TABLE OF CONTENTS

Chapter 1

The Standard Model of Particle Physics

If you wish to make an apple pie from scratch, you must first invent the universe.

–Carl Sagan, *Cosmos: A Personal Voyage*

As it stands, what has become known as the 'Standard Model (SM) of Particle Physics' is nothing less than one of the greatest achievments of mankind, due to both the magnitude by which it has changed our perception of the underlying nature of the universe and to the clever methods and tinkerings by which this nature was unveiled by many clever physicists whose history has become veritable lore. In terms of imagination and insight, it is second only to the special and general theories of relativity – though the fields are nevertheless intricately intertwined.

Not considering the scientific progress made in the 18^{th} and 19^{th} centuries, and ignoring the ancient Greeks despite their fabled invention of atomic theory, the physical insights and major work that led to the current picture of elementary particle physics described by the SM began with the *annus mirabilis* papers of Albert Einstein in the year 1905 [1, 2, 3]. In these papers, Einstein was able to shed light on the quantization of electromagnetic radiation (building off of the seminal work of Max Planck [4]) and introduce the special theory of relativity. These works laid the conceptual and philosophical groundwork for the major breakthroughs in fundamental physics of 20^{th} century physics: from the 'old quantum theory' of Bohr and Sommerfeld in the early 1900's to the equivalent wavefunction and matrix-mechanics formulations of Schrödinger and Heisenberg that coalesced into 'modern' quantum mechanics in the mid-1920's. The modern approach, non-relativistic at its heart, provided a sufficient mathematical and interpretable framework in which to work and match predictions to observed phenomena, old and new. It has for the most part remained unchanged and is the quantum mechanics that is taught to students at both the undergraduate and graduate level to this very day. It is the theory that has since revolutionised all aspects of the physical sciences and technologies that dictate our everyday-lives. In the mid-1920's, however, despite large efforts put forth by the forbears of modern quantum mechanics, the quantum-mechanical world had yet to be made consistent with Einstein's theory of relativity — a requirement that must be met for all consistent physical theories of nature. It was the insight of Paul Dirac who was finally able to successfully marry the theory of the quantum with that of relativity when he introduced his relativistic quantum-mechanical treatment of the electron in 1927 and 1928 [5, 6].[1] This work provided the starting point for a decades-long search of a consistent quantum-mechanical and relativistic treatment of electrodynamics, known as *quantum electrodynamics* (QED). The search for QED ended at the end of the 1940's with the groundbreaking work of Dyson, Feynman, Schwinger, and Tomanaga [9, 10, 11, 12, 13, 14, 15, 16] that introduced the covariant and gauge invariant formulation of QED — the first such relativistic quantum field theory (QFT). QED allowed the physcists to make predictions that agreed with observation at unprecedented levels of accuracy and has since led to the adoption of its language and mathematical toolkit as the foundational framework in which to construct models that accurately describe nature.[2] The SM is no less than an ultimate conclusion of these works: a consistent set of relativistic quantum field theories, using the language developed by Feynman et al., that describes essentially all aspects of the known particles and forces that make up the observed universe.

1.1 Particles and Forces

There are four known fundamental forces at work in the universe: electromagnetism, the weak interaction, the strong interaction, and gravity. Our understanding of the existence of each of these forces has essentially been arrived at empirically, with physicists following experimental clues, and their basic behaviors deduced after long trials of effort. The SM encompasses all of these forces except for gravity, which currently is only described by the classical (i.e. not quantum) theory of geometrodynamics, or general relativity. The gravitational interaction is incredibly weak in comparison to the others, however, and is not relevant to the types of particle interactions that we are currently sensitive to in experiments.

[1] A complete history of the people and ideas involved in the development of the modern theory of Quantum Mechanics can be found in references [7, 8], and the references therein.

[2] For a complete discussion of the developments leading up to QED, see the fabulous book by S. Schweber [17].

