Classical Mechanics 3

The general vector version of newton's second law is:

$$m\ddot{r} = F(r, \dot{r}, t, \ldots)$$

this is only for one particle. It is both a differential equation and a vector equation.

This is equivalent to three 1-dimensional equations

$$m\ddot{x} = F_x(x, y, z, \dot{x}, \dot{y}, \dot{z}, t, \ldots)$$
$$m\ddot{y} = F_y(x, y, z, \dot{x}, \dot{y}, \dot{z}, t \ldots)$$
$$m\ddot{z} = F_z(x, y, z, \dot{x}, \dot{y}, \dot{z}, t \ldots)$$

These can only be solved analytically in very simple examples. Often we have to use computers to solve them. But we all also have to deal with chaos theory.

Fortunately, we like simple examples and we can still learn a lot from them.

Motion in 1D

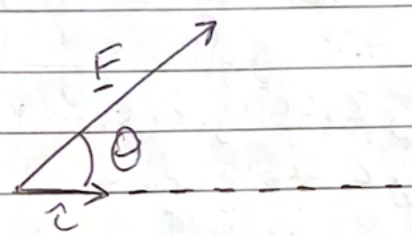
The velocity v can be both positive and negative, but the speed $|v|$ is only positive.

From now on, we rarely assume acceleration to be constant.

Analysis in 1D can be more useful than we first think.

examples we can use 1D analysis for
- → falling under gravity
- → trains on tracks
- → head on collisions
- → circular motion (only variable is θ)

We can resolve 3D motion along any axis.



$$F_x = F\cos\theta = \underline{F} \cdot \hat{i}$$

Only the x direction of the force affects the x acceleration.

Impulse

Suppose a time-dependent force $F(t)$ acts on mass m from time t_i to time t_f.

$$m\frac{dv}{dt} = F(t)$$

$$m\int_{t_i}^{t_f} \frac{dv(t)}{dt} = \int_{t_i}^{t_f} F(t)$$

$$m\left[v(t)\right]_{t_i}^{t_f} = \int_{t_i}^{t_f} F(t)$$

$$\boxed{\text{change in momentum} = \text{impulse}}$$

This equation works even when the force is not constant.

Impulse is useful for collisions. You may not know enough to work out $F(t)$ in detail, but the impulse can be found from the change in momentum, which is easier to measure.

Collisions do not usually conserve kinetic energy, ~~the~~ but the impulse = Δ(momentum) is.

1D Motion with a velocity-dependent force

$F = F(v)$ • eg. air resistance, fluid friction

$$m\frac{dv}{dt} = F(v) \quad \text{(Law 2)}$$

Example - Fluid friction

$F(v) = -\alpha v$ ok approx. when v is small

$$m\frac{dv}{dt} = -\alpha v$$

$$\frac{1}{v}\frac{dv}{dt} = -\frac{\alpha}{m}$$

we've used a mathmatical trick

$$\frac{1}{v}dv = -\frac{\alpha}{m}dt$$

$$\int_{v_0}^{v_1}\frac{1}{v}dv = \int_0^{t_a}-\frac{\alpha}{m}dt'$$

$$\ln v_1 - \ln v_0 = -\frac{\alpha}{m}t$$

$$\ln\frac{v_1}{v_0} = -\frac{\alpha}{m}t$$

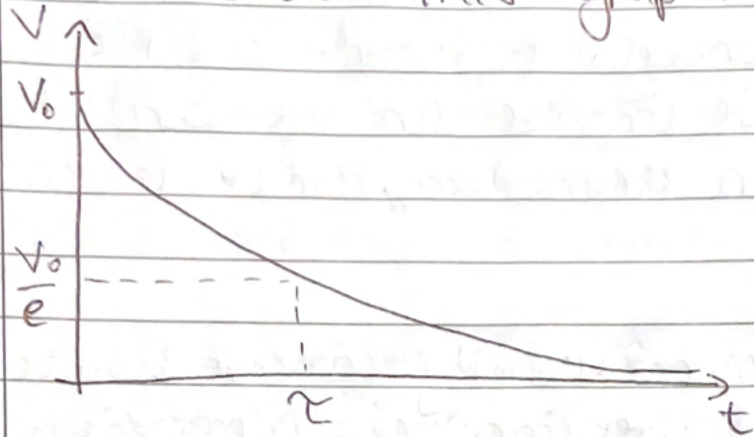
$$\boxed{v_1 = v_0 e^{-\frac{\alpha}{m}t}}$$

characteristic time.

let $\tau = m/\alpha$

$$\boxed{v_1 = v_0 e^{-\frac{t}{\tau}}}$$

lets sketch this graph: $V = V_0 e^{-t/\tau}$



What is the range of the boat?

$$\frac{dx}{dt} = V_0 e^{-t/\tau}$$

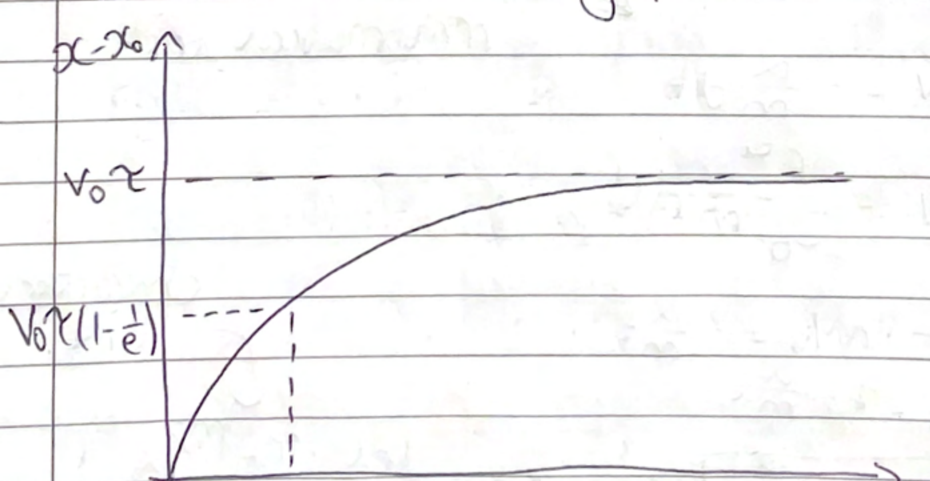
$$dx = V_0 e^{-t/\tau} dt$$

$$\int_{x'=x_0}^{x_*} dx' = \int_{t'=0}^{t'=t} V_0 e^{-t'/\tau} dt'$$

$$x - x_0 = \left[-V_0 \tau e^{-t/\tau} + V_0 \tau\right]_0^t$$

$$x - x_0 = V_0 \tau (1 - e^{-t/\tau})$$

lets draw this graph:



range of boat is $V_0 \tau$ m.