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```
clear; close all; clc;
```

## MIP Hardware

```
Vb = 7.4; %V Battery Voltage
wf = 1643.5; %rad/s motor shaft free speed
sbar = .0036021; %Nm motor stall torque
G = 143; %gearRatio
Im = 6.5085e-8; %kg*m^2 motor inertia
r = 50e-3; %m wheel radius
mw = 40e-3; %kg wheel mass
mb = 276e-3; %kg body mass
l = 50e-3; %m length from wheel axis to body centroid
Ib = 0.0016; %kg*m^2 body inertia
g = 9.81; % gravity
k = 2.1918e-06; % motor torque constant
Iw = 2*((mw*r^2)/2 + (Im * G^2)); % wheel inertia

sampleRate1 = 100;
sampleRate2 = 10;
```

## Linearization Coefficients

```
A = (Iw + (mw+mb)*r^2);
B = mb*r*l;
C = (Ib + mb*l^2);
D = mb*g*l;
E = 2*G*sbar;
F = 2*G^2*k;

J1 = B - (A*(B+C))/(A+B);
J2 = F + (F*(B+C))/(A+B);
J3 = (A*D)/(A+B);
J4 = (D*F)/(A+B);

R1 = (A+B);
R2 = (B+C);
```

---

```

R3 = D;

s = tf('s'); %% s-domain variable

Theta = (E/J1)*s; %%define theta

U = s^3 - s^2*(J2/J1) + s*(J3/J1) + (J4/J1); %%define U

Phi = R3/R1 - s^2*(R2/R1); %%define Phi

G1 = minreal(Theta/U); %%define G1
G2 = minreal(Phi/s^2); %%define G2

```

## Inner Loop Body Angle Control

```

G1poles = pole(G1);
G1zeros = zero(G1);

%lead Controller z < p
z1lead = 5;
p1lead = 70;

%lag Controller z > p
z1lag = 55;
p1lag = 0;

K1 = -6; %%negative gain
D1lead = (s+z1lead)/(s+p1lead);
D1lag = (s+z1lag)/(s+p1lag);

%include pade approximation to account for DAC h/2 delay
d1 = (1/sampleRate1)/2;
padeApprox1 = (1-(d1*s/2) + ((d1*s)^2)/12)/(1+(d1*s/2) + ((d1*s)^2)/12);

D1 = K1 * D1lead * D1lag *padeApprox1;

L1 = minreal(G1*D1);
[GM_L1,PM_L1,WCg_L1,WCp_L1] = margin(L1);

[y1,t1] = step((L1)/(1+L1));

%calculate prescalar P from last 25 terms
P1 = 1/mean(y1(end-25 : end));
T1 = (G1*D1)/(1+G1*D1);

```

## Outer Loop Wheel Position Control

```

G2poles = pole(G2);
G2zeros = zero(G2);

%lead Controller z < p
z2lead = .3;

```

---

```

p2lead = 7;

%lag Controller z < p
z2lag = .4;
p2lag = 0;

K2 = .5; %% gain
D2lead = (s+z2lead)/(s+p2lead);
D2lag = (s+z2lag)/(s+p2lag);

d2 = (1/sampleRate2)/2;
padeApprox2 = (1-(d2*s/2)+ ((d2*s)^2)/12)/(1+(d2*s/2)+ ((d2*s)^2)/12);

D2 = K2 * D2lead * D2lag *padeApprox2;

L2 = minreal(G2*D2);

[GM_L2,PM_L2,WCg_L2,WCp_L2] = margin(L2);

T2 = minreal((G2*D2)/(1+G2*D2));

```

## Successive Loop Closure

```

L3 = minreal(D2*T1*G2);

Tsystem = minreal((D2*P1*T1*G2)/(1 + D2*P1*T1*G2));
[GM_Ts,PM_Ts,WCg_Ts,WCp_Ts] = margin(Tsystem);

```

## Discrete Time Controller

```

h1 = 1 / sampleRate1;

% Convert to discrete time using tustin approximation with prewarping
%perwarp frequency designed around crossover frequency
opt1 = c2dOptions('Method','tustin','PrewarpFrequency',abs(WCp_L1));
Gz1 = c2d(G1,h1,'zoh');
Dz1 = c2d(D1,h1,opt1);
Tz1 = (Dz1*Gz1)/(1 + Dz1*Gz1);

