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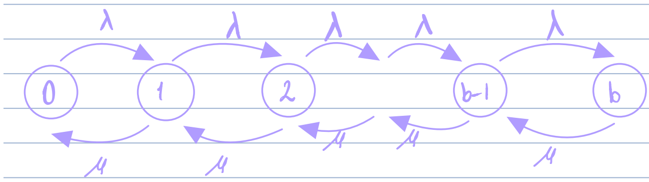
Markov Chains

M/M/1/b



$$\sum_{k=0}^{b} \pi_k = 1$$

- Analize state 0
  - Rate of leaving : $\pi_0 \lambda$
  - Rate of entering : $\pi_1 \mu$

$$\pi_0 \lambda = \pi_1 \mu$$

$$\pi_1 = \pi_0 \frac{\lambda}{\mu}$$

- Balance equations for states 1 to b-1

Leaving state 1: $(\lambda + \mu) \pi_1$

Entering state 1: $\lambda \pi_0 + \mu \pi_2$

$$(\lambda + \mu)\pi_1 = \lambda \pi_0 + \mu \pi_2$$

$$\lambda \pi_1 + \mu \pi_1 = \lambda \pi_0 + \mu \pi_2$$

$$\lambda \left(\frac{\lambda}{\mu}\right) \pi_0 + \mu \left(\frac{\lambda}{\mu}\right) \pi_0 = \lambda \pi_0 + \mu \pi_2$$

$$\lambda \left(\frac{\lambda}{\mu}\right) \pi_0 + \cancel{\lambda \pi_0} = \cancel{\lambda \pi_0} + \mu \pi_2$$

$$\left(\frac{\lambda}{\mu}\right)^2 \pi_0 = \pi_2$$

$$\pi_2 = \upsilon^2 \pi_0$$

$$\sum_{k=0}^{b} \upsilon^k \pi_0 = 1$$

$$\pi_0 \sum_{k=0}^{b} \upsilon^k = 1 \qquad \pi_0 \frac{1-\upsilon^{b+1}}{1-\upsilon} = 1 \qquad \pi_0 = \frac{1}{\frac{1-\upsilon^{b+1}}{1-\upsilon}} = \frac{1-\upsilon}{1-\upsilon^{b+1}}$$

- Balance for state b

$$\lambda \pi_{b-1} = \mu \pi_b$$

$$\pi_b = \frac{\lambda}{\mu} \pi_{b-1}$$

- Use total probability to solve for π_0

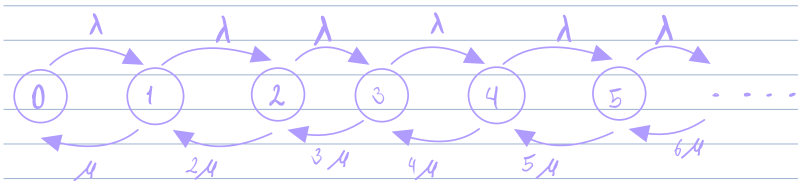
$$\pi_0 = \frac{1}{\frac{1-\rho^{b+1}}{1-\rho}} = \frac{1-\rho}{1-\rho^{b+1}}$$

In general:

$$\pi_k = \rho^k \pi_0$$

$$\boxed{\pi_k = \rho^k \left(\frac{1-\rho}{1-\rho^{b+1}} \right)}$$

M/M/∞



State 0:

L = E

$$\lambda \pi_0 = \mu \pi_1 \qquad \pi_1 = \frac{\lambda}{\mu} \pi_0$$

State 1:

Entering state 1: $\lambda \pi_0 + 2\mu \pi_2$

Leaving state 1: $\lambda \pi_1 + \mu \pi_1$

$$\lambda \pi_0 + 2\mu \pi_2 = \lambda \pi_1 + \mu \pi_1$$

$$\pi_1 = \frac{\lambda}{\mu} \pi_0$$

$$\lambda \pi_0 + 2\mu \pi_2 = \lambda \frac{\lambda}{\mu} \pi_0 + \cancel{\mu} \frac{\lambda}{\cancel{\mu}} \pi_0$$

$$\cancel{\lambda \pi_0} + 2\mu \pi_2 = \lambda \frac{\lambda}{\mu} \pi_0 + \cancel{\lambda \pi_0}$$

$$2\mu \pi_2 = \lambda \frac{\lambda}{\mu} \pi_0$$

$$2 \pi_2 = \left(\frac{\lambda}{\mu}\right)^2 \pi_0$$

$$\pi_2 = \frac{\left(\frac{\lambda}{\mu}\right)^2 \pi_0}{2}$$

State 2:

Entering state 2: $\lambda \pi_1 + 3\mu \pi_3$

Leaving state 2: $\lambda \pi_2 + 2\mu \pi_2$

$$\lambda \pi_1 + 3\mu \pi_3 = \lambda \pi_2 + 2\mu \pi_2$$

$\pi_1 = \frac{\lambda}{\mu} \pi_0$ $\quad \pi_2 = \frac{\left(\frac{\lambda}{\mu}\right)^2 \pi_0}{2}$

$$\lambda \left(\frac{\lambda}{\mu}\right) \pi_0 + 3\mu \pi_3 = \lambda \left(\frac{\left(\frac{\lambda}{\mu}\right)^2 \pi_0}{2}\right) + \cancel{2}\mu \left(\frac{\left(\frac{\lambda}{\mu}\right)^2 \pi_0}{\cancel{2}}\right)$$

$$\lambda \left(\frac{\lambda}{\mu}\right) \pi_0 + 3\mu \pi_3 = \lambda \left(\frac{\left(\frac{\lambda}{\mu}\right)^2 \pi_0}{2}\right) + \cancel{\mu} \left(\frac{\lambda}{\cancel{\mu}}\right)^2 \pi_0$$

$$\cancel{\frac{\lambda^2}{\mu} \pi_0} + 3\mu \pi_3 = \lambda \left(\frac{\left(\frac{\lambda}{\mu}\right)^2 \pi_0}{2}\right) + \cancel{\frac{\lambda^2}{\mu} \pi_0}$$

$$3\mu \pi_3 = \lambda \left(\frac{\left(\frac{\lambda}{\mu}\right)^2 \pi_0}{2}\right)$$

$$\pi_3 = \frac{\frac{\lambda}{1} \left(\frac{\left(\frac{\lambda}{\mu}\right)^2 \pi_0}{2}\right)}{3\mu} \left(\frac{1}{3\mu}\right)$$

$$\pi_3 = \frac{\lambda \left(\frac{\lambda}{\mu}\right)^2 \pi_0}{6\mu}$$

$$\pi_3 = \frac{\left(\frac{\lambda}{\mu}\right)^3 \pi_0}{6}$$

$3 \times 2 \times 1$

In general: $\pi_k = \frac{\upsilon^k \pi_0}{k!}$

$$\sum_{k=0}^{\infty} \frac{\upsilon^k \pi_0}{k!} = 1$$

$$\pi_0 \underbrace{\sum_{k=0}^{\infty} \frac{\upsilon^k}{k!}}_{e^{\upsilon}} = 1$$

$$\pi_0 e^{\upsilon} = 1$$

$$\pi_0 = \frac{1}{e^{\upsilon}}$$

$$\boxed{\pi_k = \frac{\upsilon^k \frac{1}{e^{\upsilon}}}{k!}}$$