1. Wizard People Dear Reader?

A = she's a witch
B = not receiving a letter

$P(A) = 0.75$
$P(!A) = 1 - P(A) = 0.25$
$P(B|A) = 0.03$
$P(B|!A) = 0.99$

$P(B)$ is not given so we must solve for it

$$P(B) = P(B|A)P(A) + P(B|!A)P(!A)$$
$$= 0.03 \cdot 0.75 + 0.99 \cdot 0.25$$
$$= 0.0225 + 0.2475$$
$$= 0.27$$

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)} = \frac{0.75 \cdot 0.03}{0.27} = 0.08\overline{33} = 8.33\%$$

$$\boxed{P(A|B) = 0.08\overline{33}}$$

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2. Chocolate Frogs

Amount of frogs to buy: $\frac{30}{30} + \frac{30}{29} + \frac{30}{28} + \frac{30}{\dots} + \frac{30}{1}$

$$E(x) = \sum_{n=0}^{29} \frac{30}{(30-n)} = 119.849614 = \boxed{120}$$

Hermione cannot buy a fraction of a frog

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3. Hat Problem

S = Slytherin
E = Evil

$P(S|E) = 1.00$
$P(S|!E) = 0.2$
$P(E) = 0.10$
$P(!E) = 0.90$

$P(S)$ is not given so we must solve for it

$$P(S) = P(S|E)P(E) + P(S|!E) \cdot P(!E)$$
$$= 1.00 \cdot 0.10 + 0.20 \cdot 0.90$$
$$= 0.28$$

$$P(E|S) = \frac{P(E) \cdot P(S|E)}{P(S)} = \frac{0.10 \cdot 1.00}{0.28} = 0.3571 = 35.71\%$$

$$\boxed{P(E|S) = 0.3571}$$

4. Dumblevator

Assumptions: At 1st floor elevator always goes up
" 15th " " " " down.
At any moment the elevator is equally likely to be in any floor going either direction

15	1/1
14	2/2
13	1/2
12	0/2
...	0/2 ← x9
2	0/2
1	0/1

} 4 cases out of 28 states where the elevator will be going down at 13th floor

$\frac{4}{28} = \frac{1}{7} = P(\text{down}) = 0.142857 = 14.29\%$

# of times elevator arrives in a floor
# of times elevator will go down by floor 13

P(down) = 0.142857

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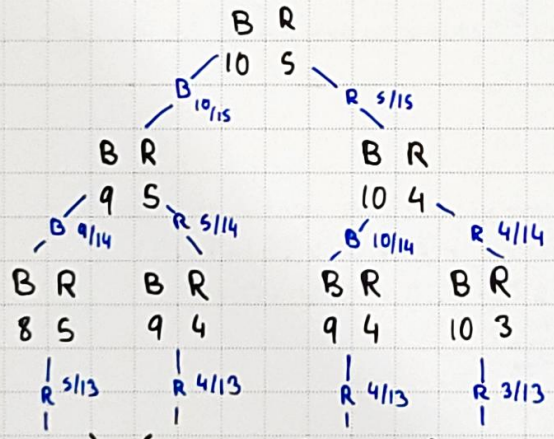
5. Urn While You Learn

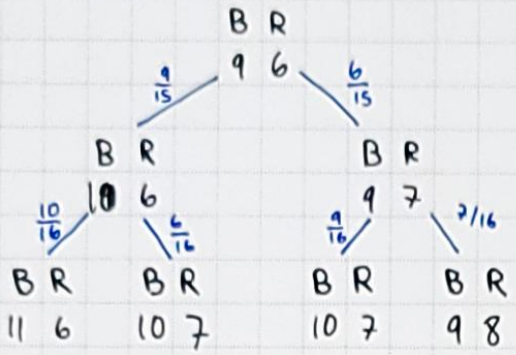
B = 10
R = 5



$$\left(\frac{5}{13}\cdot\frac{9}{14}\cdot\frac{10}{15}\right)+\left(\frac{4}{13}\cdot\frac{5}{14}\cdot\frac{10}{15}\right)+\left(\frac{4}{13}\cdot\frac{10}{14}\cdot\frac{5}{15}\right)+\left(\frac{3}{13}\cdot\frac{4}{14}\cdot\frac{5}{15}\right) = 0.\overline{33} = 33.33\%$$

P(3rd draw is red) = $0.\overline{33}$

6. Pólya's Urn



$$B = 11\left(\frac{10}{16}\cdot\frac{9}{15}\right) + 10\left(\frac{6}{16}\cdot\frac{9}{15}\right) + 10\left(\frac{9}{16}\cdot\frac{6}{15}\right) + 9\left(\frac{7}{16}\cdot\frac{6}{15}\right) = 10.2$$

$$R = 6\left(\frac{10}{16}\cdot\frac{9}{15}\right) + 7\left(\frac{6}{16}\cdot\frac{9}{15}\right) + 7\left(\frac{9}{16}\cdot\frac{6}{15}\right) + 8\left(\frac{7}{16}\cdot\frac{6}{15}\right) = 6.8$$

$E[\text{Red}] = 6.8$
$E[\text{Blue}] = 10.2$
→ Expected values of Red and Blue balls.

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7. Arithmancy

$p(x) = \frac{1}{n}$ → each outcome is equally likely

$$E[X] = \sum_x x \cdot p(x) = \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \frac{\cdots}{n} + \frac{n}{n}$$

$$= \frac{1}{n}(1+2+3+\ldots+n) \rightarrow \sum^{n} k = \frac{n(n+1)}{2}$$

$$= \frac{1}{n}\cdot\frac{n(n+1)}{2} = \frac{n+1}{2}$$

$E[X] = \frac{n+1}{2}$

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8. Birthday Attack

$$P(\text{n students are born in different days}) = \frac{365-0}{365} * \frac{365-1}{365} * \frac{365-\ldots}{365} * \frac{365-n}{365}$$

using python script:

```
total = 1
for x in range 40:
    total *= (365-x)/365
print total
> 0.108768
```

The probability no students in Hermione's 40-person classroom share a birthday is of 0.108765 or 10.87%.