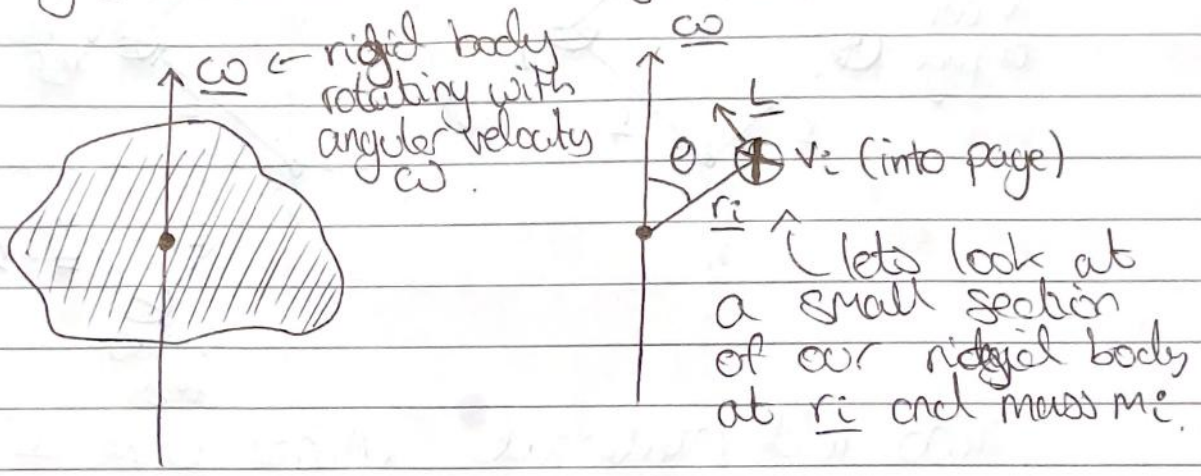
Classical Mechanics 18

Rigid Bodies

- Ridgid bodies can move and rotate, but the relative positions of the particles of which they are composed stay fixed.

Angular Momentum of Ridgid Bodies



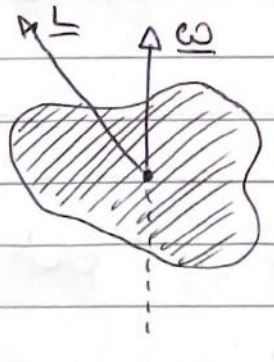
→ Because the body is rigid, every m_i rotates with the same angular velocity ω.

→ $\underline{v_i} = \underline{\omega} \times \underline{r_i} = \omega r_i \sin\theta$ into page $= \omega r_i \sin\theta \, \hat{\phi}$.

→ $\underline{L_i} = \underline{r_i} \times m_i \underline{v_i}$

The angular momentum is not parallel to the angular velocity.

The same holds for wonky-shaped rigid bodies (made of lots of point masses m_i.

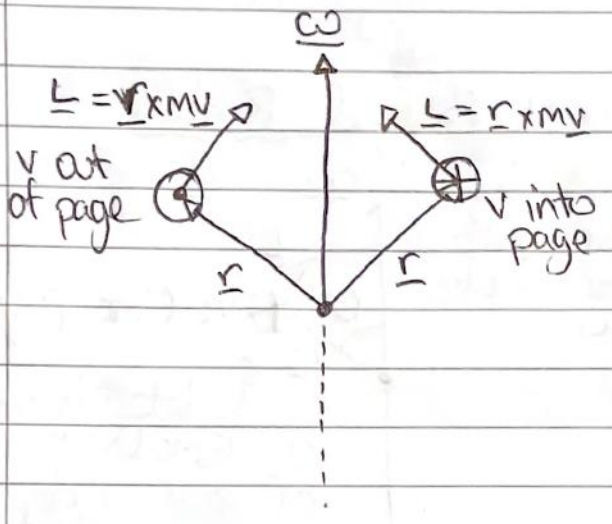


In the absence of external torques:

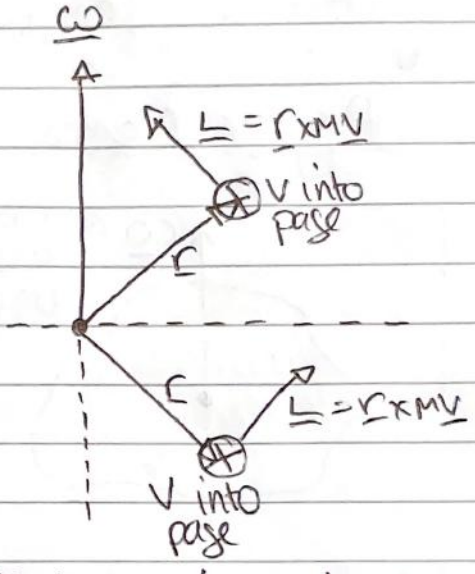
- $\underline{L}$ is conserved
- $\underline{\omega}$ is not

This is makes thing complicated!

