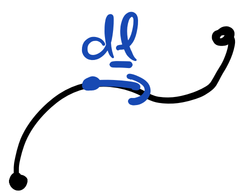
Forces on a Wire

We need to be careful with our definition of number density. For a volume it no. charges per m^3 moving at velocity v. $\underline{j} = nq\underline{v}$ (a vector). For a thin wire, we use the number of charges per m moving at speed v, then $I = nqv$ (a scalar).

Consider a small vector length $d\underline{l}$, there will be $n|d\underline{l}|$ charges moving at velocity $v\,d\hat{l}$. For a single charge the force is $\underline{F} = q\underline{v}\times\underline{B}$, so the total force over $d\underline{l}$ will be $F = nqv\,d\underline{l}\times B$.

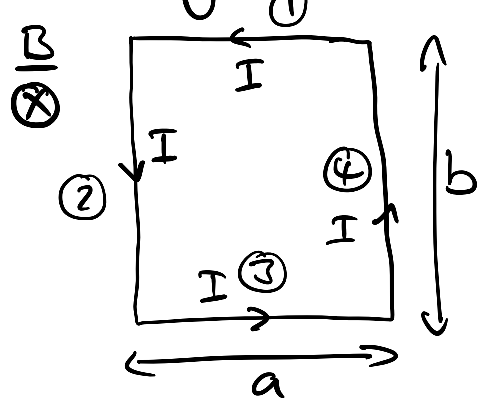


$\therefore$ The total force on the wire

$$\underline{F} = \int_0^L nqv\,d\underline{l}\times\underline{B} = I\int_0^L d\underline{l}\times B$$

(when $\underline{B}\perp d\underline{l}$ - $F = BIL$)

Consider a rectangular loop in a uniform magnetic field.



$\underline{F_1} = -BIL\hat{y}$ $\quad \underline{F_2} = BIL\hat{x}$

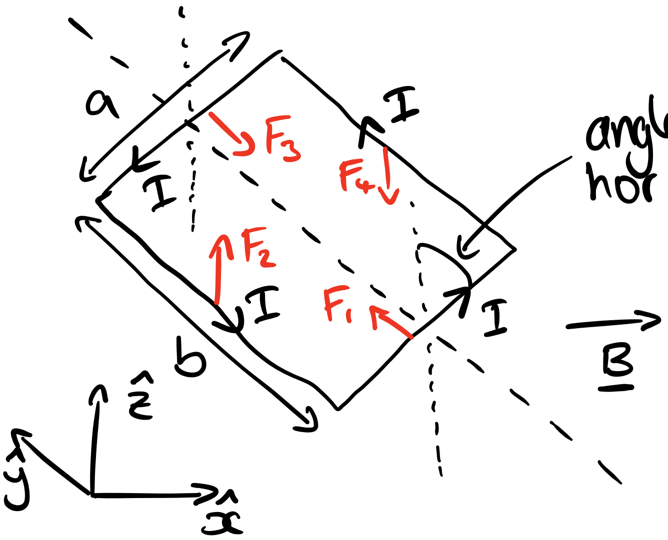
$\underline{F_3} = BIL\hat{y}$ $\quad \underline{F_4} = -BIL\hat{x}$

$\underline{F} = \underline{F_1} + \underline{F_2} + \underline{F_3} + \underline{F_4} = 0$

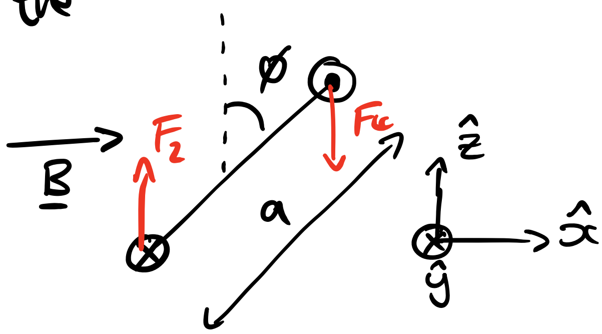
There is no net force on any current loops in a uniform magnetic field.

$$\underline{F} = \oint I\,d\underline{l} \times \underline{B} = 0$$

Torques on Current Loops



angle of ϕ to the horizontal.



$$F_2 = BIb\,\hat{z}$$

Although there is no net force, there is a torque around the central axis.

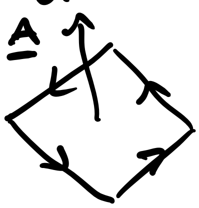
$$\text{Torque} = \text{force} \times \text{distance}$$
$$\Gamma_2 = BIb \times \frac{a}{2}\sin\phi$$
$$\Gamma = \Gamma_2 + \Gamma_4 = BIab\sin\phi$$

Area(A) = ab

$$\Gamma = ABI\sin\phi$$

We define $\underline{A}$ to be $|A|$ = area of ab and to be perp. to the loop.

$\therefore$



right-hand rule

$$\Gamma = I\underline{A} \times \underline{B}$$

Now if we define the <u>magnetic dipole moment</u> $\underline{\mu} = I\underline{A}$, a vector associated with the current (I) loop around the area $\underline{A}$.

$$\underline{\Gamma} = \underline{\mu} \times \underline{B}$$

Magnetic fields can exert torques on current loops. N.B. if we have n loops then $\Gamma = n\underline{\mu} \times \underline{B}$.