Sprint 3 Deliverables.

1. The Newton - Pepys Problem.

- This is a binomial random variable because
  - Trials (dice rolls) are independent.
  - We know the number of trials → determined by the number of dice
  - It's talking about how many sixes we get successfully

- Six fair dice are tossed and get at least one six
  ↳ we can use total probability 1 and subtract the probability of getting zero six.
  X: number of six appears.

$P(X \geqq 1) = 1 - P(X=0) = 1-(\frac{5}{6})^6 = 0.66512.$

- Twelve fair dice $\underbrace{\text{independently}}$ Tossed and get at least 2 six:
  ↳ Again we could use the total probability of 1 subtract the sum of probabilities below:
  $P(X=0)$:= no six. $P(X=1)$: one six appears.
  $P(X=0) = (\frac{5}{6})^{12}$ $P(X=1) = {}_{12}C_1 \cdot (\frac{5}{6})^{11} (\frac{1}{6})$

Therefore $P(X \geqq 2)$: Getting at least two sixes.

$$P(X \geqq 2) = 1 - \{P(X=0) + P(X=1)\}$$
$$= 1 - \{(\frac{5}{6})^{12} + {}_{12}C_1 (\frac{5}{6})^{11} (\frac{1}{6})\}$$
$$= 0.618667.$$

- Eighteen fair dice are tossed independently and get at least 3 sixes.
  ↳ Same approach with the twelve die proposition

$$P(X \geqq 3) = 1 - \{P(X=0) + P(X=1) + P(X=2)\}$$
$$= 1 - \{(\frac{5}{6})^{18} + {}_{18}C_1 (\frac{5}{6})^{17} (\frac{1}{6}) + {}_{18}C_2 \cdot (\frac{5}{6})^{16} (\frac{1}{6})^2\}$$
$$= 0.597346.$$

Therefore the first proposition obtains the greatest chance of success.

2. Geometric Urn.

We can use the Expected Value of Geometric Distibution and that was said to be 20, from the question desc ription.
We proved in class that the formula of expected value of geometric distiibution is.

$E[X] = \frac{1}{p}$ and this is 20 so

$\frac{1}{p} = 20 \quad p = \frac{1}{20}$

We are looking for the number of black and red balls.
Assume x = number of red balls, $100 - x$ = number of black balls.

Probability of getting a red ball is fraction of redballs iso.

$P(\text{Red}) = \frac{x}{x + (100 - x)} = \frac{1}{20}$ ... so we can solve for x

$\frac{x}{100} = \frac{1}{20}$

$20x = 100$

$x = 5$

number of black balls
$100 - 5 = 95$

Therefore there are 5 red balls and 95 black balls.

3. Dragon Dice X = galleons she wins.

There are 4 outcomes possible in total based on X and 4 possible probability.

$P(X = -1)$: probability that she loses 1 galleons.
$P(X = 1)$: probability that she wins 1 galleon.
$P(X = 2)$: probability that she wins 2 galleon
$P(X = 3)$ probability that she wins 3 galleon.

No.

Date

The die rolls are independent events and we know how many dice we are using so this is a Binomial distiribution so

$P(X=-1) = (\frac{5}{6})^3$
$P(X=1) = {}_3C_1 \cdot (\frac{5}{6})^2 \cdot (\frac{1}{6})$
$P(X=2) = {}_3C_2 \cdot (\frac{5}{6}) \cdot (\frac{1}{6})^2$
$P(X=3) = (\frac{1}{6})^3$

probabilities and

$E[X] = \sum_k k \cdot p(k)$

so.

$$E[X] = \sum_{k=0}^{3} k \cdot p(X=k) = -1 \cdot (\frac{5}{6})^3 + 1 \cdot \{3 \cdot (\frac{5}{6})^2 (\frac{1}{6})\} + 2 \cdot \{3 \cdot (\frac{5}{6}) \cdot (\frac{1}{6})^2\} + 3 \cdot (\frac{1}{6})^3.$$

$$= -.078704. \quad \text{or} \quad -\frac{17}{216}.$$

The martingale