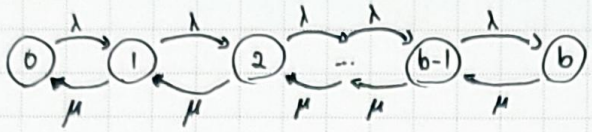
Challenge Project 3: Markov Chains

M/M/1/b Queue:



$$\pi_0 \lambda = \pi_1 \mu \quad \rightarrow \quad \pi_1 = \frac{\lambda}{\mu} \pi_0 = u \pi_0$$

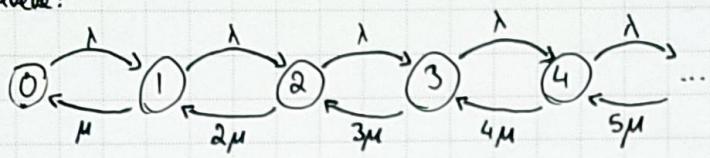
$$\pi_1 \lambda + \pi_1 \mu = \pi_0 \lambda + \pi_2 \mu \quad \rightarrow \quad \pi_1 \lambda = \pi_2 \mu \quad \rightarrow \quad \pi_2 = \frac{\lambda}{\mu} \pi_1 = \frac{\lambda^2}{\mu^2} \pi_0$$

$$\pi_k = \pi_0 \left(\frac{\lambda^k}{\mu^k} \right) = \pi_0 u^k$$

$$\sum_{k=0}^{b} \pi_0 u^k = 1 \quad \rightarrow \quad \pi_0 \underbrace{\sum_{k=0}^{b} u^k}_{\frac{1-u^{b+1}}{1-u}} = 1 \quad \rightarrow \quad \pi_0 \frac{1-u^{b+1}}{1-u} = 1$$

$$\pi_0 = \frac{1-u}{1-u^{b+1}} \quad \rightarrow \quad \frac{\pi_k}{u^k} = \frac{1-u}{1-u^{b+1}} \quad \rightarrow \quad \boxed{\pi_k = \frac{1-u}{1-u^{b+1}} u^k}$$

M/M/∞ Queue:



0→ $\pi_0 \lambda = \pi_1 \mu \rightarrow \pi_1 = \frac{\lambda}{\mu} \pi_0 = u \pi_0$

1→ $\pi_1 \lambda + \pi_1 \mu = \pi_0 \lambda + \pi_2 2\mu \rightarrow \pi_1 \lambda = \pi_2 2\mu \rightarrow \pi_2 = \frac{\lambda}{2\mu} \pi_1 = \frac{\lambda^2}{2\mu^2} \pi_0$

$\rightarrow \pi_k = \frac{1}{k!} v^k \pi_0$

2→ $\pi_2 2\mu + \pi_2 \lambda = \pi_1 \lambda + \pi_3 3\mu \rightarrow \pi_2 \lambda = 3\mu \pi_3 \rightarrow \pi_3 = \frac{\lambda}{3\mu} \pi_2 = \frac{\lambda^3}{6v^3} \pi_0$

$\sum_{k=0}^{\infty} \frac{1}{k!} v^k \pi_0 = 1 \rightarrow \pi_0 \underbrace{\sum_{k=0}^{\infty} \frac{1}{k!} u^k}_{e^u} = 1 \rightarrow \pi_0 e^u = 1 \rightarrow \pi_0 = \frac{1}{e^u}$

$\pi_k = \frac{v^k}{k!} \cdot \pi_0 = \frac{v^k}{k!} \cdot \frac{1}{e^u} = \frac{v^k e^{-u}}{k!} \rightarrow \boxed{\pi_k = \frac{v^k e^{-v}}{k!}}$ ← M/M/∞ queue is poisson