# HW2

AUTHOR
Dan Schumacher

PUBLISHED
September 20, 2023

```r
library(ggplot2)
library(tidyverse)
```

```
── Attaching core tidyverse packages ──────────────────────── tidyverse 2.0.0 ──
✔ dplyr     1.1.2     ✔ readr     2.1.4
✔ forcats   1.0.0     ✔ stringr   1.5.0
✔ lubridate 1.9.2     ✔ tibble    3.2.1
✔ purrr     1.0.1     ✔ tidyr     1.3.0
── Conflicts ──────────────────────────────────────────── tidyverse_conflicts() ──
✖ dplyr::filter() masks stats::filter()
✖ dplyr::lag()    masks stats::lag()
ℹ Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```r
library(DescTools)
library(MASS)
```

```

Attaching package: 'MASS'

The following object is masked from 'package:dplyr':

    select
```

```r
library(car)
```

```
Loading required package: carData

Attaching package: 'car'

The following object is masked from 'package:DescTools':

    Recode

The following object is masked from 'package:dplyr':

    recode

The following object is masked from 'package:purrr':

    some
```

# Algorithms 1 Homework 2

## Exercise 1: Analysis of Variance

The heartbpchol.csv data set contains continuous cholesterol (Cholesterol) and blood pressure status (BP_Status) (category: High/ Normal/ Optimal) for alive patients.

For the heartbpchol data set, consider a one-way ANOVA model to identify differences between group cholesterol means. The normality assumption is reasonable, so you can proceed without testing normality.

Perform a one-way ANOVA for Cholesterol with BP_Status as the categorical predictor. Comment on statistical significance of BP_Status, the amount of variation described by the model, and whether or not the equal variance assumption can be trusted.

- *Set Up*

```
df_hearts <- read.csv('heartbpchol.csv', header = T)
head(df_hearts)
```

```
  Cholesterol BP_Status
1         221   Optimal
2         188      High
3         292      High
4         319    Normal
5         205    Normal
6         247      High
```

```
df_hearts$BP_Status = as.factor(df_hearts$BP_Status)
head(df_hearts)
```

```
  Cholesterol BP_Status
1         221   Optimal
2         188      High
3         292      High
4         319    Normal
5         205    Normal
6         247      High
```

- *Is our data balanced?*

```
# it isn't balanced but the tests stay the same for 1-way ANOVA
table(df_hearts$BP_Status)
```

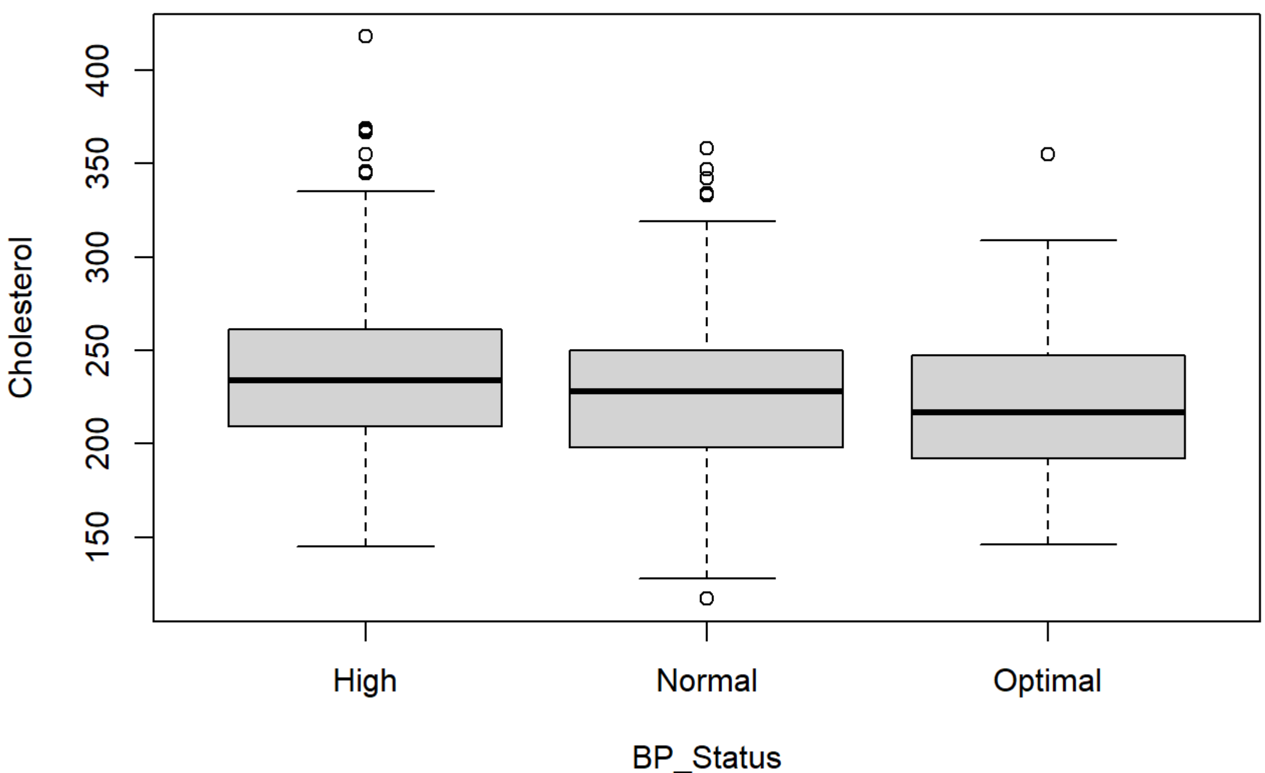
```
   High  Normal Optimal 
    229     245      67 
```

- *Assumptions of ANOVA*
  - *~~Variable types~~: 1 quantitative, 1 qualitative*
  - *~~Independence~~: representative random sample*
  - *~~Normality~~: We are told in the instructions to trust (normal qqplot looked good)*
  - *~~Equal Variances~~: We will test with a box plot and3 (passed with Levene*
- *ANOVA Hypothesis*

  *H0:* $\mu_H = \mu_N = \mu_O$

  *Halt: @ least 1 not equal.*
- *with Levene's p-val of 0.8332 we cannot reject Levene's H0: Equal variance amongst groups*
- *With an ANOVA p-value of .00137, we can reject H0, and accept Halt: At least one group has a different mean.*

