Binding Energy

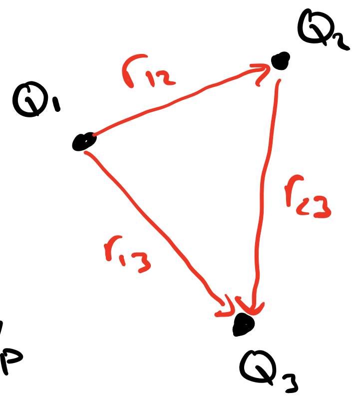
A point charge q forms the potential V at distance r. Now lets move point charge Q from ∞ to r', the change in potential energy of Q is

$$\Delta U = \int_{\infty}^{P} \underline{F}_{ext} \cdot d\underline{l} = -\int_{\infty}^{P} \underline{F}_{Q} \cdot d\underline{l} = -Q \int_{\infty}^{P} \underline{E} \cdot d\underline{l} = QV$$

Setting $U=0$ at $r=\infty$ lets $\Delta U = U$.

$$U = Q\left(\frac{q}{4\pi\varepsilon_0 r'}\right) = QV(r')$$

We call U the 'binding energy', it the amount of energy needed to move Q from $r' \to \infty$.



Multiple Charges

Now lets add a 3^{rd} charge Q',

$$\Delta U = -\int_{\infty}^{P} \underline{F}_{Q} \cdot d\underline{l} = -Q_3 \int_{\infty}^{P} \underline{E} \cdot d\underline{l} = Q_3 V_P$$

where V_P is the potential at point P due to charges Q_1 and Q_2.

$$V_P = \frac{Q_1}{4\pi\varepsilon_0 r_{13}} + \frac{Q_2}{4\pi\varepsilon_0 r_{23}}$$

'old' potential energy due to only Q_1 & Q_2

'new' potential energy due to Q3

$$\therefore \quad U = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r_{12}} + \Delta U$$

$$U = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r_{12}} + \frac{Q_1 Q_3}{4\pi\varepsilon_0 r_{13}} + \frac{Q_2 Q_3}{4\pi\varepsilon_0 r_{23}}$$

without proof, it can be shown that

$$U = \tfrac{1}{2}Q_1V_1 + \tfrac{1}{2}Q_2V_2 + \tfrac{1}{2}Q_3V_3$$

$$U = \sum_{i=1}^{n} \tfrac{1}{2}Q_iV_i$$

Equipotential Surface

$\underline{E}$ is perpendicular to an equipotential surface.

$$dV_{AB} = -\underline{E}\cdot d\underline{l} = 0$$

E & V

consider two points P_1 & P_2 at x and $x+\Delta x$. In one dimension we find that

$$V(P_2) = V(x+\Delta x) = Vx + \Delta x\frac{\partial V}{\partial x} + \dots \simeq V(P_1) + \Delta x\frac{\partial V}{\partial x}$$

In 2D, $\Delta\underline{l} = \Delta x\hat{i} + \Delta y\hat{j}$ so

$$V(P_2) = V(P_1) + \Delta x\frac{\partial V}{\partial x} + \Delta y\frac{\partial V}{\partial y}$$

in 3D:

$$V(P_2) = V(P_1) + \Delta x\frac{\partial V}{\partial x} + \Delta y\frac{\partial V}{\partial y} + \Delta z\frac{\partial V}{\partial z}$$

$$\nabla V = -\underline{E} \cdot d\underline{l} = -E_x \Delta x - E_y \Delta y - E_z \Delta z$$

$$\therefore \quad E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

$$\underline{E} = -\nabla V \qquad \left(\nabla \times (\nabla A) = 0 \right)$$