AND 444 ✓

BUS 343 Dirty Derivatives Problems — LOTS OF CHAIN RULE PRODUCT RULE QUOTIENT RULE

Find the partial derivatives to the following problems with respect to x.

1. $f(x,y) = x^3y^2 \cdot \cos\left(e^{x^2y^3}\right)$

2. $f(x,y) = \frac{7}{3}\left[\sin(\ln(x^4y^2)) \cdot e^{xy^3}\right]^3$

3. $f(x,y) = \ln\left(\ln\left(x^4y \cdot e^{x^5y^3}\right)\right)$

4. $f(x,y) = 5\left[\sin(\sin(x^5y^4 \cdot \sin(x)))\right]^3$

5. $f(x,y,z) = e^{x^2y^2z^2} \cdot \cos\left(\ln(x^3y^2z^2)\right)$

6. $f(x,y) = 6\left[\ln(3x^2y) \cdot \sin\left(4\frac{1}{y}x^3\right)\right]^4$

NOTE: ONE OF THE SIX SOLUTIONS HAS A MISTAKE... CAN YOU FIND IT?

BUS 343 Dirty Derivatives Problems - Answers

Find the partial derivatives to the following problems with respect to x.

1. $f(x,y) = x^3y^2 \cdot \cos(e^{x^2y^3})$

$g(x,y) = x^3y^2$

$h(i) = \cos(i)$

$i(j) = e^j$

$j(x,y) = x^2y^3$

$$\underbrace{f(x,y) = g(x,y) \cdot \underbrace{h(i(j(x,y)))}_{\text{CHAIN RULE}}}_{\text{PRODUCT RULE}}$$

NOW EVALUATE:

$$\frac{df}{dx} = g \cdot \frac{dh}{dx} + h \cdot \frac{dg}{dx} = g \cdot \frac{dh}{di} \cdot \frac{di}{dj} \cdot \frac{dj}{dx} + h \cdot \frac{dg}{dx}$$

SUB IN FORMULAS + X AND Y

$$\frac{df}{dx} = f(x,y)_x = x^3y^2 \cdot (-\sin(e^{x^2y^3})) \cdot (e^{x^2y^3}) + \cos(e^{x^2y^3}) \cdot 3x^2y^2$$

2. $f(x,y) = \frac{7}{3}\left[\sin(\ln(x^4y^2)\cdot e^{xy^3}\right]^3$

$$f(x,y) = g(\underbrace{\overbrace{h(i(j(x,y)))}^{\text{CHAIN RULE}}\cdot \overbrace{K(m(x,y))}^{\text{CHAIN RULE}}}_{\text{PRODUCT RULE}})$$

$j(x,y) = x^4y^2$ $\qquad$ $m(x,y) = xy^3$

$i(j) = \ln(j)$ $\qquad$ $K(m) = e^m$

$h(i) = \sin(i)$ $\qquad$ $g(h,K) = \frac{7}{3}[h\cdot K]^3$

NOW EVALUATE:

$$\frac{df}{dx} = 7[h\cdot K]^2\cdot\left[h\cdot\frac{dK}{dm}\cdot\frac{dm}{dx} + K\cdot\frac{dh}{di}\cdot\frac{di}{dj}\cdot\frac{dj}{dx}\right]$$

NOW SUB IN X AND Y:

$$f(x,y)_x = \frac{df}{dx} =$$

$$= 7\left[\sin(\ln(x^4y^2))\cdot e^{xy^3}\right]^2\cdot\left[\sin(\ln(x^4y^2))\cdot e^{xy^3}\cdot y^3 + e^{xy^3}\cdot\cos(\ln(x^4y^2))\cdot\frac{1}{x^4y^2}\cdot 4x^3y^2\right]$$

3. $f(x,y)=\ln(\ln(x^4y\cdot e^{x^5y^3}))$

$g(x,y)=x^4y$

$h(x,y)=x^5y^3$

$i(h)=e^h$

$j(i,g)=\ln(g\cdot i)$

$K(j)=\ln(j)$

$$f(x,y)=K(\underbrace{\underbrace{j(g(x,y)}_{\text{CHAINRULE}}\cdot\underbrace{i(h(x,y)))}_{\text{CHAIN RULE}}}_{\text{PRODUCT RULE}})$$

NOW EVALUATE:

$$\frac{df}{dx}=\frac{df}{dK}\cdot\frac{dK}{dj}\cdot\frac{dj}{dx}=\frac{df}{dK}\cdot\frac{dK}{dj}\cdot\left(g\cdot\frac{di}{dh}\cdot\frac{dh}{dx}+i\cdot\frac{dg}{dx}\right)$$

$$=\frac{1}{j}\cdot\frac{1}{g\cdot i}\cdot(g\cdot e^h\cdot 5x^4y^3+e^h\cdot 4x^3y)$$

