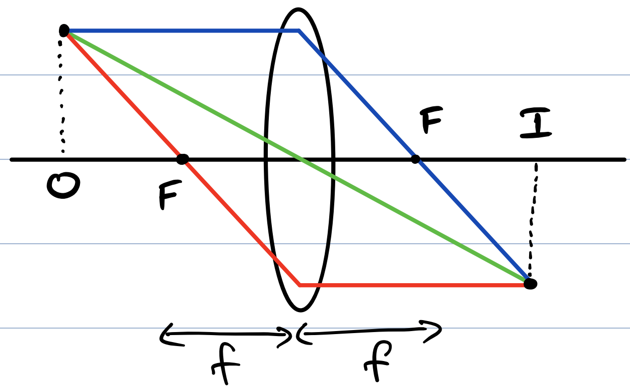
Convergent Lens (Convex)

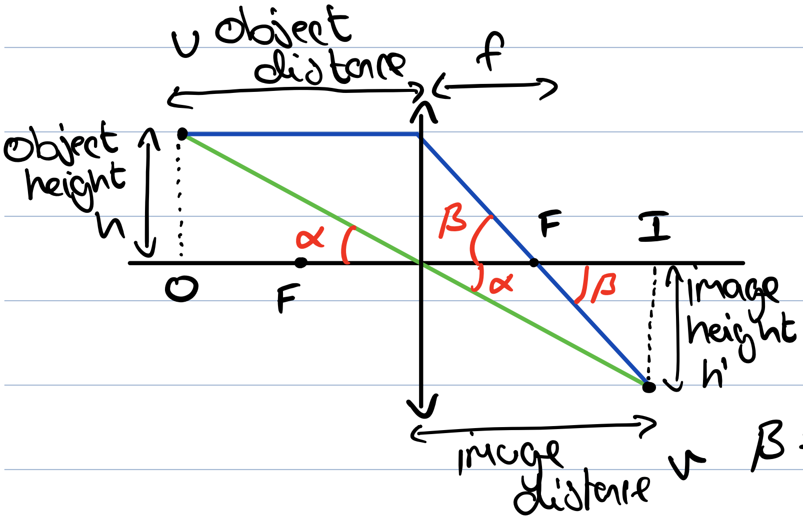
3 principle rays:

Ray parallel to axis

Ray through centre of lens

Ray through focal point





paraxial approx. ∴ $\alpha = \tan\alpha$

$$\alpha = \frac{h}{u} = \frac{h'}{v} \Rightarrow \frac{h'}{h} = \frac{v}{u}$$

$$\beta = \frac{h}{f} = h'/_{v-f} \Rightarrow \frac{h'}{h} = \frac{v-f}{f} = \frac{v}{f} - 1$$

This leads to the thin lens formula ($\frac{v}{u} = \frac{v}{f} - 1$):

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

In reality the image appears upside down so we use negative (transverse) magnification.

$$M = \frac{h'}{h} = -\frac{v}{u}$$

Divergent Lens

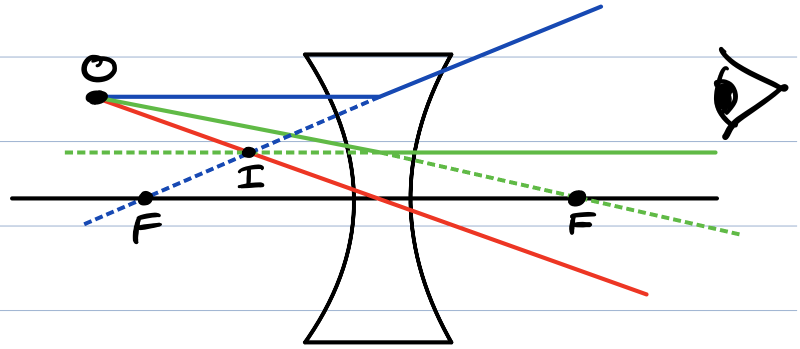
In this scenario the image is virtual, and the image forms where the rays appear to cross.

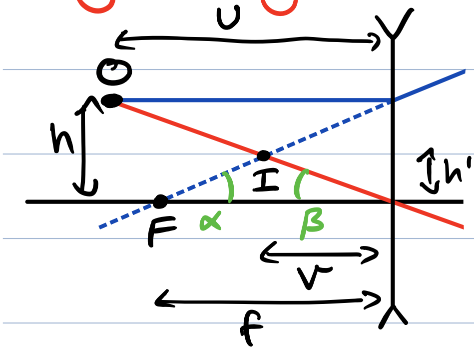
3 principle rays:

Ray parallel to axis

Ray towards focal point

Ray through centre of lens





$$\alpha = \frac{h'}{f-v} = \frac{h}{f} \Rightarrow \frac{h'}{h} = \frac{f-v}{f} = 1 - \frac{v}{f}$$

$$\beta = \frac{h'}{v} = \frac{h}{u} \Rightarrow \frac{h'}{h} = \frac{v}{u}$$

This leads to the thin lens formula ($1 - \frac{v}{f} = \frac{v}{u}$):

$$\frac{1}{u} - \frac{1}{v} = -\frac{1}{f}$$

and magnification will be positive (upright image).

