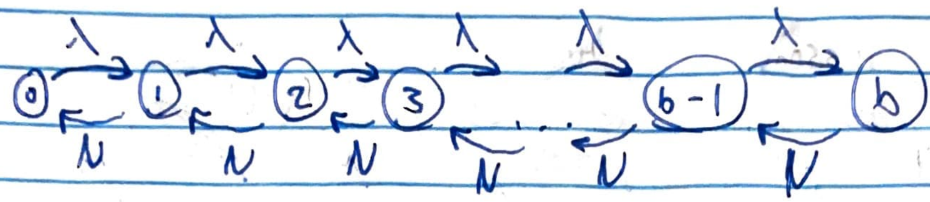
Challenge Project Markov chains

M | M | 1 | B



$$\pi_0 \lambda = \pi_1 N = \pi_0 \frac{\lambda}{N}$$

$$\pi_1 \lambda + \pi N = \pi_0 \lambda + \pi_2 N$$

$$\pi_0 \frac{\lambda}{M} \lambda + \cancel{\pi_0 \frac{\lambda}{M} M} = \cancel{\pi_0 \lambda} + \pi_2 N$$

$$\pi_0 \frac{\lambda}{M} \lambda = \pi_2 N$$

$$\pi_1 \lambda = \pi_2 M = \frac{\pi_1 \lambda}{M} = \pi_2 = \frac{\pi_0 \lambda^2}{M^2} = \pi_2$$

$$\pi_k = \pi_0 \left(\frac{\lambda}{M}\right)^k \qquad \boxed{\pi_k = \frac{1-u}{1-u^{b+1}} \cdot u^k}$$

$$\sum_{k=0}^{b} \pi_0 \left(\frac{\lambda}{M}\right)^k = 1$$

$$\pi_0 \sum_{k=0}^{b} \left(\frac{\lambda}{M}\right)^k = 1$$

$$= \pi_0 \frac{1-\left(\frac{\lambda}{M}\right)^{b+1}}{1-\frac{\lambda}{M}} = 1$$

$$\pi_0 = \frac{1-\frac{\lambda}{M}}{1-\left(\frac{\lambda}{M}\right)^{b+1}} = \frac{1-u}{1-u^{b+1}}$$

M|M|∞

markov chain =

$$\begin{tikzcd} 0 \arrow[r, bend left, "\lambda"] & 1 \arrow[l, bend left, "\mu"] \arrow[r, bend left, "\lambda"] & 2 \arrow[l, bend left, "2\mu"] \arrow[r, bend left, "\lambda"] & 3 \arrow[l, bend left, "3\mu"] \arrow[r, bend left, "\lambda"] & 4 \arrow[l, bend left, "4\mu"] \arrow[r, bend left, "\lambda"] & {} \arrow[l, bend left, "5\mu"] \end{tikzcd}$$

$$\pi_0 \lambda = \pi_1 \mu$$

$$\frac{\pi_0 \lambda}{\mu} = \pi_1$$

$$\pi_1 \lambda = \pi_2 2\mu$$

$$\frac{\pi_1 \lambda}{2\mu} = \pi_2 = \frac{\pi_0 \lambda^2}{\mu} \cdot \frac{1}{2\mu} = \frac{\pi_0 \lambda^2}{2\mu^2} = \frac{\pi_0}{2}\left(\frac{\lambda}{\mu}\right)^2$$

$$\pi_1 \lambda + \pi_1 \mu = \pi_0 \lambda + \pi_2 2\mu$$

$$\frac{\pi_0 \lambda}{\mu} \lambda + \cancel{\frac{\pi_0 \lambda}{\mu} \mu} = \cancel{\pi_0 \lambda} + \pi_2 2\mu$$

$$\frac{\pi_0 \lambda}{\mu} \lambda = \pi_2 2\mu$$

$$\pi_2 2\mu + \pi_2 \lambda = \pi_3 3\mu + \pi_2 2\mu$$

$$\frac{\pi_1 \lambda}{\cancel{2\mu}} \cancel{2\mu} + \frac{\pi_1 \lambda}{2\mu} \lambda = \pi_3 3\mu + \frac{\pi_1 \lambda}{\cancel{2\mu}} \cancel{2\mu}$$

$$\frac{\pi_0 \lambda}{\mu} \lambda + \frac{\pi_0}{2}\left(\frac{\lambda}{\mu}\right)^2 \lambda = \pi\, 3\mu + \frac{\pi_0 \lambda}{\mu} \lambda$$

$\pi_2 \Lambda = \pi_3 \, 3m$

$$\frac{\pi_2 \Lambda}{3m} = \pi_3 = \frac{\frac{\pi_0}{2}\left(\frac{\Lambda}{m}\right)^2 \Lambda}{3m} = \frac{\pi_0}{2}\left(\frac{\Lambda}{m}\right)^2 \Lambda \cdot \frac{1}{3m} =$$

$$\frac{\pi_0}{6} \cdot \left(\frac{\Lambda}{m}\right)^3$$

$$\sum_{k=0}^{\infty} \frac{\pi_0}{k!} \cdot \left(\frac{\Lambda}{m}\right)^k = 1$$

$$\pi_0 \sum_{k=0}^{\infty} \frac{1}{k!} \cdot \left(\frac{\Lambda}{m}\right)^k = 1$$

$$\pi_0 \sum_{k=0}^{\infty} \frac{\left(\frac{\Lambda}{m}\right)^k}{k!} = 1$$

$$\pi_0 \, e^{\frac{\Lambda}{m}} = 1$$

$$\pi_0 \, e^{4} = 1$$

$$\pi_0 = \frac{1}{e^4}$$

$$\pi_k = \frac{\frac{1}{e^4}}{k!} \cdot \left(\frac{\Lambda}{m}\right)^k = \pi_k = \frac{\frac{1}{e^4}}{k!} \cdot 4^k =$$

$$\boxed{\frac{e^{-4} \, 4^k}{k!}}$$