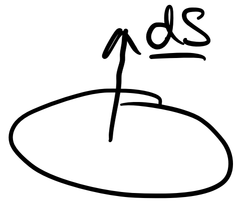
Surface Integrals

On a surface in 3D, we define the infintesimal element of area $\underline{dS}$, a vector of area $|\underline{dS}|$ with direction normal to the surface.



Surface integrals are used to find:

- Area $\iint_S |\underline{dS}|$ ← 3D surface
- total of a scalar (eg. charge) $\iint_S \sigma |\underline{dS}|$ σ is surface density
- 'flux' through surface $\iint_S \underline{F} \cdot \underline{dS}$ ← vector field

General Method

The position vector $\underline{r}$ of a surface can be expressed using only two coordinates.

position vector: $\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$

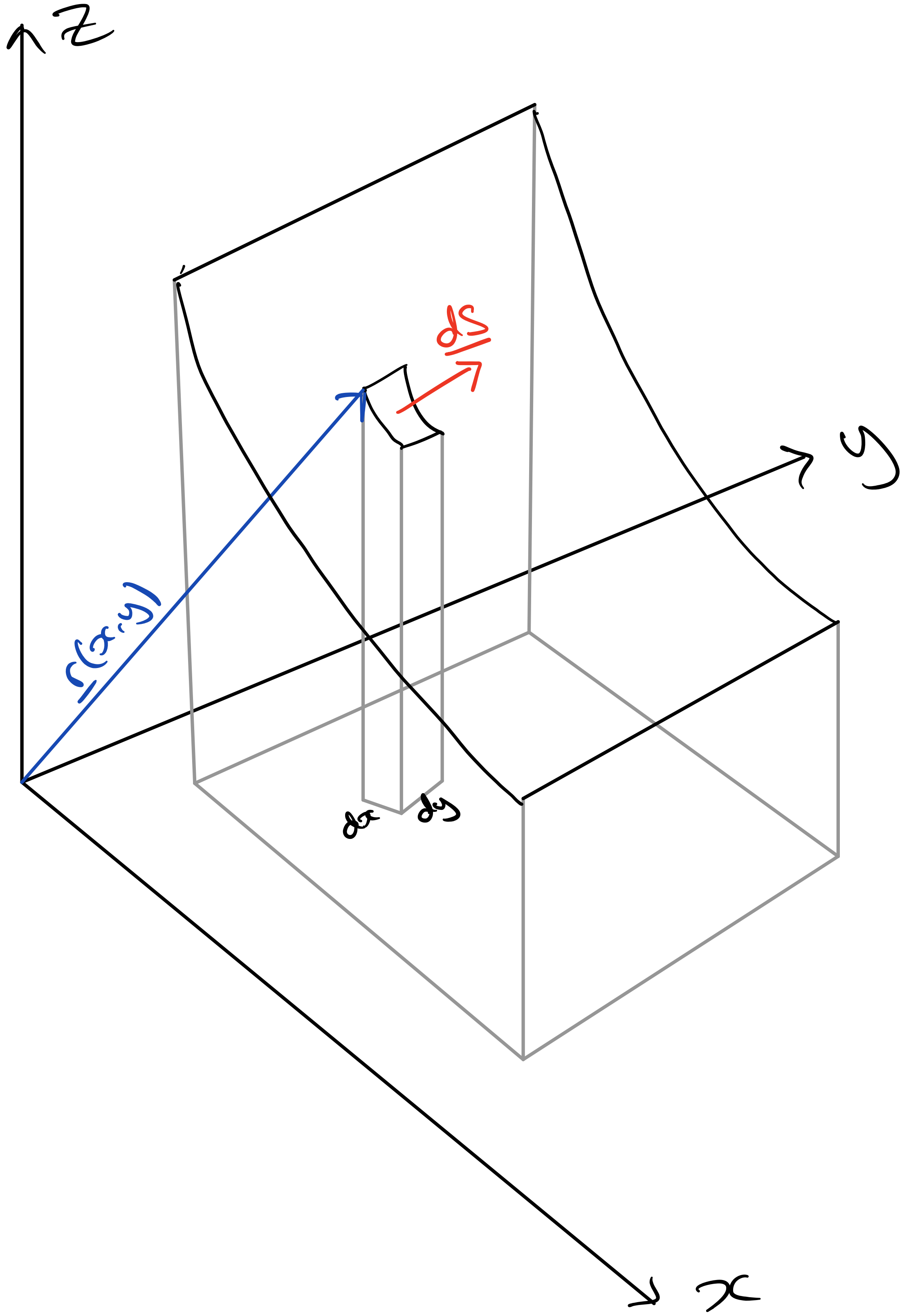
surface: $z = f(x,y)$ (from eg. $f(x,y,z) = c$)

$\therefore$ vector to the surface

$$\underline{r}(x,y) = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\underline{dS} = d\underline{r}_x \times d\underline{r}_y = \left(\frac{\partial \underline{r}}{\partial x} \times \frac{\partial \underline{r}}{\partial y}\right) dx\,dy = \underline{N}\,dx\,dy$$

$$d\underline{r}_x = \frac{\partial \underline{r}}{\partial x} dx \qquad d\underline{r}_y = \frac{\partial \underline{r}}{\partial y} dy$$



$$\underline{dS} = \left(\frac{\partial \underline{r}}{\partial x} \times \frac{\partial \underline{r}}{\partial y}\right) dx\,dy$$

Choose the coordinate system for convience.