SIMULATION & EXPECTED VALUES

401

WEEK 4

▷ EXPECTED VALUES:

ASSUME: X = SCALAR

$E[x]$ = THE EXPECTED VALUE OF X
(THINK OF AS THE MEAN)

$E[x|y]$ = EXPECTED VALUE OF X GIVEN y
AKA THE CONDITIONAL EXPECTATION
OF X GIVEN y.

X COULD BE A VECTOR OR MATRIX, BUT FOR NOW LETS ASSUME IT'S A SCALAR.

ASSUME X IS DISTRIBUTED ACCORDING TO SOME pdf (ie $x \sim pdf(x)$)

IS DISTRIBUTED AS

THE PROBABILITY DISTRIBUTION FUNCTION OF X ie. pdf(x)

FACT:

$$E[x] = \int_{-\infty}^{\infty} x \cdot pdf(x) \cdot dx$$

FACT:

$$(*) \quad E[f(x)] = \int_{-\infty}^{\infty} f(x) \cdot \underbrace{pdf(x)} \, dx$$

IMPORTANT

THIS IS THE pdf OF X, NOT THE pdf OF $f(x)$!

THE "MEAN" & "EXPECTED VALUE" TERMINOLOGY ARE SIMILAR BUT NOT THE SAME. THE MEAN TYPICALLY REFERS TO THE AVERAGE VALUE IN A SET OF (HISTORICAL) DATA, WHILE YOU TYPICALLY USE THE TERM "EXPECTED VALUE" TO REFER TO THE "FORECASTED MEAN" - I.E WHAT WE EXPECT A DATA GENERATING MECHANISM WILL PRODUCE IN THE FUTURE - ON AVERAGE.

▷ SOME EXPECTED VALUE RULES ARE:

ASSUME; a & b ARE CONSTANTS (SCALARS)
$\tilde{x}$ IS A RANDOM VARIABLE "
$\tilde{y}$ " " " " "

RULES:

- $E[a\tilde{x}] = a \cdot E[x]$ (PULL CONSTANTS OUT FRONT)
- $E[\tilde{x}+\tilde{y}] = E[x] + E[y]$
- $E[a\tilde{x}+b\tilde{y}] = a \cdot E[x] + b \cdot E[y]$

NOTE

- $\underbrace{E[\tilde{x} \cdot \tilde{y}]} \neq E[x] \cdot E[y] \leftarrow$ *

BE CAREFUL w/ THIS MULTIPLICATIVE CASE!

SOME SUMMATION RULES:

$\boxed{\text{FACT}}$

$$\frac{\sum_{i=1}^{N} \text{CONST}}{N} = \text{CONST}$$

eg

$$\frac{\sum_{i=1}^{4} e}{4} = \frac{e+e+e+e}{4} = \frac{4e}{4} = e$$

$\boxed{\text{ALSO}}$

$$\frac{\sum_{i=1}^{N} (\text{CONST}) \cdot \tilde{X}_i}{N} = \text{CONST} \cdot \frac{\sum \tilde{X}_i}{N}$$

$$= \text{CONST} \cdot \overline{X}$$

eg

$$\frac{\sum_{i=1}^{4} e \cdot x_i}{N} = \frac{e \cdot x_1 + e \cdot x_2 + e \cdot x_3 + e \cdot x_4}{4}$$

$$= e \cdot \left\{ \frac{x_1 + x_2 + x_3 + x_4}{4} \right\}$$

$$= e \cdot \overline{x}$$

▷ THE EXPECTED VALUE ANALOGY OF THE TWO ABOVE "FACTS" IS:

(1) $E[\text{CONST}] = \text{CONST}$

(2) $E[\text{CONST} \cdot \tilde{X}] = \text{CONST} \cdot E[\tilde{X}]$

▷ THE MEAN IS AKA THE FIRST MOMENT
" VARIANCE " " SECOND "
" SKEWNESS " " THIRD "
" KURTOSIS " " FOURTH "

▷

SO NOW LETS LOOK AT THE SECOND MOMENT OF $\tilde{X}$ (IE THE VARIANCE OF $\tilde{X}$)

FROM "BASIC STATS" YOU'VE SEEN

$$VAR(x) = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N}$$

SOMETIMES WRITTEN AS N-1.

