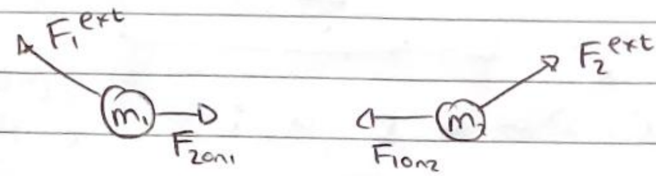
Classical Mechanics 7

Centre of Mass

Think about two bodies exerting forces on each other.



Starting from Newton's second law for both particles.

$$m_1\ddot{\underline{r}}_1 = F_{2on1} + F_1^{ext}$$
$$m_2\ddot{\underline{r}}_2 = F_{1on2} + F_2^{ext}$$

adding them together

$$m_1\ddot{\underline{r}}_1 + m_2\ddot{\underline{r}}_2 = \underbrace{F_{1on2} + F_{2on1}}_{=0\ (N3)} + F_1^{ext} + F_2^{ext}$$

$$m_1\ddot{\underline{r}}_1 + m_2\ddot{\underline{r}}_2 = F^{ext}$$

$$(m_1+m_2)\left(\frac{m_1\ddot{\underline{r}}_1 + m_2\ddot{\underline{r}}_2}{m_1+m_2}\right) = F^{ext}$$

$$(m_1+m_2)\frac{d}{dt}\left(\frac{m_1\underline{r}_1 + m_2\underline{r}_2}{m_1+m_2}\right) = F^{ext}$$

We define $\boxed{\underline{R} = \dfrac{m_1\underline{r}_1 + m_2\underline{r}_2}{m_1+m_2}}$

we can write this in terms of newtons second law.

$$m\ddot{\underline{R}} = F$$

this is a position! not a mass.

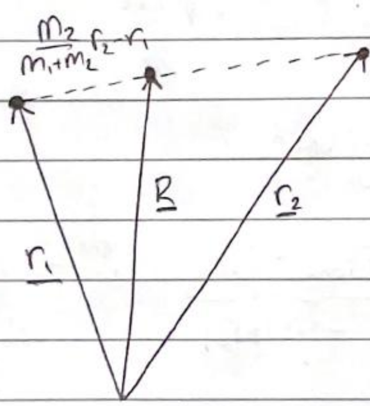
It's easy to show that

$$\boxed{\underline{R} = \frac{\sum_{i=1}^{N} m_i \underline{r}_i}{\sum_{i=1}^{N} m_i}}$$

We can rewrite $\underline{R}$ as: (add + subtract $m_2 r_1$ in numerator)

$$\underline{R} = \underline{r}_1 + \frac{m_2}{m_1+m_2}(r_2 - r_1)$$

We can be shown diagramatically as:



- $\underline{R}$ lines on the line from $\underline{r}_1$ to $\underline{r}_2$
- if $m_1 > m_2$, $\underline{R}$ is closer to $\underline{r}_1$
- if $m_1 < m_2$, $\underline{R}$ is closer to $\underline{r}_2$
- if $m_1 = m_2$, $\underline{R}$ is midway between $\underline{r}_1$ & $\underline{r}_2$

Reduced Mass

When we have two (generally more but two for now) masses being acted on by the same external force. We can simplify our problem from a two body problem to a single body problem.