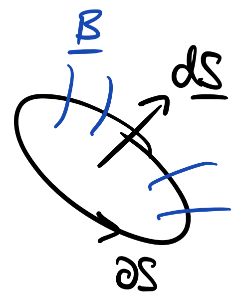
So far we've only looked at constant magnetic fields, what happens when they vary with time?

Faraday's Law

The EMF induced in a current loop is equal to the rate of change of magnetic flux through it.

$$\mathcal{E} = -\frac{d\Phi}{dt}$$



where $\Phi = \iint_S \underline{B} \cdot d\underline{S}$. We recall that

$$\mathcal{E} = \oint \underline{E} \cdot d\underline{l}$$

around any closed loop (from electrostatics). We can now use Stoke's theorem

$$\iint_R \nabla \times \underline{F} \cdot d\underline{S} = \oint_{\partial R} \underline{F} \cdot d\underline{l}$$

Which allows us to write

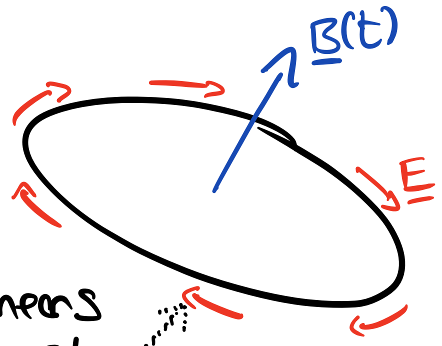
$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

This is Faraday's Law in differential form. A change in the magnetic field induces curl in the electric field.

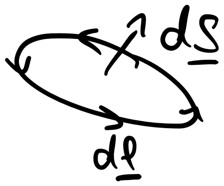
Lenz's law

$$\mathcal{E} = \oint \underline{E}\cdot d\underline{l} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\iint \nabla\times\underline{B}\cdot d\underline{S}$$

What is the significance of the -ve sign? N.B. $d\underline{S}$ & $d\underline{l}$ are defined by the right-hand rule.



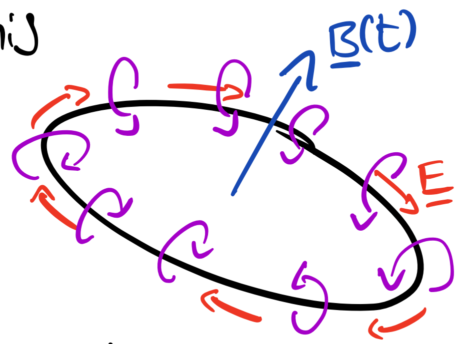
Consider the case where $\frac{\partial\Phi}{\partial t} > 0$



$\nabla\times\underline{E} = -\frac{\partial B}{\partial t}$ -ve sign means opposite of RHR!

If mobile charges are present this drives an a current.

This produces a magnetic field (Ampere's law) which opposes the original magnetic field. This acts against the increasing Φ.



This leads to Lenz's law: when a changing magnetic field induces a current, the direction of the current flow is such to produce a magnetic field which opposes the original change.

The electrostatic field does not form closed loops. A magnetic field B(t) produces an electric field formed of closed loops.