Homework 2

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Exercise 1.

Proof. Since $f \neq 0$, there exists a radius r, on which $\int_{B_r(0)} |f| = c > 0$. For $|x| > r$, we have $B_r(0) \subseteq B_{|x|+r}(x)$ and therefore

$$Mf(x) \geq \frac{1}{\mu(B_{|x|+r}(x))} \int_{B_{|x|+r}(x)} |f| \geq \frac{c}{(|x|+r)^n}$$

So that

$$\int_{\mathbb{R}^n} |Mf(x)| \geq \int_{|x|>r} \frac{c}{(|x|+r)^n} = \infty$$

Thus, $Mf \notin L^1(\mathbb{R}^n)$. $\square$

Exercise 2.

Proof. $\square$

Exercise 3.

Proof. $\square$