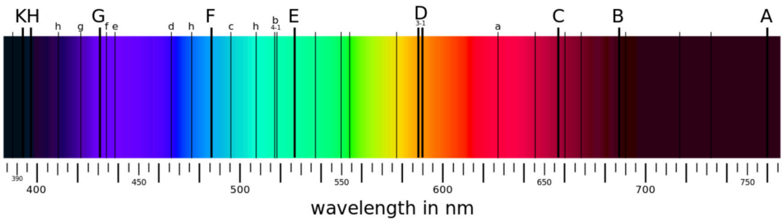
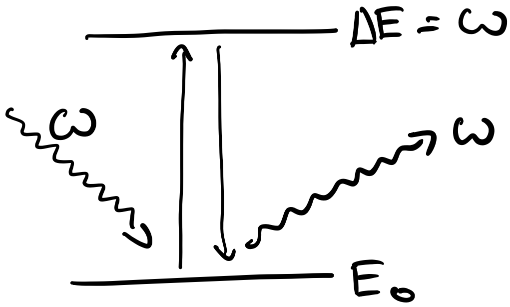
Resonance Decay

The electron in a H atom can be exited into higher energy states. The photons produce a star spectra.





An electron in an exited state will fall into a lower energy level with probability given by

$$P(t) = \begin{cases} 0 & t<0 \\ e^{-t/\tau} & t>0 \end{cases}$$

τ = timescale: lifetime

In quantum mechanics, the wavefunction is given by

$$\psi(t) = \begin{cases} 0 & t<0 \\ e^{-i\omega_0 t} e^{t/2\tau} & t>0 \end{cases}$$

$P(t) = |\psi(t)|^2$

where $\omega_0 = E/\hbar$ with E the exited energy. We can now fourier transform this to get the distribution of angular frequencies.

$$\Phi(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(t) e^{i\omega t} dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-i\omega_0 t} e^{-t/2\tau} e^{i\omega t}\, dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{i[\omega-\omega_0 + i/2\tau]t}\, dt$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{1}{i[\omega-\omega_0 + i\Gamma/2]} e^{i[\omega-\omega_0+i/2\tau]t} \right]_0^\infty$$

$\Gamma = 1/\tau$ 'decay rate'.

$$= \frac{1}{\sqrt{2\pi}} \left[0 - \frac{1}{i[\omega-\omega_0+\frac{i\Gamma}{2}]} \right]$$

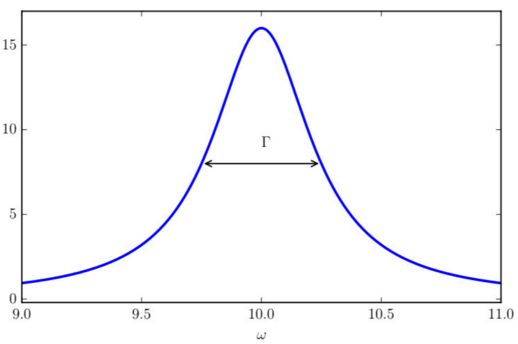
$$\Phi(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \frac{i}{\omega-\omega_0+i\Gamma/2}$$

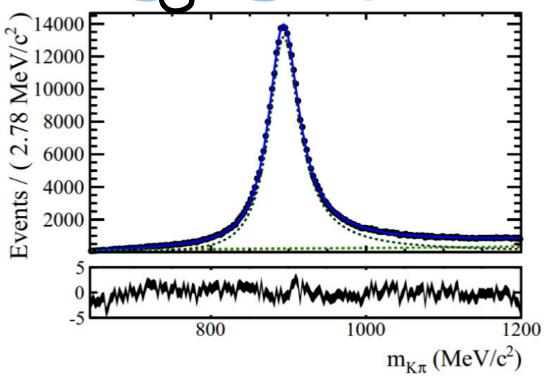
Think of as an amplitude function in terms of energy.

We can form an intensity function by squaring our amplitude function.

$$I(\omega) = |\Phi(\omega)|^2 = \frac{1}{2\pi} \cdot \frac{1}{(\omega-\omega_0)^2 + \frac{\Gamma^2}{4}}$$

This is known as the Breit-Wigner distribution.





Γ is the full-width-half-maximum of the distribution.
The shorter the lifespan the less well defined the

energy.

Multidimension Transforms

We often need to do fourier transforms in multiple dimensions (eg. image analysis).

Given a function $f(x,y)$, a multidimension fourier transform is just the transform on each variable.

$$g(k_x, k_y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{ik_x x} e^{ik_y y} \, dx dy$$

We now have two wavenumbers insted of one.

Using vectors, we can form the wave-vector

$$\underline{k} = k_x \hat{i} + k_y \hat{j}$$

$$g(\underline{k}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\underline{r}) e^{i\underline{k}\cdot\underline{r}} \, d^2\underline{r}$$

The basis functions become wave with a direction of $\hat{k}$ w.r.t cartesian coordinates.

$$f(\underline{r}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(\underline{k}) e^{-i\underline{k}\cdot\underline{r}} \, d^2\underline{k}$$

We can generalise this to n dimensions

$$g(\underline{k}) = \frac{1}{(2\pi)^{n/2}} \int_{-\infty}^{\infty} f(\underline{r})\, e^{i\underline{k}\cdot\underline{r}}\, d^n\underline{r}$$