Sprint 2 - Deliverables

Wizard People, Dear Reader?

$$P(A/B) = \frac{P(A)\,P(B/A)}{P(B)}$$

P(A) = She's a witch = 0.75
P(B) = Not recieving a letter = ?
P(C) = Not a witch = 0.25

P(B/A) = 0.03
P(B/C) = 0.99

$$P(A/B) = \frac{(0.75)(0.03)}{P(B)}$$

$$P(B) = P(B/A)P(A) + P(B/C)P(C)$$
$$= (0.03)(0.75) + (0.99)(0.25)$$
$$= 0.27$$

$$P(A/B) = \frac{0.0225}{0.27} = 0.083 \rightarrow \boxed{8.3\%}$$

Chocolate Frogs

- 30 famous Witch/Wizard cards.
- every frog 1/30
- $E[\text{frogs}] \rightarrow ?$

- $E[\text{frogs for 1 card}] = 30/30$
  $E[\text{" " 2 cards}] = 30/29$
  $\vdots$
  $E[\text{" " 29 cards}] = 30/2$
  $E[\text{" " 30 " }] = 30/1$

$\rightarrow E[\text{frogs to get all cards}] = 1 + 30/29 + 30/28 + 30/27 + 30/26 + 30/25 + 30/24 + 30/23 + 30/22 + 30/21 + 30/20 + 30/19 + 30/18 + 30/17 + 30/16 + 30/15 + 30/14 + 30/13 + 30/12 + 30/11 + 30/10 + 30/9 + 30/8 + 30/7 + 30/6 + 30/5 + 30/4 + 30/3 + 30/2 + 30$

Hat Problem

1) 20·1· → Slytherin (evil)
   40·1· → Hufflepuff
   20·1· → Gryffindor
   20·1· → Ravenclaw

If 10·1· of new students are evil, what's the prob. that a randomly chosen slytherin is evil? → P(evil/Slytherin)

$$P(\text{Evil}/\text{Slyth}) = \frac{P(\text{Evil} \cap \text{Slyth})}{P(\text{Slyth})} = \frac{P(E)\,P(S/E)}{P(S)}$$

$P(E) = 10\cdot1\cdot$, $P(S) = 20\cdot1\cdot$

- since all evil students go to Slytherin, the prob. that a chosen evil studen is a sytherin is 100·1·
  ↳ $P(S/E) = 1$

$$\therefore P(E/S) = \frac{(0.1)(1)}{0.2} = 0.5 = \boxed{50\cdot1\cdot}$$

Dumbledevator

- 15 floors
- continuously moves ↑ & ↓
- Hermione ends class @ 5 on floor 13
  ↳ wants to go to floor 1
- What is the prob. that Dumbledevator is ↓ when it arrives to the 13th floor

* H must be going ↓ if elevator was on floor 15 previously

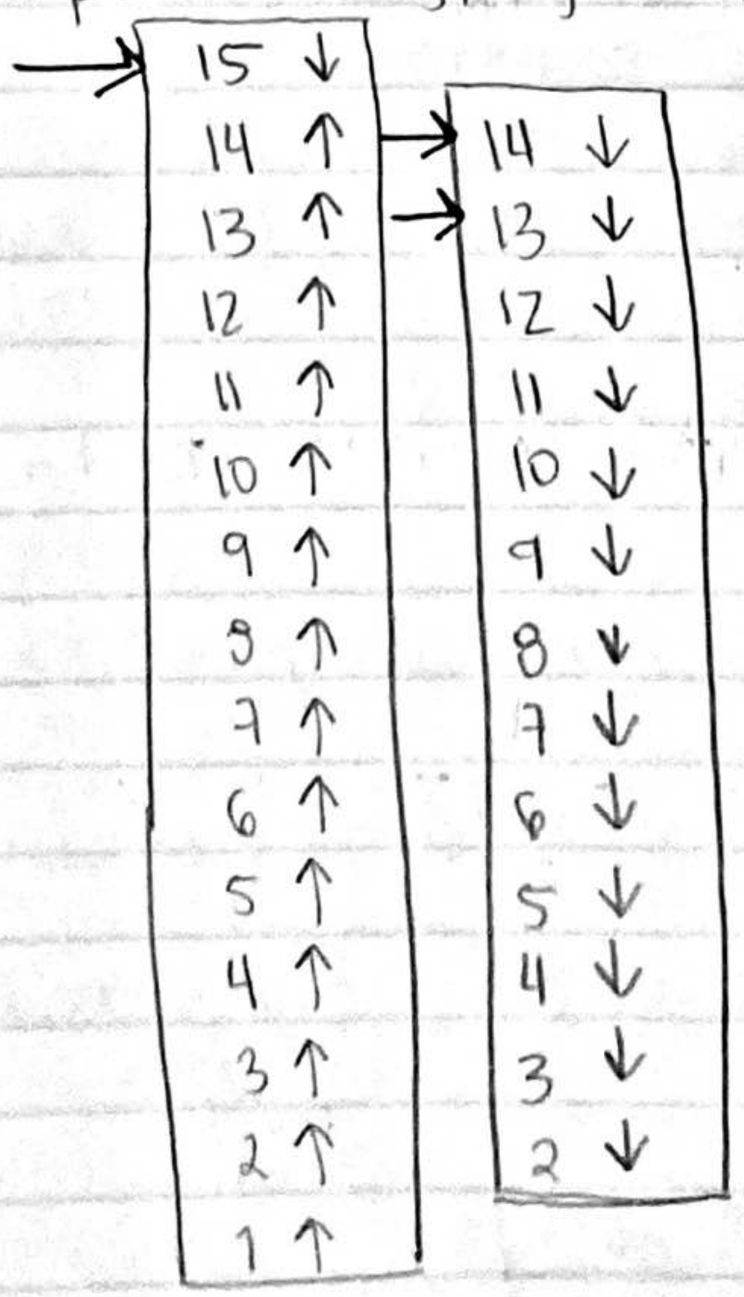
$$P(\underbrace{\text{elevator being on floor 15}}) = \frac{1}{28}$$

