Excercises for Sect. 1.1

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1.

(a)

$V = \{(a, b) \in \mathbb{R}^2 : 2a + 3b = 0\}, \mathbb{F} = \mathbb{R}$.
Closure under addition: $(a, b) + (c, d) = 2a + 3b + 2c + 3d = 2(a + c) + 3(b + d)$.
Closure under multiplication: $\alpha(a, b) = \alpha(2a + 3b) = 2\alpha a + 3\alpha b$.
V is a vector space

(b)

$V = \{(a, b) \in \mathbb{R}^2 : a + b = 1\}, \mathbb{F} = \mathbb{R}$.
NOT closed under addition: $(a, b) + (c, d) = 2$.
V is NOT a vector space.

(c)

$V = \{(a, b) \in \mathbb{R}^2 : ab = 0\}, \mathbb{F} = \mathbb{R}$.
NOT closed under addition: $(1, 0) + (0, 1) = (1, 1)$, $1(1) \neq 0$.
V is NOT a vector space.

2.

(a)

$\alpha(a, b) := (a, 0)$
Does NOT satisfy Definition 1.1.8:
$1(a, b) = (a, 0) \neq (a, b)$
V is NOT a vector space.

(b)

$\alpha(a, b) := (b, \alpha a)$
Does NOT satisfy Definition 1.1.6:

$(\alpha + \beta)(a, b) = (b, (\alpha + \beta)a)$
$\alpha(a, b) + \beta(a, b) = (b, \alpha a) + (b, \beta a) = (2b, (\alpha + \beta)a)$ V is NOT a vector space.

(c)

$\alpha(a, b) := (\alpha a, -\alpha b)$
Does NOT satisfy Definition 1.1.7:
$(\alpha\beta)(a, b) = (\alpha\beta a, -\alpha\beta b)$
$\alpha(\beta(a, b)) = \alpha(\beta a, -\beta b) = (\alpha\beta a, (-\alpha) - \beta b) = (\alpha\beta a, \alpha\beta b)$
V is NOT a vector space.

(d)

$\alpha(a, b) := (\alpha a, \frac{b}{\alpha})$
Does NOT satisfy Definition 1.1.6:
$(\alpha + \beta)(a, b) = ((\alpha + \beta)a, (\frac{1}{\alpha+\beta}b))$
$\alpha(a, b) + \beta(a, b) = (\alpha a, \frac{1}{\alpha}b) + (\beta a, \frac{1}{\beta}b) = ((\alpha + \beta)a, (\frac{1}{\alpha} + \frac{1}{\beta})b)$
V is NOT a vector space.

8.

Door prize to anyone who can solve this.

9.

Closure under multiplication:
$(\alpha + \beta)x = x, \alpha x + \beta x = x + x = x$
No additive inverse $x + (-x) = x + (-1)x = x + x = x \neq 0$
V is NOT a vector space.