Wizard people, dear reader?

P(She's a witch) = .75
P(Not receiving a letter | She's a witch) = .03
P(Not receiving a letter | She's not a witch) = .99

Under these assumptions, what is the probability that Hermione really is a witch even though she didn't get a letter?

$P(A|B) = \frac{P(A)\,P(B|A)}{P(B)}$, where:

A= she's a witch
B= not receiving a letter

P(B) = the total probability of not receiving a letter

$$P(B) = P(B|A)\,P(A) + P(B|\text{not }A)\,P(\text{not }A)$$
$$= (0.03)(0.75) + (0.99)(0.25)$$
$$0.0225 + 0.2475$$
$$= 0.27$$

$$P(A|B) = \frac{(0.75)(0.03)}{0.4725} = \frac{0.0225}{0.27} = \boxed{0.083}$$

Chocolate frogs

$E[x_i] = \frac{1}{p(\text{new card})}$ by geometric distribution

$P(\text{new card}) = \frac{30-i+1}{30}$

$E[x_i] = \frac{30}{30-i+1}$ flip

X = the number of frogs Hermionie needs to open to collect every card

$$E[X] = \sum_{i=1}^{30} E[x_i]$$

where x_i is the numbers of frogs you need to open to get your i^{th} new card

$$\frac{30}{30}+\frac{30}{29}+\frac{30}{28}+\frac{30}{27}+\frac{30}{26}+\frac{30}{25}+\frac{30}{24}+\frac{30}{23}+\frac{30}{22}+\frac{30}{21}+\frac{30}{20}+\ldots+\frac{30}{1} = 119.84 \ \boxed{\sim 120 \text{ frogs}}$$

Hat problem

$P(\text{Evil}) = .10$
$P(\text{Slytherin} \mid \text{Evil}) = 1$
$P(\text{Hufflepuff} \mid \text{not evil}) = .40$
$P(\text{Gryffindor} \mid \text{not evil}) = .20$
$P(\text{Ravenclaw} \mid \text{not evil}) = .20$
$P(\text{Slytherin} \mid \text{not evil}) = .20$

What is the probability that a random chosen Slytherin is evil?
* $P(\text{Evil} \mid \text{Slytherin})$

$$P(\text{Evil} \mid \text{Slytherin}) = \frac{P(\text{Slytherin} \mid \text{Evil})\, P(\text{Evil})}{P(\text{Slytherin})}$$

$P(\text{Slytherin})$ represents the total probability of being sorted into Slytherin

$$P(\text{Slytherin}) = P(\text{Slytherin} \mid \text{Evil})\, P(\text{Evil}) + P(\text{Slytherin} \mid \text{not Evil})\, P(\text{not Evil})$$
$$= (1)(.10) + (.20)(.90)$$
$$0.1 + 0.18$$
$$= 0.28$$

$$P(\text{Evil} \mid \text{Slytherin}) = \frac{(1)(0.10)}{0.28} = \boxed{0.3571}$$

Dumblevator

The Dumblevator continually moves between the 1st floor and the 15th floor in the order 1, 2, 3 ... 13, 14, 15, 14, 13 ... 3, 2, 1, and so forth.

Hermione's last class ends at 5pm on the 13th floor and she wants to go down to the 1st floor to reach the Great Hall for dinner. What is the probability that the Dumblevator is moving down when it arrives at the 13th floor for the first time after Hermione leaves her class?

If Hermione boards a down-moving elevator at the 13th floor it must have been either on the 14th or 15th floor when she arrived to wait

At the time Hermione arrives to the elevator we can assume that the elevator is at a random floor

Hermionie, on the 13th floor has 12 floors below and 2 above $12/14$ that the elevator is below her and moving up.
$\boxed{2/14 \text{ that the elevator is above her and moving down}}$

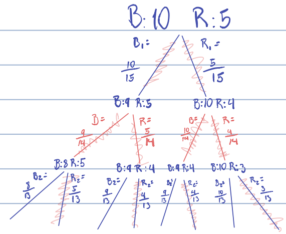
Urn while you learn

Every time Hermione takes a ball out, the urn randomly chooses and discards another ball!

Suppose the urn contains 10 black balls and 5 red balls. If Hermione draws two balls, what is the probability that the second ball is red?

