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Chapter 1

The Standard Model of Particle Physics

If you wish to make an apple pie from scratch, you must first invent the universe.
–Carl Sagan, *Cosmos: A Personal Voyage*

As it stands, what has become known as the 'Standard Model (SM) of Particle Physics' is nothing less than one of the greatest achievments of mankind, due to both the magnitude by which it has changed our perception of the underlying nature of the universe and to the clever methods and tinkerings by which this nature was unveiled by many clever physicists whose history has become veritable lore. In terms of imagination and insight, it is second only to the special and general theories of relativity – though the fields are nevertheless intricately intertwined.

Not considering the scientific progress made in the 18^{th} and 19^{th} centuries, and ignoring the ancient Greeks despite their fabled invention of atomic theory, the physical insights and major work that led to the current picture of elementary particle physics described by the SM began with the *annus mirabilis* papers of Albert Einstein in the year 1905 [1, 2, 3]. In these papers, Einstein was able to shed light on the quantization of electromagnetic radiation (building off of the seminal work of Max Planck [4]) and introduce the special theory of relativity. These works laid the conceptual and philosophical groundwork for the major breakthroughs in fundamental physics of 20^{th} century physics: from the 'old quantum theory' of Bohr and Sommerfeld in the early 1900's to the equivalent wavefunction and matrix-mechanics formulations of Schrödinger and Heisenberg that coalesced into 'modern' quantum mechanics in the mid-1920's. The modern approach, non-relativistic at its heart, provided a sufficient mathematical and interpretable framework in which to work and match predictions to observed phenomena, old and new. It has for the most part remained unchanged and is the quantum mechanics that is taught to students at both the undergraduate and graduate level to this very day. It is the theory that has since revolutionised all aspects of the physical sciences and technologies that dictate our everyday-lives. In the mid-1920's, however, despite large efforts put forth by the forbears of modern quantum mechanics, the quantum-mechanical world had yet to be made consistent with Einstein's theory of relativity — a requirement that must be met for all consistent physical theories of nature. It was the insight of Paul Dirac who was finally able to successfully marry the theory of the quantum with that of relativity when he introduced his relativistic quantum-mechanical treatment of the electron in 1927 and 1928 [5, 6].[1] This work provided the starting point for a decades-long search of a consistent quantum-mechanical and relativistic treatment of electrodynamics, known as *quantum electrodynamics* (QED). The search for QED ended at the end of the 1940's with the groundbreaking work of Dyson, Feynman, Schwinger, and Tomanaga [9, 10, 11, 12, 13, 14, 15, 16] that introduced the covariant and gauge invariant formulation of QED — the first such relativistic quantum field theory (QFT). QED allowed the physcists to make predictions that agreed with observation at unprecedented levels of accuracy and has since led to the adoption of its language and mathematical toolkit as the foundational framework in which to construct models that accurately describe nature.[2] The SM is no less than an ultimate conclusion of these works: a consistent set of relativistic quantum field theories, using the language developed by Feynman et al., that describes essentially all aspects of the known particles and forces that make up the observed universe.

1.1 Particles and Forces

There are four known fundamental forces at work in the universe: electromagnetism, the weak interaction, the strong interaction, and gravity. Our understanding of the existence of each of these forces has essentially been arrived at empirically, with physicists following experimental clues, and their basic behaviors deduced after long trials of effort. The SM encompasses all of these forces except for gravity, which currently is only described by the classical (i.e. not quantum) theory of geometrodynamics, or general relativity. The gravitational interaction is incredibly weak in comparison to the others, however, and is not relevant to the types of particle interactions that we are currently sensitive to in particle

[1] A complete history of the people and ideas involved in the development of the modern theory of Quantum Mechanics can be found in references [7, 8], and the references therein.

[2] For a complete discussion of the developments leading up to QED, see the fabulous book by S. Schweber [17].

physics experiments. Electromagnetism is by far the most familiar, as it is the force most commonly experienced and is what is at work in our everyday life (reaction forces between objects on tables and chairs, friction, wall-plugs, batteries, DNA structure, etc...) and is typically what students are first presented with in their physics studies. The weak force is responsible for things like radioactive decay, which makes possible the process of nuclear β-decay and the nuclear fission process that fuels the sun, for example. The strong force is what binds protons and neutrons together, and thus is resonsible for holding together most of the (ordinary) matter in the universe.[3]

The forces mediate the interactions between the matter particles, which we use to deduce their presence. The SM predicts fundamental, point-like particles that appear in two general classes depending on whether they have integral spin ($\mathcal{S} \in [0, 1, 2, ...)$) or half-integral spin ($\mathcal{S} \in [1/2, 3/2, ...)$); the former are referred to as *bosons* and the latter as *fermions*. In the SM, the particles that are responsible for making up matter are all spin-1/2 fermions and are either *leptons* or *quarks*; within each class there are three generations (or families) that are essentially copies of the first. The forces in the SM are interpreted as being mediated by spin-1 bosons, referred to as the *gauge bosons*. The leptons and quarks all experience the weak force, but only the quarks experience the strong interaction. All electrically charged particles interact with the electromagnetic interaction.