LETS DO A LITTLE MATH/ALGEBRA: REWRITE AS

$$\sigma_x^2 = \frac{\sum_{i=1}^{N} (x_i^2 - 2x_i \cdot \bar{x} + \bar{x}^2)}{N}$$

$$= \frac{\sum (x_i^2)}{N} - 2\frac{\sum x_i \cdot \bar{x}}{N} + \frac{\sum_{i=1}^{N} \bar{x}^2}{N}$$

$$= \frac{\sum (x_i^2)}{N} - 2\,\bar{x} \cdot \frac{\sum x_i}{N} + \bar{x}^2$$

CONST

$$\sigma_x^2 = \frac{\Sigma (x_i^2)}{N} - 2\bar{x}\cdot\bar{x} + \bar{x}^2$$

$$= \frac{\Sigma (x_i)^2}{N} - 2\bar{x}^2 + \bar{x}^2$$

$$= \frac{\Sigma (x_i)^2}{N} - \bar{x}^2$$

OFTEN SEE IN BASIC STATS BOOKS, Σ IS SOMETIMES WRITTEN AS

$$= \frac{\Sigma (x_i)^2}{N} - \left(\frac{\Sigma x_i}{N}\right)^2$$

▷ RECALL $\bar{x}$ IS A LOT LIKE $E[x]$.

THUS, $\frac{\Sigma f(x)}{N}$ IS A LOT LIKE $E[f(x)]$

THUS, THE ABOVE EQ$^{\underline{n}}$ CAN BE WRITTEN IN EXPECTED VALUE TERMS AS

$$\sigma_x^2 = E[x^2] - (E[x])^2$$

IMPORTANT EQUATION

AND SINCE

$$E[f(x)] = \int_{-\infty}^{+\infty} f(x)\cdot pdf(x)\cdot dx$$

WE CAN WRITE

$$\sigma_x^2 = E[x^2] - (E[x])^2$$

$$= \underbrace{\int_{-\infty}^{\infty} x^2 \cdot pdf(x) \cdot dx}_{E[x^2]} - \Big\{ \underbrace{\int_{-\infty}^{\infty} x \cdot pdf(x) \cdot dx}_{E[x]} \Big\}^2$$

AN EXAMPLE:

SUPPOSE X IS DISTRIBUTED <u>UNIFORM</u> ON THE RANGE 3 TO 5 (X CAN TAKE ON ONLY VALUES FROM 3 TO 5 INCLUSIVE, AND EACH NUMBER FROM 3 TO 5 HAS THE SAME CHANCE OF BEING SELECTED).

IN GENERAL, THE pdf OF A UNIFORM DIST IN THE a TO b RANGE $(a < b)$ IS WRITTEN AS

$$pdf(x) = 0 \qquad x < a$$

$$pdf(x) = \frac{1}{b-a} \qquad a \le x \le b$$

$$pdf(x) = 0 \qquad b < x$$

(NOTE, YOU SHOULD CHECK THAT THE TOTAL AREA UNDER THE pdf IS EXACTLY ONE)

LETS FIND $E[X]$:

NOTE, INTUITIVELY WE KNOW THE ANSWER IS THE MIDPOINT BETWEEN 3 & 5 $(=4)$ SINCE EVERY NUMBER IN THIS RANGE HAS THE SAME CHANCE OF BEING DRAWN.

