Oscillations & Waves 2

combing newton's second law and hooke's law:

$$m\frac{d^2\psi}{dt^2} = -k\psi$$

the solution to this equation is found by:

$$\psi = A\cos(\omega t + \phi)$$
$$\dot{\psi} = -\omega A \sin(\omega t + \phi)$$
$$\ddot{\psi} = -\omega^2 A\cos(\omega t + \phi)$$

$$-m\omega^2 A\cos(\omega t + \phi) = -kA\cos(\omega t + \phi)$$

if $\omega = \sqrt{\frac{k}{m}}$, we find SHM is a solution to hooke's law.

$$\boxed{\frac{d^2\psi}{dt^2} = -\omega^2\psi}$$

$\Rightarrow$ spring: $\omega = \sqrt{\frac{k}{m}} \rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

$\Rightarrow$ pendulum: 'spring const.' $= \frac{mg}{l} \rightarrow \omega = \sqrt{\frac{g}{l}} \rightarrow T = 2\pi\sqrt{\frac{l}{g}}$

if we look at the system $\psi = A\cos(\omega t + \phi)$ with two initial conditions:

(1) $\psi(t=0) = x_0 \Rightarrow A\cos\phi = x_0$

(2) $\dot{\psi}(t=0) = 0$

We get the solutions:

$$\psi = x_0\cos(\omega t)$$
$$\dot{\psi} = -\omega x_0 \sin(\omega t)$$
$$\ddot{\psi} = -\omega^2 x_0 \cos(\omega t) = -\omega^2\psi$$