The particles of the SM are described as quantum fields whose dynamics are described by the SM Lagrangian from which the equations of motions can be derived. The particles, and by extention the SM Lagrangian that descibes them, are found to be invariant under transformations of spacetime (space translations, rotations, Lorentz boosts) and three internal transformations described by unitary transformations: $\mathcal{P} \times \mathcal{SU}(3)_C \times \mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$. This is illustrated in Figure 1.1. The strong force is described by a local $\mathcal{SU}(3)$ symmetry that acts only on the particles that have *color charge*. The term “color” arises from the fact that the color charge is found to exist in three varieties which have been labelled as red (r), blue (b), or green (g), and due to the fact that “colorless” states are formed when all three are combined (r+g+b), just like with visible light that humans are familiar with, or when states are formed of color-anti-color pairs (r+$\bar{\text{r}}$). For this reason, the QFT describing the strong force is called Quantum *Chromo*dynamics (QCD), and is mediated by eight *gluons* (G). The particles subject to the weak force are invariant under weak-isospin $\mathcal{SU}(2)$ transformations, mediated by the three W bosons (W_1, W_2, W_3). The $\mathcal{U}(1)$ transformations, mediated by the B boson, preserve weak-hypercharge, Y. The $\mathcal{SU}(2)$ symmetry is respected only by the left-handed chiral particles (leptons or quarks), with the right-handed chiral particles not participating. There is additionally a single scalar (i.e. spin-0) field, the Higgs field, that is

[3]‘Ordinary’ to distinguish from dark matter, for example.

an $\mathcal{SU}(2)$ doublet, about which more will be described shortly. The particle content thus described is presented in detail in Table 1.1. The $\mathcal{SU}(2)$ left-handled chiral fields appear as doublets and are grouped in and "up-down" pair (e.g. $(u_{\mathrm{L}},\, d_{\mathrm{L}})$ or $(e_{\mathrm{L}},\, \nu_{e,\mathrm{L}})$) whereas the right-handed chiral fields, living in the singlet representation of $\mathcal{SU}(2)$, do not (e.g. u_{R})). Note that the SM does not allow for right-handed neutrinos (a term like ν_{R} does not appear).

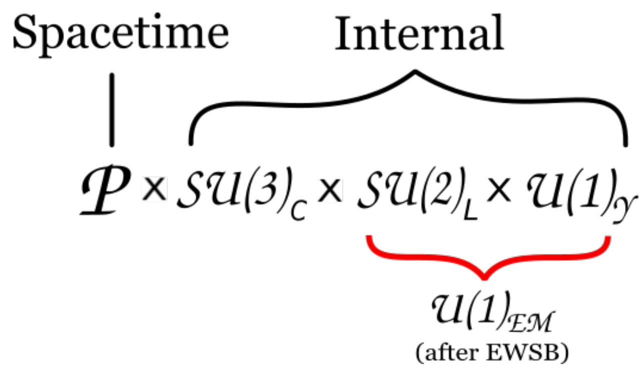

Figure 1.1: The spacetime and internal gauge structure of the SM. $\mathcal{P}$ refers to the Poincaré symmetry group. $\mathcal{SU}(3)_c$ refers to the $\mathcal{SU}(3)$ symmetry of the color sector of QCD and $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ refers to the left-handed chiral symmetry of the electroweak interaction. After spontaneous symmetry breaking due to the Higgs mechanism, the $\mathcal{SU}(2)_L \times\, \mathcal{U}(1)_Y$ symmetry reduces to the $\mathcal{U}(1)_{EM}$ symmetry of electromagnetism.

The SM Lagrangian is shown in Eqn. 1.1 and describes the complete content of the SM: encompassing all interactions between the known particles and the symmetries that they obey.

$$\mathcal{L}_{\mathrm{SM}} = -\frac{1}{4} \sum_{\text{gauge}} F^{i}_{\mu\nu} F^{i\,\mu\nu} - \sum_{f} \bar{f} \gamma^{\mu} D_{\mu} f + (D_{\mu}\phi)^{\dagger}(D^{\mu}\phi) - \mu^2 \phi^{\dagger}\phi - \lambda(\phi^{\dagger}\phi)^2 \tag{1.1}$$

