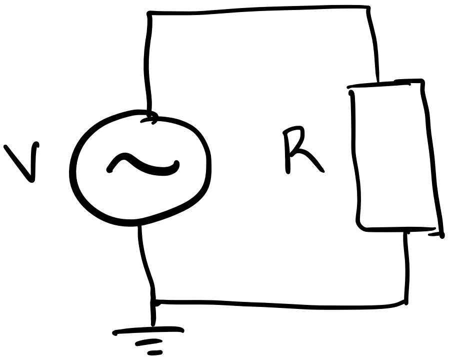
Alternating Current

$$v(t) = A\cos(\omega t + \phi)$$

amplitude

$V_{pp} = 2A \qquad V_{RMS} = \frac{A}{\sqrt{2}}$

$RMS = \sqrt{\langle v^2 \rangle}$



$V = V_0 \cos(\omega t)$

$i = \frac{V_0}{R}\cos(\omega t)$

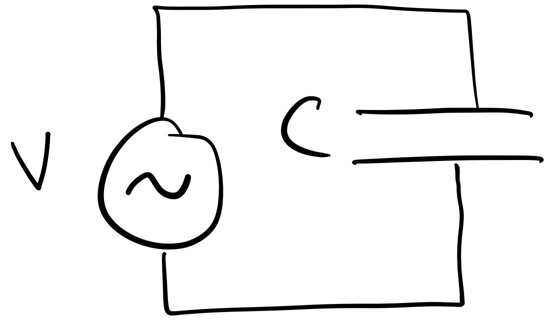
$P_R = Vi = \frac{V_0^2}{R}\cos(\omega t)$

$\langle P_R \rangle = \frac{V_0^2}{2R}$

$P_{avg} = V_{RMS} I_{RMS} \cos(\phi)$

Phase difference between V & I.

$V_{RMS} = \frac{V_0}{\sqrt{2}} \qquad I_{RMS} = \frac{I_0}{\sqrt{2}} = \frac{V_0}{R\sqrt{2}}$



$$V = V_0 \cos(\omega t)$$

$$i = C\frac{dv}{dt}$$

$$= -\omega C V_0 \sin(\omega t)$$

$$= \omega C V_0 \cos(\omega t + \pi/2)$$

current $\pi/2$ rads out of phase with the voltage

$$P_{avg} = V_{RMS} I_{RMS} \cos\phi = 0$$

$$P_C = Vi = V_0\cos(\omega t)\,\omega C V_0 \cos(\omega t + \pi/2)$$

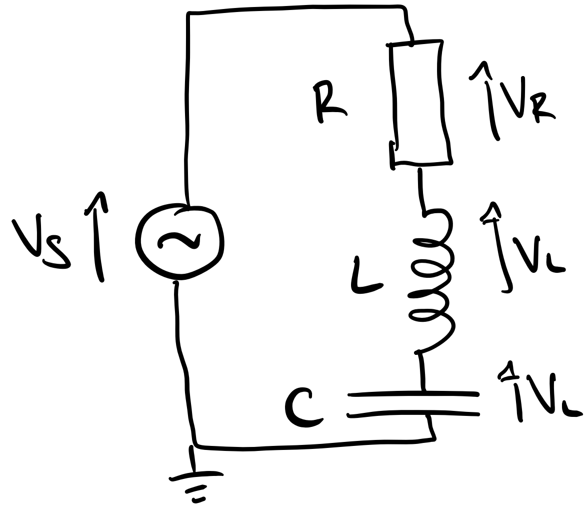
$$= -\omega C V_0^2 \cos(\omega t)\sin(\omega t) = -\tfrac{1}{2}\omega C V_0^2 \sin(2\omega t)$$

$$\langle P_C \rangle = 0$$

Driven LCR circuit

$$V_S = V_R + V_L + V_C$$

$$V_0\cos(\omega t) = Ri + L\frac{di}{dt} + \frac{1}{C}q$$



$$\frac{V_0}{L}\cos(\omega t) = \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q$$

$$\frac{V_0}{L}\cos(\omega t) = \ddot{q} + \gamma\dot{q} + \omega_0^2 q \qquad \gamma = \frac{R}{L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

let q be complex $\tilde{q}$.

in electronics we use j instead of i to be $\sqrt{-1}$.

$$\ddot{\tilde{q}} + \gamma\dot{\tilde{q}} + \omega_0^2\tilde{q} = \frac{V_0}{L}e^{j\omega t}$$

