Geometric Optics: Labs 1A and 1B

6.2370 Modern Optics Project Laboratory
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I. Introduction

This experiment aims explore how light interacts with convex lenses to produce images. Using geometric optics, one is able to predict the propagation direction of light passing through a lens system. Imaging with lenses has many important modern applications, from microscopes to cameras. In manipulating the focal lengths and distances between adjacent lenses, one is able to produce images with certain magnifications and image plane distances. Lab 1A explores the working principles of geometric optics, while Lab 1B expands on this idea through an analysis of a terrestrial microscope capable of magnifying a far-away seeing-eye chart.

II. Approach: Lab 1A

Fig 1 demonstrates a schematic of the setup for Lab 1A. A white light lamp passes light through a transparent glass slide, which has a visual acuity test etched in black about its center. A biconvex lens, with an unknown focal length *f*, is positioned at a certain distance *O* (known as the "object distance") from the slide. The height of the lens is positioned such that the bottom of the center of the slide passes through the center of the lens. This is done to maintain an optical axis that maps the bottom of the slide to the top of the image obtained.

A white sheet is then placed at a specific distance *I* (known as the "image distance") from the convex lens. When the lamp is turned on, a blurry version of the acuity test is visible on the sheet. The white sheet's distance from the lens is adjusted until a sharp image appears on the sheet. The vertical length of the image projected on the sheet is then measured.

Fig 2 demonstrates a method of calculating the focal length of the convex lens. Using the lens equation:

$$\frac{1}{f} = \frac{1}{I} + \frac{1}{O} \tag{1}$$

It is possible to write $\frac{1}{I}$ as a function of $\frac{1}{O}$:

$$\frac{1}{I} = \frac{1}{f} - \frac{1}{O} \tag{2}$$

The focal length of the lens is a constant parameter, so the y-intercept of the best-fit line gives $\frac{1}{f}$. Thus, the relation below is true:

$$f = (y\ \ intercept)^{-1} \tag{3}$$

Finally, the magnification *M* of the image is calculated by dividing the vertical length of the image by the vertical length of the physical slide. The formula is demonstrated below:

$$|M| = |\frac{height\ \ of\ \ image\ \ on\ \ sheet}{height\ \ of\ \ physical\ \ slide}| \tag{4}$$

This is repeated for 5 different object distances. The resulting measurements are compiled in Fig 3.

III. Results: Lab 1A

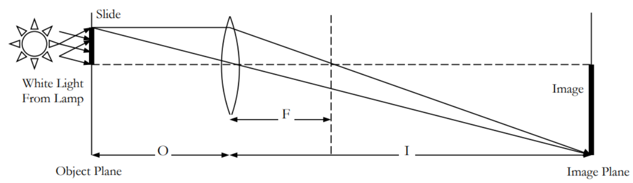

Fig. 1: Lab Setup Schematic

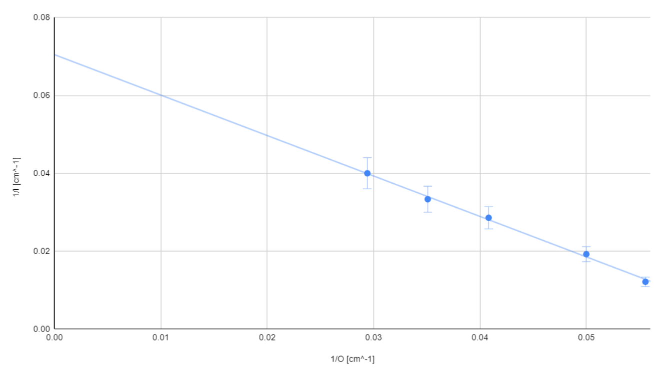

Fig. 2: Inverse Image Distance (1/I) vs. Inverse Object Distance (1/O)

object distance (O) [cm]	image distance (I) [cm]	size of sharp image [cm]	magnification
18	82.5	10.5	3.818181818
20	52	6	2.181818182
24.5	35	3.25	1.181818182
28.5	30	2.5	0.9090909091
34	25	1.75	0.6363636364

Fig. 3: Measurements With a Glass Slide of Height 2.75[cm]

IV. Discussion: Lab 1A

This section discusses the post-lab questions attached to Lab 1A and an analysis of the obtained data.