SUBSTITUTE IN X AND Y:

$$=\frac{1}{\ln(x^4y\cdot e^{x^5y^3})}\cdot\frac{1}{x^4y\cdot e^{x^5y^3}}\cdot\left(x^4y\cdot e^{x^5y^3}\cdot 5x^4y^3+e^{x^5y^3}\cdot 4x^3y\right)$$

$$=\frac{x^4y\cdot e^{x^5y^3}\cdot 5x^4y^3+e^{x^5y^3}\cdot 4x^3y}{\ln(x^4y\cdot e^{x^5y^3})\cdot x^4y\cdot e^{x^5y^3}}$$

4. $f(x,y) = 5[\sin(\sin(x^5y^4 \cdot \sin(x)))]^3$

$f(g) = 5g^3$ $\qquad$ $i(j,k) = j \cdot k$

$g(h) = \sin(h)$ $\qquad$ $j(x,y) = x^5y^4$

$h(i) = \sin(i)$ $\qquad$ $K(x) = \sin(x)$

$$f(x,y) = g(\underbrace{h(\underbrace{j(x,y) \cdot K(x)}_{\text{PRODUCT RULE}}))}_{\text{CHAIN RULE}}$$

NOW EVALUATE:

$$\frac{df}{dx} = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{di} \cdot \frac{di}{dx} = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{di} \cdot \left(j \cdot \frac{dK}{dx} + K \cdot \frac{dj}{dx} \right)$$

$$= 15[\sin(\sin(x^5y^4 \cdot \sin x))]^2 \cdot \cos(\sin(x^5y^4 \cdot \sin x))$$
$$\cdot \cos(x^5y^4 \cdot \sin x)$$
$$\cdot (x^5y^4 \cos x + 5x^4y^4 \sin x)$$

5. $f(x,y,z) = e^{x^2y^2z^2} \cdot \cos(\ln(x^3y^2z^2))$

$f(h,i) = h \cdot i$ $\qquad$ $i(K) = \cos(K)$

$h(j) = e^{j}$ $\qquad$ $K(g) = \ln(g)$

$j(x,y,z) = x^2y^2z^2$ $\qquad$ $g(x,y,z) = x^3y^2z^2$

$$f(x,y,z) = f(h(j(x,y,z)) \cdot i(K(g(x,y,z)))$$

NOW EVALUATE:

$$\frac{\partial f}{\partial x} = h \cdot \frac{di}{\partial K} \cdot \frac{dK}{\partial g} \cdot \frac{dg}{\partial x} + i \cdot \frac{dh}{\partial j} \cdot \frac{dj}{\partial x}$$

$$= e^{x^2y^2z^2} \cdot (-\sin(\ln(x^3y^2z^2))) \cdot \frac{1}{x^3y^2z^2} \cdot 3x^2y^2z^2$$

$$+ \cos(\ln(x^3y^2z^2)) \cdot e^{x^2y^2z^2} \cdot 2xy^2z^2$$

6. $f(x,y) = 6\left[\ln(3x^2y)\cdot\sin\left(4\tfrac{1}{y}x^3\right)\right]^4$

$$g(x,y) = 3x^2y \qquad j(i) = \sin(i)$$
$$h(g) = \ln(g) \qquad K(h,j) = h\cdot j$$
$$i(x,y) = 4\tfrac{1}{y}x^3 \qquad f(K) = 6K^4$$

$$f(x,y) = \overbrace{K\Big(\underbrace{\underbrace{h(g(x,y))}_{\text{CHAIN RULE}}\cdot\underbrace{j(i(x,y))}_{\text{CHAIN RULE}}}_{\text{PRODUCT RULE}}\Big)}^{\text{CHAIN RULE}}$$

NOW EVALUATE

$$\frac{df}{dx} = \frac{df}{dK}\cdot\left[h\cdot\frac{dj}{di} + j\frac{dh}{dg}\right] = \frac{df}{dK}\cdot\left[h\cdot\frac{dj}{di}\cdot\frac{di}{dx} + j\frac{dh}{dg}\cdot\frac{dg}{dx}\right]$$

$$= 24K^3\left[h\cdot\cos(i)\cdot 12\tfrac{1}{y}x^2 + j\cdot\left(\tfrac{1}{g}\right)\cdot 6xy\right]$$

SUBSTITUTE IN X AND Y:

$$\frac{df}{dx} = 24\left[\ln(3x^2y)\cdot\sin\left(4\tfrac{1}{y}x^3\right)\right]^3\cdot$$
$$\left[\ln(3x^2y)\cdot\cos\left(4\tfrac{1}{y}x^3\right)\cdot 12\tfrac{1}{y}x^2 + \sin\left(4\tfrac{1}{y}x^3\right)\cdot\frac{1}{3x^2y}\cdot 6xy\right]$$