MIT 6.2400: Introduction to Quantum Systems Engineering

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Abtract: In this paper, I explore the practical application of quantum computation techniques in solving the antenna array thinning problem. The inverse quantum Fourier transform (IQFT), optimization techniques, and gate errors are used to assess the practicality of current endeavors towards solving this issue.

1. Introduction

Antennas are essential modules in modern-day communication and measurement systems. Devices such as cell phones, satellites, and RADAR systems use specialized antenna assortments and designs to effectively communicate information. One common approach to constructing these systems is an antenna array. Using an assortment of antennas with known input currents, lengths, radiation wavelengths, and physical antenna spacing, one can create systems with particularly strong and weak reception areas such as in Fig.1.

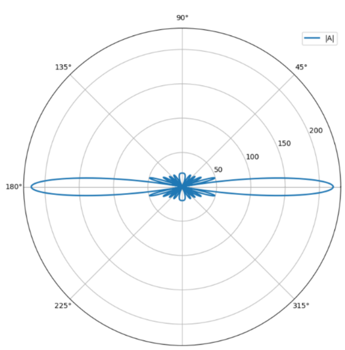

FIG. 1: Antenna array radiation pattern example

In complex system designs, factors such as manufacturing cost, energy usage, and physical space are great limiting factors to antenna setups. To address this, antenna array thinning can be employed. Antenna array thinning seeks to determine the minimum number of antennas that can be removed from a setup such that desired radiation feature are still closely maintained.

While antenna array thinning can be explored through classical techniques, research groups [1] have explored a quantum approach in hopes of speeding up the optimization process. Classical Fourier transform methods use the fast Fourier Transform (FFT), which has a time complexity of $O(nlog(n))$, while the quantum Fourier Transform has a time complexity of $O(log^2(n))$. While this is an initially-intriguing quality of quantum computation, it does not encapsulate the full complexity of quantum systems. One striking downside to current quantum systems is gate error rates, which potentially deny certain array sizes from undergoing a successful thinning process. This paper seeks to explore the practicality of quantum methods towards the array thinning problem.

2. Approach

To simulate the radiation pattern from an antenna array, the electric far-fields generated by each antenna must first be derived. For a single dipole antenna, the electric far-field can be given as:

$$\underline{E}_{ff} = \underline{w}e^{-jkr} \tag{1}$$

where $\underline{w}$ represents an antenna scaling factor that generalizes the non-complex-exponential terms of $\underline{E}_{ff}$. In this paper, it will be assumed that this term is constant. This term can be modified in antenna systems through manipulating the propagation medium, antenna height, and input current.

By definition, $k = \frac{2\pi}{\lambda}$. We can also rewrite r as $dsin(\theta)$, where d represents the spacing of an antenna from another antenna and θ represents an observer's angle from the array. Summing the far-fields, we get a parameter known as the array factor:

$$\underline{A} = \sum_{n=0}^{N-1} \underline{w_n} e^{-j2\pi \frac{d}{\lambda} sin(\theta)} \tag{2}$$

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This formula assumes uniform spacing between adjacent antennas. For this paper, a spacing of $\frac{\lambda}{2}$ will be used.

From Eq.3, it is seen that the array factor is simply a discrete-time Fourier Transform (DTFT) of $\underline{w}$. Thus,:

$$\underline{A} = DTFT\{\underline{w}\} \tag{3}$$

Therefore, $\underline{\{w\}}$ can be given as:

$$\underline{w} = IDTFT\{\underline{A}\} \tag{4}$$

Knowing the $\underline{w}$ parameter allows for an analysis of the relative strengths of each antenna. With this information, it is possible to identify antennas that contribute less to the original radiation power and, therefore, can be turned off.

