Classical Mechanics 5

Potential Energy Function

If the applied force F depends upon position only (not time/velocity), the work W_{if} done moving a body from x_i to x_f is always the same.

$$W_{if} = \int_{x_i}^{x_f} F(x)\,dx$$

It does not depend on when the body left x_i, or how fast it travels. or if it went past and then returned.

The same is not true for 3D in general, but all of the forces we are dealing with are radial, we will later come to a proof that shows it does hold.

N.B. A <u>conservative force</u> is a force with the property work done is independent of path taken.

It follows that W_{if} can be viewed as a function of x_i, x_f.

$$W_{if} = W(x_i, x_f)$$

The potential energy function is defined by the equation.

$$\boxed{F(x) = -\frac{dU(x)}{dx}}$$

Adding a constant to U has to effect on the equation, so $U(x)$ is only defined "to within an arbitary constant".

It is usual to pick a convient point (x_0) and then set $U(x_0)$ to zero.

Intergrating the equation from x_0 to x gives

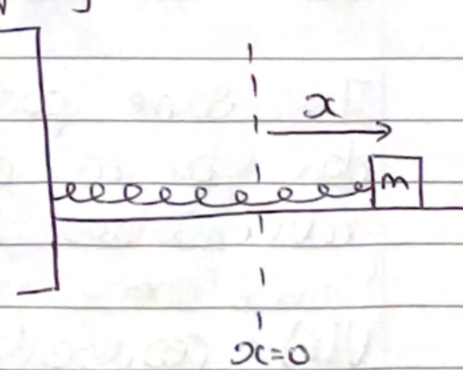
$$\int_{x_0}^{x} \frac{dU(x')}{dx'}\,dx' = -\int_{x_0}^{x} F(x')\,dx'$$

$$U(x) - \underbrace{\cancel{x}U(x_0)}_{(=0)} = -\int_{x_0}^{x} F(x')\,dx'$$

$$\boxed{U(x) = -\int_{x_0}^{x} F(x')\,dx'}$$

N.B. x_0 is the point where $U(x) = 0$! important!

<u>Example</u> Potential Energy in Spring



chose $x_0 = 0$.

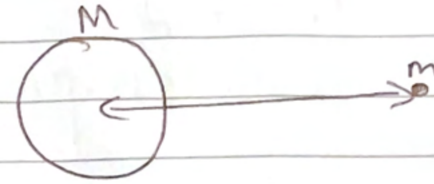
$F(x) = -Sx$

$V(x) = \int_0^x Sx'\,dx' = \left[\frac{1}{2}Sx^2\right]_0^x$

$U(x) = \frac{1}{2}Sx^2$

Example GPE from earth

$$F = -\frac{GMm}{r^2}$$



Attempt 1
$r_0 = 0$.

$$U(r) = -\int_0^r \frac{GMm}{r'^2}\,dr' = \left[-\frac{GMm}{r'}\right]_0^r = ???$$

Attemp 2
$r_0 = \infty$.

$$U(r) = -\int_\infty^r \frac{GMm}{r'^2}\,dr' = \left[-\frac{GMm}{r'}\right]_\infty^r = -\frac{GMm}{r}$$

Do not place x_0 at a point where the potential is infinite. !!

The positive number $(-U(r))$ is the work done on m by the gravitational field as m moves in from ∞ to r.

The same positive number $(-U(r))$ is the work done by m on the gravitational field as the particle moves from r to ∞.

$U(r)$ represents the amount of energy required to move m from r to ∞

A note on scaler & vectors:

- The potential $U(x)$ is a scaler
- The force $F(x) = -\frac{d}{dx}U(x)$ is a vector.

In 3D:

$$\underline{F}(\underline{r}) = -\nabla U(\underline{r}) = \left(-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right)$$

Conservation of Energy

$$W_{if} = \int_{x_i}^{x_f} F(x)\,dx = -\int_{x_i}^{x_f} \frac{dU(x)}{dx}\,dx = -\left[U(x_f) - U(x_i)\right]$$

Work done $= -\Delta$ potential $\mathcal{E}$.

Why the minus sign?
Because the potential energy is stored in the system in the system (springs, petrol, gravitational field, electric field) that applies the force.

- If the body moves in the direction of the applied force, the work done on the body is positive. Some of the stored PE is converted into KE.

- If the body moves against the direction of the applied force, work done on the body is negative. Some of the bodies KE is converted into PE.

$$W_{if} = K_f - K_i = U(x_i) - U(x_f)$$
$$K_f + U_f = K_i + K_f$$

This is the conservation of energy.

Conservative Force Field

Any force $F(x)$ that depends only on position is called a conservative force field. Any conservative $F(x)$ has a $U(x)$.

$F(x)$ (the whole function) is often called a force field; $U(x)$ is the potential function. Motion in a conservative force field conserves total energy.

Since $K + U(x) = \text{const}$, it implies that KE is also a function of position only. $K(x) = K$.

!! Many force fields are not conservative !!

We've proved energy conservation for conservative force fields in Newtonian mechanics, but the idea of energy conservation goes way beyond this.

It is an empirical observation that the TOTAL energy is always conserved!

Over time, we found other forms of energy:

- heat (jiggling atoms).
- chemical energy (bonds)
- field energy
- mass