The Magnetic Field

We define a magnetic field over all space (with unit Teslas (T)).

hospital MRI	1 T ~ a lot
earth's magnetic field	~30,000 nT
Field 1m away from 1A	~200 nT

We define magnetic flux $\underline{\Phi}$ through a surface S

$$\underline{\Phi} = \iint_S \underline{B} \cdot d\underline{S}$$

The sum of a magnetic field through a closed surface is always zero.

$$\oiint \underline{B} \cdot d\underline{S} = 0$$

ie. every field line that enters a closed volume also leaves that closed volume.

Magnetic field lines do not have a beginning or end - every one is a closed loop - this is very different to an electric field.

Now lets use the divergence theorem and apply it to

Magnetic fields.

$$\iiint_V \nabla \cdot \underline{F} \cdot dV = \oiint_S \underline{F} \cdot d\underline{S}$$

Using the relation that $\oiint \underline{B} \cdot d\underline{S} = 0$ we get

$$\iiint_V \nabla \cdot \underline{B} \, dV = 0$$

Which leads to

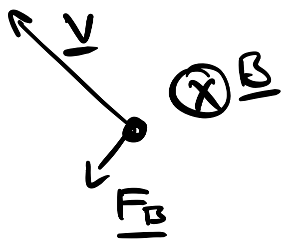
$$\nabla \cdot B = 0$$

This is one of maxwell's equations.

An important idea to evolve from this is that there are no magnetic monopoles. But then how do we generate a magnetic field? We <u>move electric charges</u>!

<u>Forces on Moving Charges</u>

Charges in motion experience a force dictated via Lorentz force.



$$\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$$

As $\underline{F}$ depends upon $\underline{v}$ it is <u>not</u> a conservative force. $\underline{F}$ is perpendicular to $\underline{B}$ so $\underline{B}$ does not represent a line of force (unlike $\underline{F} = k\underline{E}$).

Work done is equal to

$$dW = \underline{F} \cdot d\underline{r}$$

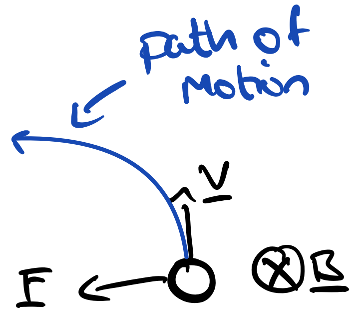
We know that

$$F = \underbrace{\frac{d\underline{r}}{dt}}_{= \underline{v}} \times \underline{B}$$

This will be perpendicular to $d\underline{r}$. $\therefore$

$$dW = F \cdot dr = 0$$

The work done by the magnetic field is zero! This implies there is no change to the kinetic energy of the particle. However the particle can accelerate. It will undergo circular motion.



$$F = q|\underline{v}||\underline{B}| = mr\omega^2 \qquad (v = r\omega)$$

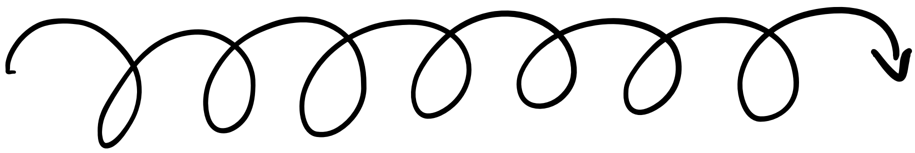
$$\Rightarrow \omega = \frac{qB}{m}$$

This does not depend upon $\underline{v}$ or $\underline{r}$, all particles of the same charge & mass have the same period of gyration. This is the basis for cyclotrons!

The radius of gyration (also called the Larmor radius) is found by

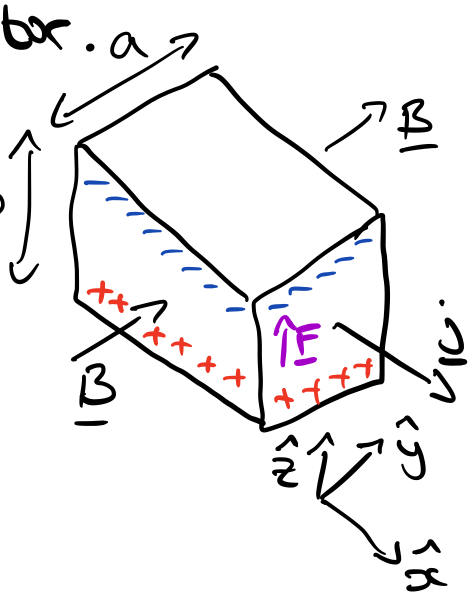
$$r = \frac{mv}{qB}$$

Normally, we would only see motion in the x-y plane, but if there was motion in the z direction, we would get <u>helical</u> motion.



Hall Effect

Charges in conductors subject to a magnetic field undergo the lorentz force. This has the effect of inducing a voltage across the conductor.



current density: $\underline{j} = \frac{I}{A}\hat{x}$ $\left[\frac{A}{m^2}\right]$

cross-sectional area: $A = ab$

number density: n $[m^{-3}]$

$$\underline{j} = n\underline{v}q$$

The lorentz force deflects charges in $\hat{z}$ direction.

This increases the no. of electrons on one side which induces a magnetic field which acts against the deflected electrons.

There is an equalibrium when there is no net force in $\hat{z}$ direction.

$$\underline{F} = q(\underline{E} + \underline{v} \times \underline{B}) = 0 \quad \Rightarrow \quad \underline{E} = -\underline{v} \times \underline{B} = \frac{IB}{Anq}\hat{z}$$

There is a voltage across the conductor due to the applied magnetic field

$$V = E \cdot b$$