MINMEAN - Results

August 21, 2016

1 Tables of Friedman, Bonferroni-Dunn, Holm, Hochberg and Hommel Tests

Table 1: Average Rankings of the algorithms

| ${ m Algorithm}$ | Ranking |
|------------------|---------------------|
| IRS | 6.6 |
| EUCLIDEAN | 3.80000000000000003 |
| CHEBYSHEV | 6.0 |
| KULLBACKLEIBLER | 3.0 |
| HELLINGER | 1.4 |
| TOTALVARIATION | 1.799999999999998 |
| CHISQUARE | 5.4 |

Friedman statistic considering reduction performance (distributed according to chi-square with 6 degrees of freedom: 27.171428571428553. P-value computed by Friedman Test: 1.3448112365843645E-4.

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Iman and Davenport statistic considering reduction performance (distributed according to F-distribution with 6 and 24 degrees of freedom: 38.424242424215.

P-value computed by Iman and Daveport Test: 3.743913634508992E-11.

Table 2: Holm / Hochberg Table for $\alpha = 0.05$

| i | algorithm | $z = (R_0 - R_i)/SE$ | p | Holm/Hochberg/Hommel |
|---|-----------------|----------------------|-----------------------|----------------------|
| 6 | IRS | 3.8060102844992794 | 1.4122651246579272E-4 | 0.008333333333333333 |
| 5 | CHEBYSHEV | 3.3668552516724395 | 7.603058428726023E-4 | 0.01 |
| 4 | CHISQUARE | 2.9277002188455996 | 0.003414791178117856 | 0.0125 |
| 3 | EUCLIDEAN | 1.7566201313073602 | 0.0789825792637829 | 0.01666666666666666 |
| 2 | KULLBACKLEIBLER | 1.1710800875382399 | 0.24156658696897293 | 0.025 |
| 1 | TOTALVARIATION | 0.2927700218845599 | 0.7696979437812898 | 0.05 |

Hochberg's procedure rejects those hypotheses that have a p-value ≤ 0.0125 .

Table 3: Holm / Hochberg Table for $\alpha = 0.10$

| | | - ' / ' - ' | 3 - 3 | |
|---|-----------------|----------------------|-----------------------|----------------------|
| i | algorithm | $z = (R_0 - R_i)/SE$ | p | Holm/Hochberg/Hommel |
| 6 | IRS | 3.8060102844992794 | 1.4122651246579272E-4 | 0.0166666666666666 |
| 5 | CHEBYSHEV | 3.3668552516724395 | 7.603058428726023E-4 | 0.02 |
| 4 | CHISQUARE | 2.9277002188455996 | 0.003414791178117856 | 0.025 |
| 3 | EUCLIDEAN | 1.7566201313073602 | 0.0789825792637829 | 0.0333333333333333 |
| 2 | KULLBACKLEIBLER | 1.1710800875382399 | 0.24156658696897293 | 0.05 |
| 1 | TOTALVARIATION | 0.2927700218845599 | 0.7696979437812898 | 0.1 |

Hochberg's procedure rejects those hypotheses that have a p-value < 0.025.

Nemenyi's procedure rejects those hypotheses that have a p-value $\leq 0.002380952380952381$.

Holm's procedure rejects those hypotheses that have a p-value $\leq 0.0029411764705882353$.

Table 4: Adjusted p-values

| i | algorithm | unadjusted p | p_{Bonf} | p_{Holm} | p_{Hoch} | p_{Homm} |
|---|-----------------|-----------------------|----------------------|-----------------------|-----------------------|------------------|
| 1 | IRS | 1.4122651246579272E-4 | 8.473590747947563E-4 | 8.473590747947563E-4 | 8.473590747947563E-4 | 8.4735907479475 |
| 2 | CHEBYSHEV | 7.603058428726023E-4 | 0.004561835057235614 | 0.0038015292143630114 | 0.0038015292143630114 | 0.00380152921436 |
| 3 | CHISQUARE | 0.003414791178117856 | 0.020488747068707135 | 0.013659164712471424 | 0.013659164712471424 | 0.0136591647124 |
| 4 | EUCLIDEAN | 0.0789825792637829 | 0.4738954755826974 | 0.2369477377913487 | 0.2369477377913487 | 0.236947737791 |
| 5 | KULLBACKLEIBLER | 0.24156658696897293 | 1.4493995218138376 | 0.48313317393794586 | 0.48313317393794586 | 0.4831331739379 |
| 6 | TOTALVARIATION | 0.7696979437812898 | 4.618187662687738 | 0.7696979437812898 | 0.7696979437812898 | 0.769697943781 |