Knowing which w values contribute the most and least to a given radiation pattern, a cost function can be applied to analyze the effects of turning antennas off. The cost function to be used is given as:

$$\int_0^{2\pi} |A_{ref} - New_{ref}|^2 d\theta \tag{5}$$

Where A_{ref} is the normalized original radiation pattern and New_{ref} is the normalized radiation pattern of a setup with antennas turned off. The function sweeps the entire radiation pattern to assess how close the new radiation pattern matches the original radiation pattern. Using this function, it is possible to define a threshold value for the new radiation pattern.

To extend these relations to a physical quantum application, there are a few tradeoffs to be recognized. First, the quantum Fourier Transform gives transformation results as probabilities rather than exact numbers, meaning numerous measurements will have to be made to determine the probability. Second, gate errors must be accounted for. In this application, I will consider bit flip errors and phase flip errors. Bit flip errors occur at around a 0.2% probability [2] and add an unwanted X gate to the circuit. Phase flip errors occur at around a 0.5% probability [3] and add an unwanted Z gate to the circuit. Finally, input qubits must be prepared such that their measured probabilities cannot exceed 1. This can be done by normalizing the input array factor. Recognizing these restraints, the following procedure is followed:

1. An array factor is defined and then normalized.

2. Qubits are prepared through the array factor. In this application, one antenna is represented by one qubit.

3. Computation errors are introduced by random chance after an intended gate interaction is complete.

4. The "strengths" of the antennas are measured from the output of the IQFT. Since relative strength is the main concern, the $|0\rangle$ state of each qubit will be measured and compared with other qubits. The simulation is run numerous times to account for the real-world necessity to obtain state probabilities through numerous trials.

5. The IQFT outputs of each qubit are ordered by probability.

6. The antenna tied to the highest-probability qubit is turned on, and the new radiation pattern is put into the cost function. If the cost threshold is not met, the antenna tied to the next-highest probability qubit is turned on. This process continues until the cost function is met. This process terminates after the threshold is met or all antennas are turned on. For this paper, the threshold is defined as 80% accurate. Thus, created radiation patterns must be 80% or more similiar to the original radiation pattern.

To better analyze the impact of errors on the system, four unideal scenarios in a single reading of the quantum system are also presented. They are as follows:

a. A bit-flip error is randomly encountered.

b. A phase-flip error is randomly encountered.

c. A bit-flip error is applied to all qubits.

d. A phase-flip error is applied to the 6th qubit.

3. Results

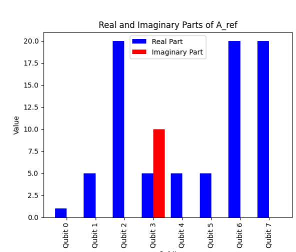

FIG. 2: Step 1: array factor with associated qubit assignments (not normalized)

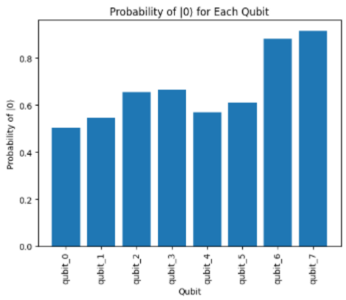

FIG. 3: Step 4: Measuring $|0\rangle$ state

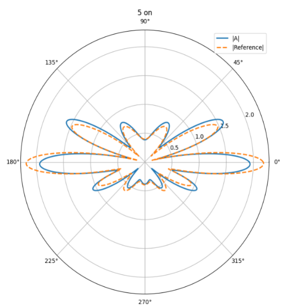

FIG. 4: Normalized radiation pattern for no error, random phase-flip error, and random bit-flip error

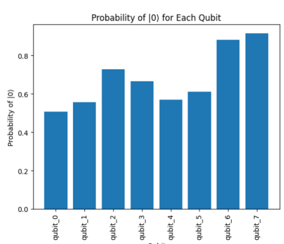

FIG. 5: Error a: $|0>$ state measurements from random bit-flip error on qubit_2

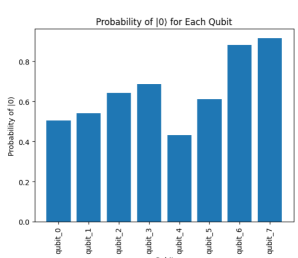

FIG. 6: Error b: $|0>$ state measurements from phase-flip error applied to qubit_4

