Antenna Array Design

6.2400 Quantum Systems Engineering

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Approach

Define a qubit as an antenna

- N qubits → N antennas
- All start at |0>

IFT result → “strength” of signal

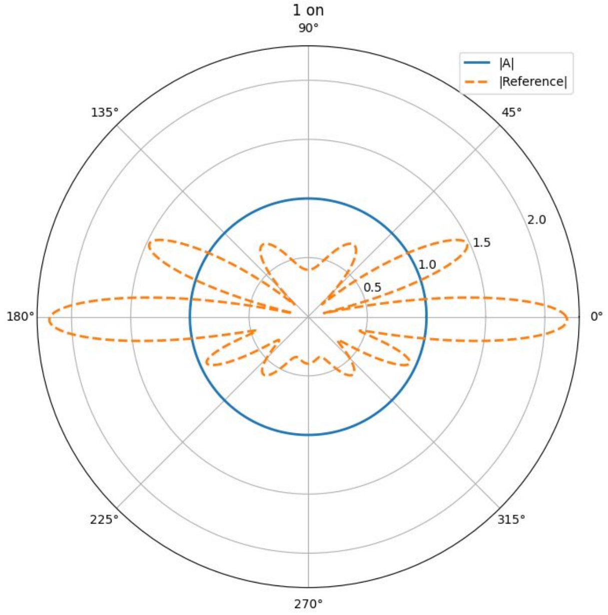
Apply cost function

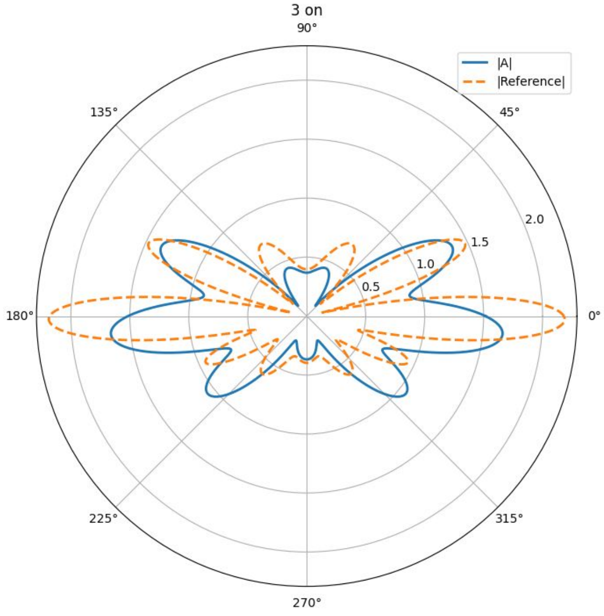
- Checks error between original and new structure

Prepare Qubits with w → Perform IQFT with errors → Measure probabilities on |0> state → Turn on high “strengths” until cost function threshold met

Cost Function

$$\Psi(\mathbf{B}) = \int_{-1}^{1} \left| \mathbb{G}\{\mathcal{A}(u \mid \mathbf{B})\} - \mathbb{G}\left\{\mathcal{A}^{ref}(u)\right\} \right|^2 du.$$





IQFT In Action

QFT example:

$$|110\rangle \rightarrow \frac{1}{\sqrt{8}}\left(|0\rangle + \underbrace{e^{2\pi i\, 0.0}}_{=1}|1\rangle\right)\left(|0\rangle + e^{2\pi i\, 0.10}|1\rangle\right) \ldots \left(|0\rangle + e^{2\pi i\, 0.110}|1\rangle\right)$$

