

Sprint - 4 Deliverables

1. Yet More lightbulbs.

average life time of lightbulb = 2000 hrs.

the lightbulb follows exponential distribution.

and we are looking for the probability that two independent bulb survive for more than 3000 hours so $X = \text{lightbulbs life time}$.

$$P(X > 3000) = e^{-\frac{1}{2000} \cdot 3000} \\ = e^{-\frac{3}{2}}$$

Expected value was 2000 hrs in this problem so that's why $\lambda = \frac{1}{2000}$

And there are two independent bulb so

$$P(\text{two lightbulb survives} > 3000) = e^{-\frac{1}{2000} \cdot 3000} \cdot e^{-\frac{1}{2000} \cdot 3000} \\ = e^{-\frac{3}{2}} \cdot e^{-\frac{3}{2}} = e^{-3} = 0.04978$$

2. The non-Persistence of Memory

We are using the same lightbulb that we used in Question 1.

The first bulb has already been used for 1000 hrs.

The second one has already been used for 2500 hrs.

We are looking for probability that both bulbs survives more than 3000 hrs.

We can use the timeless property

t : already observed time

s : time to exceed 3000

1.

$$t_1 = 1000, \quad s = 2000$$

$$t_2 = 2500, \quad s = 500$$

So for the first light bulb

$$\begin{aligned} P(X > 1000 + s \mid X > 1000) &= \frac{P(X > 1000 + 2000)}{P(X > 1000)} \\ &= \frac{e^{-\frac{1000}{2000}} \cdot e^{-\frac{2000}{2000}}}{e^{-\frac{1000}{2000}}} = e^{-1} = 0.3678 \end{aligned}$$

For the second.

$$\begin{aligned} P(X > 2500 + s \mid X > 2500) &= \frac{P(X > 2500 + 500)}{P(X > 2500)} = (e^{-\frac{2500}{2000}} \cdot e^{-\frac{500}{2000}}) \cdot \frac{1}{e^{-\frac{2500}{2000}}} \\ &= e^{-\frac{500}{2000}} \end{aligned}$$

$$\begin{aligned} P(\text{both bulb survive longer than } 3000 \text{ hrs}) &= e^{-1} \cdot e^{-\frac{500}{2000}} \\ &= 0.2865047 \end{aligned}$$

Check My Math

- Ave service time for a disk access = 5ms
- Ave number of disc access = 2
- Ave number of jobs = 120
- Ave residence time = 1

Using little's Law we calculate the through put.
we know $\bar{R} = 1$, $\bar{N} = 120$.

$$120 = \lambda \cdot 1$$

$$120 = \lambda$$

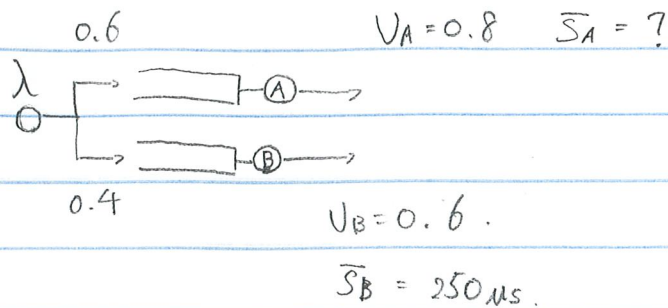
Before we use utilization Law we need to calculate the service time for a job first.

$$\begin{aligned} \text{service time for a job} &= 2 \cdot \frac{5}{1000} \\ &= \frac{10}{1000} \rightarrow \text{we are thinking in terms of second.} \end{aligned}$$

$$\begin{aligned} U &= 120 \cdot \frac{10}{1000} \\ &= 1.2 \end{aligned}$$

this is wrong because utilization is supposed to be some value between $0 \leq U < 1$

Unbalanced Server load



First, we want to get the throughput ΔB

$$0.6 = \lambda_B \cdot 250$$

$$\frac{0.6}{250} = \lambda_B$$

So now we know the arrival rate of B,
the arrival rate of the entire system
and arrival rate is related as:

$$\lambda_B = 0.4 \lambda$$

$$\frac{0.6}{250} = 0.4 \lambda$$

$$\frac{0.6}{250} \cdot \frac{10}{4} = \lambda$$

and λ_A is a fraction of
 λ so

$$\lambda \cdot 0.6 = \lambda_A$$

$$\frac{0.6}{250} \cdot \frac{10}{4} \cdot 0.6 = \lambda_A$$

By utilization law $U_A = \lambda_A \cdot \bar{S}_A$

$$0.8 = \frac{0.6}{250} \cdot \frac{1}{0.4} \cdot 0.6 \cdot \bar{S}_A$$

$$0.8 = \frac{0.6 \cdot 0.6}{100} \cdot \bar{S}_A$$

$$\frac{0.8 \cdot 100}{0.6 \cdot 0.6} = \bar{S}_A$$

$$\bar{S}_A = 222.222$$

$$\frac{0.4 \cdot 50}{0.3 \cdot 0.3} = \bar{S}_A$$

$$222.222 = \bar{S}_A$$