23/9/2020

Calculation :

FLAT – Unit II

Regex:-

Operators of regular expression:-

→ Operations on languages:-

1) Union of languages:-

$L, M \rightarrow 2$ languages

$L \cup M \rightarrow$ set of strings in either L or M or both.

Ex:-

$L = \{00, 11, \epsilon\}$ $M = \{01, 111\}$ $L \cup M = \{00, 01, 111\}$

$L \cup M = \{00, \cancel{11}, \epsilon, 01, 111\}$

2) Concatenation of languages:-

$L = \{0, 11\}$ $M = \{\epsilon, 00, 11\}$

$L.M$ or $LM = \{0, 000, 011, 11, 1100, 1111\}$

3) Closure of a language (star or kleene closure):-

$L \rightarrow$ Language, $L^* \rightarrow$ Closure of L

$L = \{0, 1\}$ $L^* \Rightarrow$ set of all strings of any length containing 0's, 1's

Ex: $L^* = \{\epsilon, 0, 1, 00, 01, 111, 000, 01111, \ldots\ldots\}$

$L^* = L^0 \cup L^1 \cup L^2 \cup \ldots\ldots$

$L^1 = L$

$L^2 = L^1 . L$

$L^3 = L^2 . L^1$

⋮

G.S.S. COLLEGE, BELAGAVI
DEPARTMENT OF PHYSICS

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~~Aim:~~ Building Regular Expressions:-

- Basis:-
  a) ϵ, ϕ denote $\{\epsilon\}$, ϕ

Apparatus :
  b) a is any symbol, then it denotes $\{a\}$ (Language $L=\{a\}$)
  c) A is a variable, used to denote language (like L), but in complex regex's, A is also used as symbol.

Formula with S. I. unit :

- Induction:-
  a) If E & F are regular expressions, then so is E+F. E+F denotes $L(E) \cup L(F)$. $L(E+F) = L(E) \cup L(F)$.
  b) If E & F are regular expressions, then so is E.F.

Principle :
  E.F or EF denotes $L(E).L(F)$. $L(E.F) = L(E).L(F)$
  c) If E is a regex, then E^* is also a regex denoting closure of $L(E)$. $L(E^*) = (L(E))^*$.
  d) If E is a regex, (E) is also a regex denoting the same language as that of E. $L((E)) = L(E)$.

Regex closely matches with NFA, because for a given problem we can have many NFA's & Regex's, but limited DFA's.

Diagram :

Precedence of operators:-

1) *
2) .
3) +

Examples:-

Observation : Write a RE for any number of a's.

Ans) a*

Write a RE for a or b.
Ans) a+b or (a+b)

Write a RE for any no's of a's & b's.
Ans) (a+b)*

Write a RE for the sequence 01.
Ans) 0.1 or 01

Write a RE for any no. of 0's followed by any no. of 1's.
Ans) 0*.1* or 0*1*

Write a RE for 0101010101......
Ans) (01)*

Write a RE for sequence of a's & b's such that the last symbol is a.

Ans) $(a+b)^* \cdot a$ or $(a+b)^* a$

Write a RE for the strings of a's & b's such that the substring ab is present.

Ans) $(a+b)^* ab (a+b)^*$

Write a RE for $\Sigma = \{a, b\}$ such that strings end with ab.

Ans) $(a+b)^* ab$

Beginning with ab.

Ans) $ab(a+b)^*$

Begins or ends with ab.

Ans) $(a+b)^* ab + ab(a+b)^*$

RE for even number of a's.

Ans) $(aa)^*$

RE for ~~string~~ whose length is 2, $\Sigma = \{a, b\}$

Ans) $(a+b)(a+b)$

Result :

Calculation :

RE for odd length, $\Sigma = \{a\}$

Ans) $a(aa)^*$ or $(aa)^*a$

[illegible]	[illegible]							
1	2	7	5					A[illegible] in
2	9	1	5	3	4	[illegible]		ascending order
3	4	[illegible]	[illegible]	[illegible]	7			
4	[illegible]	8	2					
5	3							

RE for a language of strings of a's & b's such that the strings end with abba.

→) $(a+b)^*abba$

RE for strings over $\Sigma = \{a, b\}$, length <=2.

→) $\epsilon + a + b + aa + ab + ba + bb$ or $(\epsilon + a + b)(\epsilon + a + b)$.

RE whose 4^{th} symbol from the end is a and 2nd symbol is b.

→) $\cancel{(\epsilon + a + b)(\epsilon + a + b)(\epsilon + a + b)(\epsilon + a + b)}\ a\ (a+b)\ b\ (a+b)$

$(a+b)^*a(a+b)b(a+b)$

RE for any no. of a's followed by any no. of b's followed by any no. of c's

→) $a^*b^*c^*$

RE for atleast one a followed by atleast one b followed by atleast one c.

→) $aa^*bb^*cc^*$

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Aim :

RE that either ends with a or bb.

→ $(a+b)^*(a+bb)$

Apparatus : RE that should not end with aa.

→ ~~(a+b)* ab*~~ $(a+b)^*(ab+ba+bb)$

RE for $L = \{w \mid |w| \bmod 3 = 0,\ w \in \{a,b\}^*\}$

Formula with S. I. unit :

→ $((a+b)(a+b)(a+b))^*$

RE which has atleast 3 consecutive 0's, $\Sigma = \{0,1\}$

→ $(0+1)^* 000 (0+1)^*$

Principle :

RE begins with ab & ends with ba.

→ $ab(a+b)^* ba$

RE for strings of a's & b's whose length is either even or multiple of 3.

→ $((a+b)(a+b))^* + ((a+b)(a+b)(a+b))^*$

Diagram : RE for $L = \{ a^{2n} b^{2n} / n >= 0 \}$

$\rightarrow$ $(aa)^*(bb)^*$

RE for $L = \{ a^n b^m / n >= 4, m <= 3 \}$

$\rightarrow$ $aaaaa^*(\epsilon + b + bb + bbb)$

RE for strings of a's & b's such that a's are divisible by 3.

$\rightarrow$ $\cancel{b^*(aaa)^*b^*}$ $(b^* a b^* a b^* a b^*)^*$

Observation : RE for $L = \{ a^n b^m / m+n$ is even $\}$

$\rightarrow$ $(aa)^*(bb)^* + a(aa)^* b(bb)^*$

RE for strings of a's & b's such that every block of 4 ~~contains~~ consecutive symbols contain atleast 2 a's, a can occur anywhere.

