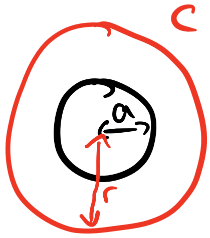
Faraday's Law & The Electric Field

Let's consider the solenoid of N turns per metre, radius a and current I. The field inside is $B = \mu_0 I N$ & outside $B = 0$.



Consider the contour C with radius r, the flux through C will be $\Phi = \pi a^2 \cdot B = \pi a^2 \mu_0 I N$ and $\dot{\Phi} = \pi a^2 \mu_0 N \dot{I}$.

$$\underbrace{\oint \underline{E} \cdot d\underline{l}}_{2\pi r E} = -\frac{d\Phi}{dt} \quad \Rightarrow 2rE = a^2 \mu_0 N \dot{I}$$

$$E = \frac{\mu_0 N}{2} \cdot \frac{a}{r^2} \cdot \frac{dI}{dt}$$

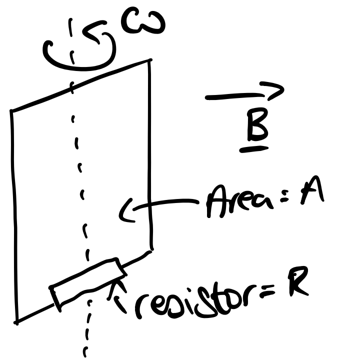
This is a circulating electric field. The field lines do not have a start or end point.

If we place a wire on C we would get a transformer.

Generators

A generator converts mechanical energy to electrical power.

Rotating a loop through a magnetic field $\underline{B}$ will change the amount of flux flowing through the loop.



$$\Phi = \iint \underline{B} \cdot d\underline{S} = BA\cos\theta = BA\cos(\omega t)$$

for N loops $\Phi_T = N\Phi = BAN\cos(\omega t)$.

$$\mathcal{E} = -\frac{d\Phi}{dt} = \omega BAN\sin(\omega t)$$

$$I = \frac{\omega BAN}{R}\sin(\omega t) \qquad P = \frac{(\omega BAN)^2}{R}\sin^2(\omega t)$$

We do not get this power for free! We notice that the current loop will form a magnetic moment $\underline{\mu}$ where $\mu = NIA = \frac{\omega BA^2N^2}{R}\sin(\omega t)$. The torque on the loop will be $\Gamma = \underline{\mu} \times \underline{B} = \mu B\sin\theta$.

The power P will be $P = \Gamma\omega = \frac{(\omega BAN)^2}{R}\sin(\omega t)$. $\therefore$ conservation of energy holds! Woohoo!