Complex Impedance

For any circuit component, there will be current $i(t)$ and a voltage $v(t)$, they will oscillate at the same frequency.

The impedance of a component is the complex equivalent of resistance.

$$\tilde{Z} = \frac{\tilde{V}}{\tilde{I}}$$

Inductor:

$$\left.i = I_0 \cos(\omega t)\right\} \text{AC current}$$

$$\left.\begin{aligned} v_L &= L\frac{di}{dt} = -I_0\omega L \sin\omega t \\ &= \underbrace{I_0\omega L}_{\text{scaled by } \omega L}\cos(\omega t + \underbrace{\pi/2}_{\text{phase difference of } \pi/2}) \end{aligned}\right\} \text{AC voltage}$$

at the time $t=0$,

$$\tilde{I} = I_0 e^{j\omega t} = I_0 e^{j\omega 0} = I_0$$

$$\tilde{V}_L = I_0\omega L e^{j(\omega t + \pi/2)} = I_0 \omega L e^{j\pi/2}$$

$$\tilde{Z}_L = \frac{\tilde{V}_L}{\tilde{I}} = \omega L e^{j\pi/2} = j\omega L$$

Capacitor:

$$V_C = V_0 \cos(\omega t)$$

$$i = C\frac{dV_C}{dt} = -V_0 C \sin(\omega t)$$

$$= \underbrace{V_0 C}_{\text{amplitude shifted by } C}\cos(\omega t + \underbrace{\tfrac{\pi}{2}}_{\text{phase shift of } 2\pi.})$$

$$\tilde{V}_c = V_0 e^{j(\omega t)} \qquad \tilde{I} = V_0 \omega C e^{j(\omega t + \pi/2)}$$

$$\tilde{Z}_c = \frac{\tilde{V}_c}{\tilde{I}} = \frac{\cancel{V_0 e^{j\omega t}}}{\cancel{V_0} \omega C \cancel{e^{j\omega t}} e^{j\pi/2}} = \frac{1}{\omega C j} = \frac{-j}{\omega C}$$

Resistor:

$$\tilde{V}_R = V_0 e^{j\omega t} \qquad \tilde{i} = \frac{\tilde{V}}{R} = \frac{V_0}{R} e^{j\omega t}$$

$$\tilde{Z} = \frac{\tilde{V}_R}{\tilde{I}} = \frac{V_0}{V_0/R} = R$$

Equivalent Impedance

Series:

$$\tilde{Z}_T = \tilde{Z}_1 + \tilde{Z}_2 + \dots$$

Parallel:

$$\frac{1}{\tilde{Z}_T} = \frac{1}{\tilde{Z}_1} + \frac{1}{\tilde{Z}_2} + \dots$$

AC Analysis

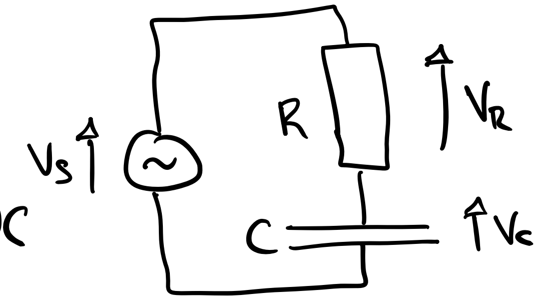
All potential differences & currents will oscillate at the same frequency.

The phase difference between the voltage & current can be found by $\arg(\tilde{Z})$. The absolute value is the ratio of the amplitudes.

$$|\tilde{Z}| = \frac{|\tilde{V}|}{|\tilde{I}|}$$

Series RC Circuit

KVL: $\tilde{V}_S = \tilde{V}_R + \tilde{V}_C$



When we have an idealised AC voltage source, we can use

$$V_S(t) = V_0 \cos(\omega t + \phi)$$

As this signal exists for all time, there are no transient affects, we're only looking at the steady-state effects.

The circuit impedance is $$\tilde{Z} = \tilde{Z}_R + \tilde{Z}_C$$
$$= R - j/\omega C$$

$$\tilde{I} = \frac{\tilde{V}_S}{\tilde{Z}}$$

$$\tilde{V}_C = \tilde{I}\tilde{Z}_C = \frac{\tilde{V}_S}{\tilde{Z}_R + \tilde{Z}_C}\tilde{Z}_C = \frac{\tilde{Z}_C}{\underbrace{\tilde{Z}_R + \tilde{Z}_C}_{\text{impedance divider}}}\tilde{V}_S$$

$$\tilde{V}_C = \tilde{V}_S \times \frac{-j/\omega C}{R - j/\omega C}$$

$$\tilde{V}_R = \tilde{V}_S - \tilde{V}_C$$