sprint 2 - hermione granger and the distributions of probability 0/3

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① wizard people, dear reader?

A - she's a witch
B - not receiving a letter

* P(A) = .75 → total probability
P(!A) = .25
* P(B|A) = .03
* P(B|!A) = .99
* given

? what is the probability that hermione really is a witch even though she didn't get a letter?

P(A|B)

bayes' rule:

$$P(A|B) = \frac{P(A) * P(B|A)}{P(B)}$$

$$P(A|B) = \frac{(.75)(.03)}{P(B)}$$

$$P(A|B) = (.0225) / P(B)$$

$$P(B) = P(B|A)P(A) + P(B|!A)P(!A)$$
$$P(B) = (.03)(.75) + (.99)(.25)$$
$$P(B) = .27$$ (total probability of not recieving a letter)

$$P(A|B) = .08\overline{3}$$

② chocolate frogs

? how many chocolate frogs will hermione need, to collect a full set of 30 unique, randomly distributed cards? (1 card per frog)

$$E(X) = 1 + \frac{30}{29} + \frac{30}{28} + \frac{30}{27} + \frac{30}{26} + \frac{30}{25} + \dots + 30$$

$$E(X) = \sum_{n=0}^{29} 30/(30-n)$$

ex = sum([(30/(30-n)) for n in range(30)])

= 119.8496 → round up* → = 120 frogs

*hermione must buy an even # of frogs

③ hat problem

E - student is evil

H - sorted to hufflepuff
G - sorted to gryffindor
R - sorted to ravenclaw
S - sorted to slytherin

* P(E) = .1 P(!E) = .9
* given
* P(S|E) = 1 P(G|E) = 0
* P(S|!E) = .2 * P(G|!E) = .2
P(H|E) = 0 P(R|E) = 0
* P(H|!E) = .4 * P(R|!E) = .2

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③ hat problem (cont.)

? what is the chance a random slytherin student is evil?

bayes' rule

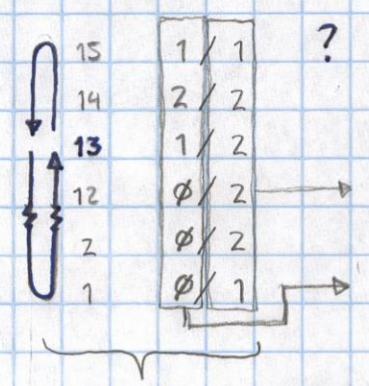
$$P(E|S) = \frac{P(S|E) * P(E)}{P(S)} = \frac{(1)(.1)}{(.28)} = .3571428$$

$$P(S) = (P(S|E) * P(E) + P(S|!E) * P(!E)) = (1)(.1) + (.2)(.9)$$

↑ (total probability a student is sorted into slytherin)

$$\boxed{P(E|S) = .3571}$$

④ dumblevator



? what is the chance the elevator car is heading down the next time it arrives at the 13th floor?

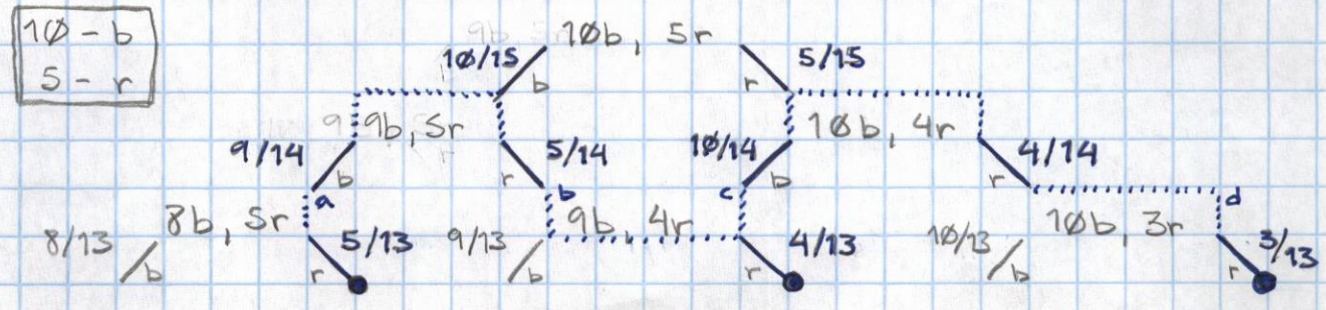
# of times per cycle the elevator car arrives at each floor