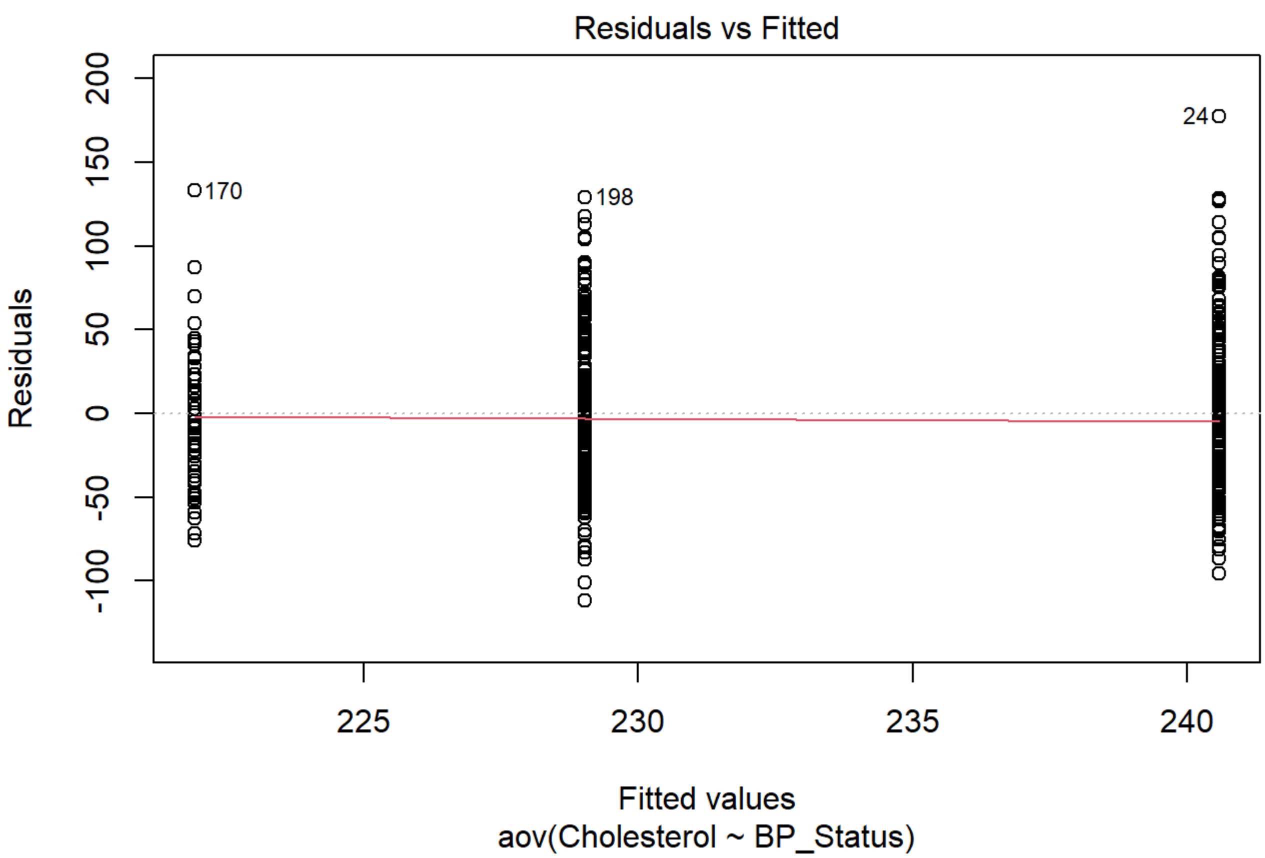
```
# The Variance looks pretty equal, Optimal is a little smaller.
boxplot(Cholesterol ~ BP_Status, data = df_hearts)
```