Question 1: Since the lenses are close together, it is reasonable to assume that the distance light propagates between the lenses is negligible. The ABCD matrix for a lens, as discussed in class, is as follows:

$$M_{lens} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \tag{5}$$

Assigning the focal length of the concave lens as $f_{concave}$ and the convex lens as f_{convex}, and placing the concave lens before the convex lens relative to the light's propagation path, the ABCD matrix of the system can be written as:

$$M_{system} = \begin{pmatrix} 1 & 0 \\ -1/f_{convex} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_{concave} & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ -1/f_{convex} - 1/f_{concave} & 1 \end{pmatrix}$$

This essentially creates a new lens with focal length determined by f_{convex} and $f_{concave}$. Using the C entry to solve for the new focal length:

$$-1/f_{system} = -1/f_{convex} - 1/f_{concave}$$
$$\implies f_{system} = \frac{f_{convex} f_{concave}}{f_{convex} + f_{concave}}$$

If you know the focal length of the convex lens and repeat the approach detailed in Section I, you could solve Equation (7) for $f_{concave}$

Question 2: As discussed in Section II, Fig 2 can be used to determine the focal length of the lens. Looking at the graph, the y-intercept appears to be about 0.07 $[cm]^{-1}$. Using equation (3):

$$f = \frac{1}{0.07} = 14.29[cm] \tag{6}$$

Question 3: Using the lens-maker's formula:

$$\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2}) \tag{7}$$

And noting that, for a biconvex lens, $R = R_1 = -R_2$ (by symmetry):

$$\frac{1}{f} = (n-1)(\frac{2}{R}) \implies R$$
$$= 2f(n-1) = (2)(14.29 * 10^{-2})(1.52 - 1) = 14.86[cm]$$

Question 4: A sharp image is not seen because, when the object distance is less than the focal length, the light rays do not converge after passing the lens. Instead, a virtual image is formed behind the lens. This is represented by my ray tracing diagram in Fig 4. As seen, to the left, a virtual image is formed, but to the right, the light diverges. Therefore, we would not expect to see a sharp image.

Data Analysis: Fig 2 demonstrates a relatively consistent linear trend, with the only notable margin of error being from the first point. This was likely an error of human

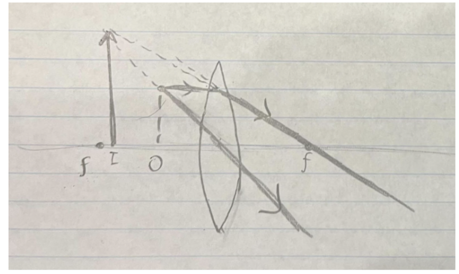

Fig. 4: Virtual Image Formation when $O < f$

measurement, though it is also possible that a artifacts (i.e. dust) or a slight scratch in the lens may have interfered with the results. The lens have also not been parallel to the slide holder, causing some slight deviation in our image distance measurement compared to other points which may or may not have been parallel with the slide holder. The object distance and image distance seem to have an inverse relationship, which is consistent with the lens equation. Object distance also seems to be inversely proportional to magnification. This is consistent with the magnification formula, $|M| = |\frac{I}{O}|$, as increasing O decreases I, which is demonstrated in my data.

V. Approach: Lab 1B

This portion of the lab involved a 3-lens setup (Fig 8) and a 4-lens setup (Fig 10). Markers with optimal lens positions were presented to us, with great attention to the distance between adjacent lenses. While the formulas from Lab 1A could be used to describe numerous-lens setups, the required algebra is lengthy. To make it more concise, it is common to use the ray vector:

$$\bar{v} = \begin{pmatrix} r \\ \theta \end{pmatrix}$$

Where r is the height of the ray and θ is the angle the ray makes relative to the optical axis.