Electromagnetism is by far the most familiar, as it is the force most commonly experienced and is what is at work in our everyday life (reaction forces between objects on tables and chairs, friction, wall-plugs, batteries, DNA structure, etc...) and is typically what students are first presented with in their physics studies. The weak force is responsible for things like radioactive decay, which makes possible the process of nuclear β-decay and the nuclear fission process that fuels the sun, for example. The strong force is what binds protons and neutrons together, and thus is resonsible for holding together most of the (ordinary) matter in the universe.[3]

The forces mediate the interactions between the matter particles, which we use to deduce their presence. The SM predicts fundamental, point-like particles that appear in two general classes depending on whether they have integral spin ($\mathcal{S} \in [0, 1, 2, ...)$) or half-integral spin ($\mathcal{S} \in [1/2, 3/2, ...)$); the former are referred to as *bosons* and the latter as *fermions*. In the SM, the particles that are responsible for making up matter are all spin-1/2 fermions and are either *leptons* or *quarks*; within each class there are three generations (or families) that are essentially copies of the first. The forces in the SM are interpreted as being mediated by spin-1 bosons, referred to as the *gauge bosons*. The leptons and quarks all interact via the weak interaction, but only the quarks interact via the strong interaction. All electrically charged particles interact with the electromagnetic interaction.

The particles of the SM are described as quantum fields whose dynamics are described by the SM Lagrangian under which the equations of motions can be derived. The particles, and by extention the SM Lagrangian that descibes them, are found to be invariant under transformations of spacetime (space translations, rotations, Lorentz boosts) and three internal transformations described by unitary transformations: $\mathcal{P} \times \mathcal{SU}(3)_C \times \mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$. This is illustrated in Figure 1.1. The strong force is described by a local $\mathcal{SU}(3)$ symmetry that acts only on the particles that have *color charge*. The term "color" arises from the fact that the color charge is found to exist in three varieties which have been labelled as red (r), blue (b), or green (g), and due to the fact that "colorless" states are formed when all three are combined (r+g+b), just like with visible light that humans are familiar with, or when states are formed of color-anti-color pairs (r+$\bar{\text{r}}$). For this reason, the QFT describing the strong force is called Quantum *Chromo*dynamics (QCD), and is mediated by eight *gluons* (G). The particles subject to the weak force are invariant under weak-isospin $\mathcal{SU}(2)$ transformations, mediated by the three $\mathcal{W}$ bosons ($\mathcal{W}_1$, $\mathcal{W}_2$, $\mathcal{W}_3$). The $\mathcal{U}(1)$ transformations, mediated by the $\mathcal{B}$ boson, preserve weak-hypercharge, Y. The $\mathcal{SU}(2)$ symmetry is respected only by the left-handed chiral particles (leptons or quarks), with the right-handed chiral particles not participating. There is additionally a single scalar (i.e. spin-0) field, the Higgs field, that is

[3]'Ordinary' to distinguish from dark matter, for example.

an $\mathcal{SU}(2)$ doublet, about which more will be described shortly. The particle content thus described is presented in detail in Table 1.1. The $\mathcal{SU}(2)$ left-handled chiral fields appear as doublets and are grouped in and "up-down" pair (e.g. $(u_{\mathrm{L}},\, d_{\mathrm{L}})$ or $(e_{\mathrm{L}},\, \nu_{e,\mathrm{L}})$) whereas the right-handed chiral fields, living in the singlet representation of $\mathcal{SU}(2)$, do not (e.g. u_{R})). Note that the SM does not allow for right-handed neutrinos (a term like ν_{R} does not appear).

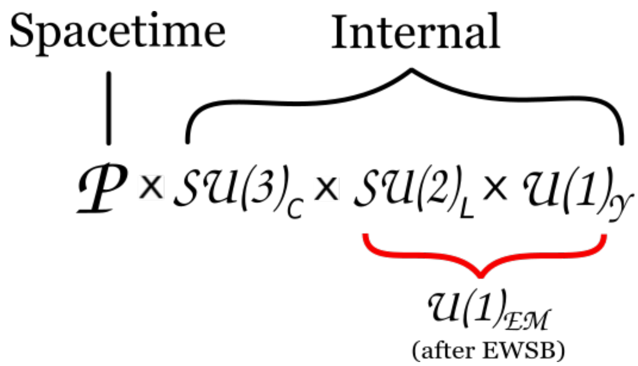

Figure 1.1: The spacetime and internal gauge structure of the SM. $\mathcal{P}$ refers to the Poincaré symmetry group. $\mathcal{SU}(3)_c$ refers to the $\mathcal{SU}(3)$ symmetry of the color sector of QCD and $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ refers to the left-handed chiral symmetry of the electroweak interaction. After spontaneous symmetry breaking due to the Higgs mechanism, the $\mathcal{SU}(2)_L \times\, \mathcal{U}(1)_Y$ symmetry reduces to the $\mathcal{U}(1)_{EM}$ symmetry of electromagnetism.