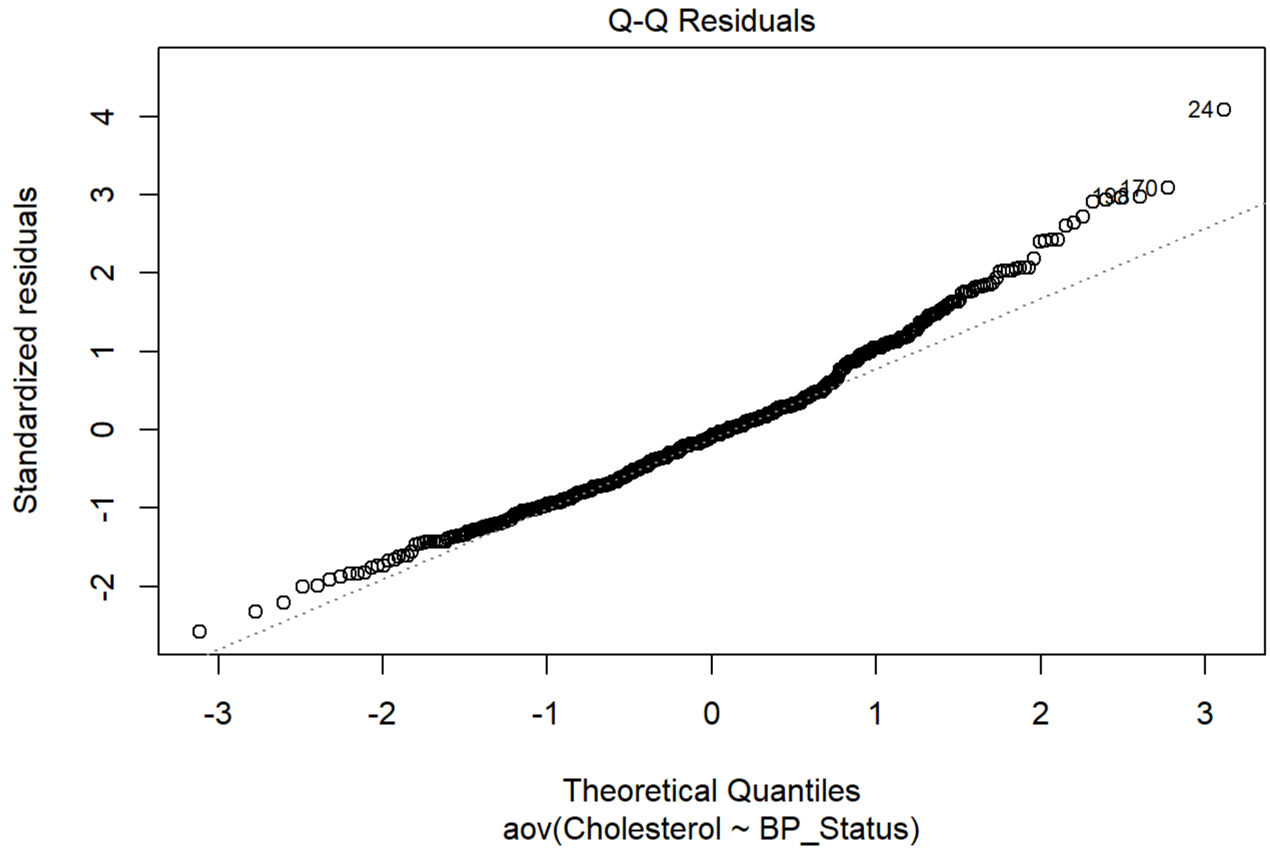


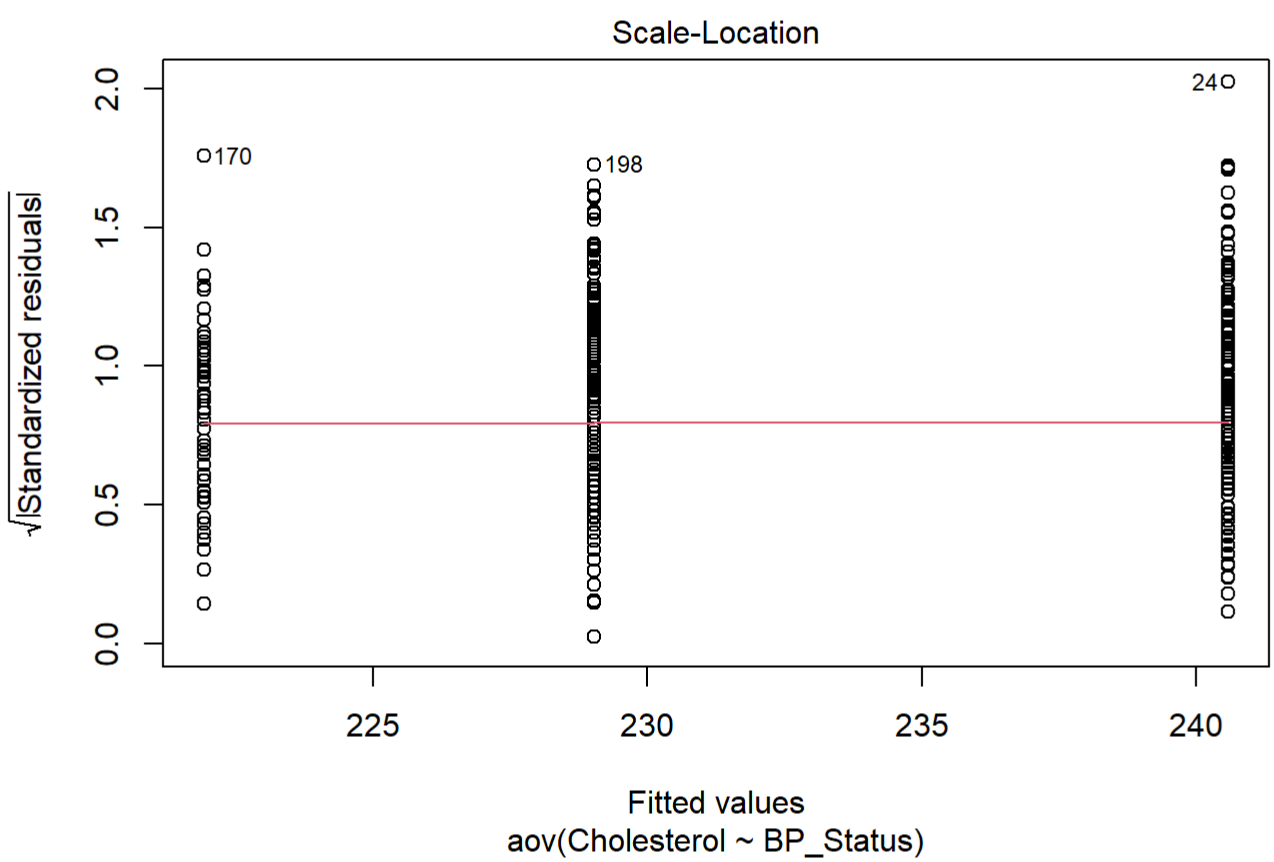
```
# what does this step do?
# we use the anova model before verify assumptions to take advantage of the LT test
# this generates anova output
aov.hearts = aov(Cholesterol ~ BP_Status, data = df_hearts)

# tests for equal var (use for 1-way ANOVA or 2-way Balanced*** maybe)
# p-val = 0.8332 can't reject H0: Equal variance amongst groups
LeveneTest(aov.hearts)
```

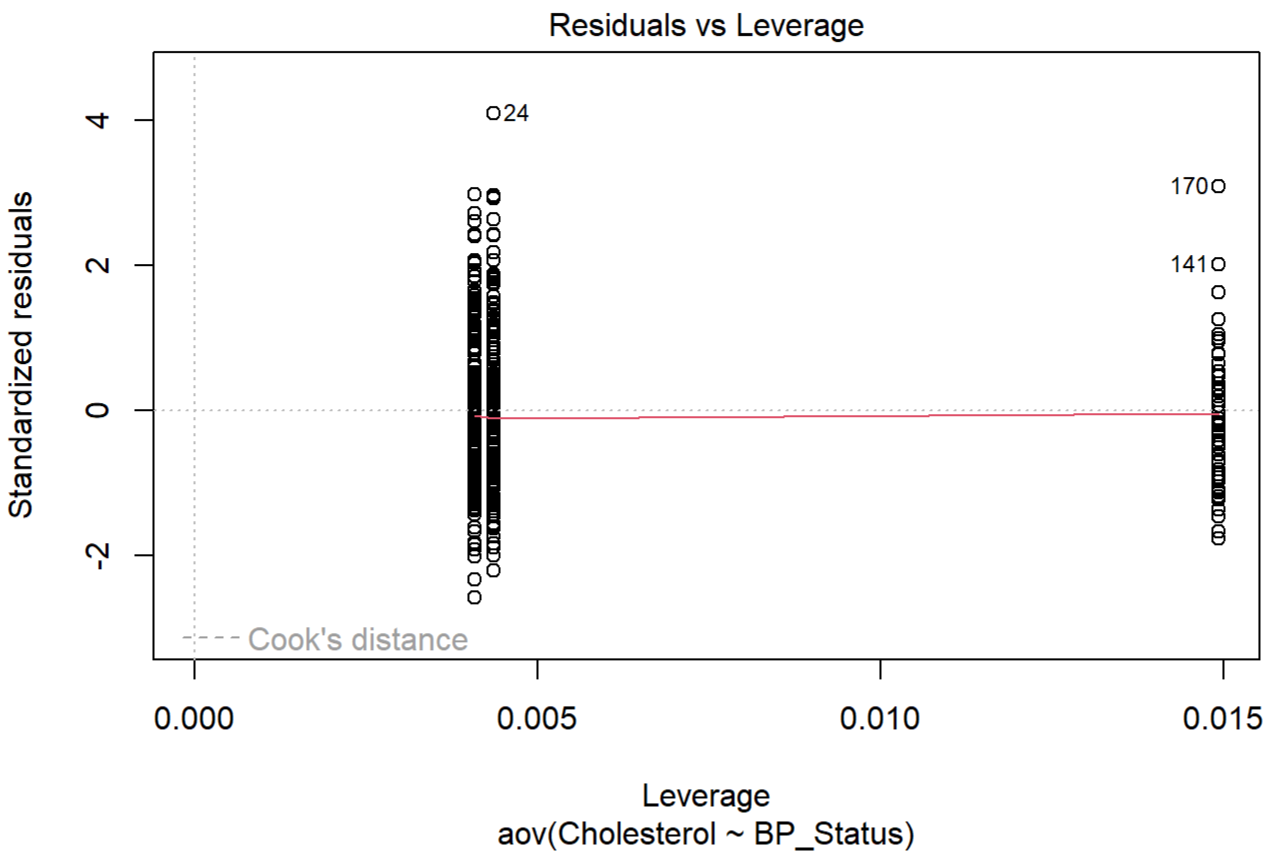
```
Levene's Test for Homogeneity of Variance (center = median)
       Df F value Pr(>F)
group   2  0.1825 0.8332
      538
```

```
# visually test equal var Larger than 2 way just use visual
plot(aov.hearts)
```