From geometric optics, we know the following matrices describe light propagating in air and light propagating through a lens respectively:

$$M_{air} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \tag{8}$$

$$M_{lens} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \tag{9}$$

Where d is the horizontal distance the light travels for a specified region and f is the focal length of a specified lens. Using these matrices, one can describe the angle and height a light ray will follow. Multiplying these matrices according to a setup can thus give a compact description of light's behavior throughout the system.

A notable use of this matrix is that, after light passes through a lens, a certain d can be chosen such that row 1, column 1 (known as "B") of the multiplied-out matrix is zero. This result essentially dictates that, at said distance d, all rays, independent of their angle relative to the optical axis, will converge to a particular height. The significance of this value is that the value d to get B=0 is the image distance. If this value is negative, a virtual image is formed. If this value is positive, a real image is formed. The telescope aims to produce a virtual image.

To determine parameters for these matrices, we first measured the distance between all lenses in the pre-determined setups (3 lens setup: Fig 6, 4 lens setup [trial 1]: Fig 9). To find the focal length (Fig 11, we kept one lens on the optical rail at a time. We then put a piece of paper behind the lens and moved it around until we could see a sharp image of the eye chart on the paper. This was determined to be the focal length since, given the far distance the image was from the lenses, the image plane was approximately the focal point. Once the sharp image was found, we measured the length between the lens and the paper. Finally, to measure how far the eye test paper was from our setup, we counted the number of floor tiles between the optical setup and paper. We then multiplied this count by the length of the floor tiles (in centimeters). The floor tiles were square and uniform. These results are cumulated in Fig 5.

After this, we looked through L_C for both setups. We then guessed how far the virtual image seemed to be (3 lens setup: Fig 7, 4 lens setup: Fig 12).

VI. Results: Lab 1B

Distance from image to L1 [cm]	LA focal length [cm]	LB focal length [cm]	LC focal length [cm]	LBnew focal length [cm]
899.16	66	6.5	2	18.5

Fig. 5: Estimated Initial Object Distance and Focal Lengths

TRIAL	LA to LB[cm]	LB to LC[cm]
1	80	17

Fig. 6: Three Lens Distance Setup with Sharp Virtual Image

Daniel [cm]	Brenna [cm]	Maggie [cm]	Average [cm]
42	36	35	37.67

Fig. 7: Three Lens Setup Virtual Image Distance Guesses

VII. Discussion: Lab 1B

Fig 13 represents the magnification and object distance results my Python script (attached on email submission) came up with for both setups. The results were determined by first guessing a final distance value d such that B=0. In this form, the "A" entry of the matrix corresponds to the magnification of the image. These code results deviate tremendously from what we saw in the lab. Based on my script results, we should have seen a real image with an extremely tiny magnification.

