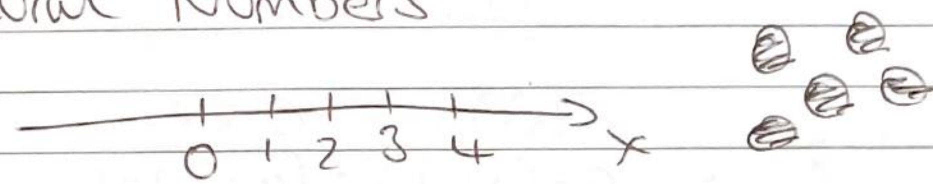
11/10/21

Complex Analysis

* complex numbers
* functions of complex variables
* intro into ordinary diff. eq^n.

1) Natural Numbers



3 - 5 = ?

- add
- subtract
- multiply
- divide

2) negative numbers

12 ÷ 3 = 4

3 ÷ 5 = ?

3) rational numbers

$\sqrt{2} =? \frac{p}{q}$

Proof

assume $\sqrt{2} = \frac{p}{q}$

$2 = \frac{p^2}{q^2}$

$p^2 = 2q^2$

p^2 is even $\Rightarrow$ p is even

$\therefore$ q is also even

assumption must be wrong

$\sqrt{2}$ is "irrational"

4) irrational numbers

$\pi \quad e \quad \sqrt{2}$ ...

$$\sqrt{2} = \frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \ldots$$

getting closer to true value

The Rules

$$a+b = b+a$$ commutative

$$a+(b+c) = (a+b)+c$$ associative

$$ab = ba$$

$$a(bc) = (ab)c$$

$$a(b+c) = ab+ac$$ distributive

5) complex numbers

$$\sqrt{-1} = i \qquad i^2 = -1$$

$$z = x + iy$$

$$x = Re(z) \qquad y = Im(z)$$

$$\begin{aligned} z + z' &= (x+iy) + (x'+iy') \\ &= (x+x') + i(y+y') \\ &= z' + z \end{aligned}$$

$$\begin{aligned} z \cdot z' &= (x+iy)(x'+iy') \\ &= (xx' - yy') + i(yx' + xy') \end{aligned}$$

$$\frac{z}{z'} = \frac{x+iy}{x'+iy'} \cdot \frac{x'-iy'}{x'-iy'} = \frac{(xx'-yy')+i(yx'+xy')}{x'^2 + y'^2}$$

$$\frac{1}{i} \times \frac{-i}{-i} = \frac{-i}{-i^2} = \underline{\underline{-i}}$$

$z = x + iy$ $x = Re(z)$

$y = Im(z)$

$z = (x, y)$ kinda like a vector...

~~Complex~~

	Complex	Vector
addition	✓	✓
multiple by real number	✓	✓
multiplecation	✓	✗

doesn't follow 'the rules'

$\therefore$ vectors are good analogy but $\mathbb{C}$ are not vectors

<u>Result</u>: we've found a way to add and multiple a pair of numbers

<u>An example</u>:

$(a, b) + (a', b') = (ab' + a'b, bb')$

$(a, b) \cdot (a', b') = (aa', bb')$

$\frac{1}{(a, b)} = (b, a)$ $(a, a) = (b, b)$

we find that $(a,b) = \frac{a}{b}$ (fraction)

what we do with with complex numbers making them obey a set of rules is the same as what we've done with rational numbers.

Complex Conjugate:

Def

for $z = x + iy$

$$z^* = x - iy$$

$$\left.\begin{array}{l} (z \pm z')^* = z^* \pm z'^* \\ (z \cdot z')^* = z^* \cdot z'^* \\ \left(\frac{z}{z'}\right)^* = \frac{z^*}{z'^*} \end{array}\right\} \text{properties of C.C.}$$

Proofs

$$(z \cdot z')^* = [(x+iy)(x'+iy')]^*$$
$$= [xx' - yy' + i(xy' + x'y)]^*$$
$$= (xx' - yy') - i(xy' + x'y)$$

Same.

$$z^* \cdot z'^* = (x - iy)(x' - iy')$$
$$= xx' - yy' - i(xy' + x'y)$$

$$z + z^* = x + iy + x - iy = 2x = 2\mathrm{Re}(z)$$
$$z - z^* = x + iy - x + iy = 2iy = 2i\mathrm{Im}(z)$$