Table 5: Holm / Shaffer Table for $\alpha=0.05$

| i | algorithms | $z = (R_0 - R_i)/SE$ | p | Holm | Shaffer |
|----|------------------------------------|----------------------|-----------------------|-----------------------|--------------------|
| 21 | IRS vs. HELLINGER | 3.8060102844992794 | 1.4122651246579272E-4 | 0.002380952380952381 | 0.002380952380952 |
| 20 | IRS vs. TOTALVARIATION | 3.5132402626147194 | 4.4267698634329765E-4 | 0.0025 | 0.0033333333333333 |
| 19 | CHEBYSHEV vs. HELLINGER | 3.3668552516724395 | 7.603058428726023E-4 | 0.002631578947368421 | 0.0033333333333333 |
| 18 | CHEBYSHEV vs. TOTALVARIATION | 3.07408522978788 | 0.0021114910066706385 | 0.00277777777777778 | 0.0033333333333333 |
| 17 | HELLINGER vs. CHISQUARE | 2.9277002188455996 | 0.003414791178117856 | 0.0029411764705882353 | 0.0033333333333333 |
| 16 | TOTALVARIATION vs. CHISQUARE | 2.63493019696104 | 0.008415458738002115 | 0.003125 | 0.0033333333333333 |
| 15 | IRS vs. KULLBACKLEIBLER | 2.6349301969610397 | 0.008415458738002124 | 0.003333333333333333 | 0.0033333333333333 |
| 14 | CHEBYSHEV vs. KULLBACKLEIBLER | 2.1957751641342 | 0.028108040147151837 | 0.0035714285714285718 | 0.003571428571428 |
| 13 | IRS vs. EUCLIDEAN | 2.0493901531919194 | 0.04042397933690863 | 0.0038461538461538464 | 0.003846153846153 |
| 12 | EUCLIDEAN vs. HELLINGER | 1.7566201313073602 | 0.0789825792637829 | 0.0041666666666666667 | 0.00416666666666 |
| 11 | KULLBACKLEIBLER vs. CHISQUARE | 1.7566201313073602 | 0.0789825792637829 | 0.004545454545454546 | 0.004545454545454 |
| 10 | EUCLIDEAN vs. CHEBYSHEV | 1.6102351203650798 | 0.10734653699381101 | 0.005 | 0.005 |
| 9 | EUCLIDEAN vs. TOTALVARIATION | 1.4638501094228002 | 0.14323490752466958 | 0.00555555555555556 | 0.00555555555555 |
| 8 | EUCLIDEAN vs. CHISQUARE | 1.1710800875382399 | 0.24156658696897293 | 0.00625 | 0.00625 |
| 7 | KULLBACKLEIBLER vs. HELLINGER | 1.1710800875382399 | 0.24156658696897293 | 0.0071428571428571435 | 0.007142857142857 |
| 6 | KULLBACKLEIBLER vs. TOTALVARIATION | 0.8783100656536801 | 0.379775474840949 | 0.008333333333333333 | 0.008333333333333 |
| 5 | IRS vs. CHISQUARE | 0.8783100656536794 | 0.3797754748409493 | 0.01 | 0.01 |
| 4 | EUCLIDEAN vs. KULLBACKLEIBLER | 0.5855400437691202 | 0.5581846494226572 | 0.0125 | 0.0125 |
| 3 | IRS vs. CHEBYSHEV | 0.4391550328268397 | 0.660549205201673 | 0.01666666666666666 | 0.01666666666666 |
| 2 | CHEBYSHEV vs. CHISQUARE | 0.4391550328268397 | 0.660549205201673 | 0.025 | 0.025 |
| 1 | HELLINGER vs. TOTALVARIATION | 0.2927700218845599 | 0.7696979437812898 | 0.05 | 0.05 |
| | | | | | |