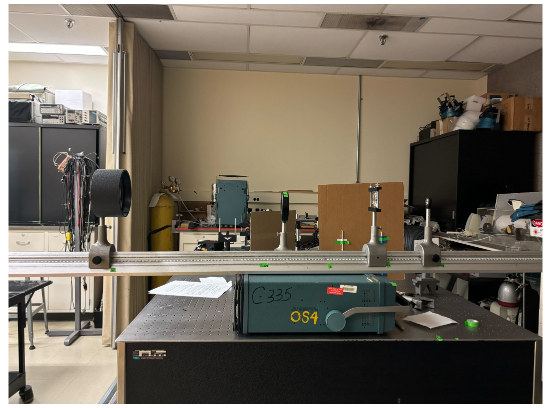

Fig. 8: Three Lens Setup. Lenses Left to Right: L_A, L_B, L_C

TRIAL	LA to LBnew[cm]	LBnew to LB[cm]	LB to LC[cm]
1	48	20	30.5
2	58	18	21.5

Fig. 9: Four Lens Distance Setups With Sharp Virtual Image

Fig. 10: Four Lens Setup. Lenses Left to Right: L_A, L_{Bnew}, L_B, L_C

Fig. 11: Focal Length Test Setup

Daniel [cm]	Brenna [cm]	Maggie [cm]	Average [cm]
27	26	24	25.67

Fig. 12: Four Lens Setup Virtual Image Guesses

However, in the lab, we saw a sharp, nicely magnified virtual image. One explanation for this strong deviation could be improper alignment of the lenses with the optical axis and their centers. If the centers of the lenses are not well-aligned with each other, the ABCD-matrix method of finding the magnification and image distance falls apart, as the method is built upon this assumption being true. Another cause of this deviation could be that we did not measure the focal lengths of the lenses properly. It is unlikely that we miscalculated the distances between the lenses and the distance the eye paper was to our optical setup, as it is very easy to double check that from the Figures attached. It is likely that we simply misrecorded a value, causing a large interruption in our matrix calculations. Some other sources of error in our optical setup may have included an unknown scratch or specs of dust on lenses, causing a virtual image to be seen experimentally but not theoretically. Finally, as realized in lab, extremely tiny changes in lens distances can throw off image focus. These tiny imperfections may have carried onto my matrix calculations for our measured data, resulting in an avalanche of wrong answers. Ultimately, I should not have expected my code to conclude that a real image would form.

	image distance [cm]	magnification
4 lens system	2.54	-0.016
3 lens system	1.603	0.04

Fig. 13: Code-Produced Image Distance and Magnification

Overall, the physical telescope worked extremely well. In the lab, we were able to see the letter ”E” extremely well. The image had little distortion and aberration. To prevent aberration and/or distortion, slightly nudging L_C backwards or forwards could put the image in better clarity. It might be the case that the lens is slightly off from its optimal position. If that fails, you could also shift the other lenses slightly.

Inserting a lens at the location of the first intermediate image has almost no effect on the system performance because, at the intermediate point, light has already converged. Since we are assuming the lens is thin, the lens has little distance to impact the direction of the rays, essentially nullifying its light-bending abilities.

Fig 14 and Fig 15 represent ray tracing diagrams for what I would have expected my code to produce. This diagram is more consistent with what I observed in the lens setups.

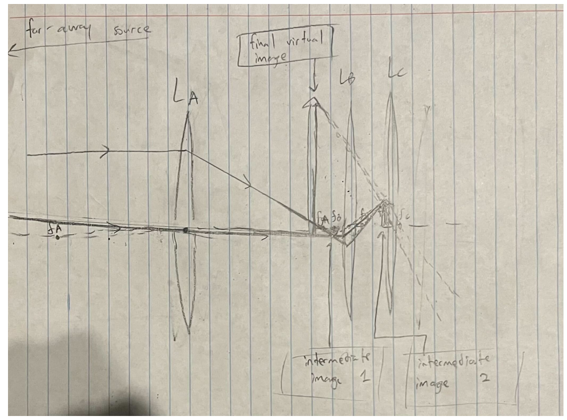

Fig. 14: Ideal Three Lens System Diagram with Virtual Image

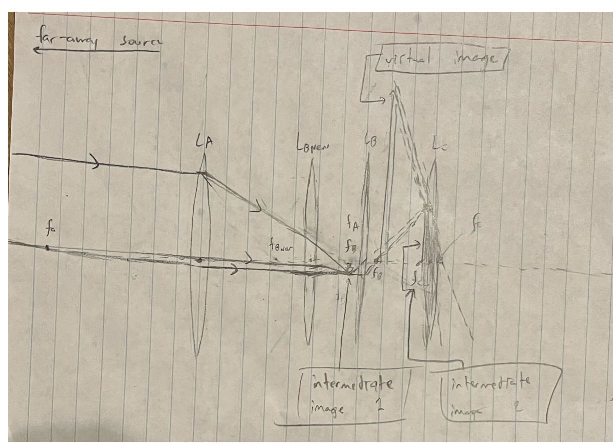

Fig. 15: Ideal Four Lens System Diagram with Virtual Image