Yet More Lightbulbs

$E[X] = 2000$

$\Lambda = \frac{1}{E[X]} = \frac{1}{2000}$

$P(X > 3000) = e^{-\lambda x}$ $\quad \lambda = \frac{1}{2000}$ $\quad x = 3000$

$= e^{-\frac{1}{2000} 3000}$

$= e^{-\frac{3}{2}}$

$= 0.223130$

We have two independent bulbs, so probability of interest is $P(X>3000)^2 = 0.223130^2$

$= 0.049787$

The Non-Persistence of Memory

Bulb 1

$\lambda = \frac{1}{2000}$

$x = 3000 - 1000 = 2000$

$P(X > 2000) = e^{-\frac{1}{2000} \cdot 2000} = e^{-1} = 0.36788$

Bulb 2

$\lambda = \frac{1}{2000}$

$x = 3000 - 2500 = 500$

$P(X > 500) = e^{-\frac{1}{2000} \cdot 500} = e^{-\frac{1}{4}} = 0.77880$

$P(\text{Bulb 1}) \cdot P(\text{Bulb 2}) = 0.36788 \cdot 0.77880 = 0.28650$

Check My Math

- Avg service time for disk access: 5ms
- Avg number of disk accesses per job: 2
- Avg number of jobs in the system: 120 → $\bar{N}$
- Avg residence time of a job: 1s → $\bar{R}$

Little's Law: $\bar{N} = \Lambda \bar{R}$

$$\Lambda = \frac{\bar{N}}{\bar{R}} = \frac{120}{1} = 120$$

Utilization Law: $u = \lambda \bar{s}$, $0 \leq u \leq 1$

$\lambda = \Lambda = 120$ by Forced Flow Law

$\bar{s} = 5ms \cdot 2 = 10ms = \frac{1}{100} s$

(Avg. time for disk access · Avg. disk access per job)

$u = 120 \cdot \frac{1}{100} = 1.2$ ← violates utilization law as $u \leq 1$

A mistake has been made, as the given numbers indicate a utilization of 1.2 which violates the range allowed by the utilization law. Therefore, one of the given parameters for the system must be wrong.

Unbalanced Server Loads

$u_A = 0.8$ $\quad a_B = 0.4$

$u_B = 0.6$ $\quad a_A = 0.6$

$\bar{s}_B = 250\mu s$

$$\lambda_B = \frac{u_B}{\bar{s}_B} = \frac{0.6}{250} = 0.0024$$

$\lambda_B = \lambda \cdot a_B$

$\lambda =$ [illegible]

$$\frac{\lambda_B}{a_B} = \frac{0.0024}{0.4} = 0.006$$

$$\lambda_A = \lambda \cdot a_A = 0.6 \cdot 0.006 = 0.0036$$

$u_A = \lambda_A \bar{s}_A$

$$\bar{s}_A = \frac{u_A}{\lambda_A} = \frac{0.8}{0.0036} = 222\mu s$$

The average service time at A is 222μs