%calculate discrete prescalar using last 5 terms
[yz1,tz1] = step(Tz1);
dz1Lim = find(tz1 >= .5);
Pz1 = 1/mean(yz1(dz1Lim(1)-5 : dz1Lim(1)));

h2 = 1/sampleRate2;

opt2 = c2dOptions('Method','tustin','PrewarpFrequency',abs(WCp_L2));
Gz2 = c2d(G2,h2,'zoh');
Dz2 = c2d(D2,h2,opt2);
Tz2 = (Dz2*Gz2)/(1 + Dz2*Gz2);

TzSystem = c2d(Tsystem,h2,'zoh');

```

---

# Display Results

G1  
G2  
D1  
P1  
D2  
Dz1  
Pz1  
Dz2  
T1  
T2  
Tz1  
Tz2  
Tsystem

G1 =

$$\frac{-570.7 s}{s^3 + 84.54 s^2 - 62.79 s - 1585}$$

Continuous-time transfer function.

G2 =

$$\frac{-0.7025 s^2 + 6.24e-16 s + 31.91}{s^2}$$

Continuous-time transfer function.

D1 =

$$\frac{-0.0072 s^4 + 8.208 s^3 - 2940 s^2 - 204984 s - 950400}{0.0012 s^4 + 1.524 s^3 + 676.8 s^2 + 40320 s}$$

Continuous-time transfer function.

P1 =

$$0.8819$$

D2 =

$$\frac{0.06 s^4 - 7.158 s^3 + 283 s^2 + 200.7 s + 34.56}{-----}$$

---


$$0.12 s^4 + 15.24 s^3 + 676.8 s^2 + 4032 s$$

Continuous-time transfer function.

Dz1 =

$$\frac{-2.157 z^4 - 3.486 z^3 + 3.281 z^2 + 5.173 z - 3.125}{z^4 - 0.3149 z^3 - 0.8703 z^2 + 0.007581 z + 0.1777}$$

Sample time: 0.01 seconds

Discrete-time transfer function.

Pz1 =

$$0.8770$$

Dz2 =

$$\frac{0.1418 z^4 + 0.171 z^3 - 0.3438 z^2 - 0.3249 z + 0.357}{z^4 - 0.318 z^3 - 0.8686 z^2 + 0.009047 z + 0.1776}$$

Sample time: 0.1 seconds

Discrete-time transfer function.

T1 =

$$0.004931 s^{12} + 1.058 s^{11} - 2291 s^{10} - 5.058e05 s^9 + 1.1e09 s^8$$

$$+ 2.428e11 s^7 + 1.749e13 s^6 + 4.406e14 s^5 + 1.296e15 s^4$$

$$- 9.429e15 s^3 - 3.466e16$$

s^2

---


$$\frac{1.44e-06 s^{14} + 0.003901 s^{13} + 4.58 s^{12} + 2854 s^{11} + 9.715e05 s^{10}}{s^{10}}$$

---


$$+ 1.674e08 s^9 + 1.6e10 s^8 + 8.923e11 s^7 + 2.8e13 s^6$$

$$+ 4.039e14 s^5 + 8.787e14 s^4 - 8.969e15 s^3 - 3.057e16 s^2$$

Continuous-time transfer function.

T2 =

$$-0.5415 s^6 + 64.6 s^5 - 2529 s^4 - 4746 s^3 + 1.157e05 s^2 + 8.229e04 s$$

$$1.417e04 \quad +$$

-----

$$s^6 + 260.4 s^5 + 6165 s^4 + 4.705e04 s^3 + 1.157e05 s^2 + 8.229e04 s$$

$$1.417e04 \quad +$$

Continuous-time transfer function.

