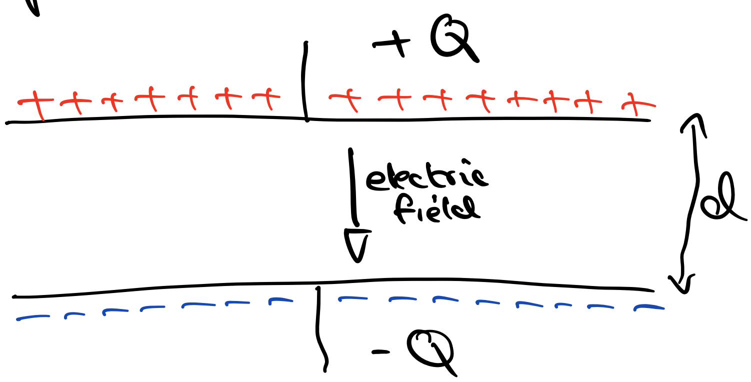
Capacitance



electric field strength $\rightarrow$ $E = \frac{V}{d}$ $\leftarrow$ change in voltage, $\leftarrow$ distance

capacitance $\rightarrow$ $C = \frac{Q}{V}$ $\leftarrow$ charge, $\leftarrow$ voltage

In a parallel plate capacitor

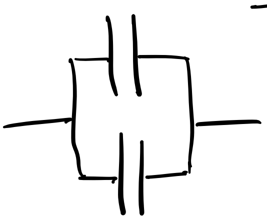
$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

A = area (m^2)
ε_0 = permittivity of free space (F/m)
ε_r = relativity permittivity
C = capacitance

ε_r is also called the dielectric constant.

Equivalent Capacitance

Parallel



$$C_T = C_1 + C_2$$

Series

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

Energy stored

$$U = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

$(Q = CV)$

during the charging process: $q = Cv$. to increase charge from q to $q + dq$: $du = v\,dq = \frac{q}{c}dq$

$$\frac{du}{dq} = \frac{q}{c} = v$$

As charge is conserved, a capacitor is always overall neutral. $(+Q - Q = 0)$

Current / Voltage Relationship

N.B.
CAPITALS = constant
lower-case = variable

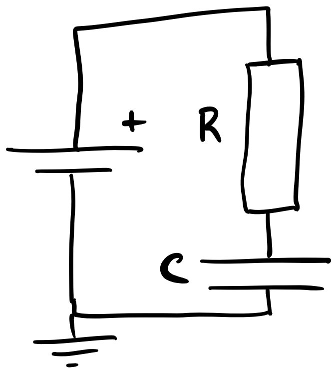
$$q = Cv$$

$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

no charge moves between the plates (the dielectric is an insulator). All charges move through the voltage source (eg. battery).

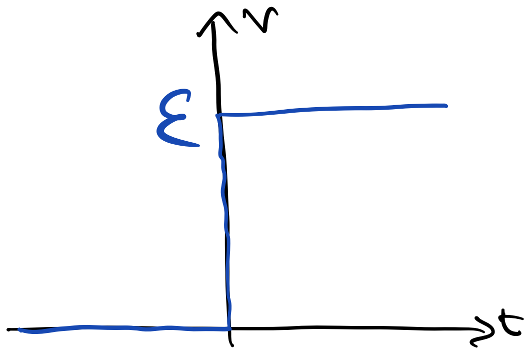
RC circuit

the voltage source is switched on at time $t = 0$.



Charging the Capacitor:

$$\mathcal{E} = iR + \frac{q}{C} \leftarrow \text{Kirchoff's voltage law}$$



$$\mathcal{E} = R\frac{dq}{dt} + \frac{1}{C}q$$

$$\mathcal{E}C = RC\frac{dq}{dt} + q$$

$$\mathcal{E}C - q = RC\frac{dq}{dt}$$

$$\frac{dq}{\mathcal{E}C - q} = \frac{dt}{RC} \Rightarrow \int_0^q \frac{dq'}{q - \mathcal{E}C} = \int_0^t \frac{-dt'}{RC}$$

$$\Rightarrow \Big[\ln(q - \mathcal{E}C)\Big]_0^q = \Big[\frac{-t}{RC}\Big]_0^t \Rightarrow \ln\left[\frac{q - \mathcal{E}C}{-\mathcal{E}C}\right] = \frac{-t}{RC}$$

$$\frac{q-\mathcal{E}C}{-\mathcal{E}C} = e^{-\frac{t}{RC}} \Rightarrow q = \mathcal{E}C\left(1-e^{-t/RC}\right)$$

Similarly for current: $i = \frac{\mathcal{E}}{R}e^{-\frac{t}{RC}}$

and for voltage:

across the capacitor

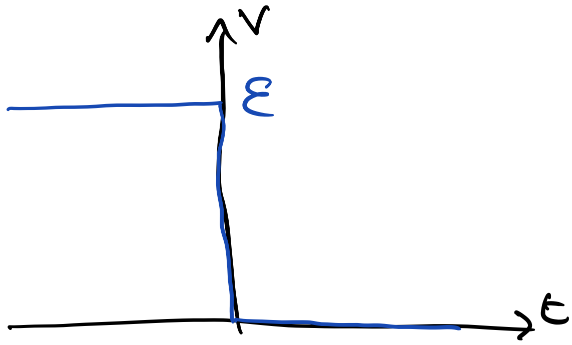
$$V_c = \mathcal{E}\left(1-e^{-\frac{t}{RC}}\right)$$

across the resistor

$$V_R = \mathcal{E}e^{-\frac{t}{RC}}$$

τ is the time constant. $\tau = RC$ (units = seconds). It is the time taken for something to fall by a factor of e^{-1} $\approx 37\%$.

Discharging a Capacitor:



$$0 = V_c + R_R = iR + \frac{q}{c}$$

$$= R\frac{dq}{dt} + \frac{1}{c}q = RC\frac{dq}{dt} + q$$

$$\Rightarrow \int_0^t -\frac{1}{RC}\,dt' = \int_0^q \frac{dq'}{q'}$$

$$\left[-\frac{t}{RC}\right]_0^t = \left[\ln(q')\right]_{C\mathcal{E}}^q \Rightarrow \frac{-t}{RC} = \ln\left[\frac{q}{C\mathcal{E}}\right] \Rightarrow q = C\mathcal{E}e^{-\frac{t}{RC}}$$

for current: $i = -\frac{\mathcal{E}}{R}e^{-t/RC}$

$P_R = V_R i = i^2 R$

capacitor voltage

$V_R = -\mathcal{E}e^{-t/RC}$ positive ~~power~~ $= \mathcal{E}e^{-t/RC}$

resistor voltage

resistor dissipating

Energy & Power

Charging

$P_R = V_R i = i^2 R$

- positive power
- resistor dissipating

$P_C = V_C i$

- positive power
- capacitor storing energy

$P_S = -V_S i = -\mathcal{E} i$

- negative
- source providing energy

Discharging

$P_R = V_R i = i^2 R$

- positive power
- resistor dissipating

$P_C = V_C i$

- negative
- realising stored energy

$P_S = -V_S i = 0$

- no source power

$$\sum P_n = 0$$