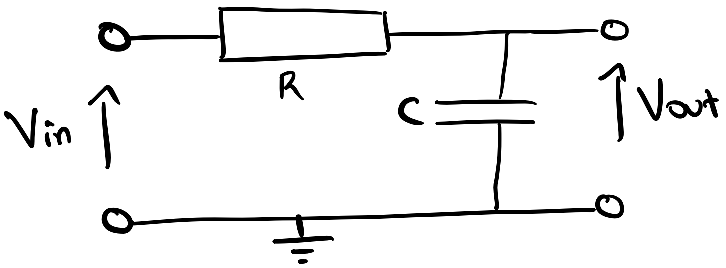
Low-Pass RC Circuit



- capacitor impedance depends on frequency.
- used to filter the noise, remove a range of frequencies from a signal.

$$\tilde{V}_{out} = \tilde{V}_{in} \times \frac{\tilde{Z}_c}{\tilde{Z}_R + \tilde{Z}_c}$$

$$= \tilde{V}_{in} \times \frac{-j/\omega C}{R - j/\omega C} \quad = \tilde{V}_{in} \times \frac{1}{1 + j\omega CR}$$

Gain

Gain is the output to input ratio. It is complex & shows how the filter changes the amplitude & phase of the input.

$$\tilde{G} = \frac{\tilde{V}_{out}}{\tilde{V}_{in}} = \frac{1}{1 + j\omega CR} = Ge^{j\phi}$$

$$G = |\tilde{G}| = \frac{1}{\sqrt{1 + \omega^2 C^2 R^2}} = \frac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|}$$

$$\phi = \arg(\tilde{G}) = -\tan(\omega CR)$$

The magnitude of gain is often given in decibels.

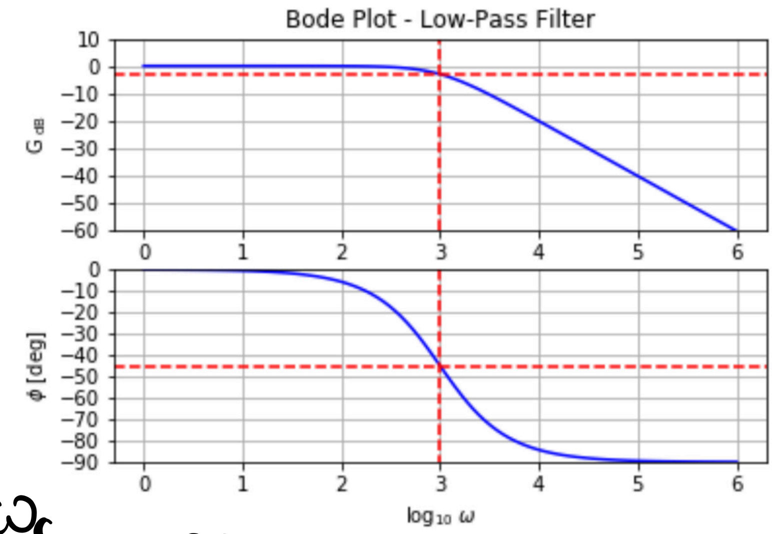
$$G_{dB} = 20 \log_{10} G$$

Bode Plot

cut-off frequency ω_c.

$$\omega_c = \frac{1}{RC} = \frac{1}{\tau}$$

$$\omega = \omega_c = \frac{1}{RC}$$



$g_{dB} = -3$
or
$g = \frac{1}{\sqrt{2}}$

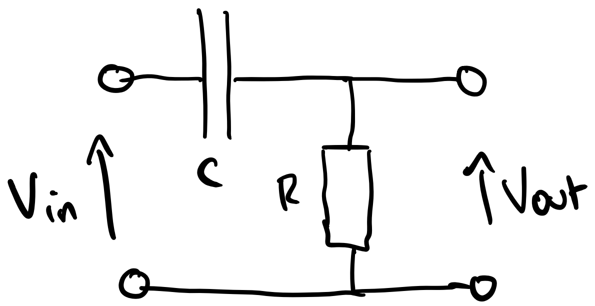
$\phi = -45°$
or
$\phi = -\frac{\pi}{4}$ rad

pass band ← ⋮ → stop band

for frequencies $\omega \ll \omega_c$, are passed through the filter.

for frequences $\omega \gg \omega_c$, are stopped by the filter.

High-Pass LC Filter



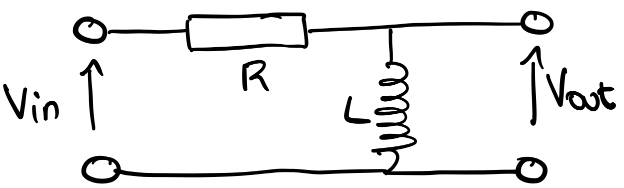
take the output across the resistor.

$$G = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} \qquad \tan\phi = \frac{1}{\omega RC}$$

$$\omega_c = \frac{1}{RC}$$

for frequencies below the cut-off frequency, the gain tends to zero. for frequencies above the cut off frequency, the gain tends to 1.

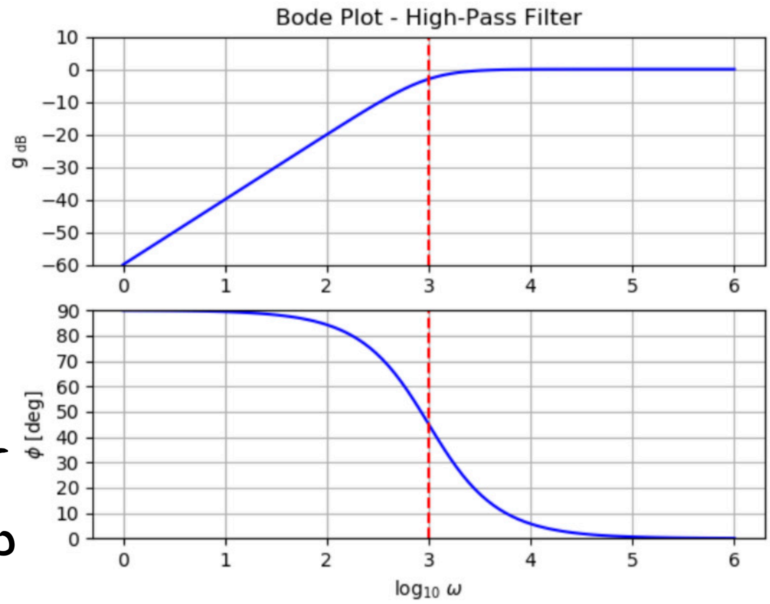
High-Pass RL Filter



$$\tan\phi = \frac{\omega_c}{\omega} \qquad \omega_c = \frac{R}{L}$$

$$G = \frac{\omega/\omega_c}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$$

when $\omega_0 = \frac{1}{RC}$, this takes the form of the RC filter.



$$\tilde{Z}_C \propto \frac{1}{\omega}$$

$$\tilde{Z}_L \propto \omega$$

you can swap the resistor and inductor to get a low pass filter.