Intro to Math Thinking Fall 2024: Assignment 6

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1. ($\Rightarrow$) Assume $\neg(\exists x A(x))$. If it is not the case that there exists an x, $A(x)$, then all x must fail to satisfy $A(x)$. So, $\forall x$, $A(x)$ must be false. Thus, $\forall x[\neg A(x)]$.

   ($\Leftarrow$) Assume $\forall x[\neg A(x)]$. If for all x fail $A(x)$, then there does not exists an x, $A(x)$. So, $\exists x A(x)$ must be false. Thus, $\neg[\exists x A(x)]$

2. Suppose there is an even prime $n \in \mathbb{N}$ bigger than 2. By definition of even, there exists an $a \in \mathbb{N}$ such that $n = 2a$. By definition of prime, n must only be divisible by 1 and itself. n is also divisible by 2, which is a contradiction.

3. (a) $\forall x[Student(x) \Rightarrow LikesPizza(x)]$
   (b) $\exists x[Friend(x) \Rightarrow \neg Car(x)]$
   (c) $\exists x[Elephant(x) \Rightarrow \neg LikesMuffins(x)]$
   (d) $\forall x[Triangle(x) \Rightarrow Isosceles(x)]$
   (e) $\exists x[StudentInClass(x) \Rightarrow \neg HereToday(x)]$
   (f) $\forall x[(\exists y)(Loves(x,y)]$
   (g) $\neg\exists x[(\forall y)(Loves(x,y)]$
   (h) $(\forall x)[(Man(x) \wedge Comes(x)) \Rightarrow (\forall x)(Woman(x) \Rightarrow Leaves(x))]$
   (i) $\forall x[Tall(x) \vee Short(x)]$
   (j) $(\forall x)[Tall(x)] \vee (\forall x)[Short(x)]$
   (k) $\neg\forall x[Precious(x) \Rightarrow Beautiful(x)]$
   (l) $\forall x[\neg LovesMe(x)]$
   (m) $\exists x[AmericanSnake(x) \Rightarrow Poisonous(x)]$
   (n) $\exists x[(Snake(x) \wedge American(x)) \Rightarrow Poisonous(x)]$

4. (a) $\exists x[Student(x) \wedge \neg LikesPizza(x)]$
   There is a student who does not like pizza
   (b) $\forall x[Freind(x) \wedge Car(x)]$
   All of my friends have a car
   (c) $\forall x[Elephants(x) \wedge LikesMuffins(x)]$
   All elephants like muffins
   (d) $\exists x[Triangle(x) \wedge \neg Isoceles(x)]$
   There is a triangle that is not isosceles
   (e) $\forall x[StudentInClass(x) \wedge HereToday(x)]$
   All students in the class are here today
   (f) $\exists x[(\exists y)(\neg Loves(x,y)]$
   There exists someone who loves no one

(g) $\exists x[(\forall y) Loves(x, y)]$
There is someone who loves everyone

(h) $\exists x[(Man(x) \wedge Comes(x)) \wedge (\exists x)(Woman(x) \wedge \neg Leaves(x))]$
There is a man who comes and a woman who doesn't leave

(i) $\exists x[\neg Tall(x) \wedge \neg Short(x)]$
There is someone who is neither tall nor short

(j) $(\exists x)[\neg Tall(x)] \wedge (\exists x)[\neg Short(x)]$
There is someone who isn't tall, and there is someone who isn't short

(k) $\forall x[Precious(x) \Rightarrow Beautiful(x)]$
All precious stones are beautiful

(l) $\exists x[LovesMe(x)]$
There is someone who loves me

(m) $\forall x[AmericanSnake(x) \wedge \neg Poisonous(x)]$
All American snakes are not poisonous

(n) $\forall x[(\neg Snake(x) \vee \neg American(x)) \wedge \neg Poisonous(x)]$
All animals are not poisonous, and either not snakes or not American

5. (a) False ($x = \frac{2}{3} \notin \mathbb{N}$)
   (b) False ($x = \pm\sqrt{2} \notin \mathbb{Q}$)
   (c) True (square root)
   (d) True (square root)
   (e) False (If $x = 0$, then trivially true. If $x \neq 0$, then $y = z$, but cannot find 1 y such that $y = z$ $\forall z$.)
   (f) False (simplify to $y = z$, but cannot find 1 prime y such that $y = z$, for all prime z)
   (g) False (choose $x = -1$)
   (h) True (antecedent always false)

6. (a) $(\forall x \in \mathbb{N})[2x + 3 \neq 5x + 1]$
   (b) $(\forall x \in \mathbb{Q})[x^2 \neq 2]$
   (c) $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})[y \neq x^2]$
   (d) $(\exists x \in \mathbb{N})(\forall y \in \mathbb{N})[y \neq x^2]$
   (e) $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R})[xy \neq xz]$
   (f) $(\exists x \in \mathbb{P})(\forall y \in \mathbb{P})(\exists z \in \mathbb{P})[xy \neq xz]$
   (g) $(\exists x \in \mathbb{R})[(x < 0) \wedge (\forall y \in \mathbb{R})(y^2 \neq x)]$
   (h) $(\exists x \in \mathbb{R}^+)[(x < 0) \wedge (\forall y \in \mathbb{R}^+)(y^2 \neq x)]$

7. (a) $(\exists x \in \mathbb{N})(\forall y \in \mathbb{N})[x + y \neq 1]$
   (b) $(\exists x > 0)(\forall y < 0)[x + y \neq 0]$
   (c) $(\forall x \in \mathbb{R})(\exists \epsilon > 0)[(x \leq -\epsilon) \vee (x \geq \epsilon)]$
   (d) $(\exists x \in \mathbb{N})(\exists y \in \mathbb{N})(\forall z \in \mathbb{N})[x + y \neq z^2]$

8. OG: $(\exists t)(\forall p)[Fool(p,t)] \wedge (\exists p)(\forall t)[Fool(p,t)] \wedge (\neg\forall p)(\forall t)[Fool(p,t)]$

   Neg: $(\forall t)(\exists p)[\neg Fool(p,t)] \vee (\forall p)(\exists t)[Fool(p,t)] \vee (\forall p)(\forall t)[Fool(p,t)]$
   You can't fool some of the people all of the time, or for every person, you can't always fool them, or you can fool all the people all of the time

9. f is *discontinuous* at a: $(\exists \epsilon > 0)(\forall \delta > 0)(\exists x)[(|x - a| < \delta) \wedge (|f(x) - f(a)| \geq \epsilon)]$