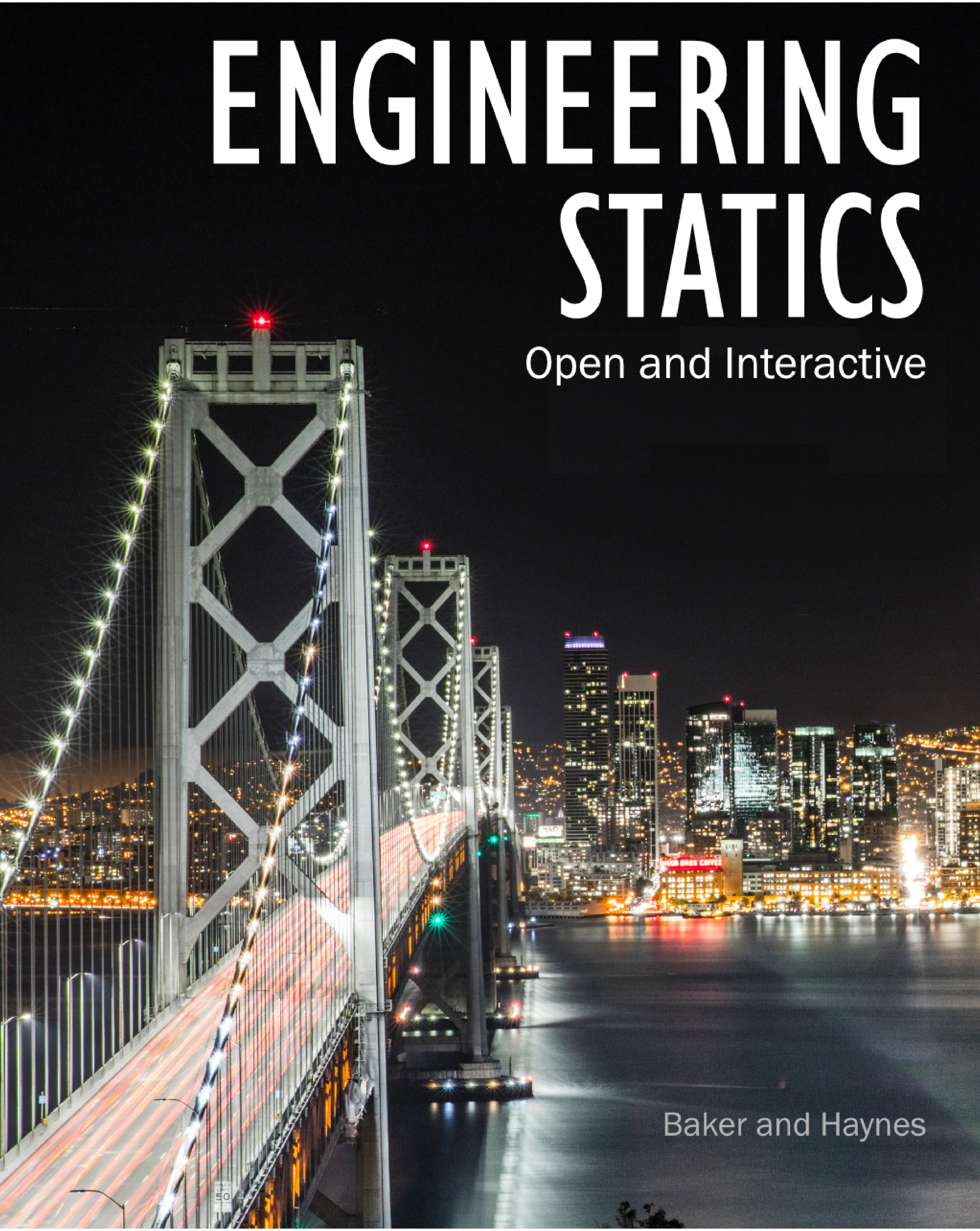
ENGINEERING STATICS

Open and Interactive



Baker and Haynes

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Website: https://engineeringstatics.org

About this Book

Engineering Statics: Open and Interactive is a free, open-source textbook for anyone who wishes to learn more about vectors, forces, moments, static equilibrium, and the properties of shapes. Specifically, it is appropriate as a textbook for Engineering Mechanics: Statics, the first course in the Engineering Mechanics series offered in most university-level engineering programs.

