Classical Mechanics II

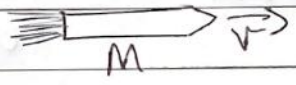
The Rocket Equation

Rockets work by ejecting mass at speed. To conserve the total momentum of the rocket and fuel, the rocket has to accelerate.

The rocket loses mass, $\frac{dM}{dt} < 0$.

Since $dt > 0$, $dM < 0$.

time t



time $t + \Delta t$

Conservation of Momentum

$$(M+dM)(v+dv) + (-dM)(-u+v) = Mv$$

$$Mv + Mdv + v\,dM + dM\,dv + u\,dM - v\,dM = Mv$$

$$\boxed{M\,dv = -u\,dM}$$

$$\Rightarrow dv = -u\frac{dM}{M}$$

$$\int_{v(0)}^{v(t)} dv' = -u\int_{M(0)}^{M(t)} \frac{dM'}{M'}$$

$$v(t) - v(0) = -u\left[\ln M'\right]_{M(0)}^{M(t)}$$

$$\boxed{v(t) = v(0) - u\ln\left|\frac{M(t)}{M(0)}\right|}$$

$$\boxed{v(t) = v(0) + u\ln\left|\frac{M(0)}{M(t)}\right|}$$