Tz1 =

$$0.04734 z^{13} - 0.06521 z^{12} - 0.2218 z^{11} + 0.3608 z^{10} + 0.3603 z^9$$

$$- 0.8446 z^8 + 0.03327 z^7 + 0.6586 z^6 - 0.2541 z^5 - 0.1966 z^4$$

$$+ 0.1268 z^3 + 0.01343 z^2 - 0.02225 z + 0.003954$$

---


$$\begin{aligned}
 & z^{14} - 5.451 z^{13} + 11.01 z^{12} - 7.671 z^{11} - 5.304 z^{10} + 11.47 z^9 \\
 & - 3.378 z^8 - 5.024 z^7 + 3.891 z^6 + 0.3236 z^5 - 1.233 z^4 \\
 & + 0.3 z^3 + 0.1276 z^2 - 0.07223 z + 0.009773
 \end{aligned}$$

Sample time: 0.01 seconds  
Discrete-time transfer function.

Tz2 =

$$\begin{aligned}
 & -0.09964 z^{12} + 0.3327 z^{11} + 0.1197 z^{10} - 1.468 z^9 + 0.908 z^8 \\
 & + 2.332 z^7 - 3.271 z^6 + 0.3495 z^5 + 1.523 z^4 - 0.6714 z^3 \\
 & - 0.2185 z^2 + 0.1976 z - 0.03441
 \end{aligned}$$

---


$$\begin{aligned}
 & 0.9004 z^{12} - 4.303 z^{11} + 7.027 z^{10} - 2.169 z^9 - 6.543 z^8 \\
 & + 7.12 z^7 - 0.3594 z^6 - 3.031 z^5 + 1.31 z^4 + 0.3267 z^3 \\
 & - 0.3506 z^2 + 0.07467 z - 0.002886
 \end{aligned}$$


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---

Sample time: 0.1 seconds  
Discrete-time transfer function.

Tsystem =

$$-1061 s^{22} - 2.842e06 s^{21} - 2.771e09 s^{20} - 6.14e11 s^{19} + 7.033e14 s^{18}$$

$$+ 2.082e17 s^{17} - 4.584e20 s^{16} - 4.14e23 s^{15} - 1.427e26 s^{14}$$

$$- 2.007e28 s^{13} - 8.193e29 s^{12} + 5.595e31 s^{11} + 3.539e33 s^{10}$$

$$- 3.173e35 s^9 - 3.262e37 s^8 - 9.449e38 s^7 - 8.052e39 s^6$$

$$+ 3.384e39 s^5 + 3.624e41 s^4 + 1.79e42 s^3 + 3.193e42 s^2$$

$$+ 1.636e42 s + 2.488e41$$

-----

$$s^{24} + 5420 s^{23} + 1.371e07 s^{22} + 2.122e10 s^{21} + 2.223e13 s^{20}$$

$$+ 1.651e16 s^{19} + 8.878e18 s^{18} + 3.473e21 s^{17} + 9.863e23 s^{16}$$

$$+ 2.026e26 s^{15} + 3.031e28 s^{14} + 3.341e30 s^{13} + 2.742e32 s^{12}$$

$$+ 1.674e34 s^{11} + 7.474e35 s^{10} + 2.354e37 s^9 + 4.976e38 s^8$$



---


$$\begin{aligned}
 &+ 6.745e39 s^7 + 5.924e40 s^6 + 3.387e41 s^5 + 1.232e42 s^4 \\
 &+ 2.699e42 s^3 + 3.193e42 s^2 + 1.636e42 s + \\
 &2.488e41
 \end{aligned}$$

*Continuous-time transfer function.*

## Plotting

### ROOT LOCUS PLOTS