This book's content should prepare you for subsequent classes covering Engineering Mechanics: Dynamics and Mechanics of Materials. At its core, *Engineering Statics* provides the tools to solve static equilibrium problems for rigid bodies. The additional topics of resolving internal loads in rigid bodies and computing area moments of inertia are also included as stepping stones for later courses. We have endeavored to write in an approachable style and provide many questions, examples, and interactives for you to engage with and learn from.

Feedback. Feedback and suggestions can be provided directly to the lead author Dan Baker via email at dan.baker@colostate.edu, by clicking the feedback button in the html footer. When reporting errors, please include a bit of the surrounding text to help locate the problem area in the source. The EngineeringStaticsGoogleGroup is a good place to ask the authors and users questions about the book. Please join the group and say "Hi" if you are using the book for teaching purposes.

Access. The entire book is available for free as an interactive online ebook at https://engineeringstatics.org. While the interactive version works best on larger screens, it will also work smartphones but with some limitations due to limited screen width. A non-interactive PDF version, suitable for printing or offline reading on a tablet or computer, is available at https://engineeringstatics.org/pdf/statics.pdf. The PDF is searchable and easy to navigate using embedded links.

The source files for this book are available on GitHub at https://github.com/dantheboatman/EngineeringStatics.

End of Chapter exercises. The randomized end-of-chapter exercises were made using the Numbas open-source assessment system and Geogebra for the dynamic diagrams. Exercises in the EngineeringStaticsRepository can be freely remixed into your own homework sets or online exams using the NumbasEditor. You can also use the editor to write new questions or modify existing ones.

The exercises can be integrated into your institution's Virtual Learning Environment to set deadlines and automatically record grades. See the NumbasDocumentation for more information. To fully take advantage of all the features, you may need the support of your institution's IT department to install the NumbasLTIprovider.

Please ask questions about Numbas integration and share any good problems you write with the Engineering Statics Group.

On the Cover. Photo of the San Francisco–Oakland Bay Bridge and city skyline, taken from Yerba Buena Island by Artur Westergren.

Image source: https://unsplash.com/photos/Rx92z9dU-mA

Acknowledgements

This book is the vision of a handful of instructors who wanted to create a free and open Engineering Statics textbook filled with dynamic, interactive diagrams to encourage visualization and engineering intuition.

Dr. Baker brought together a team of volunteers from large public universities, small private colleges, and community colleges across the United States to write the text and create the interactive elements. Some content was adapted with permission from Jacob Moore's *Mechanics Map - Open Textbook Project*. `http://mechanicsmap.psu.edu/`. After two years of development the book was released to the public in 2020.

The book continues to evolve thanks to the contributions, suggestions, and corrections made by users of the text, both professors and students. The original authors are listed below, and others who have contributed are acknowledged in the source code on GitHub.

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Contents

Chapter 1

Introduction to Statics

Engineering Statics is the gateway into *engineering mechanics*, which is the application of Newtonian physics to design and analyze objects, systems, and structures with respect to motion, deformation, and failure. In addition to learning the subject itself, you will also develop skills in the art and practice of problem solving and mathematical modeling, skills that will benefit you throughout your engineering career.

The subject is called "statics" because it is concerned with particles and rigid bodies that are in equilibrium, and these will usually be stationary, i.e. static.

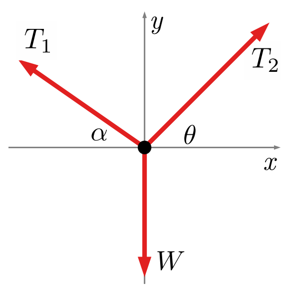
The chapters in this book are:

Moments of Inertia— an important property of geometric shapes used in many applications.

Your statics course may not cover all of these topics, or may move through them in a different order.

