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Deliverables Sprint 2 CMS_380

# Wizard People Dear Reader

$P(\overset{A}{\text{She's a witch}}) = .75$ $\quad P(\text{She's }\overset{C}{\text{Not}}\text{ a witch})$

$P(\overset{B}{\text{Not recieving a letter}} \mid \overset{A}{\text{She's a witch}}) = .03$

$P(\overset{B}{\text{Not recieving a letter}} \mid \overset{C}{\text{she's not a witch}}) = .99$

Goal: $P(\overset{A}{\text{Hermione is a witch}} \mid \overset{B}{\text{Not recieving a Letter}})$

Bayes Rule: $P(A|B) = \dfrac{P(A) \cdot P(B|A)}{P(B)}$

$P(A) = .75$

$P(B|A) = .03$

$P(B) = (B|A) \cdot P(A) + P(B|C) \cdot P(C)$

$(.03 \times .75) + (.99 \times .25)$

$.0225 + .2475$

$P(B) = .27$

$P(A|B) = \dfrac{.75 \times .03}{.27} = \dfrac{.0225}{.27} = .083$

$P(A|B) = 8.3\%$ (Approx)

# Choclate Frogs

30 total cards

P( picking any one card) $= \frac{1}{30} = .03 = 3\%$ chance

Goal = Number of cards to get all 30 unique cards

Expected value of getting 1st card +
Expected value of getting 2nd card + ... +
Expected value of getting 30th card

Expected value for getting a new card $= \frac{1}{P}$
Where P is the probability of getting a new card

$$\frac{1}{\frac{30}{30}} + \frac{1}{\frac{29}{30}} + \frac{1}{\frac{28}{30}} + \frac{1}{\frac{27}{30}} + \frac{1}{\frac{26}{30}} + \frac{1}{\frac{25}{30}} + \frac{1}{\frac{24}{30}} + \frac{1}{\frac{23}{30}} + \dots + \frac{1}{\frac{1}{30}} =$$

$$\frac{30}{30} + \frac{30}{29} + \frac{30}{28} + \frac{30}{27} + \dots + \frac{30}{1} = 119.8496$$

Frogs

# Hat Problem

20% of good students get sorted into Slythern
100% of Evil students get sorted into slythern
10% of new students are Evil

Goal: Find the probability of a random slythern Student being evil

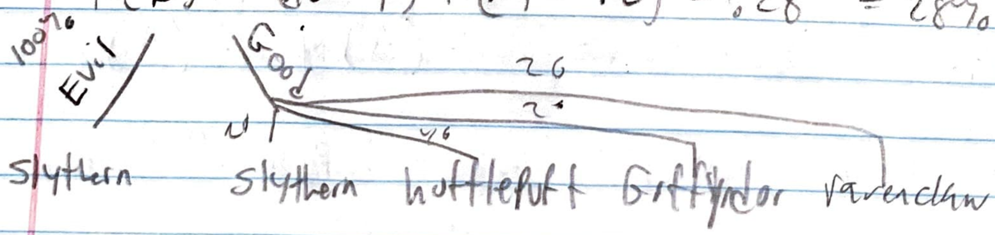
Find $P(\overset{A}{\text{Evil}} \mid \overset{B}{\underline{\text{Slythern}}})$

What we know:

$P(\overset{B}{\text{Slythern}} \mid \overset{C}{\text{Not Evil}}) = .20$
$P(\text{Slythern} \mid \text{Evil}) = 1$
$P(\text{Evil student}) = .10$
$P(\text{Good}) = .90$

Bayes therom $= (A \mid B) = \dfrac{\overset{.10}{P(A)}\ \overset{1}{P(B \mid A)}}{P(B)}$

$P(B) = (.10 \cdot 1) + (.9 \cdot .2) = .28 = 28\%$ All Students are in Slythern




$\dfrac{.10 \cdot 1}{.28} = \dfrac{.10}{.28} = .357$ = 36% chance of being Evil

Elevator is moving upwards $\frac{1}{2}$ and down $\frac{1}{2}$ of te time

# Dumblevator

Goal: Find te probability te elevator is moving down when Hermione arrives at te elevator.

