

Deliverable Problems

1. Wizard People Dear Reader?

Based on Bayes' Rule

$$P(\text{Witch} | \text{No letter}) = \frac{P(\text{Witch}) \times P(\text{No letter} | \text{Witch})}{P(\text{No letter})}$$

Total probability of No letter can be:

$$\begin{aligned} P(\text{No letter}) &= P(\text{No letter} | \text{Witch}) * P(\text{Witch}) \\ &\quad + P(\text{No letter} | \text{Not Witch}) * P(\text{Not Witch}) \\ &= 0.03 \times 0.75 + 0.99 \times 0.25 \\ &= 0.27 \end{aligned}$$

Therefore

$$P(\text{Witch} | \text{No letter}) = \frac{0.75 \times 0.03}{0.27} = \frac{0.0225}{0.27} = 0.08333 \approx 0.083$$

2. Chocolate Frogs

This uses the idea of geometric random variables when we calculate Expected value,

* Expected Value of Geometric Random variable

Suppose $P(X) = p$ $\sum_x x p(x) = \frac{1}{p}$

This is number of trials to get certain cards.

So really the probability to get all of the card will be.

$\frac{30}{30}, \frac{29}{30}, \frac{28}{30}, \dots, \frac{1}{30}$ and we are using the

$$\sum_x x p(x) = \frac{1}{p} \rightarrow \frac{1}{\frac{1}{30}} = \frac{30}{1}$$

$$\begin{aligned} E[X] &= \frac{30}{30} + \frac{30}{29} + \frac{30}{28} + \dots + \frac{30}{1} \\ &= 1 + 1.03448276... + 1.071428... + \dots + 30 \\ &= 119.8496 \end{aligned}$$

She would have to open approximately 120 Chocolate Frogs.

3. Hat Problem

Uses Conditional Probability

$P(\text{Slytherin} | \text{Evil}) = 1$

$P(\text{Evil}) = 0.1 \leftarrow \text{from the question}$

$P(\text{Slytherin} | \text{Not Evil}) = 0.2$

By Bayes' rule

$$P(\text{Evil} \mid \text{Slytherin}) = \frac{P(\text{Slytherin} \mid \text{Evil}) \times P(\text{Evil})}{P(\text{Slytherin})}$$

$$\begin{aligned} P(\text{Slytherin}) &= P(\text{Slytherin} \mid \text{Evil}) \times P(\text{Evil}) + P(\text{Slytherin} \mid \text{Not Evil}) \times P(\text{Not Evil}) \\ &= 1.00 \times 0.1 + 0.2 \times 0.9 \\ &= 0.28 \end{aligned}$$

$$P(\text{Evil} \mid \text{Slytherin}) = \frac{1.00 \times 0.1}{0.28} = 0.357142857$$

4. Dumblevator.		Possible pattern of elevator stop
15		1 (down)
14		2 (up/down)
13		2 (up/down)
12		2 (up/down)
11	sum	2 (up/down)
⋮	up	⋮
⋮	to	⋮
2	28	2 (up/down)
1		1 up

Each floor other than 15th and 1st floor two possible patterns that elevator stops and leaves. going up or down but at 15th floor, it stops and it's only going down, for 1st floor it stops and it is going up. So there are 28 possible patterns.

When Hermione is at 13th floor, and if she wanted to go down, elevator has to be either at 14th floor or 15th floor or 13th floor when elevator is moving down, so there's 4 possible outcomes there fore

E: Elevator moving down when it arrives at Thirteenth floor for the first time after Hermione leaves her class.

$$P(E) = \frac{4}{28} = \frac{1}{7}$$

5. Urn while your learn (done on different paper).
6. Polya's urn done on different paper.
7. Arithmancy done on different paper.