Q-Q Residuals
Standardized residuals
Theoretical Quantiles
aov(Cholesterol ~ BP_Status)
24
170
198


Scale-Location
√|Standardized residuals|
Fitted values
aov(Cholesterol ~ BP_Status)
24
170
198




```
# now we can interpret aov results
# with a p-val .00137 we reject H0 and accept Halt: at least 1 group has ***
summary(aov.hearts)
```

```
             Df  Sum Sq Mean Sq F value  Pr(>F)   
BP_Status     2   25211   12605   6.671 0.00137 **
Residuals   538 1016631    1890                   
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Comment on any significantly different cholesterol means as determined by the post-hoc test comparing all pairwise differences. Specifically explain what that tells us about differences in cholesterol levels across blood pressure status groups, like which group has the highest or lowest mean values of Cholesterol.

- $\mu_H > \mu_N$ *High > Normal*
- $\mu_H > \mu_O$ *High > Optimal*
- *The high blood pressure group has a higher average cholesterol than both the normal and optimal group.*

  ```
  ScheffeTest(aov.hearts)
  ```

```
  Posthoc multiple comparisons of means: Scheffe Test
```

```
        95% family-wise confidence level

    $BP_Status
                          diff     lwr.ci     upr.ci    pval
    Normal-High     -11.543481 -21.35092 -1.736038 0.0159 *
    Optimal-High    -18.646679 -33.46702 -3.826341 0.0089 **
    Optimal-Normal   -7.103198 -21.81359  7.607194 0.4958

    ---
    Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## Exercise 2: Analysis of Variance

For this problem use the bupa.csv data set. [Check UCI Machine Learning Repository]() for more information. The mean corpuscular volume and alkaline phosphates are blood tests thought to be sensitive to liver disorder related to excessive alcohol consumption. We assume that normality and independence assumptions are valid.

```r
df_bupa <- read.csv('bupa.csv')
head(df_bupa)
```

```
  mcv alkphos drinkgroup
1  85      92          1
2  85      64          1
3  86      54          1
4  91      78          1
5  87      70          1
6  98      55          1
```

```r
#change drinkgroup to factor var
df_bupa$drinkgroup <- as.factor(df_bupa$drinkgroup)
head(df_bupa)
```

```
  mcv alkphos drinkgroup
1  85      92          1
2  85      64          1
3  86      54          1
4  91      78          1
5  87      70          1
6  98      55          1
```

| Variable Name | Description |
|---|---|
| mcv | mean corpuscular volume |
| alkphos | alkaline phosphatase |
| drinkgroup | categorization of the half-pint equivalents of alcoholic beverages drunk per day: |

| Variable Name | Description |
|---|---|
| | **group 1 :** less than 1 drink. |
| | **group 2 :** at least 1 but fewer than 3 drinks. |
| | **group 3 :** at least 3 but fewer than 6 drinks. |
| | **group 4 :** at least 6 but fewer than 9 drinks. |
| | **group 5 :** 9 or more drinks. |

Perform a one-way ANOVA for mcv as a function of drinkgroup. Comment on significance of the drinkgroup, the amount of variation described by the model, and whether or not the equal variance assumption can be trusted.

- *with Levene's p-val of .87 we cannot reject Levene's H0: Equal variance amongst groups*
- *With an ANOVA p-value of 7.43e-08, we can reject H0, and accept Halt: At least one group has a different mean.*

```
aov.mcv = aov(mcv ~ drinkgroup , data = df_bupa)
LeveneTest(aov.mcv)
```

```
Levene's Test for Homogeneity of Variance (center = median)
       Df F value Pr(>F)
group   4  0.3053 0.8744
      340
```

```
summary(aov.mcv)
```

```
             Df Sum Sq Mean Sq F value   Pr(>F)    
drinkgroup    4    733  183.29   10.26 7.43e-08 ***
Residuals   340   6073   17.86                     
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Perform a one-way ANOVA for alkphos as a function of drinkgroup. Comment on statistical significance of the drinkgroup, the amount of variation described by the model, and whether or not the equal variance assumption can be trusted.

- *With Levene's p-val of .52 we cannot reject Levene's H0: Equal variance amongst groups*

- *With an ANOVA p-val of .00495, we can reject H0 and accept Halt: At least one group has a different mean.*

```
aov.alkphos = aov(alkphos ~ drinkgroup , data = df_bupa)
LeveneTest(aov.alkphos)
```

```
Levene's Test for Homogeneity of Variance (center = median)
       Df F value Pr(>F)
group   4  0.8089 0.5201
      340
```

```
summary(aov.alkphos)
```

```
             Df Sum Sq Mean Sq F value  Pr(>F)
drinkgroup    4   4946  1236.4   3.792 0.00495 **
Residuals   340 110858   326.1
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## Post Hoc

Perform post-hoc tests for models in mcv and alkphos.

### MCV Post Hoc

- $\mu_4 > \mu_1$
  *People that drink between 6 and 8 drinks a day have an average MCV of 3.74 higher than people who have less than 1 drink a day.*

- $\mu_5 > \mu_1$
  *People that drink 9 or more drinks a day have an average MCV of 3.75 higher than people who have less than 1 drink a day.*

- $\mu_4 > \mu_2$
  *People that drink between 6 and 8 drinks a day have an average MCV of 2.50 higher than people who 1 or 2 drinks per day.*