complex constant CAPITAL

$$\tilde{q} = \tilde{Q}e^{j\omega t}$$

from complex analysis course

$$\tilde{Q} = \frac{V_0/L}{\omega_0^2 - \omega^2 + j\omega\gamma}$$

Amplitude of Oscillation $= |\tilde{Q}|$ ← absolute value

Phase of Oscillation $= \arg(\tilde{Q})$

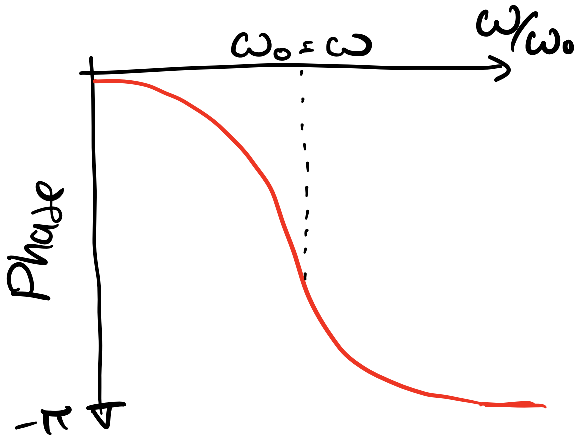
Our steady-state solution is

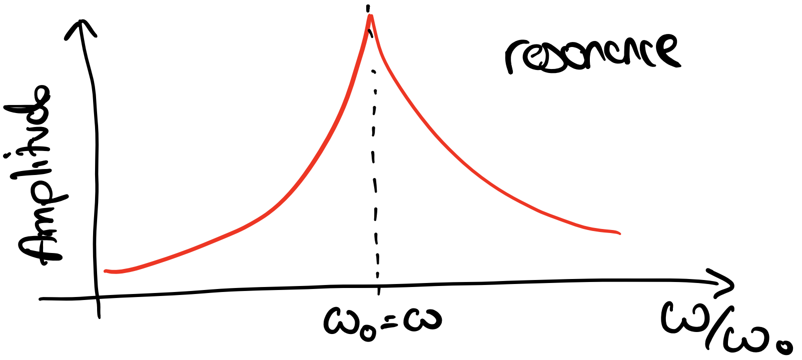
$$q(t) = \mathrm{Re}(\tilde{q}) = |\tilde{Q}|\cos(\omega t + \arg(\tilde{Q}))$$

see complex analysis

$$q(t) = \frac{V_0/L}{\sqrt{(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2}}\cos(\omega t + \phi)$$

$$\tan\phi = \frac{-\omega\gamma}{\omega_0^2 - \omega^2}$$





Phasors

We can represent any oscillating signal using complex quantities.

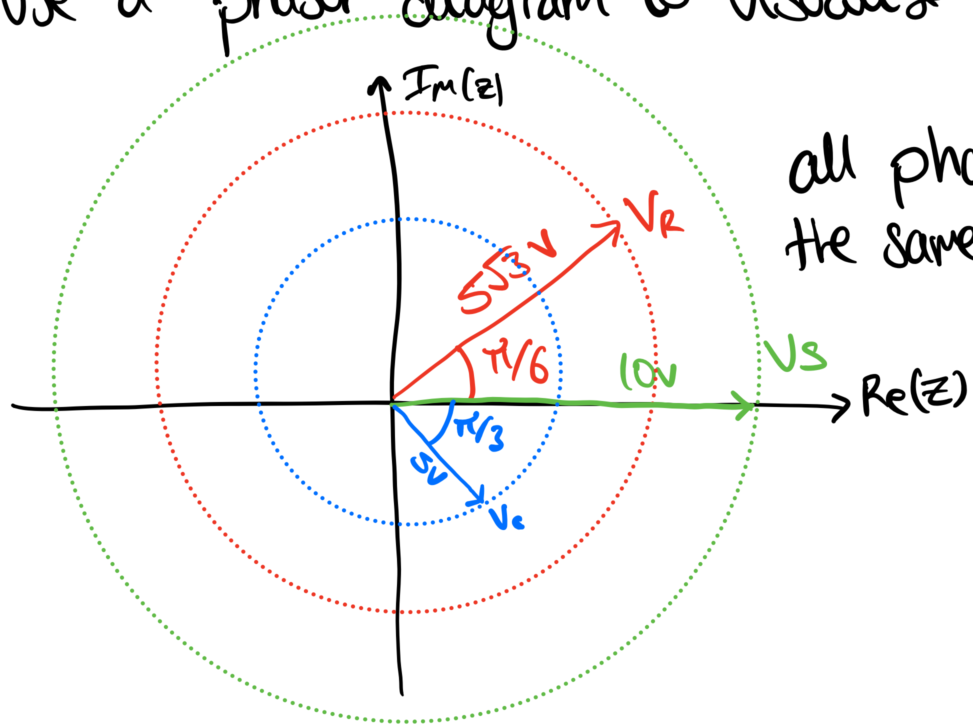
$$\tilde{q} = \tilde{Q} e^{j\omega t} \qquad \tilde{q} = Qe^{j(\omega t+\phi)} = Qe^{j\phi}e^{\omega t}$$

$\tilde{Q}$ is the phasor such that $|\tilde{Q}|$ is the amplitude & $\arg(\tilde{Q})$ is the phase. $\quad \tilde{Q} = Qe^{j\phi} \leftarrow$ phasor

Kirchoff's voltage law works for complex numbers too.

$$\tilde{V}_S = \tilde{V}_L + \tilde{V}_C + \tilde{V}_R$$

We can use a phasor diagram to visualise our phasors:



all phasors have the same angular frequency.