# of times per cycle the next time the elevator car arrives at the 13th floor it will be going down at each floor

$$\frac{2(\#\text{floors} - \text{start floor})}{2(\#\text{floors}) - 2} = 4/28 = \boxed{1/7} = P(\text{going down})$$

⑤ urn while you learn

? what is the chance the 2nd ball drawn by hermione will be red? (since the urn discards one ball each time hermione draws - consider it the third draw (abstract "players"))



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⑤ urn while you learn

$P(a) = (5/13)(9/14)(10/15)$
$P(b) = (4/13)(5/14)(10/15)$
$P(c) = (4/13)(10/14)(5/15)$
$P(d) = (3/13)(4/14)(5/15)$

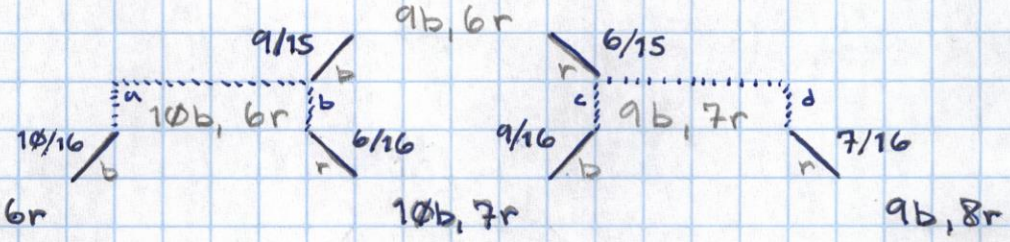
$P(R_3) = P(a) + P(b) + P(c) + P(d)$
$= (.1648) + (.0733) + (.0733) + (.0220)$

$P(R_3) = .\overline{33}$ **1/3**

⑥ pólya's urn

? how many black and red balls will be in the urn after hermione draws a random ball and places two balls of the same color back in the urn, twice?

9 - b
6 - r



$P(11b, 6r) = ((10/16)(9/15))^a = .375$
$P(10b, 7r) = ((6/16)(9/15))^b + ((9/16)(6/15))^c = .45$
$P(9b, 8r) = ((7/16)(6/15))^d = .175$

*no fractional balls in urn

$E[B] = 11(.375) + 10(.45) + 9(.175) = 10.2$ → round* → **E[B] = 10**
$E[R] = 6(.375) + 7(.45) + 8(.175) = 6.8$ → **E[R] = 7**

⑦ arithmancy

? prove that the expected value of a uniform distribution over a finite set of integers is: $E[X] = (n+1)/2$

$E[X] = \sum_x x\,p(x) = \sum_{x=1}^{n} x(1/n) = \{1(1/n) + 2(1/n) + 3(1/n) + \ldots + n(1/n)\}$
$p(x) = 1/n$
factor out $(1/n)$

$E[X] = (1/n)\sum_{x=1}^{n}\{1 + 2 + 3 + \ldots + n\}$

$\sum_{k=1}^{n} k = 1/2\,n(n+1)$ ← sum of natural #s 1 to n

$E[X] = (1/n)(n/2)(n+1)$
$E[X] = (n^2+n)/(2n)$
$E[X] = \boxed{(n+1)/2}$

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⑧ birthday attack

? what is the chance none of the students in hermione's year share the same birthday?

X - no shared birthdays * - there are 40 students in hermione's year

$P(X) = (1)(364/365)(363/365)(362/365) \ldots (326/365)$

∴ product of each students probability to share a birthday with a previous student

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px = math.prod([((365-n)/365) for n in range(40)])
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P(X) = .1088