The Newton-Pepys Problem

Which of the following three propositions has the greatest chance of success? The first one, six fair dices are tossed and at least one six appears.

- Six fair dice are tossed independently and at least one six appears.
- Twelve fair dice are tossed independently and at least two sixes appear.
- Eighteen fair dice are tossed independently and at least three sixes appear.

-Use discrete probability distribution

$p = 1/6 \to$ Success

$1 - 1/6 \to$ Failure

Use Binomial Distribution to find the probability of obtaining k successes of n trials

$P(X=k) \binom{n}{k} (1-p)^{n-k} p^k$

- Six fair dice are tossed independently and at least one six appears

k= at least 1 , so we use total probability

n=6

$P(X \geq 1) = 1 - P(X=0)$

$P(X=0) = \binom{6}{0} (1-1/6)^{6-0} (1/6)^0$

$\binom{6}{0} = \frac{6!}{(6-0)!\,0!} = \frac{1}{1} = 1$

$P(X=0) = 1 \cdot (0.833)^6 \cdot 1 = 0.335$

$P(X \geq 1) = 1 - 0.335 = 0.665$ $\approx 66.5\%$ chance of at least one six appearing

- Twelve fair dice are tossed independently and at least two sixes appear

k= at least 2, we use total probability

n= 12

$P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$P(X=0) = \binom{12}{0} (1-1/6)^{12-0} (1/6)^0$

$\binom{12}{0} = \frac{12!}{(12-0)!\,0!} = \frac{1}{1} = 1$

$P(X=0) = (1)(5/6)^{12} (1/6)^0$

$P(X=0) = (1)(0.11215)(1) = 0.11215$

$P(X=1) = \binom{12}{1} (1-1/6)^{12-1} (1/6)^1$ $\qquad \binom{12}{1} = \frac{12!}{(12-1)!\,1!} = \frac{12!}{(11)!\,1!} = \frac{1}{1} = 1$

$P(X=1) = (12)(0.1346)(1/6) = 0.2691$

$P(X \geq 2) = 1 - 0.11215 - 0.2691 = 0.61875 \approx 61.87\%$ chance of at least two sixes appearing

- Eighteen fair dice are tossed independently and at least three sixes appear.

$$P(x=0) = \binom{18}{0}(1-1/6)^{18-0}(1/6)^0$$
$$= (1)(5/6)^{18}(1)$$
$$= 0.0376$$

$$P(x=1) = \binom{18}{1}(5/6)^{17}(1/6)^1$$
$$= (18)(0.04507)(1/6)$$
$$= 0.1352$$

$$\binom{18}{1} = \frac{18!}{(18-1)!\,1!} = \frac{18!}{(17!)1!} = \frac{18}{1} = 18$$

$$P(x=2) = \binom{18}{2}(5/6)^{16}(1/6)^2$$
$$= (153)(0.054)(0.0277)$$
$$= 0.229$$

$$\binom{18}{2} = \frac{18!}{16!\,2!} = \frac{18\times17}{2!} = \frac{18\times17}{2} = 153$$

$$P(X \geq 3) = 1 - 0.0376 - 0.1352 - 0.229 = 0.5982$$

$\approx 59.82\%$ chance of at least 3 sixes appearing

Geometric Urn

We use expected value to reason about the number of red and black balls

$E[X] = 20$ → expected number of draws to get one red ball is 20

$$\frac{1}{p} = 20$$

$$20p = 1$$

$p = 1/20$ = probability of getting a red ball

$\frac{1}{20} \times \frac{5}{5} = \frac{5}{100}$ 5 red balls in total means that there are 95 black balls

Dragon Dice

X = outcome of the game as number of galleons

Possible outcomes:

$x = -1, 1, 2, 3$

$k = 0, 1, 2, 3$

→ probability of success

$p = 1/6$, rolling a die with the chosen number

$n = 3$, number of trials

k → number of successes, $k = 0, 1, 2, 3$

$$E[X] = \sum_x x\,p(x)$$

When little $x = -1$ and $k = 0$ → $P(X=0)$ no successes ($P(X=k)$)

$$P(X=0) = \binom{3}{0}(1-1/6)^3 (1/6)^0$$
$$= (1)(5/6)^3(1)$$
$$= 0.5787$$

$$P(X=1) = \binom{3}{1}(5/6)^2(1/6)^1$$
$$= 3(0.694)(0.167)$$
$$= 0.347$$

$$P(X=2) = \binom{3}{2}(5/6)^1(1/6)^2$$
$$= 3 \cdot (0.833)(0.027)$$
$$= 0.0694$$

$$P(X=3) = \binom{3}{3}(5/6)^0(1/6)^3$$
$$= 1 \cdot 1 \cdot (0.0046)$$
$$= 0.0046$$

$$E[X] = (-1)(0.5787) + (1)(0.347) + (2)(0.0694) + (3)(0.0046)$$
$$-0.5787 + 0.347 + 0.1388 + 0.0138 = -.0791$$

Hermionie is expected to lose 0.791 galleons per game

Leo Poisson

Simulate binomial process with $n = 1000$ and $p = 0.025$

Perform 100 random coin flips where each coin comes up heads with probability 0.020
Record the number of heads that occur

$P(X=k) = \binom{n}{k} (1-p)^{n-k} p^k$ ← pmf

100 balls

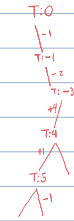
G: find the number of black and red balls

On average, it takes 20 draws to get the first red ball

Expect 20 draws to get the first red ball

$E[X] = 20 = 1/p \quad p = 1/20 = 0.05$

5% of the balls in the urn are red



if you lose:
Double previous bet
else:
bet 1