$0.10_{(BIN)} = 0.5_{(DEC)}$ $\quad$ $0.11_{(BIN)} = 0.75_{(DEC)}$

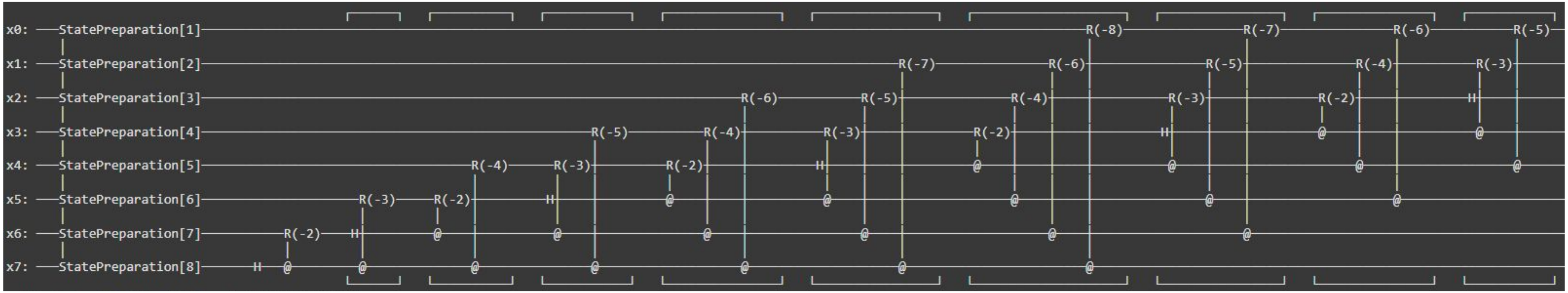
QFT High-Level Use [CNOT]

Encodes qubit data in phase

IQFT does opposite

ULTIMATELY: can find weaker and stronger antenna contributions

Thinning IQFT Circuit



$$R_k = \begin{pmatrix} 1 & 0 \\ 0 & e^{i2\pi/2^k} \end{pmatrix}$$

Implementation

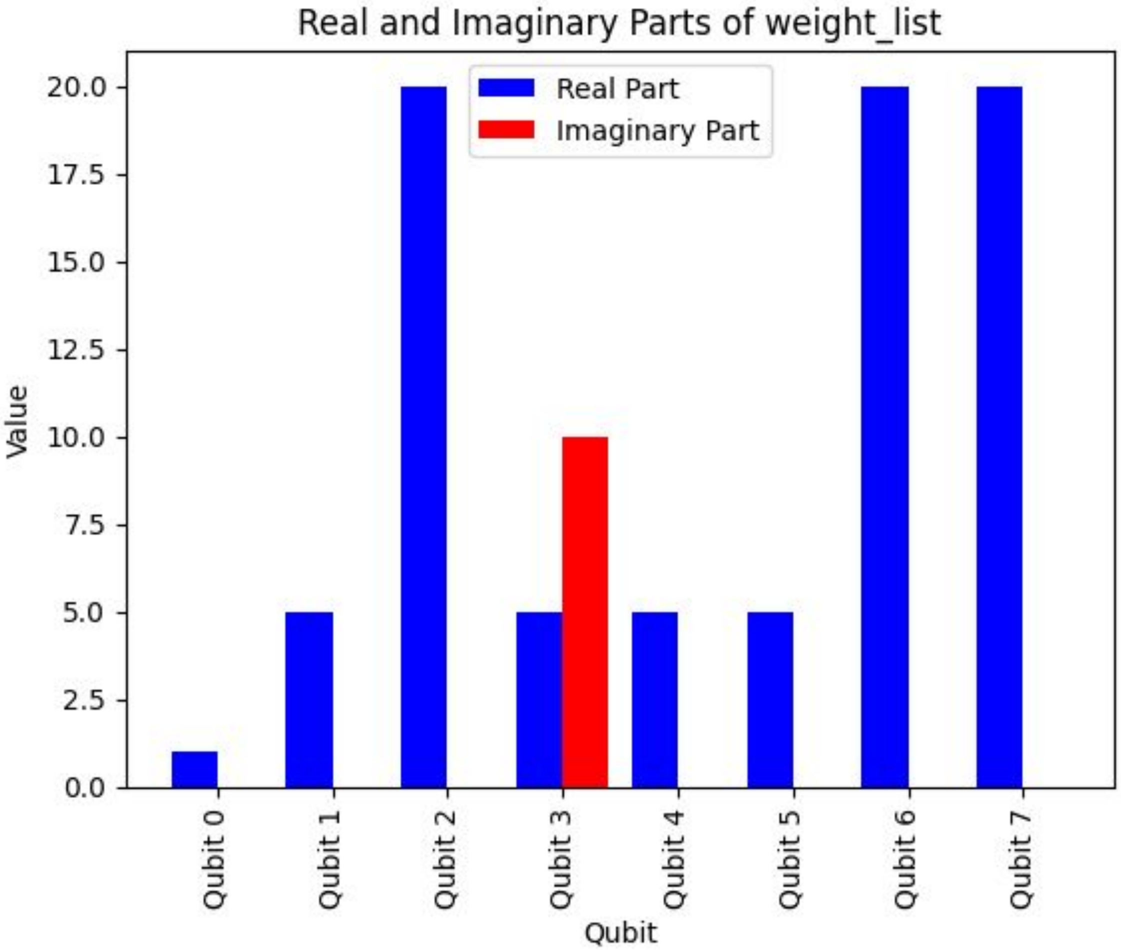
From literature…

- Bit flip rate: 0.2%
- Phase flip rate: 0.5%

Other parameters:

