Sprint 3 - Deliverables

The Newton Pepys Problem

To solve this problem we use the binomial distribution to tell us how many 6's we expect to roll in our n independent Bernoulli trials.

$P(X \geq 1)$ $k \geq 1$ $n = 6$
$P(X \geq 2)$ $k \geq 2$ $n = 12$
$P(X \geq 3)$ $k \geq 3$ $n = 18$

$$P(X \geq 1) = 1 - P(X=0)$$

$$P(X=0) = \frac{6!}{(6-0)!\,0!}\left(1-\frac{1}{6}\right)^{6-0}\frac{1}{6}^{0} =$$

$$1 \cdot \left(1-\frac{1}{6}\right)^{6-0}\frac{1}{6}^{0}$$

$$1 \cdot (0.833)^6 \cdot 1 =$$

$$(0.833)^6 = 0.335$$

$$1 - .335 = \approx 0.665 \approx 66.5\% \text{ chance } P(X \geq 1)$$

$$P(X \geq 2) = 1 - (P(X=0) + P(X=1))$$

$$P(X=0) = \frac{12!}{(12-0)!\,0!}\left(1-\frac{1}{6}\right)^{12-0} p^0 = .1121$$

$$P(X=1) \quad \frac{12!}{(12-1)!\,1!}\left(1-\frac{1}{6}\right)^{12-1} p^1 =$$

$$12 \cdot .1345 \cdot \frac{1}{6} = 0.2692$$

$$1 - .1121 - .2692 = .6187 \text{ or } 61.87\% \text{ chance}$$

$P(X \geq 3) = 1 - (P(X=0) + P(X=1) + P(X=2))$

$P(X=0) = \frac{18!}{(18-0)!\,0!} \left(1 - \frac{1}{6}\right)^{18-0} \frac{1}{6}^{0}$

$= 1 \cdot 0.0375 \cdot 1 = .0375$

$P(X=1) = \frac{18!}{(18-1)!\,1!} \cdot \left(1 - \frac{1}{6}\right)^{18-1} \frac{1}{6}^{1}$

$18 \cdot .0456 \cdot \frac{1}{6} = 0.1352$

$P(X=2) \; \frac{18!}{(18-2)!\,2!} \cdot \left(1 - \frac{1}{6}\right)^{18-2} \frac{1}{6}^{2}$

$153 \cdot .0540 \cdot \frac{1}{36} = .2295$

$1 - .0375 - .1352 - .2295 = .5978 \approx 60\%$

Six fair dice tossed independently with at least one 6 appearing has the highest probability of success at 66.5%.

Geometric Urn

To find a reasonable estimate for the number of balls we can use the expected value formula for a geometric distribution and work backwards

$\frac{1}{p} = 20$ For the red balls

$1 = 20p = \frac{1}{20}$ probability of drawing a red ball

There are 100 balls in the urn

$\frac{1}{20} \times \frac{5}{5} = \frac{5}{100}$ Probability of choosing a Red ball

100 total balls
5 red balls
95 black balls

Dragon Dice

We use the Binomial distribution to measure the success of each dice

Probability of landing on Hermione's number $\frac{1}{6}$

P(0)
P(1)
P(2)
P(3)

$$P(0) = \frac{3!}{(3-0)!\,0!} \cdot \left(1-\frac{1}{6}\right)^{3-0} \cdot \frac{1}{6}^{0}$$

$1 \cdot .57870 \cdot 1 =$ 57.9% chance no numbers come up

$$P(1) = \frac{3!}{(3-1)!\,1!} \cdot \left(1-\frac{1}{6}\right)^{3-1} \cdot \frac{1}{6}^{1}$$

$3 \cdot .6944 \cdot \frac{1}{6}$ 34% Chance one number comes up

$$P(2) = \frac{3!}{(3-2)!\,2!} \cdot \left(1-\frac{1}{6}\right)^{3-2} \cdot \frac{1}{6}^{2}$$

$3 \cdot .83333 \cdot \frac{1}{36}$ 7% chance 2 numbers are drawn

$$P(3) = \frac{3!}{(3-3)!\,3!} \cdot \left(1-\frac{1}{6}\right)^{3-3} \cdot \frac{1}{6}^{3}$$

$1 \cdot 1 \cdot \frac{1}{216} =$ .4% chance 3 numbers are drawn

Calculate the probability of success
take weighted average of the four possible outcomes

$E[X] = xP(x)$

$P(-1) = .57$
$P(1) = .34$
$P(2) = .07$
$P(3) = .04$

$= -1(.57) + 1(.34) + 2(.07) + 3(.004) =$

$E[x] = -.08$ we expect to lose .08 galleons from playing dragon dice