$\rightarrow$ $(aa(a+b)(a+b) + a(a+b)a(a+b) + a(a+b)(a+b)a + (a+b)aa(a+b) + (a+b)(a+b)aa$ $+ (a+b)a(a+b)a)$. (same thing as in previous brackets)*

RE for $L = \{ a^n b^m / m >= 1, n >= 1, nm >= 3 \}$

$\rightarrow$ $aaaa^* bb^* + aa^* bbbb^* + aaaa^* bbb^*$

RE for strings of a's & b's containing not more than 3 a's.

$\rightarrow$ ~~$((\epsilon + a + b)(\epsilon + a + b)(\epsilon + a + b) b^*) + ((\epsilon + a + b)(\epsilon + a + b) b^* (\epsilon + a + b) b^*)$ $+ ((\epsilon + a + b) b^* (\epsilon + a + b) b^* (\epsilon + a + b) b^*)$~~ $+ (b^* (\epsilon + a) b^* (\epsilon + a) b^* (\epsilon + a) b^*)$

RE for $L = \{ vuv / u, v$ belong to $\{a, b\}^*$ and $|v| = 2 \}$

$\rightarrow$ ~~(a+b)(a+b)(a+b)*(a+b)(a+b)~~

$aa(a+b)^* aa + ab(a+b)^* ab + ba(a+b)^* ba + bb(a+b)^* bb$

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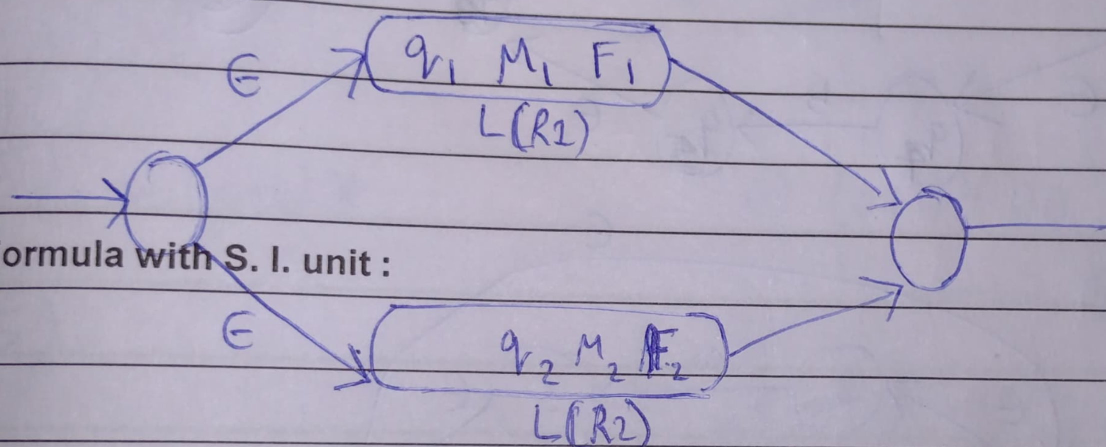
Aim : RE continued :-

Obtaining ϵ-NFA for given RE:-

Case 1 : $R = R1 + R2$

Then $L = L(R1) + L(R2) = L(R1+R2)$

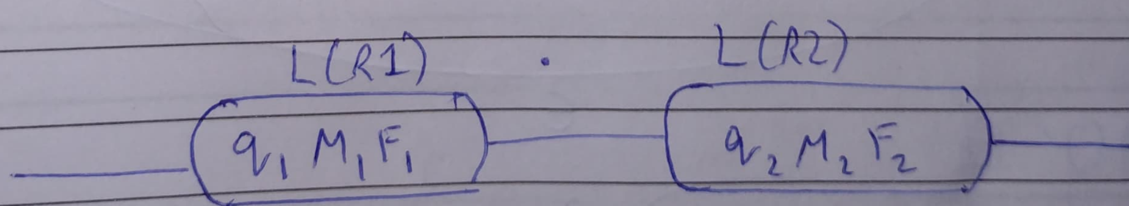
Apparatus :



Formula with S. I. unit :

Case 2 : $R = R1 . R2$

Principle : Then $L = L(R1).L(R2) = L(R1.R2)$



Case 3 : $R = R1^*$

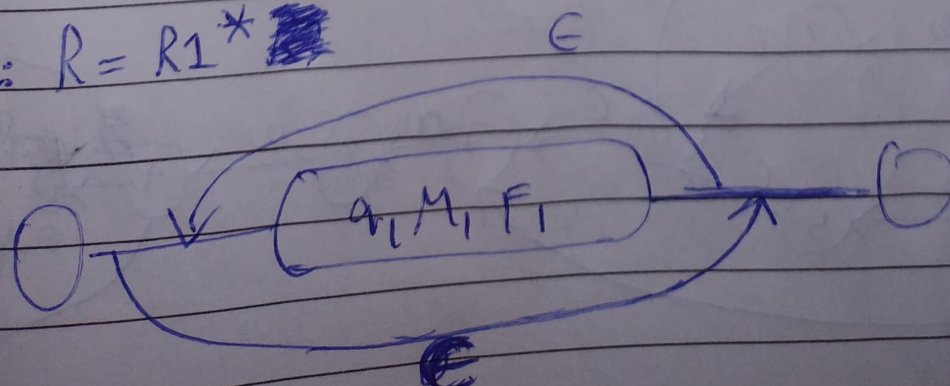


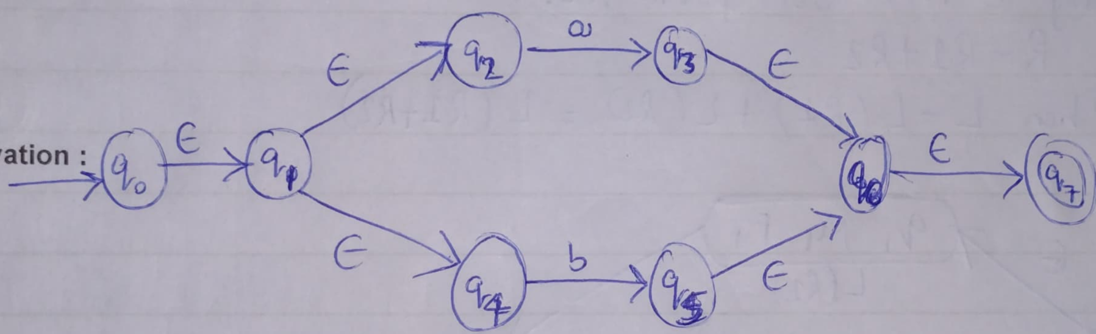
Diagram :