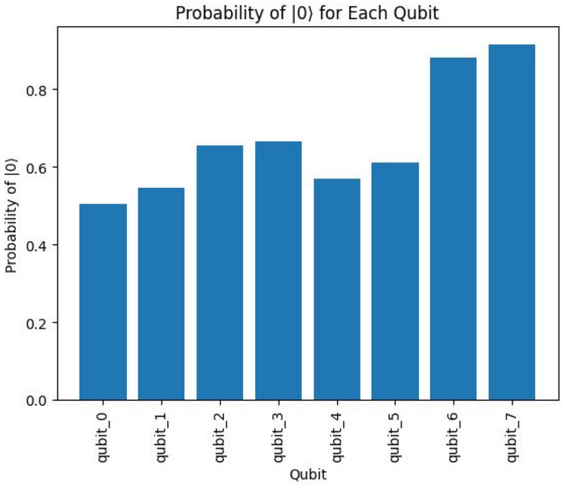
- Threshold: 20% deviation

Test Case: random pattern

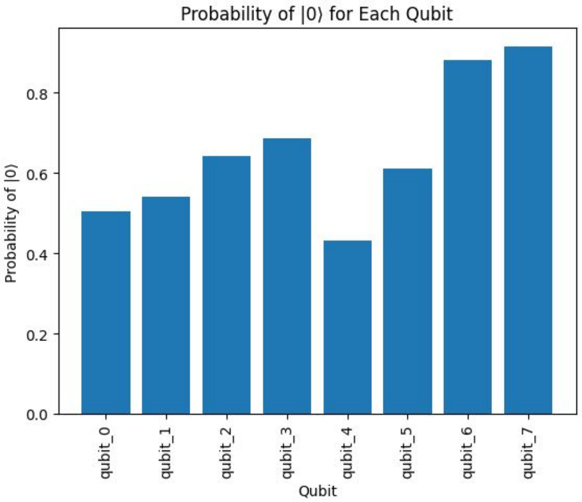


Results: Random Runs

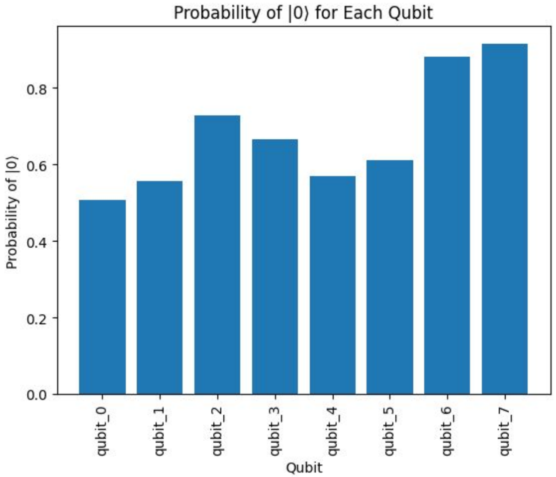
Random runs



No Errors

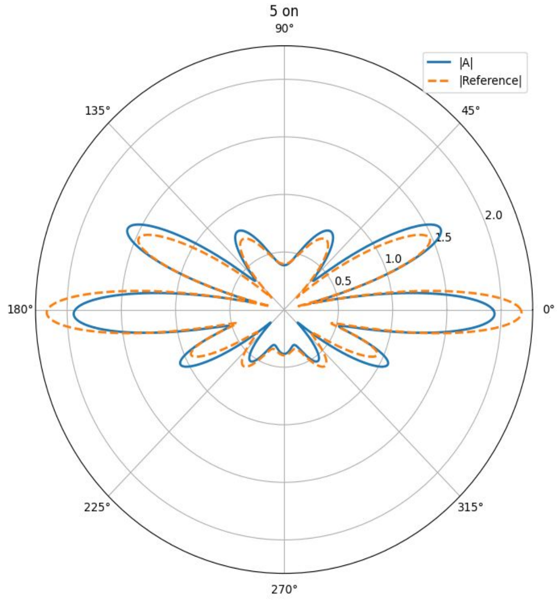


Phase error on Qubit 4 (extra Z gate)