The first term of Eqn. 1.1 is a sum over the three internal gauge goups, and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$, where A_μ is one of the three gauge fields, g is the associated gauge coupling parameter, and a sum over i is implied. The f^{abc} are the so-called *structure constants* of the gauge group. For Abelian groups like $\mathcal{U}(1)$, $f^{abc} = 0$. For non-Abelian gauge groups like $\mathcal{SU}(2)$ and $\mathcal{SU}(3)$, $f^{abc} \neq 0$. For example, for $\mathcal{SU}(2)$ the structure constants are nothing more than the Levi-Civita totally anti-symmetric tensor, ε_{ijk}, giving for the weak gauge force:

$$\boldsymbol{\mathcal{W}}_{\mu\nu} = \partial_\mu \boldsymbol{\mathcal{W}}_\nu - \partial_\nu \boldsymbol{\mathcal{W}}_\mu - g_2 \boldsymbol{\mathcal{W}}_\mu \times \boldsymbol{\mathcal{W}}_\nu \tag{1.2}$$

where $\boldsymbol{\mathcal{W}}_\mu$ is the vector of the three weak gauge fields (W_1, W_2, and W_3) and g_2 is their associated gauge coupling. The non-zero f^{abc} of non-Abelian gauge groups means that the gauge bosons of the weak and strong interactions can interact with themselves due to terms appearing in Eqn. 1.1 that contain only the gauge bosons. add Feynman diagram? – showing what the squared term of $W_{\mu\nu}$ representing triple and quartic couplings

The second term of Eqn. 1.1 describes the lepton and quark kinetic energies and their interactions with the gauge fields. The f refer to the fermion fields (quarks and leptons) and the corresponding sum is over all species of fermion. D_μ is the gauge covariant derivative, and for the SM is given by:

$$D_\mu = \partial_\mu - ig_1 \frac{Y}{2}\mathcal{B}_\mu - ig_2 \frac{\tau^i}{2}\mathcal{W}^i_\mu - ig_3 \frac{\lambda^a}{2} G^a_\mu \tag{1.3}$$

where g_1, g_2, and g_3 are the gauge coupling constants for $\mathcal{U}(1)_Y$, $\mathcal{SU}(2)_L$, and $\mathcal{SU}(3)_C$, respectively, that give the overall strength of the associated coupling. Summation over repeated indices is implied and the τ^i (λ^a) are the three (eight) generators of the $\mathcal{SU}(2)$ ($\mathcal{SU}(3)$) gauge group, with $i \in [1, 2, 3]$ ($a \in [1, .., 8]$), and are typically represented by the Pauli (Gell-Mann) matrices. Note that the form of Eqn. 1.3 is strictly mandated by the requirement that the theory be *gauge invariant*, i.e. that transformations of the fields under the internal symmetries of Fig. 1.1 leave the action of $\mathcal{L}_{\mathrm{SM}}$ unchanged. This is described in detail in Appendix XXX.

The last three terms in Eqn. 1.1 are all terms including the Higgs field, ϕ, and will be discussed in detail in Section 1.3.

Inspection of Eqn. 1.1 will reveal two things. The first thing that one may notice is that it does not appear to describe electromagnetism as it does not have a term representing the photon, the familiar mediator of the electromagnetic interaction. The second, and perhaps more immediately obvious, thing is that no mass terms appear in $\mathcal{L}_{\mathrm{SM}}$: all fields appear to have zero mass! Both of these facts are counter to our everday experience: we know electromagnetism is real and that matter, at the very least, is massive. In the next few sections we will see how these apparent issues are resolved.

Table 1.1: Fix quantum numbers The particle content of the SM and their transformation properties under the SM gauge groups, prior to electroweak symmetry breaking. The representations of each of the gauge groups are shown in the three-right columns. The $\mathcal{U}(1)$symmetry of weak-hypercharge transformations is one-dimensional and the column gives the weak-hypercharge $\mathcal{Y}$ associated with each field. For $\mathcal{SU}(3)$and $\mathcal{SU}(2)$, **1** refers to the field belonging to the associated singlet representation, **2** to the doublet representation, **3** to the triplet representation, and **8** to the octet representation.

