

Pepys Problem

Binomial distribution

One 6 appears from 6 dice, $1 - p^{k=0}(\text{no successes})$

$$1 - \sum_{k=0}^0 (1 - (1/6))^{6-k} (1/6)^k \approx .6651$$

Two 6's appears from 12 dice, $1 - p^{k=1}(\text{1 success and no successes})$

$$1 - \sum_{k=0}^1 \binom{12}{k} (1/6)^k (1 - (1/6))^{12-k} \approx .6187$$

Three 6's appears from 18 dice, $1 - p^{k=2}(\text{2 successes, 1 success and no successes})$

$$1 - \sum_{k=0}^2 \binom{18}{k} (1/6)^k (1 - (1/6))^{18-k} \approx .5973$$

The first proposition has the greatest chance of success.

Geometric Urn

$$E[\text{draw 1 red}] = 20 = 1/p \Rightarrow p = 1/20$$

probability of drawing a red ball = $1/20$

therefore in an Urn of 100 balls

we would reasonably expect 5 red balls and 95 black balls