Modeling Blood Flow in Macrocirculatory System

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Notes pertaining to mathematical models of blood flow in the macrocirculatory system.

Contents

1 Preliminaries

Mathematical Notation

$\mathbb{R}$	set of real numbers
$\mathbb{R}^+$	set of positive real numbers
$\mathbb{R}^-$	set of negative real numbers
$\mathbb{R}^n$	n-dimensional real vector space
$\Omega \subset \mathbb{R}^n$	a connected open subset of $\mathbb{R}^n$
$\overline{\Omega}$	the closure of Ω
$\partial\Omega$	the boundary of Ω
dx	Lebesgue measure on $\mathbb{R}^n$
dS	surface measure on $\partial\Omega$
∇	gradient operator
Δ	Laplace operator
div	divergence of a vector field
div	divergence of a tensor
$\boldsymbol{v}_i$	i-th component of vector $\boldsymbol{v}$
$\langle \cdot, \cdot \rangle_X$	inner product on vector space X
$\langle \boldsymbol{u}, \boldsymbol{v} \rangle$	inner product of vectors $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^n$
$\dfrac{\partial}{\partial \hat{\boldsymbol{n}}} = \langle \nabla, \hat{\boldsymbol{n}} \rangle$	normal derivative on $\partial\Omega$
$\Vert \cdot \Vert$	L^2-norm
$C^k(\Omega)$	space of k times continuously differentiable functions on Ω
$C_0^k(\Omega)$	space of k times continuously differentiable functions with compact support in Ω
$C_0^k(\overline{\Omega})$	space of k times continuously differentiable functions which have bounded and uniformly continuous derivatives up to order k with compact support in Ω
$C_0^\infty(\Omega)$	space of smooth functions with compact support in Ω
$L^p(\Omega)$	Lebesgue space of p-integrable functions on Ω

Symbols and Abbreviations

$\therefore$	consequently
$\because$	because
$\implies$	implies
$\iff$	if and only if
$:=$	defines
$\equiv$	equivalent
s.t.	such that
w.r.t.	with respect to
r.t.a.	referred to as
a.e.	almost everywhere
wlog	without loss of generality
i.e.	”id est” (that means)
e.g.	”exempli gratia” (for example)
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
IC	Initial Condition
BC	Boundary Condition
0D	Zero dimensional
1D	One dimensional
2D	Two dimensional
3D	Three dimensional
FSI	Fluid-Structure Interaction
WHO	World-Health Organization
SB	Stenotic Bloodflow
bpm	beats per minute

Parameters and Units

ρ	density of blood $\left[\frac{kg}{m^3}\right]$
η	dynamic viscosity $\left[Pa \cdot s\right]$
μ	kinematic viscosity $\left[\frac{m^2}{s}\right]$
τ	shear stress
$\dot{\gamma}$	shear rate
W_0	Wormsley number $\left[-\right]$
Re	Reynolds number $\left[-\right]$
Pe	Peclet number $\left[-\right]$

2 Physiological Introduction

The circulatory system consists of a human's heart, vascular network, lungs and organs. The heart is the source, transporting Oxygen-rich blood to the organs and deoxygenated (and carbon dioxide enriched) blood back to the lungs; which discharge $CO2$ and enrich blood with Oxygen. We refer to these respective processes as the *pulmonary circulation* and the *systemic circulation* (resp.). The *macrocirculatory system*, consists of the heart and the large vessels in the systemic circulation. Particularly, the arteries of the macrocirculatory system transport oxygenated blood from the heart, driving the return of deoxygenated blood in large vessels back to the heart. Hemodynamics refers to the study of blood flow in the circulatory system.

Cardiovascular disease (CVD) is the leading cause of death in developed nations. According to the World Health Organization (WHO), CVD accounts for approximately 31% of all global deaths, making it a significant public health concern. Understanding the hemodynamics of the macrocirculatory system is crucial for diagnosing, treating, and preventing CVD. We aim to develop mathematical models that accurately describe blood flow in the macrocirculatory system. Our motivation is to use these models to simulate and analyze various cardiovascular conditions, such as arterial stenosis, aneurysms, and heart valve disorders.

