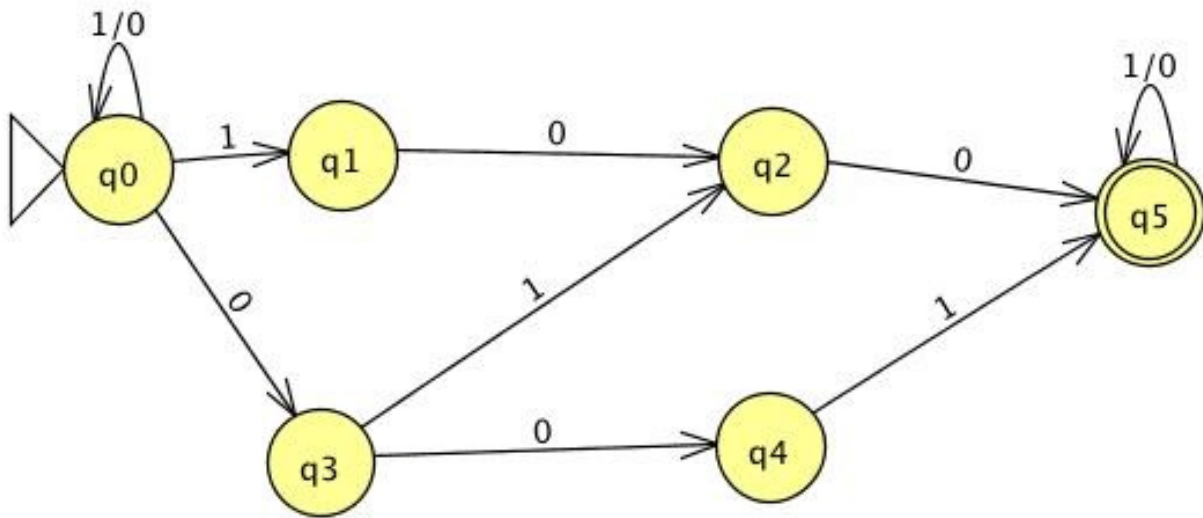


Q1:

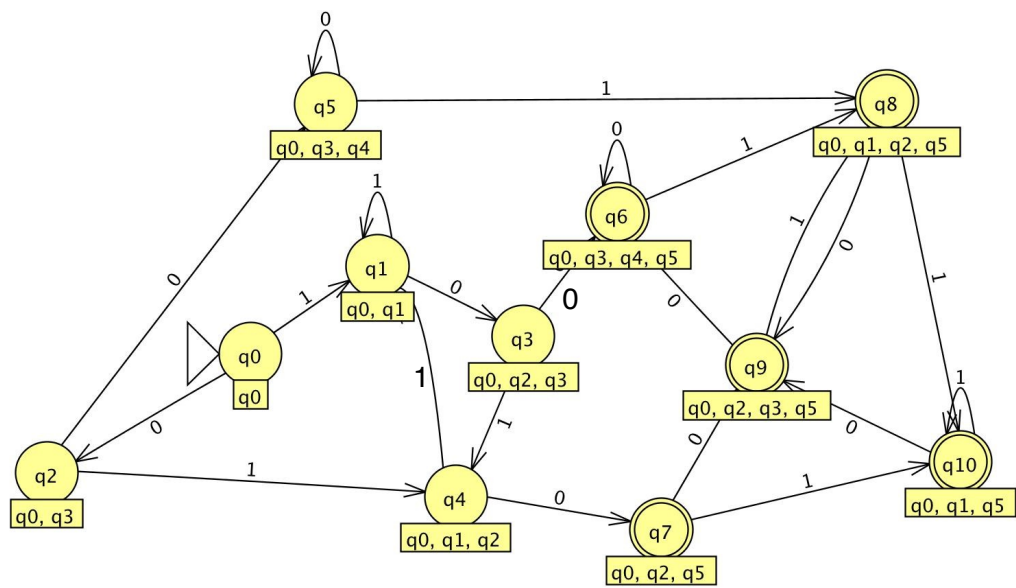
The NFA for the RE is:



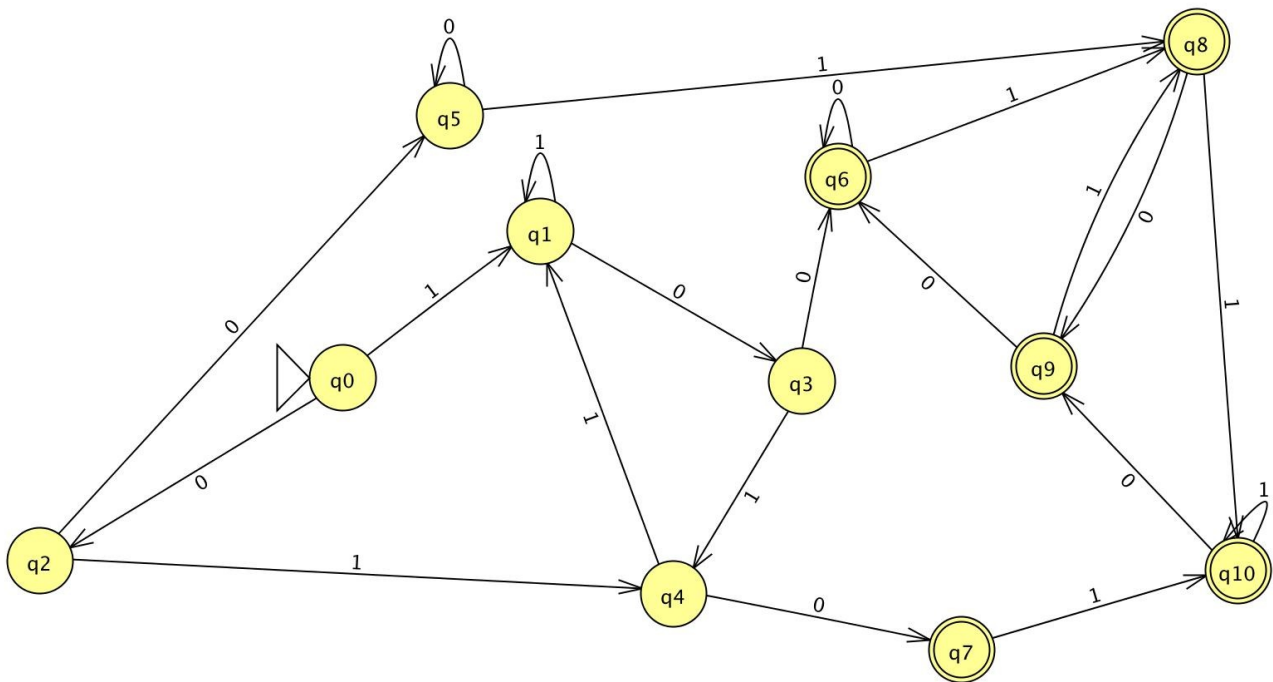
The  $\mathcal{E}$ -closure( $q_0$ )= $\{q_0\}$ , therefore, we have the following table:

State	1	0
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0, q_3\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3, q_4\}$
$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3, q_4, q_5\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_5\}$
$\{q_0, q_3, q_4\}$	$\{q_0, q_1, q_2, q_5\}$	$\{q_0, q_3, q_4\}$
$\{q_0, q_3, q_4, q_5\}$	$\{q_0, q_1, q_2, q_5\}$	$\{q_0, q_3, q_4, q_5\}$
$\{q_0, q_2, q_5\}$	$\{q_0, q_1, q_5\}$	$\{q_0, q_2, q_3, q_5\}$
$\{q_0, q_1, q_2, q_5\}$	$\{q_0, q_1, q_5\}$	$\{q_0, q_2, q_3, q_5\}$
$\{q_0, q_2, q_3, q_5\}$	$\{q_0, q_1, q_2, q_5\}$	$\{q_0, q_3, q_4, q_5\}$
$\{q_0, q_1, q_5\}$	$\{q_0, q_1, q_5\}$	$\{q_0, q_2, q_3, q_5\}$

Therefore, the DFA for this RE is:



DFA without description:



Q2:

We prove by contradiction, assume that  $L'$  is a regular language, then since RLs are closed under complement, so the complement of  $L'$  is also a RL, but the complement of  $L'$  is  $L = \{0^n 1^n \mid n \geq 0\}$ , which is known to be a non regular language, this brings a contradiction.

Therefore,  $L'$  is not a regular language. Q.E.D.

Q3:

The DFA specifies a regular language defined as follow: a language that has odd number of a and even number of b, or has even number of a and odd number of b.

A simplified version is: a language constructed by  $\{a, b\}^*$ , of which each string has an odd number of characters.

Therefore, the RE for the language is:  $(alb)((aalbb)^*(ablba)^*)^*$

Q4:

Proof:

because the keene-star of  $L$  is defined as:  $L^* = \{\epsilon\} \cup L \cup L^2 \cup \dots$ , where  $L_{i+1} = \{wv \mid w \in L_i, v \in L\}$

and  $L_0^* = \{\epsilon\} \cup L_0 \cup L_0^2 \cup \dots$ , where  $L_{0(i+1)} = \{w'v' \mid w' \in L_{0i}, v' \in L_0\}$

since  $L_0 \subseteq L$ ,  $L$  contains all strings that are in  $L_0$ , and hence  $w' \in L^*$ ,  $v' \in L^*$ , and so does  $w'v'$

$\therefore L^*$  also contains  $L_0^*$

$\therefore L_0^* \subseteq L^*$  Q.E.D.

Q5:

$L_P^*$  is a regular language. Proof:

Claim 1:  $L_P^* = \{a^n \mid n = 0, 2, 3, 4, \dots\}$

proof: if  $a^n$  has a length of an even number, then  $a^n = (a^2)^i$ , where  $i = 0, 1, 2, 3, \dots$

if  $a^n$  has a length of an odd number which is greater than 1, then  $a^n = (a^3)(a^2)^i$ ,

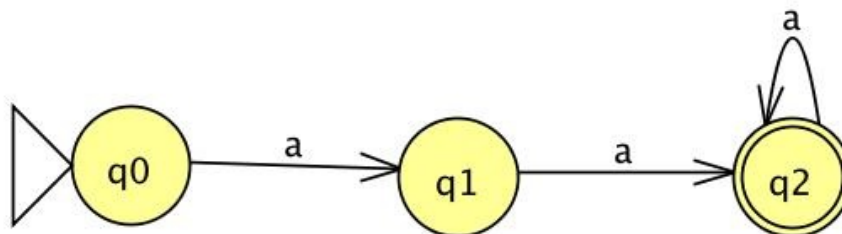
where  $i = 0, 1, 2, 3, \dots$

Therefore:  $L_P^* = \{a^n \mid n = 2, 3, 4, \dots\}$

Claim 2:  $L_P^*$  can be expressed by a DFA

proof:  $L_P^* = \{a^n \mid n = 2, 3, 4, \dots\}$  can be expressed by a 3-state DFA:

The following DFA accepts  $L_P^* = \{a^n \mid n = 0, 2, 3, 4, \dots\}$



Since a language can be expressed by a DFA is equivalent to the language is regular, then therefore  $L_P^*$  is regular. Q.E.D.