Write ε-NFA for following RE's:-

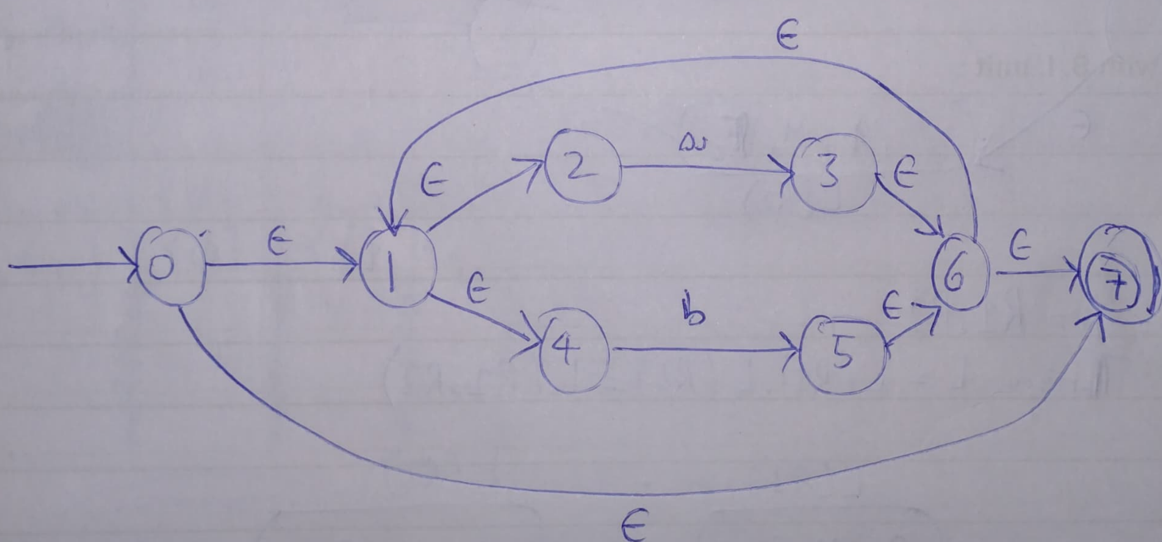
1) $a+b$
2) $(a+b)^*$
3) $(a+b)^* aa (a+b)^*$
4) $a^*+b^*+c^*$

1)

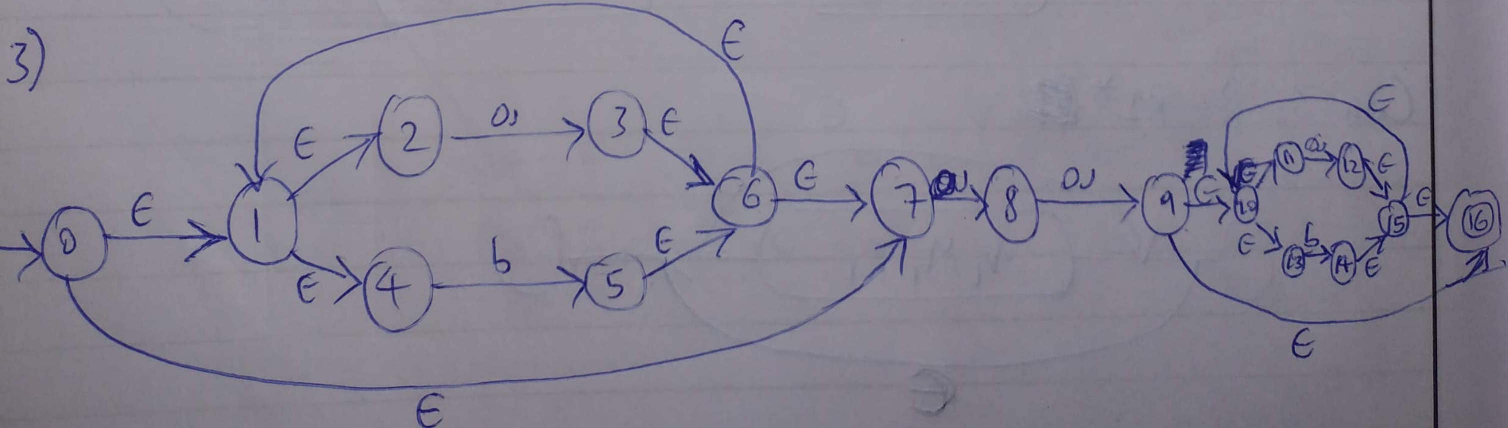
Observation :



2)

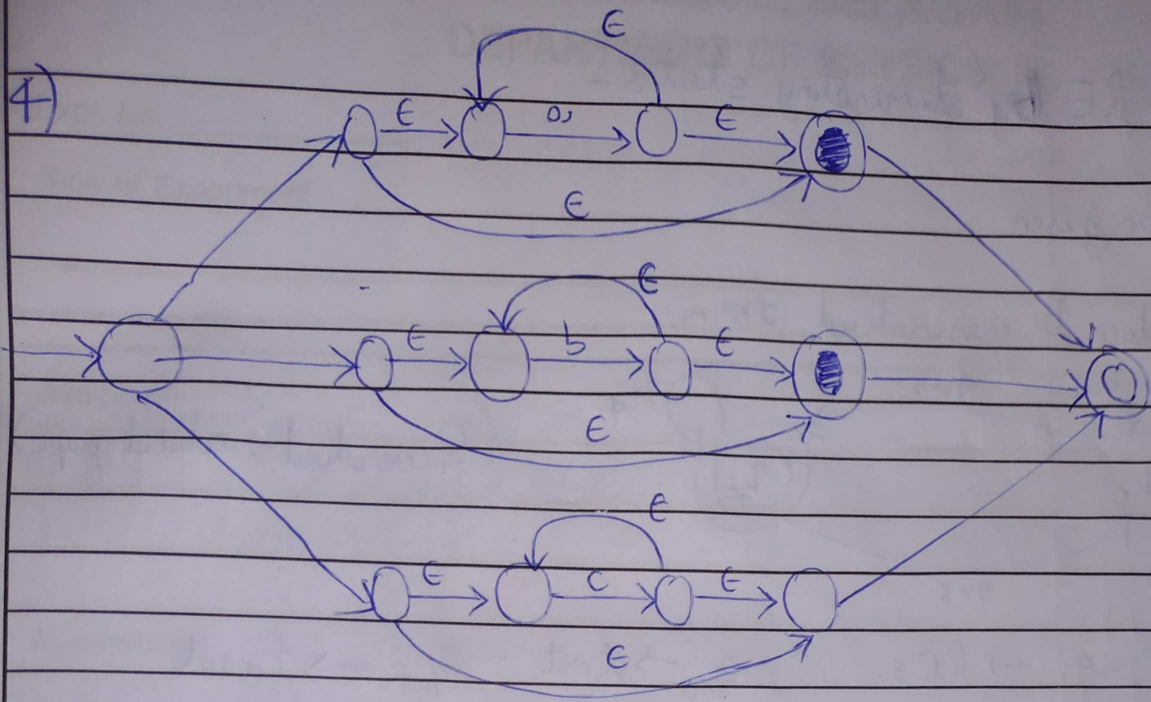


3)



