

## PePy's Problem

- 6 dice are tossed @ least 1 is 6

$$P(X \geq 1) = 1 - P(X = \emptyset)$$

$$P(X = \emptyset) = \binom{6}{0} \left(1 - \frac{1}{6}\right)^6 \left(\frac{1}{6}\right)^0 = \left(\frac{5}{6}\right)^6$$

$$1 - \left(\frac{5}{6}\right)^6 = \boxed{0.665} \star$$

- 12 dice, @ least 2 are 6

$$P(X \geq 2) = 1 - P(X = \emptyset) - P(X = 1)$$

$$P(X = \emptyset) = \binom{12}{0} \left(1 - \frac{1}{6}\right)^{12} \left(\frac{1}{6}\right)^0 = \left(\frac{5}{6}\right)^{12} = 0.112$$

$$P(X = 1) = \binom{12}{1} \left(1 - \frac{1}{6}\right)^{11} \left(\frac{1}{6}\right)^1 = 12 \left(\frac{5}{6}\right)^{11} \left(\frac{1}{6}\right) = 0.269$$

$$1 - 0.112 - 0.269 = \boxed{0.619}$$



= 18 fair dice, 3 6's

$$P(X \geq 3) = 1 - P(X=0) - P(X=1) - P(X=2)$$

$$P(X=0) = \binom{18}{0} \left(\frac{5}{6}\right)^{18} \left(\frac{1}{6}\right)^0 = \left(\frac{5}{6}\right)^{18} = 0.0376$$

$$P(X=1) = \binom{18}{1} \left(\frac{5}{6}\right)^{17} \left(\frac{1}{6}\right)^1 = 0.135$$

$$P(X=2) = \binom{18}{2} \left(\frac{5}{6}\right)^{16} \left(\frac{1}{6}\right)^2 = 0.230$$

$$1 - 0.0376 - 0.135 - 0.230 = \boxed{0.597}$$

\* The first proposition has the greatest chance of success.

### Geometric Urn

- 100 balls, some are red & others black
- draw w/ replacement
- # of draws needed to get first red ball
- $E(X) = 20 = 1/p$
- $p = 0.05 \rightarrow 5\%$  of balls are red
- reasonable estimate:

5 red balls.

95 black balls.



# Dragon Dice

- pick # 1-6
- rolls 3 dice
- $K = 1, 2$ , or 3 times, she wins  $K$  galleons
- loses a galleon if the # doesn't come up
- Expected outcome?

$$P(1 \text{ success}) = +1 \text{ galleon}$$

$$P(2 \text{ " }) = +2 \text{ "}$$

$$P(3 \text{ " }) = +3 \text{ "}$$

$$P(0 \text{ " }) = -1 \text{ "}$$

- Probability that die 1 has the # =  $1/6$
- " " die 2 " " =  $1/6$
- " " die 3 " " =  $1/6$
- " " die 1 & 2 have the # =  $(\frac{1}{6})(\frac{1}{6})$
- " " die 1 & 2 & 3 " " =  $(\frac{1}{6})^3$
- " " no die has the # =  $(\frac{5}{6})^3$

$$• P(1 \text{ success}) = 1/6 = 0.167$$

$$• P(2 \text{ " }) = 1/36 = 0.0278$$

$$• P(3 \text{ " }) = 1/216 = 0.0046$$

$$• P(0 \text{ " }) = 0.579$$

$$\begin{aligned} E[\text{galleons}] &= (1)(0.167) + (2)(0.0278) + (3)(0.0046) \\ &\quad - (1)(0.579) \\ &= -0.343 \text{ galleons} \end{aligned}$$