The SM Lagrangian is shown in Eqn. 1.1 and describes the complete content of the SM: encompassing all interactions between the known particles and the symmetries that they obey.

$$\mathcal{L}_{\mathrm{SM}} = -\frac{1}{4}\sum_{\mathrm{gauge}} F^{i}_{\mu\nu}F^{i\,\mu\nu} - \sum_{f} \bar{f}\gamma^{\mu}D_{\mu}f + (\mathcal{D}_{\mu}\phi)^{\dagger}(\mathcal{D}^{\mu}\phi) - \mu^2\phi^{\dagger}\phi - \lambda(\phi^{\dagger}\phi) \tag{1.1}$$

The first term of Eqn. 1.1 is a sum over the three internal gauge goups, and $F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} + gf^{abc}A^{b}_{\mu}A^{c}_{\nu}$, where A_{μ} is one of the three gauge fields, g is the associated gauge coupling parameter, and a sum over i is implied. The f^{abc} are the so-called *structure constants* of the gauge group. For Abelian groups like $\mathcal{U}(1)$ $f^{abc} = 0$, and for non-Abelian gauge groups like $\mathcal{SU}(2)$ and $\mathcal{SU}(3)$, $f^{abc} \neq 0$. For example, for $\mathcal{SU}(2)$, the structure constants are nothing more than the Levi-Civita totally anti-symmetric tensor, ε_{ijk}, giving for the weak gauge force:

$$\boldsymbol{\mathcal{W}}_{\mu\nu} = \partial_{\mu}\boldsymbol{\mathcal{W}}_{\nu} - \partial_{\nu}\boldsymbol{\mathcal{W}}_{\mu} - g_2\boldsymbol{\mathcal{W}}_{\mu} \times \boldsymbol{\mathcal{W}}_{\nu} \tag{1.2}$$

where $\boldsymbol{\mathcal{W}}_{\mu}$ is the vector of the three weak gauge fields ($\mathcal{W}_1$, $\mathcal{W}_2$, and $\mathcal{W}_3$) and g_2 is the gauge coupling of the weak force. The non-zero f^{abc} of non-Abelian gauge groups means that the gauge bosons of the weak and strong interactions can interact with themselves due to terms appearing in Eqn. 1.1 that contain only the gauge bosons. add Feynman diagram?

The second term of Eqn. 1.1 describes the lepton and quark kinetic energies and their interactions with the gauge fields. The f refer to the fermion fields (quarks and leptons) and the corresponding sum is over all species of fermion. D_μ is the gauge covariant derivative, and for the SM is given by:

$$D_\mu = \partial_\mu - ig_1 \frac{Y}{2}\mathcal{B}_\mu - ig_2 \frac{\tau^i}{2}\mathcal{W}^i_\mu - ig_3 \frac{\lambda^a}{2} G^a_\mu \tag{1.3}$$

where g_1, g_2, and g_3 are the gauge coupling constants for $\mathcal{U}(1)_Y$, $\mathcal{SU}(2)_L$, and $\mathcal{SU}(3)_C$, respectively, that give the overall strength of the associated coupling. Summation over repeated indices is implied and the τ^i (λ^a) are the three (eight) generators of the $\mathcal{SU}(2)$ ($\mathcal{SU}(3)$) gauge group, with $i \in [1, 2, 3]$ ($a \in [1, .., 8]$), and are typically represented by the Pauli (Gell-Mann) matrices. Note that the form of Eqn. 1.3 is strictly mandated by the requirement that the theory be *gauge invariant*, i.e. that transformations of the fields under the internal symmetries of Fig. 1.1 leave the action of $\mathcal{L}_{\text{SM}}$ unchanged. This is described in detail in Appendix XXX.