The probability that the 2nd ball is red:

$P(R_1 R_2) + P(B_1 R_2)$



$P(R_1 R_2) = P(R_1 B R_2) + P(R_1 R R_2) = \frac{2}{21}$

$P(R_1 B R_2) = \frac{5}{15} \cdot \frac{10}{14} \cdot \frac{4}{13} = \frac{20}{273}$

$P(R_1 R R_2) = \frac{5}{15} \cdot \frac{4}{14} \cdot \frac{3}{13} = \frac{2}{91}$

$P(B_1 R_2) = P(B_1 B R_2) + P(B_1 R R_2) = \frac{5}{21}$

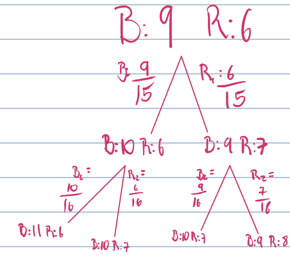
$P(B_1 B R_2) = \frac{10}{15} \cdot \frac{9}{14} \cdot \frac{5}{13} = \frac{15}{91}$

$P(B_1 R R_2) = \frac{10}{15} \cdot \frac{5}{14} \cdot \frac{4}{13} = \frac{20}{273}$

$P(R_1 R_2) + P(B_1 R_2) = 7/21 = \boxed{1/3}$

Pólya's urn

Expected number of red balls after Hermione makes 2 draws?



$$E[X] = \sum_x xp(x)$$

$$P(6\text{ Red}) = \frac{9}{15} \cdot \frac{10}{16} = \frac{3}{8}$$

$$P(7\text{ Red}) = \frac{9}{15} \cdot \frac{6}{16} + \frac{6}{15} \cdot \frac{9}{16} = \frac{9}{20}$$

$$P(8\text{ Red}) = \frac{6}{15} \cdot \frac{7}{16} = \frac{7}{40}$$

$$E[X] = \frac{3}{8} \cdot 6 + \frac{9}{20} \cdot 7 + \frac{7}{40} \cdot 8$$
$$= 2\tfrac{1}{4} + 3\tfrac{3}{20} + 1\tfrac{2}{5}$$
$$= \boxed{6\tfrac{4}{5}}$$

Expected number of black balls after Hermione makes 2 draws?

$$E[X] = \sum_x xp(x)$$

$$E[X] = \frac{3}{8} \cdot 11 + \frac{9}{20} \cdot 10 + \frac{7}{40} \cdot 9$$
$$= 4\tfrac{1}{8} + 4\tfrac{1}{2} + 1\tfrac{23}{40}$$
$$= \boxed{10\tfrac{1}{5}}$$

$$P(11\text{ Black}) = \frac{9}{15} \cdot \frac{10}{16} = \frac{3}{8}$$

$$P(10\text{ Black}) = \frac{9}{15} \cdot \frac{6}{16} + \frac{6}{15} \cdot \frac{9}{16} = \frac{9}{20}$$

$$P(9\text{ Black}) = \frac{6}{15} \cdot \frac{7}{16} = \frac{7}{40}$$

Arithmacy

$$E[X] = \sum_x xp(x)$$

Each of the n values is equally likely to be observed: $\frac{1}{n}$ probability

pmf:

$$p(x) = \frac{1}{n},\ x = 1, 2, 3, \ldots, n$$

$$E[X] = \sum_{x=1}^{n} x \cdot \frac{1}{n}$$

$$= \frac{1}{n} \sum_{x=1}^{n} x$$

$$= \frac{1}{n} \cdot (1 + 2 + 3 + \ldots + n)$$

sum of n natural numbers

$$\sum_{a=1}^{n} 1 + 2 + 3 + \ldots + (n-2) + (n-1) + n =$$

$$\begin{array}{r} 1 + 2 + 3 + \ldots + (n-2) + (n-1) + n \\ + \quad n + (n-1) + (n-2) + \ldots + 3 + 2 + 1 \\ \hline (n+1) + (n+1) + (n+1) + \ldots + (n+1) + (n+1) + (n+1) \end{array}$$

reverse and add

n number of $(n+1)$ terms $= n(n+1)$

we only want half the numbers so we divide by 2 $= \frac{n(n+1)}{2}$

$$= \frac{1}{n} \cdot \frac{n(n+1)}{2}$$

$$E[X] = \boxed{\frac{n+1}{2}}$$

Birthday attack

There are a total of 40 students in Hermione's year at Hogwarts.

What's the probability that no two students in Hermione's year share the same birthday?

- Assume there are 365 possible birthdays and that wizards are equaly likely to be born on any day of the year
- Suppose there are only 2 students: Hermione and Victoria

P(Both are born on two different days) = $\frac{365}{365} \cdot \frac{364}{365}$

P(3 students are born on two different days) = $\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365}$

For more students we would follow the same pattern, where n is the number of total students:

$$\frac{365-n+1}{365}$$

P(All 40 students are born on on different days)

$$\left(\frac{365}{365}\right) \cdot \left(\frac{364}{365}\right) \cdot \left(\frac{363}{365}\right) \cdot \left(\frac{362}{365}\right) \cdot \left(\frac{361}{365}\right) \cdot \ldots \cdot \left(\frac{326}{365}\right)$$

$$\prod_{n=1}^{40} \frac{365-n+1}{365} = \boxed{0.108768} \qquad 10.87\%$$