosis Prediction Does Not Mean Coronary Artery Stenosis Obstruction". In: *European Heart Journal* (2021). DOI: 10.1093/eurheartj/ehab332. URL: https://watermark.silverchair.com/ehab332.pdf.

Code Listings

Code listings

Code 1: Algorithm 16.5

```
1        function foo()
2            println("Hello World)
3        end
```