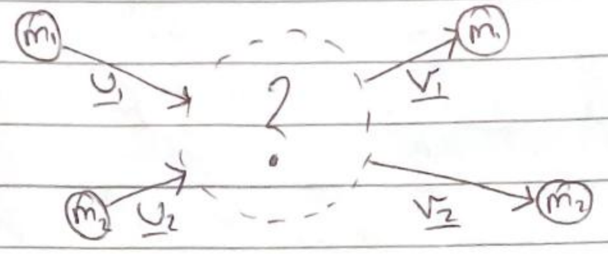
Classical Mechanics 8

Collisions

Brief interactions of two or more bodies via collisions.



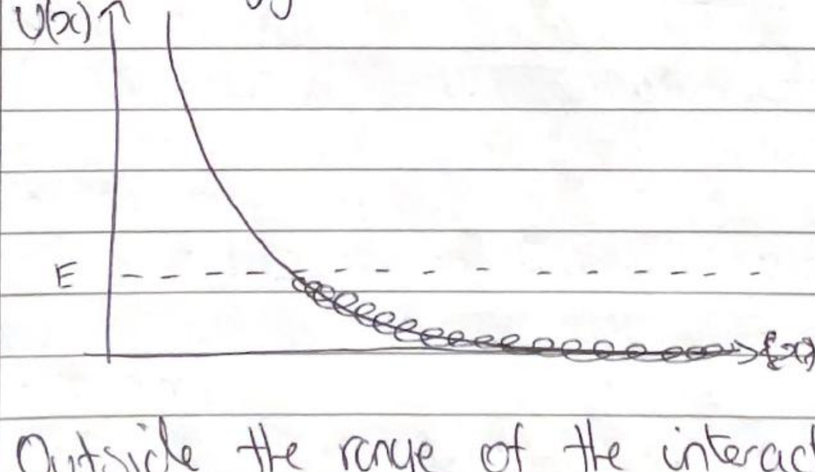
$$m_1\dot{v}_1 = F_{2on1} = -F_{1on2} = -m_2\dot{v}_2$$

$$m_1\dot{v}_1 + m_2\dot{v}_2 = \frac{d}{dt}(m_1 v_1 + m_2 v_2) = 0$$

The total momentum is conserved before, during and after the collision.

Elastic Collisions

If the interaction is conservative $\left(F(r) = -\frac{dU(r)}{dr}\right)$, the total energy is conserved.



Outside the range of the interaction, $U(r) = \text{const}$, usually set to 0, so KE before = KE after.

We call this collision elastic. Most real world collisions are inelastic. major exception: particle physics.

If the forces involved are conservative, the all collisions must be elastic.

Inelastic Collisions

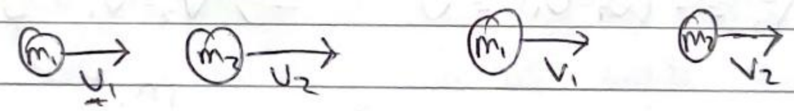
When the kinetic energy after the collision is less than the kinetic energy before the collision

Inelastic collisions are not conservative as energy becomes heat.

Collisions in 1D

Elastic

Before: After:



- For a collision to occur: $u_2 < u_1$
- To avoid m_1 passing through m_2: $v_2 > v_1$

What we know: Conservation!

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2$$

$$\Downarrow$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$\frac{u_1^2 - v_1^2}{u_1 - v_1} = \frac{v_2^2 - u_2^2}{v_2 - u_2}$$

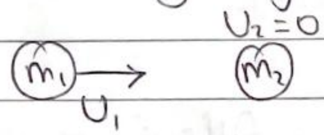
$$u_1 + v_1 = u_2 + v_2$$

This can be equivalently,

$$\underbrace{V_2 - V_1}_{\text{relative velocity after collision}} = -\underbrace{(U_2 - U_1)}_{\text{relative velocity before collision}}$$

- Relative speed stays the same, relative velocity changes sign.
- This is generally true (even for vectors) for elastic collisions.

<u>Example</u> (stationary target)



$$V_2 - V_1 = -(\cancel{U_2} - U_1) = U_1 \quad \Rightarrow V_2 - V_1 = U_1$$

$$m_1 V_1 + m_2 V_2 = m_1 U_1 \quad \Rightarrow V_1 + \frac{m_2}{m_1} V_2 = U_1$$

$$\left(1 + \frac{m_2}{m_1}\right) V_2 = 2U_1 \quad \Rightarrow \boxed{V_2 = \frac{2m_1}{m_1 + m_2} U_1}$$

$$V_1 = \frac{2m_1}{m_1 + m_2} U_1 - U_1 \quad \Rightarrow \boxed{V_1 = \frac{m_1 - m_2}{m_1 + m_2} U_1}$$

- $m_1 = m_2$

  $V_1 = 0$, $V_2 = U$. Projectile stops, target picks up entire momentum & KE.
- $m_1 > m_2$

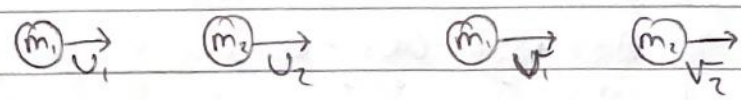
  V_1 and V_2 both positive, after collision particles move to the right
- $m_1 < m_2$

  $V_1 < 0$ and $V_2 > 0$, incoming particle moves back to the left, target moves to the right.

• $m_1 << m_2$

$v_1 \approx -u$, $v_2 \approx 0$. Incoming particle bounces off target, reversing momentum. final speed of target very small.

Inelastic Collisions in 1D



Emperically, in many inelastic collisions,

$$v_2 - v_1 = -e(u_2 - u_1)$$

'e' is the 'coefficient of restitution' and is roughly constant. When $e = 1$, we have an elastic collision. When $e = 0$, the particles stick together.

If $e > 1$, $KE_{after} > KE_{before}$, this could be due to an explosion (for example).

$$\boxed{v_1 = \frac{m_1 - em_2}{m_1 + m_2} u_1} \qquad \boxed{v_2 = \frac{(1+e)m_1}{m_1 + m_2} u_1}$$