- $\mu_4 > \mu_3$
  *People that drink between 6 and 8 drinks a day have an average MCV of 2.81 higher than people who 3 to 5 drinks per day.*

```
ScheffeTest(aov.mcv)
```

```
  Posthoc multiple comparisons of means: Scheffe Test
    95% family-wise confidence level

$drinkgroup
            diff      lwr.ci    upr.ci    pval
```

```
2-1  1.241452991 -0.94020481 3.423111  0.5410
3-1  0.938131313 -0.90892674 2.785189  0.6495
4-1  3.744610282  1.73913894 5.750082 1.9e-06 ***
5-1  3.746031746  0.64379565 6.848268  0.0081 **
3-2 -0.303321678 -2.59291786 1.986275  0.9966
4-2  2.503157290  0.08395442 4.922360  0.0380 *
5-2  2.504578755 -0.87987039 5.889028  0.2646
4-3  2.806478969  0.68408993 4.928868  0.0025 **
5-3  2.807900433 -0.37116998 5.986971  0.1151
5-4  0.001421464 -3.27222796 3.275071  1.0000

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

### Alkphos Post Hoc

- $\mu_5 > \mu_2$
  *People that drink 9 or more drinks a day have a 15.22 higher average alkaline phosphatase level than people who 1 or 2 drinks per day.*

- $\mu_5 > \mu_3$
  *People that drink 9 or more drinks a day have a 16.62 higher average alkaline phosphatase level than people who 3 to 5 drinks per day.*

```r
ScheffeTest(aov.alkphos)
```

```
  Posthoc multiple comparisons of means: Scheffe Test
    95% family-wise confidence level

$drinkgroup
         diff      lwr.ci    upr.ci   pval
2-1 -2.645299 -11.9663647  6.675766 0.9419
3-1 -4.056138 -11.9476367  3.835360 0.6389
4-1 -1.148743  -9.7170578  7.419571 0.9965
5-1 12.572650  -0.6815582 25.826857 0.0734 .
3-2 -1.410839 -11.1930681  8.371390 0.9953
4-2  1.496556  -8.8394138 11.832525 0.9952
5-2 15.217949   0.7579944 29.677903 0.0329 *
4-3  2.907395  -6.1604467 11.975236 0.9117
5-3 16.628788   3.0463078 30.211268 0.0069 **
5-4 13.721393  -0.2651729 27.707959 0.0578 .

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

### Comparing MCV and Alkphos Post Hocs

Comment on any similarities or differences you observe from their results.

- *There was never a significant difference in means (in either the MCV or alkphos test) when comparing a group that drank <u>less</u> to a group that drank <u>more</u>. The significant difference always showed a heavier drinking group with a larger mean than the lighter drinkers.*

## Exercise 3

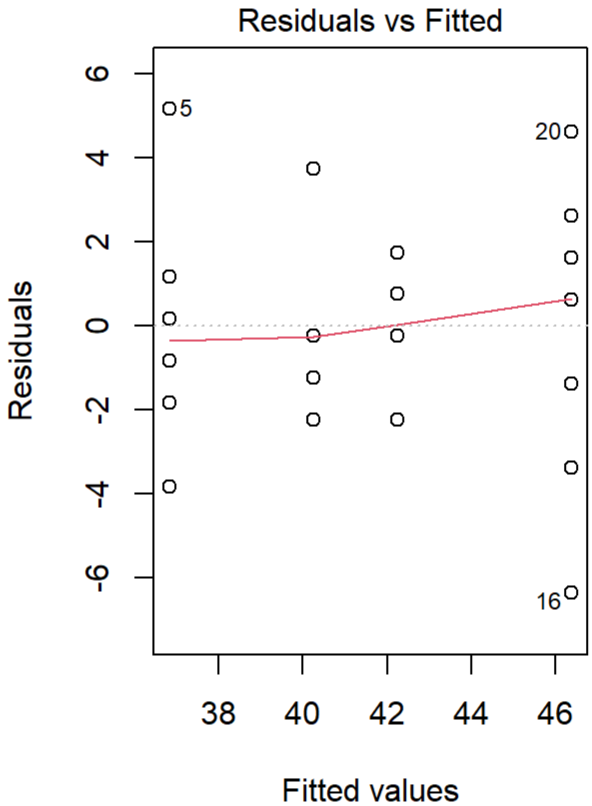
The psychology department at a hypothetical university has been accused of underpaying female faculty members. The data represent salary (in thousands of dollars) for all 22 professors in the department. This problem is from Maxwell and Delaney (2004).

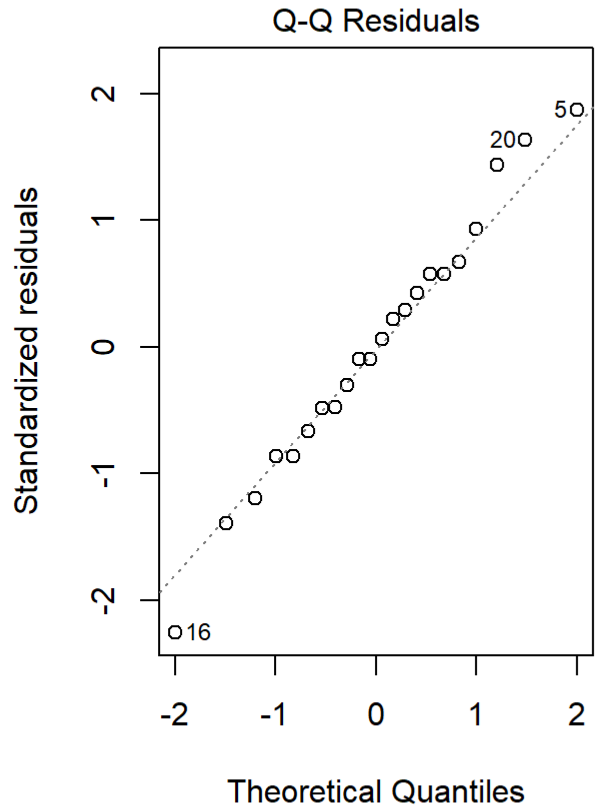
```
df_psych <- read.csv('psych.csv')
df_psych$sex <- as.factor(df_psych$sex)
df_psych$rank <- as.factor(df_psych$rank)
head(df_psych,22)
```

```
   sex   rank salary
1    F Assist     33
2    F Assist     36
3    F Assist     35
4    F Assist     38
5    F Assist     42
6    F Assist     37
7    M Assist     39
8    M Assist     38
9    M Assist     40
10   M Assist     44
11   F  Assoc     42
12   F  Assoc     40
13   F  Assoc     44
14   F  Assoc     43
15   M  Assoc     43
16   M  Assoc     40
17   M  Assoc     49
18   M  Assoc     47
19   M  Assoc     48
20   M  Assoc     51
21   M  Assoc     48
22   M  Assoc     45
```

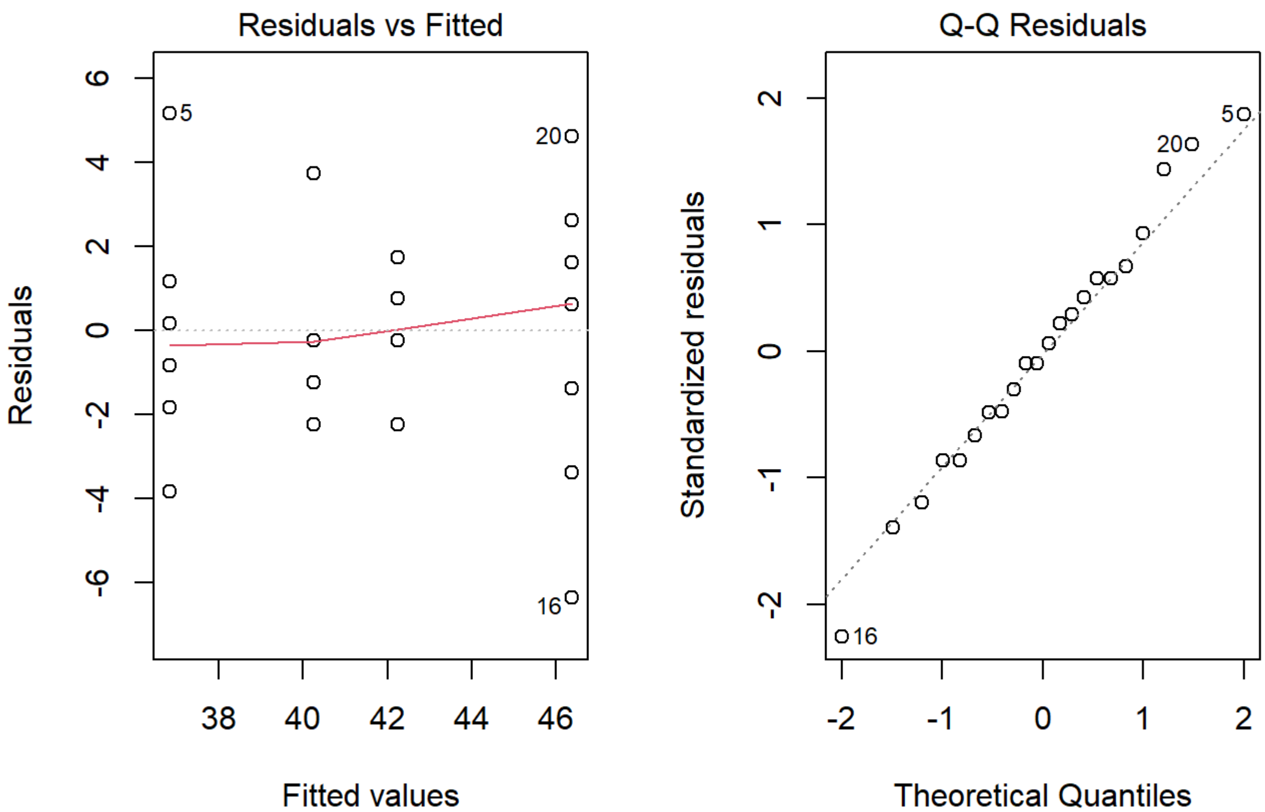
Fit a two-way ANOVA model including sex (F, M) and rank (Assistant, Associate) and the interaction term.

```
# Make ANOVA model
# We have an unbalanced data set. so lets set up Sex based off of rank and rank based off of sex
aov.psych.sr <- aov(salary ~ sex * rank, data = df_psych)
aov.psych.rs <- aov(salary ~ rank * sex, data = df_psych)

#Check for Variance with 2-way, just use visual
par(mfrow = c(1,2)) ; plot(aov.psych.sr, c(1,2))
```




```
par(mfrow = c(1,2)) ; plot(aov.psych.rs, c(1,2))
```




```
#looks good!

# These are Type 1 tests sr = sex -> rank. rs is reverse
summary(aov.psych.sr)
```

```
            Df Sum Sq Mean Sq F value   Pr(>F)
sex          1 155.15  155.15  17.007 0.000637 ***
rank         1 169.82  169.82  18.616 0.000417 ***
sex:rank     1   0.63    0.63   0.069 0.795101
Residuals   18 164.21    9.12
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(aov.psych.rs)
```

```
            Df Sum Sq Mean Sq F value   Pr(>F)
rank         1 252.22  252.22  27.647 5.33e-05 ***
sex          1  72.76   72.76   7.975   0.0112 *
rank:sex     1   0.63    0.63   0.069   0.7951
Residuals   18 164.21    9.12
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Anova(aov.psych.rs, type = 3)
```

```
Anova Table (Type III tests)

Response: salary
            Sum Sq Df  F value  Pr(>F)    
(Intercept) 8140.2  1 892.2994 < 2e-16 ***
rank          70.4  1   7.7189 0.01240 *  
sex           28.0  1   3.0711 0.09671 .  
rank:sex       0.6  1   0.0695 0.79510    
Residuals    164.2 18                     
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

What do the Type 1 and Type 3 sums of squares tell us about significance of effects? Is the interaction between sex and rank significant? Also comment on the variation explained by the model.

*I ran the following tests and got the following results:*

- *Type 1 sex * rank*
  - *Tells us that there is a significant difference in mean salary based on both rank and sex not off the interaction effect of the two.*
- *Type 1 rank * sex*
  - *Tells us the same as above ^^^*
- *Type 3*
  - *Tells us that only rank has a significant difference in mean salary.*

*Variation Explained:*

- *All of the tests above had a high f-value for rank. And both the Type 1 tests had a high f-value for sex. This means that our model explains the variance extremely well in those cases.*

Refit the model without the interaction term.

```
aov.psych.no_interaction_term <-
  aov(salary ~ sex + rank, data = df_psych)

summary(aov.psych.no_interaction_term)
```

```
            Df Sum Sq Mean Sq F value   Pr(>F)    
sex          1  155.2  155.15   17.88 0.000454 ***
rank         1  169.8  169.82   19.57 0.000291 ***
Residuals   19  164.8    8.68                     
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
Anova(aov.psych.no_interaction_term, type = 3)
```

```
Anova Table (Type III tests)

Response: salary
             Sum Sq Df   F value    Pr(>F)    
(Intercept) 10227.6  1 1178.8469 < 2.2e-16 ***
sex            72.8  1    8.3862 0.0092618 ** 
rank          169.8  1   19.5743 0.0002912 ***
Residuals     164.8 19                        
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Comment on the significance of effects and variation explained. Report and interpret the Type 1 and Type 3 tests of the main effects. Are the main effects of rank and sex significant?

- *Type 1*
  - *Tells us that there is a significant difference in mean salary based on both rank and sex.*
- *Type 3*
  - *Tells us the same as above ^^^*

Obtain model diagnostics to validate your Normality assumptions. Choose a final model based on your results from parts (a) and (b). Comment on any significant group differences through the post-hoc test.

```r
# I have already checked for Normality above.

# I picked the aov.psych.no_interaction_term as my model because we are primarily concered about 

ScheffeTest(aov.psych.no_interaction_term)
```

```

  Posthoc multiple comparisons of means: Scheffe Test 
    95% family-wise confidence level

$sex
        diff   lwr.ci   upr.ci   pval    
M-F 5.333333 1.986138 8.680529 0.0018 ** 

$rank
                 diff   lwr.ci   upr.ci   pval    
Assoc-Assist 5.377778 2.030582 8.724973 0.0017 ** 

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

State the differences in salary across different main effect groups and interaction (if included) between them.

- Males make $5,340 more on average than females.

- Associates make $5,378 more on average than assistants

## Exercise 4

### Set Up

```
df_cars <- read.csv('cars_new.csv')
df_cars$cylinders <- as.factor(df_cars$cylinders)
df_cars$origin <- as.factor(df_cars$origin)
df_cars$type <- as.factor(df_cars$type)
head(df_cars)
```

```
    type origin cylinders mpg_highway
1  Sedan   Asia         4          31
2  Sedan   Asia         4          29
3  Sedan   Asia         6          28
4  Sedan   Asia         6          24
5  Sedan   Asia         6          24
6 Sports   Asia         6          24
```

```
#our data is unbalanced
table(df_cars$cylinders);table(df_cars$origin);table(df_cars$type)
```

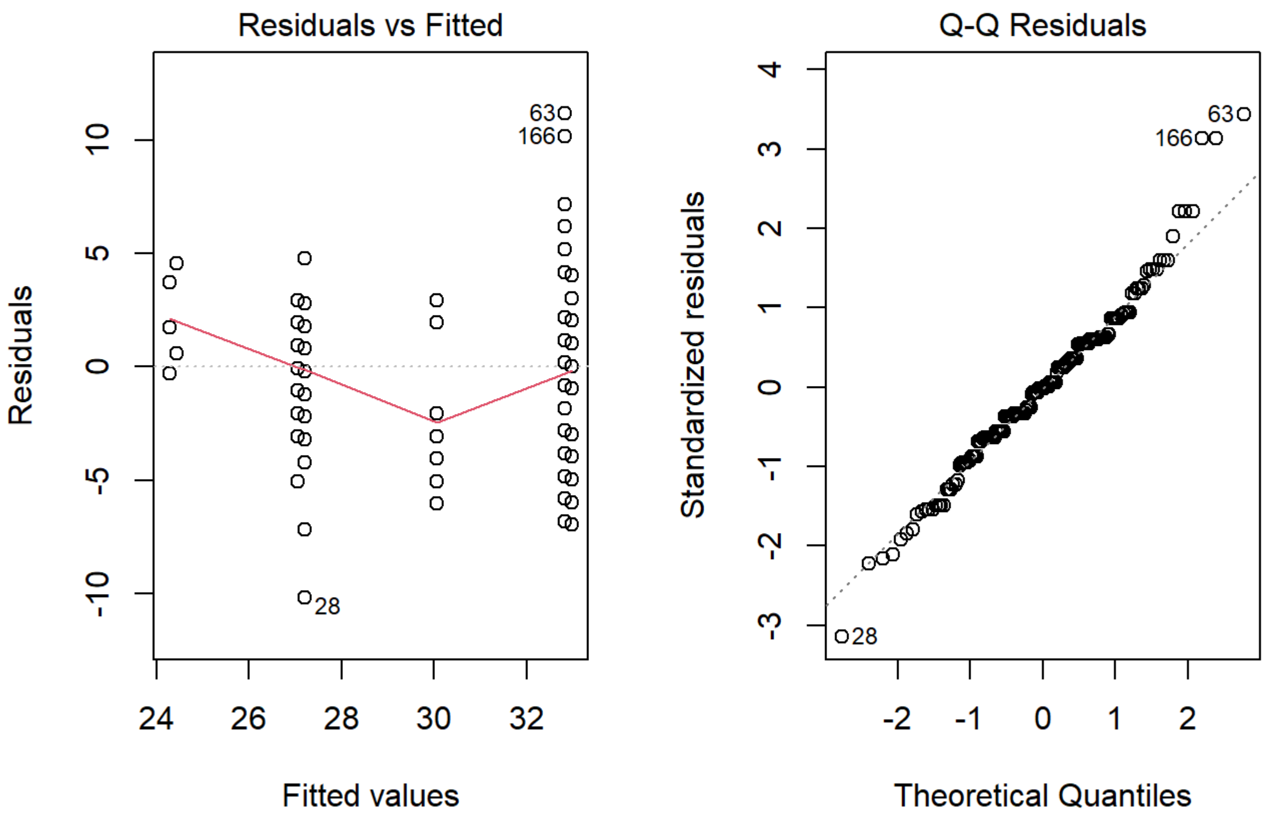
```
 4  6
86 94
```

```
Asia  USA
 104   76
```

```
 Sedan Sports
   164     16
```

a) Start with a three-way main effects ANOVA and choose the best main effects ANOVA model for mpg_highway as a function of cylinders, origin, and type for the cars in this set. Comment on which terms should be kept in a model for mpg_highway and why based on Type 3 SS.

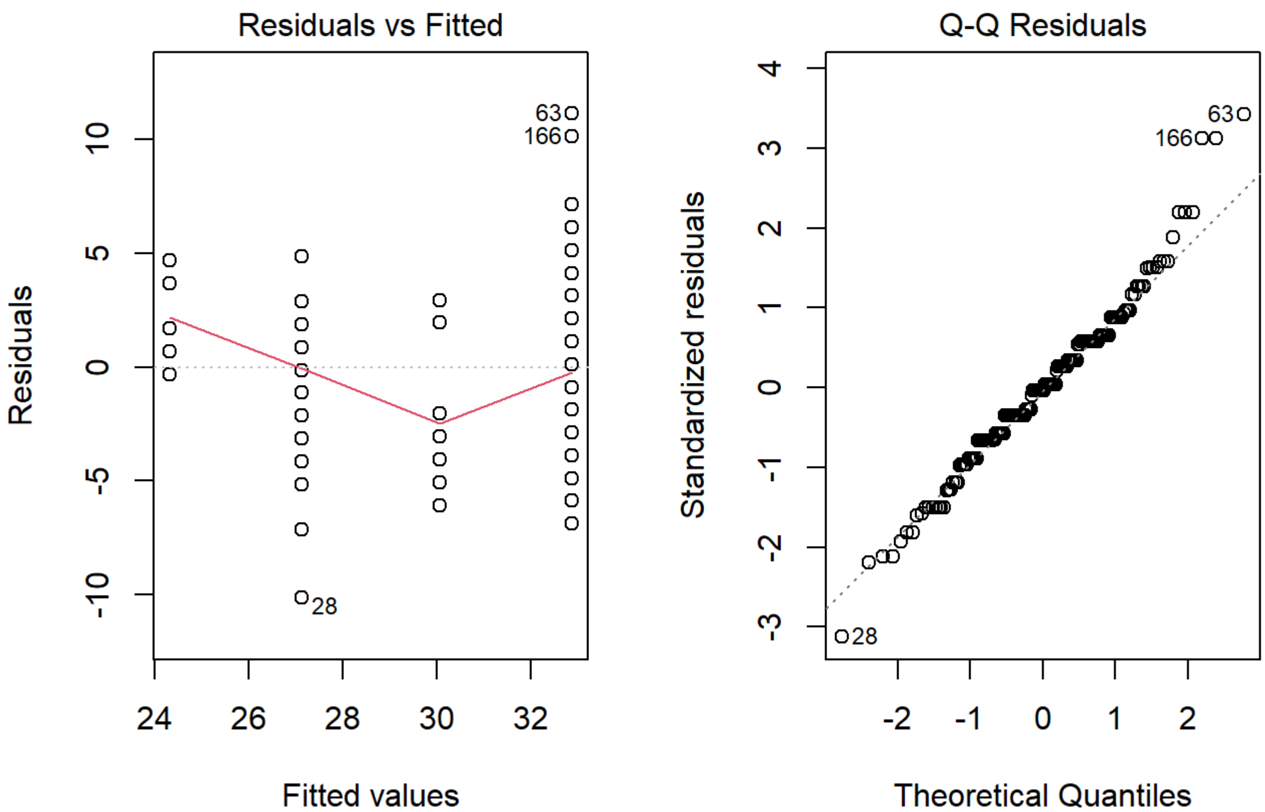
```
aov.cars <-
  aov(mpg_highway ~ cylinders + origin + type,
      data = df_cars)