$$E[X] = \int_{-\infty}^{\infty} x \cdot pdf(x) \cdot dx$$

$$= \int_{-\infty}^{3} x \cdot 0 \cdot dx + \int_{3}^{5} x \cdot \left(\frac{1}{5-3}\right) dx + \int_{5}^{\infty} x \cdot 0 \cdot dx$$

$$= 0 + \int_{3}^{5} x \cdot \left(\frac{1}{5-3}\right) dx + 0$$

$$= \frac{1}{2} \int_{3}^{5} x \, dx = \frac{1}{2} \cdot \frac{1}{2} x^2 \Big|_3^5 = \frac{1}{4} x^2 \Big|_3^5$$

$$= \frac{1}{4}(5)^2 - \frac{1}{4}(3)^2$$

$$= \frac{25}{4} - \frac{9}{4} = \frac{16}{4} = 4 \leftarrow \text{AS WE THOUGHT!}$$

$\boxed{\text{NOTE:}}$ IN GENERAL, IF $X \sim \text{UNIFORM}(a, b)$ THEN $E[X] = \frac{a+b}{2}$

$\boxed{\text{EX}}$ $X \sim \text{UNIFORM}(3,5)$ & $E[X] = \frac{3+5}{2} = 4$

LETS FIND THE VARIANCE OF X:

RECALL

$$\sigma_x^2 = \int_{-\infty}^{\infty} x^2 \cdot pdf(x) \cdot dx - \left\{ \int_{-\infty}^{\infty} x \cdot pdf(x) \cdot dx \right\}^2$$

$$= \int_3^5 x^2 \frac{1}{5-3}\, dx - (E[x])^2$$

$$= \frac{1}{2} \int_3^5 x^2\, dx - (4)^2$$

$$= \frac{1}{2} \cdot \frac{1}{3} x^3 \Big|_3^5 - 16$$

$$= \frac{1}{6} x^3 \Big|_3^5 - 16 = \frac{1}{6}\left\{ 5^3 - 3^3 \right\} - 16$$

$$= \frac{125-27}{6} - 16 = 16\tfrac{1}{3} - 16$$

$$= \frac{1}{3}$$

YOU'LL KNOW YOU MADE A MISTAKE IF THIS IS NEGATIVE. VARIANCES CANNOT BE NEGATIVE!

$\sigma^2 = \frac{(R-L)^2}{12}$ FOR A UNIFORM (L,R) DIST^{3}

Q: WHAT OCCURS IF WE TREAT $E[X^2]$ AS

$E[X] \cdot E[X]$?

THAT IS, GIVEN $f(x) = x^2$

SUPPOSE WE WANT TO KNOW $E[f(x)]$

$= E[X^2]$

Q: CAN WE JUST COMPUTE $E[X]$ AND SQUARE IT?
A: NO!

$\boxed{EX}$ SUPPOSE $f(x) = x^2$ & $X \sim \text{UNIFORM}(3,5)$

WHAT IS $E[f(x)] \stackrel{?}{=} E[X^2]$?
IS IT $E[X] \cdot E[X]$?
$(= 4 \cdot 4 = 16)$?

LETS SEE:

$$E[X^2] = \int_3^5 x^2 \cdot \frac{1}{5-3} \cdot dx$$

$$= \frac{1}{2} \int x^2\, dx = \frac{1}{2} \cdot \frac{1}{3} x^3 \Big|_3^5$$

$$= \frac{1}{6}(125 - 27) = 16\frac{1}{3} \neq 16 \;!$$

$\triangleright$ LETS LOOK @ $g(x) = x^5$

SUPPOSE $X \sim \text{UNIFORM}(3,5)$

Q: IS $E[X^5] = (E[X])^5$?

$$= (4)^5$$
$$= 1024 \quad ?$$

LET'S SEE:

$$E[X^5] = \int_3^5 x^5 \cdot \frac{1}{5-3} \cdot dx$$

$$= \frac{1}{2}\frac{1}{6} x^6 \Big|_3^5$$

$$= \frac{1}{12}(15{,}625 - 729) = 1241\tfrac{1}{3}$$

NOT THE SAME!