```

figure(1)
subplot(2,1,1)
rlocus(G1)
title('G1')

subplot(2,1,2)
rlocus(G2)
title('G2')
sgtitle('      Plant Root Locus Plot')

figure(2)
subplot(2,1,1)
rlocus(L1)
title('L1=G1(s)*D1(s)')

subplot(2,1,2)
rlocus(L2)
title('L2=G2(s)*D2(s)')

sgtitle('      Loop-Gain Root Locus Plot')

figure(3)
rlocus(Tsystem,'b')
grid on
title('System Transfer Function Bode Plot')

% BODE PLOTS
figure(4)
subplot(2,1,1)
bode(G1)
grid on
title('L1=G1(s)*D1(s)')

subplot(2,1,2)
bode(G2)

```

---

```

grid on
title('L2=G2(s)*D2(s)')

sgtitle('      Plant Bode Plot')

figure(5)
subplot(2,1,1)
bode(L1)
grid on
title('L1=G1(s)*D1(s)')

subplot(2,1,2)
bode(L3)
grid on
title('L3=G2(s)*D2(s)*T1')

sgtitle('      Controller Bode Plot')

figure(6)
margin(Tsystem)
hold on
bode(Tsystem,'b')
grid on
title('System Transfer Function Bode Plot')

% NYQUIST PLOTS
figure(7)
subplot(2,1,1)
nyquist(L1)
title('L1=G1(s)*D1(s)')
subplot(2,1,2)
nyquist(L3)
xlim([-1.5 .8])
title('L2=G2(s)*D2(s)')
sgtitle('      Loop-Gain Nyquist Plot')

% STEP RESPONSES
figure(8)
step(P1 * T1,'b',Pz1 * Tz1,'r',1)
hold on
grid on
legend('Continuous','Discrete')
title('Inner-Loop Transfer Function Step Response')

figure(9)
step(T2,'b',Tz2,'r',5)
hold on
grid on
legend('Continuous','Discrete')
title('Outer-Loop Transfer Function Step Response')