Procedure :

4)

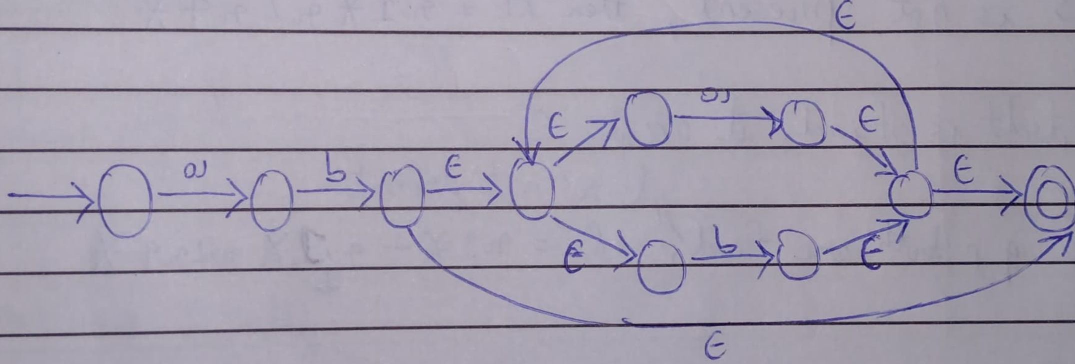


5) ab(a+b)*

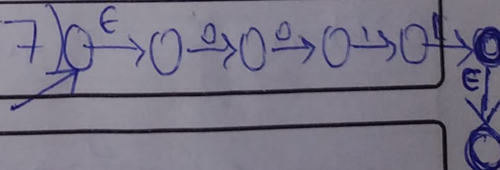
6) (0+1)*00

7) 0011

5)

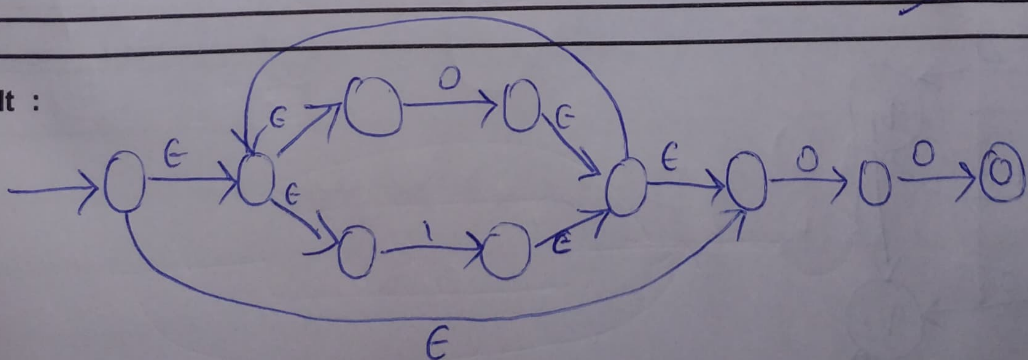


7)



6)

Result :

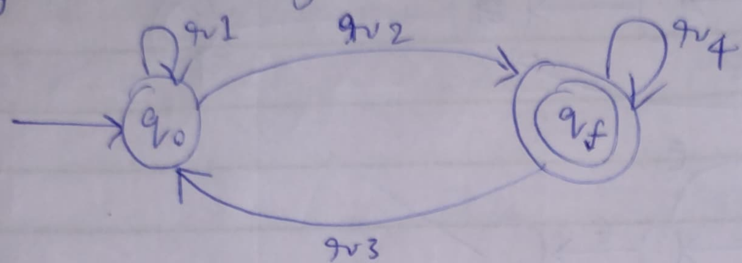


Calculation :

Obtaining RE by eliminating states:-

DFA will be given.

Try to reduce to generalised form:



(Generalised reduced graph)

r1, r2, r3, r4 → RE's. q_0 → start q_f ⇒ final

$$RE = r1^* r2 (r4 + r3\, r1^* r2)^*$$

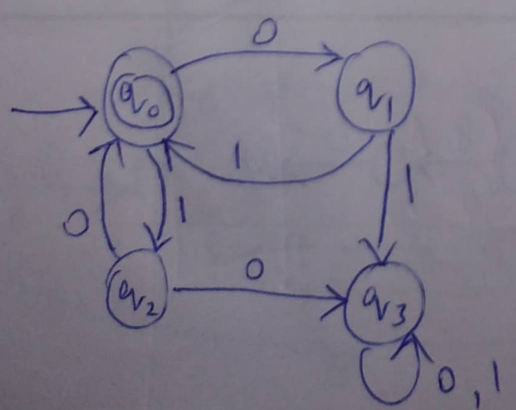
If r3 is not present, then $RE = r1^* r2\, r4^*$

If q_0 itself is final, it accepts ∈.

If q_0 & q_f both are final, & r3 not present, $RE = r1^* + r1^* r2 r4^*$

<u>Ex:-</u>

1) Obtain RE for given transn. diagram:-



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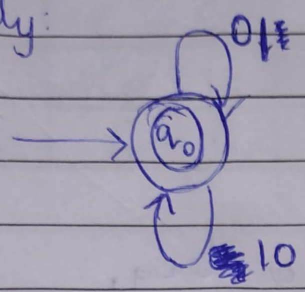
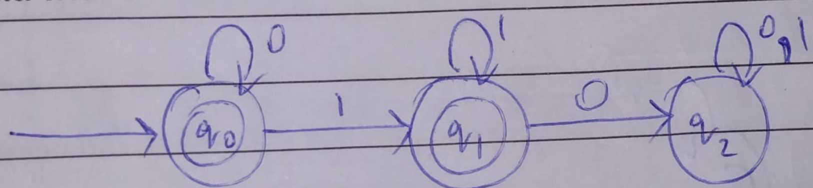
Aim : Finally:



