Machine Learning from Data -IDC
HW5–Theory+ SVM

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Question 1)

a)

Let K, L be two kernels (operating on the same space)and let α, β be two positive scalars

Prove that $\alpha K + \beta L$ is a kernel.

b)

Provide (two different) examples of non-zero kernels K, L (operating on the same space), so that:

i. $K - L$ is a kernel

ii. $K - L$ is not a kernel

Question 2)

Use Lagrange Multipliers to find the maximum and minimum values of the function subject to the given constraints:

Function: $f(x, y, z,) = x^2 + y^2 + z^2$.

Constraint: $g(x, y, z) = \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} + \frac{z^2}{\beta^2} = 1$, where $\alpha > \beta > 0$

$$\bigtriangledown f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} = 2 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\bigtriangledown g = \begin{bmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \\ \frac{\partial g}{\partial z} \end{bmatrix} = 2 \begin{bmatrix} \frac{x}{\alpha^2} \\ \frac{y}{\beta^2} \\ \frac{z}{\beta^2} \end{bmatrix}$$

$$\bigtriangledown f = \lambda \bigtriangledown g$$

$$\iff \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \lambda \begin{bmatrix} \frac{x}{\alpha^2} \\ \frac{y}{\beta^2} \\ \frac{z}{\beta^2} \end{bmatrix} = 0$$

$$\iff \begin{bmatrix} x(1 - \frac{\lambda}{\alpha^2}) \\ y(1 - \frac{\lambda}{\beta^2}) \\ z(1 - \frac{\lambda}{\beta^2}) \end{bmatrix} = 0$$

Notice that if $x \neq 0 \Rightarrow \lambda = \alpha^2$ and if $y \neq 0 \bigvee z \neq 0 \Rightarrow \lambda = \beta^2$. Therefore: $\alpha = \beta$ contradicting our constraint.

Consider $x \neq 0$:

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} + \frac{z^2}{\beta^2} = \frac{x^2}{\alpha^2} = 1 \iff x = \pm\alpha$$

Consider $x = 0$:

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} + \frac{z^2}{\beta^2} = \frac{y^2}{\beta^2} + \frac{z^2}{\beta^2} = 1$$

$$y^2 + z^2 = \beta^2$$

Notice: $\alpha > \beta > 0 \Rightarrow \alpha^2 > \beta^2$

Therefore we can define the maxima $f(\pm\alpha, 0, 0) = \alpha^2$

And minima $f(0, x, z) = y^2 + z^2 = \beta^2$

Question 3)

Let $X = \mathbb{R}^2$. Let $C = H = \{h(a, b, c) = \{(x, y, z) \mid |x| \leq a, |y| \leq b, |z| \leq c\} \mid a, b, c \in \mathbb{R}^+\}$ the set of all origin centered boxes.

Describe the polynomial sample complexity algorithm L that learns C using H. State the time complexity and sample complexity of your suggested algorithm. Prove all your steps.