→ see my drawing below as to why its 1/28

the elevator being on floor 15 guarantees it will be going down, which is what we want

* If it is in floor 14, it could be going ↑ or ↓

* possible stages of the elevator & its direction:



→ thus gives us 28 possible "outcomes"

→ there are only 3 occasions where we know the elevator will be going down @ floor 13 (marked with →)

→ So the probability of the elevator to be going down when it reaches floor 13 is $\boxed{3/28}$

Urn While You Learn

- Hermione takes a ball, the urn randomly discards another one
- 10 black balls & 5 red balls
- If Hermione draws 2 balls, prob. that the 2nd one is red?

* You can have: 2 red balls or 1 black & 1 red to "succeed"

2 red balls

Ball 1:

$$P(R_1) = \frac{5}{15} = \frac{1}{3}$$

Ball 2: We don't know if a red or black ball was discarded

$$P(R_2) = \frac{5}{14} \text{ or } \frac{4}{14}$$

$\frac{5}{14}$ ↓ black ball is gone

$\frac{4}{14}$ ↓ red ball is gone

$$P(R_1 R_2) = \left(\frac{1}{3}\right)\left(\frac{5}{14}\right) + \left(\frac{1}{3}\right)\left(\frac{4}{14}\right) = \frac{9}{42}$$

1 black ball & 1 red ball

Ball 1:

