Griffin Lehrer

Deliverables Sprint 2 CMS_380

Wizard People Dear Reader

A

P(She's a witch) = .75

C

P(She's Not a witch)

B | A

P(Not recieving a letter | She's a witch) = .03

B | C

P(Not recieving a letter | She's not a witch) = .99

A | B

Goal: P(Hermione is a witch | Not recieving a Letter)

Bayes Rule: $P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}$

$P(A) = .75$

$P(B|A) = .03$

$P(B) = (B|A) \cdot P(A) + P(B|C) \cdot P(C)$

$(.03 \times .75) + (.99 \times .25)$

$.0225 + .2475$

$P(B) = .27$

$P(A|B) = \frac{.75 \times .03}{.27} = \frac{.0225}{.27} = .083$

$P(A|B) = 8.3\%$ (Approx)

Choclate Frogs

30 total cards

P(picking any one card) $= \frac{1}{30} = .03 = 3\%$ Chance

Goal = Number of cards to get all 30 unique cards

Expected value of getting 1st card +
Expected value of getting 2nd card + ... +
Expected value of getting 30th Card

Expected value for getting a new card $= \frac{1}{P}$
Where P is the probability of getting a new card

$$\frac{1}{\frac{30}{30}} + \frac{1}{\frac{29}{30}} + \frac{1}{\frac{28}{30}} + \frac{1}{\frac{27}{30}} + \frac{1}{\frac{26}{30}} + \frac{1}{\frac{25}{30}} + \frac{1}{\frac{24}{30}} + \frac{1}{\frac{23}{30}} \ldots + \frac{1}{\frac{1}{30}} =$$

$$\frac{30}{30} + \frac{30}{29} + \frac{30}{28} + \frac{30}{27} + \ldots + \frac{30}{1} = 119.8446$$

frogs

Hat Problem

20% of good students get sorted into Slythern
100% of Evil students get sorted into slytern
10% of new students are Evil

Goal: Find the probability of a random slytern student being evil

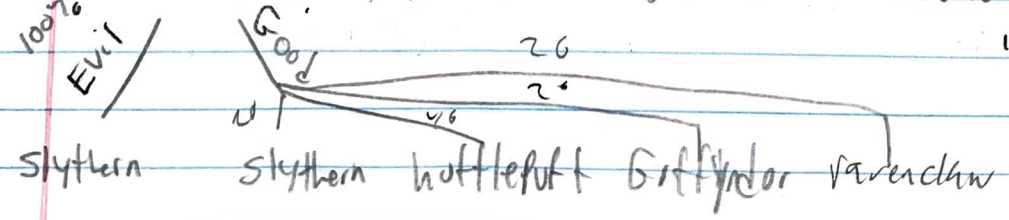
Find $P(\underset{A}{\text{Evil}} \mid \underset{B}{\text{Slytern}})$

What we know:

$P(\underset{B}{\text{Slytern}} \mid \underset{C}{\text{Not Evil}}) = .20$
$P(\text{Slytern} \mid \text{Evil}) = 1$
$P(\text{Evil student}) = .10$
$P(\text{Good}) = .90$

Bayes theorem $= (A|B) = \dfrac{P(A)\,P(B|A)}{P(B)}$ ($P(A)$: .10, $P(B|A)$: 1)

$P(B) = (.10 \cdot 1) + (.9 \cdot .2) = .28 = 28\%$ All students are in Slytern



$\dfrac{.10 \cdot 1}{.28} = \dfrac{.10}{.28} = .357 = 36\%$ chance of being Evil

Elevator is moving upwards $\frac{1}{2}$ and down $\frac{1}{2}$ of the time

Dumblevator

Goal: Find the probability the elevator is moving down when Hermione arrives at the elevator.

The Elevator moves up in sequential order 1, 2, 3, 4, ...
When the elevator reaches floor 15 it moves down in sequential order 15, 14, 13...

The Elevator will be moving down when it is on the 15^{th} or 14^{th} floors

Probability the Elevator will be on any given floor $= \frac{1}{15}$

On the 13^{th} floor the Dumblevator has a $\frac{1}{3}$ chance of moving up or down when Hermione arrives.

