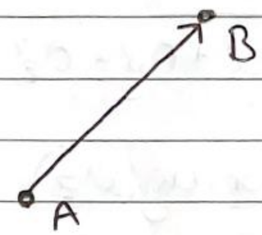
Vectors

12/10/21

A vector is a quantity that has a "direction" as well as "length".

In general, a vector is a quantity with multiple components.

You must declare a quantity to be a vector. Commonly $\underline{a}$ or $\vec{a}$ or bold.



A vector between two points - direction vector - often written $\overrightarrow{AB}$

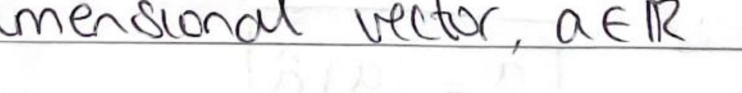
An n-component vector has n-dimensions,

$\underline{a} = (a_1, a_2, a_3, \ldots a_n)$ "row vector"

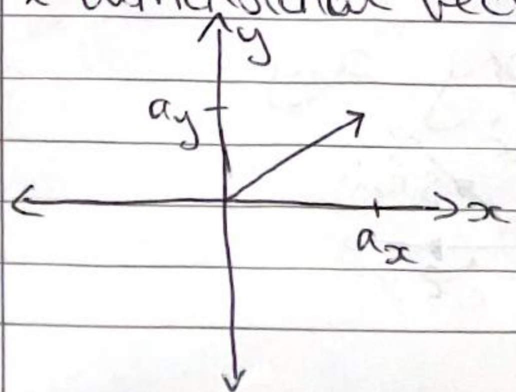
$\underline{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix}$ "column vector"

Visual Rep.

1-dimensional vector, $a \in \mathbb{R}$



2-dimensional vector, $a \in \mathbb{R}^2$



$\underline{a} = (a_x, a_y)$

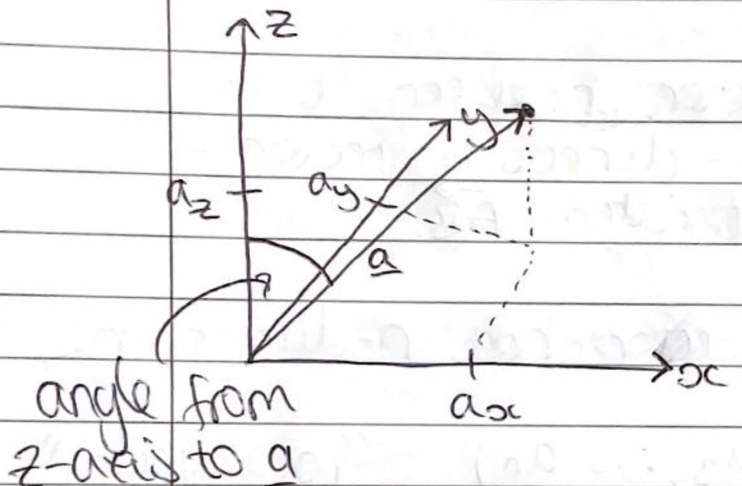
Length of $\underline{a}$ often written as $|\underline{a}|$

$$|\underline{a}| = \sqrt{a_x^2 + a_y^2} = \text{modulus of } a$$

$$\theta = \arctan\left(\frac{a_y}{a_x}\right) = \arg(\underline{a})$$

$$\cos\theta = \frac{a_x}{|\underline{a}|} \qquad \sin\theta = \frac{a_y}{|\underline{a}|}$$

3D vector, $\underline{a} \in \mathbb{R}^3$



$$|\underline{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\underline{a} = (a_x, a_y, a_z)$$

$$\cos\theta = \frac{a_z}{|\underline{a}|}$$

$$\cos\theta_n = \frac{a_n}{|\underline{a}|}$$

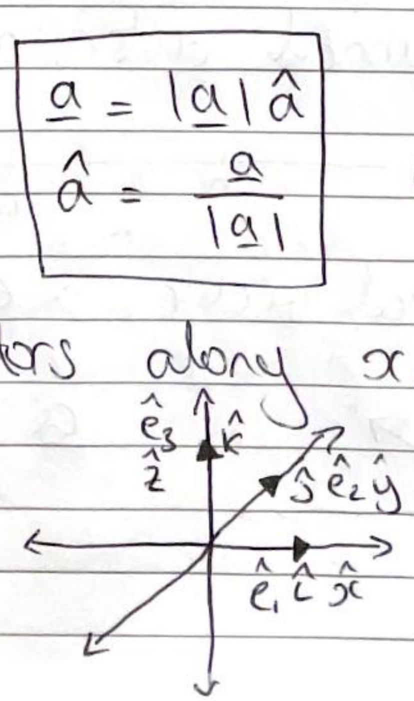
Basis Vectors

The vector of unit length in the direction of vector $\underline{a}$ is called a normalised vector $\hat{a}$.

$$\boxed{\begin{aligned} \underline{a} &= |\underline{a}|\,\hat{a} \\ \hat{a} &= \frac{\underline{a}}{|\underline{a}|} \end{aligned}}$$

def.

The unit vectors along x, y, z



Other basis vectors exist! (don't have to be orthogonal)

Cyclinder coordinates:

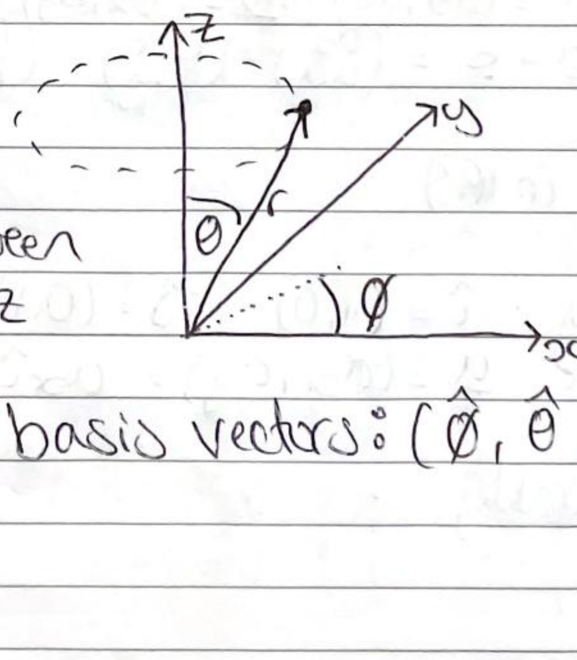
z: height of cyclinder

r: radius of cyclinder

ø: angle between x axis an vector projection

basis vectors: $(\hat{\phi}, \hat{z}, \hat{r})$

Sphereical coordinates:



θ: angle between vector and z axis

basis vectors: $(\hat{\phi}, \hat{\theta}, \hat{r})$