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7,5

① Definimos a matriz ampliada do sistema como:

$$A = \left|\begin{array}{ccc:c} 2 & -3 & 0 & -4 \\ 0 & 3 & 5 & 21 \\ 1 & 1 & -1 & 0 \end{array}\right|$$

Resolvendo através do MEG com pivoteamento parcial:

$L_3 \to L_3 - 1/2\, L_1$

$L_1 \to L_1$

$L_2 \to L_2$

$$A = \left|\begin{array}{ccc:c} 2 & -3 & 0 & -4 \\ 0 & 3 & 5 & 21 \\ 0 & 2{,}5 & -1 & 2 \end{array}\right|$$

Pivô e maior elemento na coluna = 2

C

$L_3 \to L_3 - \frac{2{,}5}{3} L_2$

$L_1 \to L_1$

$L_2 \to L_2$

$$A = \left|\begin{array}{ccc:c} 2 & -3 & 0 & -4 \\ 0 & 3 & 5 & 21 \\ 0 & 0 & \frac{-15{,}5}{3} & -15{,}5 \end{array}\right|$$

Pivô e maior elemento na coluna = 3

C

Resolvendo o sistema:

$$S: \begin{cases} 2j_1 - 3j_2 = -4 \\ 3j_2 + 5j_3 = 21 \\ -\frac{19,5}{3} j_3 = -19,5 \end{cases}$$

$\rightarrow j_1 = \frac{-4 + 3(2)}{2} = 1$

$\rightarrow j_2 = \frac{21 - 5(3)}{3} = 2$

$\rightarrow j_3 = 3$

Portanto:

$$\boxed{S = \begin{cases} j_1 = 1 \\ j_2 = 2 \\ j_3 = 3 \end{cases}}$$

C

② Definimos a matriz ~~ampliada~~ dos coeficientes do sistema:

$$A = \begin{bmatrix} 20 & 7 & 9 \\ 7 & 30 & 8 \\ 9 & 8 & 30 \end{bmatrix}$$

como a matriz A é simétrica, assumimos que ela é definida positiva e utilizamos a fatoração Cholesky.

$$A = G^t G \rightarrow \begin{bmatrix} 20 & 7 & 9 \\ 7 & 30 & 8 \\ 9 & 8 & 30 \end{bmatrix} = \begin{bmatrix} g_{11} & 0 & 0 \\ g_{12} & g_{22} & 0 \\ g_{13} & g_{23} & g_{33} \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ 0 & g_{22} & g_{23} \\ 0 & 0 & g_{33} \end{bmatrix}$$

1ª coluna:

$$\begin{bmatrix} 20 \\ 7 \\ 9 \end{bmatrix} = \begin{bmatrix} g_{11} & 0 & 0 \\ g_{12} & g_{22} & 0 \\ g_{13} & g_{23} & g_{33} \end{bmatrix} \begin{bmatrix} g_{11} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{20} \\ 7/\sqrt{20} \\ 9/\sqrt{20} \end{bmatrix}$$

C

Contas?

Temos:

$$G = \begin{bmatrix} \sqrt{20} & 7/\sqrt{20} & 9/\sqrt{20} \\ 0 & g_{22} & g_{23} \\ 0 & 0 & g_{33} \end{bmatrix}$$

2ª coluna:

$$\begin{bmatrix} 7 \\ 30 \\ 8 \end{bmatrix} = \begin{bmatrix} \sqrt{20} & 0 & 0 \\ 7/\sqrt{20} & g_{22} & 0 \\ 9/\sqrt{20} & g_{23} & g_{33} \end{bmatrix} \begin{bmatrix} 7/\sqrt{20} \\ g_{22} \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{551/20} \\ \dfrac{1940}{20\sqrt{551}} \end{bmatrix}$$

(-2,5) Contas?

não apresentou as contas completas

Digitalizado com CamScanner

assim:

$$G = \begin{bmatrix} \sqrt{20} & 7/\sqrt{20} & 9/\sqrt{20} \\ 0 & \sqrt{551/20} & \dfrac{97}{20\sqrt{\frac{551}{20}}} \\ 0 & 0 & g_{33} \end{bmatrix}$$

