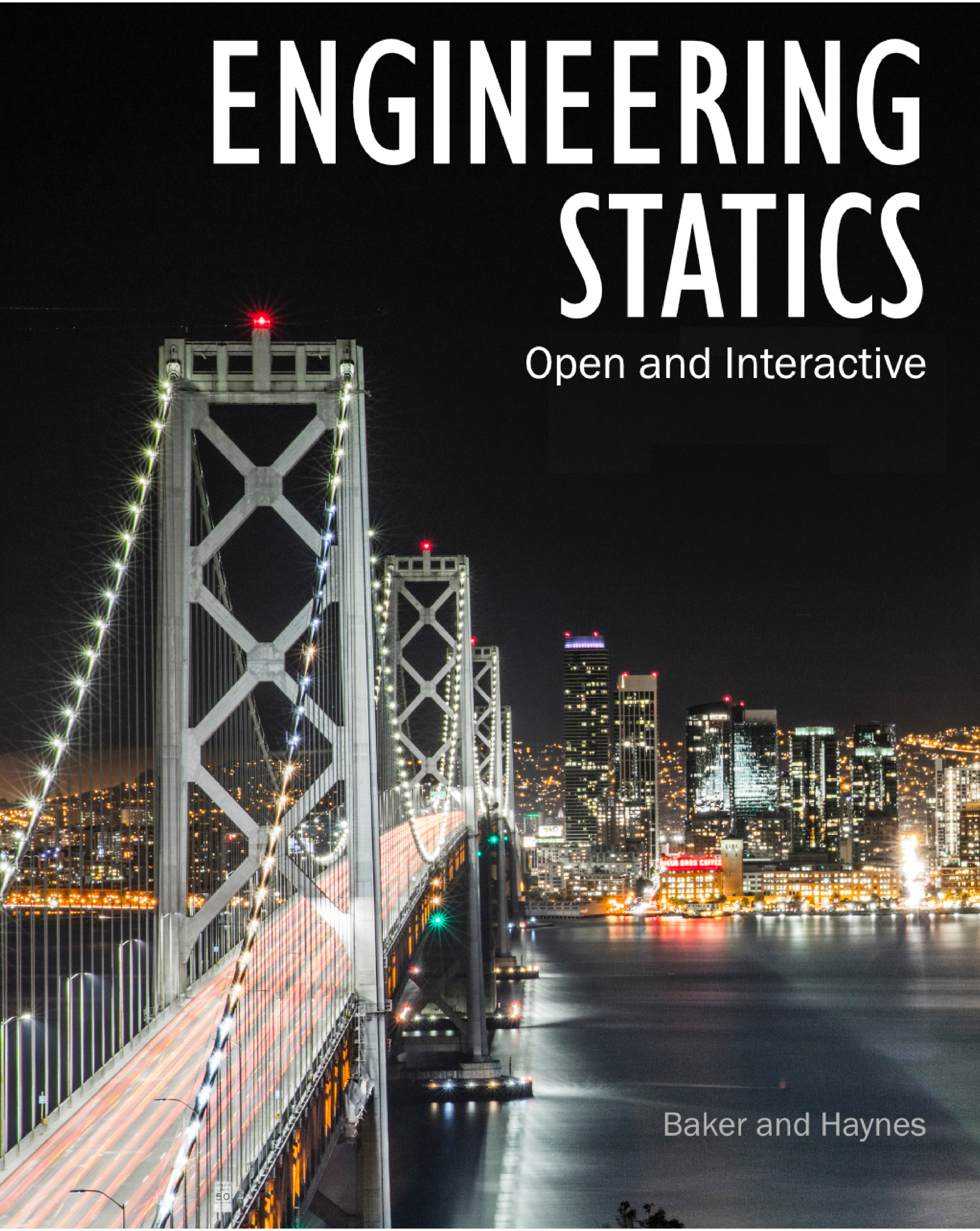
ENGINEERING STATICS

Open and Interactive



Baker and Haynes

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Preface

Engineering Statics is a free, open-source textbook appropriate for anyone who wishes to learn more about vectors, forces, moments, static equilibrium, and the properties of shapes. Specifically, it has been written to be the textbook for Engineering Mechanics: Statics, the first course in the Engineering Mechanics series offered in most university-level engineering programs.