figure(10)
step(Tsystem,'b',TzSystem,'r')

```

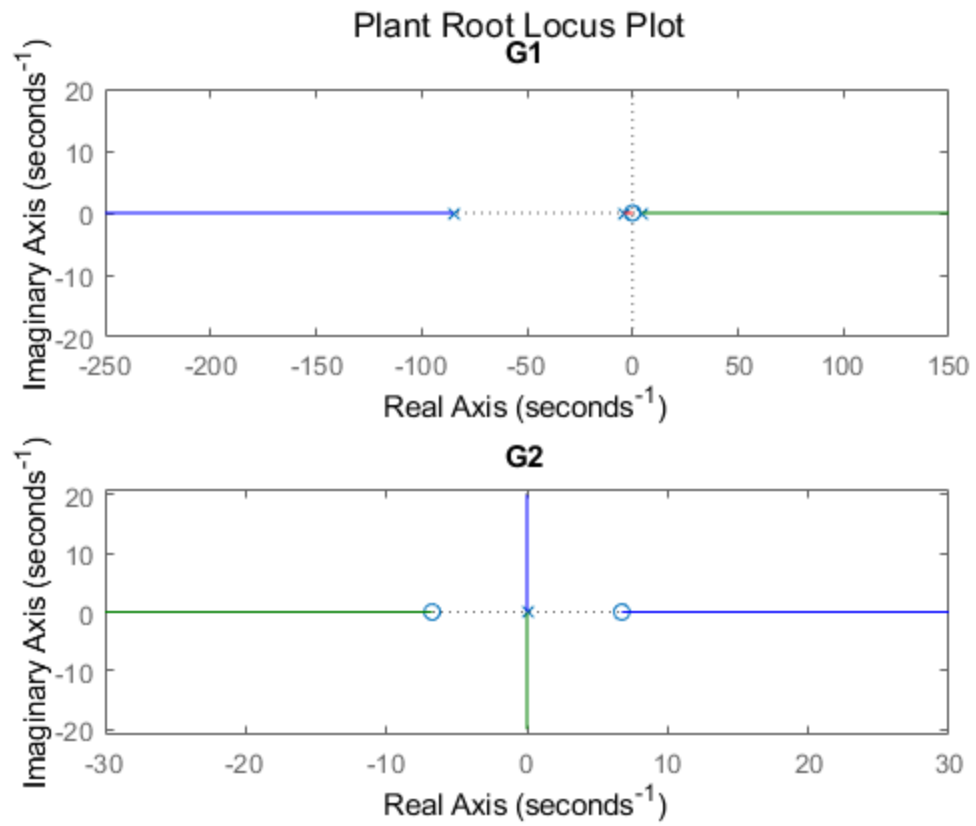
---