	Field Label	Content	Spin	$\mathcal{U}(1)$ $(=\mathcal{Y})$	$\mathcal{SU}(2)$	$\mathcal{SU}(3)$
Quarks	Q_i	$(u_\mathrm{L},\, d_\mathrm{L}),\, (c_\mathrm{L},\, s_\mathrm{L}),\, (t_\mathrm{L},\, b_\mathrm{L})$	1/2	1/6	**2**	**3**
	$u_{\mathrm{R},i}$	u_R	1/2	2/3	**1**	**3**
	$d_{\mathrm{R},i}$	d_R	1/2	$-1/3$	**1**	**3**
Leptons	L_i	$(e_\mathrm{L},\, \nu_{e,\mathrm{L}}),\, (\mu_\mathrm{L},\, \nu_{\mu,\mathrm{L}}),\, (\tau_\mathrm{L},\, \nu_{\tau,\mathrm{L}})$	1/2	1/2	**2**	**1**
	$e_{\mathrm{R},i}$	$e_\mathrm{R},\, \mu_\mathrm{R},\, \tau_\mathrm{R}$	1/2	-1	**1**	**1**
Gauge Fields	B	B	1	0	**1**	**1**
	W	$(W_1,\, W_2,\, W_3)$	1	0	**3**	**1**
	G	$G_a,\, a \in [1, .., 8]$	1	0	**1**	**8**
Higgs Field	ϕ	$(\phi^+,\, \phi^0)$	0	1/2	**2**	**1**

1.2 The Electroweak Theory

It was the work of Glashow, Weinberg, and Salam (GWS) that ultimately put forth a consistent picture of the chiral weak force and its unification with electromagnetism [18, 19, 20]. As a result, the theory of particles and fields that respect the $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ gauge invariance of the SM is sometimes referred to as 'GWS theory', but is more typically known as the electroweak theory. Since all matter particles are subject to the electroweak interaction, but only a subset of the particles that have color charge (the quarks) are subject to the strong interaction described by QCD, the study of the SM can essentially be partitioned into two parts: the part that deals with the dynamics and interactions of colored objects (the 'QCD part', $\mathcal{L}_\mathrm{QCD}$) and the part that deals with electroweak interactions, including the Higgs (the 'Electroweak part', $\mathcal{L}_\mathrm{Electroweak}$). Given the broad reach of the electroweak interaction, in the early days GWS theory was considered the heart of the SM and why GWS were awarded the Nobel prize in 1979.[4] In this section we will focus on the $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ portion of $\mathcal{L}_\mathrm{SM}$.

[4] Actually, the acceptance of the GWS theory as the de-facto SM of the time was not widely held until some years after its publication, when t'Hooft proved that it was renormalizable [21, 22]. Such a complete

The first thing to remember is that the electroweak theory is *chiral*, i.e., it distinguishes between left- and right-chiral fermion fields. For conceptual clarity, it can be useful to take the massless (relativistic) limit of fermions to get an idea of what chirality represents. For a massless fermion field, the chirality is equivalent to the perhaps more-familiar *helicity*, defined as the projection of its spin onto its momentum (direction of motion). The helicity of left-handed (right-handed) massless fermions is positive (negative), meaning that their spin is parallel (anti-parallel) to its momentum. Fermion fields, then, are commonly defined inclusive of their handedness, with the left- and right-handed components projected out using the P_L and P_R projection operators,

$$f_{\mathrm{L}} = P_L f = \frac{1}{2}(1-\gamma_5)f \quad \text{and} \quad f_{\mathrm{R}} = P_R f = \frac{1}{2}(1+\gamma_5)f. \tag{1.4}$$

Focusing only on the first generation of the leptons (the discussion holds equally well for the second and third generations, as well as for the quarks), we can gather the $\mathcal{U}(1)$ terms of Eqn. 1.1,

$$\begin{aligned} -\mathcal{L}_{\mathrm{ferm}}(\mathcal{U}(1), \mathrm{leptons}) &= \bar{L} i\gamma^\mu (i g_1 \frac{Y_L}{2}\mathcal{B}_\mu) L + \bar{e}_R i\gamma^\mu (i g_1 \frac{Y_R}{2} B_\mu) e_R \\ &= \frac{g_1}{2}[Y_L(\bar{\nu}_L\gamma^\mu \nu_L + \bar{e}_L\gamma^\mu e_L) + Y_R \bar{e}_R \gamma^\mu e_R] B_\mu, \end{aligned} \tag{1.5}$$

where $L = (\nu_L, e_L)$ is used in going from the first to second line. Likewise, gathering the associated $\mathcal{SU}(2)$ terms and noting that $\tau^i W^i$ is a 2×2 matrix since the τ^i are $\mathcal{SU}(2)$ generators (e.g. the Pauli matrices) gives,

$$\begin{aligned} -\mathcal{L}_{\mathrm{ferm}}(\mathcal{SU}(2), \mathrm{leptons}) &= \bar{L}\, i\gamma^\mu [i g_2 \frac{\tau^i}{2} W^i_\mu]\, L \\ &= -\frac{g_2}{2}\left[\bar{\nu}_L\gamma^\mu\nu_L W^0_\mu - \sqrt{2}\bar{\nu}_L\gamma^\mu e_L W^+_\mu - \sqrt{2}\bar{e}_L\gamma^\mu\nu_L W^-_\mu - \bar{e}_L\gamma^\mu e_L W^0_\mu\right], \end{aligned} \tag{1.6}$$

where we have used the following re-definitions of the $\mathcal{SU}(2)$ gauge fields,

$$W^+_\mu = \frac{1}{\sqrt{2}}\left(-W^1_\mu + iW^2_\mu\right) \qquad W^-_\mu = \frac{1}{\sqrt{2}}\left(-W^1_\mu - iW^2_\mu\right) \qquad W^0_\mu = W^3_\mu. \tag{1.7}$$