The last three terms in Eqn. 1.1 are all terms including the Higgs field, ϕ, and will be discussed in detail in Section 1.3.

Inspection of Eqn. 1.1 will reveal two things. The first thing that one may notice is that it does not appear to describe electromagnetism as it does not have a term representing the photon, the familiar mediator of the electromagnetic interaction. The second, and perhaps more immediately obvious, thing is that no mass terms appear in $\mathcal{L}_{\text{SM}}$: all fields appear to have zero mass! Both of these facts are counter to our everday experience: we know electromagnetism is real and that matter, at the very least, is massive. In the next few sections we will see how these apparent issues are resolved.

1.2 The Electroweak Theory

It was the work of Glashow, Weinberg, and Salam (GWS) that ultimately put forth a consistent picture of the chiral weak force and, ultimately, its unification with electromagnetism [18, 19, 20]. As a result, the theory of particles and fields that respect the $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ gauge invariance of the SM is sometimes referred to as 'GWS theory', but is more typically known as the electroweak theory, or interaction. Since all matter particles are subject to the electroweak interaction, but only a subset of the particles that have color charge are subject to the strong interaction described by QCD, the study of the SM can essentially be partitioned into two parts: the part that deals with the dynamics and interactions of colored objects (the

Table 1.1: The particle content of the SM and their transformation properties under the SM gauge groups, prior to electroweak symmetry breaking. The representations of each of the gauge groups are shown in the three-right columns. The $\mathcal{U}(1)$symmetry of weak-hypercharge transformations is one-dimensional and the column gives the weak-hypercharge $\mathcal{Y}$ associated with each field. For $\mathcal{SU}(3)$and $\mathcal{SU}(2)$, **1** refers to the field belonging to the associated singlet representation, **2** to the doublet representation, **3** to the triplet representation, and **8** to the octet representation.

	Field Label	Content	Spin	$\mathcal{U}(1)$ $(=\mathcal{Y})$	$\mathcal{SU}(2)$	$\mathcal{SU}(3)$
Quarks	Q_i	$(u_{\mathrm{L}},\, d_{\mathrm{L}}),\, (c_{\mathrm{L}},\, s_{\mathrm{L}}),\, (t_{\mathrm{L}},\, b_{\mathrm{L}})$	1/2	1/6	**2**	**3**
	$u_{\mathrm{R},i}$	u_{R}	1/2	2/3	**1**	**3**
	$d_{\mathrm{R},i}$	d_{R}	1/2	$-1/3$	**1**	**3**
Leptons	L_i	$(e_{\mathrm{L}},\, \nu_{e,\mathrm{L}}),\, (\mu_{\mathrm{L}},\, \nu_{\mu,\mathrm{L}}),\, (\tau_{\mathrm{L}},\, \nu_{\tau,\mathrm{L}})$	1/2	1/2	**2**	**1**
	$e_{\mathrm{R},i}$	$e_{\mathrm{R}},\, \mu_{\mathrm{R}},\, \tau_{\mathrm{R}}$	1/2	-1	**1**	**1**
Gauge Fields	$\mathcal{B}$	$\mathcal{B}$	1	0	**1**	**1**
	$\mathcal{W}$	$(\mathcal{W}_1,\, \mathcal{W}_2,\, \mathcal{W}_3)$	1	0	**3**	**1**
	G	$G_a,\, a \in [1, .., 8]$	1	0	**1**	**8**
Higgs Field	ϕ	$(\phi^+,\, \phi^0)$	0	1/2	**2**	**1**

'QCD part', $\mathcal{L}_{\mathrm{QCD}}$) and the part that deals solely with electroweak interactions, including the Higgs (the 'Electroweak part', $\mathcal{L}_{\mathrm{Electroweak}}$). Given the ubiquity and broad reach into all observable phenomena of the electroweak interaction, in the early days the GWS theory was thought of as the heart of the SM and why, ultimately, GWS were awarded the Nobel prize in 1979.[4] In this section, then, we will focus closely on a part of the $\mathcal{SU}(2)_L \times \mathcal{U}(1)_Y$ portion of $\mathcal{L}_{\mathrm{SM}}$.

