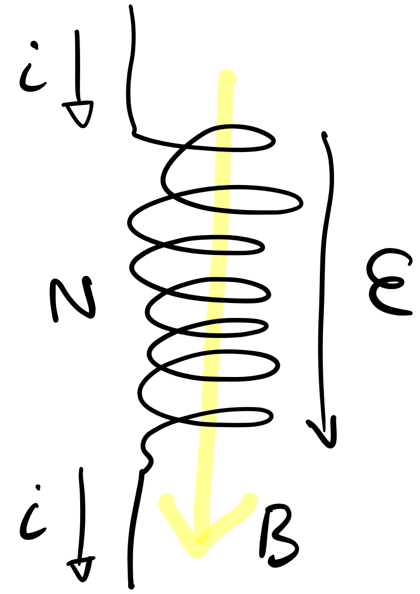
Inductance

A cyclindrical coil of N turns with current i.



It stores energy in a magnetic fields when electric current flows through it.

When the current flowing through the coil changes, the time-varying magnetic field induces an electromotive force (*e.m.f.*) (voltage) in the conductor, described by Faraday's law of induction. According to Lenz's law, the induced voltage has a polarity (direction) which opposes the change in current that created it. As a result, inductors oppose any changes in current through them.

induced EMF

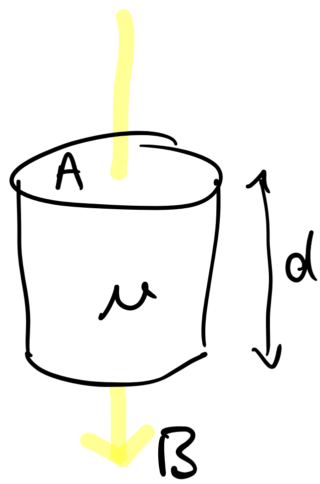
$$\mathcal{E} = -N\frac{d\phi}{dt}$$

magnetic flux

Lenz's law

number of coils

The magnetic flux can be found by:



relative permeability

permeability of free space

number of coils

magnetic flux

current

$$\phi = BA = \frac{\mu_0 \mu_r N A i}{d}$$

magnetic field strength

cross sectional area.

It is often easier to use the inductance (L) instead of the magnetic field.

current

$$\mathcal{E} = -L\frac{di}{dt}$$

induced EMF

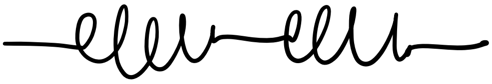
inductance

inductance can be found by:

$$L = \frac{\mu_0 \mu_r N^2 A}{d}$$

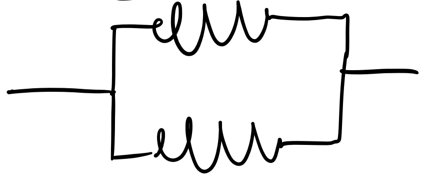
Equivalent Inductance

Series



$$L_{eq} = L_1 + L_2 + \ldots$$

Parallel



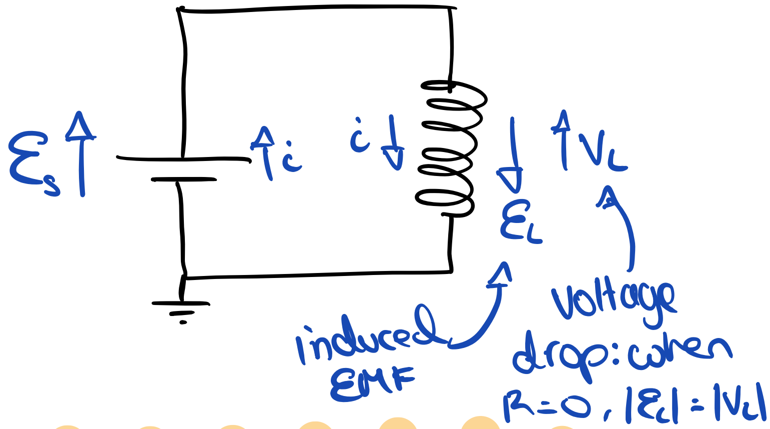
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots$$