This book's content should prepare you for subsequent classes covering Engineering Mechanics: Dynamics and Mechanics of Materials. At its core, *Engineering Statics* provides the tools to solve static equilibrium problems for rigid bodies. The additional topics of resolving internal loads in rigid bodies and computing area moments of inertia are also included as stepping stones for later courses. We have endeavored to write in an approachable style and provide many questions, examples, and interactives for you to engage with and learn from.

Feedback. Please contact our team with feedback and suggestions that can be provided directly to the lead author Dan Baker via email at `dan.baker@colostate.edu`, or through the Engineering Statics Google Group. We would also appreciate knowing if you are using the book for teaching purposes. There are currently no instructor resources available beyond the textbook itself.

Access. The entire book is available for free as an interactive online ebook at `https://engineeringstatics.org`. This should work well on all screen sizes, including smartphones. A free PDF version, suitable for reading on a tablet or computer, is available for offline use at `https://engineeringstatics.org/pdf/statics.pdf`. The PDF is searchable and easy to navigate using embedded links. The source files for this book are available on GitHub at `https://github.com/dantheboatman/EngineeringStatics`.

History. This book began as the vision of a handful of the authors to create a book filled with interactives instead of static figures. Spurred into action by funding opportunities in the Fall of 2018, the team worked on the book through 2019 and launched this working draft version in Fall 2020. Over the two years of creating the book, our team grew to a coast-to-coast representation of large public universities, small private colleges, and community colleges.

On the Cover. A photo by Artur Westergren from Yerba Buena Island across the San Francisco bridge of the San Francisco, California skyline. Image source: https://unsplash.com/photos/Rx92z9dU-mA

Contents

Chapter 1

Introduction to Statics

Engineering Statics is the gateway into *engineering mechanics*, which is the application of Newtonian physics to design and analyze objects, systems, and structures with respect to motion, deformation, and failure. In addition to learning the subject itself, you will also develop skills in the art and practice of problem solving and mathematical modeling, skills that will benefit you throughout your engineering career.

The subject is called "statics" because it is concerned with particles and rigid bodies that are in equilibrium, and these will usually be stationary, i.e. static.

The chapters in this book are:

Introduction to Statics— an overview of statics and an introduction to units and problem solving.

`[cross-reference to target(s) "Chapter_02" missing or not unique]`— basic principles and mathematical operations on force and position vectors.

`[cross-reference to target(s) "Chapter_03" missing or not unique]`— an introduction to equilibrium and problem solving.

`[cross-reference to target(s) "Chapter_04" missing or not unique]`— the rotational tendency of forces, and simplification of force systems.

`[cross-reference to target(s) "Chapter_05" missing or not unique]`— balance of forces and moments for single rigid bodies.

`[cross-reference to target(s) "Chapter_06" missing or not unique]`— balance of forces and moments on interconnected systems of rigid bodies.

`[cross-reference to target(s) "Chapter_07" missing or not unique]`— an important geometric property of shapes and rigid bodies.

`[cross-reference to target(s) "Chapter_08" missing or not unique]`— forces and moments within beams and other rigid bodies.

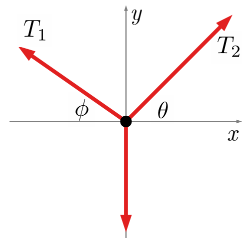
`[cross-reference to target(s) "Chapter_09" missing or not unique]`— equilibrium of bodies subject to friction.

[cross-reference to target(s) "Chapter_10" missing or not unique]—an important property of geometric shapes used in many applications.

Your statics course may not cover all of these topics, or may move through them in a different order.

Below are two examples of the types of problems you'll learn to solve in statics. Notice that each can be described with a picture and problem statement, a free-body diagram, and equations of equilibrium.

Equilibrium of a particle: A 140 lb person walks across a slackline stretched between two trees. If angles α and θ are known, find the tension is in each end of the slackline.