$$M = \frac{h'}{h} = +\frac{v}{u}$$

∴ We use the following sign convention $\left[\frac{1}{u} + \frac{1}{v} = \frac{1}{f}\right]$

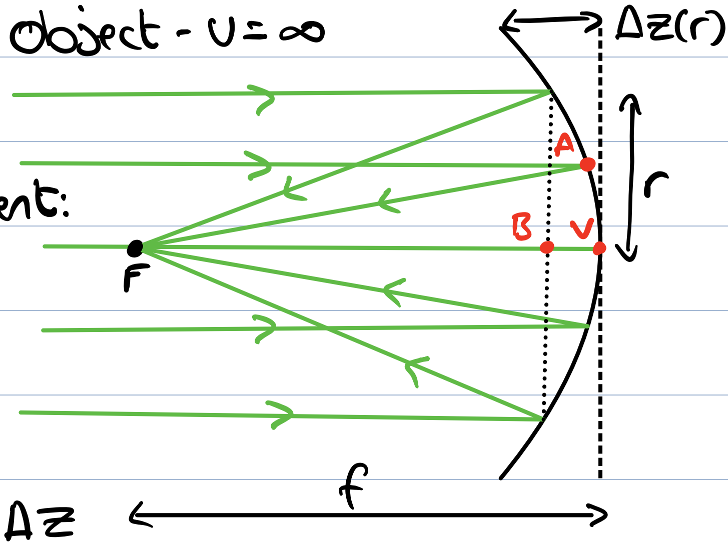
u : +ve (real) on left, -ve (virtual) on right

v : +ve (real) on right, -ve (virtual) on left

f : +ve converging, -ve diverging

Mirror Imaging

By fermats imaging requirement:



$$OPL(AF) = OPL(BVF)$$

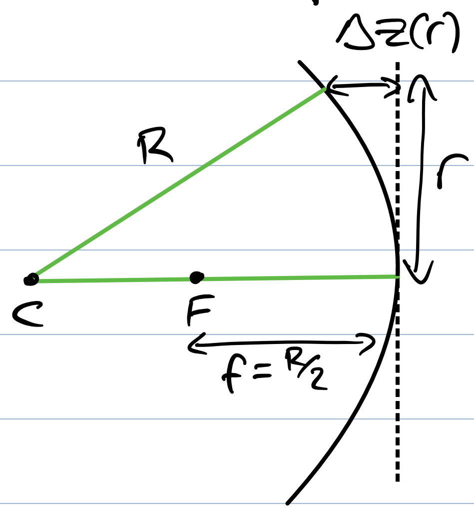
$AF = \sqrt{(f-\Delta z)^2 + r^2}$ $\quad BVF = f + \Delta z$

$$AF = BVF \Rightarrow (AF)^2 = (BVF)^2 = (f-\Delta z)^2 + r^2 = (f+\Delta z)^2$$

$$f^2 - 2f\Delta z + \Delta z^2 + r^2 = f^2 + 2f\Delta z + \Delta z^2 \quad = r^2 = 4f\Delta z$$

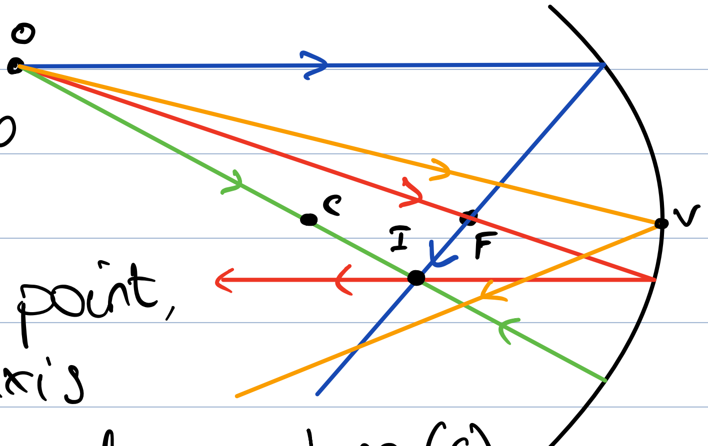
parabolic mirror $\quad \Delta z = \dfrac{r^2}{4f}$ $\quad$ valid for all r!

Spherical Mirrors



$$\Delta z = \frac{r^2}{2R} \qquad f = \frac{R}{2}$$

Principle Rays for Mirror



1) parralel to axis, passes through focal point F
2) passes through focal point, returns parralel to axis
3) passes through centre of curvature (C), returns on same path
4) incident at mirror vertex, reflects at equal angle

Without derivation, it can be shown that the following equations are true for mirror imaging.

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{R}{2}$$

f/R are +ve concave, -ve convex

$$M = \frac{-h'}{h} = -\frac{v}{u}$$

Magnification is negative, the image will appear inverted/upside down.

Two Lens Imaging

We use the image of the first lens as the object of the second lens.

The total magnification can easily be found by multiplying the magnification of the individual lenses.

When the lenses are in close proximity (the distance between them is zero), we can get the total focal length via

$$\frac{1}{f_T} = \frac{1}{f_1} + \frac{1}{f_2}$$