# Contents

# 1   Example 1: Foo

Operations:
1 - Assignment, Addition, Return
0 - For loop overhead

$$i = 1, 2, 4, 8, \ldots, n$$

$$f(i) = 0, 1, 2, 3, 4, \ldots, ? = \log_2 i$$

$$n = 9 : i = 1, 2, 4, 8, 16$$

$$0, 1, 2, 3, \ldots, \lceil \log_2 n \rceil$$

$$1 + \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 2 + \sum_{j=0}^{n-1} \left( 2 + \sum_{k=0}^{j-1} (2) \right) \right) + 1$$

$$2 + \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 2 + \sum_{j=0}^{n-1} \left( 2 + \sum_{k=0}^{j-1} (2) \right) \right)$$

$$2 + \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 2 + \sum_{j=0}^{n-1} \left( 2 + 2 \cdot \sum_{k=0}^{j-1} (1) \right) \right)$$

$$2 + \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 2 + 2 \cdot \sum_{j=0}^{n-1} \left( 1 + \sum_{k=0}^{j-1} (1) \right) \right)$$

$$2 + 2 \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 1 + \sum_{j=0}^{n-1} \left( 1 + \sum_{k=0}^{j-1} (1) \right) \right)$$

Rules for Summations:

$$\sum_{k=a}^{b} 1 = b - a + 1$$

$$\sum_{k=0}^{n} k = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$0 + 1 + 2 + \cdots + n = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=a}^{b} (f(k) + g(k)) = \sum_{k=a}^{b} (f(k)) + \sum_{k=a}^{b} (g(k))$$

Back to the problem:

$$2 + 2 \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 1 + \sum_{j=0}^{n-1} \left( 1 + \sum_{k=0}^{j-1} (1) \right) \right)$$

$$2 + 2 \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 1 + \sum_{j=0}^{n-1} (1 + ((j-1) - (0) + 1)) \right)$$

$$2 + 2 \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 1 + \sum_{j=0}^{n-1} (1 + (j)) \right)$$

$$2 + 2 \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 1 + \sum_{j=0}^{n-1} (1) + \sum_{j=0}^{n-1} (j) \right)$$

$$2 + 2 \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 1 + (n - 1 - 0 + 1) + \sum_{j=0}^{n-1} (j) \right)$$

$$2 + 2 \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 1 + (n) + \sum_{j=0}^{n-1} (j) \right)$$

$$2 + 2 \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 1 + (n) + \frac{(n-1)(n+1-1)}{2} \right)$$

$$2 + 2 \sum_{i=0}^{\lceil \log_2 n \rceil} \left( 1 + (n) + \frac{(n-1)n}{2} \right)$$

$$2 + 2 \left( \sum_{i=0}^{\lceil \log_2 n \rceil} (1) + \sum_{i=0}^{\lceil \log_2 n \rceil} (n) + \sum_{i=0}^{\lceil \log_2 n \rceil} \left( \frac{(n-1)n}{2} \right) \right)$$

$$2 + 2 \left( (\lceil \log_2 n \rceil - 0 + 1) + \sum_{i=0}^{\lceil \log_2 n \rceil} (n) + \sum_{i=0}^{\lceil \log_2 n \rceil} \left( \frac{(n-1)n}{2} \right) \right)$$

$$2 + 2 \left( (\lceil \log_2 n \rceil + 1) + \sum_{i=0}^{\lceil \log_2 n \rceil} (n) + \sum_{i=0}^{\lceil \log_2 n \rceil} \left( \frac{(n-1)n}{2} \right) \right)$$

$$2 + 2 \left( (\lceil \log_2 n \rceil + 1) + n \cdot \sum_{i=0}^{\lceil \log_2 n \rceil} (1) + \sum_{i=0}^{\lceil \log_2 n \rceil} \left( \frac{(n-1)n}{2} \right) \right)$$

$$2 + 2 \left( (\lceil \log_2 n \rceil + 1) + n \cdot \sum_{i=0}^{\lceil \log_2 n \rceil} (1) + \frac{(n-1)n}{2} \cdot \sum_{i=0}^{\lceil \log_2 n \rceil} (1) \right)$$

$$2 + 2 \left( (\lceil \log_2 n \rceil + 1) + n \cdot (\lceil \log_2 n \rceil + 1) + \frac{(n-1)n}{2} (\lceil \log_2 n \rceil + 1) \right)$$

$$O(n^2 \log n)$$

Rule for Logarithms, why Log-Base doesn't matter in Big O.

$$\frac{\log n}{\log 2} = \log_2 n$$

$$\frac{1}{\log 2} = c$$