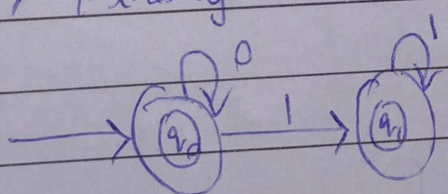
Apparatus :

$RE = (01+10)^*$

Formula with S. I. unit :

2) 

Principle : → Finally:



$RE = 0^* 1 1.^* + 0^*$

Diagram :

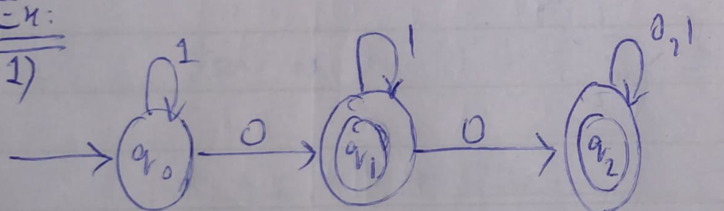
Table filling algorithm :-

1) Indistinguishable states (pairs)
2) Distinguishable states
3) Transition table / diagram
4) Minimised DFA.

1) Indistinguishable / Equivalent states :-

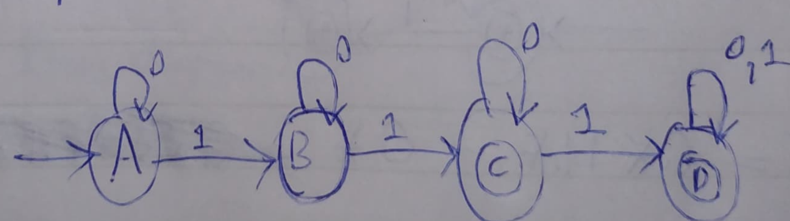
Observation : If p & q are 2 states & w is a string,
$\{\delta(p,w) \in F$ & $\delta(q,w) \in F\}$ or $(\delta(p,w) \notin F$ or $\delta(q,w) \notin F)$
Then p & q are final states. or p & q both are non-final.
Hence p & q are equivalent.

Ex:
1)



(q_1, q_2) Indistinguishable pair.

2)



(A, B)
(C, D) } Indistinguishable pairs

Procedure :

δ	0	1
→A	B	F
B	G	C
*C	A	C
D	C	G
E	H	F
F	C	G
G	G	E
H	G	C

TFA :-

B	X						
C	X	X					
D	X	X	X				
E		X	X	X			
F	X	X	X		X		
G	X	X	X	X	X	X	
H	X	X	X	X	X	X	X
	A	B	C	D	E	F	G

Vertically start with 2nd state
Horizontally start with 1st state

X → DP's.

First mark X in row & column of final state.

(A,C) (B,C) (D,C) (E,C) (F,C) (G,C) (H,C) → DP's

	0	1
~~(A,B)~~		
~~(A,D)~~		
~~(A,E)~~		
~~(A,F)~~		
~~(A,G)~~		
~~(A,H)~~		

	0	1
~~(B,D)~~		
~~(B,E)~~		
~~(B,F)~~		
~~(B,G)~~		
~~(B,H)~~		

Result :

Calculation :

S1)

		0	1
✓	(A,B)	(B,G)	(F,C)
✓	(A,D)	(B,C)	(F,G)
	(A,E)	(B,H)	(F,F)
✓	(A,F)	(B,C)	(F,G)
	(A,G)	(B,G)	(F,E)
✓	(A,H)	(B,G)	(F,C)
✓	(B,D)	(G,C)	(C,G)
✓	(B,E)	(G,H)	(C,F)
✓	(B,F)	(G,C)	(C,G)
✓	(B,G)	(G,G)	(C,E)
	(B,H)	(G,G)	(C,C)
✓	(D,E)	(C,H)	(G,F)
	(D,F)	(C,C)	(G,G)
✓	(D,G)	(C,G)	(G,E)
✓	(D,H)	(C,G)	(G,C)
✓	(E,F)	(H,C)	(F,G)
	(E,G)	(H,G)	(F,E)
✓	(E,H)	(H,G)	(F,C)
✓	(F,G)	(C,G)	(G,E)
✓	(F,H)	(C,G)	(G,C)
✓	(G,H)	(G,G)	(E,C)

S2)

		0	1
	(A,E)	(B,H)	(F,F)
✓	(A,G)	(B,G)	(F,E)
	(B,H)	(G,G)	(C,C)
	(D,F)	(C,C)	(G,G)
✓	(E,G)	(H,G)	(F,E)

S3)

	0	1
(A,E)	(B,H)	(F,F)
(B,H)	(G,G)	(C,C)
(D,F)	(C,C)	(G,G)

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Aim : (A, E), (B, H), (D, F) are indistinguishable pairs.
C & G are distinguishable.

Apparatus :

	0	1
→(AE)	(BH)	(DF)
(BH)	G	C
(DF)	C	G
*C	(AE)	C
G	G	(AE)

→ don't write FF since both D & F are equivalent, write FF.

G & C ← Cuz available on left side

Formula with S. I. unit :

If we get only A or E or like that, write complete Indistinguishable pair (AE).

Principle :

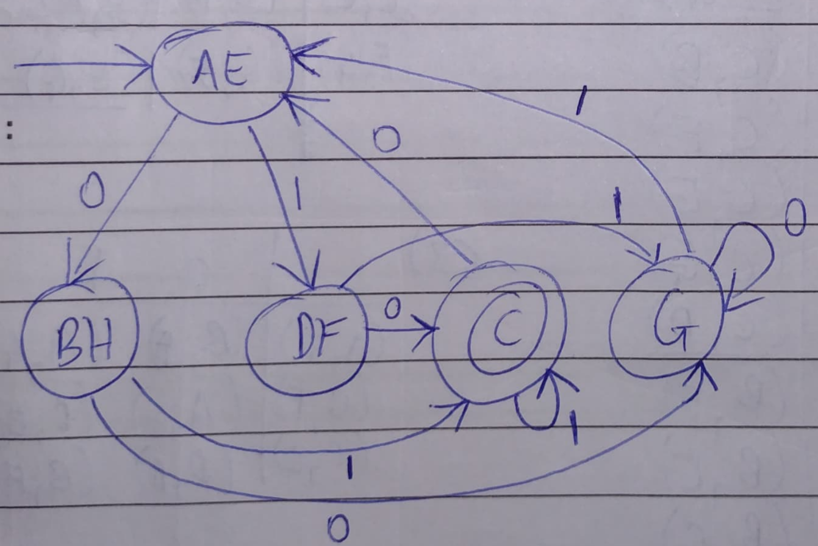


Diagram :

