This is a theoretical exercise and musy be submitted **<u>individually</u>**. Submissions in pairs will **<u>NOT</u>** be accepted.

<u>Question 1</u>

In this question we will prove that rotations around any point in two dimensions with the operation of composition are a group. First, we will show it for rotations around the origin.

Note: A group is an algebraic structure composed of a set and an operation s.t. the four conditions that we are going to prove in a-d hold.

a) Closure: Let R_1, R_2 be two rotations around the origin, prove that $R_1 \circ R_2$ is also a rotation around the origin.

b) Associativity: Let R_1, R_2, R_3 be three rotations around the origin, prove that $(R_1 \circ R_2) \circ R_3 = R_1 \circ (R_2 \circ R_3)$

c) Identity element: Prove that there exists rotation around the origin R_{id} s.t. for all rotations around the origin R it holds that $R \circ R_{id} = R_{id} \circ R = R$

d) Inverse element: Prove that for every rotation around the origin R there exists R' which is also a rotation around the origin s.t. $R \cdot R' = R' \cdot R = R_{id}$. Given R, what is R'?

e) Now, show that all of the above are true for rotations around any single point (For example all the rotations around (1,1) are a group).

<u>Question 2</u>

Write the matrices representing the following transformations. For each transformation right its specific type (linear/rigid/similarity/affine/projective)

a) Rotate $30°$ counterclockwise around the line $l(t) = (1, -2, 1) + t\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$ and uniformly scale by 1.5.

b) Scale by 5 in the direction of $(3, 1, -4)$ then shear by a factor of 0.2 in the Y direction (meaning after the shearing $x' = x + 0.2y$m $z' = z + 0.2y$

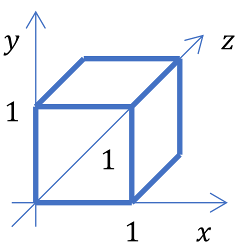
c) Reflect around the yz plane then translate in the $(1,5,2)$ direction by a factor of 0.5

d) Scale uniformly by a factor of 0.3 then project (cavalier projection) on the XY plain with an angle of 45°

<u>Question 3</u>

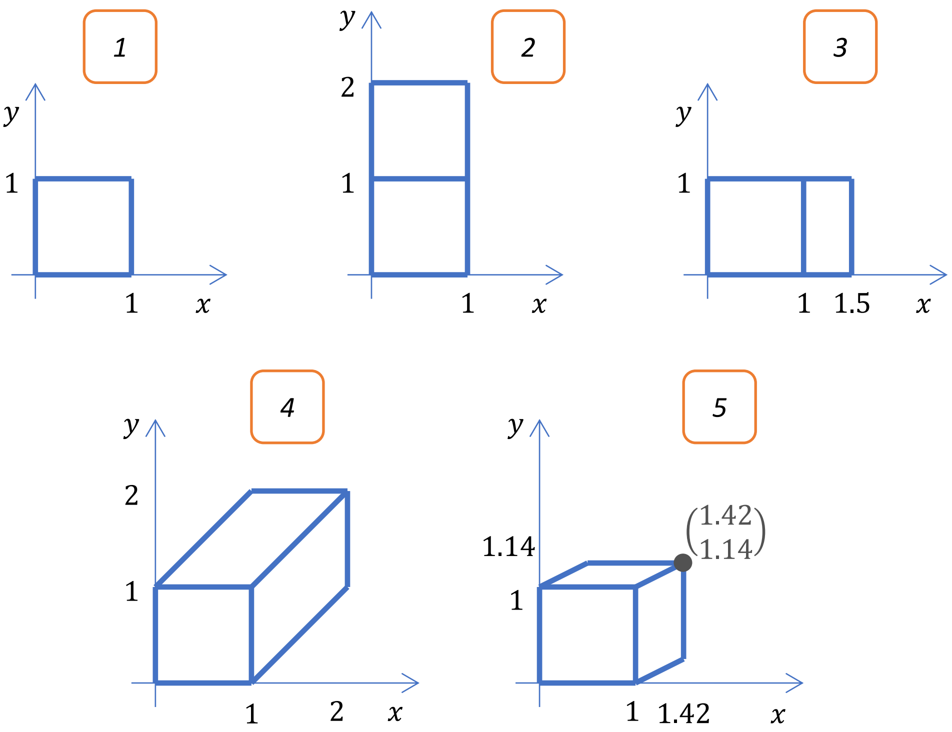
Given a cube with vertices at:

$(0,0,0), (0,0,1), (1,0,0), (0,1,0), (1,1,0), (1,0,1), (0,1,1), (1,1,1)$



For each of the following diagrams, find the parallel projection matrix
that will project the cube upon the given shape, in the xy plane (in which

$z = 0$).



Question 4

a) Project (perspective) a scene with C.O.P=$(0,0,0)$ and a viewing plane $z = -3$. The line $l = (14,2,-7) + t(1,3,0)$. What is the parametric representation of the line after the projection, l_p?

b) We project (perspective) a scene with C.O.P = (0,0,0) and a viewing plane z=-2. We know that two intersecting lines in our scenes, l_1, l_2, have two different vanishing points, $(1,1,-2)$, $(2,4,-2)$ accordingly. What is the angle θ between l_1 and l_2?

c) Find the matrix representing the perspective projection where the COP=$(1,-3,2)$ and projection plane with implicit representation $-2x - y + 3z = 0$.