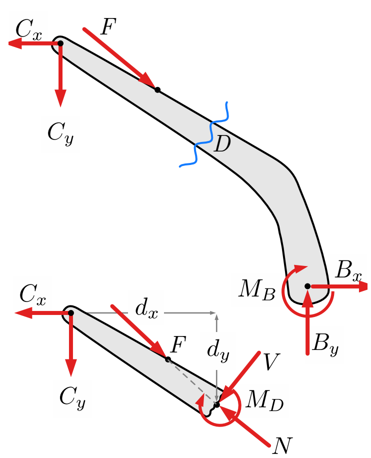


Person's point of contact to slackline:

$$\Sigma F_x = 0$$
$$T_1 \cos\alpha + T_2 \cos\theta = 0$$

$$\Sigma F_y = 0$$
$$T_1 \sin\alpha + T_2 \sin\theta - W = 0$$

Equilibrium of a rigid body: Given the interaction forces at point C on the upper arm of the excavator, find the internal axial force, shear force, and bending moment at point D.



Section cut FBD:

$$\Sigma F_x = 0$$
$$-C_x + F_x + V_x + N_x = 0$$

$$\Sigma F_y = 0$$
$$-C_y + F_x + V_y - N_y = 0$$

$$\Sigma M_D = 0$$
$$-(d_y)C_x + (d_x)C_y - M_D = 0$$

The knowledge and skills gained in Statics will be used in your other engineering courses, in particular in Dynamics, Mechanics of Solids (also called Strength or Mechanics of Materials), and in Fluid Mechanics. Statics will be a foundation of your engineering career.

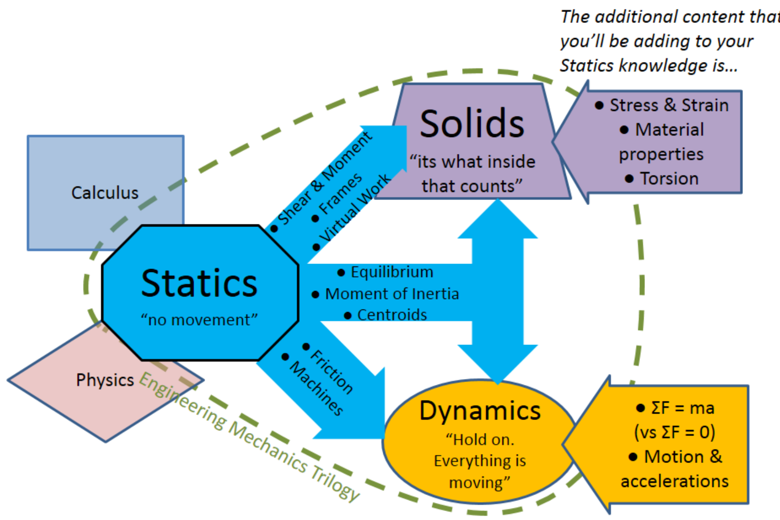


Figure 1.0.1 Map of how Statics builds upon the prerequisites of Calculus and Physics and then informs the later courses of Mechanics of Solids and Dynamics.

1.1 Newton's Laws of Motion

Key Questions

- What are the two types of motion?
- What three relationships do Newton's laws of motion define?
- What are physical examples for each of Newton's three laws of motion?

The English scientist Sir Issac Newton established the foundation of mechanics in 1687 with his three laws of motion, which describe the relation between forces, objects and motion. Motion can be separated into two types:

> **Translation**— where a body changes position without changing its orientation in space, and

> **Rotation**— where a body spins about an axis fixed in space, without changing its average position.

Some moving bodies are purely translating, others are purely rotating, and many are doing both. Conveniently, we can usually separate translation and rotation and analyze them individually with independent equations.

Newton's three laws and their implications with respect to translation and rotation are described below.

1.1.1 Newton's 1st Law

Newton's first law states that

> *an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force.*

This law, also sometimes called the "law of inertia," tells us that bodies maintain their current velocity unless a net force is applied to change it. In other words, if an object is at rest it will remain at rest until an unbalanced force changes its velocity, and if an object is moving at a constant velocity, it will hold that velocity unless a force makes it change. Remember that velocity is a vector quantity which includes both speed and direction, so an unbalanced force may cause an object to speed up, slow down, or change direction.

