CSCI3136
Assignment 2

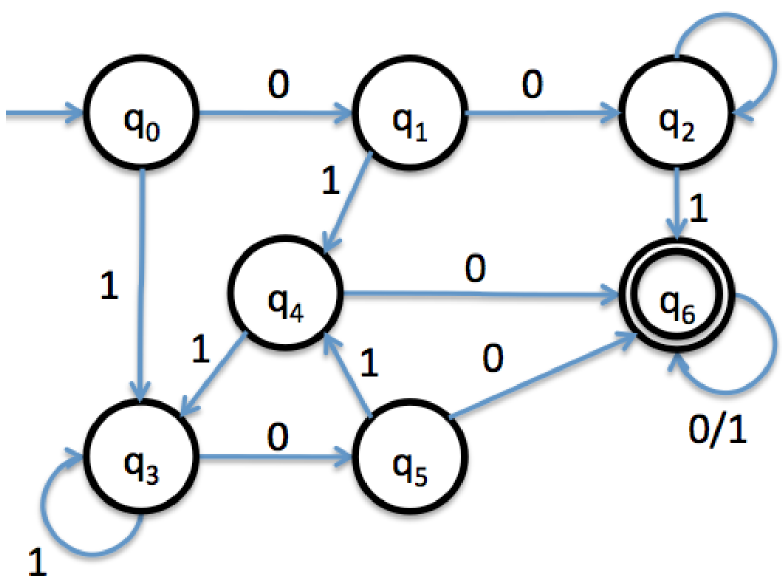
Instructor: Alex Brodsky

Due: 3:00pm, Monday, January 27, 2014

1. **[10 marks]** Construct a DFA for the language of binary strings specified by the following regular expression:

$$(1|0)*(100|010|001)(0|1)*$$

Note that the standard approach is to first construct an NFA and transform it to a DFA.



Note: To understand how this DFA works, consider what each state represents:

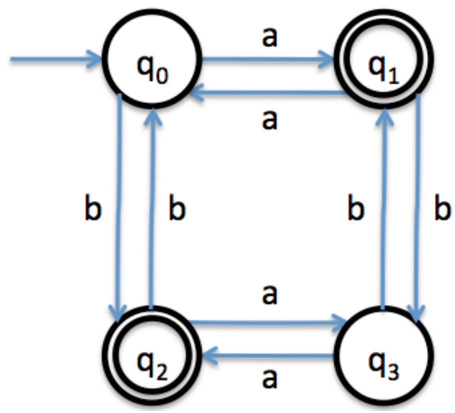
q_0 : initial state, no input yet
q_1 : last char read 0
q_2 : last 2 chars read 00
q_3 : last char read 1
q_4 : last 2 chars read 01
q_5 : last 2 chars read 10
q_6 : final state, have read one of (100, 010, or 001)

This is all the DFA needs to keep track off.

2. **[10 marks]** Recall that $L = \{0^n1^n | n \geq 0\}$ is not regular. Prove, using the properties of regular languages that, that $L' = \{0^i1^j | i \neq j\}$ is also not regular.

   Proof by contradiction. Assume that L' is regular. Hence, if L' is regular, then so is its complement $\overline{L'}$. Let $L_{0*1*} = \{0^i1^j | i \geq 0, j \geq 0\}$, which we know from class is regular. Hence, the intersection $L'' = \overline{L'} \cap L_{0*1*}$ must also be regular. But, L'' is the set of binary strings not containing L' where all the strings are a sequence of 0s followed by a sequence of 1s, i.e., where the number of 0s must equal the number of 1s. But then $L'' = L$, which we know is not regular! Contradiction.

3. **[10 marks]** Give a regular expression that specifies the language recognized by the following DFA.



   After reduction to a two state GNFA, the four REs are

   R_1 : $a(bb) * a|((a(bb) * ba)|ba)(aa|ab(bb) * ba) * (ab(bb) * a|b)$
   R_2 : $a(bb) * a|((a(bb) * ba)|ba)(aa|ab(bb) * ba)*$
   R_3 : $\emptyset$
   R_4 : $\emptyset$

   Hence, the final RE is simply: $R_1 * R_2$

4. **[10 marks]** Prove that if $L_0 \subseteq L$, then $L_0^* \subseteq L^*$.

   We show that if $\sigma \in L_0^*$ then $\sigma \in L^*$, implying that $L_0^* \subseteq L^*$. Choose any $\sigma \in L_0^*$. If $\sigma \in L_0^*$, then by definition of Kleene-* $\sigma = \sigma_1\sigma_2 \ldots \sigma_k$, where $k \geq 0$ and $\sigma_i \in L_0$. If $\sigma_i \in L_0$ and $L_0 \subseteq L$, then $\sigma_i \in L$. Since $\sigma_i \in L$ for $i = 0 \ldots k$, $\sigma \in L^*$. hence $L_0^* \subseteq L^*$.

5. **[10 marks]** We know from our discussion that the language $L_P = \{a^p | p \textit{ is prime}\}$ is not regular. Is the language L_P^* regular? Be sure to prove your answer. Note: 1 is not a prime number.

L_P^* is regular. Proof: Let $L_2 = \{aa\}$. Since 2 is a prime, $L_2 \subset L_P$, and hence, $L_2^* \subset L_P^*$. Let $L_3 = \{aaa\}$. Since 3 is also a prime, $L_3 \subset L_P$ and hence, $L_3L_2^* \subset L_P^*$. Observe that L_2^* is the set of all even length strings and that $L_3L_2^*$ is the set of all odd length strings of length greater than 1. Hence $L_2^* \cup L_3L_2^*$ is the set of all strings except a. Since $a \notin L_P^*$, because 1 is not a prime, $L_2^* \cup L_3L_2^* = L_P^*$. Since L_2 is regular, L_2^* is regular. And, since L_3 is regular, $L_3L_2^*$ is regular, and hence $L_2^* \cup L_3L_2^* = L_P^*$ is regular.

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Winter 2014

Student Name	Login ID	Student Number	Student Signature

	Mark
Question 1	/10
Question 2	/10
Question 3	/10
Question 4	/10
Question 5	/10
Total	**/50**

Comments:

Assignments are due by 3:00pm on the due date before class and must include this cover page. Assignment *must* be submitted into the assignment boxes on the second floor of the Goldberg CS Building (by the elevators).

Plagiarism in assignment answers will not be tolerated. By submitting their answers to this assignment, the authors named above declare that its content is their original work and that they did not use any sources for its preparation other than the class notes, the textbook, and ones explicitly acknowledged in the answers. Any suspected act of plagiarism will be reported to the Facultys Academic Integrity Officer and possibly to the Senate Discipline Committee. The penalty for academic dishonesty may range from failing the course to expulsion from the university, in accordance with Dalhousie Universitys regulations regarding academic integrity.