$\triangle$

THUS, IF ASKED TO FIND THE EXPECTED VALUE OF A FUNCTION, BE CAREFULL TO NOT DO THIS

$$g(x) = A + Bx + Cx^2$$

OK $\longrightarrow$ $E[g(x)] = A + B \cdot E[x] + C \cdot E[x^2]$

<u>NOT</u> OK $\longrightarrow$ $= A + B \cdot E[x] + C \cdot \underbrace{(E[x])^2}_{\text{NO!}}$

THAT IS $\left(E[x]\right)^2 \neq E[x^2]$

$\triangle$

WHAT IF WE DID NOT KNOW HOW TO INTEGRATE AT ALL (OR THE INTEGRAL IS DIFFICULT TO EVALUATE)

Q: WHAT TO DO?

A: MONTE CARLO SIMULATION WOULD WORK!

$\triangle$

SIMULATION:

MONTE CARLO SIMULATION TYPICALLY MEANS TO DRAW A RANDOM VARIABLE (OR A SET OF RV'S) FROM SOME ASSUMED pdf (WHICH BY ASSUMPTION APPEARS TO BE GENERATING DATA) AND THEN GIVEN THE INDIVIDUAL DRAWS FROM THE pdf, EMPLOY THEM IN DETERMINING THE DISTRIBUTION OF SOME OTHER FUNCTION (WHICH HAS OUR RANDOM VARIABLE(S) AS INPUT(S)).

EXAMPLE:

ASSUME $X \sim U(0,1)$ ← UNIFORM $(0,1)$

ie X IS PULLED RANDOMLY FROM THE INTERVAL 0 TO 1.00.

SUPPOSE WE WANTED TO FIND THE EXPECTED VALUE OF $g(x) = e^{(x^2)}$

ALSO SUPPOSE WE DID NOT HAVE A CALCULUS BACKGROUND... WHAT TO DO?

DO THE FOLLOWING:

1. RANDOMLY DRAW AN X FROM THE $U(0,1)$ DISTRIBUTION, SAY $.372 = X_1$
2. PLUG THE VALUE OF X ($=.372$) INTO THE FUNCTION (ie EVALUATE $\exp(.372^2) = 1.1484$) AND SAVE THIS VALUE.
3. REPEAT STEPS 1 AND 2 A LARGE # OF TIMES, SAY, 10,000 (USING A COMPUTER PROGRAM)
4. COMPUTE THE MEAN & VARIANCE OF THE 10,000 VALUES OF $g(x)$. THIS PROVIDES YOU WITH AN ESTIMATE OF THE EXPECTED VALUE OF $g(x)$ ($= E[g(x)]$) AND THE VARIANCE OF $g(x)$.

ALSO, GIVEN THE 10,000 VALUES OF $g(x)$ THAT YOU GENERATED, YOU COULD DETERMINE THE UPPER & LOWER 5% TAILS. THAT IS THE LARGEST 500 OF THE 10,000 DRAWS (5%) RESIDE IN THE UPPER TAIL WHILE THE SMALLEST 500 OF THE 10,000 DRAWS COMPRISES THE LOWER TAIL.

THIS IDEA OF CONSTRUCTING THE EMPIRICAL DISTRIBUTION OF $g(x)$ ALLOWS FOR HYPOTHESIS TESTING, THE COMPUTATION OF P-VALUES ETC...

EXAMPLE # 2

SUPPOSE YOU ARE EVALUATING A PROPOSED PROJECT (ie CAPITAL BUDGETING) — AND WE KNOW WE WANT TO ACCEPT THE PROPOSAL IF THE EXPECTED VALUE IS POSITIVE.

SUPPOSE OUR ENGINEERS HAVE BEEN ABLE TO ESTIMATE THE FIXED & VARIABLE PRODUCTION COSTS, (INCLUDING THE UP-FRONT CAPITAL CONSTRUCTION COSTS AND THE OPERATING COSTS THEREAFTER).

THE PRIMARY REMAINING QUESTION IS, WHAT PRICE DO WE EXPECT TO CHARGE (AND THIS MIGHT BE A RANDOM VARIABLE AS MARKET FORCES CHANGE) AND WHAT QUANTITY DO WE EXPECT TO SELL?