Figure 1.1.1 This rock is at rest with zero velocity and will remain at rest until a unbalanced force causes it to move.

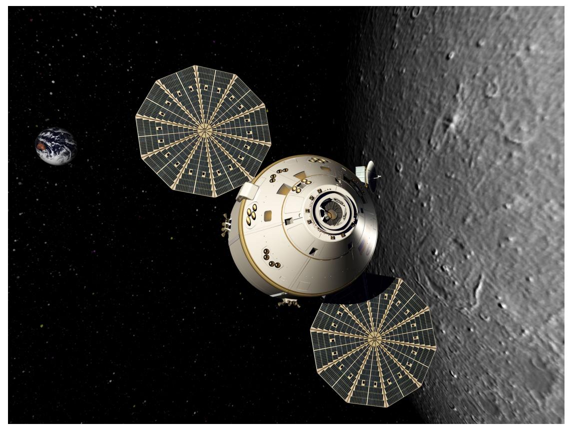


Figure 1.1.2 In the absence of friction in space, this space capsule will maintain its current velocity.

Newton's first law also applies to angular velocities, however instead of force, the relevant quantity which causes an object to rotate is called a **torque** by physicists, but usually called a **moment** by engineers. A moment, as you will learn in `[cross-reference to target(s) "Chapter_04" missing or not unique]`, is the rotational tendency of a force. Just as a force will cause a change in linear velocity, a moment will cause a change in angular velocity. This can be seen in things like tops, flywheels, stationary bikes, and other objects that spin on an axis when a moment is applied, but eventually stop because of the opposite moment produced by friction.

Figure 1.1.3 A spinning top demonstrates rotatory motion.

In the absence of friction this top would spin forever, but the small frictional moment exerted at the point of contact with the table will eventually bring it to a stop.

1.1.2 Newton's 2nd Law

Newton's second law is usually succinctly stated with the familiar equation

$$\mathbf{F} = m\mathbf{a} \tag{1.1.1}$$

where $\mathbf{F}$ is net force, m is mass, and $\mathbf{a}$ is acceleration.

You will notice that the force and the acceleration are in bold face. This means these are vector quantities, having both a magnitude and a direction. Mass on the other hand is a scalar quantity, which has only a magnitude. This equation indicates that a force will cause an object to accelerate in the direction of the net force, and the magnitude of the acceleration will be proportional to the net force but inversely proportional to the mass of the object.

In this course, Statics, we are only concerned with bodies which are *not* accelerating which simplifies things considerably. When an object is not accelerating $a = 0$, which implies that it is either at rest or moving with a constant velocity. With this restriction Newton's Second Law for translation simplifies to

$$\sum \mathbf{F} = 0 \tag{1.1.2}$$

where $\sum \mathbf{F}$ is used to indicate the *net* force acting on the object.

Newton's second law for rotational motions is similar

$$\mathbf{M} = I\ . \tag{1.1.3}$$

This equation states that a net moment $\mathbf{M}$ acting on an object will cause an angular acceleration proportional to the net moment and inversely proportional to I, a quantity known as the **mass moment of inertia.** Mass moment of inertia for rotational acceleration is analogous to ordinary mass for linear acceleration. We will have more to say about the moment of inertia in `[cross-reference to target(s) "Chapter_10" missing or not unique]`.

Again, we see that the net moment and angular acceleration are vectors, quantities with magnitude and direction. The mass moment of inertia, on the other hand, is a scalar quantity and has only a magnitude. Also, since Statics deals only with objects which are *not* accelerating $= 0$, they will always be at rest or rotating with constant angular velocity. With this restriction Newton's second law implies that the net moment on all static objects is zero.

$$\sum \mathbf{M} = 0 \tag{1.1.4}$$

1.1.3 Newton's 3rd Law

Newton's Third Law states

> *For every action, there is an equal and opposite reaction.*

The actions and reactions Newton is referring to are **forces**. Forces occur whenever one object interacts with another, either directly like a push or pull, or indirectly like magnetic or gravitational attraction. Any force acting on one body is always paired with another equal-and-opposite force acting on some other body.

