Functions 1

13/10/21

If two variables, x and y, follow a rule:

"when x is given, then y is determined as..."

we write $y = f(x)$

x - independent variable
y - dependent variable

Example:

circle of radius r

Area: $A = \pi r^2$

Circumfrence: $C = 2\pi r$

C ↑ dependent, r ↑ independent

Domain & Range

For a set of values of x ('domain'), there is a corresponding set of values of y ('range')

Common notation:

we often write $y = f(x)$

eg. $y = x + x^2$

Common Functions

Polynomial:

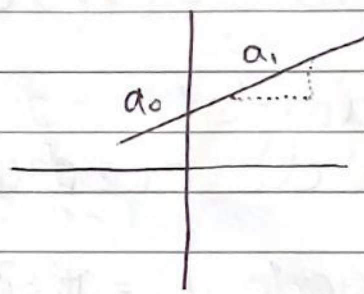
$$y = a_0 + a_1x + a_2x^2 + \dots a_nx^n$$

$$= \sum_{r=0}^{n} a_rx^r$$

"of ~~greater~~ degree n"

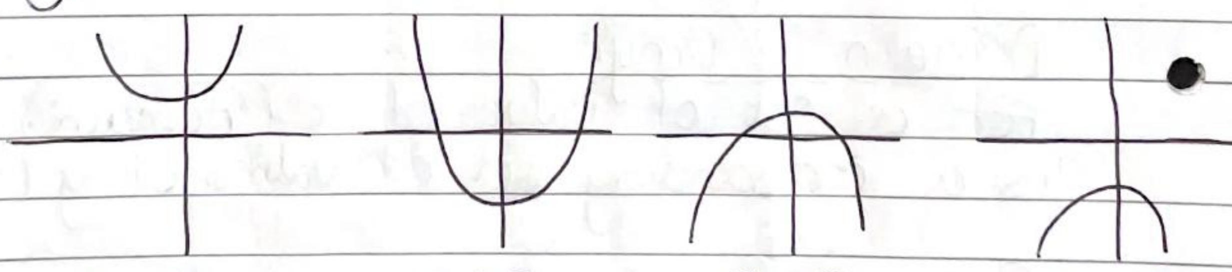
Linear function:

$$y = a_0 + a_1x$$



Quadratic functions:

$$y = a_0 + a_1x + a_2x^2$$



$\underbrace{a_2 > 0}_{\text{no real roots}}$ $\underbrace{a_2 > 0 \quad a_2 < 0}_{\text{contain roots}}$ $\underbrace{a_2 < 0}_{\text{no real roots}}$

Completing The Square

$\underline{a_2 > 0}$ $$a_2x^2 + a_1x + a_0 = \left(\sqrt{a_2} + \frac{a_1}{2\sqrt{a_2}}\right)^2 - \frac{a_1^2}{4a_2} + a_0$$

$\underline{a_2 < 0}$

$$a_2x^2 + a_1x + a_0 = -\left(|a_2|x^2 - a_1x - a_0\right)$$

method as above.

AG-GM Inequality

$x^2 - 2\sqrt{b}\,x + b$ with $b \geq 0$

$\equiv (x - \sqrt{b})^2$

hence $(\sqrt{a} - \sqrt{b})^2 \geq 0$ ← from inequality above replacing x with $\sqrt{a}$ with $\sqrt{a} \geq 0$

$a + b - 2\sqrt{a}\sqrt{b} \geq 0$

$\frac{a+b}{2} \geq \sqrt{ab}$ AM-GM Inequality

Arithmetic Mean Geometric Mean

does this work for $n = 4$?

$$\frac{\left(\frac{a_1+a_2}{2}\right) + \left(\frac{a_3+a_4}{2}\right)}{2} \geq \sqrt{\left(\frac{a_1+a_2}{2}\right)\left(\frac{a_3+a_4}{2}\right)}$$

$$\frac{a_1 + a_2 + a_3 + a_4}{4} \geq \sqrt{\frac{a_1a_3 + a_1a_4 + a_2a_3 + a_2a_4}{4}}$$

$$\geq \sqrt{\sqrt{a_1a_2}\sqrt{a_3a_4}}$$

Result: true for $n = 2, 4, 8, 16 \ldots$

⊛ true for all n but not as easy..

CAUCHY-SCHWARZ INEQUALITY

$$\sum_{j=1}^{n} (a_j x - b_j)^2 \geq 0$$

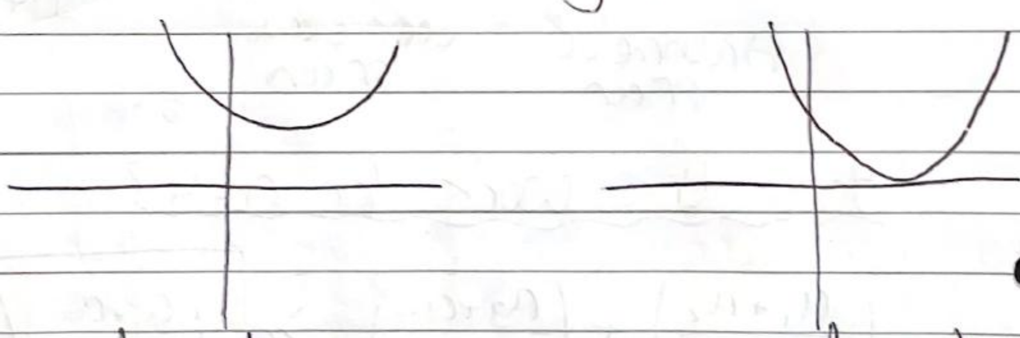
$$\sum_{j=1}^{n} (a_j)^2 x - 2a_j b_j x + (b_j)^2 \geq 0$$

$$\left(\sum_{j=1}^{n} a_j^2\right) x^2 - 2\left(\sum_{j=1}^{n} a_j b_j\right) x + \left(\sum_{j=1}^{n} b_j^2\right) \geq 0$$

→ at most there will be one real root when

$$x = \frac{b_n}{a_n}$$

otherwise no real roots. graph will either be:



no real roots   one real root

for these graphs, the discriminant must be zero or negative.

$$b^2 - 4ac \leq 0$$

$$b^2 \leq 4ac$$

$$4\left(\sum_{j=1}^{n} a_j b_j\right)^2 \leq 4\left(\sum_{j=1}^{n} a_j^2\right)\left(\sum_{j=1}^{n} b_j^2\right)$$

!! $$\left(\sum_{j=1}^{n} a_j^2\right)\left(\sum_{j=1}^{n} b_j^2\right) \geq \left(\sum_{j=1}^{n} a_j b_j\right)^2$$!!