Below are two examples of the types of problems you'll learn to solve in statics. Notice that each can be described with a picture and problem statement, a free-body diagram, and equations of equilibrium.

Equilibrium of a particle: A 140 lb person walks across a slackline stretched between two trees. If angles α and θ are known, find the tension in each end of the slackline.

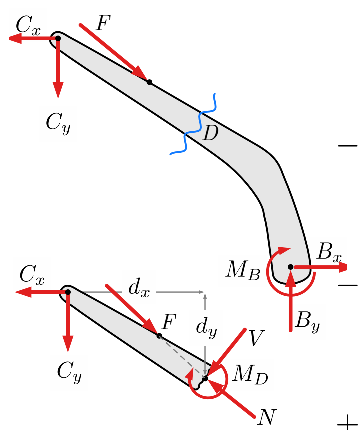


Person's point of contact to slackline:

$$\Sigma F_x = 0$$
$$-T_1 \cos\alpha + T_2 \cos\theta = 0$$

$$\Sigma F_y = 0$$
$$T_1 \sin\alpha + T_2 \sin\theta - W = 0$$

Equilibrium of a rigid body: Given the interaction forces at point C on the upper arm of the excavator, find the internal axial force, shear force, and bending moment at point D.



Section cut FBD:

$$\Sigma F_x = 0$$
$$-C_x + F_x - V_x - N_x = 0$$

$$\Sigma F_y = 0$$
$$-C_y - F_y - V_y + N_y = 0$$

$$\Sigma M_D = 0$$
$$+(d_y)C_x + (d_x)C_y - M_D = 0$$

The knowledge and skills gained in Statics will be used in your other engineering courses, in particular in Dynamics, Mechanics of Solids (also called Strength or Mechanics of Materials), and in Fluid Mechanics. Statics will be a foundation of your engineering career.

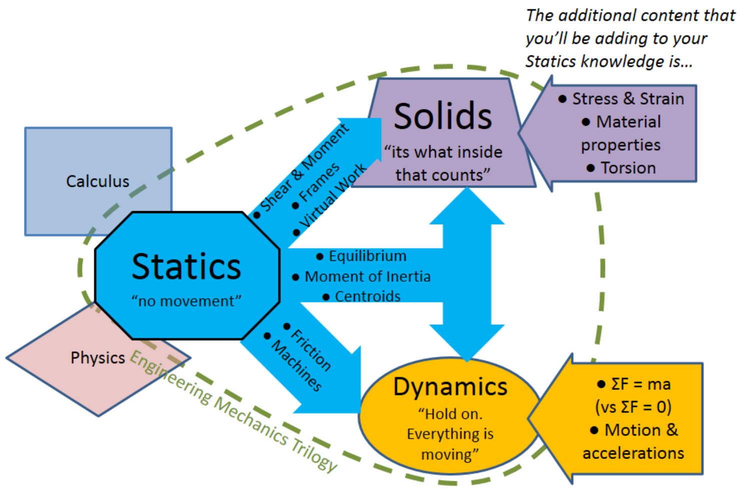


Figure 1.0.1 Map of how Statics builds upon the prerequisites of Calculus and Physics and then informs the later courses of Mechanics of Solids and Dynamics.

1.1 Newton's Laws of Motion

Key Questions

- What are the two types of motion?
- What three relationships do Newton's laws of motion define?
- What are physical examples for each of Newton's three laws of motion?

The English scientist Sir Issac Newton established the foundation of mechanics in 1687 with his three laws of motion, which describe the relation between forces, objects and motion. Motion can be separated into two types:

> **Translation**— where a body changes position without changing its orientation in space, and
>
> **Rotation**— where a body spins about an axis fixed in space, without changing its average position.

Some moving bodies are purely translating, others are purely rotating, and many are doing both. Conveniently, we can usually separate translation and rotation and analyze them individually with independent equations.

Newton's three laws and their implications with respect to translation and rotation are described below.

