Potential Energy

consider a charge Q. A test charge q experiences force $\underline{F}_Q$. Now lets apply a force $\underline{F}_{ext}$ to move q from A to B at a constant speed.

$$\underline{F}_{ext} + \underline{F}_Q = 0$$

Elemental work done by $\underline{F}_{ext}$ on q is

$$dW = \underline{F}_{ext} \cdot d\underline{l} = -\underline{F}_Q \cdot d\underline{l}$$

The total work done by $\underline{F}_{ext}$ is found by integrating:

$$W = \int_A^B \underline{F}_{ext} \cdot d\underline{l} = -\int_A^B \underline{F}_Q \cdot d\underline{l}$$

Work done is equal to change in potential energy.

$$\Delta U = -\int_A^B \underline{F}_Q \cdot d\underline{l}$$

We know that $F_Q = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{r^2} \hat{r}$, $\therefore$

$$dW = -\frac{1}{4\pi\varepsilon_0} \frac{qQ}{r^2} \hat{r} \cdot d\underline{l}$$

due to it being a conservative field, the path doesn't matter.

$$W = \Delta U_{AB} = -\int_A^B \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\,dr$$

$$= \frac{-qQ}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

For multiple charges, we can use the superposition principle.

$$\Delta U_{AB} = -\int_A^B \underline{F}_{Q1}\cdot d\underline{l} - \int_A^B \underline{F}_{Q2}\cdot d\underline{l} - \dots - \int_A^B \underline{F}_{QN}\cdot d\underline{l}$$

Potential Difference

path integral

$$\boxed{\Delta V_{AB} = \frac{\Delta U_{AB}}{q} = -\int_A^B \underline{E}\cdot d\underline{l}}$$

ΔU_{AB} is path independent, and is independent of q. SI units of JC^{-1}. We can define where $V=0$, often chosen to be when $r \to \infty$.

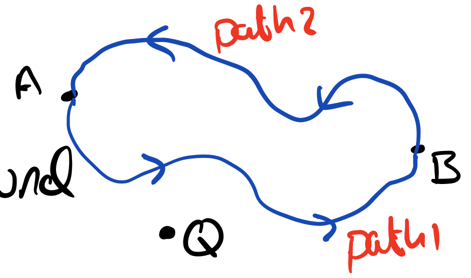
The potential (V) at point P is defined as the work needed to bring a +1C charge from $r=\infty$ to P, at a constant speed.

scalar field

$$\boxed{V = -\int_\infty^P \underline{E}\cdot d\underline{l}}$$

For a point charge Q, $V = \frac{Q}{4\pi\varepsilon_0 r}$.

Circulation law



Consider the potential difference around a closed loop.

$$\Delta U_{AB} = -\int_A^B \underline{E}\cdot d\underline{l} - \int_B^A \underline{E}\cdot d\underline{l} = \int_A^B \underline{E}\cdot d\underline{l} - \int_A^B \underline{E}\cdot d\underline{l} = 0$$

$$\hookrightarrow \qquad \oint \underline{E}\cdot d\underline{l} = 0$$

This is only true for electrostatics! We can use Stokes' theorem to say that

$$\oint_{\partial R} \underline{E}\cdot d\underline{l} = \iint_R (\nabla\times\underline{E})\cdot d\underline{S} = 0$$

For an infinitesimal surface element we get

$dS \neq 0$

$$\nabla\times\underline{E}\cdot d\underline{S} = 0 \quad \Rightarrow \quad \nabla\times\underline{E} = 0$$

This equation must be satisfied at all points in the electrostatic field.