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Contents

1 Example 1: Foo

Operations:
1 - Assignment, Addition, Return
0 - For loop overhead

$$i = 1, 2, 4, 8, \ldots, n$$

$$f(i) = 0, 1, 2, 3, 4, \ldots, ? = \log_2 i$$

$$n = 9 : i = 1, 2, 4, 8, 16$$

$$0, 1, 2, 3, \ldots, \lceil \log_2 n \rceil$$

$$1 + \sum_{i=0}^{\lceil \log_2 n \rceil} \left(2 + \sum_{j=0}^{n-1} \left(2 + \sum_{k=0}^{j-1} (2) \right) \right) + 1$$

$$2 + \sum_{i=0}^{\lceil \log_2 n \rceil} \left(2 + \sum_{j=0}^{n-1} \left(2 + \sum_{k=0}^{j-1} (2) \right) \right)$$

$$2 + \sum_{i=0}^{\lceil \log_2 n \rceil} \left(2 + \sum_{j=0}^{n-1} \left(2 + 2 \cdot \sum_{k=0}^{j-1} (1) \right) \right)$$

$$2 + \sum_{i=0}^{\lceil \log_2 n \rceil} \left(2 + 2 \cdot \sum_{j=0}^{n-1} \left(1 + \sum_{k=0}^{j-1} (1) \right) \right)$$

$$2 + 2 \sum_{i=0}^{\lceil \log_2 n \rceil} \left(1 + \sum_{j=0}^{n-1} \left(1 + \sum_{k=0}^{j-1} (1) \right) \right)$$

Rules for Summations:

$$\sum_{k=a}^{b} 1 = b - a + 1$$

$$\sum_{k=0}^{n} k = \sum_{k=1}^{n} k = \frac{n\,(n+1)}{2}$$

$$0 + 1 + 2 + \cdots + n = 1 + 2 + \cdots + n = \frac{n\,(n+1)}{2}$$

$$\sum_{k=a}^{b}(f(k)+g(k)) = \sum_{k=a}^{b}(f(k)) + \sum_{k=a}^{b}(g(k))$$

Back to the problem:

$$2+2\sum_{i=0}^{\lceil \log_2 n \rceil}\left(1+\sum_{j=0}^{n-1}\left(1+\sum_{k=0}^{j-1}(1)\right)\right)$$

$$2+2\sum_{i=0}^{\lceil \log_2 n \rceil}\left(1+\sum_{j=0}^{n-1}(1+((j-1)-(0)+1))\right)$$

$$2+2\sum_{i=0}^{\lceil \log_2 n \rceil}\left(1+\sum_{j=0}^{n-1}(1+(j))\right)$$

$$2+2\sum_{i=0}^{\lceil \log_2 n \rceil}\left(1+\sum_{j=0}^{n-1}(1)+\sum_{j=0}^{n-1}(j)\right)$$

$$2+2\sum_{i=0}^{\lceil \log_2 n \rceil}\left(1+(n-1-0+1)+\sum_{j=0}^{n-1}(j)\right)$$

$$2+2\sum_{i=0}^{\lceil \log_2 n \rceil}\left(1+(n)+\sum_{j=0}^{n-1}(j)\right)$$

$$2+2\sum_{i=0}^{\lceil \log_2 n \rceil}\left(1+(n)+\frac{(n-1)(n+1-1)}{2}\right)$$

$$2+2\sum_{i=0}^{\lceil \log_2 n \rceil}\left(1+(n)+\frac{(n-1)n}{2}\right)$$

$$2+2\left(\sum_{i=0}^{\lceil \log_2 n \rceil}(1)+\sum_{i=0}^{\lceil \log_2 n \rceil}(n)+\sum_{i=0}^{\lceil \log_2 n \rceil}\left(\frac{(n-1)n}{2}\right)\right)$$

$$2+2\left((\lceil \log_2 n \rceil-0+1)+\sum_{i=0}^{\lceil \log_2 n \rceil}(n)+\sum_{i=0}^{\lceil \log_2 n \rceil}\left(\frac{(n-1)n}{2}\right)\right)$$

$$2+2\left((\lceil \log_2 n \rceil+1)+\sum_{i=0}^{\lceil \log_2 n \rceil}(n)+\sum_{i=0}^{\lceil \log_2 n \rceil}\left(\frac{(n-1)n}{2}\right)\right)$$

$$2+2\left((\lceil \log_2 n \rceil+1)+n\cdot\sum_{i=0}^{\lceil \log_2 n \rceil}(1)+\sum_{i=0}^{\lceil \log_2 n \rceil}\left(\frac{(n-1)n}{2}\right)\right)$$

$$2+2\left((\lceil \log_2 n \rceil+1)+n\cdot\sum_{i=0}^{\lceil \log_2 n \rceil}(1)+\frac{(n-1)n}{2}\cdot\sum_{i=0}^{\lceil \log_2 n \rceil}(1)\right)$$

$$2+2\left((\lceil \log_2 n \rceil+1)+n\cdot(\lceil \log_2 n \rceil+1)+\frac{(n-1)n}{2}(\lceil \log_2 n \rceil+1)\right)$$

$$O(n^2\log n)$$

Rule for Logarithms, why Log-Base doesn't matter in Big O.

$$\frac{\log n}{\log 2} = \log_2 n$$

$$\frac{1}{\log 2} = c$$