$$P(B_1) = \frac{10}{15} = \frac{2}{3}$$

Ball 2: We don't know if red or black was discarded

$$P(R_2) = \frac{5}{14} \text{ or } \frac{4}{14}$$

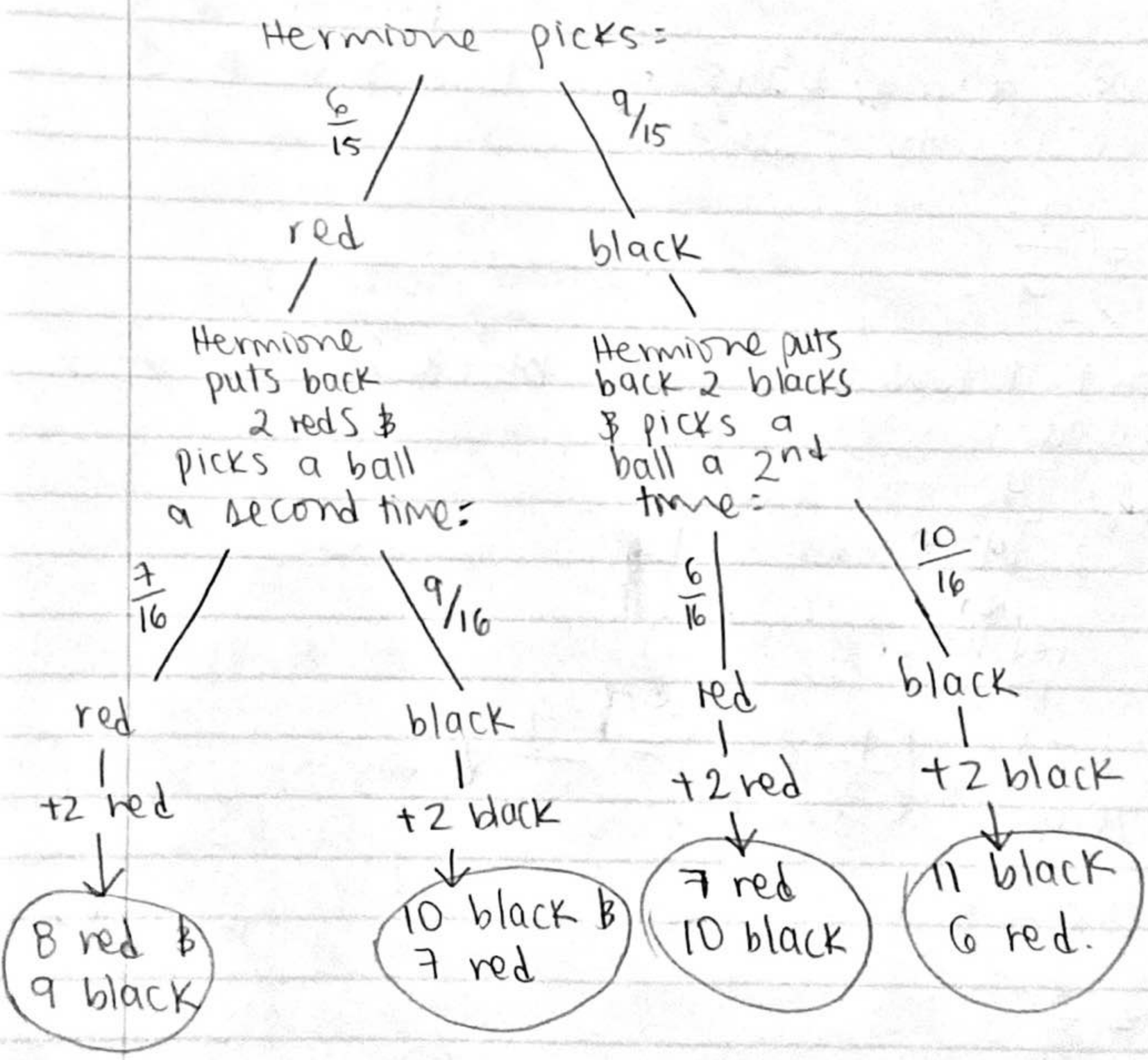
$$P(B_1 R_2) = \left(\frac{2}{3}\right)\left(\frac{5}{14}\right) + \left(\frac{2}{3}\right)\left(\frac{4}{14}\right) = \frac{18}{42}$$

$$P(\text{second ball is red}) = \frac{9}{42} + \frac{18}{42} = \frac{27}{42} = \boxed{\frac{9}{14}}$$

Pólya's Urn

*science undergraduate internship programs

- 9 black & 6 red balls
- Hermione picks a ball, returns it, & adds a ball of = color
- She does it twice
- Expected # of red & black balls?



$E[red] = 8(1/4) + 7(1/4) + 7(1/4) + 6(1/4)$

$\boxed{E[red] = 7}$

$E[black] = 9(1/4) + 10(1/4) + 10(1/4) + 11(1/4)$

$\boxed{E[black] = 10}$

Arithmancy

- Prove $\sum_{k=1}^{n} \frac{1}{n} K = \frac{n+1}{2}$ → right hand side

→ left hand side

$$E[x] = \sum_{k=1}^{n} \frac{1}{n} K$$

$$= \frac{1}{n} \sum_{k=1}^{n} K = \frac{n+1}{2} \rightarrow \sum_{k=1}^{n} = \frac{n(n+1)}{2}$$

- If die has n+1 sides (left hand side):

$$E[x] = \frac{1}{n+1} \sum_{k=1}^{n+1} K$$

$$= \frac{1}{n+1} \sum_{k=1}^{n} K + n + 1$$

→ substitute with

$$= \frac{1}{n+1} \left[\left(\frac{n(n+1)}{2} \right) + n + 1 \right]$$

$$= \frac{1}{n+1} \left(\frac{n^2+n}{2} + \frac{2n}{2} + \frac{2}{2} \right)$$

$$= \frac{1}{n+1} \left(\frac{n^2+3n+2}{2} \right)$$

$$= \frac{1}{n+1} \left[\frac{(n+2)(n+1)}{2} \right] = \frac{n+2}{2}$$

- If die has n+1 sides (right hand side):

$n = n+1$

$$\frac{(n+1)+1}{2} = \frac{n+2}{2}$$

$\frac{n+2}{2} = \frac{n+2}{2}$

$\therefore \sum_{k=1}^{n} \frac{1}{n} K = \frac{n+1}{2}$

Birthday Attack

- 40 students
- What is the probability that no student share the same bday?

- 365 possible bdays

- Hermione + friend

$$P(\neq \text{bdays}) = \left(\frac{365}{365}\right)\left(\frac{364}{365}\right)$$

- + friend

$$P(\neq \text{bdays}) = \left(\frac{365}{365}\right)\left(\frac{364}{365}\right)\left(\frac{363}{365}\right)$$

- Hermione + 39 friends (40 students)

$$P(\neq \text{bdays}) = \left(\frac{365}{365}\right)\left(\frac{364}{365}\right) * \cdots * \left(\frac{327}{365}\right)\left(\frac{326}{365}\right)$$