Shaffer's procedure rejects those hypotheses that have a p-value $\leq 0.002380952380952381$. Bergmann's procedure rejects these hypotheses:

- IRS vs. HELLINGER
- IRS vs. TOTALVARIATION
- CHEBYSHEV vs. HELLINGER
- CHEBYSHEV vs. TOTALVARIATION
- HELLINGER vs. CHISQUARE

Table 6: Holm / Shaffer Table for $\alpha = 0.10$

| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | / | | | |
|---|----|------------------------------------|----------------------|-----------------------|--|--|
| RS vs. TOTALVARIATION 3.5132402626147194 4.4267698634329765E-4 0.005 0.0066666666666667 0.006666666666667 0.006666666666667 0.006666666666667 0.00666666666666666666666666666666666 | i | | $z = (R_0 - R_i)/SE$ | p | Holm | Shaffer |
| CHEBYSHEV vs. HELLINGER 3.366852516724395 7.603058428726023E-4 0.005263157894736842 0.00666666666666666666666666666666666 | 21 | IRS vs. HELLINGER | 3.8060102844992794 | 1.4122651246579272E-4 | 0.004761904761904762 | 0.004761904761904762 |
| 18 | 20 | IRS vs. TOTALVARIATION | 3.5132402626147194 | 4.4267698634329765E-4 | 0.005 | 0.00666666666666667 |
| HELLINGER vs. CHISQUARE | 19 | CHEBYSHEV vs. HELLINGER | 3.3668552516724395 | 7.603058428726023E-4 | 0.005263157894736842 | 0.00666666666666667 |
| 16 TOTALVARIATION vs. CHISQUARE 2.63493019696104 0.008415458738002115 0.00625 0.006666666666666666666666666666666666 | 18 | CHEBYSHEV vs. TOTALVARIATION | 3.07408522978788 | 0.0021114910066706385 | 0.00555555555555556 | 0.00666666666666667 |
| 15 IRS vs. KULLBACKLEIBLER 2.6349301969610397 0.008415458738002124 0.006666666666666666666666666666666666 | 17 | HELLINGER vs. CHISQUARE | 2.9277002188455996 | 0.003414791178117856 | 0.0058823529411764705 | 0.00666666666666667 |
| 14 CHEBYSHEV vs. KULLBACKLEIBLER 2.1957751641342 0.028108040147151837 0.0071428571428571435 0.0071428571428571435 13 IRS vs. EUCLIDEAN vs. HELLINGER 2.0493901531919194 0.04042397933690863 0.007692307692307693 0.007692307692307693 12 EUCLIDEAN vs. HELLINGER 1.7566201313073602 0.0789825792637829 0.0083333333333333333333333333333333333 | 16 | TOTALVARIATION vs. CHISQUARE | 2.63493019696104 | 0.008415458738002115 | 0.00625 | 0.00666666666666667 |
| 13 IRS vs. EUCLIDEAN 2.0493901531919194 0.04042397933690863 0.0076923076923076993 0.007692307692307693 12 EUCLIDEAN vs. HELLINGER 1.7566201313073602 0.0789825792637829 0.003333333333333333333333333333333333 | 15 | IRS vs. KULLBACKLEIBLER | 2.6349301969610397 | 0.008415458738002124 | 0.00666666666666667 | 0.00666666666666667 |