1.1.1 Newton's 1st Law

Newton's first law states that

> *an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force.*

This law, also sometimes called the "law of inertia," tells us that bodies maintain their current velocity unless a net force is applied to change it. In other words, an object at rest it will remain at rest and a moving object will hold its current speed and direction unless an unbalanced force causes a velocity change. Remember that velocity is a vector quantity that includes both speed and direction, so an unbalanced force may cause an object to speed up, slow down, or change direction.

Figure 1.1.1 This rock, likely a glacial erratic dropped here in the last ice age, is at rest with zero velocity and will remain at rest until an unbalanced force (like a crane or another ice age) causes it to move.

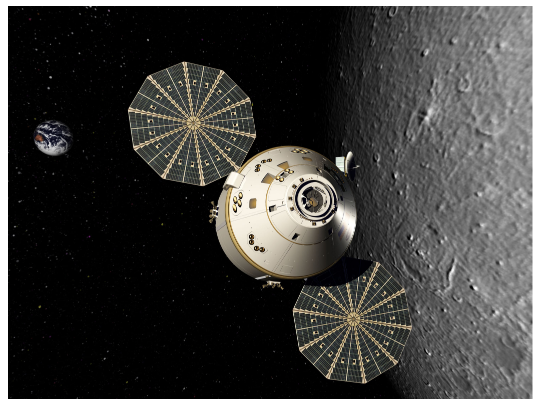


Figure 1.1.2 In deep space, where friction and gravitational forces are negligible, an object moves with constant velocity; near a celestial body gravitational attraction continuously changes its velocity.

Newton's first law also applies to angular velocities, however instead of force, the relevant quantity which causes an object to rotate is called a **torque** by physicists, but usually called a **moment** by engineers. A moment, as you will learn in Chapter **??**, is the rotational tendency of a force. Just as a force will cause a change in linear velocity, a moment will cause a change in angular velocity. This can be seen in things like tops, flywheels, stationary bikes, and other objects that spin on an axis when a moment is applied, but eventually stop because of the opposite moment produced by friction.

Figure 1.1.3 A spinning top demonstrates rotary motion.

In the absence of friction this top would spin forever, but the small frictional moment exerted at the point of contact with the table will eventually bring it to a stop.

1.1.2 Newton's 2nd Law

Newton's second law is usually succinctly stated with the familiar equation

$$\mathbf{F} = m\mathbf{a} \tag{1.1.1}$$

where $\mathbf{F}$ is net force, m is mass, and $\mathbf{a}$ is acceleration.

You will notice that the force and the acceleration are in bold face. This means these are vector quantities, having both a magnitude and a direction. Mass on the other hand is a scalar quantity, which has only a magnitude. This equation indicates that a force will cause an object to accelerate in the direction of the net force, and the magnitude of the acceleration will be proportional to the net force but inversely proportional to the mass of the object.

When studying Statics we are only concerned with bodies which are *not* accelerating which simplifies things considerably. When an object is not accelerating $a = 0$, which implies that it is either at rest or moving with a constant velocity. With this restriction Newton's Second Law for translation simplifies to

$$\Sigma\mathbf{F} = 0 \tag{1.1.2}$$

where $\Sigma\mathbf{F}$ is read as "the sum of the forces" and used to indicate the *net* force acting on the object.

Newton's second law for rotational motions is similar

$$\mathbf{M} = I\boldsymbol{\alpha}. \tag{1.1.3}$$

This equation states that a net moment $\mathbf{M}$ acting on an object will cause an angular acceleration $\boldsymbol{\alpha}$ proportional to the net moment and inversely proportional to I, a quantity known as the **mass moment of inertia.** Mass moment of inertia for rotational acceleration is analogous to ordinary mass for linear acceleration. We will have more to say about the moment of inertia in Chapter **??**.

Again, we see that the net moment and angular acceleration are vectors, quantities with magnitude and direction. The mass moment of inertia, on the other hand, is a scalar quantity and has only a magnitude. Also, since Statics deals only with objects which are *not* accelerating $\boldsymbol{\alpha} = 0$, they will always be at rest or rotating with constant angular velocity. With this restriction Newton's second law implies that the net moment on all static objects is zero.

$$\Sigma \mathbf{M} = 0 \tag{1.1.4}$$

1.1.3 Newton's 3rd Law

Newton's Third Law states

> *For every action, there is an equal and opposite reaction.*

The actions and reactions Newton is referring to are **forces**. Forces occur whenever one object interacts with another, either directly like a push or pull, or indirectly like magnetic or gravitational attraction. Any force acting on one body is always paired with another equal-and-opposite force acting on some other body.

