Reality Condition

$$f(x) = \sum_{-\infty}^{\infty} c_n e^{inx} \qquad f(x) \in \mathbb{C}$$

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx \qquad c_n \in \mathbb{C}$$

What about when $f(x) \in \mathbb{R}$?

Generally, $f(x) = f^{R}(x) + i f^{I}(x)$ but if $f(x) \in \mathbb{R}$ then $f(x) = f^{*}(x)$. But:

$$c_n = -\frac{1}{2\pi} \int_{-\pi}^{\pi} \underset{\mathbb{R}}{\underset{\uparrow}{f(x)}} \, \underset{\mathbb{C}}{\underset{\uparrow}{e^{-inx}}} \, dx \quad \Rightarrow \therefore c_n \in \mathbb{C}$$

Let's consider the complex conjugate of c_n.

$$c_n^{*} = \frac{1}{2\pi} \int_{-\pi}^{\pi} f^{*}(x) e^{inx} dx = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-i(-n)x} = c_{-n}$$

All of the negative n coefficients are defined by the complex conjugate of positive n.

$\therefore$ for $f(x) \in \mathbb{R}$, we only need to specify positive n. All coefficients less than zero are not independent

This is called the reality condition.

Dirichlet Conditions

When does this equality hold? $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$

* period over interval 2π
* single valued
* finite number of extrema in the interval
* finite number of discontinuities in the interval
* integral of the absolute value is finite.

$$\int_{-\pi}^{\pi} |f(x)|\,dx < \infty.$$

If these conditions are met:

* the series converges to $f(x)$ at all points where $f(x)$ is continuous.
* series converges to the midpoint between the values of $f(x)$ from the left & right at points of discontinuity.

Example 1

$$f(x) = \frac{1}{1+e^{\sin x}}$$

does this satisfy the fith condition?

$$\frac{1}{1+e^{-1}} > \frac{1}{1+e^{\sin x}} > \frac{1}{1+e}$$

$$\frac{e}{e+1} > \frac{1}{1+e^{\sin x}} > \frac{1}{1+e} \implies \left|\frac{1}{1+e^{\sin x}}\right| < 1$$

$$\Rightarrow \int_{-\pi}^{\pi} 1\,dx = 2\pi < \infty. \quad :)$$

<u>Example 2</u> $f(x) = 1/x$

$$\int_{-\pi}^{\pi} \left|\frac{1}{x}\right| dx = 2\int_{0}^{\pi} \frac{1}{x}\,dx = \left[2\ln(x)\right]_0^{\pi} = 2\ln(\pi) - \ln(0)$$
$$= 2\ln(\infty) = \infty.$$

$\therefore$ dirichlet conditions not satisfied

<u>Example 3</u> $f(x) = 1/\sqrt{x}$

$$\int_{-\pi}^{\pi} \left| 1/\sqrt{x} \right| dx = 2\int_{0}^{\pi} 1/\sqrt{x}\,dx = 2\left[2x^{1/2}\right]_0^{\pi} = 4\sqrt{\pi}$$

<u>Excersize 4.1</u> $f(x) = \sin(\ln|x|) \quad -\pi \leq x \leq \pi$

infintie no. of maxima in the interval. Fails!