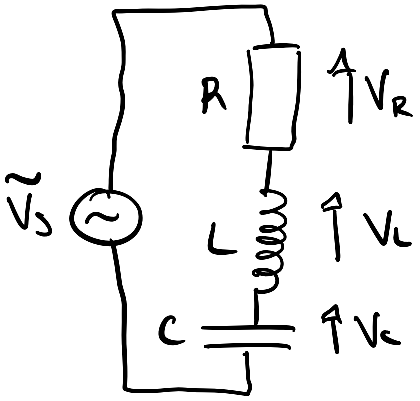
Series LCR Circuit

$V_S = V_0 \cos(\omega t)$

$\tilde{V}_S = V_0 \qquad \left(\mathrm{Re}\{\tilde{V}_S e^{j\omega t}\} = V_0 \cos(\omega t) \right)$



$$\tilde{Z} = \tilde{Z}_R + \tilde{Z}_L + \tilde{Z}_C = R + j\omega L - \frac{j}{\omega C}$$

$$\tilde{I} = \frac{\tilde{V}}{\tilde{Z}} = I_0 e^{j\phi} \qquad I_0 = \frac{V_0}{\sqrt{\tilde{Z}\tilde{Z}^*}} = \frac{V_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$\tan\phi = \frac{-(\omega L - 1/\omega C)}{R}$$

(steady state solution, no transient effects)

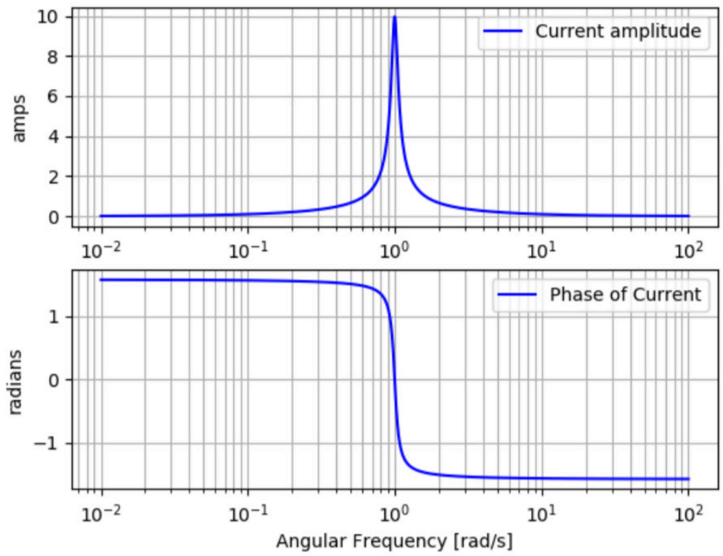
$$\tilde{V}_C = \tilde{I}\tilde{Z}_C \qquad \tilde{V}_R = \tilde{I}R$$

Resonance:

occurs when $\omega = \omega_0$.

$\tilde{Z}_C = -\tilde{Z}_L \Rightarrow \tilde{Z} = L$

$I = V_0/R \qquad \phi = 0$



max power! At resonance,

$(\omega L - 1/\omega C) = 0. \Rightarrow \omega_0 = 1/\sqrt{LC}$

$$P_R(t) = V_R i = i^2 R = I_0^2 \cos^2(\omega t + \phi) R$$

$$P_{avg} = \langle P_R(t) \rangle = \frac{I_0^2 R}{2} \qquad \left(\langle \cos^2(x) \rangle = 1/2 \right)$$

Bandwidth

Both LC & RL have a single cut off frequency ω_c. However an LCR circuit has two cut of frequencies ω_ℓ & ω_h. The definition is the same: when the power falls to half of the maximum.

We define the bandwidth $\Delta\omega$ as $\Delta\omega = \omega_h - \omega_\ell$.

$$\Delta\omega = \omega_h - \omega_\ell = \frac{R}{L} = \gamma$$

Q Factor

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{\gamma}$$

A circuit with a high Q Factor has a narrow peak. It is related to the rate of loss of energy.