| 12 EUCLIDEAN vs. HELLINGER 1.7566201313073602 0.0789825792637829 0.0083333333333333333333333333333333333 | 14 | CHEBYSHEV vs. KULLBACKLEIBLER | 2.1957751641342 | 0.028108040147151837 | 0.0071428571428571435 | 0.0071428571428571435 |
| 11 KULLBACKLEIBLER vs. CHISQUARE 1.7566201313073602 0.0789825792637829 0.0090909090909090909090909090909090909 | 13 | IRS vs. EUCLIDEAN | 2.0493901531919194 | 0.04042397933690863 | 0.007692307692307693 | 0.007692307692307693 |
| 10 EUCLIDEAN vs. CHEBYSHEV 1.6102351203650798 0.10734653699381101 0.01 0.01 0.01 9 EUCLIDEAN vs. TOTALVARIATION 1.4638501094228002 0.14323490752466958 0.01111111111111111 0.0111111111111111 8 EUCLIDEAN vs. CHISQUARE 1.1710800875382399 0.24156658696897293 0.0125 0.0125 7 KULLBACKLEIBLER vs. HELLINGER 1.1710800875382399 0.24156658696897293 0.014285714285714287 0.014285714285714287 6 KULLBACKLEIBLER vs. TOTALVARIATION 0.8783100656536801 0.379775474840949 0.016666666666666 0.0166666666666666666666666666666666666 | 12 | EUCLIDEAN vs. HELLINGER | 1.7566201313073602 | 0.0789825792637829 | 0.008333333333333333 | 0.008333333333333333 |
| 9 EUCLIDEAN vs. TOTALVARIATION 1.4638501094228002 0.14323490752466958 0.01111111111111111 0.0111111111111111 8 EUCLIDEAN vs. CHISQUARE 1.1710800875382399 0.24156658696897293 0.0125 0.0125 7 KULLBACKLEIBLER vs. HELLINGER 1.1710800875382399 0.24156658696897293 0.0142857142857142877 6 KULLBACKLEIBLER vs. TOTALVARIATION 0.8783100656538691 0.379775474840949 0.01666666666666666 0.01666666666666666 5 IRS vs. CHISQUARE 0.878310065538794 0.3797754748409499 0.025 0.025 4 EUCLIDEAN vs. KULLBACKLEIBLER 0.5855400437691202 0.5581846494226572 0.025 0.025 3 IRS vs. CHEBYSHEV 0.4391550328268397 0.660549205201673 0.033333333333333 0.03333333333333 2 CHEBYSHEV vs. CHISQUARE 0.4391550328268397 0.660549205201673 0.05 | 11 | KULLBACKLEIBLER vs. CHISQUARE | 1.7566201313073602 | 0.0789825792637829 | 0.009090909090909092 | 0.009090909090909092 |
| 8 EUCLIDEAN vs. CHISQUARE 1.1710800875382399 0.24156658696897293 0.0125 0.0125 7 KULLBACKLEIBLER vs. HELLINGER 1.1710800875382399 0.24156658696897293 0.0142857142877 0.0142857142877 6 KULLBACKLEIBLER vs. TOTALVARIATION 0.8783100656536801 0.379775474840949 0.016666666666666666 0.0166666666666666 5 IRS vs. CHISQUARE 0.8783100656536794 0.3797754748409493 0.02 0.02 4 EUCLIDEAN vs. KULLBACKLEIBLER 0.5855400437691202 0.5581846494226572 0.025 0.025 3 IRS vs. CHEBYSHEV 0.4391550328268397 0.660549205201673 0.033333333333333 0.03333333333333 2 CHEBYSHEV vs. CHISQUARE 0.4391550328268397 0.660549205201673 0.05 0.05 | 10 | EUCLIDEAN vs. CHEBYSHEV | 1.6102351203650798 | 0.10734653699381101 | 0.01 | 0.01 |
