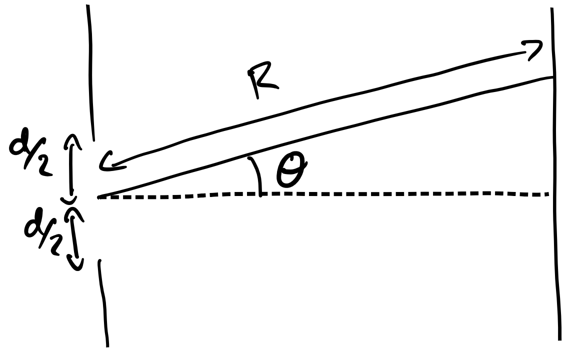
Fraunhofer Diffraction

Light passing through a small aperture forms a diffraction pattern.



The plane wave arriving at x over the infinitesimal area dx will contribute $d\Psi(\theta)$ to the amplitude radiation field.

$$d\Psi(\theta) = a(x)\, e^{i\left[\frac{2\pi}{\lambda}(R + x\sin\theta)\right]} dx$$

relative amplitude — path length — path difference

This works in the approximation $\lambda \ll d$ & $R \gg d$. We can now drop the common phase, as we only need the pattern on the screen.

$$d\Psi(\theta) \sim a(x)\, e^{i\left[\frac{2\pi}{\lambda} x\sin\theta\right]} dx$$

Assuming small angles ($\ell \sim \sin\theta$)

$$d\Psi(\ell) \sim a(x)\, e^{i\left[\frac{2\pi}{\lambda} \ell x\right]} dx$$

We can now integrate to get the total contribution

$$\Psi(l) = A\int_{-d/2}^{d/2} a(x)\, e^{i\frac{2\pi}{\lambda}lx}\, dx$$

aperture function

where A is the overall normalisation.

let $\underline{k} = \frac{2\pi l}{\lambda}$ we can see that

$$\Psi(l) = A\int_{-d/2}^{d/2} a(x)\, e^{ikx}\, dx$$

Which is just a fourier transform of our aperture function $a(x)$.

- ☆ A very narrow slit can be represented by the delta function so the diffraction pattern is smoothly spread in all directions.
- ☆ A very wide slit will produce a delta function as the diffraction pattern.
- ☆ To get the diffraction pattern for two slits we just fourier transform a single slit and add it to the same transform shifted by an amount.

This is how X-ray crystallography works. (Bragg).