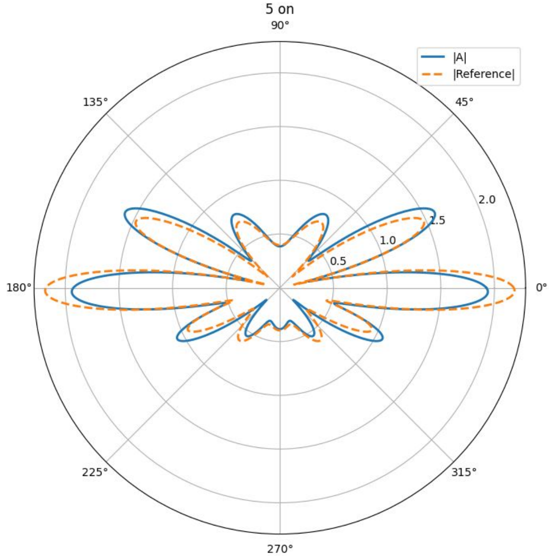


Bit flip error on Qubit 2 (extra X gate)

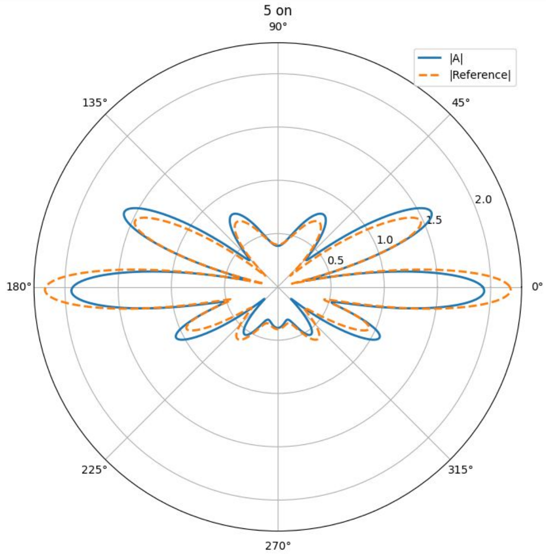
Results: Random Runs



No Errors



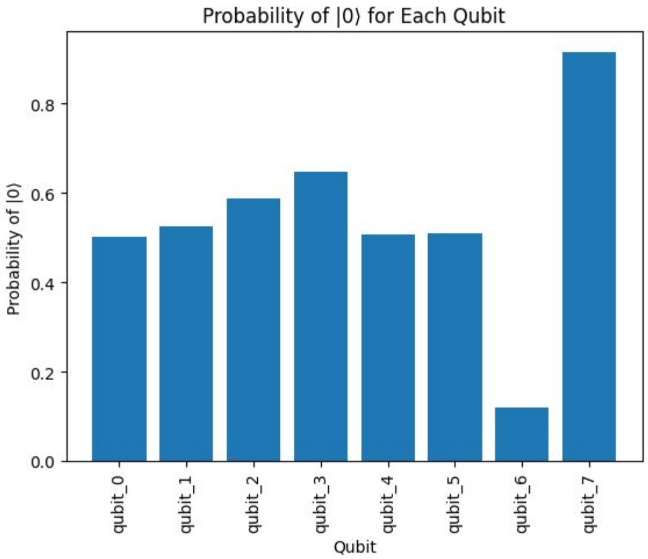
Phase error (extra Z gate)



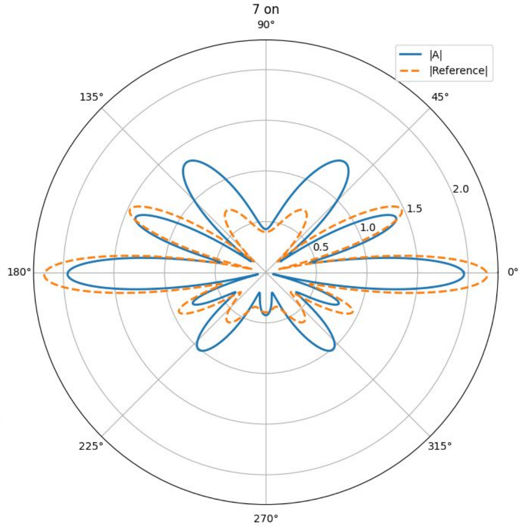
Bit flip error (extra X gate)

Results: Extreme Case (Z)