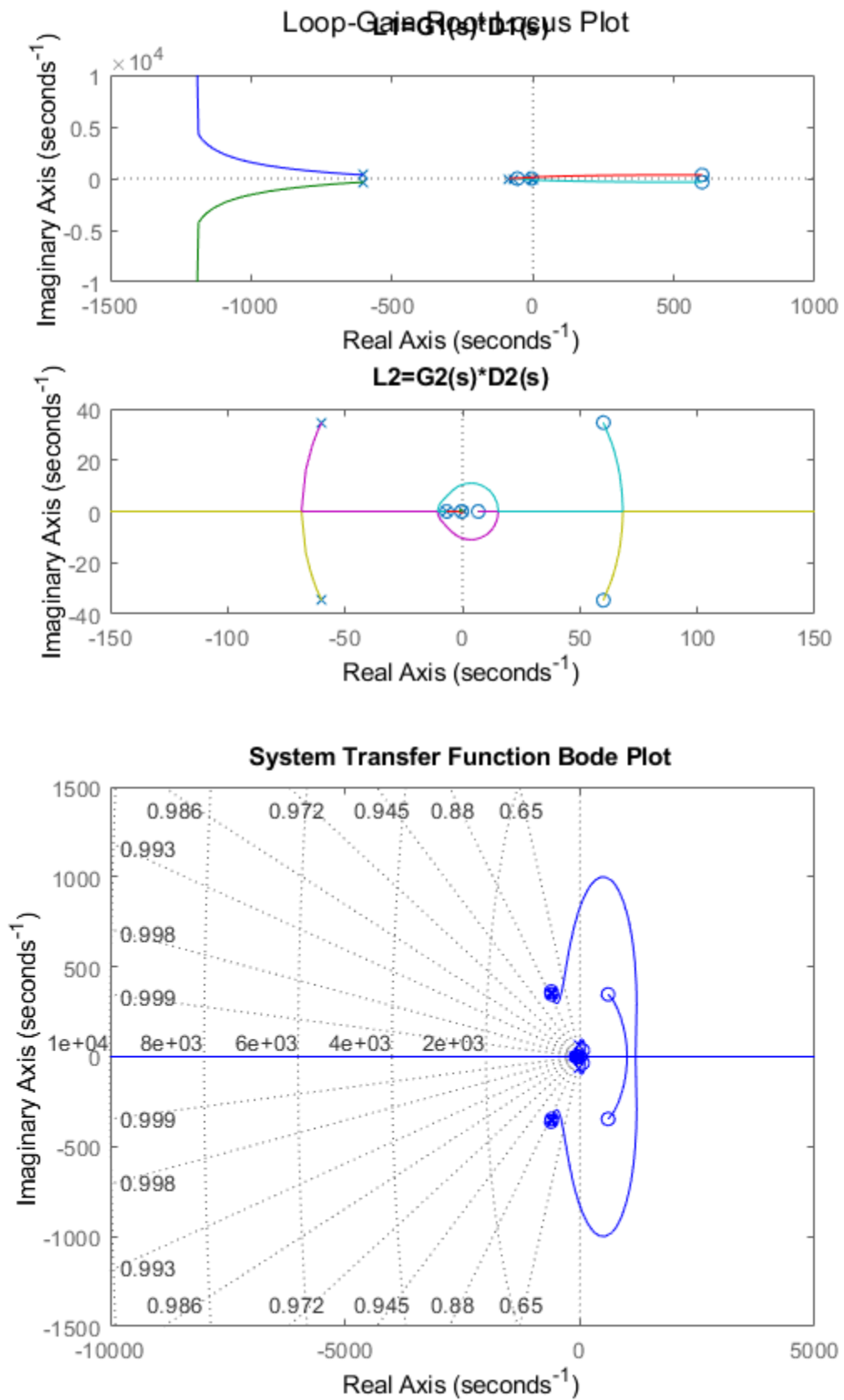
---

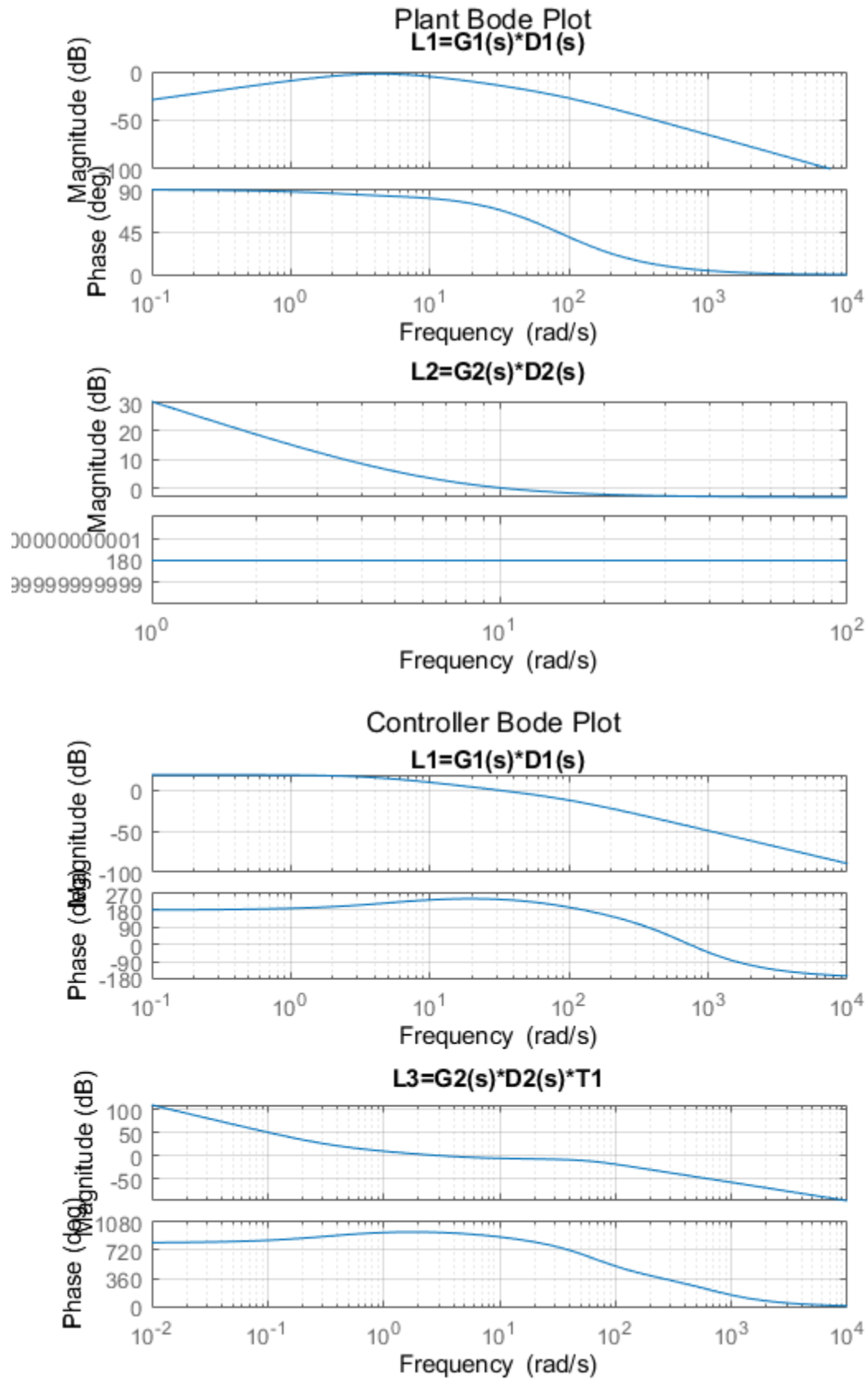
```

