## Functions 12 Cartesians - D Polar I both are othergonal systems. $v(x,y) \equiv \bar{v}(r,\theta)$ $y = r\cos\theta$ $r = (x^2 + y^2)^{\frac{1}{2}}$ $y = r\sin\theta$ $\theta = \arctan(y/x)$ We need to be racker! $\frac{\partial c}{\partial x} = \frac{\partial c}{\partial x} =$ y constant. Keeping O Constant We should not be tempted The cortesion and polar versions of our functions have the same value but are described differently. Chair Rule $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} =$ (u(r,0) $=(\cos \theta)\frac{\partial \theta}{\partial c} + (-\frac{\sin \theta}{c})\frac{\partial \theta}{\partial \theta}$

We have therefore partial differential opperators!

$$\frac{\partial}{\partial x} = \cos \theta \frac{\partial}{\partial r} + \frac{\sin \theta}{r} \frac{\partial}{\partial \theta}$$

Which relates the rates of drage in the two of different coordinate systems.

Example  $u(\alpha_{1}y) = \alpha^{2} - y^{2} = r^{2}(\cos^{2}\theta - \sin^{2}\theta) = \bar{u}(r,\theta)$ 

$$\frac{\partial y}{\partial x} = 200 = 20000$$

$$\frac{\partial v}{\partial x} = -2r \sin \theta$$

Justing our above Ou - - 2y = -2rsin0 results we can check

Example We con express the following equation in place polar coordinates.

$$(\frac{\partial x}{\partial y})^2 + (\frac{\partial y}{\partial y})^2$$

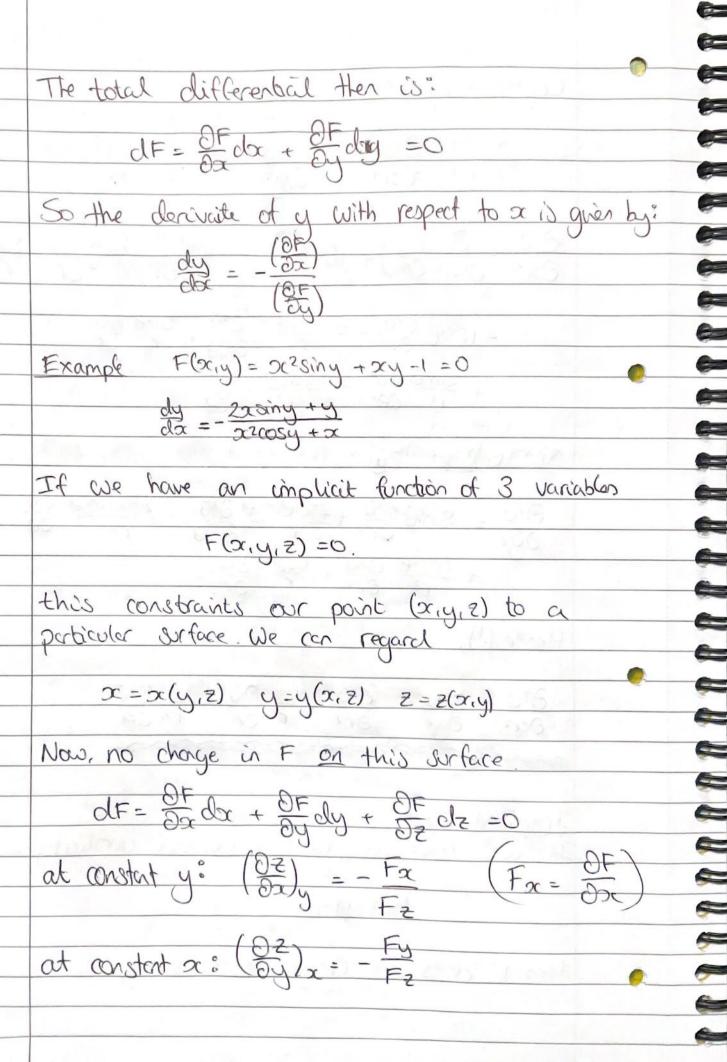
$$\left[\cos\theta \frac{\partial \sigma}{\partial r} - \sin\theta \frac{\partial \sigma}{\partial \theta}\right]^2 + \left[\sin\theta \frac{\partial \sigma}{\partial r} + \cos\theta \frac{\partial \sigma}{\partial \theta}\right]^2$$

$$\equiv \left(\frac{\partial \overline{\partial}}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial \overline{\partial}}{\partial \theta}\right)^2$$

Laplace's Equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

$$\frac{\partial^2 U}{\partial x^2} - \frac{\partial}{\partial x} \left(\frac{\partial U}{\partial x}\right) = \left(\frac{\partial U}{\partial x}\right) \frac{\partial}{\partial x} - \frac{\partial}{\partial x} \left(\frac{\partial U}{\partial x}\right) \frac{\partial}{\partial x} - \frac{\partial}{\partial x} \frac{\partial}{\partial x} \frac{\partial}{\partial x} + \frac{\partial}{\partial x} \frac{$$



at constant z: (Dx) = N.B.  $\left(\frac{\partial z}{\partial x}\right)_y = \left(\frac{\partial x}{\partial z}\right)_y$ As we're holding the same variable constant, they are recipricals of one another. Ideal Gas Law PV = nRT (76) (V6) (96) (96) 9 (76) T (V6)