Probability the Dumblevator will be moving down =

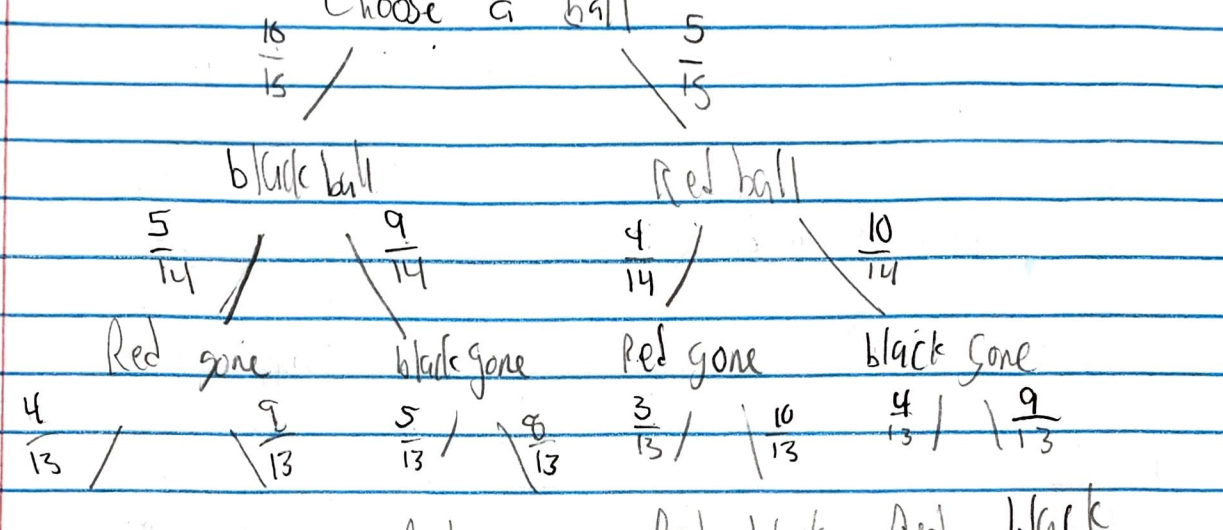
Prob 14^{th} floor + prob 15^{th} floor + $\frac{1}{2}$ Prob 13^{th} floor =

$\frac{1}{15} + \frac{1}{15} + \frac{1}{15} \cdot \frac{1}{2} = \frac{1}{15} + \frac{1}{15} + \frac{1}{30} = \frac{2}{30} + \frac{2}{30} + \frac{1}{30} = \frac{3}{30} =$

$\frac{1}{10}$ Probability the elevator will be moving down when Hermione arrives

Urn While You Learn

Goal: Find the Probability the second ball chosen is red



$$\left(\frac{10}{15}\cdot\frac{5}{14}\cdot\frac{4}{13}\right)+\left(\frac{10}{15}\cdot\frac{9}{14}\cdot\frac{5}{13}\right)+\left(\frac{5}{15}\cdot\frac{4}{14}\cdot\frac{3}{13}\right)+\left(\frac{5}{15}\cdot\frac{10}{14}\cdot\frac{4}{13}\right)$$

$= .33$ or 33% chance of the second ball being red

Pólya's Urn

Pick a ball from the urn

$\frac{9}{15}$ black $(10_b, 6_r)$ $\quad$ $\frac{6}{15}$ Red $(9_b, 7_r)$

$\frac{10}{16}$ black $\quad$ $\frac{6}{16}$ red $\quad$ $\frac{9}{16}$ black $\quad$ $\frac{7}{16}$ red

$11_b 6_r$ black $\quad$ red $10_b 7_r$ $\quad$ $10_b 7_r$ black $\quad$ red $9_b 8_r$

$E[x]$ = Expected value = $\sum_x x P(x)$

Expected value of the red balls =

$$6\left[\frac{9}{15}\cdot\frac{10}{16}\right) + 7\left(\frac{9}{15}\cdot\frac{6}{16}\right) + 7\left(\frac{6}{15}\cdot\frac{9}{16}\right) + 8\left(\frac{6}{15}\cdot\frac{7}{16}\right)$$

Expected value of the black balls

$$11\left(\frac{9}{15}\cdot\frac{10}{16}\right) + 10\left(\frac{9}{15}\cdot\frac{6}{16}\right) + 10\left(\frac{6}{15}\cdot\frac{9}{16}\right) + 9\left(\frac{6}{15}\cdot\frac{7}{16}\right)$$

$E[R] = 6.8$ $\quad$ $E[B] = 10.2$

Arithmancy

Six sided die example

Sample Space = $\{1, 2, 3, 4, 5, 6\}$
Prob of each number appearing: $\frac{1}{6}$

expected value = $\sum_x x P(x) = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) = 0 + \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = \frac{7}{2}$

$= \frac{n+1}{2}$

Arbitrary Example

Prove the expected value of a discrete uniform distribution with $a=1$ and $b=n$ is $E[X] = \frac{n+1}{2}$

$E[X] = xP(x)$

$E[x] = \sum_{k=1}^{n} \frac{1}{n} \cdot k = \frac{1}{n} \sum_{k=1}^{n} k = 1+2+3+\ldots+n-1+n$

$= \frac{1}{n} \sum_{k}^{n} = 1+2+3+\ldots+n = \frac{n(n+1)}{2} = \frac{1}{n} \cdot \frac{n(n+1)}{2} =$

$1 \cdot \frac{(n+1)}{2} = \frac{(n+1)}{2}$

Birthday Attack

Goal find the probability that No two students have the same birthday

40 total students 365 possible birthdays

P(Everyone has a different birthday) = (365/365) x (364/365) x (363/365) ... x (325/365)

= .1087 = 10.87% chance No two classmates have the same birthday