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Chapter 1

The Standard Model of Particle Physics

If you wish to make an apple pie from scratch, you must first invent the universe.

–Carl Sagan, *Cosmos: A Personal Voyage*

As it stands, what has become known as the ‘Standard Model (SM) of Particle Physics’ is nothing less than one of the greatest achievements of mankind, due to both the magnitude by which it has changed our perception of the underlying nature of the universe and to the clever methods and tinkering by which this nature was unveiled by many clever physicists whose history has become veritable lore. In terms of imagination and insight, it is second only to the special and general theories of relativity – though the fields are nevertheless intricately intertwined.

Not considering the scientific progress made in the 18th and 19th centuries, and ignoring the ancient Greeks despite their fabled invention of atomic theory, the physical insights and major work that led to the current picture of elementary particle physics described by the SM began with the *annus mirabilis* papers of Albert Einstein in the year 1905 [1, 2, 3]. In these papers, Einstein was able to shed light on the quantization of electromagnetic radiation (building off of the seminal work of Max Planck [4]) and introduce the special theory of relativity. These works laid the conceptual and philosophical groundwork for the major breakthroughs in fundamental physics of 20th century physics: from the ‘old quantum theory’ of Bohr and Sommerfeld in the early 1900’s to the equivalent wavefunction and matrix-mechanics formulations of Schrödinger and Heisenberg that coalesced into ‘modern’ quantum mechanics in the mid-1920’s. The modern approach, non-relativistic at its heart,

provided a sufficient mathematical and interpretable framework in which to work and match predictions to observed phenomena, old and new. It has for the most part remained unchanged and is the quantum mechanics that is taught to students at both the undergraduate and graduate level to this very day. It is the theory that has since revolutionised all aspects of the physical sciences and technologies that dictate our everyday-lives. In the mid-1920's, however, despite large efforts put forth by the forbears of modern quantum mechanics, the quantum-mechanical world had yet to be made consistent with Einstein's theory of relativity — a requirement that must be met for all consistent physical theories of nature. It was the insight of Paul Dirac who was finally able to successfully marry the theory of the quantum with that of relativity when he introduced his relativistic quantum-mechanical treatment of the electron in 1927 and 1928 [5, 6].¹ This work provided the starting point for a decades-long search of a consistent quantum-mechanical and relativistic treatment of electrodynamics, known as *quantum electrodynamics* (QED). The search for QED ended at the end of the 1940's with the groundbreaking work of Dyson, Feynman, Schwinger, and Tomonaga [9, 10, 11, 12, 13, 14, 15, 16] that introduced the covariant and gauge invariant formulation of QED — the first such relativistic quantum field theory (QFT). QED allowed the physicists to make predictions that agreed with observation at unprecedented levels of accuracy and has since led to the adoption of its language and mathematical toolkit as the foundational framework in which to construct models that accurately describe nature.² The SM is no less than an ultimate conclusion of these works: a consistent set of relativistic quantum field theories, using the language developed by Feynman et al., that describes essentially all aspects of the known particles and forces that make up the observed universe.

1.1 Particles and Forces

There are four known fundamental forces at work in the universe: electromagnetism, the weak interaction, the strong interaction, and gravity. Our understanding of the existence of each of these forces has essentially been arrived at empirically, with physicists following experimental clues, and their basic behaviors deduced after long trials of effort. The SM encompasses all of these forces except for gravity, which currently is only described by the classical (i.e. not quantum) theory of geometrodynamics, or general relativity. The gravitational interaction is incredibly weak in comparison to the others, however, and is not relevant to the types of particle interactions that we are currently sensitive to in experiments.

¹ A complete history of the people and ideas involved in the development of the modern theory of Quantum Mechanics can be found in references [7, 8], and the references therein.

² For a complete discussion of the developments leading up to QED, see the fabulous book by S. Schweber [17].

Electromagnetism is by far the most familiar, as it is the force most commonly experienced and is what is at work in our everyday life (reaction forces between objects on tables and chairs, friction, wall-plugs, batteries, DNA structure, etc...) and is typically what students are first presented with in their physics studies. The weak force is responsible for things like radioactive decay, which makes possible the process of nuclear β -decay and the nuclear fission process that fuels the sun, for example. The strong force is what binds protons and neutrons together, and thus is responsible for holding together most of the (ordinary) matter in the universe.³

The forces mediate the interactions between the matter particles, which we use to deduce their presence. The SM predicts fundamental, point-like particles that appear in two general classes depending on whether they have integral spin ($\mathcal{S} \in [0, 1, 2, \dots]$) or half-integral spin ($\mathcal{S} \in [1/2, 3/2, \dots]$); the former are referred to as *bosons* and the latter as *fermions*. In the SM, the particles that are responsible for making up matter are all spin-1/2 fermions and are either *leptons* or *quarks*; within each class there are three generations (or families) that are essentially copies of the first. The forces in the SM are interpreted as being mediated by spin-1 bosons, referred to as the *gauge bosons*. The leptons and quarks all interact via the weak interaction, but only the quarks interact via the strong interaction. All electrically charged particles interact with the electromagnetic interaction.

