Pepys Problem

1 6 appears from 6 dice:

$$\binom{6}{1}(1-1/6)^5(1/6)^1$$

$$6 \times (5/6)^5 \times 1/6 = .4018 = 40.18\%$$

2 6's appear from 12 dice:

$$\binom{12}{1}(1-1/6)^{11}(1/6)^1$$

$$12 \times (5/6)^{11} \times 1/6 = .2691 = 26.91\%$$

3 6's appear from 18 dice

$$\binom{18}{1}(1-1/6)^{17}(1/6)^1$$

$$18 \times (5/6)^{17}(1/6) = .135 = 13.5\%$$

1 six appearing from 6 dice rolls, has the highest probability of $p = .4018$. The odds for 2 from 12 and 3 from 18 are .2691 and .135, respectively.

Geometric Urn

$E[\text{draw 1 red}] = 20 = 1/p \quad p = 1/20$

probability of drawing red = 1/20

Therefore, in an urn of 100 balls we would expect there to be 5 red and 95 black balls