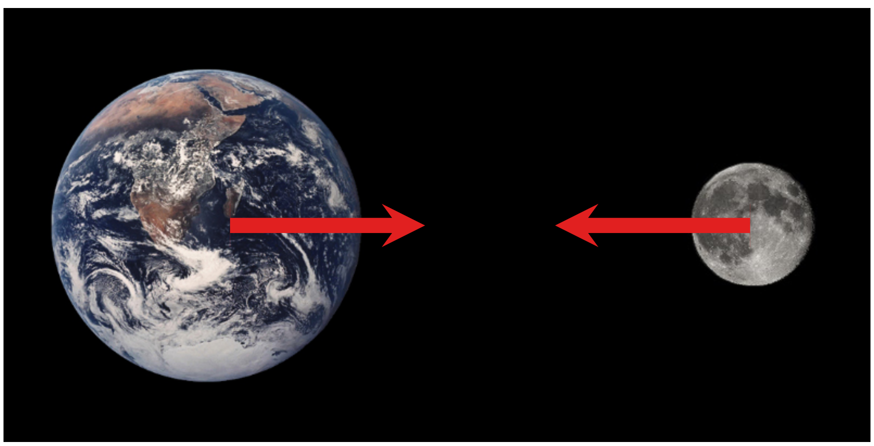


Figure 1.1.4 The earth exerts a gravitational force on the moon, and the moon exerts an equal and opposite force on the earth.

These equal-and-opposite pairs can be confusing, particularly when there are multiple interacting bodies. To clarify, we always begin solving statics problems by drawing a **free-body diagram** — a sketch where we isolate a body or system of interest and identify the forces acting *on* it, while ignoring any forces exerted *by* it on interacting bodies.

Consider the situation in figure Figure 1.1.5. Diagram (a) shows a book resting on a table supported by the floor. The weights of the book and table are placed at their centers of gravity. To solve for the forces on the legs of the table, we use the free-body diagram in (b) which treats the book and the table as a single system and replaces the floor with the forces of the floor *on* the table. In diagram (c) the book and table are treated as independent objects. By separating them, the equal-and-opposite interaction forces of the book *on* the table and the table *on* the book are exposed.

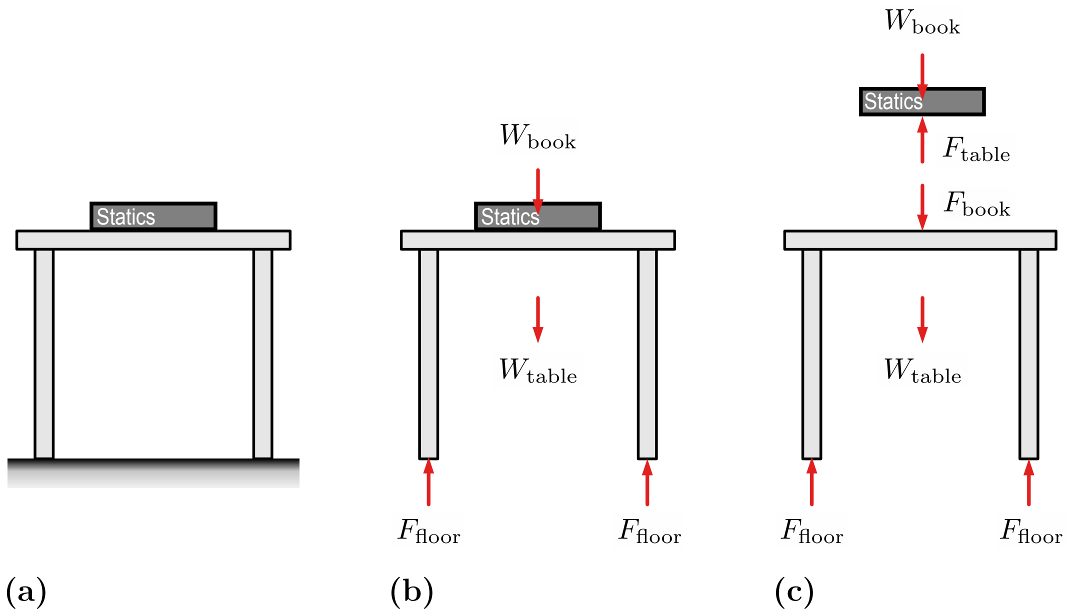


Figure 1.1.5 Free-body diagrams are used to isolate objects and identify relevant forces and moments.