	0	1
→A	B	A
B	A	C
C	D	B
*D	D	A
E	D	F
F	G	E
G	F	G
H	G	D

B	X						
C	X	X					
D	X	X	X				
E	X	X		X			
F	X		X	X	X		
G		X	X	X	X	X	
H	X	X	X	X	X	X	X
	A	B	C	D	E	F	G

Observation :

S1)

		0	1
	(A,B)	(B,A)	(A,C)
✓	(A,C)	(B,D)	(A,B)
✓	(A,E)	(B,D)	(A,F)
	(A,F)	(B,G)	(A,E)
	(A,G)	(B,F)	(A,G)
✓	(A,H)	(B,G)	(A,D)
✓	(B,C)	(A,D)	(C,B)
✓	(B,E)	(A,D)	(C,F)
	(B,F)	(A,G)	(C,E)
	(B,G)	(A,F)	(C,G)
✓	(B,H)	(A,G)	(C,D)
	(C,E)	(D,D)	(B,F)
✓	(C,F)	(D,G)	(B,E)
✓	(C,G)	(D,F)	(B,G)
✓	(C,H)	(D,G)	(B,D)
✓	(E,F)	(D,G)	(F,E)
✓	(E,G)	(D,F)	(F,G)
✓	(E,H)	(D,G)	(F,D)
	(F,G)	(G,F)	(E,G)
✓	(F,H)	(G,G)	(E,D)
✓	(G,H)	(F,G)	(G,D)

S2)

		0	1
✓	(A,B)	(B,A)	(A,C)
✓	(A,F)	(B,G)	(A,E)
	(A,G)	(B,F)	(A,G)
	(B,F)	(A,G)	(C,E)
✓	(B,G)	(A,F)	(C,G)
	(C,E)	(D,D)	(B,F)
✓	(F,G)	(G,F)	(E,G)

S3)

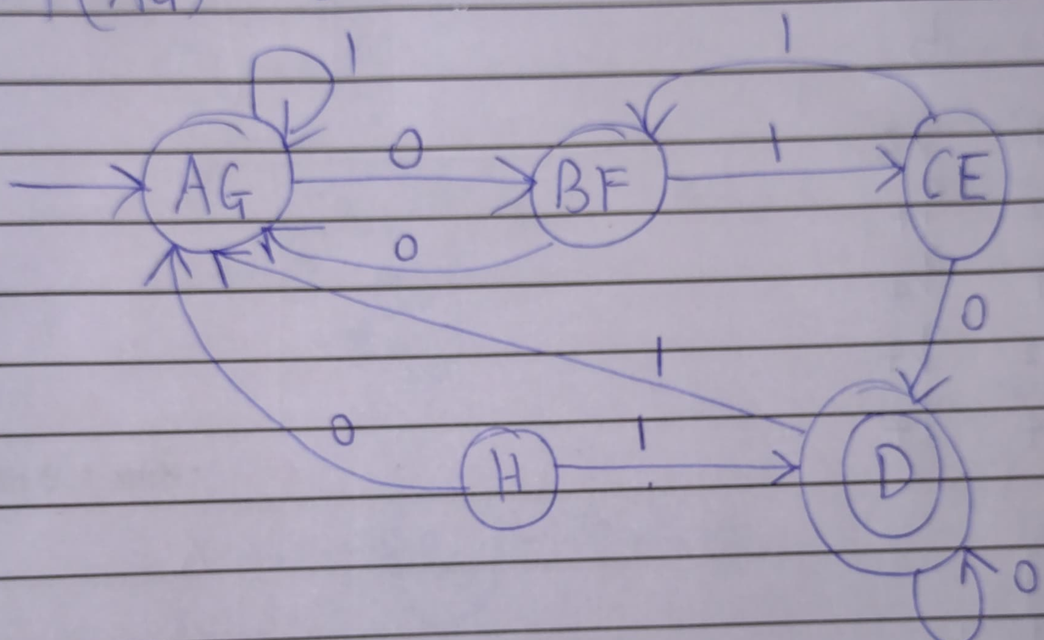
	0	1
(A,G)	(B,F)	(A,G)
(B,F)	(A,G)	(C,E)
(C,E)	(D,D)	(B,F)

∴ (A,G) (B,F) (C,E) — IDP

D, H ⟶ Distinguishable

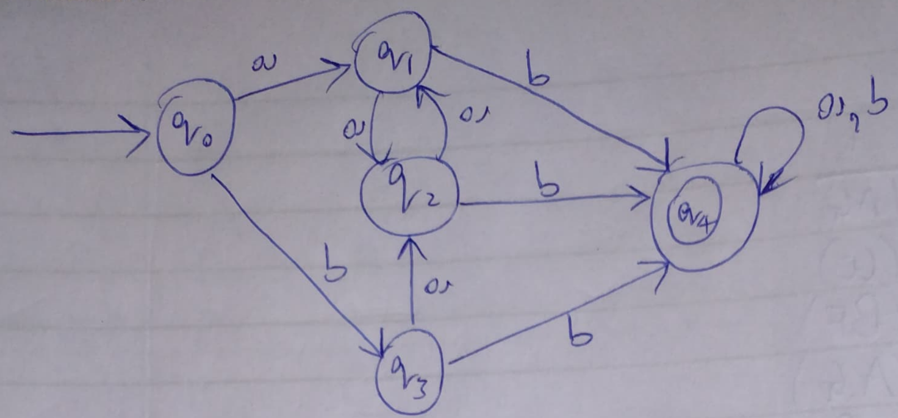
T T (Final) :-

	0	1
→ (A, G)	(B, F)	(AG)
(B, F)	(AG)	(CE)
(C, E)	D	(BF)
* D	D	(AG)
H	(AG)	D



Result :

Calculation : Obtain minimised DFA.