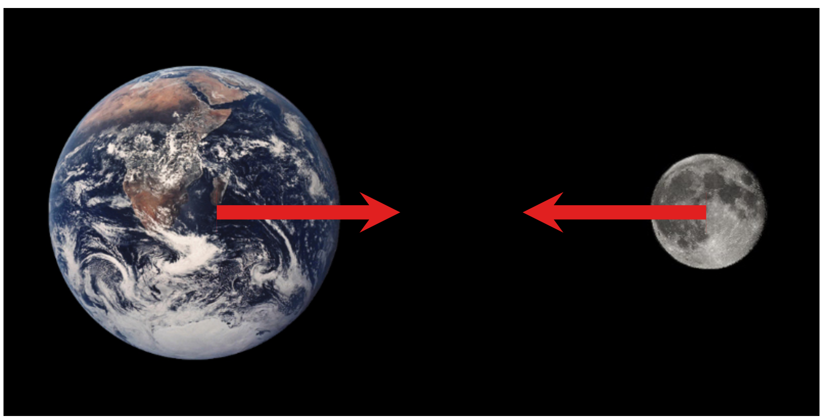


Figure 1.1.4 The earth exerts a gravitational force on the moon, and the moon exerts an equal and opposite force on the earth.

These equal-and-opposite pairs can be confusing, particularly when there are multiple interacting bodies. To clarify, we always begin solving statics problems by drawing a **free-body diagram**— a sketch where we isolate a body or system of interest and identify the forces acting *on* it, while ignoring any forces exerted *by* it on interacting bodies.

Consider the situation in Figure 1.1.5. Diagram (a) shows a book resting on a table supported by the floor. The weights of the book and table are placed at their centers of gravity. To solve for the forces on the legs of the table, we use the free-body diagram in (b) which treats the book and the table as a single system and replaces the floor with the forces of the floor *on* the table. In diagram (c) the book and table are treated as independent objects. By separating them, the equal-and-opposite interaction forces of the book *on* the table and the table *on* the book are exposed.

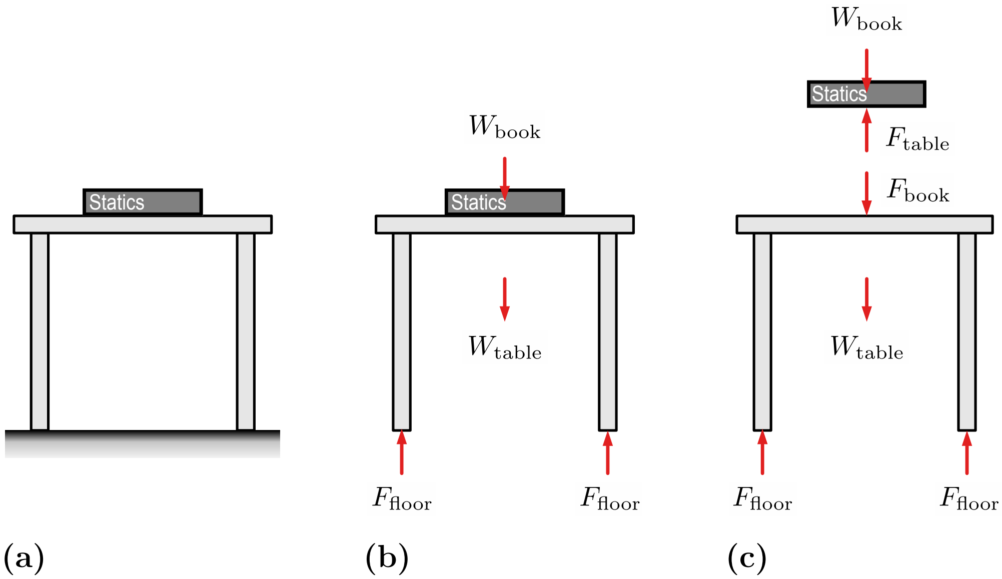


Figure 1.1.5 Free-body diagrams are used to isolate objects and identify relevant forces and moments.