Te Elevator moves up in sequential order 1, 2, 3, 4, ...
when te elevator reaches floor 15 it moves down in sequental order 15, 14, 13...

Te Elevator will be moving down when it is on te $15^{th}$ or $14^{th}$ floors

Probability te Elevator will be on any given floor $= \frac{1}{15}$

On te $13^{th}$ floor te Dumblevator has a $\frac{1}{30}$ chance of moving up or down when hermione arrives.

probability te Dumblevator will be moving down =

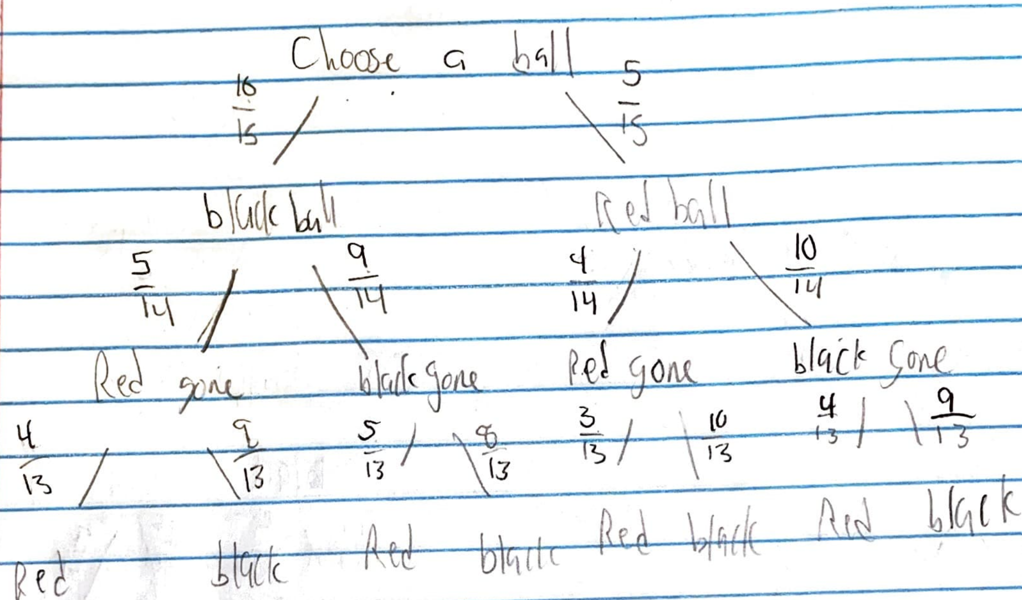
Prob $14^{th}$ floor + Prob $15^{th}$ floor + $\frac{1}{2}$ Prob $13^{th}$ floor =

$$\frac{1}{15} + \frac{1}{15} + \frac{1}{15} \cdot \frac{1}{2} = \frac{1}{15} + \frac{1}{15} + \frac{1}{30} = \frac{2}{30} + \frac{2}{30} + \frac{1}{30} = \frac{3}{30} =$$

$\frac{1}{10}$ Probability te elevator will be moving down when Hermione arrives

# Win While You Learn

Goal: Find the Probability the second ball chosen is red




$$\left(\frac{10}{15}\cdot\frac{5}{14}\cdot\frac{4}{13}\right)+\left(\frac{10}{15}\cdot\frac{9}{14}\cdot\frac{5}{13}\right)+\left(\frac{5}{15}\cdot\frac{4}{14}\cdot\frac{3}{13}\right)+\left(\frac{5}{15}\cdot\frac{10}{14}\cdot\frac{4}{13}\right)$$

= .33 or 33% chance of the second ball being red

# Pólya's Urn

Pick a ball from the Urn

$\frac{9}{15}$ $\qquad\qquad$ $\frac{6}{15}$

black $(10_b, 6_r)$ $\qquad\qquad$ Red $(9_b, 7_r)$

$\frac{10}{16}$ $\frac{6}{16}$ $\qquad\qquad$ $\frac{9}{16}$ $\frac{7}{16}$

black $\quad$ red $\qquad\qquad$ black $\quad$ red

$\frac{11}{17}$ $\frac{6}{17}$ $\quad$ $\frac{10}{17}$ $\frac{7}{17}$ $\qquad\qquad$ $\frac{10}{17}$ $\frac{7}{17}$ $\quad$ $\frac{10}{17}$ $\frac{9}{17}$

