Sprint 3 - Ryan King

Newton - Pepys Problem

6 dice rolled, at least 1 is 6

$$1-\binom{6}{0}\left(1-\frac{1}{6}\right)^{6-0}\left(\frac{1}{6}\right)^0 = 1-\left(\frac{5}{6}\right)^6$$

12 dice rolled, at least 2 are 6

$$1-\binom{12}{1}\left(1-\frac{1}{6}\right)^{12-1}\left(\frac{1}{6}\right)^1-\binom{12}{0}\left(1-\frac{1}{6}\right)^{12-0}\left(\frac{1}{6}\right)^0$$

$$=1-12\left(\frac{5}{6}\right)^{11}\left(\frac{1}{6}\right)-\left(\frac{5}{6}\right)^{12}$$

18 dice rolled, at least 3 are 6

$$1-\binom{18}{2}\left(1-\frac{1}{6}\right)^{18-2}\left(\frac{1}{6}\right)^2-\binom{18}{1}\left(1-\frac{1}{6}\right)^{18-1}\left(\frac{1}{6}\right)^1-\binom{18}{0}\left(1-\frac{1}{6}\right)^{18}\left(\frac{1}{6}\right)^0$$

$$=1-153\left(\frac{5}{6}\right)^{16}\left(\frac{1}{6}\right)^2-18\left(\frac{5}{6}\right)^{17}\left(\frac{1}{6}\right)-\left(\frac{5}{6}\right)^{18}$$

Geometric Urn

Since it takes 20 draws to get a red on average, we can say that $P(\text{red ball}) = \frac{1}{20}$. Since there are 100 balls in total, the most reasonable estimate for the number of red balls is $\frac{100}{20} = 5$.

Dragon Dice

$$P(3 \text{ times}) = \binom{3}{3}\left(1-\frac{1}{6}\right)^{3-3}\left(\frac{1}{6}\right)^3 = \left(\frac{1}{6}\right)^3$$

$$P(2 \text{ times}) = \binom{3}{2}\left(1-\frac{1}{6}\right)^{3-2}\left(\frac{1}{6}\right)^2 = 3\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^2$$

$$P(1 \text{ time}) = \binom{3}{1}\left(1-\frac{1}{6}\right)^{3-1}\left(\frac{1}{6}\right)^1 = 3\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)$$

$$P(0 \text{ times}) = \binom{3}{0}\left(1-\frac{1}{6}\right)^3\left(\frac{1}{6}\right)^0 = \left(\frac{5}{6}\right)^3$$

$$E[x] = 3\cdot\left(\frac{1}{6}\right)^3 + 2\cdot 3\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^2 + 1\cdot 3\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + -1\cdot\left(\frac{5}{6}\right)^3$$