The particles of the SM are described as quantum fields whose dynamics are described by the SM Lagrangian under which the equations of motions can be derived. The particles, and by extension the SM Lagrangian that describes them, are found to be invariant under transformations of spacetime (space translations, rotations, Lorentz boosts) and three internal transformations described by unitary transformations: $\mathcal{P} \times SU(3)_C \times SU(2)_L \times U(1)_Y$. This is illustrated in Figure 1.1. The strong force is described by a local $SU(3)$ symmetry that acts only on the particles that have *color charge*. The term “color” arises from the fact that the color charge is found to exist in three varieties which have been labelled as red (r), blue (b), or green (g), and due to the fact that “colorless” states are formed when all three are combined (r+g+b), just like with visible light that humans are familiar with, or when states are formed of color-anti-color pairs (r+ \bar{r}). For this reason, the QFT describing the strong force is called Quantum *Chromodynamics* (QCD), and is mediated by eight *gluons* (G). The particles subject to the weak force are invariant under weak-isospin $SU(2)$ transformations, mediated by the three \mathcal{W} bosons ($\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3$). The $U(1)$ transformations, mediated by the \mathcal{B} boson, preserve weak-hypercharge, Y . The $SU(2)$ symmetry is respected only by the left-handed chiral particles (leptons or quarks), with the right-handed chiral particles not participating. There is additionally a single scalar (i.e. spin-0) field, the Higgs field, that is

³‘Ordinary’ to distinguish from dark matter, for example.

an $SU(2)$ doublet, about which more will be described shortly. The particle content thus described is presented in detail in Table 1.1. The $SU(2)$ left-handed chiral fields appear as doublets and are grouped in an “up-down” pair (e.g. (u_L, d_L) or $(e_L, \nu_{e,L})$) whereas the right-handed chiral fields, living in the singlet representation of $SU(2)$, do not (e.g. u_R). Note that the SM does not allow for right-handed neutrinos (a term like ν_R does not appear).

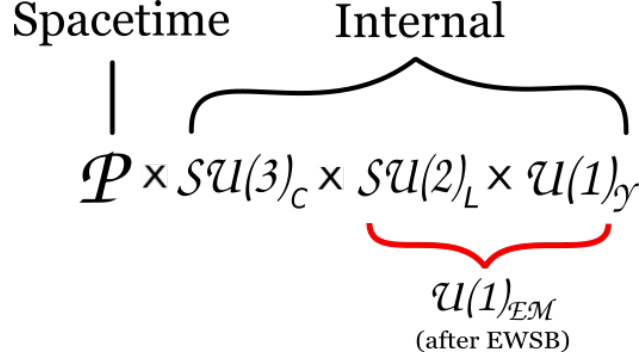


Figure 1.1: The spacetime and internal gauge structure of the SM. \mathcal{P} refers to the Poincaré symmetry group. $SU(3)_c$ refers to the $SU(3)$ symmetry of the color sector of QCD and $SU(2)_L \times U(1)_Y$ refers to the left-handed chiral symmetry of the electroweak interaction. After spontaneous symmetry breaking due to the Higgs mechanism, the $SU(2)_L \times U(1)_Y$ symmetry reduces to the $U(1)_{EM}$ symmetry of electromagnetism.

The SM Lagrangian is shown in Eqn. 1.1 and describes the complete content of the SM: encompassing all interactions between the known particles and the symmetries that they obey.

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4} \sum_{\text{gauge}} F_{\mu\nu}^i F^{i\mu\nu} - \sum_f \bar{f} \gamma^\mu D_\mu f + (\mathcal{D}_\mu \phi)^\dagger (\mathcal{D}^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \quad (1.1)$$

The first term of Eqn. 1.1 is a sum over the three internal gauge groups, and $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$, where A_μ is one of the three gauge fields, g is the associated gauge coupling parameter, and a sum over i is implied. The f^{abc} are the so-called *structure constants* of the gauge group. For Abelian groups like $U(1)$ $f^{abc} = 0$, and for non-Abelian gauge groups like $SU(2)$ and $SU(3)$, $f^{abc} \neq 0$. For example, for $SU(2)$, the structure constants are nothing more than the Levi-Civita totally anti-symmetric tensor, ε_{ijk} , giving for the weak gauge force:

$$\mathcal{W}_{\mu\nu} = \partial_\mu \mathcal{W}_\nu - \partial_\nu \mathcal{W}_\mu - g_2 \mathcal{W}_\mu \times \mathcal{W}_\nu \quad (1.2)$$

where \mathcal{W}_μ is the vector of the three weak gauge fields \mathcal{W}_1 , \mathcal{W}_2 , and \mathcal{W}_3 and g_2 is the weak coupling constant. The non-zero f^{abc} of non-Abelian gauge groups means that the gauge bosons of the weak and strong interactions can interact with themselves due to terms

appearing in Eqn. 1.1 that contain only the gauge bosons. add Feynman diagram?