In principle, Equations 1.5 and 1.6 describe completely all electroweak interactions between matter and the gauge fields of $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$. We would like to make the correspondence between these equations and what we know to empirically exist: the electromagnetic interaction and the presence of a massive, charged mediator of the weak nuclear force re-

understanding in the QCD sector would not come until almost a decade later, in the late 1970'sWoijkec etc CITE.

sponsible for nuclear β-decay, for example. From the theory of QED, it is a-priori known what the form of the interaction of the neutral photon and the electron should look like. Inspecting all charge-preserving (i.e. neutral) terms of Eqn. 1.5 and 1.6, it can be seen that the B_μ and ${W^0}_\mu$ fields have this expected fermion coupling, suggesting a re-definition as follows,

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W^0_\mu \end{pmatrix}, \tag{1.8}$$

where we have used $Y_L = -1$ and define the relations between the $\mathcal{SU}(2)$ and $\mathcal{U}(1)$ couplings as,

$$\sin\theta_W = \frac{g_1}{\sqrt{g_1^2+g_2^2}} \qquad \cos\theta_W = \frac{g_2}{\sqrt{g_1^2+g_2^2}} \qquad e = \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}}. \tag{1.9}$$

The angle θ_W is known as the *Weinberg angle*. It quantifies the amount of *gauge mixing* that occurs between the neutral $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ gauge fields, B_μ and ${W^0}_\mu$.

The above algebra allows to re-write the electroweak Lagrangian, now describing interactions between the fermions and the newly defined A_μ, Z, and $W^\pm$, as,

$$\begin{aligned} \mathcal{L}_{\text{ferm, first-gen.}} = & \underbrace{\sum_{f\in\nu_e,e,u,d} eQ_f\left(\bar{f}\gamma^\mu f\right)A_\mu}_{\text{Neutral, }\sim\text{ EM}} \\ & \underbrace{+\frac{g_2}{\cos\theta_W}\sum_{f\in\nu_e,e,u,d}\left[\ \bar{f}_L\gamma^\mu f_L\left(T_f^3 - Q_f\sin^2\theta_W\right) + \bar{f}_R\gamma^\mu f_R\left(-Q_f\sin^2\theta_W\right)\ \right] Z_\mu}_{\text{Neutral weak interaction}} \\ & \qquad\qquad \underbrace{+\frac{g_2}{\sqrt{2}}\left[\ \left(\bar{u}_L\gamma^\mu d_L + \bar{\nu}_{e,L}\gamma^\mu e_L\right) W_\mu^+ + h.c.\ \right]}_{\text{Charged weak interaction}} \end{aligned} \tag{1.10}$$

The first term of Eqn. 1.10 has the expected form expected from QED, describing the interaction between a neutral gauge boson and fermions and allows us to interpret the parameter e, introduced in Eqn. 1.9, as the coupling of electromagnetism (electric charge) with Q_f as the fermion's electric charge quantum number (in units of e). The A_μ arrived at via the gauge mixing of Eqn. 1.8 then must correspond to the photon of electromagnetism. The second term of Eqn. 1.10 predicts the existence of an additional neutral gauge boson, the Z boson, with its couplings to the left- and right-handed fermions dictated by the $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ gauge mixing. The quantity T_f^3 is the fermion field's quantum number associated with the third component of weak-isospin ($\mathcal{SU}(2)$). The third term of Eqn. 1.10 involves charged weak-interactions involving the ${W^\pm}_\mu$ gauge bosons that transform the up- and down-type fields of the left-handed $\mathcal{SU}(2)$ doublet fields into each other.

The terms involving ${W^\pm}_\mu$ in Eqn. 1.10 are of the form $\bar{\nu}_L\gamma^\mu e_L$ which, using the chiral projection operators (Eqn. 1.4), can be written as follows,

$$\bar{\nu}_L \gamma \mu e_L = \frac{1}{2}\bar{\nu}\gamma^\mu(1-\gamma_5)e, \tag{1.11}$$

showing that the charged weak interactions involving ${W^\pm}_\mu$ are the coherent sum of vector (γ^μ) and axial-vector ($\gamma^\mu\gamma_5$) bilinear covariants; this is the famous *V-A* charged-current interaction of Fermi's nuclear β-decay. It is this *V-A* form that results in the charged interactions of the weak force not being invariant under chiral transformations ($f_R \leftrightarrow f_L$): they involve only the left-chiral fermion fields. For this reason, *parity* is said to be maximally violated by the weak interaction.[5] This result, as presented in the above, is due to our having injected it into our assumption on the field content in the first place out of hindsight. There is no first-principles reason why the weak interactions should be this way, however, and historically it was arrived at empirically.

