

① the newton-pepys problem

$$P(X \geq 1 \text{ 6 in 6 rolls}) = 1 - P(B)^6$$

↑ total probability

$$P(X \geq 1 \text{ 6 in 6 rolls}) = .6651020233196159$$

A - getting a six
B - getting anything else
 $P(A) = 1/6$
 $P(B) = 5/6$

$$P(X \geq 2 \text{ 6 in 12 rolls}) = 1 - P(X=0) + P(X=1)$$
$$= 1 - P(B)^{12} + \binom{12}{1} * P(B)^{11} * P(A)^1$$

$$P(X \geq 2 \text{ 6 in 12 rolls}) = .6186673737$$

$$P(X \geq 3 \text{ 6 in 18 rolls}) = 1 - P(X=0) + P(X=1) + P(X=2)$$
$$= 1 - P(B)^{18} + \binom{18}{1} * P(B)^{17} * P(A)^1 + \binom{18}{2} * P(B)^{16} * P(A)^2$$

$$P(X \geq 3 \text{ 6 in 18 rolls}) = .5973456859$$

(*) The proposition with the greatest chance of success is the first:
getting at least one 6 in 6 rolls

② geometric urn

$$E[X] = 1/p = 20$$

↑ expected number of draws to draw first red ball

total number of balls = 100

$$E[X] = 1/20 = \text{redBalls}/100$$

$$100 * (1/20) = (\text{redBalls}/100) * 100$$

$$5 = \text{redBalls}$$

$$\text{blackBalls} = \text{totalBalls} - \text{redBalls}$$

$$\text{blackBalls} = 95$$

$$\text{estimated number of red balls} = 5$$

$$\text{estimated number of black balls} = 95$$

③ dragon dice

$$P(0) = P(B)^3$$

$$P(1) = \binom{3}{1} * P(B)^2 * P(A)^1$$

$$P(2) = \binom{3}{2} * P(B)^1 * P(A)^2$$

$$P(3) = P(A)^3$$

↑ number of die that match hermlone's pick

$$P(0) = .5787037037$$

$$P(1) = .3472222222$$

$$P(2) = .0694444444$$

$$P(3) = .0046296296$$

$$E[X] = (-1)(P(0)) + (1)(P(1)) + (2)(P(2)) + (3)(P(3))$$

↑ expected outcome (amount won) for playing dragon dice

$$E[X] = -.0787037037$$

A - match

-1 amount for no match

B - no match

1 amount for one match

P(A) - 1/6

2 amount for two matches

P(B) - 5/6

3 amount for three matches