TYPICALLY, FINANCE TEXTBOOKS WILL TELL YOU TO SIMPLY INSERT E[PRICE] & E[QUANTITY] INTO YOUR ANALYSIS (PROBABLY BEING DONE VIA A SPREADSHEET) AND COMPUTE THE EXPECTED VALUE. BUT IS THAT THE BEST WAY? <u>NO</u>!

LETS LOOK AT SEVERAL WAYS TO IMPROVE UPON THIS TYPICAL APPROACH:

FIRST OF ALL, WE ARE NOW TALKING ABOUT 2 VARIABLES $\tilde{X}$ <u>AND</u> $\tilde{Y}$ ($\tilde{p}$ <u>AND</u> $\tilde{q}$) EACH OF WHICH COMES FROM SOME pdf.

JUST LIKE IT IS <u>NOT</u> OK TO WRITE

$$E[X^2] = E[X] \cdot E[X]$$

IT IS TYPICALLY <u>NOT</u> OK TO WRITE

$$E[XY] = E[X] \cdot E[Y] \quad (414.1)$$

IN OUR CASE, THIS MEANS WE SHOULD <u>NOT</u> ASSUME ~~THAT~~ EXPECTED REVENUE $E[p \cdot q]$ IS EQUAL TO $E[p]$ TIMES $E[q]$

$\boxed{\text{NOTE,}}$ EQ^N (414.1) IS OK <u>IF</u> X IS INDEPENDENT OF Y, OR IF THE COVARIANCE BETWEEN X & Y IS ZERO.

HOWEVER VIA SUPPLY/DEMAND FORCES q IS A FUNCTION OF PRICE! THUS

$$E[p \cdot q] \neq E[p] \cdot E[q]$$

IT'S IMPORTANT TO BE AWARE THAT THIS POINT COMES FROM THE EXPRESSION:

$$COV(X,Y) = E[X \cdot Y] - E[X] \cdot E[Y]$$

<u>IF</u> $COV(X,Y) = 0$ THEN

$$E[X \cdot Y] = E[X] \cdot E[Y]$$

OTHERWISE IS WRONG!

THUS, GIVEN THE DEPENDENCE OF q ON P IT IS BETTER TO FIND THE MEAN (EXPECTED) NPV OF THE PROJECT IN A MORE CAREFULL MANNER, — SUCH AS VIA MONTE CARLO SIMULATION.

SO LETS ASSUME p & q ARE DISTRIBUTED VIA A MULTI-VARIATE NORMAL DISTRIBUTION. A MULTI-VARIATE (MV) NORMAL DIST^{B} IS SIMILAR IN IDEA TO THE UNIVARIATE NORMAL THAT YOU ARE ALREADY FAMILIAR WITH, EXCEPT IT DESCRIBES HOW A SET (IN OUR CASE A SET OF 2) OF RANDOM VARIABLES IS GENERATED.

RECALL:

UNIVARIATE pdf $\sim N(\mu, \sigma^2)$

$$\text{pdf}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

A MULTIVARIATE NORMAL pdf IS WRITTEN AS: (IN MATRIX FORM)

$$f(\vec{x}) = (2\pi)^{-\frac{N}{2}} \, \text{DET}(\Sigma)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\vec{x}-\vec{\mu})' \Sigma^{-1} (\vec{x}-\vec{\mu})\right\}$$

N×1 VECTOR (2×1 IN OUR CASE)

WHERE

WHERE:

$\vec{X}$ = $N \times 1$ RANDOMLY DRAWN VECTOR

$\vec{\mu}$ = $N \times 1$ VECTOR CONSISTING OF THE EXPECTED VALUES OF THE ELEMENTS COMPRISING $\vec{X}$

Σ = $N \times N$ VAR-COV MATRIX DESCRIBING THE "SECOND MOMENTS" OF THE N RANDOM VARIABLES.

