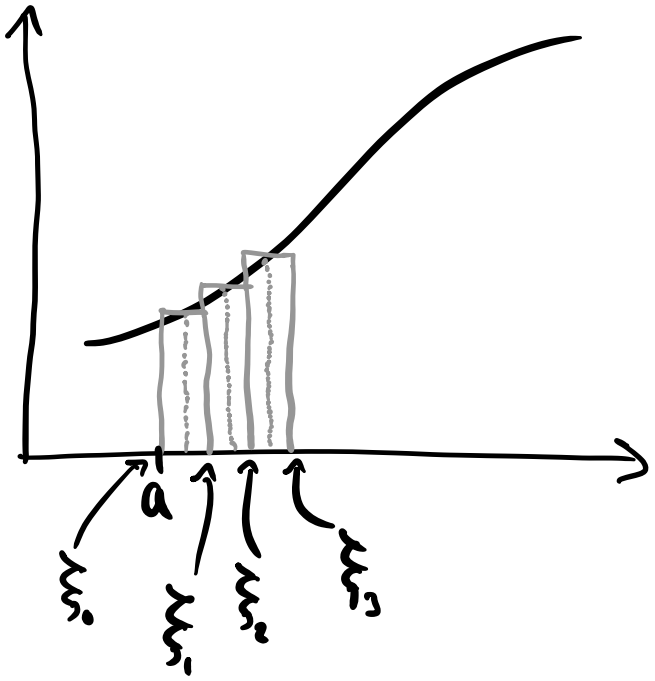
1D Integration



An integral over the interval a to b of f(x) is the limit of the Reimann sum.

$$S = \sum_{i=1}^{n} f(x_i)[\xi_i - \xi_{i-1}]$$

✡ x_i is any point between ξ_i and ξ_{i-1}.

$\int_a^b f(x)dx$ is the limit of S as $n \to \infty$ and $[\xi_i - \xi_{i-1}] \to 0$.

The widths do not have to be uniform.

A 1D integral is often viewed as the area under a curve, but its better to consider the 'weighted sum' as there are many different used for intergration.

The fundamental theorem of calculus provides a connection between integration and differentiation.

$$F(b) - F(a) = \int_a^b f(x)dx \quad \text{where} \quad f(x) = \frac{dF}{dx}$$

F is the anti-derivative of f. There are many possible different functions which differ by a constant.

Often we don't need to find the sum - which is hard to do. We just need to find a function which differentiates to f(x).

2D Integrals

The 2D integral of $f(x,y)$ over a region R

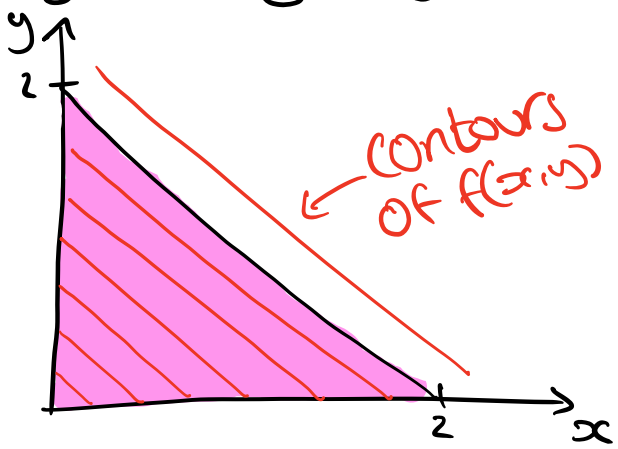
$$S = \sum_{p=1}^{n} f(x_p, y_p)\Delta A_p \implies \iint_R f(x,y)dA$$

is the limit of the Reimann sum as $n \to \infty$ and $\Delta A_p \to 0$. The shape and relative size of ΔA_p do not matter.

A 2D integral is like 'volume under surface', but generally is a weighted sum. We can use a uniform grid $\Delta x \Delta y$. Then as $n \to \infty$, $dA = dxdy$.

Example $f(x,y) = x + y$

R is the region bounded by $x=0, y=0, y=2-x$

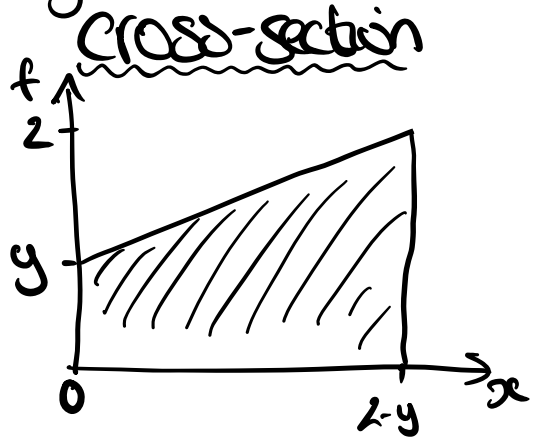


Consider the sum for a single slice for a fixed y.

$$dV = dy \sum f(x,y)\,dx$$

y is const.

$$= dy \int_0^{2-y} (x+y)\,dx$$



$$dV = dy\left[\frac{x^2}{2} + xy\right]_0^{2-y}$$

$$= dy\left[\frac{(2-y)^2}{2} + (2-y)y\right]$$

$$= dy\left[2 - \tfrac{1}{2}y^2\right]$$

area of a slice of const. y.

$$V = \sum dV = \int_0^2 \left(2 - \tfrac{1}{2}y^2\right) dy = \left[2y - \tfrac{1}{6}y^3\right]_0^2 = 4 - \frac{4}{3} = \frac{8}{3}$$

more compactly:

$$V = \int_{y=0}^{2} \left[\int_{x=0}^{2-y} (x+y)\,dx \right] dy$$

"iterated integral"

limits are given explicitly!!

convention is to do the inner integral first

It does not matter whether we integrate x or y first. do easiest!