Fortunately, for bodies with enough symetry, the components of $\underline{L}$ perpendicular to $\underline{\omega}$ cancel.

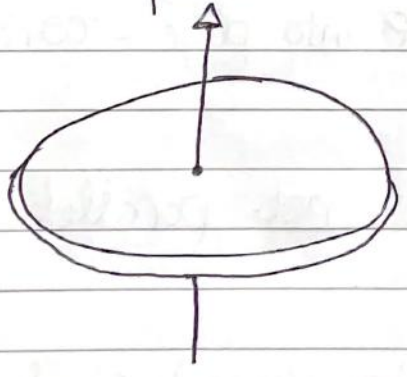


Two fold rotational symetry about $\underline{\omega}$.

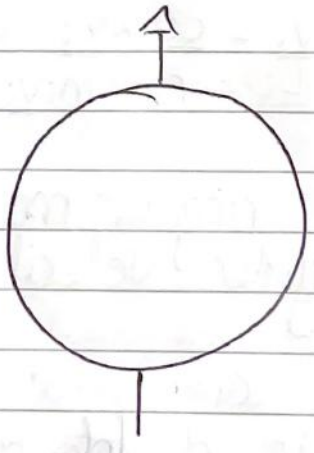


Mirror plane ⊥ to $\underline{\omega}$
Origin in plane

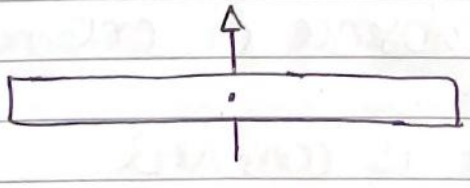
$\underline{L}$ is parallel to $\underline{\omega}$ in all of these examples:



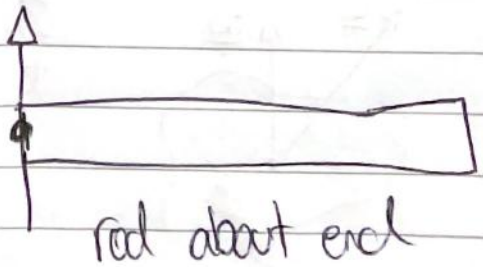
Wheel about an axel.



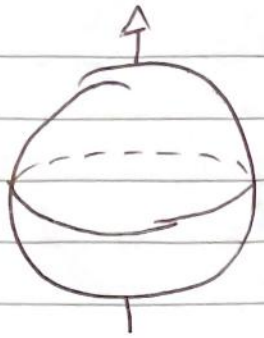
disc about its diameter



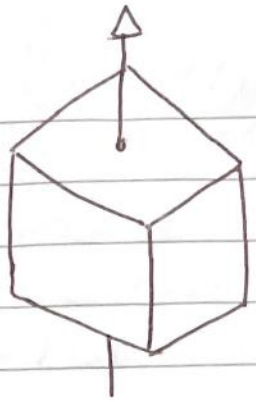
rod about centre



rod about end



Sphere about diameter



cube about axis thrgh centre of face.

For any object, no matter how 'wonky' it is, you can always find 3 perpendicular axis for which $\underline{L}$ is parallel to $\underline{\omega}$. They are called the principle axis.

We showed last time that the motion of a rigid body separates into motion of the CoM, treated as a point particle, and the rotation about the CoM.

For bodies mounted on an axle, the rotation is defined by the axle, which need not pass through the CoM.

The axle applies any force & torque required to keep the body in place and aligned.

The only thing an axle cannot do is exert a torque about its own axis. In that $\parallel$ direction only:

$$\frac{d\underline{L}_{\parallel}}{dt} = \underline{G}_{\parallel}^{ext}$$

$\underline{L}$ may have $\perp$ components, but they don't matter.