Classical Mechanics

12/10/21

Classical mechanics studies the motion of bodies subject to forces.

- -> no quantum effects
- -> no special or general relativity ($v << c$)
- -> no fluid mechanics
- -> no lagrangian or hamiltonian mechanics

Assumptions:

* universality of time: all observers agree on the time interval between two events.
* space is homogeneous, isotropic and Euclidean

homogeneous: looks the same from all origins

isotropic: looks the same in all directions

Euclidean: $a^2 = b^2 + c^2$

These assumptions are pretty good on earth for $v << c$.

A particle is a point-like object on which forces act.

A body is a particle or assembly of particles bound together. The centre of mass of a body responds to forces as if it were a point particle. Bodies may rotate and deform.

A reference frame is a set of coordinate axes used to measure position:

Position is a vector

$$\underline{r} = (x, y, z) = \begin{cases} x\hat{x} + y\hat{y} + z\hat{z} \\ x\hat{i} + y\hat{j} + z\hat{k} \end{cases}$$

When writing by hand, underline vectors.

Hats denote unit vectors.

Velocity is the rate of change of position with respect to time

A scalar is something which doesn't change (the numerical value) when in a different frame of reference.

The velocity $\underline{v} = d\underline{r}/dt$ is a vector.

The speed $|\underline{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

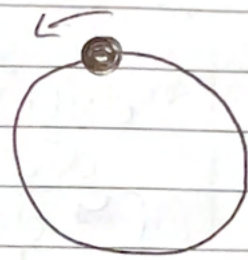
How to differentiate vectors:

$$\frac{d\underline{r}}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

Average velocity:

$$\underline{v} = \frac{\underline{r}(t_2) - \underline{r}(t_1)}{t_2 - t_1} = \frac{\Delta \underline{r}}{\Delta t}$$

in a circular orbit:



$\underline{v} = 0$ (no total displacement)

but $|v| > 0$, the avg. speed doesn't equal to zero. $|v| \neq v$

<u>Acceleration:</u>

$$a = \frac{dv}{dt} = \frac{dx^2}{dt^2} \quad \text{in 1D}$$

$$\underline{a} = \frac{d\underline{v}}{dt} = \frac{d\underline{x}^2}{dt^2} \quad \text{in 3D}$$

<u>Notation:</u>

Newton's: $\dot{r} = \frac{dr}{dt}$ <u>dot for space</u>

$v' = \frac{dv}{dx}$ <u>prime for space</u>

Newton: $\dot{r}(t_0)$

Leibniz: $\left(\frac{dr}{dt}\right)(t_0)$ or $\left.\frac{dr}{dt}\right|_{t=t_0}$

Always <u>underline</u> vectors - or lose marks!

The SUVAT Eq^n 12/10/21

→ describe motion with constant acceleration

$v = u + at$

$$\frac{dv(t)}{dt} = a = \text{const.}$$

$$\int_{t=0}^{t_1} \frac{dv}{dt}\,dt = \int_{t=0}^{t_1} a\,dt$$

$$\left[v(t)\right]_0^{t_1} = \left[at\right]_0^{t_1}$$ ← limits mean no const. of intergration

$$v(t_o) - v(0) = at_1$$

$$v(t_o) = at + v(0)$$

$s = ut + \frac{1}{2}at^2$

$$\frac{dx}{dt} = u + \underset{\text{const.}}{a}t^2$$

$$\int_{t_0}^{t_1} \frac{dx}{dt} = \int_{t_0}^{t_1} u + at^2$$

$$x = ut + \frac{1}{2}at^2$$