CMS 380 / Sprint 2 Deliverables

Christian Hubes

Wizard People

P(she's a witch) = P(A) = 0.75

P(no letter) = P(B)

$P(B \mid A) = 0.03$

$P(B \mid \bar{A}) = 0.99$

Use Baye's rule to get P(B):

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(B)}$$

$$= \frac{0.75 \cdot 0.03}{P(B)}$$

$$= \frac{0.0225}{P(B)}$$

$$P(B) = P(B \mid A)P(A) + P(B \mid \bar{A})P(\bar{A})$$

$$= 0.03 \cdot 0.75 + 0.99 \cdot 0.25$$

$$= 0.27$$

$$P(A \mid B) = \frac{0.0225}{0.27}$$

$$= 0.083$$

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Chocolate Frogs

30 different frogs, each equally likely to get. All frogs are unique.

What is the expected number of frogs that needs to be opened to get all unique frogs?

$$E[x] = 1 + \frac{30}{29} + \frac{30}{28} + \frac{30}{27} + \dots + 30$$

$$\sum_{k=0}^{29} \left(\frac{30}{30-k}\right) = 119.84967393$$ → Hermione needs to open 120 chocolate frogs.

Hat Problem

All evil students are sorted to Slytherin, everybody else into the other Houses.

Gryffindor = G → P(N|G) = 0.2

Hufflepuff = H → P(N|H) = 0.4

Ravenclaw = R → P(N|R) = 0.2

Slytherin = S → P(N|S) = 0.2

Evil = E → P(E) = 0.1

Not Evil = N = $\bar{E}$

P(E|S) = 1

What is the probability that a randomly chosen Slytherin is evil?

$$P(S) = P(N|S)\,P(N) + P(E|S) \cdot P(E)$$
$$= 0.2 \cdot 0.9 + 0.1 \cdot 1$$
$$= 0.28$$

} P(Randomly chosen student to be in Slytherin)

$$P(E) = 0.1$$

↳ $P(A|B) = \frac{B}{A}$, Therefore:

$$P(S|E) = \frac{0.1}{0.28}$$

$$= 0.357143$$

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Dumblevator

Elevator circles through floors 1–15. What is the probability that the first observation on floor 13 is the elevator going downwards? → P(X)

① Floor:	② Stops / trip	③ Stops before 13
15	1	1
14	2	2
13	2	1
12	2	0
11	2	0
⋮	⋮	⋮
2	2	0
1	1	0
Total:	28	4

① Floor number

② Number of stops at that floor per dumblevator cycle

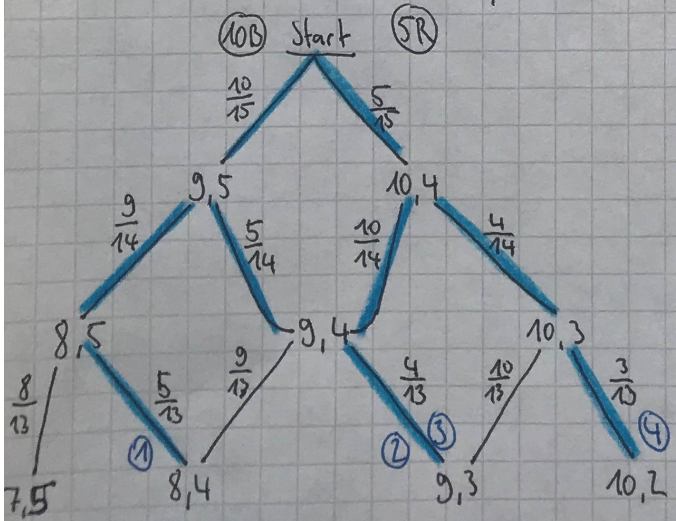
③ Number of stops per floor before stopping at Fl. 13 to go downwards

$$\rightarrow P(X) = \frac{4}{28} = \frac{1}{7}$$

Urn While You Learn

Urn: 10 black, 5 red balls. 2 Draws w/out replacement, Urn discards another ball after each draw.

