Excercises for Sect. 1.3

Student

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1.

 $(e_1 + e_2, e_2 + e_3, e_3 + e_1) = (e_1, e_2, e_3) + (e_2, e_3, e_1)$ and (e_1, e_2, e_3) spans \mathbb{R}^3 so by Theorem 1.17.4 then $(e_1 + e_2, e_2 + e_3, e_3 + e_1)$ also spans \mathbb{R}^3 .

3.

There are only 3 vectors for a 4 dimensional space so can not fully span the space.

For example the element $\frac{13t^3}{6} + \frac{7t^2}{12} - \frac{2t}{3} + 1$ cannot be created from any combination of the original elements.

6.

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9.

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12.

 \mathbf{a}

(1,0,0,0), (-1,1,0,0), (0,-1,0,0), (0,0,1,1)

 \mathbf{b}

First setting a=(1,0,0,0) and b=(0,1,0,0) yields c=(-5,-2,0,0) and d=(-7,-4,0,0)

 \mathbf{c}

Setting $(a,b,c)=(e_1,e_2,e_3)$ yields $\mathbf{d}=(1/4,-1/2,3/4,0)$