This will be discussed further in `[cross-reference to target(s) "Chapter_03" missing or not unique]` and `[cross-reference to target(s) "Chapter_05" missing or not unique]`.

1.2 Units

Key Questions

- What are the similarities and differences between the SI and US Customary Unity Systems?
- How do you convert a value into different units?
- When a Statics problem lists the pounds [lb] of a body, is this referring to pounds-force [lbf] or pounds-mass [lbm]?

Most quantities used in engineering consist of a numeric value and an associated unit. The value by itself is meaningless, unless, except when the quantity is unitless.

In the United States there are two primary unit systems in use. The International System of Units, SI, abbreviated from the French *Système international (d'unités)* is the modern form of the metric system and is the most widely used system of measurement. It comprises a coherent system of units of measurement built on seven base units: the second, meter, kilogram, ampere, kelvin, mole, candela. In statics, the only the first three base units are used. All other units required are derived from combinations of the base units. Prefixes to unit names are used to specify the base-10 multiple of the original unit.

The other unit system in use is the United States customary system. This system was developed from the measurement system in use in the British Empire before the US became an independent country. However, the United Kingdom's system of measures was overhauled in 1824 to create the Imperial system, changing the definitions of some units. Therefore, while many US units are similar to their Imperial counterparts, there are significant differences between the systems. The base units in the customary system for time, distance, and mass are the second, foot, and slug.

The magnitude of a force is measured in units of mass $[m]$ times length $[L]$ divided by time $[t]$ squared

$$[F = mL/t^2].$$

In metric units, the most common force unit is the newton, abbreviated N, where one newton is a kilogram multiplied by a meter per second squared. This means that a one-newton force would cause a one-kilogram object to accelerate at a rate of one-meter-per-second-squared. In English units, the most common unit is the pound-force $[\mathrm{lb}_f]$, or pound [lb] for short, where one pound is the force which can accelerate a mass of one slug at one foot per second squared. Many physics texts use pounds mass $[\mathrm{lb}_m]$ exclusively instead of slugs, where 1 slug = 32.174 lb_m. This text will use slugs as they are the standard mass unit in US customary system and so are analogous to kilograms in the SI system.

The unit of force for the two unit systems in terms of the base units are

$$1\ \mathrm{N} = 1\frac{[\mathrm{kg}][\mathrm{m}]}{[\mathrm{s}^2]} \text{ in SI units, and}$$

$$1\ \mathrm{lb} = 1\frac{[\mathrm{slug}][\mathrm{ft}]}{[\mathrm{s}^2]} \text{ in US customary units.}$$

When you find the weight of an object from its mass you are applying Newton's Second Law.

Table 1.2.1 Fundamental Units

Unit System	Force	Mass	Length	Time	g (Earth)
SI	newton [N]	kilogram [kg]	meter [m]	second [s]	9.81 m/s^2
US Customary	pound [lb]	slug [slug]	foot [ft]	second [s]	32.2 ft/s^2
US lb_m	pound-force $[\mathrm{lb}_f]$	pound-mass $[\mathrm{lb}_m]$	foot [ft]	second [s]	1 ft/s^2

Table 1.2.1 shows the name and abbreviation of the standard units for weight, mass, length, time, and gravitational acceleration in SI and US unit systems. When in doubt always convert to these units.

Take care to consider the difference between mass and weight.

$$W = mg. \tag{1.2.1}$$

Gravitational acceleration g varies up to about 0.5% across the earth's surface due to factors including latitude and elevation, but for the purpose of this course the values in Table 1.2.1 are sufficiently accurate.