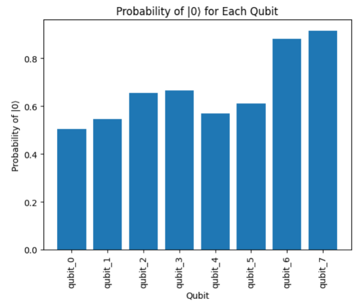

FIG. 7: Error c: $|0>$ state measurements from bit-flip error applied to all qubits

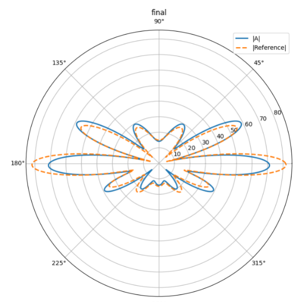

FIG. 8: Error c: Radiation pattern after applying bit-flip error to all qubits

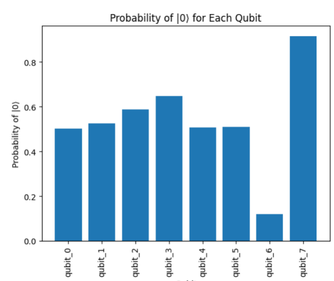

FIG. 9: Error d: $|0>$ state measurements from phase-flip error applied to qubit_6

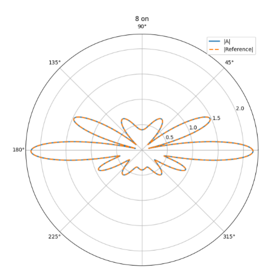

FIG. 10: Error d: Radiation pattern from phase-flip error applied to qubit_6

4. Discussion

The system was tested at its maximum capacity (N=8 antennas). This cutoff was determined from my computer's RAM capabilities.

Overall, for small antenna array systems, the quantum algorithm successfully thinned the array with the 80% threshold. While errors occassionally occurred, the error chance was small that their overall affect on my system was mostly negligible.

Bit-flip errors appear to not have affected the system too much. Fig.8's final radiation pattern turns on the same antennas and amount of antennas as the no errors case in Fig.4. I predict this is because the IQFT uses a Hadamard gate before using rotation gates. Since the rotation gates are applied to the $|0>$ and $|1>$ state regardless, the overall effect of a bit flip is not too concerning.

Phase-flip errors appear to greatly affect measurement results when acting on qubits associated with stronger antennas, causing the radiation pattern in Fig.10 to report no thinning solution (all antennas had to be turned on to satisfy the cost function). I predict this occurs because the rotation gate acts on the $|1>$ state, and since this state acts as both a controller and controlled qubit by the rotation gates, this could have seriously affected the qubit's transform development. If the phase flip occurs on a qubit associated with an intermediate strength, however, the impact is less impactful, as seen on Fig.4. Regardless, its impact is much more pronounced than that of the bit-flip errors.

5. Conclusion

I have demonstrated the practicality of antenna array thinning for a small number of antennas in an array. To simulate large numbers of antennas, one could get a more powerful computing system. For future applications, three-dimensional arrays could be explored. This algorithm currently only considers a straight line of antennas, but more complex systems, such as starlink, use a circular grid to transmit in numerous directions. Despite these findings, it may ultimately still be impractical to use quantum computing over classical techniques for this application. Measuring the states numerous times is time-consuming, and for a small array, it is likely faster to stick with classical methods.

References

[1] IEEE Xplore Full-Text PDF (2024), accessed 12 Dec. 2024.
[2] S. S. Tannu and M. K. Qureshi, Mitigating measurement errors in quantum computers by exploiting state-dependent bias (2019).
[3] Google Quantum, Exponential suppression of bit or phase flip errors with repetitive error correction (n.d.).