

How to Implement a Sudoku Solver

Seri
A High-Level User Language for SMT Queries

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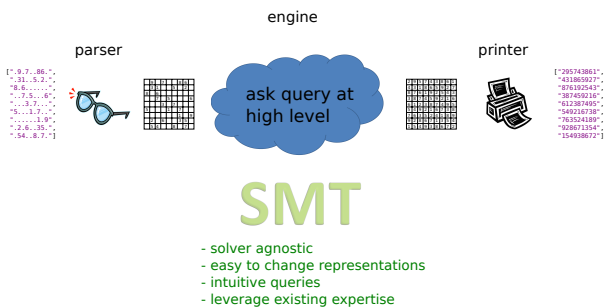
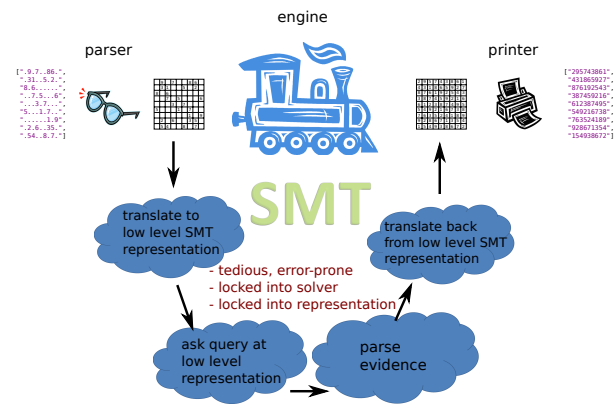
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```
diabolical :: [[Char]]
diabolical =
  [".9.7..86.",
   ".31..5.2.",
   "8.6.....",
   ".7.5...6",
   "...3.7...",
   "5...1.7..",
   ".....1.9",
   ".2.6..35.",
   ".54..8.7."]
```

	9		7			8	6	
	3	1			5		2	
8		6						
		7		5				6
			3		7			
5				1		7		
						1		9
	2		6			3	5	
	5	4			8		7	

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The Parser in Seri

```
data Cell = C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9
data Board = Board [[Cell]]

readCell :: Char -> Query Cell
readCell '1' = return C1
readCell '2' = return C2
...
readCell '9' = return C9
readCell '.' = free
readCell c = error ("readCell:" ++ [c])

readRow :: [Char] -> Query [Cell]
readRow = mapM readCell

readBoard :: [[Char]] -> Query Board
readBoard rows = do
  brows <- mapM readRow rows
  return (Board brows)
```

algebraic data types

pattern matching

introduce a free variable

The Sudoku Constraints

```
instance Eq Cell where
  (==) C1 C1 = True
  (==) C2 C2 = True
  ...
  (==) C9 C9 = True
  (==) _ _ = False

unique :: (Eq a) => [a] -> Bool
unique [] = True
unique (x:xs) = notElem x xs && unique xs

rows :: Board -> [[Cell]]
rows (Board x) = x

cols :: Board -> [[Cell]]
cols (Board x) = transpose x

boxes :: Board -> [[Cell]]
boxes = ...

isvalid :: Board -> Bool
isvalid b = all unique (concat [rows b, cols b, boxes b])
```

ad-hoc polymorphism (overloading)

recursive functions

higher order functions

The Sudoku Solver

```
solve :: [[Char]] -> Query [[Char]]
solve input = do
  board <- readBoard input
  assert (isvalid board)
  result <- query board
  case result of
    Unsatisfiable -> return ["no solution"]
    Satisfiable v -> return (print v)

main :: Query [[Char]]
main = solve diabolical
```

- Yices1: 1m15s
- Yices2: 1.6s

No change in source code going from Yices1 to Yices2!

A Different Cell Representation

```
data Cell = Cell (Bit #9)

instance Eq Cell where
  (==) (Cell a) (Cell b) = (a == b)

freeCell :: Query Cell
freeCell = do
  x <- free
  assert (isValidCell x)
  return x

join :: [Bit #n] -> Bit #n
join [] = 0
join (x:xs) = bv_or x (join xs)

value :: Cell -> Bit #9
value (Cell x) = x

unique :: [Cell] -> Bool
unique cells = join (map value cells) == 0x1ff
```

- Yices1: 3m53s (was 1m15s)
- Yices2: 1.0s (was 1.6s)

Interactive and Reusable Queries

```
allQ :: (Eq a) => (a -> Bool) -> Query [a]
allQ p = do
  x <- free
  assert (p x)
  r <- query x
  case r of
    Unsatisfiable -> return []
    Satisfiable v -> do
      vs <- allQ (\a -> (p a) && (a /= v))
      return (v:vs)

predicate :: Integer -> Bool
predicate x = (x > 3) && (x < 6)

main :: Query [Integer]
main = allQ predicate
```

Current Status, Future Plans

Current Status

- Yices1, Yices2 supported
- All queries shown work

Future Work (Ph.D. Thesis)

- Optimize generated queries
- Add support for more solvers
- Integrate tool more seamlessly with Haskell
- Explore implementation of formal tools, such as model checkers, built in Seri with reusable library components