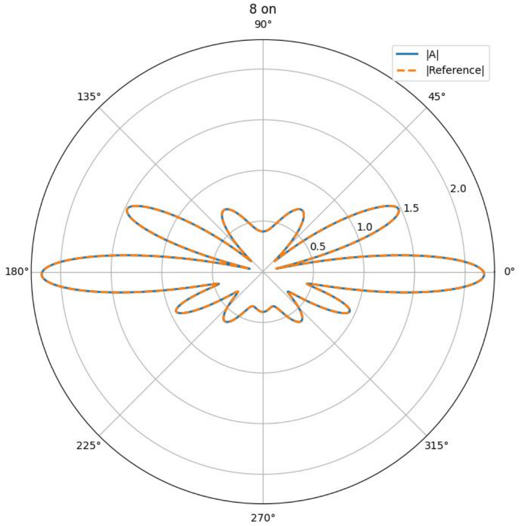
Extreme case: phase error on antenna with high w



Phase error on Qubit 6



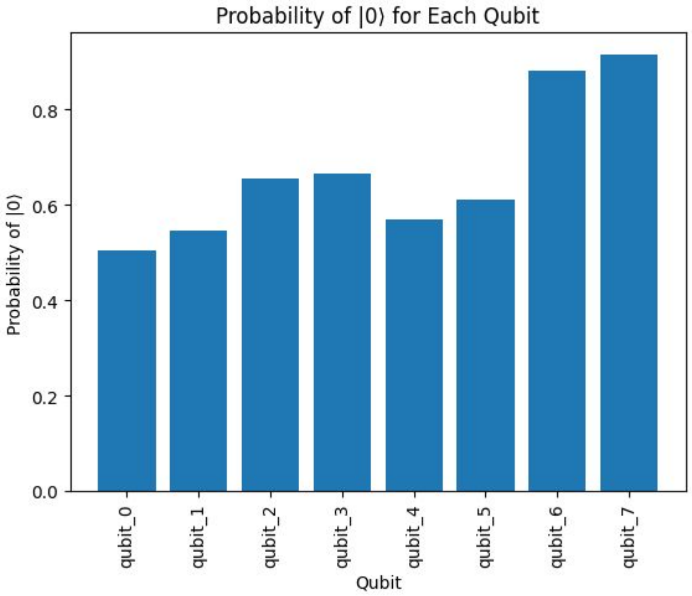
Before antenna 6 activation



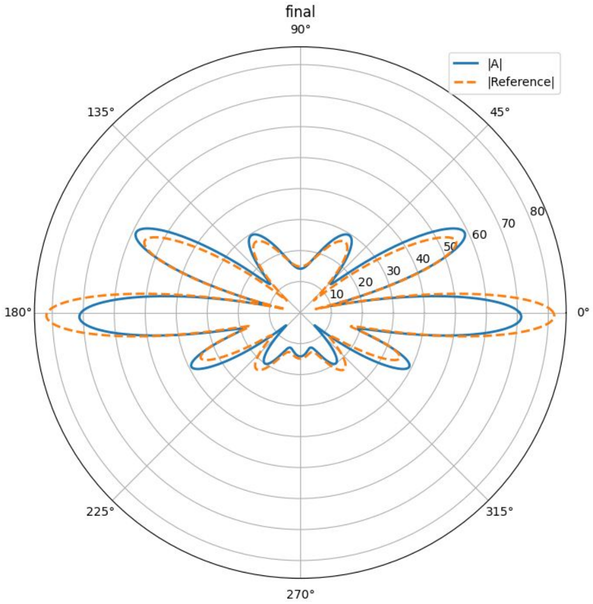
No solution (all antennas still on)

Results: Extreme Case (X)

Extreme case: bit flip error on antenna with high w



Bit flip error on all gates



Resulting radiation pattern

How did IQFT do?

Limitations

- Max of N=8: RAM limitations
  - Gate operations $\propto$ N!
  - N=30 very prone to errors

Analysis

- Error rates VERY low
  - Numerous runs to get even one error
- Z errors:
  - Acts on high probability → Failed thinning
  - Acts on intermediate probability → Decent thinning
- X errors:
  - Generally doesn't change much
- For small N, running a couple times probably gets rid of errors

Concluding Thoughts

For small N,...

- Works well!
- Error so small, just run again

For large N,...

- Cannot make assertive claim (RAM limitations), but based on trends:
  - Z error appears catastrophic
  - X error appears somewhat fine to have

Ultimately,...

- Better simulator setup could give insight on large N
- Explore more complex arrays (i.e. 3-D arrays, non-dipole antennas)