A single beat of the heart propels blood through the macrocirculatory system, the "lub-dub" sound and the sequence of successive events until the next heartbeat is referred to as the *cardiac cycle*. The cardiac cycle consists of two main phases: systole and diastole, during which the heart chamber is accumulating blood and releasing blood (resp.). The beat can be recognized as a pulse wave in large vessels, characterized by the Wormsley number W_0:

$$W_0 := D \cdot \sqrt{\frac{\omega\rho}{\eta}},$$

a dimensionless parameter comparing the frequency ω of the pulse wave to the blood viscosity η and the vessel diameter D. The Reynolds number Re characterizes flow in blood vessels:

$$Re := \frac{\rho U D}{\eta},$$

where U is the mean flow velocity. Low Re indicates laminar flow, while high Re suggests turbulent flow. Note [1, Table 1.1, p. 10, §1.1] shows $W_0 \propto D$ and $Re \propto D^{-1}$; we observe large pulses and turbulent flow in large vessels and small pulses and laminar flow in small vessels.

Assume a normal resting heart rate of $\omega = 70$ bpm, so the duration of a cardiac cycle is approximately $0.86s.$:

- Systole duration $\approx 0.15 - 0.3$ seconds. Heart contracts, propelling blood into the arteries. The "lub-dub" sound derives from the start and end of systole, where heart valves open and close.
- Diastole duration ≈ 0.7 seconds. Heart relaxes, allowing the chambers to fill with blood.

The blood volume of human is approximately 5.7-6.0 liters of blood, flowing a full cycle approximately every 70 beats. The energy driving the flow comes from oxygen and nutrients absorbed from food, creating waste products that must be removed; the *coronary artery*'s responsibility. The buildup of waste products results in Arteriosclerosis, a narrowing of the coronary artery, leading to reduced and turbulent blood flow. (Add citations here of turbulence in the presence of stenotic arteries)

3 Navier Stokes System

Our concern is the flow of blood in the macrocirculatory system. We develop mathematical models describing the hemodynamics of arterial and large venous segments, then we extend our models to macrocirculation networks.

In *continuum mechanics*, matter is modeled by *continuous fields*. For instance, to model the behavior of an *incompressible* fluid in time, one could use a **pressure field** and a **velocity field**. At *atomic scales*, this model breaks down—fluids as fluids are discrete collections of molecules, not continuous fields. However, at the macroscopic scale, one can hope that such models remain accurate.

$$\text{TODO: Add system} \tag{1}$$

3.0.1 NS-Sytem Derivation

3.1

4 Coronary Artery Stenosis (CAS)

Coronary artery stenosis (CAS) is the narrowing of the coronary arteries due to the buildup of plaque. This narrowing can restrict blood flow to the heart muscle, leading to various cardiovascular problems, including chest pain (angina), heart attacks, and other serious complications. Current methods for predicting coronary artery stenosis are rudimentary; and often prediction does mean coronary artery stenosis obstruction ([2]). . There is a need for more accurate and reliable methods to predict and assess the severity of CAS. narrowing can restrict blood flow to the heart muscle, leading to various cardiovascular problems, including chest pain (angina), heart attacks, and other serious complications. Current methods for predicting coronary artery stenosis are rudimentary; and often prediction does mean coronary artery stenosis obstruction. . There is a need for more accurate and reliable methods to predict and assess the severity of CAS.

We perform a literature survey of arterial blood flow using known methods from the literature, with the hope of understanding the computational challenges and tradeoffs of various *mathematical models*.

Make some comment about "correct terminology for describing the types of coronary arterial stenosis is "coronary artery stenosis morphology." and ref figures in DOI: 10.1056

Understand Dr. Zhou's statement: Regarding the assumption about the absence of a vortex, I cannot defini-

5 Literature Review

6 Appendix

References

[1] Luca Formaggia, Alfio Quarteroni, and Alessandro Veneziani. *Cardiovascular Mathematics. Modeling and Simulation of the Circulatory System*. Springer, 2009.

[2] Add authors here. "Add exact title here". In: *European Heart Journal* (2021). DOI: `10.1093/eurheartj/ehab332`. URL: `https://watermark.silverchair.com/ehab332.pdf`.

References

Code Listings

Optional Space for sumplementary code listings of computations done while investigating

Code 1: Algorithm 16.5

```
function foo()
    println("Hello World)
end
```