TT :-

	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_2	q_4
q_2	q_1	q_4
q_3	q_2	q_4
$*q_4$	q_4	q_4

q_1	X			
q_2	X			
q_3	X			
q_4	X	X	X	X
	q_0	q_1	q_2	q_3

S1)

	a	b
(q_0, q_1)	(q_1, q_2)	(q_3, q_4)
(q_0, q_2)	(q_1, q_1)	(q_3, q_4)
(q_0, q_3)	(q_1, q_2)	(q_3, q_4)
(q_1, q_2)	(q_2, q_1)	(q_4, q_4)
(q_1, q_3)	(q_2, q_2)	(q_4, q_4)
(q_2, q_3)	(q_1, q_2)	(q_4, q_4)

S2)

	a	b
(q_1, q_2)	(q_2, q_1)	(q_4, q_4)
(q_1, q_3)	(q_2, q_2)	(q_4, q_4)
(q_2, q_3)	(q_1, q_2)	(q_4, q_4)

$\therefore$ ~~(q_1, q_2) (q_1, q_3) (q_2, q_3) $\rightarrow$ IDP's.~~

Take (q_1, q_2, q_3) 1 state
Transition property

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Aim : q_0 & q_4 are distinguishable

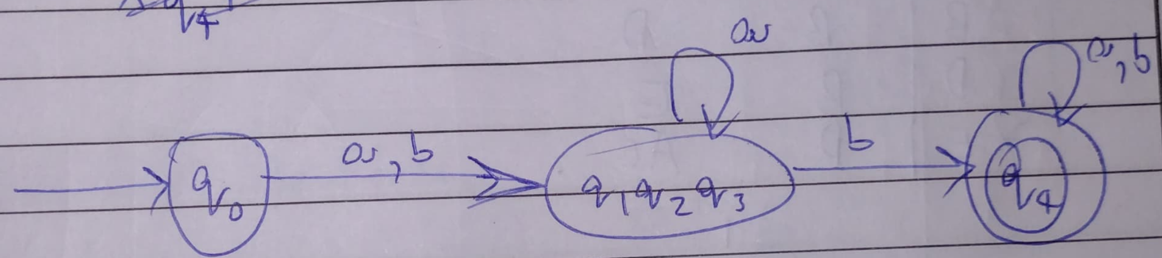
Find TT:-

	a	b
~~(q_1, q_2)~~	~~(q_2, q_1)~~	~~q_4~~
~~(q_1, q_3)~~	~~(q_2, q_1)~~	~~q_4~~
~~(q_2, q_3)~~	~~(q_2, q_1)~~	~~q_4~~
~~$\rightarrow q_0$~~		
~~$*q_4$~~		

	a	b
$q_1q_2q_3$	$q_1q_2q_3$	q_4
$\rightarrow q_0$	$q_1q_2q_3$	$q_1q_2q_3$
$*q_4$	q_4	q_4

Apparatus :

Formula with S. I. unit :



TFA also called Myhill Nerode.(MN)

Principle :

δ	0	1
$\rightarrow A$	B	C
B	B	D
C	B	C
D	B	E
$*E$	D	C

B	X			
C		X		
D	X	X	X	
E	X	X	X	X
	A	B	C	D

Diagram :

S1)

	0	1
✓ (A,B)	(B,B)	(C,D)
(A,C)	(B,B)	(C,C)
✓ (A,D)	(B,B)	(C,E)
(B,C)	(B,B)	(D,C)
✓ (B,D)	(B,B)	(D,E)
✓ (C,D)	(B,B)	(C,E)

S2)

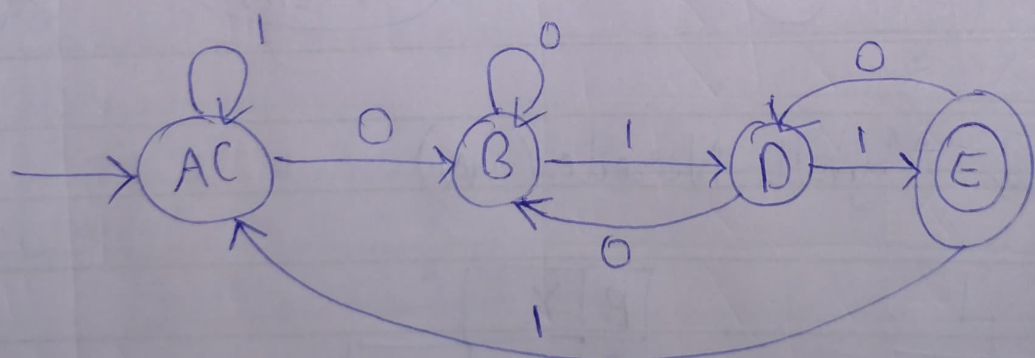
	0	1
✓ (A,B)	(B,B)	(C,D)
(A,C)	(B,B)	(C,C)
✓ (B,C)	(B,B)	(D,C)

S3)

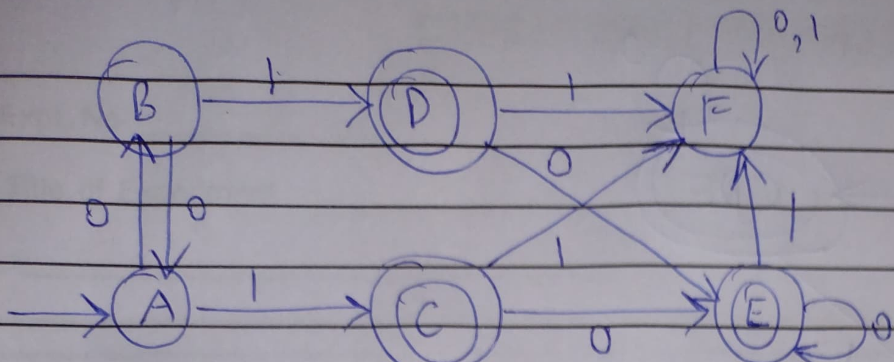
	0	1
(A,C)	(B,B)	(C,C)

Observation : (A,C) is IDP. BDE are distinguishable.

δ	0	1
→AC	B	AC
B	B	D
D	B	E
*E	D	AC



Procedure :



If we get (C, D) or (D, E) or both final states in pair, don't mark.

	0	1
→A	B	C
B	A	D
*C	E	F
*D	E	F
*E	E	F
F	F	F

B					
C	X	X			
D	X	X			
E	X	X			
F	X	X	X	X	X
	A	B	C	D	E

CD, DC not marked initially as both are final

Don't remove trap state like F here.