Awareness of units will help you prevent errors in your engineering calculations. You should always:

- Pay attention to the units of every quantity in the problem. Forces should have force units, distances should have distance units etc.
- Use the unit system given in the problem statement.
- Avoid unit conversions when possible. If you must, convert given values to a consistent set of units and stick with them.
- Check your work for unit consistency. You can only add or subtract quantities which have the same units. When multiplying or dividing quantities with units, multiply or divide the units as well. The units on both sides of the equals sign must be the equivalent.
- Develop a sense of the magnitudes of the units and consider your answers for reasonableness. A kilogram is about 2.2 times as massive as a pound-mass and a newton weighs about a quarter pound.
- Be sure to include units with every answer.

Example 1.2.2 How much does a 5 kg bag of flour weigh?

Hint. A value in kg is a mass. Weight is a force.

Answer. $W = 49.05\text{ N}$

Solution.

$$\begin{aligned} W &= mg \\ &= 5\text{ kg}(9.81\text{ m/s}^2) \\ &= 49.05\text{ N} \end{aligned}$$

□

Example 1.2.3 How much does a 5 lb bag of sugar weigh?

Hint. When someone says “pounds” they probably mean “pounds-force.”.

Even if they mean pounds-mass, $1\text{ lb}_m = 1\text{ lb}_f$ on earth.

Answer. $W = 5\text{ lb}$

Solution.

$$5\text{ lb} = 5\text{ lb}_f$$

□

1.3 Forces

Key Questions

- What are some of the fundamental types of forces used in statics?
- Why do we often simplify distributed forces with equivalent forces?

Statics is a course about forces and we will have a lot to say about them. At its simplest, a force is a “push or pull,” but forces come from a variety of sources and occur in many different situations. As such we need a specialized vocabulary to talk about them. We are also interested in forces that cause rotation, and we have special terms to describe these too.

As an example of the types of forces you will encounter in statics consider the forces affecting a box on a rough surface being pulled by a cable. The loading on the box can be represented by four different types of force. The cable causes a point force, the normal and friction forces are reaction forces, and the weight is a body force.

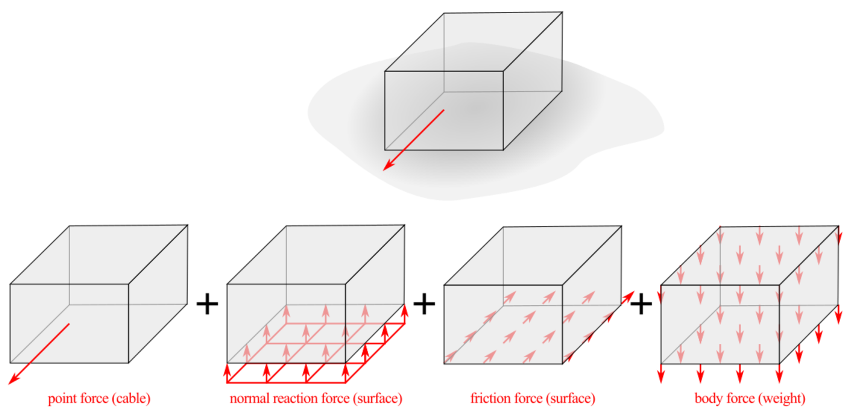


Figure 1.3.1 Forces on a box being pulled across a rough surface.

Some of the important terms used describe different types of forces are given below; others will be defined as needed later in the book.

A **point force** is a force that acts at a single point. Examples would be the push you give to open a door, the thrust of a rocket engine, or the pull of the chain suspending a wrecking ball. In reality, point forces are an idealization as all forces are distributed over some amount of area. Point forces are also called **concentrated forces**. Point forces are the easiest type to deal with computationally so we will learn some mathematical tools to represent other types as point forces.

Body forces are forces that are distributed throughout a three dimensional body. The most common body force is the weight of an object, but there are other body forces including buoyancy and forces caused by gravitational, electric, and magnetic fields. Weight and buoyancy will be the only body forces we consider in this book.

In many situations, these forces are small in comparison to the other forces acting on the object, and as such may be neglected. In practice, the decision to neglect forces must be made on the basis of sound engineering judgment; however, in this course you should consider the weight in your analysis if the problem statement provides enough information to determine it, otherwise you may ignore it.