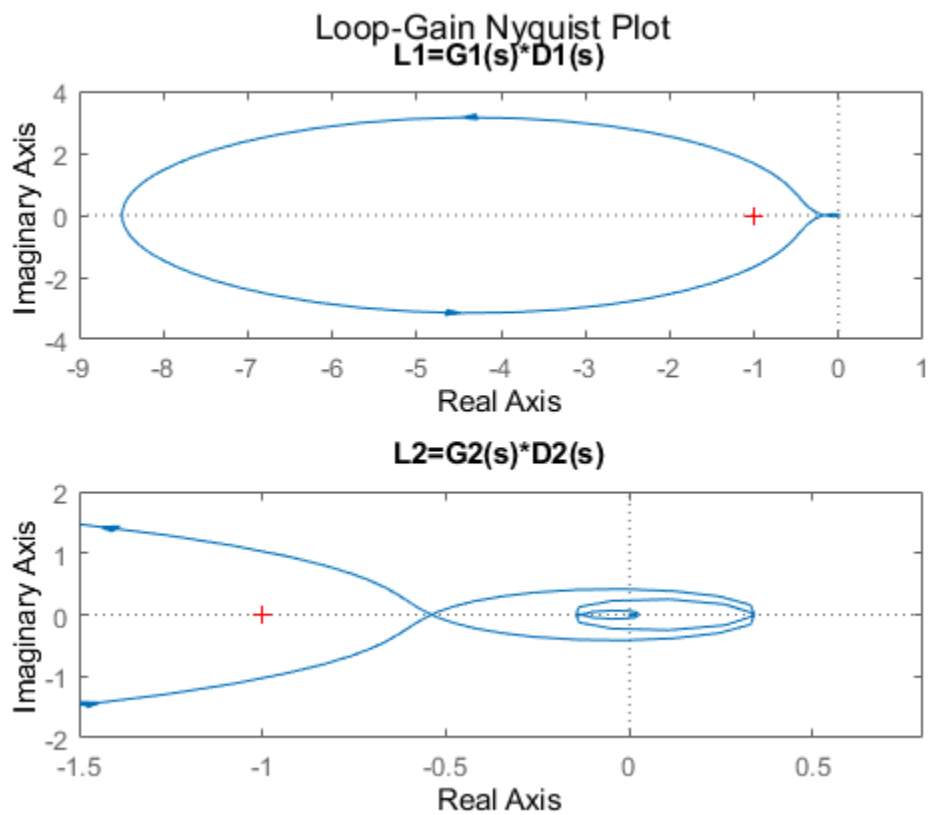
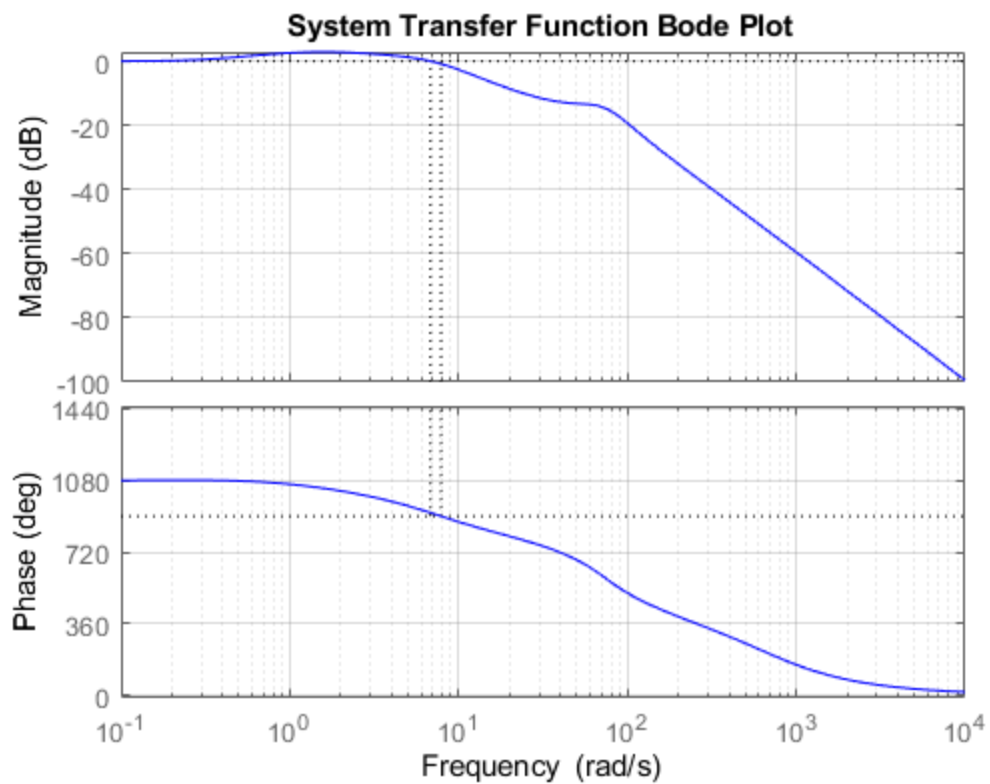
hold on
grid on
legend('Continuous','Discrete')
title('System Transfer Function Step Response')

