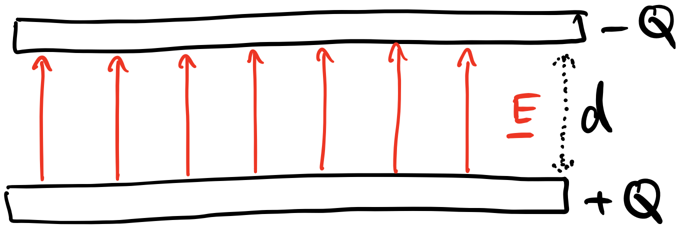
Parallel Plate Capacitor



The electric field between the two plates will be

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Now, looking at potential difference

$$\Delta V = V_A - V_B = Ed = \frac{Qd}{\epsilon_0 A}$$

Allowing us to define capacitance

$$C = \frac{\epsilon_0 A}{d}$$

Capacitors store energy, as well as charge. From earlier we know the potential energy from a set of charges is given by

$$U = \sum_i \tfrac{1}{2} Q_i V_i$$

$$U = \tfrac{1}{2} Q V_A - \tfrac{1}{2} Q V_B = \tfrac{1}{2} Q \Delta V = \tfrac{1}{2} C V^2$$

Lets explore this a little further, noting that $Q = \epsilon_0 A E$ & $V = Ed$.

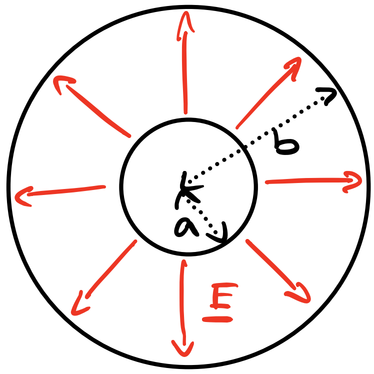
$$U = \tfrac{1}{2}\epsilon_0 AE(Ed) = \tfrac{1}{2}\epsilon_0 AdE^2$$

Ad is the volume of the capacitor. We can therefore define the energy density of $\underline{E}$

$$u_E = \tfrac{1}{2}\epsilon_0 E^2$$

Spherical Capacitors

A spherical capacitor is formed of two conductors. The first is a solid sphere of charge Q. The second is a spherical shell of charge b.



$$\underline{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

+ve -ve definition!

For a capacitor $V = \Delta V = V_a - V_b = -\int^a \underline{E}\cdot d\underline{l}$

$$V = \frac{Q}{4\pi\epsilon_0}\int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_a^b = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

We can therefore find the capacitance by

$$C = \frac{Q}{V} = \frac{4\pi\epsilon_0}{\left(\frac{1}{a}-\frac{1}{b}\right)}$$

For the single, isolated spherical capacitor ($b \to \infty$)

$$C = 4\pi\epsilon_0 a$$