Vector Algebra 12/10/21

multipling by a scalar

→ multiplying a vector by a scalar multiplies each component in turn.

eg $\underline{a} = (a_x, a_y)$, $\lambda \in \mathbb{R} \Rightarrow \lambda\underline{a} = (\lambda a_x, \lambda a_y)$

eg $\underline{a} = |\underline{a}|\,\hat{a}$ $\quad \hat{a} = \underline{a}/|\underline{a}|$

→ Adding Vectors

→ vectors are added component wise

eg $\underline{a} = (a_x, a_y)$, $\underline{b} = (b_x, b_y)$

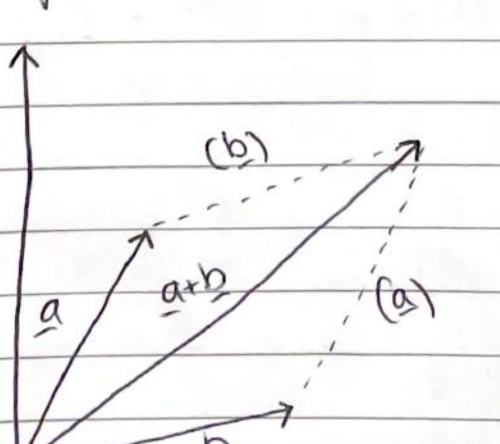
$\underline{a} + \underline{b} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} + \begin{pmatrix} b_x \\ b_y \end{pmatrix} = \begin{pmatrix} a_x + b_x \\ a_y + b_y \end{pmatrix}$

N.B (in $\mathbb{R}^2$)

using $\hat{\imath} = (1,0)$ $\quad \hat{\jmath} = (0,1)$

then $\underline{a} = (a_x, a_y) = a_x\hat{\imath} + a_y\hat{\jmath}$

Graphically,

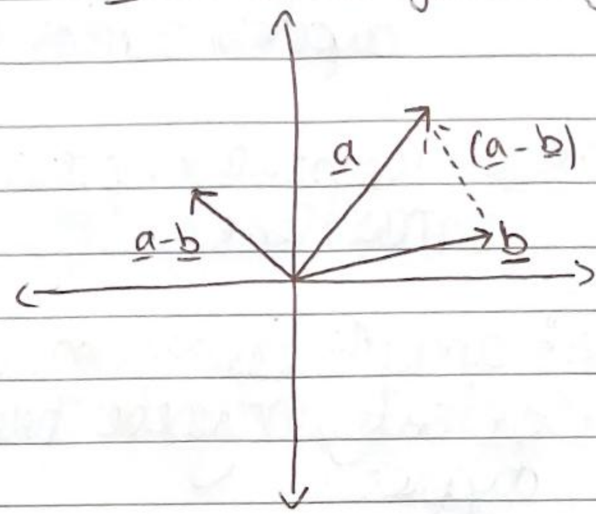


commutes

$\underline{a} + \underline{b} = \underline{b} + \underline{a}$

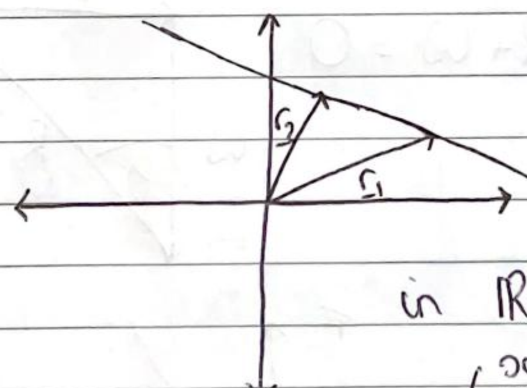
Subtration:

$$\underline{a}-\underline{b} = \underline{a}+(-\underline{b}) = \begin{pmatrix} a_x \\ a_y \end{pmatrix} + \begin{pmatrix} -b_x \\ -b_y \end{pmatrix} = \begin{pmatrix} a_x - b_x \\ a_y - b_y \end{pmatrix}$$



Equation of a line

find eqⁿ of line throug $\underline{r_1} = (x_1, y_1, z_1)$ and $\underline{r_2} = (x_2, y_2, z_2)$



$$\boxed{\underline{r} = \underline{r_1} + \lambda(\underline{r_2} - \underline{r_1})}$$

in $\mathbb{R}^3$

$$\underline{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \lambda \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix}$$

solve for λ:

$$\lambda = \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Equalibria:

common vectors: forces, direction, velocity, acceleration, electric fields, dipoles, current density, area

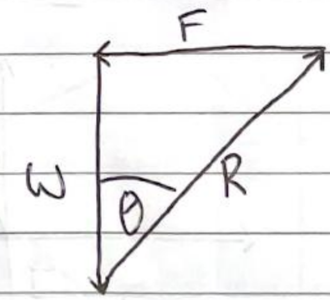
common scalars: temperature, pressure, energy, mass, time

psuedovectors: angular momentum, torque, angular velocity, magnetic fields, magnetic dipoles

Problem:

what force does the finger need to apply to hold the particle constant?

$$F + R + W = 0$$



$$\frac{F}{W} = \tan\theta$$

$$F = W\tan\theta$$

$$F = mg\tan\theta$$