What we have principally shown in this section is that, in order for the $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ content of the SM Lagrangian to correspond to what is found experimentally, it is expected that the gauge fields of the underlying symmetries mix. Specifically, the neutral $\mathcal{SU}(2)$ gauge field (${W^0}_\mu$) mixes with that of the $\mathcal{U}(1)$ gauge symmetry (B_μ) by an amount dicated by the Weinberg angle, θ_W, resulting in descriptions of interactions consistent with the experimentally observed photon and a prediction of a neutral Z-boson. The fermion electric charge is seen to be dependent on this mixing and can be related to the underlying $\mathcal{SU}(2)$ and $\mathcal{U}(1)$ gauge symmetries by the Gell-Mann-Nishijima relation,

$$Q_f = T^3_f + \frac{1}{2}Y, \tag{1.12}$$

This relation summarises well the result of the $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ gauge mixing and allows one to infer that the electromagnetism of common experience is related to the weak interaction and is in fact just one aspect of a unified electroweak interaction. Later on, we will see that (gauge) unification such as this plays a large role in our current understanding of the universe.

We have thus shown that the SM predicts the existence of the familiar electromagnetic force and potentially provides an additional mediator (the ${W^\pm}_\mu$) for the charged weak interaction that, prior to the formulation of GWS, was lacking a consistent physical description. However, it is still not evident how the SM can support the experimental fact that fermions have mass and that the predicted mediator of the charged weak-nuclear force (the ${W^\pm}_\pm$) *must* be massive given the very short range of the interaction. In order for such mass terms

[5]A parity transformation refers to inverting a field's space coordinates as $\vec{x} \to -\vec{x}$.

to be allowed in $\mathcal{L}_{\mathrm{SM}}$, we need the Higgs mechanism.

1.3 The Higgs Mechanism and Electroweak Symmetry Breaking

The missing mass-terms for the fields in $\mathcal{L}_{\mathrm{SM}}$ are provided by the Brout-Englert-Higgs (BEH) mechanism [23, 24, 25]. Before describing the specifics of the BEH mechanism, we should first describe the problem of why $\mathcal{L}_{\mathrm{SM}}$ doesn't support general mass terms for any of the fields in the first place. That is, for example, why can't a fermion term like $m\bar{f}f$ exist in $\mathcal{L}_{\mathrm{SM}}$?

Adding mass terms to $\mathcal{L}_{\mathrm{SM}}$ for the fermions explicitly breaks the underlying $\mathcal{SU}(2)$ gauge symmetry. This can be understood if we recognize the experimentally supported fact that the left-handed fermions appear as $\mathcal{SU}(2)$ doublets and that the right-handed fermions as singlets,

$$\begin{aligned} m\bar{f}f &= m\bar{f}(P_L + P_R)f \\ &= m\bar{f}P_L P_L f + m\bar{f}P_R P_R f \\ &= m\left(\bar{f}_R f_L + \bar{f}_L f_R\right),, \end{aligned} \tag{1.13}$$

where we have used identity relations of the projection operators P_L and P_R and the fact that $\bar{f}P_L = \bar{f}_R$ (and vice-versa). The last line of Eqn. 1.13 involve terms mixing $\mathcal{SU}(2)$ doublets with $\mathcal{SU}(2)$ singlets. Such a term is therefore not allowed if we wish to keep the $\mathcal{SU}(2)$ gauge symmetry intact.

Mass terms for the gauge bosons, of the form $mB_\mu B^\mu$, also do not work. For the Abelian $\mathcal{U}(1)$ symmetry, for example, gauge invariance implies invariance of $\mathcal{L}_{\mathrm{SM}}$ under transformations of the form $B'_\mu \to B_\mu - \partial_\mu \chi/g$. Such a mass term for the gauge bosons is clearly not invariant under such a transformation. Even forgoing this fact, adding such a term would quickly lead to non-renormalisable divergences appearing in the theory, due to the longitudinal field components that appear in massive field propagators, rendering $\mathcal{L}_{\mathrm{SM}}$ meaningless.