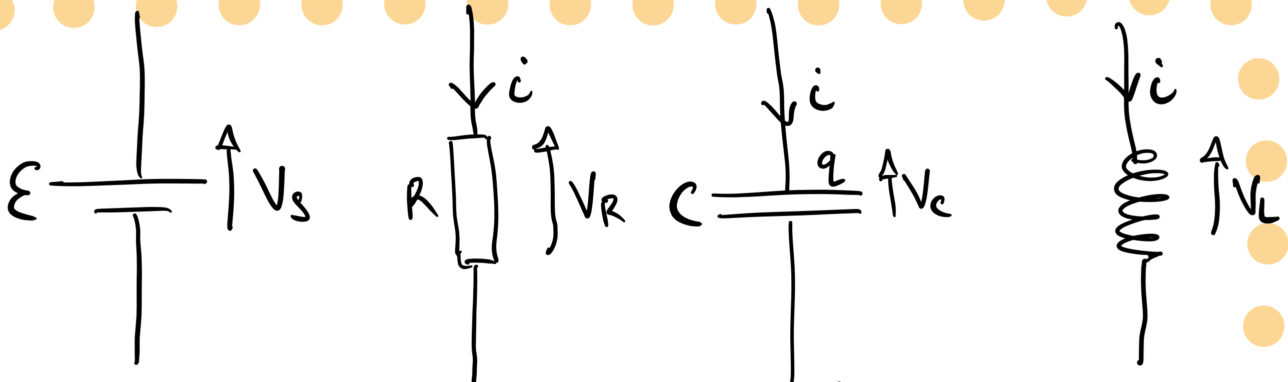
Current & Voltage

from KVL: $\mathcal{E}_s + \mathcal{E}_L = 0$

$$V_L + \mathcal{E}_L = 0$$

$$V_L = L\frac{di}{dt}$$





$V_s = \mathcal{E}$ $\quad$ $V_R = iR$ $\quad$ $i = C\frac{dV_c}{dt}$ $\quad$ $V_L = L\frac{di}{dt}$

Energy stored

inductance

current

$$\boxed{U = \frac{LI^2}{2}}$$

energy stored

$$\frac{du}{dt} = V_L i = Li\frac{di}{dt}$$

$$du = Li\,di$$

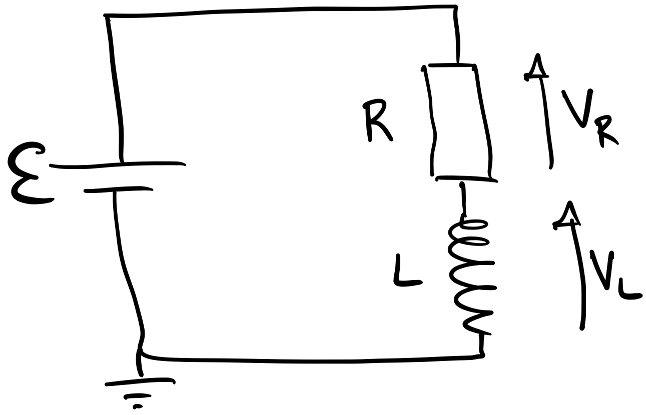
$$\int_0^U du = \int_0^I Li\,di$$

$$U = \frac{Li^2}{2}$$

RL circuit

Energising:

Source is switched on at time $t=0$.



$$\mathcal{E} = V_R + V_L$$

$$\mathcal{E} = Ri + L\frac{di}{dt}$$

time const.

$$i = \frac{\mathcal{E}}{R}\left[1 - e^{-\frac{tR}{L}}\right] \qquad \tau = \frac{L}{R}$$

$$V_R = iR = \mathcal{E}\left[1 - e^{-tR/L}\right] \qquad V_L = L\frac{di}{dt} = \mathcal{E}e^{-tR/L}$$

