Dijkstra's Algorithm

A Deep Dive and Implementation

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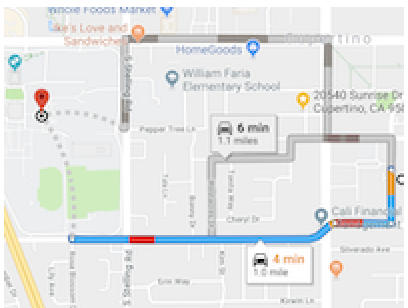
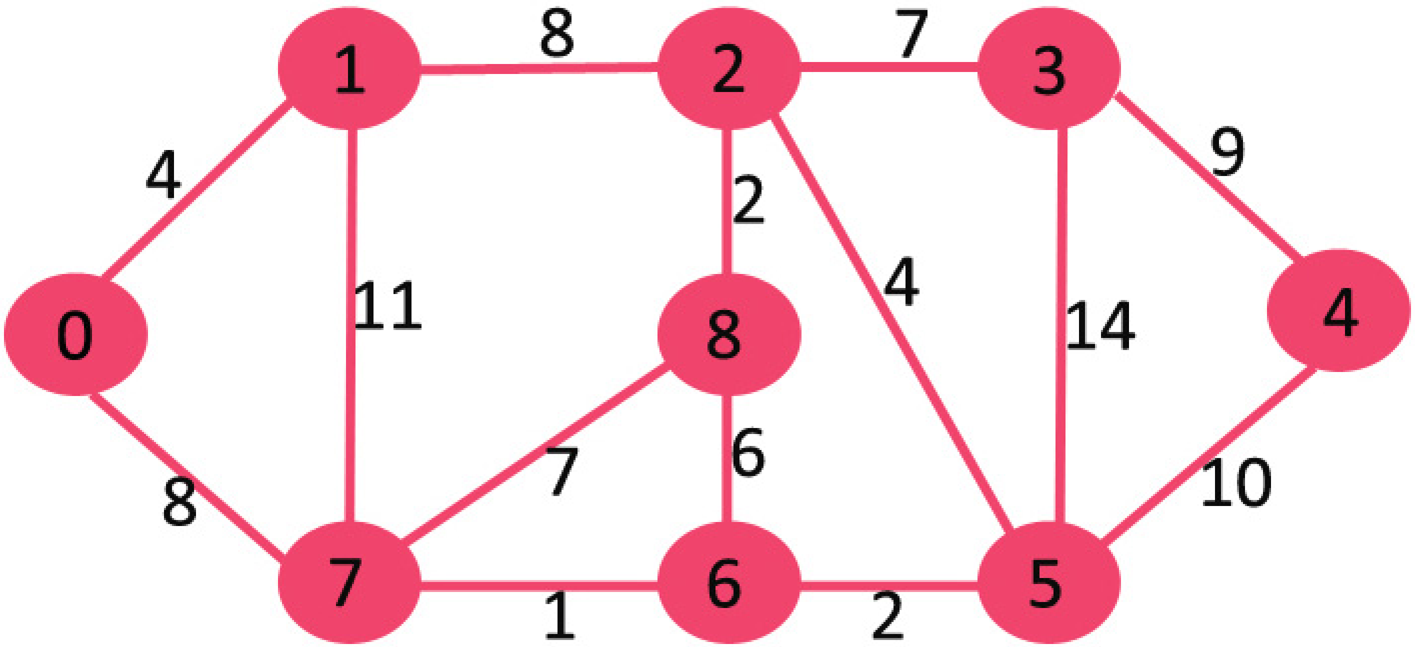
Introduction

- **"Simplicity is a prerequisite for reliability" - Edsger Dijkstra**
- 

- A search algorithm and graph data structure designed to find the shortest path between two nodes in a connected graph.
- Designed by Edsgar Dijkstra in 1956

Explanation



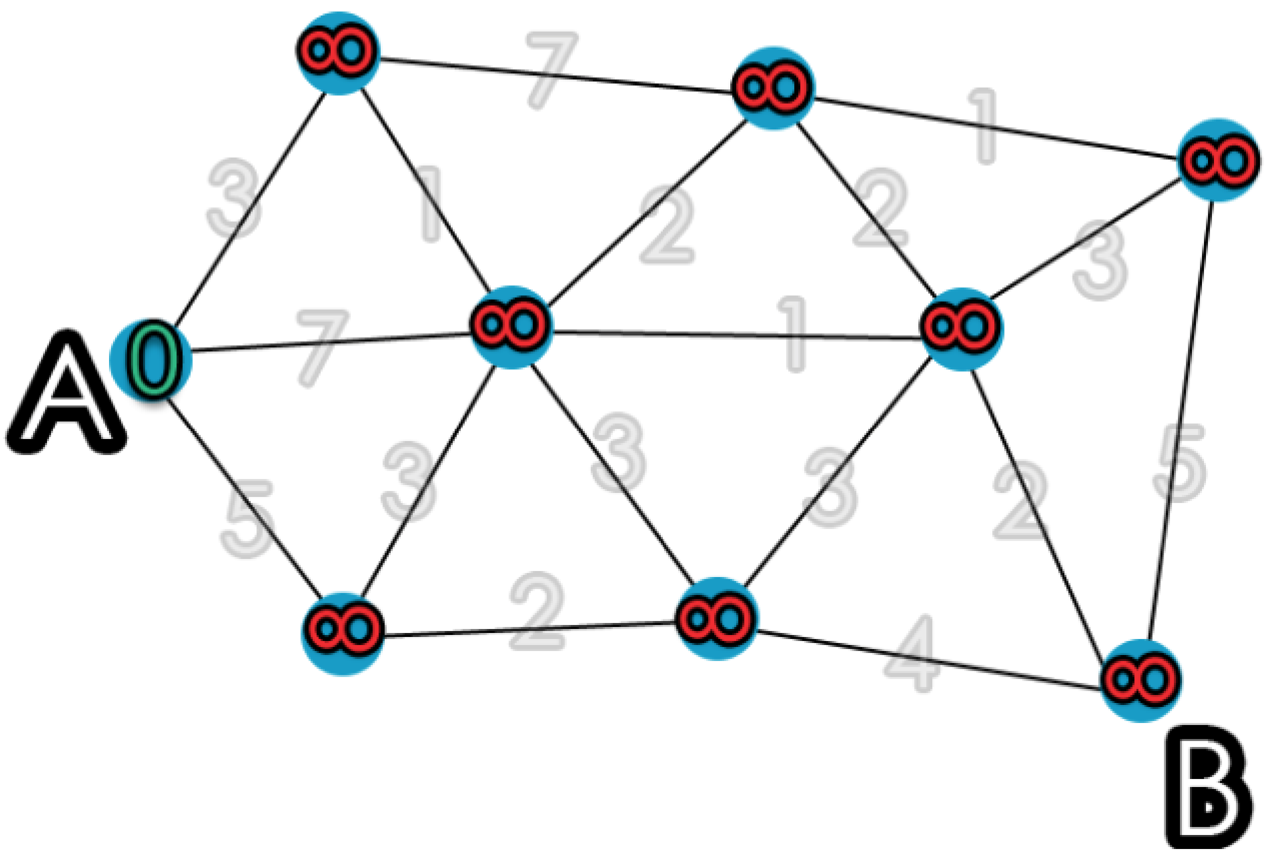
- There are nodes (or vertices)
- Weigted edges connect nodes

Explanation Continued

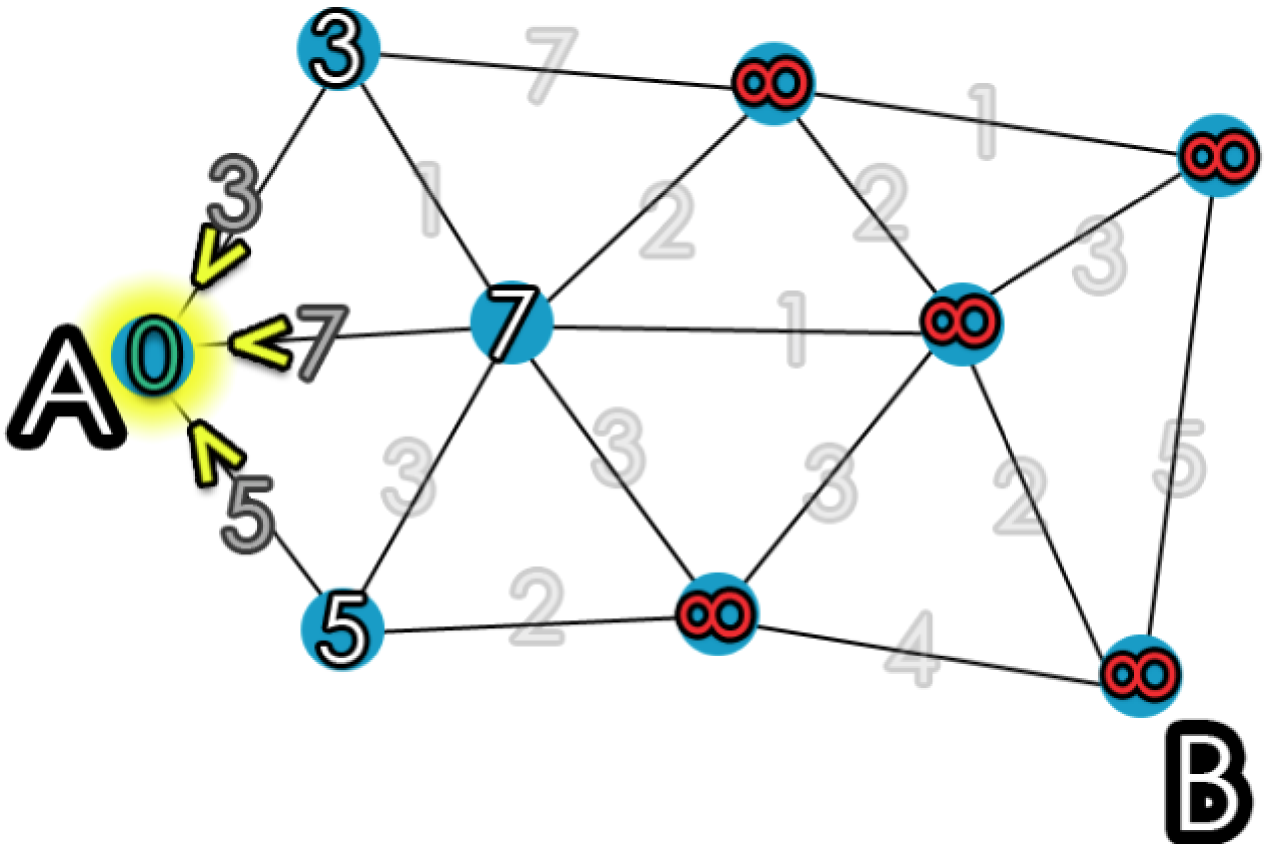
- `dist` initialized, an array to store distances from source node
    - `dist(s) = 0` since the distance from source node should be 0
    - `dist(v) = infinity` since distances to all other nodes are unknown
    - Q, a queue is used to store all nodes in the graph, Q will be empty by the time the algorithm is done
    - S represents the set of nodes the algorithm visited

Psuedo-explanation

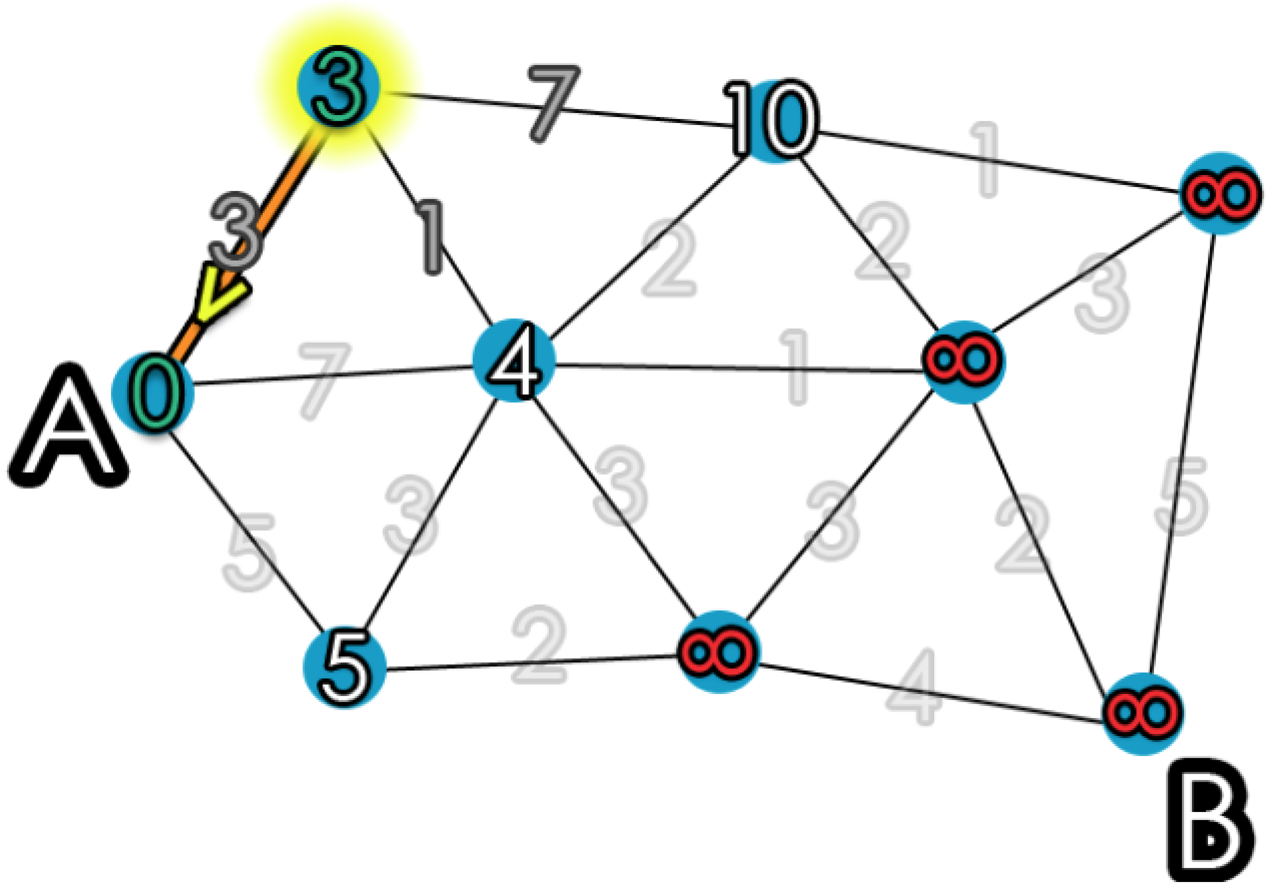
Suppose you have this connected graph:



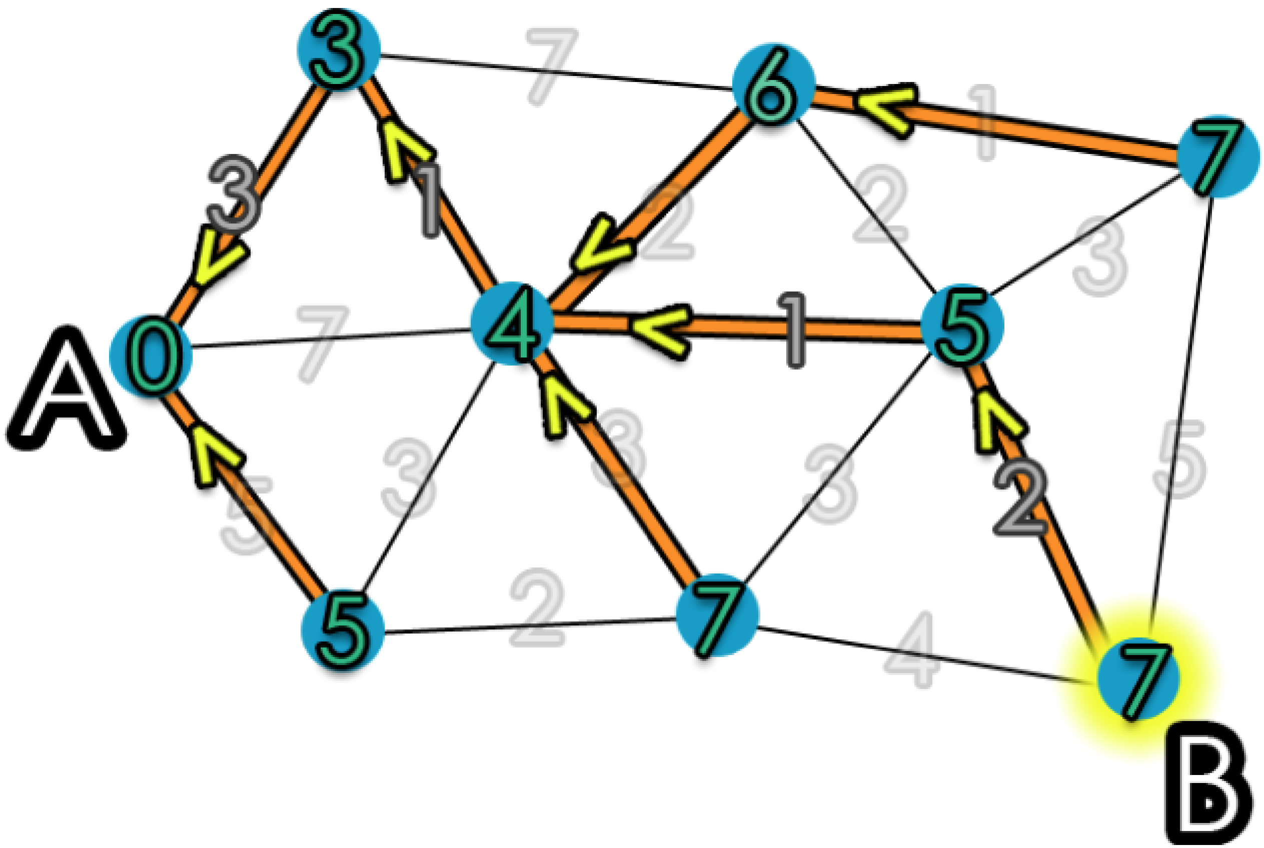
Calculate distances to adjacent nodes



Pick node closest to source node



Finally we get



Psuedocode

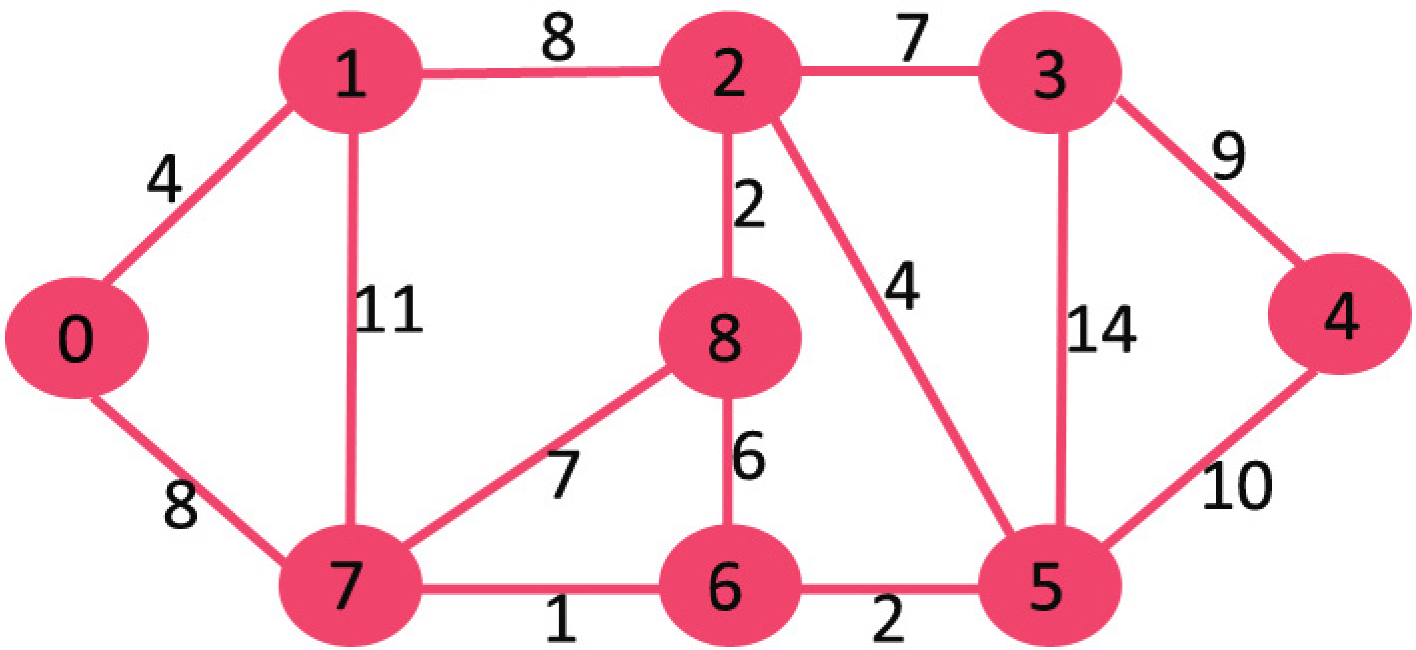
```
dist[s] = 0
for all other distances:
        dist[v] = infinity

S                                       #initialize
Q = V                                   #initialize V

while Q not null:                       #initalize Q is not null
        do u = minDistance(Q) #select element u w/ min di
        S = S union {u} # add u to list of visited vert.
        for all v in neighbors of {u}
                do if dist[v]>dist[u] + w(u,v)
                        # if new shortest path found
                        then d[v] = d[u]+w(u,v)
                    #set value of shortest path
```

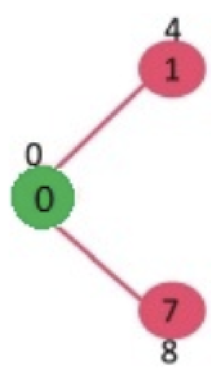
Another Implementation

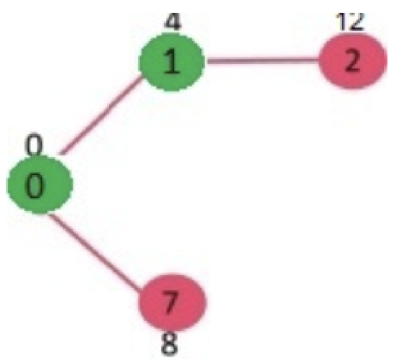
- Given a graph with weighted edges, construct a shortest-path "tree."
- Create a min heap and put in all the vertices.
    - Ordered by distance from source.
- Any node added into the SPT will be removed from this heap.

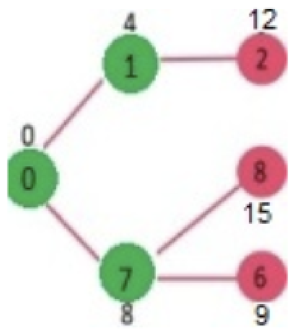


Another Implementation Continued

- Start by adding the source node into the SPT.
- Retrieve the top of the heap (a.k.a. the node closest to the source) and add it to the SPT.
- Update distances of nodes adjacent to the picked node.
- Repeat above two steps until every node is in the SPT/nothing left in the heap.

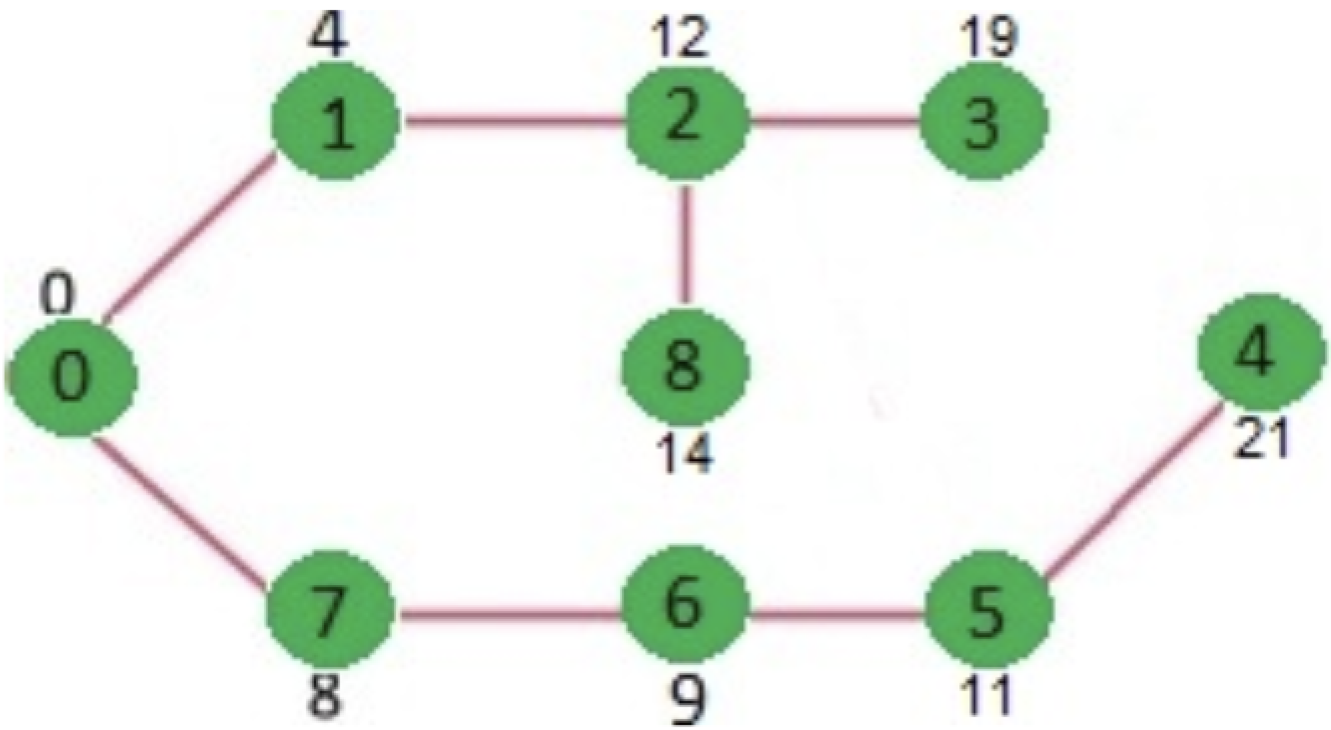






Another Implementation Extended

- Completed SPT:



Data Structures Used

- Adjacency Matrix
  - Used to store the graph.
- Arrays
  - Min heap and adjacency matrix implementation uses this.
  - ArrayLists/Vectors could have been used, but that would be excessively complicated.
- Heaps
  - Used to determine the node closest to the source to add to the SPT.
- Linked Lists
  - Used as adjacency lists for each vertex.

Big O Analysis

- Our implementation: $O(n^2)$
- Can be improved by simply storing the edges instead of the entire graph to $O(E \log V)$
    - E - Number of edges.
    - V - Number of vertices.

Implementation

Have a nice day!