The BEH mechanism provides a way out of this problem. It refers to the introduction of a spin-0 field, theHiggs field (Table 1.1), to the SM along with its corresponding interaction terms to $\mathcal{L}_{\mathrm{SM}}$: the last three terms of Eqn. 1.1. These terms make up what is referred to as the Higgs potential which can be expressed as (ignoring the kinetic terms proportional to

D_μ),

$$V(\phi) = -\mu^2\phi^2 - \lambda\phi^4 \tag{1.14}$$

The Higgs field is an $\mathcal{SU}(2)$ doublet and it can be seen that the interactions described by Eqn. 1.14 respect $\mathcal{SU}(2)$ gauge symmetry. If $\mu^2 > 0$, nothing all too interesting occurs and Eqn. 1.14 describes a self-interacting, complex scalar field. If we take $\mu^2 < 0$, however, then the classical potential described by Eqn. 1.14 has non-zero minima located at $\phi = \pm v$ with $v = \sqrt{-\mu^2/\lambda}$. This is illustrated in Fig. 1.2. We see that the stable equilibrium point of the Higgs potential, the *Higgs vacuum expectatin value* (vev), is not at $\phi = 0$ but at v,

$$\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \tag{1.15}$$

The choice of Eqn. 1.15 to represent the Higgs vacuum is motivated by the requirement that the vacuum not be electrically charged — a fact motivated very much by experiment and everyday experience — so the up-type $\mathcal{SU}(2)$ component of the Higgs field, ϕ^+ (Table 1.1), is chosen to be zero for ϕ_0. The choice of an electrically neutral vacuum sets the rest of the $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ structure of the complex Higgs field since, by the Gell-Mann-Nishijima relation (Eqn. 1.12) and charge conservation, a neutral $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ field should have down-type $\mathcal{SU}(2)$ quantum numbers and $\mathcal{U}(1)$ hypercharge $Y = 1$,

$$Q = T_3 + \frac{1}{2}Y \rightarrow Q_{\phi_0} = -\frac{1}{2} + \frac{1}{2} \times 1 = 0. \tag{1.16}$$

Note that Eqn. 1.15 states that only one component of the Higgs $\mathcal{SU}(2)$ doublet attains a non-zero vev. This clearly means that the $\mathcal{SU}(2)$ gauge symmetry is not respected by the choice of $\mu^2 < 0$ and that the electroweak $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ symmetry is *spontaneously broken*.[6] The Higgs field acquiring a non-zero vev is then referred to as the *electroweak symmetry breaking* (EWSB) of the SM.

To further examine the physical consequences of EWSB, we perturb the Higgs field about the its minimum value of v,

$$\phi(x) \propto \begin{pmatrix} 0 \\ \frac{1}{2}(v + h(x)) \end{pmatrix}, \tag{1.17}$$

[6]A symmetry of a Lagrangian is said to be 'spontaneously' broken if the Lagrangian of the underlying theory respects the symmetry but it gets broken through dynamical means or if the lowest-energy state (vacuum) does not respect the symmetry.

where $h(x)$ correspond to excitations of the Higgs field that represent the physically observable Higgs boson. If we plug Eqn. 1.15 into the $D_\mu \phi$ terms of Eqn. 1.1, one eventually works through the algebra and obtains,

$$|D_\mu \phi(x)|^2 = \frac{1}{8} v^2 g_2^2 \left[\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2 \right] + \frac{1}{8} v^2 \left(g_1 B_\mu - g_2 W_\mu^3\right)^2 . \tag{1.18}$$

mention gauge-boson-higgs interactions / diagrams? Using the field re-definitions for the W_μ, A_μ and Z_μ introduced in Section 1.2, we see that this can be re-written as (modulo factors of 2),

$$|D_\mu \phi(x)|^2 \propto \left(\frac{1}{2} v g_2\right)^2 W_\mu^+ W^{-\,\mu} + \left(\frac{1}{2} v \sqrt{g_1^2 + g_2^2}\right)^2 Z_\mu Z^\mu + (0)^2 A_\mu A^\mu, \tag{1.19}$$

which provide, clearly, mass terms for the electroweak gauge bosons:

$$M_W = \frac{1}{2} v g_2, \qquad M_Z = \frac{1}{2} v \sqrt{g_1^2 + g_2^2}, \qquad M_A = 0. \tag{1.20}$$