$(12_b, 6_r)$ black $\quad$ Red $\quad$ black $\quad$ Red $\qquad\qquad$ 11, 7, black $\quad$ Red $\quad$ black $\quad$ Red

$(11_b, 7_r)$ $\quad$ $(11_b, 7_r)$ $\quad$ $(10_b, 7_r)$ $\qquad\qquad$ $10_b\,8_r$ $\quad$ $10_b\,8_r$ $\quad$ $9_b\,9_r$

$E[X]$ = Expected value = $\sum_x x P(x)$

Expected value of the red balls =

$$6\left(\frac{9}{15}\cdot\frac{10}{16}\cdot\frac{11}{17}\right) + 7\left(\frac{9}{15}\cdot\frac{10}{16}\cdot\frac{6}{17}\right) + 7\left(\frac{9}{15}\cdot\frac{6}{16}\cdot\frac{10}{17}\right) + 7\left(\frac{9}{15}\cdot\frac{6}{16}\cdot\frac{7}{17}\right) +$$
$$7\left(\frac{6}{15}\cdot\frac{9}{16}\cdot\frac{10}{17}\right) + 8\left(\frac{6}{15}\cdot\frac{9}{16}\cdot\frac{7}{17}\right) + 8\left(\frac{6}{15}\cdot\frac{7}{16}\cdot\frac{10}{17}\right) + 9\left(\frac{6}{15}\cdot\frac{7}{16}\cdot\frac{9}{17}\right)$$

Expected value of the black balls

$$12\left(\frac{9}{15}\cdot\frac{10}{16}\cdot\frac{11}{17}\right) + 11\left(\frac{9}{15}\cdot\frac{10}{16}\cdot\frac{6}{17}\right) + 11\left(\frac{9}{15}\cdot\frac{6}{16}\cdot\frac{10}{17}\right) + 10\left(\frac{9}{15}\cdot\frac{6}{16}\cdot\frac{7}{17}\right) +$$
$$11\left(\frac{6}{15}\cdot\frac{9}{16}\cdot\frac{10}{17}\right) + 10\left(\frac{6}{15}\cdot\frac{9}{16}\cdot\frac{7}{17}\right) + 10\left(\frac{6}{15}\cdot\frac{7}{16}\cdot\frac{10}{17}\right) + 9\left(\frac{6}{15}\cdot\frac{7}{16}\cdot\frac{9}{17}\right)$$

$E[R] = 7.28$ $\qquad$ $E[b] = 10.99$

# Arithmancy

Six sided die example

Sample space $= \{1, 2, 3, 4, 5, 6\}$
Prob of each number appearing: $\frac{1}{6}$

expected value $= \sum_{x} x\,P(x) = 1(\frac{1}{6}) + 2(\frac{1}{6}) + 3(\frac{1}{6}) +$

$4(\frac{1}{6}) + 5(\frac{1}{6}) + 6(\frac{1}{6}) = 0 + \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = \frac{7}{2}$

$= \frac{n+1}{2}$

Arbatrary Example

Prove the expected value of a discrete uniform distribution with $a=1$ and $b=n$ is $E[x] = \frac{n+1}{2}$

$E[x] = x\,P(x)$

$E[x] = \sum_{k=1}^{n} \frac{1}{n} \cdot k = \frac{1}{n} \sum_{k=1}^{n} k = 1+2+3+\ldots+n-1+n$

$= \frac{1}{n} \sum_{k}^{n} = 1+2+3+\ldots+n = \frac{n(n+1)}{2} = \frac{1}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{2} >$

$\frac{1 \cdot (n+1)}{2} = \frac{(n+1)}{2}$

# Birthday Attack

Goal find the probability that No two Students have the same birthday

40 total Students     365 possible birthdays

P(Everyone has a different birthday) = (365/365) x (364/365) x (363/365) ... x (325/365)

= .1087 = 10.87% chance No two classmates have the same birthday