The first thing to re-iterate is that the electroweak theory is *chiral*, i.e., it distinguishes between left- and right-chiral fermion fields. For some conceptual clarity, in the limit of a massless fermion, the chirality is the same as the more-familiar *helicity*, which is defined by the projection of the particles spin angular momentum, $\vec{\sigma}$, onto their momentum (direction of motion): $\vec{\sigma} \cdot \vec{p}$. If the massless fermion's spin is aligned parallel to its momentum ($\vec{\sigma} \cdot \vec{p} > 0$), it is said to be right-handed. If otherwise ($\vec{\sigma} \cdot \vec{p} < 0$), it is said to be left-handed. One

[4] Actually, the acceptance of the GWS theory as the de-facto SM of the time was not widely held until some years after its publication, when t'Hooft proved that it was renormalizable [21, 22]. Such a complete understanding in the QCD sector would not come until almost a decade later, in the late 1970'sWoijkec etc CITE.

typically defines fermion fields without the chiral specification and then defines projections onto the left- and right-handed components. For example:

$$f_{\mathrm{L}} = P_L f = \frac{1}{2}(1-\gamma_5)f, \qquad f_{\mathrm{R}} = P_R f = \frac{1}{2}(1+\gamma_5)f$$

First we introduce re-defined $\mathcal{W}$ bosons as follows:

$$\begin{aligned} \mathcal{W}^+ &= \frac{1}{\sqrt{2}}(-\mathcal{W}_1 + i\mathcal{W}_2) \\ \mathcal{W}^- &= \frac{1}{\sqrt{2}}(-\mathcal{W}_2 - i\mathcal{W}_2) \\ \mathcal{W}^0 &= \mathcal{W}_3 \end{aligned} \tag{1.4}$$

Since the electroweak interaction affects the leptons and quarks similarly, and the same goes for fermions across generations (e.g. $e_{\mathrm{L}} \leftrightarrow \mu_{\mathrm{L}} \leftrightarrow \tau_{\mathrm{L}}$), we focus only on the leptons and consider for simplicity only a single generation. Taking the associated $\mathcal{U}(1)$ terms of Eqn. 1.1, we get:

$$\begin{aligned} -\mathcal{L}_{\mathrm{ferm}}(\mathcal{U}(1), \mathrm{leptons}) &= \bar{L} i\gamma^\mu (ig_1 \frac{Y_L}{2}\mathcal{B}_\mu)L + \bar{e}_R i\gamma^\mu (ig_1 \frac{Y_R}{2} B_\mu) e_R \\ &= \frac{g_1}{2}[Y_L(\bar{\nu}_L\gamma^\mu\nu_L + \bar{e}_L\gamma^\mu e_L) + Y_R\bar{e}_R\gamma^\mu e_R]B_\mu \end{aligned} \tag{1.5}$$

Where we use the fact that $L = (\nu_L, e_L)$ in going from the first to second line of Eqn. 1.5. Taking the associated $\mathcal{SU}(2)$ terms of Eqn. 1.1, and noting that, since the τ^i are the Pauli matrices, $\tau^i W^i$ is a 2×2 matrix, we get:

$$\begin{aligned} -\mathcal{L}_{\mathrm{ferm}}(\mathcal{SU}(2), \mathrm{leptons}) &= \bar{L} i\gamma^\mu [ig_2 \frac{\tau^i}{2}\mathcal{W}^i_\mu]L \\ &= -\frac{g_2}{2}(\bar{\nu}_L, \bar{e}_L)\gamma^\mu \begin{pmatrix} \mathcal{W}^3_\mu & \mathcal{W}^1_\mu - i\mathcal{W}^2_\mu \\ \mathcal{W}^1_\mu + i\mathcal{W}^2_\mu & -\mathcal{W}^3_\mu \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \\ &= -\frac{g_2}{2}\left[\bar{\nu}_L\gamma^\mu\nu_L\mathcal{W}^0_\mu - \sqrt{2}\bar{\nu}_L\gamma^\mu e_L\mathcal{W}^+_\mu - \sqrt{2}\bar{e}_L\gamma^\mu\nu_L\mathcal{W}^-_\mu - \bar{e}_L\gamma^\mu e_L\mathcal{W}^0_\mu\right] \end{aligned} \tag{1.6}$$