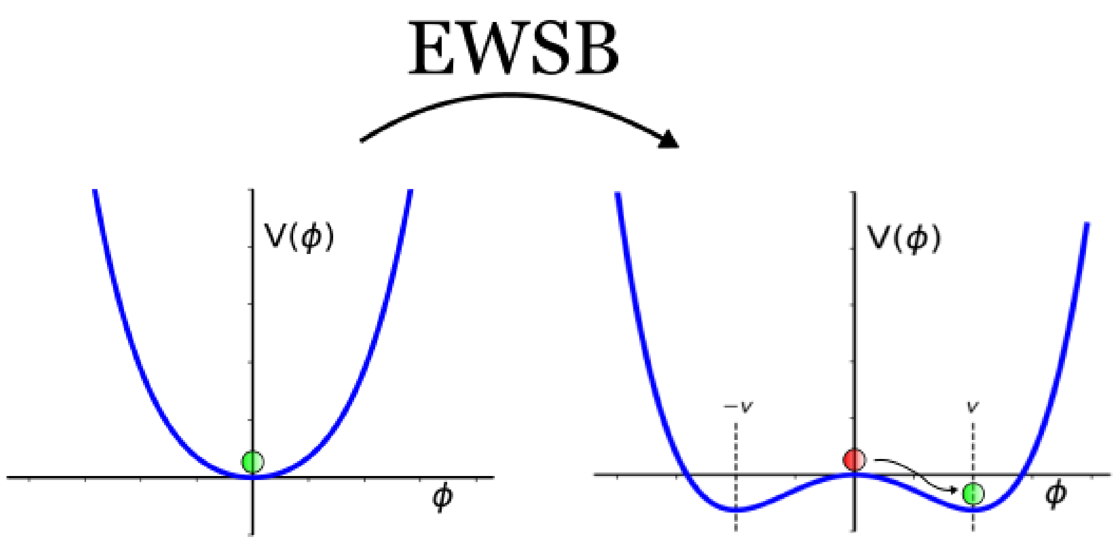

Figure 1.2: Illustration of electroweak symmetry breaking (EWSB). *Left*: Higgs potential with $\mu^2 > 0$ with stable equilibrium at $\phi = 0$. *Right*: With $\mu^2 < 0$, $\phi = 0$ is no longer a stable equilibrium and the Higgs attains a non-zero vacuum expectation value at $\pm v$ — breaking the $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ gauge symmetry of the electroweak sector of the SM.

The masses of the fermions can be obtained by adding the interaction terms such as the following:

$$\mathcal{L}_{f-h} = g_e \left(\bar{L}\phi e^-_R + \phi^\dagger \overline{e^-}_R L \right). \tag{1.21}$$

Since both L and ϕ are $\mathcal{SU}(2)$ doublets, adding the right-handed $\mathcal{SU}(2)$ singlet terms do not spoil the $\mathcal{SU}(2)$ symmetry. We can require EWSB and insert Eqn. 1.17 into Eqn. 1.21 resulting in,

$$\mathcal{L}_{f-h} = \frac{1}{\sqrt{2}} y_f v \left(\overline{f^-_L} f^-_R + \overline{f^-_R} f^-_L \right) + \frac{1}{\sqrt{2}} \left(\overline{f^-_L} f^-_R + \overline{f^-_R} f^-_L \right) h, \tag{1.22}$$

from which we can read off the fermion mass terms as,

$$m_f = y_f \frac{v}{\sqrt{2}}, \tag{1.23}$$

where the y_f are referred to as the fermion *Yukawa couplings*, and are free parameters of the SM that need to be measured. Given this form for the masses of the fermions, we can see from the second term of Eqn. 1.22 that the coupling of the Higgs to a given fermion species is directly related to the fermion's mass. To make this obvious, we re-write Eqn. 1.22 as,

$$\mathcal{L}_{f-h} = \underbrace{m_f \bar{f} f}_{f \text{ mass term}} + \underbrace{\frac{m_f}{v} \bar{f} f h}_{f-h \text{ coupling}}. \tag{1.24}$$

1.4 The Complete Standard Model, Successes and Shortcomings

Table 1.2: The particle content of the SM after the process of electroweak symmetry breaking.

	Physical Field	Q	Coupling	Mass [GeV]
Quarks	$u,\ c,\ t$	$2/3$	$(y_i =)\ 1\times10^{-5},\ 7\times10^{-3},\ 1$	$2\times10^{-3},\ 1.27,\ 173$
	$d,\ s,\ b$	$-1/3$	$(y_i =)\ 3\times10^{-5},\ 5\times10^{-4},\ 0.02$	$4\times10^{-4},\ 0.10,\ 4.18$
Leptons	$e,\ \mu,\ \tau$	-1	$(y_i =)\ 3\times10^{-7},\ 6\times10^{-4},\ 0.01$	$5\times10^{-4},\ 0.106,\ 1.777$
	$\nu_e,\ \nu_\mu,\ \nu_\tau$	0	–	–
Bosons	γ	0	$\alpha_{\rm EM}\simeq 1/137$	0
	Z	0	$\sin\theta_W\simeq 0.5$	91.2
	$(W^+,\ W^-)$	$(+1,-1)$	$\mathcal{V}_{\rm CKM}$	80.4
	G	0	$\alpha_s\simeq 0.1$	0
Higgs	h	0	$\lambda,\ \mu$	125.09

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