This will be discussed further in Chapter **??** and Chapter **??**.

1.2 Units

Key Questions

- What are the similarities and differences between the SI, British Gravitational, and English Engineering unit systems?
- How do you convert a value into different units?
- When a Statics problem lists the pounds [lb] of a body, is this referring to pounds-force [lbf] or pounds-mass [lbm]?

Engineering quantities consist of a numeric value and an associated unit (like 150 kg, 9.81 m/s^2, or 17 ft). The values by themselves (150, 9.81, or 17) are just numbers, but the units give the numbers context. When discussing a quantity, you must *always* include the associated unit. The exceptions to this rule are unitless quantities, which are typically ratios where the units all cancel out.

Related units are defined as a coherent **unit system**. All unit systems are based on seven base units, the important ones for Statics being mass, length, and time. These base units combine to form all other measurement units. For example:

- Acceleration is defined as length [L] divided by time [t] squared, so has units $a = [L/t^2]$ or

- Force is the product of mass and acceleration, as defined by Newton's second law, so the units of force are $F = [mL/t^2]$

1.2.1 Unit Systems

Multiple unit systems are generally used in engineering practice in the United States, including the SI and British Gravitational systems. A third, the English Engineering system, is commonly only used in Physics applications.

The SI system, abbreviated from the French *Système International (d'unités)* is the modern form of the metric system. The SI system is the most widely used measurement system worldwide. In the SI system, the unit of force is the *newton*, abbreviated N, and the unit of mass is the *kilogram*, abbreviated kg. The base unit of time, used by all systems, is the *second*, abbreviated s. Prefixes are added to unit names are used to specify the base-10 multiple of the original unit. One newton is equal to 1 $\text{kg} \cdot \text{m/s}^2$ because 1 N of force applied to 1 kg of mass causes the mass to accelerate at a rate of 1 m/s^2. The SI system is used for most international engineering and is also increasingly used in the U.S. in specific fields like science, medicine, electronics, and the military.

The other unit systems, British Gravitational and English Engineering, fall under the general name of "Imperial units," given their broad use in the British Empire. The British Gravitational system uses the *foot*, abbreviated ft, as the base unit of distance, the *second* for time, and the *slug* for mass. Force is expressed in the unit of *pound-force*, abbreviated lbf, or pound for short. One pound-force will accelerate a mass of one slug at 1 ft/s^2, so $1 \text{ lbf} = 1 \text{ slug} \cdot \text{ft/s}^2$. On earth, a 1 slug mass weighs 32.174 lbf.

The English Engineering system also uses *foot* as the base unit of distance and *second* for time, but, unlike the British Gravitational system, uses *pound-mass* as the base unit of mass, where $32.174 \text{ lbm} = 1 \text{ slug} = 14.6 \text{ kg}$.

While gravitational acceleration technically remains the same between the British Gravitational and English Engineering systems, a conversion factor is required to maintain unit consistency.

$$1 = \left[\frac{1 \text{ lbf} \cdot \text{s}^2}{32.174 \text{ ft} \cdot \text{lbm}}\right] = \left[\frac{1 \text{ slug}}{32.174 \text{ lbm}}\right] \tag{1.2.1}$$

The perceived advantage of the English Engineering system is that on Earth, a mass of 1 lbm weighs 1 lbf. However, this numerical equivalence often causes confusion, given the required gravitational conversion factor mentioned in the previous paragraph and the simple fact that mass and weight are different. Mass describes how much matter an object contains, while weight is a force — the effect of gravity *on* a mass.

Therefore, the U.S. engineering community primarily embraces the British Gravitational system but bypasses the mass term of slugs, instead stating material amounts as weight in pounds-force lbf, or for brevity, pounds lb.