Wave Optics

We can describe light in terms of a wave in an electric field. For example a plane wave

$$E(\underline{r},t) = A_0 e^{i(\underline{k}\cdot\underline{r}-\omega t+\phi)} = \tilde{E}_0 e^{i(\underline{k}\cdot\underline{r}-\omega t)}$$

where $\tilde{E}_0$ is the complex field potential. $\underline{k}$ is the wavevector and points in the direction the wave is travelling in.

$$|\underline{k}| = \frac{2\pi}{\lambda}$$

The intensity is given by $E = \frac{1}{2}c\varepsilon_0|E_0|^2$. For this course we only care about the distribution so we'll take

$$I = |E_0|^2$$

Spherical waves are very important in wave optics, they take the form

$$E(r,t) = \frac{A_0}{r}e^{i(kr-\omega t)} = E(r)e^{i(kr-\omega t)}$$

where $r=\sqrt{x^2+y^2+z^2}$ is the distance from the source. The intensity $I = |E(r)|^2 = \frac{A_0^2}{r^2} = P/4\pi r^2$ follows the inverse square law, given a constant power P and surface area $4\pi r^2$.

Interference is based on the principle of superposition. The result field is given by the sum of the individual fields.

$$E_p = \sum_i E_i \qquad E_i(r,t) = E_{0i} e^{i(kr-\omega t)}$$

This is assuming the light is monochromatic and all have the same wavevector.

The phase change $\Delta\phi = k(r_2 - r_1) = k\Delta r$, or more generally $\Delta\phi = kn\Delta r = k(OPL)$ given the refractive index n. The phase change constructivly interfer when $\Delta\phi = k\Delta r = \frac{2\pi}{\lambda}\Delta r = 2\pi m$. It occurs when $\Delta r = mk$.

Diffraction

The Huygens-Fresnel Principle states that every wavefront points acts as a source of secondary waves. The optical field is the superposition of these waves.

- ✡ A single narrow slit (width ~ wavelength) can be considered a single point source which produces a single spherical wave.
- ✡ Two narrow slits produce two spherical waves $E_p = E_1 + E_2$. constructive interference occurs at $\Delta r = r_2 - r_1 = m\lambda$.

✡ A plane wave can be considered as an infinite number of spherical waves. The sideways waves cancel.

✡ An aperture produces a diffraction pattern $E_P = \int_A E(x)\,dx$.

Double Slits

Two slits (width ~ wavelength) separated by distance d are illuminated by light (ang. freq. ω).

$$E_P = E_1 + E_2 = \frac{A}{r_1} e^{i(kr_1 - \omega t)} + \frac{A}{r_2} e^{i(kr_2 - \omega t)}$$

$$E_P = \left(\frac{A}{r_1} e^{ikr_1} + \frac{A}{r_2} e^{ikr_2} \right) e^{-i\omega t}$$

Near the aperture, the spherical wave amplitude varies greatly and the phase difference $k(r_2 - r_1) = k\Delta r$ can vary in a complex article.

Further away we take $r_1 \approx r_2$ we can take $\frac{A}{r_1} \approx \frac{A}{r_2} = E_0$,

$$E_P = E_0 e^{ikr_1} (1 + e^{ik\Delta r})$$

At far enough distances (covered later) we get to the point where $\Delta r = d \sin\theta$.

$$E_P = E_0 e^{ikr_1} (1 + e^{ikd\sin\theta})$$

Given that $\cos x = \frac{1}{2}(e^{ix} + e^{-ix})$, we get

$$1 + e^{ikd\sin\theta} = 2e^{\frac{ikd\sin\theta}{2}} \cos\left(\frac{kd\sin\theta}{2}\right)$$

which gives the diffraction pattern

$$E_p(\theta) = 2E_0 e^{ikr_0} \cos\left(\frac{kd\sin\theta}{2}\right)$$

And the intensity $I_p = |E_p|^2$ is given by

$$I_p(\theta) = 4I_0 \cos^2\left(\frac{kd\sin\theta}{2}\right)$$

Maxima occur when $\frac{kd\sin\theta}{2} = m\pi \Rightarrow d\sin\theta = m\lambda$. Minima occur when $d\sin\theta = (m + \frac{1}{2})\lambda$.

Near & Far Field Diffraction

For a more complicated aperture we use

$$E_p = \int_{-a/2}^{a/2} \frac{e^{ikr}}{r} dx \quad \text{(ignoring } \overset{\text{monochromatic}}{e^{-i\omega t}}\text{)}$$

The rayleigh distance is given by $z_R = a^2/2\lambda$ where a is the aperture width. (Derivation not shown - see lectures).

Beyond the rayleigh distance we get far-field diffraction also called Fraunhofer Diffraction. Within the rayleigh distance we get near-field diffraction, also called Fresnel diffraction.

At long distances $L \gg z_R$, the angular pattern doesn't change form as the path difference $\Delta r = r_2 - r_1 = a\sin\theta$ is independent of distance L.

Fraunhofer Diffraction

Given the single extended slit diffraction integral

$$E_P = \int_{-a/2}^{a/2} \frac{e^{ikr_x}}{r_x}\,dx$$

We can define a ray (r_x) that's parallel to a ray from the centre of the slit (r_0) by distance x by

$$r_x = r_0 - a\sin\theta$$

which gives the new far-field diffraction integral as

$$E_P(\theta) = \underbrace{\frac{e^{ikr_0}}{r_0}}_{\text{spherical wave}} \int_{-a/2}^{a/2} e^{-ikx\sin\theta}\,dx$$

We can extend this analysis to include any general aperture function $A(x)$.

$$E_P(\theta) = C(L)\int_{-\infty}^{\infty} A(x)e^{-ikx\sin\theta}\,dx \qquad C(L) = \frac{e^{ikL}}{L}$$

(L- distance)

For paraxial angles ($\sin\theta \approx \tan\theta \approx \frac{X}{L}$) we can simplify our integral to

$$E(X) = C(L) \int_{-\infty}^{\infty} A(X) e^{-\frac{ikXx}{L}} dx$$

Fourier Transform

We often drop the prefactor term and only look at the distribution

$$E(\theta) = \int_{-\infty}^{\infty} A(x) e^{-ikx\sin\theta} dx = \int_{-\infty}^{\infty} A(x) e^{-ik_x x} dx$$

Where $k_x = k\sin\theta$ is the x-component of the wavevector.

This leads to a key point: the far-field diffraction pattern is just the fourier transform of the aperture function.

$$E(k_x) = \int_{-\infty}^{\infty} A(x) e^{-ik_x x} dx = \mathcal{F}[A(x)]$$