3ª coluna:

$$\begin{bmatrix} 9 \\ 8 \\ 30 \end{bmatrix} = \begin{bmatrix} \sqrt{20} & 0 & 0 \\ 7/\sqrt{20} & \sqrt{\frac{551}{20}} & 0 \\ 9/\sqrt{20} & \dfrac{97}{20\sqrt{\frac{551}{20}}} & g_{33} \end{bmatrix} \begin{bmatrix} 9/\sqrt{20} \\ \dfrac{97}{20}\sqrt{551/20} \\ g_{33} \end{bmatrix} =$$

$$\begin{bmatrix} \\ \\ 5{,}00960964 \end{bmatrix}$$

Contas?

Portanto:

$$G = \begin{bmatrix} \sqrt{20} & 7/\sqrt{20} & 9/\sqrt{20} \\ 0 & \sqrt{551/20} & \frac{97}{20\sqrt{551/20}} \\ 0 & 0 & 5{,}00960964 \end{bmatrix}$$

assim escrevemos: $Ax = b$

$$\rightarrow G^{t} G x = b$$

$$\rightarrow G^{t} y = b \quad \cancel{(y = G)}$$

$$\rightarrow G x = y$$

Portanto:

$$G^{t} \cdot y = b \rightarrow \begin{bmatrix} \sqrt{20} & 0 & 0 \\ 7/\sqrt{20} & \sqrt{551/20} & 0 \\ 9/\sqrt{20} & \frac{97}{20\sqrt{551/20}} & 5{,}00960964 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 38 \\ 38 \end{bmatrix}$$

Resolvendo o sistema:

$$\begin{cases} \sqrt{20}\, y_1 = 16 \\ 7/\sqrt{20}\, y_1 + \sqrt{551/20}\, y_2 = 38 \\ 9/\sqrt{20}\, y_1 + \frac{97}{20\sqrt{551/20}}\, y_2 + 5{,}00960964\, y_3 = 38 \end{cases}$$

$$y_1 = 16/\sqrt{20} \rightarrow y_1 = 3{,}577708764$$

$$y_2 = \frac{38 - \left(7/\sqrt{20} \cdot y_1\right)}{\sqrt{551/20}} = 6{,}172828465$$

$$y_3 = \frac{38 - \left(9/\sqrt{20}\, y_1\right) - \left(\frac{97}{20\sqrt{551/20}}\, y_2\right)}{5{,}00960964} =$$

$$y_3 = \frac{38 - 7{,}2 - 5{,}703811253}{5{,}00960964} = 5{,}00960964$$

Assim:

$$Gx = y \rightarrow \begin{bmatrix} \sqrt{20} & 7/\sqrt{20} & 9/\sqrt{20} \\ 0 & \sqrt{551/20} & \frac{97}{20\sqrt{551/20}} \\ 0 & 0 & 5{,}00960964 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3{,}577708764 \\ 6{,}172828465 \\ 5{,}00960964 \end{bmatrix}$$

Obtemos o sistema

$$\begin{cases} \sqrt{20}\, x + 7/\sqrt{20}\, y + 9/\sqrt{20}\, z = 3{,}577708764 \\ \sqrt{551/20}\, y + \frac{97}{20\sqrt{551/20}}\, z = 6{,}172828465 \\ 5{,}00960964\, z = 5{,}00960964 \end{cases}$$

Resolvendo:

$$z = \frac{5,00960964}{5,00960964} = 1$$

$$y = \frac{6,172828465 - 0,924019075}{5,248809389} = 1$$

$$x = \frac{3,577708764 - \frac{9}{\sqrt{20}}z - \frac{7}{\sqrt{20}}y}{\sqrt{20}} = 0$$

Portanto:

$$\boxed{S: \begin{cases} x = 0 \\ y = 1 \\ z = 1 \end{cases}}$$