# check variance
# This does not at all look like equal variance amongst groups. However we will continue as if it
par(mfrow = c(1,2)) ; plot(aov.cars, c(1,2))
```




```
#based off of the below results, origin is not significant. Lets redefine our model by dropping o
Anova(aov.cars, type = 3)
```

```
Anova Table (Type III tests)

Response: mpg_highway
            Sum Sq  Df   F value  Pr(>F)    
(Intercept)  69548   1 6501.6715 < 2e-16 ***
cylinders     1453   1  135.8499 < 2e-16 ***
origin           1   1    0.0786 0.77948    
type           108   1   10.1018 0.00175 ** 
Residuals     1883 176                      
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
aov.cars <-
  aov(mpg_highway ~ cylinders + type,
      data = df_cars)

#this still does not look like equal variance to me.
par(mfrow = c(1,2)) ; plot(aov.cars, c(1,2))
```




```r
summary(aov.cars)
```

```
              Df Sum Sq Mean Sq F value   Pr(>F)    
cylinders      1 1470.8  1470.8  138.22 < 2e-16 ***
type           1  115.8   115.8   10.88 0.00117 ** 
Residuals    177 1883.5    10.6                    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
Anova(aov.cars, type = 3)
```

```
Anova Table (Type III tests)

Response: mpg_highway
            Sum Sq  Df F value    Pr(>F)    
(Intercept)  88449   1 8311.96 < 2.2e-16 ***
cylinders     1482   1  139.27 < 2.2e-16 ***
type           116   1   10.88  0.001175 ** 
Residuals     1883 177                      
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

For the model with just predictors you decided to keep, comment on the significant effects in the model and comment on how much variation in highway fuel efficiency the model describes.

- Both the number of cylinders and type of car have a significant effect on highway fuel efficiency. With high F values (and low p-values) in both cylinders and type categories, we can say with certainty that our model explains a great deal of the variance in highway fuel efficiency.

b) Starting with main effects chosen in part (a), find your best ANOVA model by adding in any additional interaction terms that will significantly improve the model. For your final model, comment on the significant effects and variation explained by the model.

- My final model will be aov.cars_tc. The interaction effect, now that origin is dropped, is significant. It is worth including along with the main effects.

```
# lets try building another model that has an interaction term but still leaves out origin.

aov.cars_ct <-
  aov(mpg_highway ~ cylinders * type,
      data = df_cars)

aov.cars_tc <-
  aov(mpg_highway ~ cylinders * type,
      data = df_cars)

#both type 1s and the type 3 test all show significant for both of our main effects and our inter
summary(aov.cars_ct)
```

```
                Df Sum Sq Mean Sq F value  Pr(>F)
cylinders        1 1470.8  1470.8 143.839 < 2e-16 ***
type             1  115.8   115.8  11.323 0.00094 ***
cylinders:type   1   83.9    83.9   8.201 0.00470 **
Residuals      176 1799.6    10.2
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(aov.cars_tc)
```

```
                Df Sum Sq Mean Sq F value  Pr(>F)
cylinders        1 1470.8  1470.8 143.839 < 2e-16 ***
type             1  115.8   115.8  11.323 0.00094 ***
cylinders:type   1   83.9    83.9   8.201 0.00470 **
Residuals      176 1799.6    10.2
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Anova(aov.cars, type = 3)
```

```
Anova Table (Type III tests)

Response: mpg_highway
            Sum Sq  Df F value    Pr(>F)    
(Intercept)  88449   1 8311.96 < 2.2e-16 ***
cylinders     1482   1  139.27 < 2.2e-16 ***
type           116   1   10.88  0.001175 ** 
Residuals     1883 177                      
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

c) Comment on any significant group differences through the post-hoc test. What does this tell us about fuel efficiency differences across cylinders, origin, or type groups?

```
ScheffeTest(aov.cars_tc)
```

```

  Posthoc multiple comparisons of means: Scheffe Test 
    95% family-wise confidence level

$cylinders
         diff    lwr.ci    upr.ci   pval    
6-4 -5.722662 -7.069542 -4.375783 <2e-16 ***

$type
                  diff    lwr.ci     upr.ci   pval    
Sports-Sedan -2.817931 -5.181999 -0.4538626 0.0117 *  

$`cylinders:type`
                        diff    lwr.ci    upr.ci    pval    
6:Sedan-4:Sedan   -6.1723315 -7.583670 -4.760993 < 2e-16 ***
4:Sports-4:Sedan  -5.2275641 -8.578473 -1.876655 0.00036 ***
6:Sports-4:Sedan  -6.6025641 -9.953473 -3.251655 2.7e-06 ***
4:Sports-6:Sedan   0.9447674 -2.391611  4.281146 0.88732    
6:Sports-6:Sedan  -0.4302326 -3.766611  2.906146 0.98763    
6:Sports-4:Sports -1.3750000 -5.888107  3.138107 0.86373    

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- The results above show us the following:
  - $\mu_6 < \mu_4$

    On average a 6-cylinder car's highway mileage is 6.17 mpg worse than a car with 4-cylinders.
  - $\mu_s < \mu_d$

    On average a Sport car's highway mileage is 2.18 mpg worse than a sedans.
  - There is a significant interaction effect between the following car and cylinder combinations

- 6:Sedan-4:Sedan
- 4:Sports-4:Sedan
- 6:Sports-4:Sedan