Complex Functions

$$f(z) = u(z) + iv(z) \qquad f(z), z \in \mathbb{C}$$
$$u(z), v(z) \in \mathbb{R}$$

$$u(z) = \frac{f(z) + f^*(z)}{2} \qquad v(z) = \frac{f(z) - f^*(z)}{2i}$$

$$e^{i\theta} = \cos\theta + i\sin\theta \Rightarrow e^{in\theta} = \cos(n\theta) + i\sin(n\theta)$$

for $f(x)$, $x \in \mathbb{R}$, we have

$$f'(x) = u'(x) + iv'(x)$$
$$\int f(x) = \int u(x) + i\int v(x)$$

Even & Odd Functions

$$f(x) = -f(-x) \;:\; \text{Odd} \qquad x, x^3, x^5, \sin x$$
$$f(x) = f(-x) \;:\; \text{Even} \qquad x^2, x^4, \cos x$$

We can decompose any function into even & odd parts.

$$f(x) = \underbrace{e(x)}_{\text{even}} + \underbrace{o(x)}_{\text{odd}}$$

easy to show that:

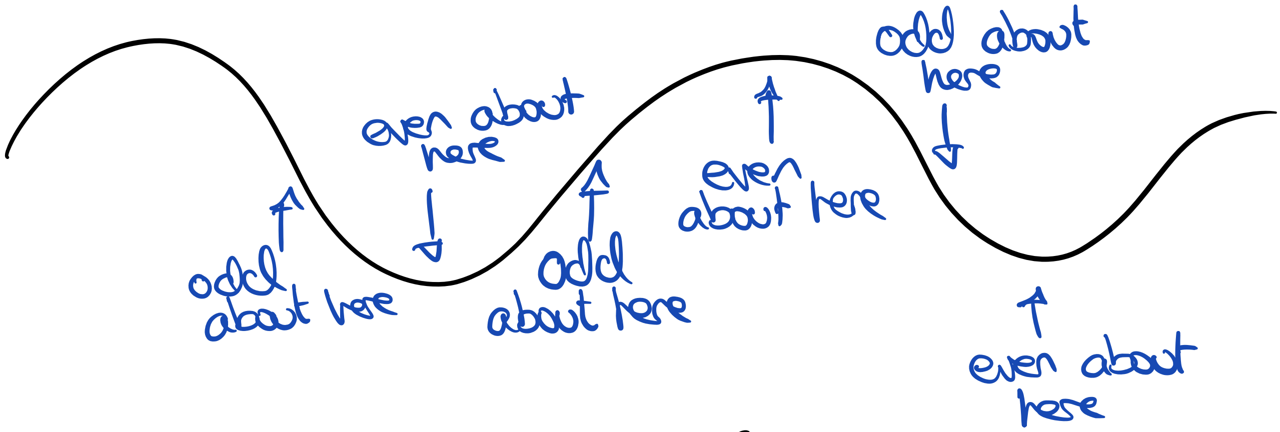
$$e(x) = \frac{f(x) + f(-x)}{2} \qquad o(x) = \frac{f(x) - f(-x)}{2}$$

The product of two odd functions is an even function. As is the product of two even functions. However, the product of an even & an odd function will be odd.

$$\int_{-R}^{R} o(x)\,dx = 0$$

$$\int_{-R}^{R} e(x)\,dx = 2\int_{0}^{R} e(x)\,dx$$

These can be combined to define parity about any point.



Example $\quad f(x) = x-3 \qquad \int_0^6 f(x)\,dx.$

let $f'(x') = f(x+3)$. $f'(x')$ is odd $(=x)$.

$$\int_0^6 f(x)\,dx = \int_{-3}^{3} f'(x')\,dx' = \int_{-3}^{3} x'\,dx' = 0$$

Exercise 1.1 $\qquad f(x) = \cos x + i\sin x$

$$e(x) = \frac{f(x)+f(-x)}{2} \qquad o(x) = \frac{f(x)-f(-x)}{2}$$

$$e(x) = \frac{\cos x + i\sin x + (\cos(-x) + i\sin(-x))}{2}$$

$$= \frac{\cos x + i\sin x + \cos x - i\sin x}{2}$$

$$= \frac{2\cos x}{2} = \cos x$$

$$o(x) = [\cos x + i\sin(x) - (\cos(-x) + i\sin(-x))]/2$$
$$= [\cos(x) + i\sin(x) - \cos x + i\sin x]/2$$
$$= \frac{2i\sin x}{2} = i\sin x$$

Exercise 1.2 $\quad f(x) = e^{x}$ on interval $(-\pi, \pi)$

$$e(x) = \frac{e^{x} + e^{-x}}{2} \qquad o(x) = \frac{e^{x} - e^{-x}}{2}$$

$$= \cosh(x) \qquad\qquad = \sinh(x)$$

What happens when we integrate a function multiplied by a sin/cos?

$$f(x) = e(x) + o(x)$$

$$\int_{-\infty}^{+\infty} [e(x) + o(x)]\overset{\text{even}}{\cos x}\,dx = \underbrace{\int_{-\infty}^{+\infty} e(x)\cos x\,dx}_{\text{even}} + \underbrace{\int_{-\infty}^{+\infty} o(x)\cos x\,dx}_{\text{odd} \,\therefore\, \int f(x) = 0}$$

$$= \int_{-\infty}^{+\infty} e(x)\cos(x)\,dx$$

$$\int_{-\infty}^{+\infty} [e(x) + \underbrace{o(x)}_{\text{odd}}]\sin(x)\,dx = \underbrace{\int_{-\infty}^{+\infty} e(x)\sin(x)\,dx}_{\text{odd}} + \underbrace{\int_{-\infty}^{+\infty} o(x)\sin(x)\,dx}_{\text{even}}$$

$\therefore \int o(x) = 0$

$$= \int_{-\infty}^{+\infty} o(x)\sin(x)\,dx$$