# Yet more light bulbs

I have two brand new lightbulbs, each rated for an average lifetime of 2000 hours.
What is the probability that both bulbs survive for more than 3000 hours?
Assume the lightbulbs are independent.

Let X be the exponential random variable representing the lifetime of the bulb

$\lambda = \frac{1}{2000} = 0.0005$

Since the lightbulbs are independent we're looking for $P(X>3000) \cdot P(X>3000)$

To find $P(X>3000)$ we use CCDF

$\bar{F}_X(x) = P(X>x) = 1 - F_X(x)$

$\bar{F}_X(x) = e^{-\lambda x}$

$\bar{F}_X(3000) = e^{-0.0005 \cdot 3000} \approx 0.22313$

$P(X>3000) \approx 0.22313$

$P(X>3000) \cdot P(X>3000) \approx 0.04978$

# The non-persistence of memory

Once again we have light bulbs with an average lifetime of 2000 hours, but these have already been in operation for a while.

- The first light bulb has already run for 1000 hours
- The second light bulb has already run for 2500 hours

What is the probability that both bulbs have a lifetime of more than 3000 hours?

Let $t$ denote the already-observed time that has passed
We'd like to reason about the probability that the lifetime exceeds some value $t+s$
$\lambda = \frac{1}{2000} = 0.0005$
For light bulb #1, $t = 1000$ $s = 2000$ additional hours
For light bulb #2, $t = 2500$ $s = 500$ additional hours

$$P(X>t+s \mid X>t) = \frac{P(X>t+s \text{ and } X>t)}{P(X>t)} = \frac{P(X>t+s)}{P(X>t)}$$

Using the exponential CCDF:

$$P(X>t+s \mid X>t) = \frac{e^{-\lambda(t+s)}}{e^{-\lambda t}} = e^{-\lambda s}$$

Light bulb #1

$P(X>1000+2000 \mid X>1000) = e^{-0.0005 \cdot 2000} \approx 0.3678$

Light bulb #2

$P(X > 2500 + 500 \mid X > 2500) = e^{-0.0005 \cdot 500} \approx 0.7788$

$P(\text{both light bulbs have a lifetime of more than 3000 hours}) = 0.2864$

## Check my math

- Average service time for a disk access = 5 ms
- Average number of disk accesses per job = 2
- Average number of jobs in the system at any moment = 120
- Average residence time of a job in the system = 1 second

Is there a mistake?

$\bar{S}$, the average service time = 5 ms = 0.005 seconds

$\bar{N}$, the average number resident in the system = 120

$\bar{R}$, the average residence time = 1 second

By Little's Law, $\bar{N} = \lambda \bar{R}$

$120 = \lambda 1$

$\lambda = \frac{120 \text{ jobs}}{1 \text{ second}}$

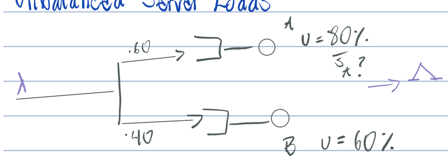
By utilization law $U = \lambda \bar{S}$

$\lambda = \frac{120 \text{ jobs}}{1 \text{ second}}$

Since average number of disk accesses per job is 2, $\bar{S} = 10\text{ ms} = 0.01$ seconds

$U = 120 \cdot 0.01 = 1.2$ → this is the mistake, in a stable system you shouldn't have a utilization of over 1.2

## Unbalanced Server Loads




$U = 60\%$

$\bar{S}_B = 250$ ms

$\lambda_B = \frac{.60}{250 \text{ ms}}$

By the utilization Law,

$U = \lambda \bar{S}$

$\lambda_B = \frac{U}{\bar{S}_B} = \frac{0.60}{250 \text{ ms}}$ customers/ms

By Forced Flow Law

$\lambda_A = 0.60 \lambda = 0.0036$

$U = \lambda \bar{S}$

$\bar{S} = \frac{U}{\lambda_A} = \frac{.80}{0.0036} = 222.22$

$\lambda_B = 0.40 \lambda$

$\lambda = \frac{\lambda_B}{0.40}$

$\lambda = \frac{.60/250}{0.40} = 0.006$

Understanding the M/M/1 Queue

| arrival_time | service_time | enter_service_time | departure_time | residence_time |
| --- | --- | --- | --- | --- |
| 1 | 3 | 1 | 4 | 3 |
| 3 | 2 | 4 | 6 | 3 |
| 5 | 4 | 6 | 10 | 5 |
| 7 | 1 | 10 | 11 | 4 |
| 8 | 1 | 11 | 12 | 4 |
| 13 | 2 | 13 | 15 | 2 |
| 14 | 1 | 15 | 16 | 2 |
| 17 | 3 | 17 | 20 | 3 |

Average residence time $(\bar{R}) = 3.25$