Where Eqn. 1.4 has been used in getting to the last line of Eqn. 1.6. Considering only those terms which are charge-neutral in Eqn. 1.5 and Eqn. 1.6, one can consider performing a second field-redefinition using $\mathcal{B}$ and $\mathcal{W}^0$:

$$\begin{aligned} A_\mu &= \frac{g_2\mathcal{B}_\mu - g_1 Y_L \mathcal{W}^0_\mu}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \qquad & Z_\mu &= \frac{g_1 Y_L \mathcal{B}_\mu + g_2 \mathcal{W}^0_\mu}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \\ \hookrightarrow B_\mu &= \frac{g_2 A_\mu + g_1 Y_L Z_\mu}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \qquad & \mathcal{W}^0_\mu &= \frac{-g_1 Y_L A_\mu + g_2 Z_\mu}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \end{aligned} \tag{1.7}$$

Using this last result for the re-defined $\mathcal{B}$ and $\mathcal{W}^0$ in terms the fields A_μ and Z_μ to re-write the terms in Eqn. 1.5 and 1.6 involving $\bar{e}_{L,R}\gamma^\mu e_{L,R}$, allows one to see that A_μ should correspond to the photon of electromagnetism if we fix,

$$-e = \frac{g_1 g_2 Y_L}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \qquad - e = \frac{g_1 g_2 Y_R}{2\sqrt{g_2^2 + g_1^2 Y_L^2}}$$

where e is the charge of the positron. Setting $Y_L = -1$ then allows to formulate the positron charge in terms of the weak couplings g_1 and g_2 simply as,

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \tag{1.8}$$

Eqn. 1.8 leads one to introduce the following relations,

$$\sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}} \qquad \cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} \tag{1.9}$$

The angle θ_W is the *Weinberg angle*, and it represents the amount of mixing occuring between the $\mathcal{SU}(2)$ and $\mathcal{U}(1)$ gauge fields $\mathcal{W}^0_\mu$ and $\mathcal{B}_\mu$, respectively. Using Eqn. 1.8 gives,

$$g_1 = \frac{e}{\cos\theta_W} \qquad g_2 = \frac{e}{\sin\theta_W} \tag{1.10}$$

This defines the gauge couplings, g_1 and g_2, of the electroweak theory purely in terms of known or measurable quantities.[5]

From the above algebra, we can re-write the portion of the electroweak Lagrangian describing the interactions of the first-generation fermions with the electroweak gauge bosons

[5]We do not describe this in detail, but θ_W can in principle be determined once the masses of the $\mathcal{W}^\pm$ and $\mathcal{Z}$ bosons are measured.

as,

$$
\begin{aligned}
\mathcal{L}_{\text{ferm, first-gen.}} = & \sum_{f\in\nu_e,e,u,d} eQ_f\left(\bar{f}\gamma^\mu f\right)A_\mu \\
& + \sum_{f\in\nu_e,e,u,d} \frac{g_2}{\cos\theta_W}\left[\bar{f}_L\gamma^\mu f_L\left(T^3_f - Q_f\sin^2\theta_W\right) + \bar{f}_R\gamma^\mu f_R\left(-Q_f\sin^2\theta_W\right)\right]Z_\mu \\
& + \frac{g_2}{\sqrt{2}}\left[\left(\bar{u}_L\gamma^\mu d_L + \bar{\nu}_L\gamma^\mu e_L\right)\mathcal{W}^+_\mu + \text{hermitian conjugate}\right]
\end{aligned}
\tag{1.11}
$$

The related terms for the second and third generations of the fermions, (ν_μ, μ, c, s) and (ν_τ, τ, t, b), respectively, are identical. In Eqn. 1.11, T^3_f (Q_f) is the third component of the weak-isospin (electric charge) of the fermion species f. These last terms are related via the Gell-Mann-Nishijima relation,

$$
Q_f = T^3_f + \frac{1}{2}Y \tag{1.12}
$$

which can be deduced by following the algebra above, and our having fixed $Y_L = -1$ for the (ν_L, e_L) $\mathcal{SU}(2)$ doublet.

Note that the terms involving $\mathcal{W}^\pm$ in Eqn. 1.11 are of the form $\bar{\nu}_L\gamma^\mu e_L$ which, given the chiral projects of Eqn. 1.2, can be re-written as follows,

$$
\bar{\nu}_L\gamma\mu e_L = \frac{1}{2}\bar{\nu}\gamma^\mu(1-\gamma_5)e \tag{1.13}
$$

which shows that the charged weak interactions involving $\mathcal{W}^\pm$ are the coherent sum of vector (γ^μ) and axial-vector bilinear covariants $(\gamma^\mu\gamma_5)$; this is the famous *V-A* charged-current interaction of Fermi's nuclear β-decay.

What we have shown in this section is that, due to mixing of the SM $\mathcal{SU}(2)_L$ and $\mathcal{U}(1)_Y$ gauge fields, we can arrive at an electroweak theory that supports the known fact that their exists an electromagnetic force that is mediated by a neutral boson A_μ (the photon) that couples to electrically charged fields: this is what is shown in the first line of Eqn. 1.11. The field re-definitions described above also introduce the $\mathcal{W}^\pm$ boson as the mediators of the charged electroweak interaction, responsible for radioactivity, and an additional neutral electroweak interaction mediated by the $\mathcal{Z}$ boson. The fact that the $\mathcal{SU}(2)_L$ and $\mathcal{U}(1)_Y$ gauge fields mix, by an amount dictated by θ_W, suggest that the weak and electromagnetic interactions can be unified into the single electroweak interaction, as mentioned at the beginning of this section. Later on, we will see that (gauge) unification such as this play a large role in modern elementary particle physics.

We have thus shown that the SM predicts the existence of the familiar electromagnetic force, which is not immediately apparent based on $\mathcal{L}_{\text{SM}}$ of Eqn. 1.1. However, it is still not evident how it can support the experimental fact that fermions have mass and that the mediators of the weak-nuclear force (the $\mathcal{W}^{\pm}$) *must* be massive. No such mass terms have been provided for in the SM Lagrangian yet. To resolve this, we need the Higgs mechanism.

1.3 The Higgs Mechanism

1.4 The Complete Standard Model, Successes and Shortcomings

Table 1.2: The particle content of the SM after the process of electroweak symmetry breaking.

	Physical Field	Q	Coupling	Mass [GeV]
Quarks	u, c, t	$2/3$	$(y_i =)\ 1\times10^{-5}$, 7×10^{-3}, 1	2×10^{-3}, 1.27, 173
	d, s, b	$-1/3$	$(y_i =)\ 3\times10^{-5}$, 5×10^{-4}, 0.02	4×10^{-4}, 0.10, 4.18
Leptons	e, μ, τ	-1	$(y_i =)\ 3\times10^{-7}$, 6×10^{-4}, 0.01	5×10^{-4}, 0.106, 1.777
	ν_e, ν_μ, ν_τ	0	–	–
Bosons	γ	0	$\alpha_{\text{EM}} \simeq 1/137$	0
	$\mathcal{Z}$	0	$\sin\theta_W \simeq 0.5$	91.2
	$(\mathcal{W}^+, \mathcal{W}^-)$	$(+1, -1)$	$\mathcal{V}_{\text{CKM}}$	80.4
	G	0	$\alpha_s \simeq 0.1$	0
Higgs	h	0	λ, μ	125.09

Bibliography

[1] A. Einstein. “Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt (On a Heuristic Point of View about the Creation and Conversion of Light)”. In: *Annalen der Physik* 322 (1905), pp. 132–148. DOI: 10.1002/andp.19053220607 (cit. on p. 1).

[2] A. Einstein. “Zur Elektrodynamik bewegter Körper (On the Electrodynamics of Moving Bodies)”. In: *Annalen der Physik* 322 (1905), pp. 891–921. DOI: 10.1002/andp.19053221004 (cit. on p. 1).

[3] A. Einstein. “Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig? (Does the Inertia of a Body Depend Upon Its Energy Content?)” In: *Annalen der Physik* 323 (1905), pp. 639–641. DOI: 10.1002/andp.19053231314 (cit. on p. 1).

[4] M. Planck. “Ueber das Gesetz der Energieverteilung im Normalspectrum (On the law of the distribution of energy in the normal spectrum)”. In: *Annalen der Physik* 309 (1901), pp. 553–563. DOI: 10.1002/andp.19013090310 (cit. on p. 1).

