Intro to Math Thinking Fall 2024: Assignment 4

1.

ϕ	ψ	$\phi \Rightarrow \psi$	$\psi \Rightarrow \phi$	$(\psi \Rightarrow \psi) \wedge (\psi \Rightarrow \phi)$	$\phi \Leftrightarrow \psi$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

2.

ϕ	ψ	$\neg\phi$	$\phi \Rightarrow \psi$	$\neg\phi \vee \psi$	$(\phi \Rightarrow \psi) \Rightarrow (\neg\phi \vee \psi)$	$(\phi \Rightarrow \psi) \Leftarrow (\neg\phi \vee \psi)$	$(\phi \Rightarrow \psi) \Leftrightarrow (\neg\phi \vee \psi)$
T	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	F	T	T	T
F	F	T	T	T	T	T	T

3.

ϕ	ψ	$\neg\psi$	$\phi \not\Rightarrow \psi$	$\phi \wedge \neg\psi$	$(\phi \not\Rightarrow \psi) \Leftrightarrow (\phi \wedge \neg\psi)$
T	T	F	F	F	T
F	T	F	F	F	T
T	F	T	T	T	T
F	F	T	F	F	T

4. (a)

ϕ	ψ	$\phi \Rightarrow \psi$	$\phi \wedge (\phi \Rightarrow \psi)$	$[\phi \wedge (\phi \Rightarrow \psi)] \Rightarrow \psi$
T	T	T	T	T
F	T	T	F	T
T	F	F	F	T
F	F	T	F	T

   (b) it is a tautology meaning that ψ always follows from knowing ψ and $\phi \Rightarrow \psi$

5. $\phi \vee \psi$ means either ϕ or ψ is true or both
   Thus $\neg(\phi \vee \psi)$ means that ϕ and ψ must both be false
   This is the same as saying $\neg\phi$ and $\neg\psi$ must both be false (def of negation)
   By def of *and*, this can be written as $(\neg\phi) \wedge (\neg\psi)$

6. (a) 34159 is not a prime number
   (b) Roses are not red or violets are not blue
   (c) There are no hamburgers but I won't have a hot-dog
   (d) Fred won't go or he will play
   (e) The number x is non-negative and less than or equal to 10
   (f) We will lose the first game and the second

7.

ϕ	ψ	$\neg\phi$	$\neg\psi$	$(\neg\phi) \Leftrightarrow (\neg\psi)$	$\phi \Leftrightarrow \psi$
T	T	F	F	T	T
F	T	T	F	F	F
T	F	F	T	F	F
F	F	T	T	T	T

8. (a)

ϕ	ψ	$\phi \Rightarrow \psi$	$\phi \Leftarrow \psi$	$\phi \Leftrightarrow \psi$
T	T	T	T	T
F	T	T	F	F
T	F	F	T	F
F	F	T	T	T

(b)

ϕ	ψ	θ	$(\psi \vee \theta)$	$\phi \Rightarrow (\psi \vee \theta)$
T	T	T	T	T
F	T	T	T	T
F	F	T	T	T
F	T	F	T	T
T	F	F	F	F
T	T	F	T	T
T	F	T	T	T
F	F	F	F	T

9.

ϕ	ψ	θ	$(\psi \wedge \theta)$	$\phi \Rightarrow (\psi \wedge \theta)$	$\phi \Rightarrow \psi$	$\phi \Rightarrow \theta$	$(\phi \Rightarrow \psi) \wedge (\phi \Rightarrow \theta)$
T	T	T	T	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	F	T	T	T	T
F	T	F	F	T	T	T	T
T	F	F	F	F	F	F	F
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
F	F	F	F	T	T	T	T

10. [$\Rightarrow$] Suppose ϕ is true
$\psi \wedge \theta$ means that both ψ and θ must be true
Now suppose if ϕ is true, the $\psi \wedge \theta$ is true
By definition of *implies*, this can be written as $\phi \Rightarrow (\psi \wedge \theta)$
Since ψ and θ are true, it follows: if ϕ, then ψ; and if ϕ, then θ
By def of *implies* and *and*, this can be written as $(\phi \Rightarrow \psi) \wedge (\phi \Rightarrow \theta)$

[$\Leftarrow$] Suppose $(\phi \Rightarrow \psi) \wedge (\phi \Rightarrow \theta)$ is true
Then ψ, θ cannot both be false when ϕ is true
By def of *and*, ψ, θ cannot both be false can be written as $(\neg\psi \vee \neg\theta)$ that is equivalent to $(\psi \wedge \theta)$
Thus by def of *implies*, the expression can be written as $\phi \Rightarrow (\psi \wedge \theta)$

11.

ϕ	ψ	$\neg\phi$	$\neg\psi$	$\phi \Rightarrow \psi$	$(\neg\psi) \Rightarrow (\neg\phi)$
T	T	F	F	T	T
F	T	T	F	T	T
T	F	F	T	F	F
F	F	T	T	T	T

12. (a) If 2 rectangles don't have the same area, they aren't congruent
    (b) If $a^2 + b^2 \neq c^2$, then the triangle with sides a, b, c (c largest) is not right-angled
    (c) If n is not prime, then $2^n - 1$ is not prime
    (d) If the Dollar does not fall, then the Yuan won't rise

13.

ϕ	ψ	$\neg\phi$	$\neg\psi$	$(\neg\psi) \Rightarrow (\neg\phi)$	$\psi \Rightarrow \phi$
T	T	F	F	T	T
F	T	T	F	T	F
T	F	F	T	F	T
F	F	T	T	T	T

14. (a) If 2 rectangles have the same area, then they are congruent
    (b) If $a^2 + b^2 = c^2$, then a triangle with sides a, b, c (c largest) is right-angled
    (c) If n is prime, then $2^n - 1$ is prime
    (d) If the Dollar falls, the Yuan will rise

1 Optional Probs

1. $\neg\psi \Rightarrow \phi$

2.

ϕ	ψ	$\phi \dot{\vee} \psi$
T	T	F
F	T	T
T	F	T
F	F	F

3. $\phi \dot{\vee} \psi$ is equivalent to $(\phi \wedge \neg\psi) \vee (\neg\phi \wedge \psi)$

4. (a) If the statement is true, then it is not false
   (b) If $x = 3$, then $x^2 = 9$
   (c) If $-3 < x < 3$, then $x^2 < 9$
   (d) If $x = 2$, then $x^2 = 4$

5.

M	N	$M \times N$	$M + N$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	0

6. (a) $\wedge$
   (b) $\dot{\vee}$
   (c) no

7. (a) $\vee$
   (b) $(M \wedge N) \vee (\neg M \wedge \neg N)$
   (c) yes

8. 2, cards B, 4

9. Suppose m, n are 2 natural numbers. If we multiply mn, then there are 3 cases:
   (a) if at least 1 of m, n is even, then mn is even
   (b) if both m, n are even, then mn is even
   (c) if both m, n are odd, then mn is odd

   Then mn is odd iff m and n are odd.

10. False, suppose m is even and n is odd, then mn is even. Refer to statements in *9.*

11. 1 face down ID, 1 7-up or vodka and tonic

12. Similar, need 2 verifications. Used contrapositive in Wason's problem and process of elimination in *11.* Setup for *11.* made it clearer.