Vectors 9

Maxtrices

If we have a system of liner equations:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots \qquad \vdots \qquad \ddots \qquad \vdots \qquad \vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots a_{nn}x_n = b_n$$

we can write this using matrix notation

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

matrix A $\underline{x}$ vector (matrix) $\underline{b}$ vector (matrix)

$$A\underline{x} = \underline{b}$$

A matrix is an array of numbers with n rows and m coloumns.

If $n=m$, we call our matrix a square matrix.

Matrix Addition

matrices are added component-wise.

$$\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1+2 & 1+3 \\ 2-1 & 1+2 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 1 & 3 \end{pmatrix}$$

Matrices can only be added if both matrices are the same shape.

Scaler multiplication

$$\lambda\begin{pmatrix}1 & 1\\ 2 & 1\end{pmatrix} = \begin{pmatrix}\lambda & \lambda\\ 2\lambda & \lambda\end{pmatrix} \quad \lambda\begin{pmatrix}2 & 0 & 1\\ 2 & 1 & -2\end{pmatrix} = \begin{pmatrix}2\lambda & 0 & \lambda\\ 2\lambda & \lambda & -2\lambda\end{pmatrix}$$

N.B. when scaler multiply determinants we only multiply by one coloumn. Not true for matrices.

Transpose

when we transpose a matrix, we swap the rows and columns.

$$\begin{pmatrix}1 & 1\\ 2 & 1\end{pmatrix}^T = \begin{pmatrix}1 & 2\\ 1 & 1\end{pmatrix} \quad \begin{pmatrix}2 & 0 & 1\\ 2 & 1 & -2\end{pmatrix}^T = \begin{pmatrix}2 & 2\\ 0 & 1\\ 1 & -2\end{pmatrix}$$

for non-square matrices, the matrix transpose will not be the same shape as the original.

Matrix Multiplication

when multiply two matrices $A\ (n_A \times m_A)$ and $B(n_B \times m_B)$. It must be the case that $m_A = n_B$.

eg. $A = \begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix} \quad \underline{x} = \begin{pmatrix}x\\ y\\ z\end{pmatrix} \quad \underline{b} = \begin{pmatrix}b_1\\ b_2\\ b_3\end{pmatrix}$

$$A\underline{x} = \underline{b}$$

$$A\underline{x} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11}x & a_{12}y & a_{13}z \\ a_{21}x & a_{22}y & a_{23}z \\ a_{31}x & a_{32}y & a_{33}z \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

N.B. two matrices are equal only if each element is equal.

eg. $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 1 & 8 \end{pmatrix}$ $\neq$ non-commutative!!

$\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 4 \\ 3 & 1 \end{pmatrix}$

$\underline{\underline{AB \neq BA}}$

eg. $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 1 & -1 \\ 6 & 1 & 0 \end{pmatrix}$

<u>Properties of matrices</u>

I) matrix multiplication is not-commutative $AB \neq BA$

II) matrix multiplication is distributive

$A(B+C) = AB + AC \qquad (A+B)C = AC + BC$

III) matrix multiplication is associative $A(BC) = (AB)C$

IV) there exists a zero matrix, all elements $= 0$

V) there exists a square unit matrix such that

$AI = A$

$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad I_3\underline{x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$