In the example above, the point force due to the cable, and the weight of the box are both called **loads**. The weight of an object and any forces intentionally applied to it are considered loads, while forces which hold a loaded object in equilibrium or hold parts of an object together are not.

R**eaction forces** or simply **reactions** are the forces and moments which hold or constrain an object or mechanical system in equilibrium. They are called the reactions because they react when other forces on the system change. If the load on a system increases, the reaction forces will automatically increase in response to maintain equilibrium. Reaction forces are introduced in `[cross-reference to target(s) "Chapter_03" missing or not unique]` and reaction moments are introduced in `[cross-reference to target(s) "Chapter_05" missing or not unique]`.

In the example above, the force of the ground on the box is a reaction force, and is distributed over the entire contact surface. The reaction force can be divided into two parts: a **normal** component which acts perpendicular to the surface and supports the box's weight, and a **tangential** friction component which acts parallel to the ground and resists the pull of the cable.

The weight, normal component, and frictional component are all examples of **distributed forces** since they act over a volume or area and not at a single point. For computational simplicity we usually model distributed forces with equivalent point forces. This process is discussed in `[cross-reference to target(s) "Chapter_07" missing or not unique]`.

1.4 Problem Solving

Key Questions

- What are some strategies to practice selecting a tool from your problem-solving toolbox?
- What is the basic problem-solving process for equilibrium?

Statics may be the first course you take where you are required to decide on your own how to approach a problem. Unlike your previous physics courses, you can't just memorize a formula and plug-and-chug to get an answer; there are often multiple ways to solve a problem, not all of them equally easy, so before you begin you need a plan or strategy. This seems to cause a lot of students difficulty.

The ways to think about forces, moments and equilibrium, and the mathematics used to manipulate them are like tools in your toolbox. Solving statics problems requires acquiring, choosing, and using these tools. Some problems can be solved with a single tool, while others require multiple tools. Sometimes one tool is a better choice, sometimes another. You need familiarity and practice to get skilled using your tools. As your skills and understanding improve, it gets easier to recognize the most efficient way to get a job done.

Struggling statics students often say things like:

> "I don't know where to start the problem."
>
> "It looks so easy when you do it."
>
> "If I only knew which equation to apply, I could solve the problem."

These statements indicate that the students think they know how to use their tools, but are skipping the planning step. They jump right to writing equations and solving for things without making much progress towards the answer, or they start solving the problem using a reasonable approach but abandon it in mid-stream to try something else. They get lost, confused and give up.

Choosing a strategy gets easier with experience. Unfortunately, the way you get that experience is to solve problems. It seems like a chicken and egg problem and it is, but there are ways around it. Here are some suggestions which will help you become a better problem-solver.

- Get fluent with the math skills from algebra and trigonometry.
- Do lots of problems, starting with simple ones to build your skills.
- Study worked out solutions, however don't assume that just because you understand how someone else solved a problem that you can do it yourself without help.
- Solve problems using multiple approaches. Confirm that alternate approaches produce the same results, and try to understand why one method was easier than the other.
- Draw neat, clear, labeled diagrams.
- Familiarize yourself with the application, assumptions, and terminology of the methods covered in class and the textbook.
- When confused, identify what is confusing you and ask questions.

The majority of the topics in this book focus on equilibrium. The remaining topics are either preparing you for solving equilibrium problems or setting you up with skills that you will use in later classes. For equilibrium problems, the problem-solving steps are:

1. Read and understand the problem.
2. Identify what you are asked to find and what is given.
3. Stop, think, and decide on an strategy.
4. Draw a free-body diagram and define variables.
5. Apply the strategy to solve for unknowns and check solutions.

6. (a) Write equations of equilibrium based on the free-body diagram.
   (b) Check if the number of equations equals the number of unknowns. If it doesn't, you are missing something. You may need additional free-body diagrams or other relationships.
   (c) Solve for unknowns.
7. Conceptually check solutions.

Using these steps does not guarantee that you will get the right solution, but it will help you be critical and conscious of your chosen strategies. This reflection will help you learn more quickly and increase the odds that you choose the right tool for the job.