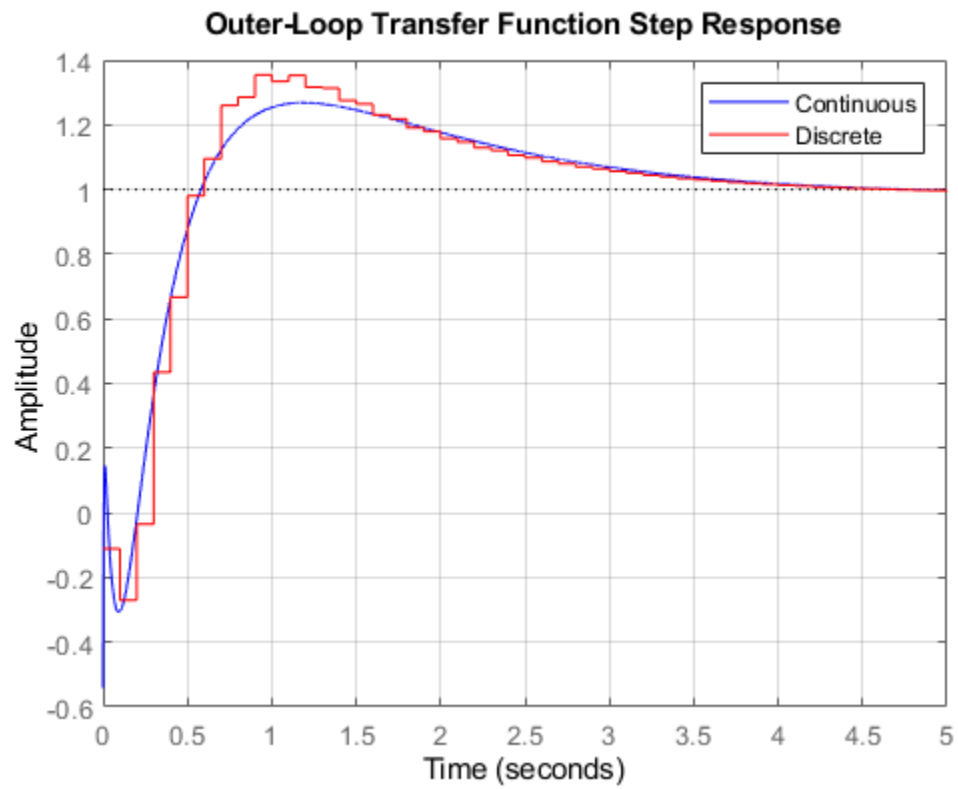
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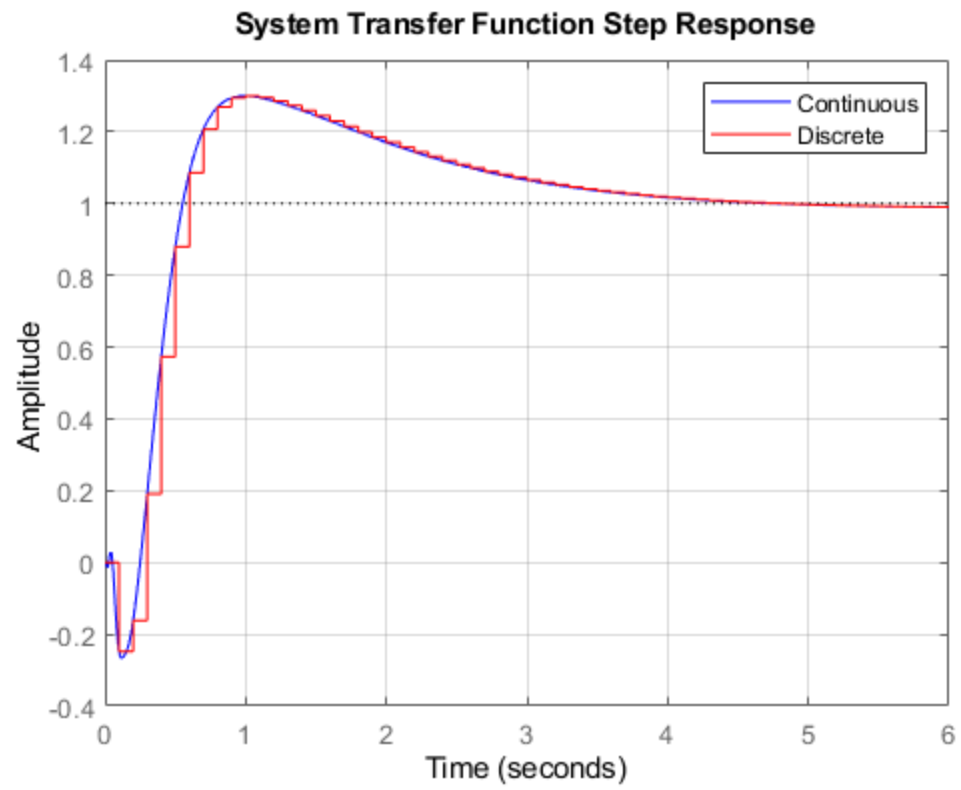












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