The second term of Eqn. 1.1 describes the lepton and quark kinetic energies and their interactions with the gauge fields. The f refer to the fermion fields (quarks and leptons) and the corresponding sum is over all species of fermion. D_μ is the gauge covariant derivative, and for the SM is given by:

$$D_\mu = \partial_\mu - ig_1 \frac{Y}{2} \mathcal{B}_\mu - ig_2 \frac{\tau^i}{2} \mathcal{W}_\mu^i - ig_3 \frac{\lambda^a}{2} G_\mu^a \quad (1.3)$$

where g_1 , g_2 , and g_3 are the gauge coupling constants for $\mathcal{U}(1)_Y$, $\mathcal{SU}(2)_L$, and $\mathcal{SU}(3)_C$, respectively, that give the overall strength of the associated coupling. Summation over repeated indices is implied and the τ^i (λ^a) are the three (eight) generators of the $\mathcal{SU}(2)$ ($\mathcal{SU}(3)$) gauge group, with $i \in [1, 2, 3]$ ($a \in [1, \dots, 8]$), and are typically represented by the Pauli (Gell-Mann) matrices.

The last three terms in Eqn. 1.1 are all terms including the Higgs field, ϕ , and will be discussed in detail in 1.3.

Inspection of Eqn. 1.1 will reveal two things.

Table 1.1: The particle content of the SM and their transformation properties under the SM gauge groups, prior to electroweak symmetry breaking. The representations of each of the gauge groups are shown in the three-right columns. The $\mathcal{U}(1)$ symmetry of weak-hypercharge transformations is one-dimensional and the column gives the weak-hypercharge \mathcal{Y} associated with each field. For $\mathcal{SU}(3)$ and $\mathcal{SU}(2)$, **1** refers to the field belonging to the associated singlet representation, **2** to the doublet representation, **3** to the triplet representation, and **8** to the octet representation.

	Field Label	Content	Spin	$\mathcal{U}(1)$ ($= \mathcal{Y}$)	$\mathcal{SU}(2)$	$\mathcal{SU}(3)$
Leptons Quarks	Q_i	$(u_L, d_L), (c_L, s_L), (t_L, b_L)$	1/2	1/6	2	3
	$u_{R,i}$	u_R	1/2	2/3	1	3
	$d_{R,i}$	d_R	1/2	-1/3	1	3
	L_i	$(e_L, \nu_{e,L}), (\mu_L, \nu_{\mu,L}), (\tau_L, \nu_{\tau,L})$	1/2	1/2	2	1
	$e_{R,i}$	e_R, μ_R, τ_R	1/2	-1	1	1
Gauge Fields	\mathcal{B}	\mathcal{B}	1	0	1	1
	\mathcal{W}	$(\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3)$	1	0	3	1
	G	$G_a, a \in [1, \dots, 8]$	1	0	1	8
Higgs Field	ϕ	(ϕ^+, ϕ^0)	0	1/2	2	1

1.2 The Electroweak Theory

1.3 The Higgs Mechanism

1.4 The Complete Standard Model, Successes and Shortcomings

Table 1.2: The particle content of the SM after the process of electroweak symmetry breaking.

	Physical Field	Q	Coupling	Mass [GeV]
Leptons Quarks	u, c, t	$2/3$	$(y_i =) 1 \times 10^{-5}, 7 \times 10^{-3}, 1$	$2 \times 10^{-3}, 1.27, 173$
	d, s, b	$-1/3$	$(y_i =) 3 \times 10^{-5}, 5 \times 10^{-4}, 0.02$	$4 \times 10^{-4}, 0.10, 4.18$
	e, μ, τ	-1	$(y_i =) 3 \times 10^{-7}, 6 \times 10^{-4}, 0.01$	$5 \times 10^{-4}, 0.106, 1.777$
	ν_e, ν_μ, ν_τ	0	—	—
Bosons	γ	0	$\alpha_{\text{EM}} \simeq 1/137$	0
	Z	0	$\sin \theta_W \simeq 0.5$	91.2
	(W^+, W^-)	$(+1, -1)$	\mathcal{V}_{CKM}	80.4
	G	0	$\alpha_s \simeq 0.1$	0
Higgs	h	0	λ, μ	125.09

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