

Intro to Math Thinking Fall 2024: Assignment 5

1. (a) $\exists x \in \mathbb{N} \mid x^3 = 27$
(b) $\exists x \in \mathbb{N} \mid x > 1000000$
(c) $(\exists p \in \mathbb{N})(\exists q \in \mathbb{N})(p > 1 \wedge q > 1 \wedge n = pq)$
2. (a) $\forall x \in \mathbb{N} \mid x^3 \neq 28$
(b) $\forall n \in \mathbb{N} \mid 0 < n$
(c) $(\forall p \in \mathbb{N})(\forall q \in \mathbb{N})((n = pq) \Rightarrow (p = 1 \vee q = 1))$
3. (a) $\forall x \exists y L(x, y)$, where $L(x, y)$ means „x loves y”
(b) $\forall x (Tall(x) \vee Short(x))$
(c) $\forall x Tall(x) \vee \forall x Short(x)$
(d) $\forall x \neg H(x)$, where $H(x)$ means is at home
(e) $Comes(John) \Rightarrow \forall x (Woman(x) \Rightarrow Leaves(x))$
(f) $\exists x (Man(x) \wedge Comes(x)) \Rightarrow \forall x (Woman(x) \Rightarrow Leaves(x))$
4. (a) $(\forall a \in \mathbb{R})(\exists x \in \mathbb{R}) \mid x^2 + a = 0$
(b) $(\forall a \in \mathbb{R})[(a < 0) \Rightarrow (\exists x \in \mathbb{R})(x^2 + a = 0)]$
(c) $(\forall x \in \mathbb{R})(\exists p \in \mathbb{N})(\exists q \in \mathbb{N})[p = qx \vee p = -qx \vee x = 0]$
(d) $(\exists x \in \mathbb{R})(\forall p \in \mathbb{N})(\forall q \in \mathbb{N})[p \neq nx \wedge p \neq -nx]$
(e) $(\forall y \in \mathbb{R})(\exists x \in \mathbb{R})[(x > y) \wedge (\forall m \in \mathbb{N})(\forall n \in \mathbb{N})(m \neq nx)]$
5. (a) $(\forall x \in C) \mid (D(x) \Rightarrow M(x))$
(b) $(\forall x \in C) \mid (\neg D(x) \Rightarrow M(x))$
(c) $(\forall x \in C) \mid (M(x) \Rightarrow D(x))$
(d) $(\exists x \in C) \mid D(x) \wedge \neg M(x)$
(e) $(\exists x \in C) \mid \neg D(x) \wedge M(x)$
6. $\forall a \forall b [(a < b) \Rightarrow \exists c (Q(x) \wedge (a < c < b))]$
7. $F(p, t)$ means „You can fool person p at time t”
 $\exists t \forall p F(p, t) \wedge \exists p \forall t F(p, t) \wedge \neg \forall p \forall t F(p, t)$
8. $\exists x \forall t A(x, t)$
9. „Every 6 seconds a driver is involved in an accident”
 $\forall t \exists x A(x, t)$