| 7 KULLBACKLEIBLER vs. HELLINGER 1.1710800875382399 0.24156658696897293 0.014285714285714287 0.014285714285714287 6 KULLBACKLEIBLER vs. TOTALVARIATION 0.8783100656536801 0.379775474840949 0.01666666666666666 0.0166666666666666 5 IRS vs. CHISQUARE 0.8783100656536794 0.3797754748409493 0.02 0.02 4 EUCLIDEAN vs. KULLBACKLEIBLER 0.5855400437691202 0.5581846494226572 0.025 0.025 3 IRS vs. CHEBYSHEV 0.4391550328268397 0.660549205201673 0.03333333333333 0.03333333333333 2 CHEBYSHEV vs. CHISQUARE 0.4391550328268397 0.660549205201673 0.05 0.05 | 9 | EUCLIDEAN vs. TOTALVARIATION | 1.4638501094228002 | 0.14323490752466958 | 0.011111111111111111111111111111111111 | 0.011111111111111111111111111111111111 |
| 6 KULLBACKLEIBLER vs. TOTALVARIATION 0.8783100656536801 0.379775474840949 0.01666666666666666 0.01666666666666666 | 8 | EUCLIDEAN vs. CHISQUARE | 1.1710800875382399 | 0.24156658696897293 | 0.0125 | 0.0125 |
| 5 IRS vs. CHISQUARE 0.8783100656536794 0.3797754748409493 0.02 0.02 4 EUCLIDEAN vs. KULLBACKLEIBLER 0.5885400437691202 0.58851846494226572 0.025 0.025 3 IRS vs. CHEBYSHEV 0.4391550328268397 0.660549205201673 0.033333333333333 0.033333333333333 2 CHEBYSHEV vs. CHISQUARE 0.4391550328268397 0.660549205201673 0.05 0.05 | 7 | KULLBACKLEIBLER vs. HELLINGER | 1.1710800875382399 | 0.24156658696897293 | 0.014285714285714287 | 0.014285714285714287 |
| 4 EUCLIDEAN vs. KULLBACKLEIBLER 0.5855400437691202 0.5581846494226572 0.025 0.025 3 IRS vs. CHEBYSHEV 0.4391550328268397 0.660549205201673 0.03333333333333 0.0333333333333 2 CHEBYSHEV vs. CHISQUARE 0.4391550328268397 0.660549205201673 0.05 | 6 | KULLBACKLEIBLER vs. TOTALVARIATION | 0.8783100656536801 | 0.379775474840949 | 0.01666666666666666 | 0.01666666666666666 |
| 3 IRS vs. CHEBYSHEV 0.4391550328268397 0.660549205201673 0.03333333333333333333333333333333333 | 5 | IRS vs. CHISQUARE | 0.8783100656536794 | 0.3797754748409493 | 0.02 | 0.02 |
| 2 CHEBYSHEV vs. CHISQUARE 0.4391550328268397 0.660549205201673 0.05 0.05 | 4 | EUCLIDEAN vs. KULLBACKLEIBLER | 0.5855400437691202 | 0.5581846494226572 | 0.025 | 0.025 |
| | 3 | | 0.4391550328268397 | 0.660549205201673 | 0.03333333333333333 | 0.03333333333333333 |
| 1 HELLINGER vs. TOTALVARIATION 0.2927700218845599 0.7696979437812898 0.1 0.1 | 2 | CHEBYSHEV vs. CHISQUARE | 0.4391550328268397 | 0.660549205201673 | 0.05 | 0.05 |
| | 1 | HELLINGER vs. TOTALVARIATION | 0.2927700218845599 | 0.7696979437812898 | 0.1 | 0.1 |