What is the probability that the second ball is red? → P(X)



① $P(8,4) = \left(\frac{10}{15}\right)\left(\frac{9}{14}\right)\left(\frac{5}{13}\right) = 0.165$

② $P(9,3) = \left(\frac{10}{15}\right)\left(\frac{5}{14}\right)\left(\frac{4}{13}\right) = 0.073$

③ $P(9,3) = \left(\frac{5}{15}\right)\left(\frac{10}{14}\right)\left(\frac{4}{13}\right) = 0.073$

④ $P(10,2) = \left(\frac{5}{15}\right)\left(\frac{4}{14}\right)\left(\frac{3}{13}\right) = 0.022$

$$\rightarrow P(X) = P(8,4) + (P(9,3)\cdot 2) + P(10,2)$$
$$= 0.165 + (2\cdot 0.073) + 0.022$$
$$= 0.33$$

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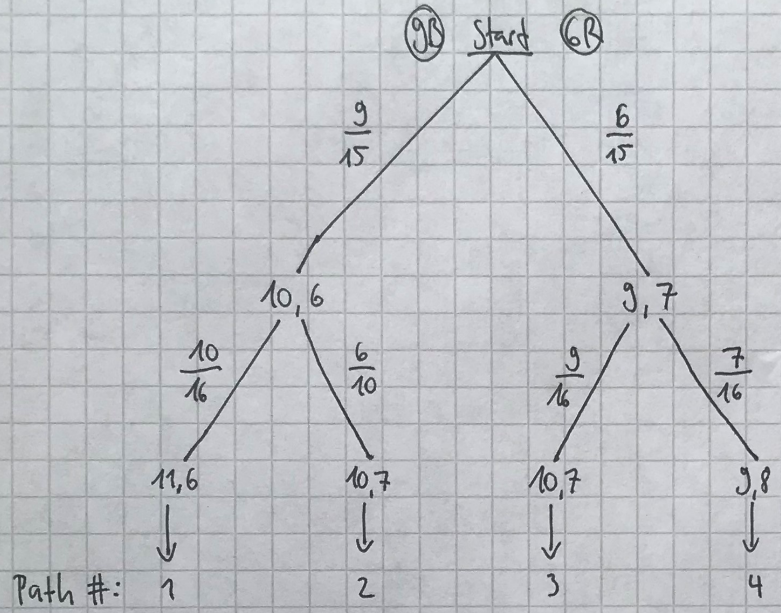
Pólya's Urn

Urn: 9 black, 6 red balls. Draw with double replacement.

What are the expected number of black and red balls?



1: $P(11,6) = \left(\frac{9}{15}\right)\left(\frac{10}{16}\right) = 0.375$

2: $P(10,7) = \left(\frac{9}{15}\right)\left(\frac{6}{10}\right)$

3: $P(10,7) = \left(\frac{6}{15}\right)\left(\frac{9}{16}\right)$

$P(10,7) = 0.225 + 0.125 = 0.450$

4: $P(9,8) = \left(\frac{6}{15}\right)\left(\frac{7}{16}\right) = 0.175$

$P(1,2,3,4) = 0.375 + 0.450 + 0.175 = 1$ ✓

→ Expected number of:

→ Black Balls: $(11 \cdot 0.375) + (10 \cdot 0.450) + (9 \cdot 0.175) = 10.20 = 11$

→ Red Balls: $(6 \cdot 0.375) + (7 \cdot 0.450) + (8 \cdot 0.175) = 6.79 = 7$

always round up to next integer

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Arithmancy

Discrete uniform distribution: $\{a, a+1, a+2, \dots, b\}$

-> The probability of each value is $\frac{1}{b}$.

Prove that expected value of a discr. unif. distr. with $a=1$; $b=n$ is

$$E[x] = \frac{n+1}{2}$$

If $a=1$ and $b=n$, $p(a) = \frac{1}{n}$.

Expected value $E[x] = \sum_x x p(x)$,

Therefore:

$$E[x] = \sum_x x p(x) = 1\frac{1}{n} + 2\frac{1}{n} + 3\frac{1}{n} + \dots + n\frac{1}{n}$$

$$= \frac{1}{n}(1+2+3+4+\dots+n)$$

$$\sum_{k=1}^{n} k = \frac{1}{2} n(n+1)$$

$$= \frac{1}{n} \cdot \frac{1}{2} n(n+1)$$

$$= \frac{1}{2} \cdot (n+1)$$

$$= \frac{n+1}{2}$$

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Birthday Attack

40 students.

What is the probability that no two students in Hermione's year share the same bday?

$$P(\text{no same birthdays}) = \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \dots \cdot \frac{326}{365}$$

$$= 0.1087682$$