De-energising:

$$0 = Ri + L\frac{di}{dt} \Rightarrow i = \frac{\mathcal{E}}{R}e^{-\frac{tR}{L}}$$

$$V_R = iR = \mathcal{E}e^{-\frac{tR}{L}} \qquad V_L = L\frac{di}{dt} = -\mathcal{E}e^{-tR/L}$$

Power

Energising

- $p_R = V_R i = i^2R$
  positive power
  dissipating energy
- $p_L = V_L i$
  positive power
  storing energy
- $p_S = -V_S i = \mathcal{E}i$
  negative power
  releasing energy

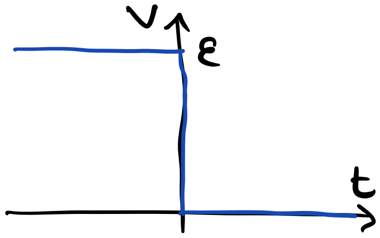
De-Energising

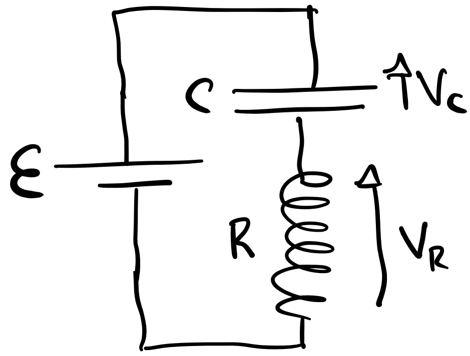
- $p_R = V_R i = i^2R$
  positive power
- $p_L = V_L i$
  negative power
  releasing stored energy
- $p_S = -V_S i = 0$
  no source power

$$P_S + P_L + P_R = 0$$

LC circuit

for $t > 0$,





$$0 = Cq + L\frac{d^2q}{dt^2}$$

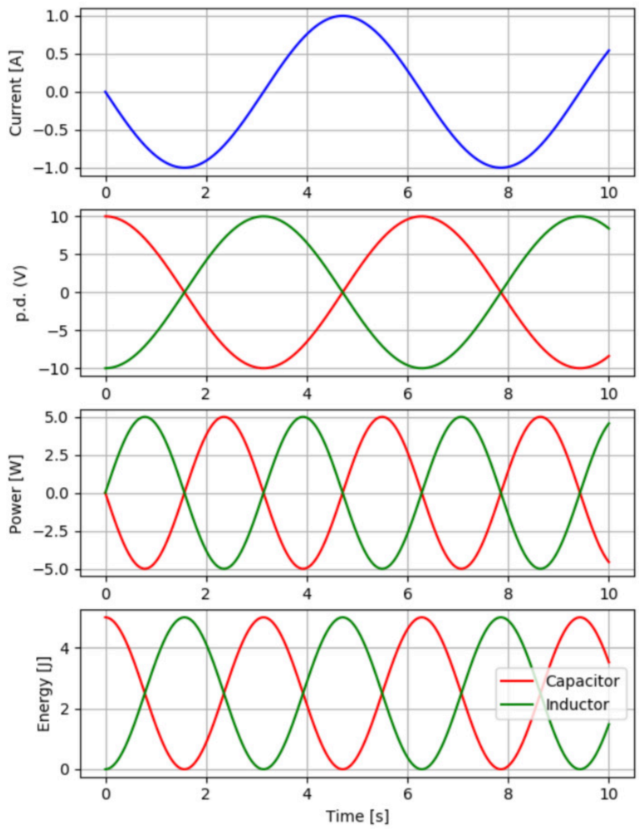
$$q = -\frac{1}{LC}\ddot{q}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\mathcal{E} = L\frac{di}{dt} = L\frac{d^2q}{dt}$$

$$q_0 = C\mathcal{E}$$

$$q = q_0\cos(\omega_0 t)$$



☆ the p.d. for the capacitor is in antiphase with the p.d. for the capacitor.