Nemenyi's procedure rejects those hypotheses that have a p-value $\leq 0.004761904761904762$. Holm's procedure rejects those hypotheses that have a p-value ≤ 0.00625 . Shaffer's procedure rejects those hypotheses that have a p-value $\leq 0.004761904761904762$. Bergmann's procedure rejects these hypotheses:

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- \bullet IRS vs. KULLBACKLEIBLER
- IRS vs. HELLINGER
- IRS vs. TOTALVARIATION
- CHEBYSHEV vs. HELLINGER
- CHEBYSHEV vs. TOTALVARIATION
- HELLINGER vs. CHISQUARE
- TOTALVARIATION vs. CHISQUARE

Table 7: Adjusted *p*-values

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|----|------------------------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| i | hypothesis | unadjusted p | p_{Neme} | p_{Holm} | p_{Shaf} | p_{Berg} |
| 1 | IRS vs .HELLINGER | 1.4122651246579272E-4 | 0.0029657567617816473 | 0.0029657567617816473 | 0.0029657567617816473 | 0.0029657567617816473 |
| 2 | IRS vs .TOTALVARIATION | 4.4267698634329765E-4 | 0.00929621671320925 | 0.008853539726865953 | 0.006640154795149465 | 0.006640154795149465 |
| 3 | CHEBYSHEV vs .HELLINGER | 7.603058428726023E-4 | 0.01596642270032465 | 0.014445811014579443 | 0.011404587643089034 | 0.011404587643089034 |
| 4 | CHEBYSHEV vs .TOTALVARIATION | 0.0021114910066706385 | 0.04434131114008341 | 0.038006838120071496 | 0.03167236510005958 | 0.021114910066706384 |
| 5 | HELLINGER vs .CHISQUARE | 0.003414791178117856 | 0.07171061474047498 | 0.05805145002800355 | 0.05122186767176784 | 0.037562702959296417 |
| 6 | TOTALVARIATION vs .CHISQUARE | 0.008415458738002115 | 0.1767246334980444 | 0.13464733980803384 | 0.12623188107003172 | 0.0589082111660148 |
| 7 | IRS vs .KULLBACKLEIBLER | 0.008415458738002124 | 0.1767246334980446 | 0.13464733980803384 | 0.12623188107003186 | 0.09257004611802336 |
| 8 | CHEBYSHEV vs .KULLBACKLEIBLER | 0.028108040147151837 | 0.5902688430901886 | 0.3935125620601257 | 0.3091884416186702 | 0.19675628103006285 |
| 9 | IRS vs .EUCLIDEAN | 0.04042397933690863 | 0.8489035660750812 | 0.5255117313798122 | 0.44466377270599494 | 0.36381581403217766 |
| 10 | EUCLIDEAN vs .HELLINGER | 0.0789825792637829 | 1.658634164539441 | 0.9477909511653948 | 0.8688083719016119 | 0.7108432133740461 |
| 11 | KULLBACKLEIBLER vs .CHISQUARE | 0.0789825792637829 | 1.658634164539441 | 0.9477909511653948 | 0.8688083719016119 | 0.7108432133740461 |
| 12 | EUCLIDEAN vs .CHEBYSHEV | 0.10734653699381101 | 2.2542772768700314 | 1.07346536993811 | 1.07346536993811 | 0.7108432133740461 |
| 13 | EUCLIDEAN vs .TOTALVARIATION | 0.14323490752466958 | 3.007933058018061 | 1.2891141677220261 | 1.2891141677220261 | 0.8594094451480174 |
| 14 | EUCLIDEAN vs .CHISQUARE | 0.24156658696897293 | 5.072898326348431 | 1.9325326957517834 | 1.6909661087828105 | 1.2078329348448646 |
| 15 | KULLBACKLEIBLER vs .HELLINGER | 0.24156658696897293 | 5.072898326348431 | 1.9325326957517834 | 1.6909661087828105 | 1.4493995218138376 |
| 16 | KULLBACKLEIBLER vs .TOTALVARIATION | 0.379775474840949 | 7.975284971659929 | 2.278652849045694 | 2.278652849045694 | 1.519101899363796 |
| 17 | IRS vs .CHISQUARE | 0.3797754748409493 | 7.975284971659936 | 2.278652849045694 | 2.278652849045694 | 1.8988773742047464 |
| 18 | EUCLIDEAN vs .KULLBACKLEIBLER | 0.5581846494226572 | 11.721877637875801 | 2.278652849045694 | 2.278652849045694 | 1.8988773742047464 |
| 19 | IRS vs .CHEBYSHEV | 0.660549205201673 | 13.871533309235133 | 2.278652849045694 | 2.278652849045694 | 1.8988773742047464 |
| 20 | CHEBYSHEV vs .CHISQUARE | 0.660549205201673 | 13.871533309235133 | 2.278652849045694 | 2.278652849045694 | 1.8988773742047464 |
| 21 | HELLINGER vs .TOTALVARIATION | 0.7696979437812898 | 16.163656819407084 | 2.278652849045694 | 2.278652849045694 | 1.8988773742047464 |
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