S1)

	0	1
(A, B)	(B, A)	(C, D)
✓ (A, C)	(B, E)	(C, F)
✓ (A, D)	(B, E)	(C, F)
✓ (A, E)	(B, E)	(C, F)
✓ (A, F)	(B, F)	(C, F)
✓ (B, C)	(A, E)	(D, F)
✓ (B, D)	(A, E)	(D, F)
(B, E)	(A, E)	(D, F)
✓ (B, F)	(A, F)	(D, F)
(C, D)	(E, E)	(F, F)
(C, E)	(E, E)	(F, F)
(D, E)	(E, E)	(F, F)

S2)

δ	0	1
(A, B)	(B, A)	(C, D)
(C, D)	(E, E)	(F, F)
(C, E)	(E, E)	(F, F)
(D, E)	(E, E)	(F, F)

AB, $\underbrace{\text{CD, CE, DE}}$ are IDP's

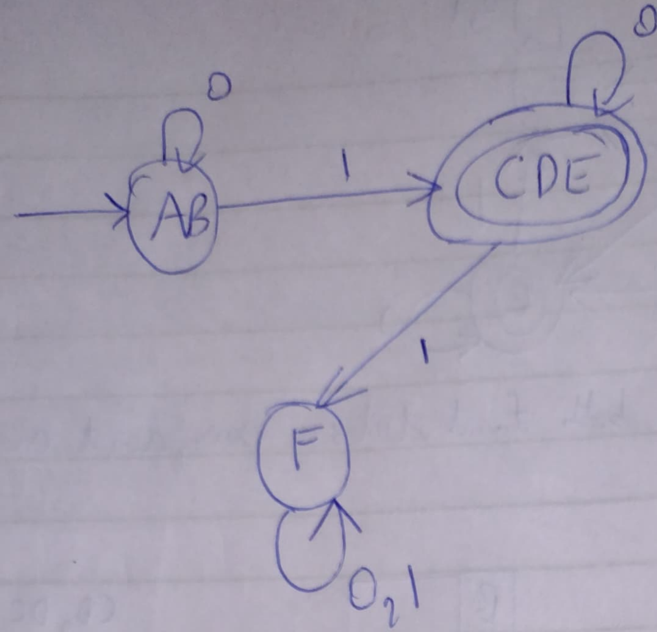
AB, CDE are IDP's.

Result :

F is DP.

δ	0	1
→ AB	AB	CDE
* CDE	CDE	F
F	F	F

Calculation :



	0	1
→A	B	E
B	C	F
*C	D	H
D	E	H
E	F	I
*F	G	B
G	H	B
H	I	C
*I	A	E

B	X							
C	X	X						
D		X	X					
E	X		X	X				
F	X	X	X	X	X			
G		X	X		X	X		
H	X		X	X		X	X	
I	X	X	X	X	X		X	X
	A	B	C	D	E	F	G	H

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DEPARTMENT OF PHYSICS

Expt. No. ____________ Date ____________

Title of Experiment ________________________________

Aim : S1)

δ	0	1
(A,B)	(B,C)	(E,F)
(A,D)	(B,E)	(E,H)
(A,E)	(B,F)	(E,I)
(A,G)	(B,H)	(E,B)
(A,H)	(B,I)	(E,C)
(B,D)	(C,E)	(F,H)
(B,E)	(C,F)	(F,I)
(B,G)	(C,H)	(F,B)
(B,H)	(C,I)	(F,C)
(C,F)	(D,G)	(H,B)
(C,I)	(D,A)	(H,E)
(D,E)	(E,F)	(H,I)
(D,G)	(E,H)	(H,B)
(D,H)	(E,I)	(H,C)
(E,G)	(F,H)	(I,B)
(E,H)	(F,I)	(I,C)
(F,I)	(G,A)	(B,E)
(G,H)	(H,I)	(B,C)

Apparatus :

Formula with S.I unit :

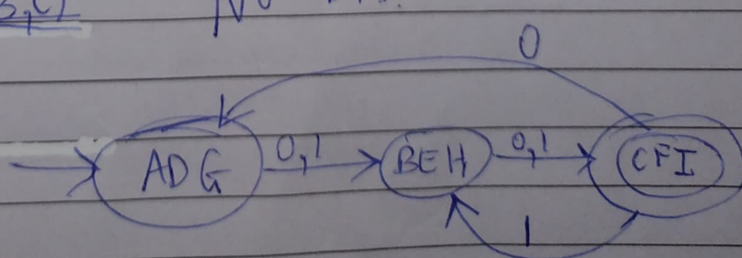
Principle :

δ	0	1
(A,D)	(B,E)	(E,H)
(A,G)	(B,H)	(E,B)
(B,E)	(C,F)	(F,I)
(B,H)	(C,I)	(F,C)
(C,F)	(D,G)	(H,B)
(C,I)	(D,A)	(H,E)
(D,G)	(E,H)	(H,B)
(E,H)	(F,I)	(I,C)
(F,I)	(G,A)	(B,E)

(A,D), (A,G), (B,E), (B,H)
(C,F), (C,I), (D,G), (E,H), (F,I) → IDP's

(A,D) (A,G) (D,G) → ADG
(B,E) (B,H) (E,H) → BEH
(C,F) (C,I) (F,I) → CFI

No DP's



δ	0	1
→ADG	BEH	BEH
BEH	CFI	CFI
*CFI	ADG	BEH

TM
$\downarrow$
LBA
$\downarrow$
PDA
$\downarrow$
FA

Diagram :

Pumping Lemma

L → Finite
  ↳ FA
  ↳ RE

If we can draw DFA or write RE for L, it is regular language, otherwise its irregular.

* Negativity test.

Observation :

<u>Pumping Lemma statement:</u> ~~Suppose~~ L is a regular language having pumping length p such that s is a string in the language such that
$|s| \geq p$ where string s can be divided into 3 parts x, y, z
→ $xyz = s$
such that following 3 conditions are true:

i) $xy^iz,\ i \geq 0 \in L$
ii) $|y| > 0$
iii) $|xy| \leq p$

Procedure :

Using PL, show that $L=\{a^n b^n \mid n \geq 0\}$ is not regular language.

→ $S = a^p b^p$, $p = 7$ (Can take any length).

$x \quad y \quad z$

$S = \underbrace{aa}_{x}\,\underbrace{aaaa}_{y}\,\underbrace{abbbbbbb}_{z}$

x, y appears in 'a' part

$x = aa$, $y = aaaa$, $z = abbbbbbb$

Let $i = 0$

$\therefore xy^i z = aaabbbbbbb \notin L$

$\therefore L$ is non regular.

Using PL ~~show that~~ check whether $L = \{a^i b^j \mid i \leq j\}$ is regular or not.

→ $S = a^p b^{p+1}$, $p = 4$

$s = aaaabbbbb$

$\underbrace{aa}_{x}\,\underbrace{a}_{y}\,\underbrace{abbbbb}_{b}$

$xy^i z$ for $i = 3$ is $aaaaaaabbbbb \notin L$

$\therefore L$ is not regular.

Using PL, show that $L = \{0^{j^2} \mid j \geq 1\}$ is Non regular.

Result :