IN OUR EXAMPLE:

$$X = \begin{bmatrix} p \\ q \end{bmatrix}_{2\times 1} \qquad \mu = \begin{bmatrix} E[p] \\ E[q] \end{bmatrix}_{2\times 1}$$

$$\Sigma = \begin{bmatrix} \sigma_p^2 & COV(p,q) \\ COV(p,q) & \sigma_q^2 \end{bmatrix}_{2\times 2}$$

SO, LETS ASSUME

$$E[p] = \$5^{00}/\text{UNIT}$$
$$E[q] = 1{,}000 \text{ UNITS}$$
$$\left\{ \begin{array}{l} \sigma_p^2 = .09 \implies \sigma_p = .3 \\ \sigma_q^2 = 360{,}000 \implies \sigma_q = 600 \end{array} \right.$$

HOW DID WE PICK THESE?

WELL...

THESE #'S SHOULD COME FROM OBSERVING HOW MUCH PRICES & QUANTITIES VARY ACROSS SIMILAR PRODUCTS. --

WE DO HAVE A LOT OF CONTROL OVER PRICES SO ITS σ RELATIVE TO ITS MEAN SHOULD BE "SMALL".

RECALL THE STATISTIC "COEFFICIENT OF VARIATION"

$$CV = \frac{\sigma}{\bar{X}}$$

IN OUR EXAMPLE, WE'LL USE $\frac{\sigma_x}{E[x]} = CV_x$

THUS FOR PRICE, WITH LOTS OF CONTROL OVER IT, WE WANT A "SMALL" CV_p, SO I PICKED $\sigma = .3$ GIVING

$$CV_p = \frac{.3}{5.00} = .06$$

HOWEVER QUANTITY SOLD IS VERY HARD TO "PIN DOWN" (IT HAS HIGH σ) SO I CHOOSE A MUCH LARGER CV FOR q.

I CHOOSE

$$CV_q = \frac{600}{1000} = .6$$ ← 10X LARGER THAN $CV_p = .06$

▷ WE NEED TO DETERMINE THE CORRELATION BETWEEN p & q.

WE KNOW THAT AS $p \uparrow$, QUANTITY SOLD WILL LIKELY DECLINE. THUS WE EXPECT

THAT THE CORRELATION BETWEEN p & q IS NEGATIVE.

WE COULD DO A LOT MORE ANALYSIS INTO ESTIMATING THE CORRELATION BETWEEN p & q, BUT FOR NOW LETS JUST ASSUME IT'S EQUAL TO -0.5

THUS, OUR CORR MATRIX LOOKS LIKE

$$\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix} \quad \text{AND w/} \quad \sigma_p^2 = .09 \quad \text{— TYPO}$$

$$\sigma_q^2 = 360{,}000$$

WE GET THE VAR-COV MATRIX OF p & q TO BE

$$\Sigma = \begin{bmatrix} .09 & -16{,}200 \\ -16{,}200 & +360{,}000 \end{bmatrix}$$

WE NOW HAVE FULLY DEFINED THE INPUT PARAMETERS (μ & Σ) TO THE MULTI-VARIATE NORMAL pdf ON THE BOTTOM OF PAGE 415.

GIVEN THIS (AND USING A COMPUTER) WE'D REPEAT THE ESSENCE OF THE ALGORITHM ON PAGE 412. THAT IS

① GIVEN A FULLY DEFINED MV-NORMAL DISTB, DRAW A VECTOR OF OBSERVATIONS FROM THE pdf.
(IN OUR CASE WE DRAW A 2×1 VECTOR CONSISTING OF A P & MATCHING q)

② USE THE JUST OBTAINED P & q TO COMPUTE THE NPV OF THE PROJECT ($NPV = f(P, q)$) AND SAVE THIS NPV.

③ REPEAT ① & ② A LARGE # OF TIMES, SAY 10,000 TIMES.

④ GIVEN THE 10,000 (DIFFERENT!) NPV'S THE EXPECTED NPV IS THE SAMPLE MEAN, WHILE THE VARIANCE OF THE NPV IS THE SAMPLE VARIANCE.

OF COURSE 5% TAILS CAN BE EASILY CONSTRUCTED AND STATEMENTS LIKE "THERE IS A 5% CHANCE NPV WILL MEET OR EXCEED $743.52".

