Question 1

Let l_1 be the line that passes through $p_1 = (2,9,8)$ and $p_2 = (1,9,9)$ and let l_2 be the line that passes through $p_3 = (1,1,1)$ and $p_4 = (2,5,4)$

a) Find out if the two lines intersect and if so, find the intersection point of l_1 and l_2

b) Let S be the sphere whose center is the intersection of l_1 and l_2 and whose radius is $r = 4$. Write the implicit representation of the sphere.

c) Find the implicit representation of the two planes P_1 and P_2 that are tangent to the sphere S at the points of intersections of l_1 towards p_2 and l_2 towards p_4, respectively.

Question 2

a) Let $l = (2,0,3) + t(1,5,2)$ be a line. Find the projection of $p = (3,-1,4)$ on l.

b) Find an implicit and a parametric representation for the plane the contains both P and l.

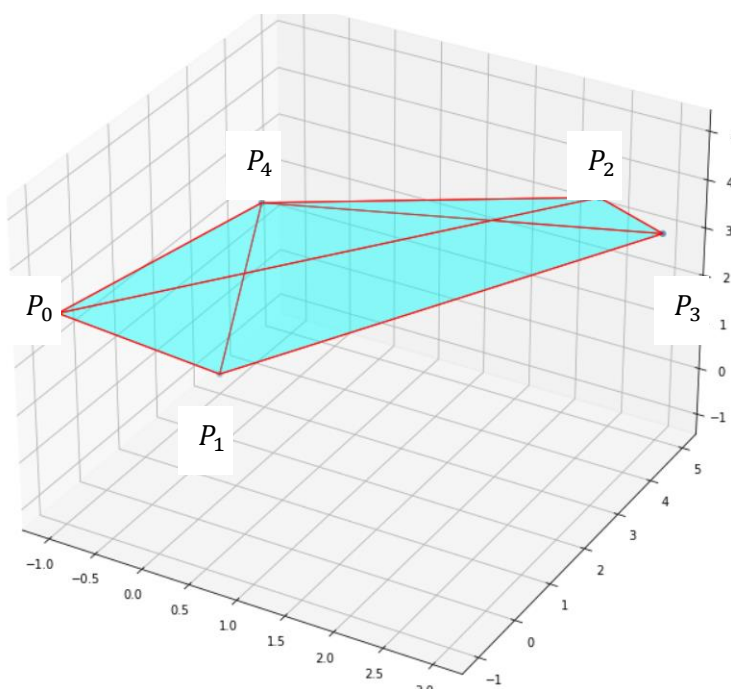**Question 3**

Given the following points:

$$P_0 = (-1,-1,3), P_1 = (-1,3,-1), P_2 = (3,3,5), P_3 = (3,5,3), P_4 = (-\frac{1}{2}, 3,3)$$

a) Show that P_0, P_1, P_2, P_3 lie on the same plane, H, and find the implicit equation of H.

A pyramid is defined by the plane H and the following triangular faces:

$$(P_0, P_2, P_4), (P_0, P_1, P_4), (P_1, P_3, P_4), (P_2, P_3, P_4)$$

This is a sketch of the shape (this is only an illustration!):

b) Determine the outwards facing unit normal vector of each triangular face.

c) Calculate the implicit representation of the planes containing each face of the shape.

d) For each of the following points determine if it is inside or outside the shape (hint: the point is inside the shape if it lies on the same side for all the planes)

  i) $\left(-\frac{1}{2}, 1, 2\right)$

  ii) $(1,0,1)$

  iii) $(3,2,4)$

Question 4

A 2D light ray is sent from point $P = (1, -1)$. It is reflected off a surface (represented by a line) at $R = (6,11)$, and reaches a receiver point at $Q = \left(25, 13\frac{2}{17}\right)$. Note that light rays hitting a surface reflect in a direction which is symmetric according to the normal.

a) Find the implicit representation of the surface s.t. its "up" is towards P (i.e. it faces the incoming ray). See the illustration below. The small black arrow is the direction the surface is facing.

b) Find the angle between the ray and the surface.

Tip: During the calculation you may encounter fractions. Do not use approximated decimal but leave them in the form of $\frac{a}{b}$.

Illustration: