FLAT :- formal language & automata theory.

1. Introduction to Automata Theory.
2. Regular language & expressions.
3. CFG → context free Grammar & languages.
4. PDA & TM → Turing machine
   PDA → push down automata.
5. Lex & Yacc

PDA → DPDA - Deterministic (one state)
PDA → NPDA - Non deterministic

* INTRODUCTION TO AUTOMATA THEORY :-

* Basic terminologies.
* NDFA → Non-deterministic finite automata.
* DFA → Deterministic finite automata
* Equivalence of DFA & NDFA
* ϵ-NFA

1. ALPHABET :→ A finite non empty set of symbols / elements (Σ)

eg $\Sigma = \{0,1\}$, $\Sigma = \{a,b,c,d ---,z\}$

$\Sigma = \{0,1\}$ → binary alphabet

$\Sigma = \{a,b,c,d ---- z\}$ → set of lower case english letters

$\Sigma = \{aa, aaa, aaaaa, ...\}$

2. STRINGS :- A finite sequence of symbols
(word)

ϵ, λ → empty string

e.g 1. 01101
2. abbabc
3. $\{\epsilon, 011010, 0011\}$

length of string (cardinality / size) :- | |

(1) $|1011011| = 5$ $|abbabc|$ $\{\epsilon, 01, 1010, 0011\}$

length: 0, 2, 4, 4

(1 zero, 1 one) (2 zero, 2 one)

* Concatenation of strings :-

x is a string having i symbols in it. y is also a string having j number of symbols in it such that

$x = a_1 a_2 a_3 \dots a_i$

$y = b_1 b_2 b_3 \dots b_j$

then concatenation of two strings $xy = a_1 a_2 a_3 \dots a_i b_1 b_2 b_3 \dots b_j$

(3) power of an alphabet :- is denoted by Σ^k

set of all strings of length k

where k is length of the string.

$\Sigma^1 = \{0, 1\}$, $\Sigma^0 = \{\epsilon\}$, $\Sigma^2 = \{00, 01\}$, $\Sigma^3 = \{000, 111\}$

power of an alphabet is also denoted with :-

$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \dots$

$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \dots$

(Non empty finite set of alphabets)

(4) Language :- set of all strings
(L)

$L = \{\epsilon, 0, 1, 00, 11, 1010, 0011, 11001, 1100100 \dots\}$

Hirarchy or classification of languages

① Type 0 → Recursively enumerable → TM (Turing machine)

② Type 1 → Context sensitive Language → LBA (Linear Bounded automata)

③ Type 2 → Context free Language → PDA (Push down automata)

④ Type 3 → Regular Language → FA (Finite automata)

∅ → empty language

Type 0 → phrase structured / Recursively enumerable → TM (Turing machine)

Type 1 → context sensitive Language → LBA → Linear Bounded automata.

Type 2 → Context free language ⇒ PDA → push down automata.

Type 3 → Regular language → finite automata.
- → DFA
- → NFA / NDFA
- → ϵ-NFA

} abstract machines / computational models

TOC → Theory of computation

abstract → Brief summary.

finite automata is defined as abstract machine, a mathematical computational model which comprises both hardware & software in reality these kind of machine don't have physical existence.

Types :→

1. DFA :→ Deterministic final automata
2. NDFA :→ Non-Deterministic final automata
3. ϵ - NFA

* Representation of finite automata :→

$$\left.\begin{array}{l}\text{states}\\ \text{Input alphabets}\\ \text{Transition}\\ \text{start state}\\ \text{final states}\end{array}\right\}$$

- TD → Transition diagram
- TT → Transition Table.

$\underbrace{q_0 \text{ or } A}_{\text{states}}$

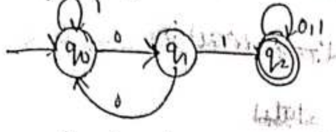
$\rightarrow q_0 \xrightarrow{1} q_1$ → Arc / edges / Transition from q_0 on 1 to q_1

The start state is represented by :

$\boxed{\rightarrow q_0}$ start state.

final state is denoted by q_f

* Example for DFA. Identify the start state, final state & transition.



start state → q_0 | q_s

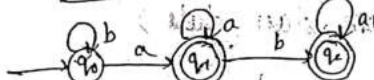
final state → q_2 | q_f

Input alphabets → $\Sigma = \{0, 1\}$

Transitions :→ $\delta(q_0, 0) = q_1$ $\quad \delta(q_0, 1) = q_0$ $\quad \delta(q_1, 1) = q_2$

$\delta(q_1, 0) = q_0$ $\quad \delta(q_2, 1) = q_2$ $\quad \delta(q_2, 0) = q_2$

* Transition Table :→



q_0 → start state

$q_2, q_1 \subseteq Q$ → final state

$\Sigma = \{a, b\}$

Transition Table

$\delta(q_0, a) = q_1$

$\delta(q_0, b) = q_0$

$\delta(q_1, a) = q_1$

$\delta(q_1, b) = q_2$

$\delta(q_2, a) = q_2$

$\delta(q_2, b) = q_2$

states

δ	a	b
→ q_0	q_1	q_0
* q_1	q_1	q_2
* q_2	q_2	q_2

Input alphabets

Here → indicates start state

* indicates final state

DEFINATION :→

DFA :→ (1) Determinstic finite automata

$M_{DFA} = (Q, \Sigma, \delta, q_0, q_f)$ M :→ Machine

Q :→ finite non empty set of states

Σ :→ finite non empty set of input alphabets

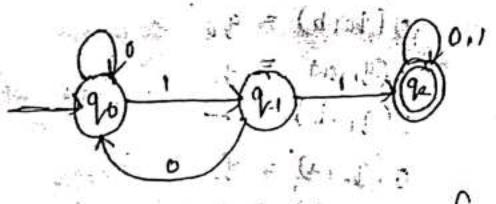
δ :→ Transition function.

$\delta : Q \times \Sigma = Q$

Transition is mapping from states to input alphabets which yields states

q_0 → start state (Only one start state)

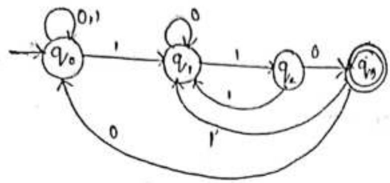
q_f → final state & is subset of Q
(i.e more than one final state)



$Q = \{q_0, q_1, q_2\}$ q_2 → final state

$\Sigma = \{0, 1\}$ q_0 → start state

* Check wether the fallowing transitions constitute DFA or not

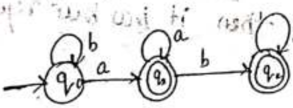
(1) 

$\delta(q_0,0) = q_0$
$\delta(q_0,1) = \{q_0, q_1\}$
$\delta(q_1,0) = q_1$
$\delta(q_1,1) = q_2$
$\delta(q_2,0) = q_3$
$\delta(q_2,1) = q_1$
$\delta(q_3,0) = q_0$
$\delta(q_3,1) = q_1$

Transition table

	δ	0	1
Q	$\rightarrow q_0$	q_0	$\{q_0, q_1\}$
	q_1	q_1	q_2
	q_2	q_3	q_1
	$* q_3$	q_0	q_1

As for same input alphabet there are multiple state it is not DFA.

(2) 

δ	a	b
$\rightarrow q_0$	q_1	q_0
$* q_1$	q_1	q_2
$* q_2$	q_2	q_2

It is DFA

(3) 

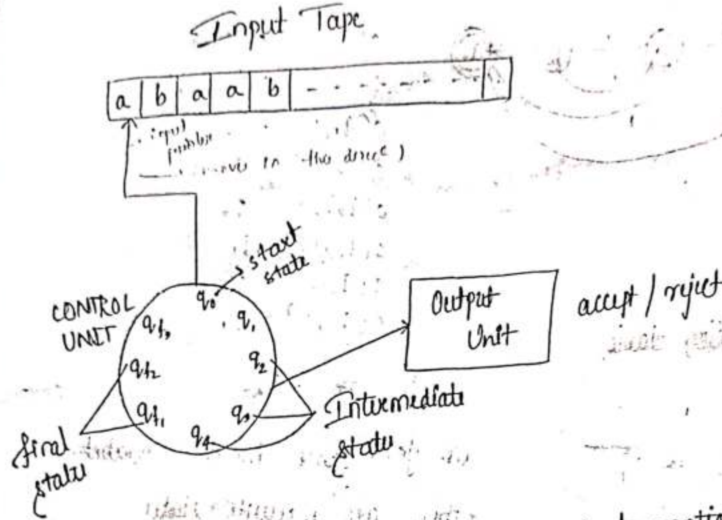
$\Sigma = \{0, 1\}$
$Q = \{q_0, q_1, q_2, q_3\}$

Transitions

$\delta(q_0,0) = q_0$
$\delta(q_0,1) = q_1$
$\delta(q_1,0) = q_1$
$\delta(q_1,1) = \{q_1, q_2\}$
$\delta(q_2,0) = \{q_2, q_3\}$
$\delta(q_2,1) = q_3$
$\delta(q_3,0) = q_3$
$\delta(q_3,1) = \phi$

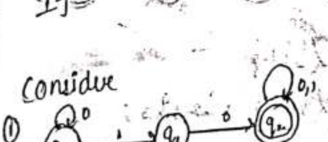
Scanned by CamScanner

* Working procedure of finite automata :-



The machine will be in start state (Assumption).
If it reaches final state then it is been accepted.
If it is in intermediate states then it has been rejected.

Consider



$W_0 = 0110110$

$\delta(q_0, 0) = q_0$

$\delta(q_0, 1) = q_1$

$\delta(q_1, 1) = q_0$

$\delta(q_0, 0) = q_0$

$\delta(q_0, 1) = q_1$

$\delta(q_1, 0) = q_2$ }

final state

$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_2$

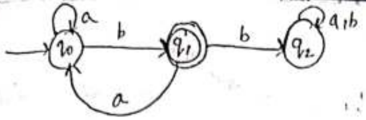
OR

$W_1 = 0110101$

$q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_2$ } final state ∴ accepted

$W_2 = 11001101$

$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_0$ } Rejected

(2) 

w = abba

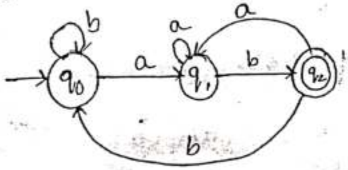
$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_2$ Rejected

w = abbabb

$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_2 \xrightarrow{b} q_2 \xrightarrow{b} q_2$

* What are the moves that are made by the following DFA to accept the following strings.

(1) abaab (2) abb (3) abaa.



Sol^n $\Rightarrow$ $Q = \{q_0, q_1, q_2\}$ $\Sigma = \{a, b\}$

$q_0 \rightarrow$ start state

$q_2 \rightarrow$ final state

$\delta(q_0, a) = q_1$ $\delta(q_1, a) = q_1$ $\delta(q_2, a) = q_1$

$\delta(q_0, b) = q_0$ $\delta(q_1, b) = q_2$ $\delta(q_2, b) = q_0$

(i) ~~ab~~ w : abaab

$\delta(q_0, a) = q_1$, $\delta(q_1, b) = q_2$ $\delta(q_2, a) = q_1$ $\delta(q_1, a) = q_1$ $\delta(q_1, b) = q_2$

Since the machine is in final state the string is accepted by DFA.

(ii) w = abb $\delta(q_0, a) = q_1$ $\delta(q_1, b) = q_2$ $\delta(q_2, b) = q_0$

Since the machine is not in final state the string is rejected or// $q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{b} q_0$

(ii) abaa

$q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_1 \xrightarrow{a} q_1$

since the machine is not in final state it is rejected.

* <u>Extended</u> <u>transition</u> <u>function</u> :-

It is denoted by $\boxed{\delta^*(q, w) = p}$

δ^* denote extended transition function.

q is the first parameter which represents current state.

w is a second parameter which represents the input string.

p is a new state obtained after transition.

Let $w = xa$ w is input string.

$\delta(\delta^*(q, x), a)$

$\delta^*(q, \epsilon) = q$

* Using extended transition function write the moves that are made by DFA for the following

(1) abaab (2) abb (3) abaa.

(1) $\delta^*(q_0, \epsilon) = q_0$

$a \rightarrow \delta^*(q_0, a) = \delta(\delta^*(q_0, \epsilon), a)$
$= \delta(q_0, a)$
$= q_1$

$\underline{ab} \rightarrow \delta^*(q_0, ab) = \delta(\delta^*(q_0, a), b)$
$= \delta(q_1, b)$
$= q_2$

for the prefix

$\underline{aba} \rightarrow \delta^*(q_0, aba) = \delta(\delta^*(q_0, ab), a)$
$= \delta(q_2, a)$
$= q_1$

$\underline{abaa} \rightarrow \delta^*(q_0, abaa) = \delta(\delta^*(q_0, aba), a)$
$= \delta(q_1, a)$
$= q_1$

$\underline{abaab} \rightarrow \delta^*(q_0, abaab) = \delta(\delta^*(q_0, abaa), b)$
$= \delta(q_1, b)$
$= q_2$

② $\delta^*(q_0, \epsilon) = q_0$

for prefix a $\quad \delta^*(q_0, a) = \delta(\delta^*(q_0, \epsilon), a)$
$= \delta(q_0, a)$
$= q_1$

for prefix b $\quad \delta^*(q_0, ab) = \delta(\delta^*(q_0, a), b)$
$= \delta(q_1, b)$
$= q_2$

for prefix b $\quad \delta^*(q_0, abb) = \delta(\delta^*(q_0, ab), b)$
$= \delta(q_2, b)$
$= q_0$

③ $\delta^*(q_0, \epsilon) = q_0$

$\delta^*(q_0, a) = \delta(\delta^*(q_0, \epsilon), a)$
$= \delta(q_0, a)$
$= q_1$

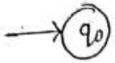
$\delta^*(q_0, ab) = \delta(\delta^*(q_0, a), b)$
$= \delta(q_1, b)$
$= q_2$

$\delta^*(q_0, aba) = \delta(\delta^*(q_0, ab), a)$
$= \delta(q_2, a)$
$= q_1$

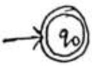
$\delta^*(q_0, abaa) = \delta(\delta^*(q_0, aba), a)$
$= \delta(q_1, a)$
$= q_1$

* Transition diagram for the following languages

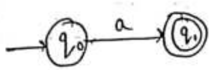
① ϕ → Empty language.



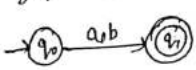
② ϵ → Empty string.



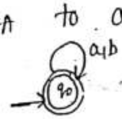
③ DFA to accept only one a



④ DFA to accept one a or one b.



⑤ DFA to accept zero or more number of a's or b's



DFA :→

① pattern recognization problem

② Divisible by k

③ Modulo k counter.

To solve a problem

STEP :→

① Identify minimum string.

② Identify input alphabet

③ Construct a base DFA, having start state & final state.

④ Identify the transitions that are not defined in the 3rd step.

⑤ Write complete transition diagram with all ~~steps~~ transitions & ~~to~~ hence write transition table.

* Construct DFA to accept strings of a having attleast one a.

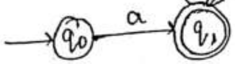
$\Rightarrow$ $L = \{a, aa, aaa, aaaa, aaaaa \ldots\ldots\}$

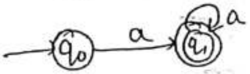
or

$L = \{w : |n_a \geq 1, w \in \{a^*\}\}$

① a

② $\Sigma = \{a\}$ 

③ 

④ 

⑤ $Q = \{q_0, q_1\}$ $\Sigma = \{a\}$ $q_0 \rightarrow$ start state $q_1 \rightarrow$ final state.

δ: $\delta(q_0, a) = q_1$, $\delta(q_1, a) = q_1$

δ	a
$\rightarrow q_0$	q_1
$* q_1$	q_1

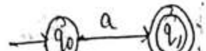
(2) Construct a DFA to accept strings of a's & b's having exactly one a.

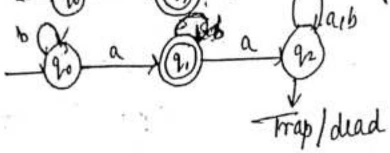
⇒ ~~L = {a, ab, abb, abbb ....}~~

L = {ba, bba, bbba, bab ....}

(1) a

(2) $\Sigma = \{a, b\}$

(3) 

(4) 

δ	a	b
$\rightarrow q_0$	q_1	q_0
$*q_1$	q_2	q_1
q_2	q_2	q_2

(3) Construct DFA to accept strings of a's & b's having atleast one a.

⇒ (1) a

L = {a, ab, abb, bab, bba, bbaa, baa, baaa, bbaa, bba ....}

(2) $\Sigma = \{a, b\}$

(3) 

$\delta(q_0, b) = ?$.
$\delta(q_1, a) = ?$.
$\delta(q_1, b) = ?$.

δ	a	b
$\rightarrow q_0$	q_0	q_1
$*q_1$	q_1	q_1

$Q = \{q_0, q_1\}$

$\Sigma = \{a, b\}$

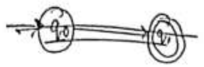
$q_0 \rightarrow$ start state

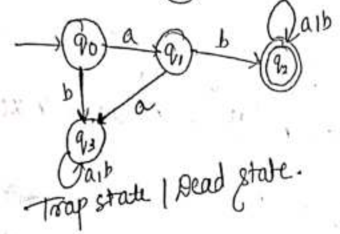
$q_1 \rightarrow$ final state.

(1) Construct a DFA of strings a & b such that sequence should begin with ab.

⇒ (1) ab

(2) $\Sigma = \{a, b\}$ $L = \{ab, ababba, abba, abbba, \ldots\}$

~~(3)~~ 

(3) 

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$q_0 \rightarrow$ start state

$q_2 \rightarrow$ final state

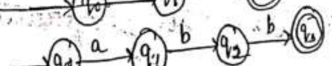
δ	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_3	q_2
$*q_2$	q_2	q_2
q_3	q_3	q_3

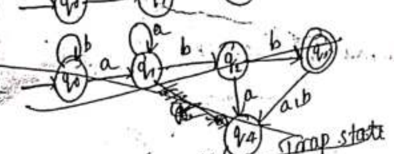
(2) ~~Construct~~ Construct DFA of strings of a & b such that each & every string should end with abb.

(1) abb $L = \{abb, ababb, aabb, babb, aababb, bbabb, aaabb, \ldots\}$

(2) $\Sigma = \{a, b\}$

(3) 

(3) 

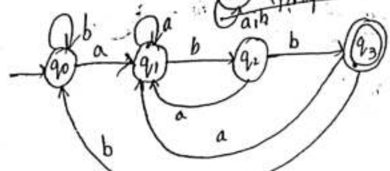
(4) 

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$q_0 \rightarrow$ start state

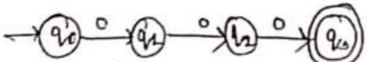
$q_3 \rightarrow$ final state

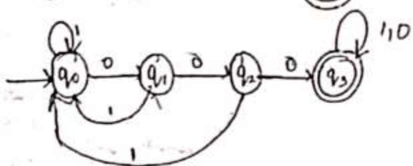
(4) 

(6) Construct a DFA of strings 0's & 1's, such that it should have three consecutive zeros.

⇒ L = {000, 100011, 10000, 00011 ... }

(2) Σ = {0, 1}   (1) 000

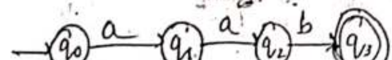
(3) 

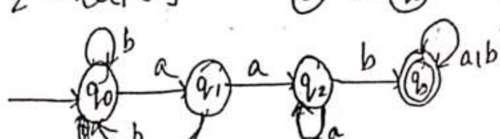
(4) 

(7) Write a DFA of strings a's & b's having a substring aab.

⇒ L { aab, baabaa, aaabb, aaaabb ... }

(1) aab.

(2) Σ = {a, b}   (3) 

(4) 

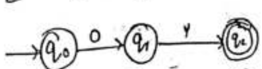
δ	a	b
$\rightarrow q_0$		
q_1		
q_2		
$*q_3$		

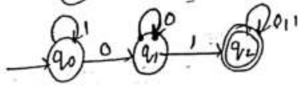
(8) Construct a DFA to accept strings of 0's & 1's such that somewhere in the sequence 01 should appear in the string.

$\Rightarrow L = \{01, 1011, 00101, 110100, 000101, 0011 \cdots\}$

(1) 01

(2) $\Sigma = \{0, 1\}$

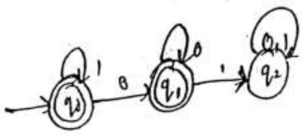
(3) 

(4) 

δ	0	1
q_0		
q_1		
q_2		

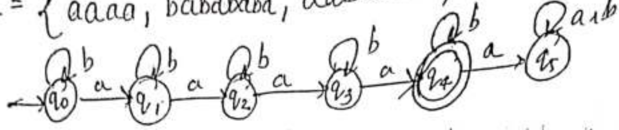
(9) Construct a DFA to accept strings of 0's & 1's such that strings should not have a substring 01.

* NOTE :- If not keyword appears change final state → Non final state
Non-final state → final state.



(10) Construct DFA ~~of~~ str having even a's for the input alphabet $\Sigma = \{a, b\}$.

⇒ L = {aaaa, babababa, aabbaabb, abababa ... }

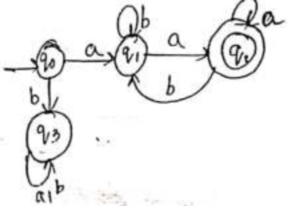


(11) Construct a DFA to accept the following language

L = {awa | w ∈ {a,b}*}.

→ Minimum string :→ aa

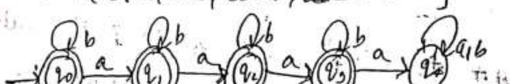
L = {aa, aba, aaa, abaa ... }



(12) a's & b's having atmost 3 a's.

⇒ ~~L = {aaa, baaa, abaa~~

L = {ε, a, aa, aaa, ao - - - }



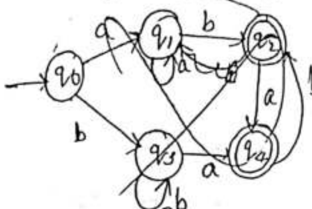
Here they are made final states because to accept zero a's, 1 a, 2a ---

(13) $L = \{ w \in a,b \mid w(ab + ba) \}$

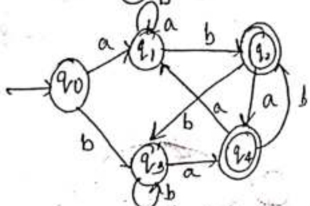
or

⇒ This implies the string should either end with ab or ba

$L = \{ aab, aba, aab, bab, bbab, aaba \ldots \}$

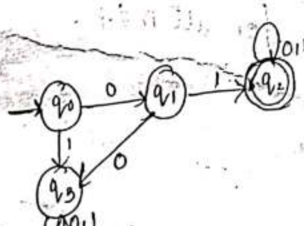


$\delta(q_1, a) =$



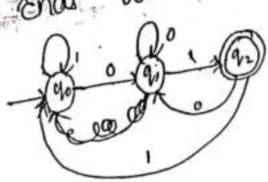
(14) Construct a DFA of strings 0's & 1's which begins with 01.

⇒ $L \{ 01, 0100, 0110 \ldots \}$



(15) Ends with 01.

⇒



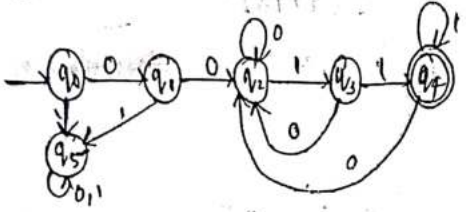
Scanned by CamScanner

(16) Construct DFA for following languages :-

(1) $L = \{ w \in (0+1)^* \mid 00(0+1)^*11 \}$

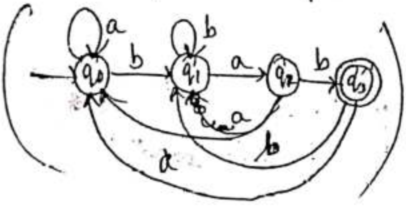
⇒ Begins with atleast two zeroes & ends with atleast two ones.

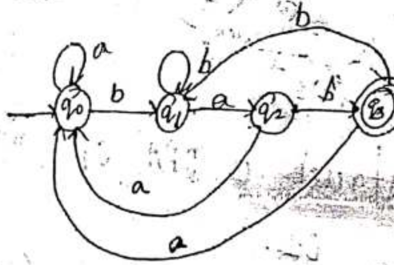
(* indicates zero or any times)



(2) $L = \{ w \in \{a,b\} \mid wbab \}$

⇒ strings ending with bab.





(17) DFA's of a's & b's which accepts all the strings of length a^n for all $n \neq 4$.

⇒

(18) Multiples of two of a's $\Sigma = \{a, b\}$

$\Rightarrow$

① DIVISIBLE BY K PROBLEMS :→

$\delta(q_i, a) = q_j$ $\quad Q \times \Sigma = Q$

$j = (r * i + d) \bmod k$

r :→ radix input

i :→ remainder obtained after dividing by k

d :→ digits

k :→ divisor

for binary radix input :→ 2

decimal radix input :→ 10

" :→ 3, {0, 1, 2}

NOTE :→ start state & final state will be same.

① Construct a DFA of strings 0's & 1's where each strings are represented as binary number which are divisible by 5.

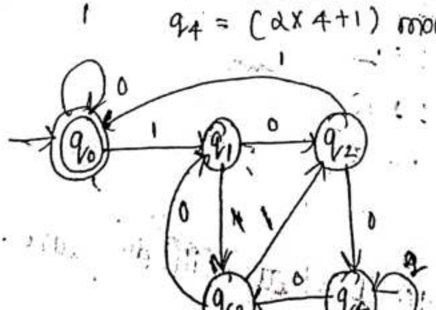
$\Rightarrow$ $r = 2$

$i = 0,1,2,3,4$

$d = \{0,1\}$

$k = 5$

i	d	$j = (r*i+d) \bmod k = j$	$\delta(q_i, a) = q_j$
$i=0$	0	$q_0 = (2*0+0) \bmod 5 = 0$	$\delta(q_0, 0) = q_0$
	1	$q_0 = (2*0+1) \bmod 5 = 1$	$\delta(q_0, 1) = q_1$
$i=1$	0	$q_1 = (2*1+0) \bmod 5 = 2$	$\delta(q_1, 0) = q_2$
	1	$q_1 = (2*1+1) \bmod 5 = 3$	$\delta(q_1, 1) = q_3$
$i=2$	0	$q_2 = (2*2+0) \bmod 5 = 4$	$\delta(q_2, 0) = q_4$
	1	$q_2 = (2*2+1) \bmod 5 = 0$	$\delta(q_2, 1) = q_0$
$i=3$	0	$q_3 = (2*3+0) \bmod 5 = 1$	$\delta(q_3, 0) = q_1$
	1	$q_3 = (2*3+1) \bmod 5 = 2$	$\delta(q_3, 1) = q_2$
$i=4$	0	$q_4 = (2 \times 4+0) \bmod 5 = 3$	$\delta(q_4, 0) = q_3$
	1	$q_4 = (2 \times 4+1) \bmod 5 = 4$	$\delta(q_4, 1) = q_4$



② Construct a DFA of strings of decimal numbers that are divisible by 3.

$\Rightarrow$ $r = 10$

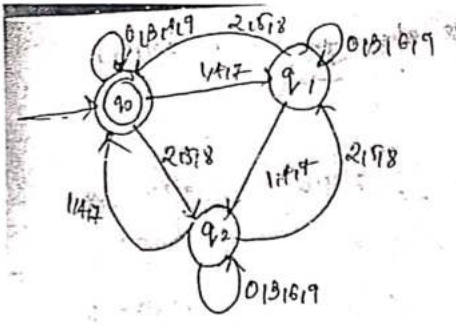
$i = 0, 1, 2$

$d = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$k = 3$

	d	$j = (r * i + d) \bmod k$	$\delta(q_i, a) = q_j$
$i = 0$	0	$q_0 = (10 * 0 + 0) \bmod 3 = 0$	$\delta(q_0, 0) = q_0$
	1	$q_0 = (10 * 0 + 1) \bmod 3 = 1$	$\delta(q_0, 1) = q_1$
	2	$q_0 = (10 * 0 + 2) \bmod 3 = 2$	$\delta(q_0, 2) = q_2$
	3	q_0	
	4	q_0	
	5	q_0	
	6	q_0	
	7	q_0	
	8	q_0	
	9	q_0	

	d	$(r * i + d) \bmod k = j$
$i = 0$	$(0, 3, 6, 9)$	$(10 * 0 + 0) \bmod 3 = q_0$
	$(1, 4, 7)$	$(10 * 0 + 1) \bmod 3 = q_1$
	$(2, 5, 8)$	$(10 * 0 + 2) \bmod 3 = q_2$
$i = 1$	$(0, 3, 6, 9)$	$(10 * 1 + 0) \bmod 3 = q_1$
	$(1, 4, 7)$	$(10 * 1 + 1) \bmod 3 = q_2$
	$(2, 5, 8)$	$(10 * 1 + 2) \bmod 3 = q_0$
$i = 2$	$(0, 3, 6, 9)$	$(10 * 2 + 0) \bmod 3 = q_2$
	$(1, 4, 7)$	$(10 * 2 + 1) \bmod 3 = q_0$
	$(2, 5, 8)$	$(10 * 2 + 2) \bmod 3 = q_1$



(3) Construct a DFA of strings of 0's & 1's which are divisible by 5 such that DFA should not accept binary number 000

(4) Construct a DFA of string of 0's, 1's & 2's which are represented as combination of 0's, 1's & 2's divisible by 4.

Divisibility problem :-

(1) divisible by 3 1's decimal number.

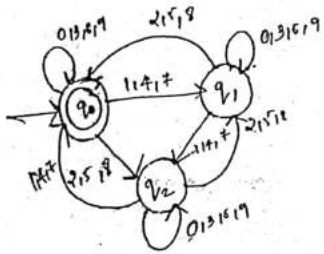
⇒ Remainders are 0,1,2

$0 \rightarrow q_0$ $\qquad$ $d = 0,1,2,3,4,5,6,7,8,9.$

$1 \rightarrow q_1$

$2 \rightarrow q_2$

δ	(0,3,6,9)	(1,4,7)	(2,5,8)
$\rightarrow q_0$	q_0	q_1	q_2
q_1	q_1	q_2	q_0
q_2	q_2	q_0	q_1



(2) Construct a DFA all strings over $\Sigma = \{0,1\}$ ends with 100.

⇒ q_0 is start state or initial state. q_1 is a state ending with one q_2 is a state ending with 10, q_3 100.

$q_1 \rightarrow 1 \qquad q_2 \rightarrow 10 \;, q_3 \rightarrow 100$

$M = (Q, \Sigma, \delta, q_0, q_f)$

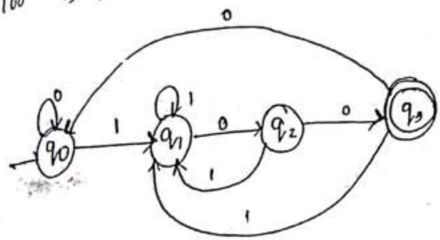
$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0,1\}$

$q_0 \rightarrow ss \quad q_3 \rightarrow fs$

	δ	0	1
ϵ	$\rightarrow q_0$	q_0	q_1
1	q_1	q_2	q_1
10	q_2	q_3	q_1
100	$*q_3$	q_0	q_1

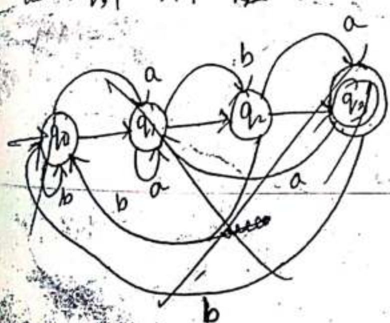
$\cancel{\phi}$0, $\cancel{\phi}$1
ϕ10, $\cancel{\phi}$11
$\cancel{1}$100, $\cancel{0}$101
1000 1001



(c) Ending with aba.

	δ	a	b
ϵ	$\rightarrow q_0$	q_1	q_0
a	q_1	q_1	q_2
ab	q_2	q_3	q_0
aba	$*q_3$	q_1	q_2

$\cancel{\phi}$a, $\cancel{\epsilon}$b
$\cancel{a}$a, ab
aba, $\cancel{ab}$b
abda, abab.



NFA :- Non deterministic finite automata

$M_{NFA} = \{Q, \Sigma, \delta, q_0, F\}$

$\delta : \Sigma \times Q = Q$ (DFA)

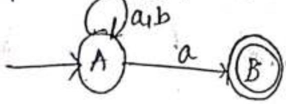
$\delta : Q \times \Sigma = 2^Q$ (power set) $\longrightarrow$ NFA.

Q is set of finite states.

power set $\longrightarrow$ set of all subsets

$A = \{a, b, c\}$

$P(A) = \{\phi, a, b, c, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$



$\delta(A, a) = A$

$\delta(A, b) = A$

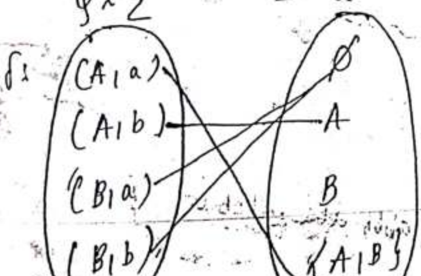
$\delta(B, a) =$

$\delta(B, b) =$

δ	a	b
$\rightarrow A$	$\{A, B\}$	A
$*B$	ϕ	ϕ

$Q \times \Sigma = 2^Q$

~~Examples of NFA~~



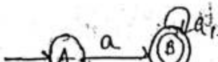
Scanned by CamScanner

EXAMPLES:-

① Obtain an NFA over strings a,b where all the strings starts with a.

→ ① L = { starts with a }

L = { a, ab, abb, aabb ---- }

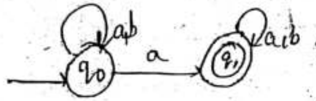


$\delta(A, a) \to B$

$\delta(B, b) \to B$

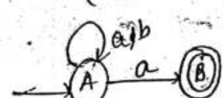
② L = { containing a } over a,b

⇒ ~~L = { abba,~~ L = { aba, bab, bba, ab, ba ... }



③ L = { ends with a } over a,b

⇒

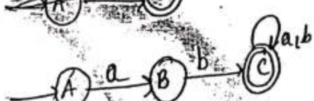


④ L = { starting with ab }

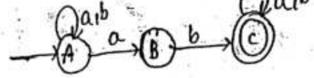
⑤ L = { containing ab }

⑥ L = { ends with ab }

④ L = { abbb, aba, aabb }

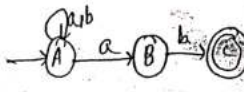


⑤



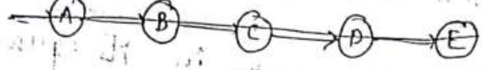
L = { abab, abbb, bbabb }

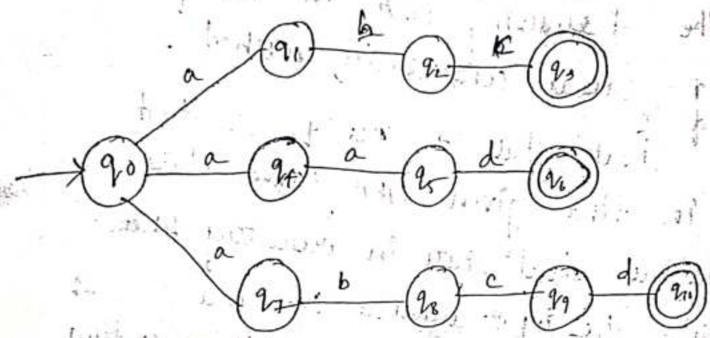
⑥



(7) In order to accept the following set of strings

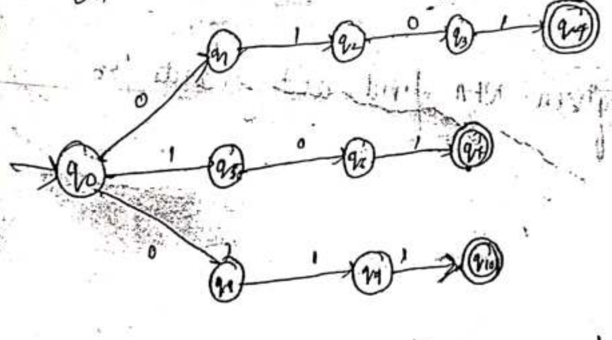
abc
aad
abcd

⇒ 



(8) Construct NFA for given set of strings over 0,1 in order to accept

0101
101
011



NOTE :→ While processing the string or any sequence in a result or in a set of states atleast one state should be accepting state or final state then the string or sequence is accepted by NFA

REGULAR EXPRESSION

ϕ :→ It is a regular expression which denotes Empty language

ϵ :→ It is a regular expression which denotes Empty string.

R is a regular expression corresponding to language L_R.

S is a regular expression then its corresponding language denoted by L_s

& the following cases holds good.

$R+S = L_R \cup L_s$

$R.S = L_R . L_s$

$R^* = (L_R)^*$

$S^* = (L_s)^*$

* closure operator (Highest priority)

\+ Union operator

. Concatenation operator (Least priority)

* Write regular expressions for the input alphabets ~~a & b~~.

① strings of any number of a's including NULL.

→ { ∈, a, aa, aaa .... }

a^*

(2) Obtain a regular expression for a string having atleast one a.

$\Rightarrow$ a^+ or aa^*

(3) strings having one atleast one a or one b

$\Rightarrow$ $(a+b)(a+b)^*$

(4) strings of a's & b's starts with a & ends with abb.

$\Rightarrow$ $a(a+b)^*abb$

(5) strings of a's & b's having aa as substring.

$\Rightarrow$ $(a+b)^* aa (a+b)^*$

(6) strings of zeros & ones should end with three consecutive zeros

$\Rightarrow$ $(0+1)^*000$

(7) Even number of a's.

$(aa)^*$

(8) Odd number of a's

~~(aa)*~~, $a(aa)^*$

(9) obtain regular expression for strings of a & b whose length should be equal to 2.

$(a+b)(a+b)$

(10) of length ≤ 2.

$\Rightarrow$ $L = \{\epsilon, a, b, ab, ba, aa, bb\}$ $\quad (\epsilon+a+b)(\epsilon+a+b)$

or $(\epsilon+a+b)^2$

Operation on Regular Expressions :-

① $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

② $A \cdot B = \{x \mid x \in A \ \& \ x \in B\}$

③ $A^* = \{x \mid x_1, x_2, x_3 \ldots\ldots x_k \mid k \geq 0\}$ $x_k \in A$

$A = \{pq, r\}$ $B = \{t, uv\}$

$A \cup B = \{pq, r, t, uv\}$

$A \cdot B = \{pqt, pquv, rt, ruv\}$

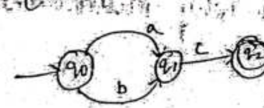
$A^* = \{pq, r, pqr, rpq, pqpq, pqpqpq, rrr \ldots\}$

$B^* = \{$

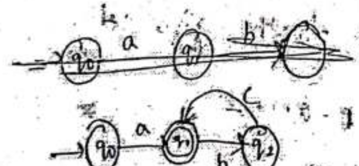
Write DFA for the following regular expressions

① ba^*b $L = \{bb, bab, baab \ldots\}$ (if * comes self loop)

② $(a+b)c$ $L = \{ac, bc\}$



③ $a(bc)^*$ $L = \{a, abc, abcbc$



Obtain regular expression for strings of a & b for the n^{th} block of 4 consecutive symbols atleast 2 a's should be there

$a(a+b)(a+b)a$

$+$

$(a+b)\,a\;\;a\,(a+b)$

$+$

$a\,(a+b)\,a\,(a+b)$

$+$

$a\,a\,(a+b)\,(a+b)$

$+$

$(a+b)\,(a+b)\,a\;\;a$

$+$

$(a+b)\,a\,(a+b)\,a$

string of a's & b's having string of length atleast 2

$\Rightarrow$ $((a+b)(a+b)$ or $(a+b)^2$ } → exact 2)

$(a+b)(a+b)(a+b)^*$ → atleast 2.

3] Write regular expression

$L = \{ a^{2n} b^{2m} \mid n, m \geq 0 \}$

$\Rightarrow (aa)^*(bb)^*$

4] odd length & multiples of 3, mod 3.

$((a+b)(a+b)(a+b))^*$

7] $|w| \geq 3$ atleast

$(a+b)^3 (a+b)^*$

8] Obtain a regular expression of strings of a's & b's such that 3 symbol from left hand should be b.

$\Rightarrow R = (a+b).(a+b).b.(a+b)^*$

$R = (a+b)^2 . b . (a+b)^*$

9] From the right hand side 28th symbol should be a.

$\Rightarrow (a+b)^* . a . (a+b)^{27}$

10]

Scanned by CamScanner

10] Write finite automata for the following regular expression.

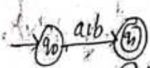
ϵ → Empty string

ϕ → Empty language

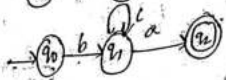
a^* :-

$(a+b)^*$

$(a+b)$



$b(c)^*a$



Minimize the DFA :-

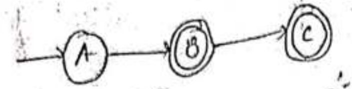
Table filling algorithm :-

* Indistinguishable / Equivalent states } Equivalence of states in FSM / FA
* Distinguishable states

Indistinguishable states :- If they are two states p & q on reading some input if the yield a final state then the pair (p,q) is called indistinguishable pair.

Then the pair (p,q) is called indistinguishable pair.

1. $\left.\begin{array}{l}\delta(p,a) \in F \\ \delta(q,a) \in F\end{array}\right\} \Rightarrow (p,q) \rightarrow$ Indistinguishable pair.
2. $\begin{array}{l}\delta(p,a) \rightarrow \notin F \\ \delta(q,a) \rightarrow \notin F\end{array} \rightarrow (p,q) \rightarrow$

p, q → states a → input F → final state e.g. 

$(B, C) \in F$ $\therefore$ IDP

~~$(A, C) \notin F$~~

* Distinguishable states :→ (DP)

$\left.\begin{array}{l}\delta(P, a) \notin F \\ \delta(Q, a) \in F\end{array}\right\}$ DP $\quad$ $\left.\begin{array}{l}\delta(P, a) \in F \\ \delta(Q, a) \notin F\end{array}\right\}$ DP

* Minimize the following ~~DFA~~ DFA ~~by~~ by applying table filling algorithm.

(F)

δ	a	b
→ A	B	F
B	G	C
* C	A	C
D	C	G
E	H	F
F	C	G
G	G	E
H	G	C

start from 2nd state.

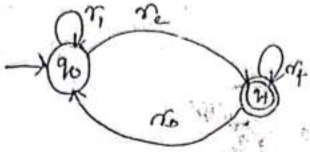
B	X						
C	X	X					
D		X	X				
E		X	X				
F	X	X	X				
G		X	X			X	
H	X		X	X	X		X
	A	B	C	D	E	F	G

start from 1st state to last but one.

δ	a	b
(A, B)	(B, G)	(F, C)
(A, D)	(B, C)	(F, G)
(A, E)	(B, H)	(F, F)
(A, F)	(B, C)	(F, G)
(A, G)	(B, G)	(F, E)
(A, H)	(B, G)	(F, C)

δ	a	b
(B, D)		
(B, E)		
(B, F)		
(B, G)		
(B, H)		

STATE ELIMINATION METHOD

(1) 

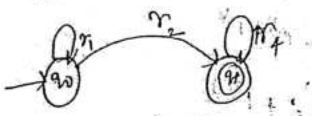
$r_1\ r_2\ r_3\ r_4$ } Regular Expressions

(RE) Regular Expression $= r_1^* r_2 (r_4 + r_3 r_1^* r_2)^*$

$q_0 \rightarrow$ start state $\quad q_f \rightarrow$ final state

(2) 

$RE = r^*$

(3) 

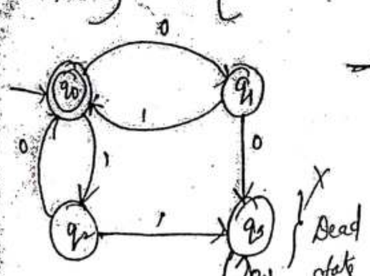
$RE = r_1^* r_2 r_4^*$

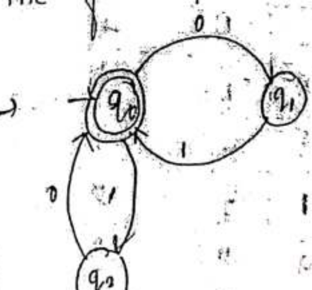
No connectivity between final & start start.

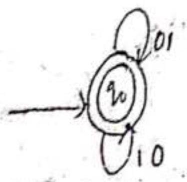
(4) When start state becomes final state Regular expression can be written as.

$RE = r_1^* + r_1^* r_2 r_4^*$

* Obtain the regular expression using state elimination method for the given finite automata.

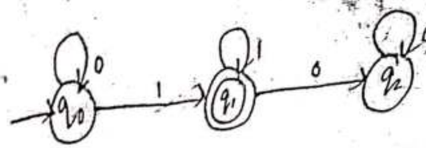


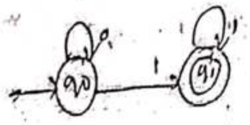




$RE = (01+10)^*$

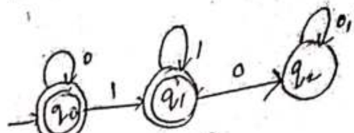
(a)

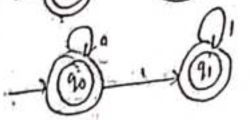




$\boxed{RE = 0^* 1 1^*}$

(b)





$0^* + 0^* 1 1^*$

$0^* (\epsilon + 1^+)$

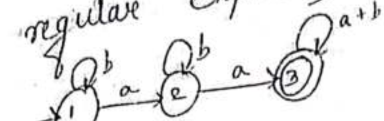
$0^* 1^*$

Using table filling algorithm minimize the given DFA.

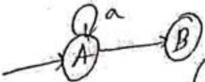
δ	a	b
→ A	B	E
B	C	F
* C	D	H
D	E	H
E	F	I
* F	G	B
G	H	B
H	I	C
* I	A	E

* Using state elimination method obtain the regular expression :→

(1)



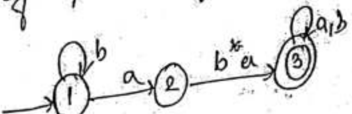
RE = a+b .

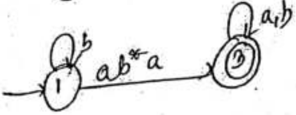


$a^* b$.

$1 \cdot (0+1)^*$

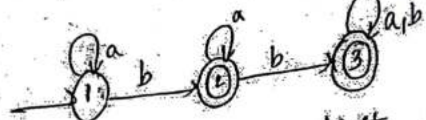
In state elimination method except start & final state the intermediate states should be eliminated by preserving the inputs .





$\boxed{RE = b^* a b^* a \, (a+b)^*}$

(2)



⇒ split to two finite atomata with different final state .

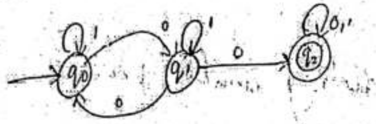
2 as an final state

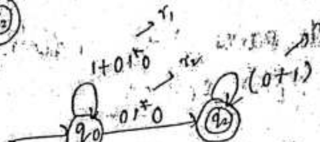
a^*ba^* ($\because$ 3 is trap state)

with 3 as an final state

$a^*ba^*b(a+b)^*$

$$\boxed{\therefore RE = a^*ba^* + a^*ba^*b(a+b)^*}$$

③ 



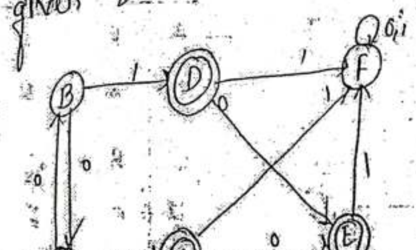
$r_3 = \phi$

$\Rightarrow$ $\cancel{1^*(0+0)1^*0(0+1)^*}$

$1^*\cancel{01^*}0(0+1)^*$

$(1+01^*0)^*01^*0(0+1)^*$

* Using table filling algorithm minimize the given DFA.

$\Rightarrow$ 

δ	0	1
$\rightarrow$A	B	C
B	A	D
*C	E	F
*D	E	F
*E	E	F
F	F	F

B					
C	X	X			
D	X	X			
E	X	X			
F	X	X	X	X	X
	A	B	C	D	E

δ	0	1
(A,B) (A,F)	(B,A) (B,F)	(C,D) (C,F)
(B,F)	(A,F)	(D,F)
(C,D) (C,E)	(E,E) (E,E)	(F,F) (F,F)
(D,E)	(E,E)	(F,F)

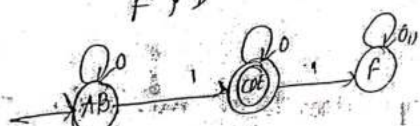
Unmarked pairs :→ Indistinguishable pairs

(AB) (DC) (EC) (ED)

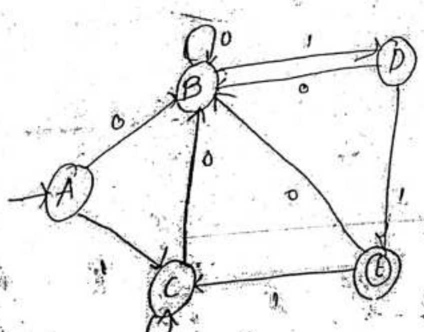
→ (AB) (CDE) } IDP

F } D

δ	0	1
→AB	AB	CDE
*CDE	CDE	F
F	F	F

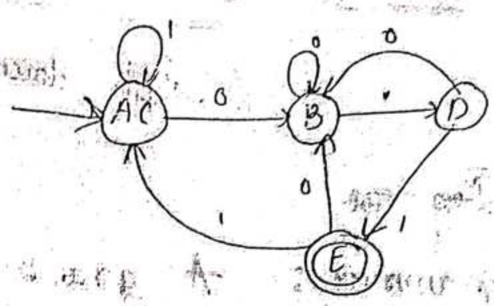


2.



δ	0	1
→A	B	C
B	B	D
C	B	C
D	B	E
*E	B	C

B	X			
C		X		
D	X	X	X	
E	X	X	X	X
	A	B	C	D



* CONVERSION FROM NFA to DFA

① subset construction method.

② Lazy evaluation method

Every DFA is NFA but vice versa is not true

* convert the following NFA in its equivalent DFA using subset construction method.

⇒ ① STEP 1] start state :→ since q_0 is start state in the given NFA same state continues as start state in resulting DFA.

② Identify the input alphabets :→ since $\Sigma = \{a, b\}$ for the given problem the same thing constitutes as input alphabets for the resulting DFA.

③ Determine final state :→ since q_2 is final state in given NFA, if q_2 presents in any of the subset the entire set is considered as final state.

④ For the given NFA find out subsets for the states -

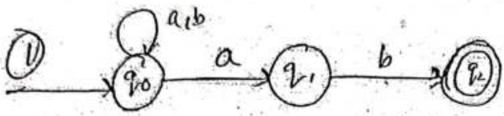
$M_N = \{q_0, q_1, q_2\}$

$M_D = \{\phi, q_0, q_1, q_2, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

(5) Identify the transition for all the subsets in the resulting DFA.

δ	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
q_1	$\emptyset$	q_2
$* q_2$	$\emptyset$	$\emptyset$

(1)



(1) $\delta(\emptyset, a) = \emptyset$ $\quad$ $\delta(\emptyset, b) = \emptyset$

(2) $\delta(q_0, a) = \{q_0, q_1\}$ $\quad$ $\delta(q_0, b) = q_0$

(3) $\delta(q_1, a) = \emptyset$ $\quad$ $\delta(q_1, b) = q_2$

(4) $\delta(q_2, a) = \emptyset$ $\quad$ $\delta(q_2, b) = \emptyset$

(5) $\delta(\{q_0, q_1\}, a)$
$= \delta(q_0, a) \cup \delta(q_1, a)$
$= \{q_0, q_1\} \cup \emptyset$
$= \{q_0, q_1\}$

$\delta(\{q_0, q_1\}, b)$
$= \delta(q_0, b) \cup \delta(q_1, b)$
$= q_0 \cup q_2$
$= \{q_0, q_2\}$

(6) $\delta(\{q_0, q_2\}, a)$
$= \delta(q_0, a) \cup \delta(q_2, a)$
$= \{q_0, q_1\} \cup \emptyset$
$= \{q_0, q_1\}$

$\delta(\{q_0, q_2\}, b)$
$= \delta(q_0, b) \cup \delta(q_2, b)$
$= q_1 \cup \emptyset$
$= q_1$

(7) $\delta(\{q_1, q_2\}, a)$
$= \delta(q_1, a) \cup \delta(q_2, a)$
$= \emptyset \cup \emptyset$
$= \emptyset$

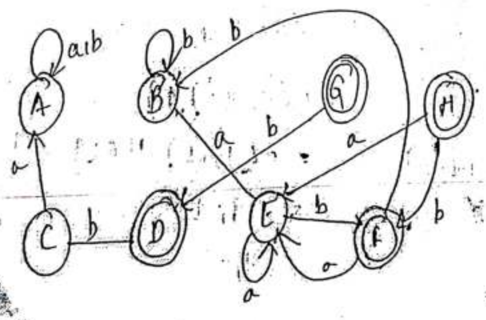
$\delta(\{q_1, q_2\}, b)$
$= \delta(q_1, b) \cup \delta(q_2, b)$
$= q_2 \cup \emptyset$
$= q_2$

⑥ $\delta(\{q_1, q_2, q_3\}, a)$ $\delta(\{q_1, q_2, q_3\}, b)$

$= \delta(q_1, a) \cup \delta(q_2, a) \cup \delta(q_3, a)$ $= \{q_0, q_2\}$

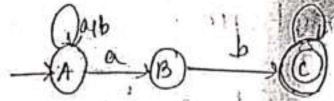
$= \{q_0, q_1\}$

	δ	a	b
A	ϕ	ϕ	ϕ
B	$\rightarrow q_0$	q_0q_1	q_0
C	q_1	ϕ	q_2
D	$* q_2$	ϕ	ϕ
	~~q_3~~	~~ϕ~~	
E	$\{q_0, q_1\}$	q_0q_1	q_0q_2
F	$* \{q_0, q_2\}$	q_0q_1	q_0
G	$* \{q_1, q_2\}$	ϕ	q_2
H	$* \{q_0, q_1, q_2\}$	q_0q_1	q_0q_2



A is nill

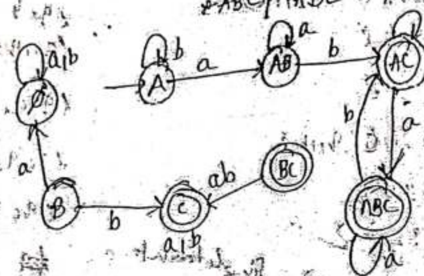
(2) Convert the NFA into its equivalent DFA.



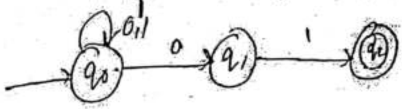
$\Longrightarrow$

δ	a	b
$\rightarrow A$	$\{A,B\}$	A
B	$\emptyset$	C
$*C$	C	C

δ	a	b
$\rightarrow \emptyset$	$\emptyset$	$\emptyset$
A	$\{A,B\}$	A
B	$\emptyset$	C
$*C$	C	C
AB	$\{AB\}$	AC
$*AC$	$\{ABC\}$	AC
$*BC$	C	C
$*ABC$	ABC	AC



* Lazy evaluation Method :-



$\delta(q_0,0) = \{q_0 q_1\}$

$\delta(q_0,1) = \{q_0\}$

$\delta(\{q_0 q_1\},0) = \delta(q_0,0) \cup \delta(q_1,0)$

$= \{q_0 q_1\} \cup \emptyset = \{q_0 q_1\}$

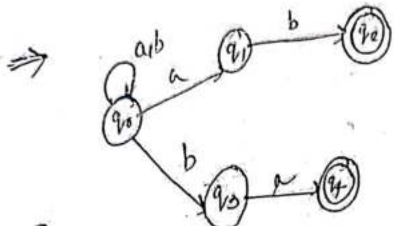
$\delta(\{q_0 q_1\},1) = \delta(q_0,1) \cup \delta(q_1,1)$

$= q_0 \cup q_2 = \{q_0 q_2\}$

$\delta(\{q_0 q_2\},0) = \{q_0 q_1\}$ $\delta(\{q_0 q_2\},1) = \{q_0\}$

① Construct an NFA that ends with ab & ba. Convert it into its equivalent DFA.

⇒



δ	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0 q_3\}$
q_1	$\emptyset$	q_2
$* q_2$	$\emptyset$	$\emptyset$
q_3	q_4	$\emptyset$
$* q_4$	$\emptyset$	$\emptyset$

① for input a

$\delta(q_0, a) = \{q_0, q_1\}$

for input b

$\delta(q_0, b) = \{q_0, q_3\}$

② $\delta(\{q_0, q_1\}, a) = \delta(q_0, a) \cup \delta(q_1, a)$

$= \{q_0, q_1\} \cup \emptyset$

$= \{q_0, q_1\}$

$\delta(\{q_0 q_1\}, b) = \delta(q_0, b) \cup \delta(q_1, b)$

$= \{q_0 q_3\} \cup q_2$

$= \{q_0 q_3 q_2\}$ → Singleton

$\{q_0\}$
$\{q_0, q_1\}$
$\{q_0 q_3 q_2\}$
$\{q_0, q_3\}$
$\{q_0\ q_1 q_4\}$

5

③ $\delta(\{q_0, q_3\}, a) = \delta(q_0, a) \cup \delta(q_3, a)$

$= \{q_0 q_1\} \cup q_4$

$= \{q_0 q_1 q_4\}$

$\delta(\{q_0 q_3\}, b) = \delta(q_0, b) \cup \delta(q_3, b)$

$= \{q_0 q_3\} \cup \emptyset$

$= \{q_0 q_3\}$

④ $\delta(\{q_0 q_3 q_2\}, a) = \delta(q_0, a) \cup \delta(q_3, a) \cup \delta(q_2, a)$

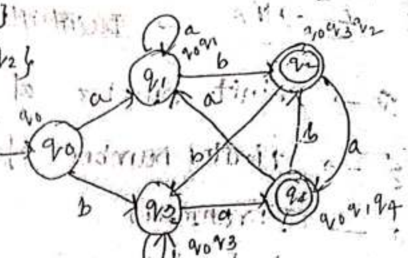
$= \{q_0 q_1\} \cup q_4 \cup \emptyset$

$= \{q_0 q_1 q_4\}$

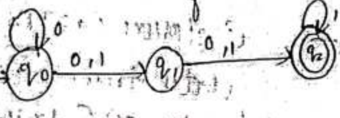
$\delta(\{q_0 q_3 q_2\}, b) = \{q_0 q_3\}$

(5) $\delta(\{q_0 q_1 q_4\}, a) = \{q_0 q_1\}$

$\Rightarrow \delta(\{q_0 q_1 q_4\}, b) = \{q_0 q_3 q_2\}$



(2) Convert the given NFA into its equivalent DFA



δ	0	1
$\rightarrow q_0$	$\{q_0 q_1\}$	q_1
q_1	q_2	q_2
$*q_2$	$\emptyset$	q_2

(1) $\delta(q_0, 0) = \{q_0 q_1\}$

$\delta(q_0, 1) = q_1 \rightarrow q_1$

(2) $\delta(\{q_0 q_1\}, 0) = \{q_0 q_1 q_2\}^*$

$\delta(\{q_0 q_1\}, 1) = \{q_1, q_2\}^* \rightarrow B$

(3) $\delta(q_1, 0) = q_2 * \rightarrow q_2$

$\delta(q_1, 1) = q_2$

(4) $\delta(q_2, 0) = \emptyset$

$\delta(q_2, 1) = q_2$

(5) $\delta(\{q_0 q_1 q_2\}, 0) = \{q_0 q_1 q_2\}$

$\delta(\{q_0 q_1 q_2\}, 1) = \{q_1 q_2\}$

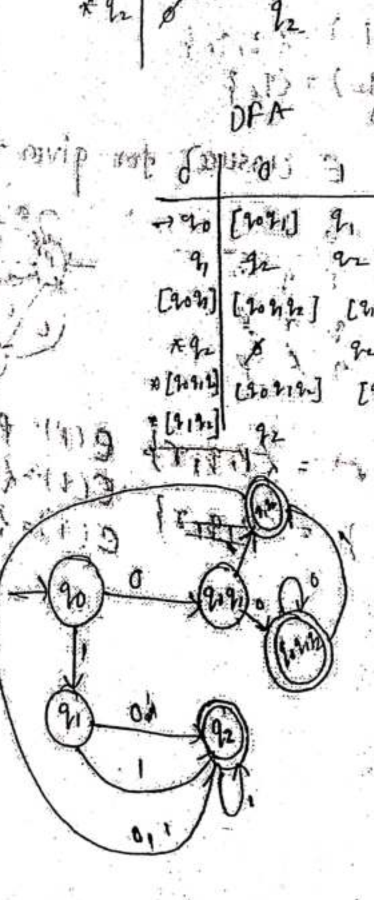
(6) $\delta(\{q_1 q_2\}, 0) = q_2$

$\delta(\{q_1 q_2\}, 1) = q_2$

DFA

δ	0	1
$\rightarrow q_0$	$[q_0 q_1]$	q_1
q_1	q_2	q_2
$[q_0 q_1]$	$[q_0 q_1 q_2]$	$[q_1 q_2]$
$*q_2$	$\emptyset$	q_2
$*[q_0 q_1 q_2]$	$[q_0 q_1 q_2]$	$[q_1 q_2]$
$*[q_1 q_2]$	q_2	q_2

square represents one state i.e. single only for DFA



$\in$ NFA :→ quintuple having $Q, \Sigma, \delta, q_0, F$

$Q \rightarrow$ finite number of states

$\Sigma \rightarrow$ finite number of input alphabets

$\delta \rightarrow$ Transitions

$q_0 \rightarrow$ start state

$F \rightarrow$ Final state

$\delta : Q \times \{\Sigma \cup \in\} = 2^Q$

$\in$ closure :→ The reachable state from a particular state on $\in$ transition

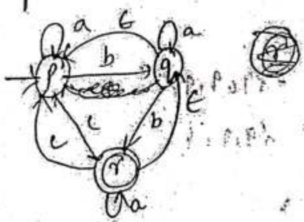
$\in(q_0) = \{q_0\, q_1\, q_2\}$

$\in(q_1) = \{q_1, q_0\}$

$\in(q_2) = \{q_2\}$

Write $\in$ closure for given transition table :→

δ	$\in$	a	b	c
→p	$\emptyset$	p	q	r
q	p	q	r	$\emptyset$
*r	q	r	$\emptyset$	p



~~$\in(p) = \{p, q, r\}$~~

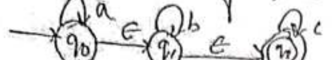
~~$\in(q) = \{p, q, r\}$~~

$\in(p) = p$

$\in(q) = \{p, q\}$

$\in(r) = \{p\, q\, r\}$

* Convert the given ϵ NFA into its equivalent DFA :-

→ 

$E(q_0) = \{q_0 q_1 q_2\}$ $\quad E(q_1) = \{q_1 q_2\}$, $E(q_2) = q_2$.

$E(q_0) = \{q_0 q_1 q_2\} = A$

$\delta(\{q_0 q_1 q_2\}, a) = q_0 = E(q_0) = \{q_0 q_1 q_2\} = A$

$\delta(\{q_0 q_1 q_2\}, b) = q_1 = E(q_1) = \{q_1 q_2\} = B$

$\delta(\{q_0 q_1 q_2\}, c) = q_2 = E(q_2) = q_2 = C$

$\delta(\{q_1 q_2\}, a) = \phi$ $\quad \cancel{= E(q_0) = \{q_0 q_1 q_2\}}$

$\delta(\{q_1 q_2\}, b) = q_1 = E(q_1) = q_1 q_2 = B$

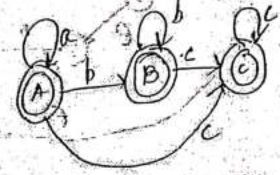
$\delta(\{q_1 q_2\}, c) = q_2 = E(q_2) = q_2 = B$

$\delta(q_2, a) = \phi$

$\delta(q_2, b) = \phi$

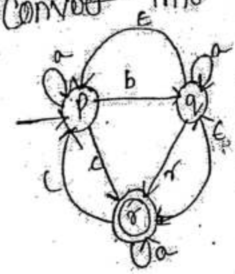
$\delta(q_2, c) = q_2 = E(q_2) = q_2 = C$

δ	a	b	c
*→A	A	B	C
*B	ϕ	B	C
*C	ϕ	ϕ	C



As q_2 is present in all state all states are final state.

* Convert into its equivalent DFA :-



⇒

δ	ϵ	a	b	c
→p	ϕ	p	q	r
q	p	q	r	ϕ
*r	q	r	ϕ	p

$E(P) = P$

$\delta(P, a) = P = E(P) = P$

$\delta(P, b) = q = E(q) = \{pq\}$

$\delta(P, c) = r = E(r) = \{pqr\}$

$\delta(\{pq\}, a) = \{pq\}$

$\delta(\{pq\}, b) = \delta(p, b) \cup \delta(q, b)$

$= q \cup r$

$= \{pq\} \cup \{pqr\}$

$= pqr$

$\delta(\{pq\}, c) = \{pqr\}$

$= \{pqr\}$

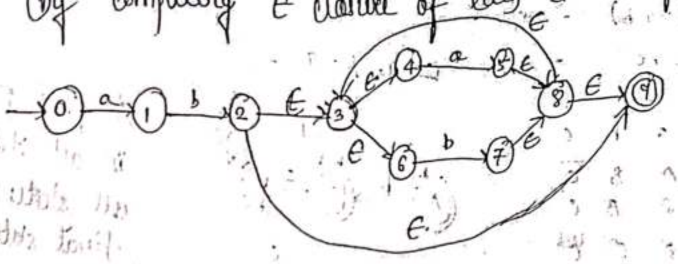
$\delta(\{pqr\}, a) = pqr$

$\delta(\{pqr\}, b) = pqr$

$\delta(\{pqr\}, c) = pqr$

δ	a	b	c
$\rightarrow p$	p	pq	pqr
pq	pq	pqr	pqr
*pqr	pqr	pqr	pqr

* Convert the given ε-NFA into its equivalent DFA by computing ε closure of each & every state.



$\Rightarrow E(0) = 0$

$E(1) = 1$

$E(2) = \{2, 3, 4, 6, 9\}$

$E(3) = \{3, 4, 6\}$

$E(4) = \{5, 8, 9, 3, 4, 6\} = \{3, 4, 5, 6, 8, 9\}$

$E(6) = 6$

$E(7) = \{3, 4, 6, 7, 8, 9\}$

$E(8) = \{3, 4, 6, 8, 9\}$

$E(9) = 9$

$\delta(0,a) = 1 = E(1)$

$\delta(0,b) = \phi$

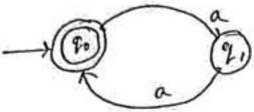
$\delta(1,a) = \phi$

$\delta(1,b) = 2 = E(2) = \{2,3,4,6,9\}$

$\delta(\{2,3,4,6,9\}, a) = \{$

① Construct a DFA where $\Sigma = \{a\}$ accepting only even number of a's.

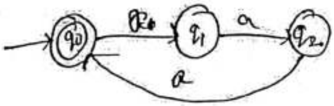
$\Rightarrow L = \{\epsilon, aa, aaaa \ldots\}$ $|w| \bmod 2 = 0$



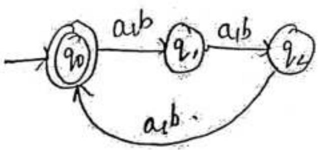
q_0 is final state to accept 0 number of a's.

② $|w| \bmod 3 = 0$

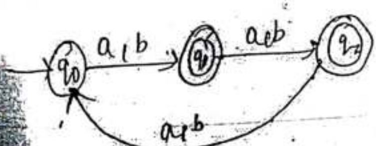
$\Sigma = \{a\}$



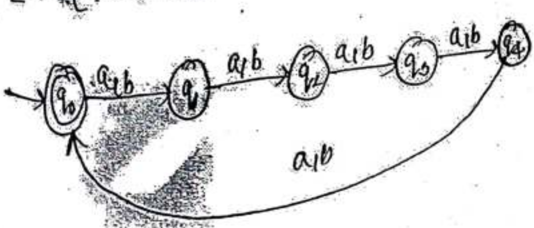
③ $|w| \bmod 3 = 0$



④ $L = \{|w| \bmod 3 \neq 0\}$ $\Sigma = \{a, b\}$



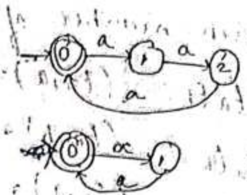
⑤ $L = \{|w| \bmod 5 = 0\}$ $\Sigma = \{a, b\}$



⑥ $L = \{ |w| \bmod 3 \geq |w| \bmod 2 \}$, $\Sigma = \{a\}$

$\Rightarrow L = \{ |w| \bmod 3 \geq |w| \bmod 2 \}$

M_1 $(0,1,2)$ M_2 $(0,1)$.



$M_1 \times M_2 = ((0,0), (0,1), (1,0), (1,1), (2,0), (2,1))$

$\delta((0,0), a) = \delta(0,a)\ (0,a) = \delta(1,1) \to q_1$

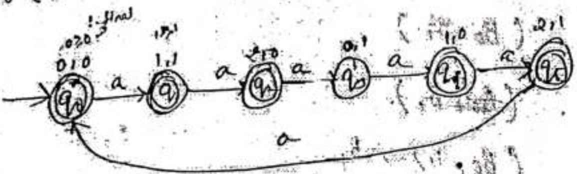
$\delta((1,1), a) = (2,0) \Rightarrow q_2$

$\delta((2,0), a) = (0,1) \longrightarrow q_3$

$\delta((0,1), a) = (1,0) \longrightarrow q_4$

$\delta((1,0), a) = (2,1) \rightarrow q_5$

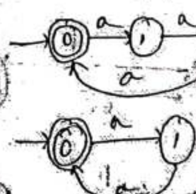
$\delta((2,1), a) = (0,0)$



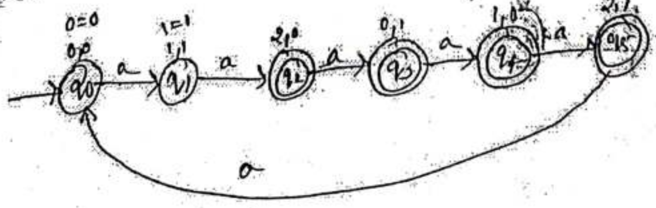
⑦ $L = \{ |w| \bmod 3 \neq |w| \bmod 2 \}$ $M_1 \to (0,1,2)$

$\Rightarrow L = \{ |w| \bmod 3 \neq |w| \bmod 2 \}$

$M_2 \to \{0,1\}$.



$M_1 \times M_2 = ((0,0), (0,1), (1,0), (1,1), (2,0), (2,1))$



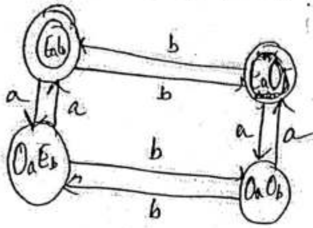
(8) Construct a DFA to have even number of a's & even number of b's over a, b.

$\Rightarrow$ $E_a E_b$ $(E_a, a) = O_a$

$E_a O_b$ $(E_b, b) = O_b$

$O_a E_b$ $(O_a, a) = E_a$

$O_a O_b$ $(O_b, b) = E_b$.



(9) $N_a|W| \bmod 3 \geq N_b|W| \bmod 2$

$\downarrow$ $\downarrow$

0, 1, 2 0, 1

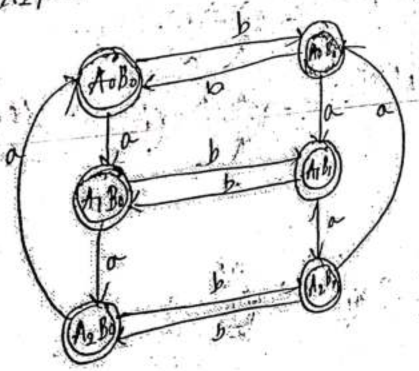
$(A_0\ A_1\ A_2)$ $(B_0\ B_1)$

~~(B_0, B_1)~~

$(A_0, a) - A_1$ $(B_0, b) \rightarrow B_1$

$(A_1, a) - A_2$ $(B_1, b) \rightarrow B_0$

$(A_2, a) - A_0$

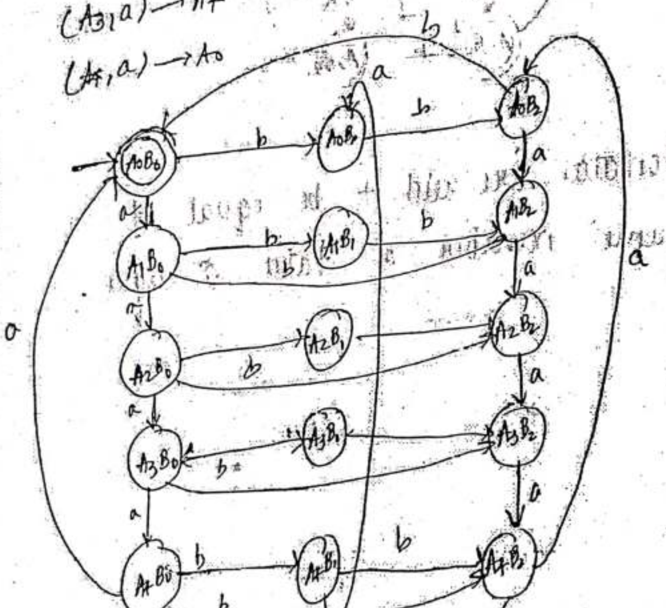


(10) $\{ w \mid N_a(w) \bmod 5 = 0$ and $N_b(w) \bmod 3 = 0 \}$

$(A_0\ A_1\ A_2\ A_3\ A_4)$ $(B_0\ B_1\ B_2)$

$\Rightarrow \{ (A_0B_0)\ (A_0B_1)\ (A_0B_2)\ (A_1B_0)\ (A_1B_1)\ (A_1B_2)\ (A_2B_0)\ (A_2B_1)$
$(A_2B_2)\ (A_3B_0)\ (A_3B_1)\ (A_3B_2)\ (A_4B_0)\ (A_4B_1)\ (A_4B_2) \}$

δ $(A_0, a) \rightarrow A_1$ $(B_0, b) \rightarrow B_1$
$(A_1, a) \rightarrow A_2$ $(B_1, b) \rightarrow B_2$
$(A_2, a) \rightarrow A_3$ $(B_2, b) \rightarrow B_0$
$(A_3, a) \rightarrow A_4$
$(A_4, a) \rightarrow A_0$



(ii) $L = \{ w \in (a+b)^* \mid n_a(w) \bmod 3 \geq n_b(w) \bmod 2 \}$

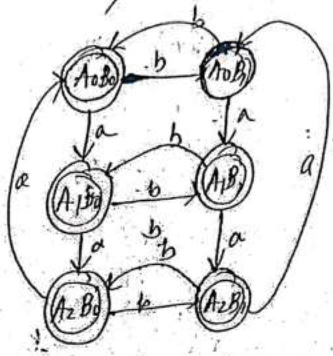
$\rightarrow$ $n_a(w) \bmod 3 \downarrow 0,1,2$; $n_b(w) \bmod 2 \downarrow 0,1$

$A_0 A_1 A_2 \times B_0 B_1$

$(A_0B_0)\ (A_0B_1)$

$(A_1B_0)\ (A_1B_1)$

$(A_2B_0)\ (A_2B_1)$



NOTE :→

Two finite automatas are said to be equal if they have same number of states & edges.