[5] P. A. M. Dirac. “The Quantum Theory of the Electron”. In: *Proceedings of the Royal Society of London Series A* 117 (Feb. 1928), pp. 610–624. DOI: 10.1098/rspa.1928.0023 (cit. on p. 2).

[6] Paul A. M. Dirac. “Quantum theory of emission and absorption of radiation”. In: *Proc. Roy. Soc. Lond.* A114 (1927), p. 243. DOI: 10.1098/rspa.1927.0039 (cit. on p. 2).

[7] Sigfrido Boffi. “The Rise of Quantum Mechanics”. In: *arXiv e-prints* (2008). arXiv: 0806.4515 [physics.hist-ph] (cit. on p. 2).

[8] Jagdish Mehra and Helmut Rechenberg. “The Historical Development of Quantum Theory”. In: (2001) (cit. on p. 2).

[9] S. Tomonaga. “On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields”. In: *Progress of Theoretical Physics* 1 (Aug. 1946), pp. 27–42. DOI: 10.1143/PTP.1.27 (cit. on p. 2).

[10] R. P. Feynman. "Space-Time Approach to Quantum Electrodynamics". In: *Physical Review* 76 (Sept. 1949), pp. 769–789. DOI: 10.1103/PhysRev.76.769 (cit. on p. 2).

[11] R. P. Feynman. "The Theory of Positrons". In: *Physical Review* 76 (Sept. 1949), pp. 749–759. DOI: 10.1103/PhysRev.76.749 (cit. on p. 2).

[12] R. P. Feynman. "Mathematical Formulation of the Quantum Theory of Electromagnetic Interaction". In: *Physical Review* 80 (Nov. 1950), pp. 440–457. DOI: 10.1103/PhysRev.80.440 (cit. on p. 2).

[13] J. Schwinger. "On Quantum-Electrodynamics and the Magnetic Moment of the Electron". In: *Physical Review* 73 (Feb. 1948), pp. 416–417. DOI: 10.1103/PhysRev.73.416 (cit. on p. 2).

[14] J. Schwinger. "Quantum Electrodynamics. I. A Covariant Formulation". In: *Physical Review* 74 (Nov. 1948), pp. 1439–1461. DOI: 10.1103/PhysRev.74.1439 (cit. on p. 2).

[15] F. J. Dyson. "The Radiation Theories of Tomonaga, Schwinger, and Feynman". In: *Physical Review* 75 (Feb. 1949), pp. 486–502. DOI: 10.1103/PhysRev.75.486 (cit. on p. 2).

[16] F. J. Dyson. "The S Matrix in Quantum Electrodynamics". In: *Physical Review* 75 (June 1949), pp. 1736–1755. DOI: 10.1103/PhysRev.75.1736 (cit. on p. 2).

[17] S. S. Schweber. *QED and the men who made it: Dyson, Feynman, Schwinger, and Tomonaga*. 1994 (cit. on p. 2).

[18] S. L. Glashow. "Partial Symmetries of Weak Interactions". In: *Nucl. Phys.* 22 (1961), pp. 579–588. DOI: 10.1016/0029-5582(61)90469-2 (cit. on p. 5).

[19] Steven Weinberg. "A Model of Leptons". In: *Phys. Rev. Lett.* 19 (1967), pp. 1264–1266. DOI: 10.1103/PhysRevLett.19.1264 (cit. on p. 5).

[20] Abdus Salam. "Weak and Electromagnetic Interactions". In: *Conf. Proc.* C680519 (1968), pp. 367–377 (cit. on p. 5).

[21] Gerard 't Hooft. "Renormalization of Massless Yang-Mills Fields". In: *Nucl. Phys.* B33 (1971), pp. 173–199. DOI: 10.1016/0550-3213(71)90395-6 (cit. on p. 6).

[22] Gerard 't Hooft. "Renormalizable Lagrangians for Massive Yang-Mills Fields". In: *Nucl. Phys.* B35 (1971). [,201(1971)], pp. 167–188. DOI: 10.1016/0550-3213(71)90139-8 (cit. on p. 6).