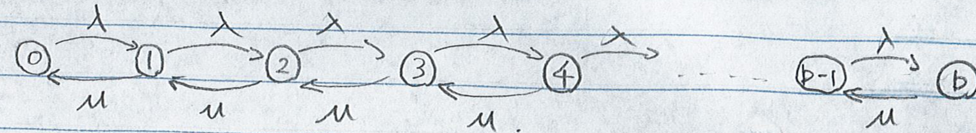
Challenge Project. M/M/1/b.



out of π_0 entering π_0

$$\lambda \pi_0 = \mu \pi_1$$

$$\frac{\lambda}{\mu}\pi_0 = \pi_1$$

out of π_1 entering π_1

$$(\lambda+\mu)\pi_1 = \lambda\pi_0 + \mu\pi_2$$

$$(\lambda+\mu)\cdot\left(\frac{\lambda}{\mu}\pi_0\right) = \lambda\pi_0 + \mu\pi_2$$

$$\frac{\lambda^2}{\mu}\pi_0 + \cancel{\lambda\pi_0} = \cancel{\lambda\pi_0} + \mu\pi_2$$

$$\frac{\lambda^2}{\mu^2}\pi_0 = \pi_2$$

by utilization law

$$U = \lambda \bar{s} = \lambda\cdot\frac{1}{\mu} = \frac{\lambda}{\mu}.$$

By utilization law

$$\pi_1 = U\cdot\pi_0$$

$$\pi_2 = U^2\cdot\pi_0$$

$$\pi_3 = U^3\cdot\pi_0$$

$$\vdots$$

$$\pi_b = U^b\cdot\pi_0$$

out of π_2

$$(\lambda+\mu)\cdot\pi_2 = \lambda\pi_1 + \mu\pi_3$$

$$\frac{\lambda^3}{\mu^2}\pi_0 + \frac{\lambda^2}{\mu}\pi_0 = \frac{\lambda^2}{\mu}\pi_0 + \mu\pi_3$$

$$\left(\frac{\lambda}{\mu}\right)^3\pi_0 = \pi_3$$

Therefore we can derive that

$$\sum_{k=0}^{b}\pi_k = 1 \quad \sum_{k=0}^{b} U^b\cdot\pi_0 = \pi_0\sum_{k=0}^{b}U^k$$

$$= \pi_0\cdot\frac{1-U^{b+1}}{1-U} = 1$$

We want to solve for π_0

$$\pi_0 = \frac{1-U}{1-U^{b+1}} = \frac{1-\left(\frac{\lambda}{\mu}\right)}{1-\left(\frac{\lambda}{\mu}\right)^{b+1}}$$

$$\downarrow$$

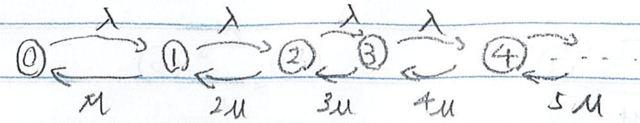
$$\frac{1-\left(\frac{\lambda}{\mu}\right)}{1-\left(\frac{\lambda}{\mu}\right)^{b+1}} = \frac{1-U}{1-U^{b+1}}$$

$$\pi_b = U^b\cdot\frac{1-U}{1-U^{b+1}} = \frac{U^b-U^{b+1}}{1-U^{b+1}}$$

or

$$\pi_b = \frac{U^b(1-U)}{1-U^{b+1}}$$

M / M / ∞



Balance equations

out of π_0 = entering π_0

$$\lambda \pi_0 = \mu \pi_1$$

$$\frac{\lambda}{\mu}\pi_0 = \pi_1$$ or by utilization law, $U\pi_0 = \pi_1$

out of π_1 = entering π_1

$$(\mu+\lambda)\cdot\pi_1 = \lambda\pi_0 + 2\mu\pi_2$$

$$(\mu+\lambda)\cdot\frac{\lambda}{\mu}\pi_0 = \lambda\pi_0 + 2\mu\pi_2$$

$$\cancel{\lambda\pi_0} + \frac{\lambda^2}{\mu}\pi_0 = \cancel{\lambda\pi_0} + 2\mu\pi_2$$

$$\frac{\lambda^2}{\mu}\pi_0 = 2\mu\pi_2$$

$$\frac{\lambda^2}{2\mu^2}\pi_0 = \pi_2$$ by utilization.

$$\frac{1}{2}\cdot U^2\pi_0 = \pi_2$$

out of π_2 = entering π_2

$$(2\mu+\lambda)\pi_2 = \lambda\pi_1 + 3\mu\pi_3$$

$$(2\mu+\lambda)\cdot\frac{\lambda^2}{2\mu^2}\pi_0 = \frac{\lambda^2}{\mu}\pi_0 + 3\mu\pi_3$$

$$\cancel{\frac{\lambda^2}{\mu}\pi_0} + \frac{\lambda^3}{2\mu^2}\pi_0 = \cancel{\frac{\lambda^2}{\mu}\pi_0} + 3\mu\pi_3$$

$$\frac{\lambda^3}{6\mu^3}\pi_0 = \pi_3$$

$$\frac{1}{6}\cdot U^3\pi_0 = \pi_3$$

out of π_3 = entering π_3

$$(3\mu+\lambda)\pi_3 = \lambda\pi_2 + 4\mu\pi_4$$

$$(\cancel{3}\mu+\lambda)\cdot\frac{\lambda^3}{\cancel{6}\mu^2}\pi_0 = \cancel{\frac{\lambda^3}{2\mu^2}}\pi_0 + 4\mu\pi_4$$

$$\frac{\lambda^4}{6\mu^3}\pi_0 = 4\mu\pi_4$$

$$\frac{\lambda^4}{24\mu^4}\pi_0 = \pi_4$$

as you go further in the process, the constant ex. $\frac{1}{2}, \frac{1}{6}, \frac{1}{24}$ gets bigger using factorial

$$\pi_2 = \frac{1}{2!}\cdot U^2\pi_0$$

$$\pi_3 = \frac{1}{3!}\cdot U^3\pi_0$$

$$\pi_4 = \frac{1}{4!}U^4\pi_0$$

$$\pi_k = \frac{1}{k!}U^k\pi_0.$$

Continues to the next page.

And therefore to calculate the total probability

$$\sum_{k=0}^{\infty} \frac{1}{k!} \cdot \left(\frac{\lambda}{\mu}\right)^k \cdot \pi_0 = \pi_0 \sum_{k=0}^{\infty} \frac{1}{k!} \cdot U^k = 1$$

By $\sum_{k=0}^{\infty} \frac{y^k}{k!} = e^y$, $\quad \pi_0 \cdot e^U = 1$

$$\pi_0 = \frac{1}{e^U}$$

and we want to know the π_k so plug in the $\pi_0 = \frac{1}{e^U}$ to the result of π_k's balance equation in previous page

$$\pi_k = \frac{U^k e^{-U}}{k!}$$

therefore the M/M/1 queue is Poisson distributed.