ALSO IF 6237 OF THE 10,000 DRAWS HAVE POS NPV, THEN WE CAN SAY THAT THERE IS A 62.37% CHANCE THAT THE PROJECT WILL BE A SUCCESS.

↓

ALSO, IF ASKED "WHAT IS THE PROBABILITY THAT THE NPV WILL BE ABOVE $1000?" THEN WE CAN SIMPLY DETERMINE HOW MANY OF THE 10,000 NPV'S WERE ABOVE $1000, AND REPORT THE OVERALL PERCENT.

ie IF 329 OF THE 10,000 NPV'S HAD NPV > $1000, THEN WE COULD ESTIMATE THAT THE PROB OF SEEING THE ACTUAL NPV BE ABOVE $1000 IS 3.29%.

▷ BY THE WAY, THIS IS THE FIRST COMPUTER PROGRAM THAT WE WRITE IN BUS-444 (FINANCIAL ENGINEERING)

▷ IN SUMMARY, MONTE CARLO SIMULATION IS A GREAT TOOL FOR EMPIRICALLY ESTIMATING COMPLEX INTEGRALS (IN ORDER TO GET EXPECTED VALUES AND VARIANCES), AND IT ALSO PROVIDES A MEANS FOR ESTIMATING THE EMPIRICAL DISTRIBUTION OF THE COMPLEX FUNCTION UNDER CONSIDERATION, $\{g(x,y)$ OR IN OUR EXAMPLE $NPV(p,q)\}$.

Q: WHAT ELSE COULD BE SIMULATED?

A1: THE EXPECTED RETURN ON A CALL OPTION GIVEN THAT THE UNDERLYING ASSET HAS SOME ASSUMED pdf FOR ITS RETURNS, — MAYBE NORMAL.

A2: THE EXPECTED RETURN ON A BOND PORTFOLIO GIVEN SOME PROCESS FOR INTEREST RATES (INVOLVES MODELS OF THE YIELD CURVE).

A3: THE EXPECTED LEVEL OF INVENTORY GIVEN SOME ASSUMED MULTI-VARIATE PROCESS (AND IMPLIED MULTI-VARIATE pdf) BETWEEN PRODUCTION & QUANTITY SOLD.

IN GENERAL, MONTE CARLO SIMULATION IS USEFUL WHEN WHAT WE WANT TO ANALYZE IS A BIT COMPLICATED, BUT WE HAVE SOME IDEA HOW THE INPUTS ARE GENERATED.

ALSO, YOU DO NOT NEED TO KNOW THE THEORITICAL pdf OF THE INPUTS. YOU COULD INSTEAD HAVE A BIG HISTORY OF HOW THE INPUTS HAVE BEHAVED IN THE PAST, AND THEN DRAW

(W/ REPLACEMENT) FROM THIS SET OF HISTORICALLY BASED INPUTS.
(THIS IS AKA A "BOOTSTRAPPING")

eg SUPPOSE WE HAVE DAILY RETURNS FOR H-P FOR THE LAST 10 YEARS (2520 OBSERVATIONS).

AND SUPPOSE WE WANT TO ESTIMATE THE $E[R]$ ON A DECEMBER CALL OPTION W/ STRIKE PRICE OF, SAY $38.

IN THIS CASE WE COULD EITHER

(I) USE THE 2520 OBS TO DETERMINE $\bar{R}$ AND σ_R^2 AND ASSUME A NORMAL DIST^{B} FOR H-P RETURNS THEN SIMULATE VIA MAKING 10,000 DRAWS FROM THIS pdf... THEN EVALUATING THE RETURN OF THE OPTION FOR EACH DRAW ... ETC

OR

(II) (1) RANDOMLY SELECT A RETURN FROM THE SET OF 2520 RETURNS AND USE IT TO EVALUATE THE $E[R]$ OF THE OPTION.

(2) REPEAT (1) 10,000 TIMES.

USES THE ACTUAL DATA — DOES <u>NOT</u> ASSUME NORMALITY. — IMPLICITLY ASSUMES THE PAST WILL REPEAT ITSELF.