

	1	2	3	4	5
2	0.6	0.6	0.6	0.6	0.6
3	1	1	1	1	1
4	0.3	0.2	0.2	0.2	0.2

$Ce(A)$  on domain  $X$ :

Ex: 2

$$C = \text{diffuse market size} = \left\{ \frac{0.3}{1}, \frac{0.5}{2}, \frac{0.6}{3}, \frac{0.6}{4}, \frac{0.5}{5}, \frac{0.3}{6} \right\}$$

$$X = \{1, 2, 3, 4\}$$

$Ce(C)$  on domain  $X$ :

	1	2	3	4	5	6
1	0.3	0.5	0.6	0.6	0.5	0.3
2	0.3	0.5	0.6	0.6	0.5	0.3
3	0.3	0.5	0.6	0.6	0.5	0.3
4	0.3	0.5	0.6	0.6	0.5	0.3

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Cartation product:

$$\text{row } A = \left\{ \frac{0.1}{100}, \frac{0.2}{200}, \frac{0.3}{300}, \frac{0.4}{400} \right\} \in X$$

$$\text{column } B = \left\{ \frac{0.6}{1}, \frac{0.5}{2}, \frac{0.8}{3}, \frac{0.9}{4} \right\} \in Y.$$

$A \times B =$

	1	2	3	4
100	0.1	0.1	0.1	0.1
200	0.2	0.2	0.2	0.2
300	0.3	0.3	0.3	0.3
400	0.4	0.4	0.4	0.4

## Composition:

- \* min - max
- \* max - max
- \* min - min
- \* max - average
- \* sum - product

$$\text{Let } A = \left\{ \frac{1}{s_1} + \frac{0.5}{s_2} + \frac{0.2}{s_3} \right\}$$

$$B = \left\{ \frac{1}{w_1} + \frac{0.5}{w_2} + \frac{0.3}{w_3} \right\}$$

$$C = \left\{ \frac{0.1}{s_1} + \frac{0.6}{s_2} + \frac{1}{s_3} \right\}$$

*composition product*

$$\text{Find } R = A \times B, \quad S = C \times B$$

Find  $S, R$  using max-min composition.

$$\Rightarrow R = A \times B = \begin{matrix} & w_1 & w_2 & w_3 \\ s_1 & 1 & 0.5 & 0.3 \\ s_2 & 0.5 & 0.5 & 0.3 \\ s_3 & 0.2 & 0.2 & 0.2 \end{matrix}$$

$$S = C \times B = \begin{matrix} & w_1 & w_2 & w_3 \\ s_1 & 0.1 & 0.1 & 0.1 \\ s_2 & 0.6 & 0.5 & 0.3 \\ s_3 & 1 & 0.5 & 0.3 \end{matrix}$$

$$T = \begin{matrix} A \\ (A) \\ B \end{matrix} = (B \times B). (C \times B)$$

$$= AXB$$

	$w_1$	$w_2$	$w_3$
$s_1$	0.5	0.5	0.3
$s_2$	0.5	0.5	0.3
$s_3$	0.2	0.2	0.2

$$s_1 w_1 = \max \left[ \min(1, 0.1), \min(0.5, 0.6), \min(0.3, 1) \right]$$

$$= \max [0.1, 0.5, 0.3]$$

$$= 0.5$$

$$s_1 w_2 = \max \left[ \min(1, 0.1), \min(0.5, 0.5), \min(0.3, 0.5) \right]$$

$$= \max [0.1, 0.5, 0.3]$$

$$= 0.5$$

$$s_1 w_3 = \max \left[ \min(1, 0.1), \min(0.5, 0.3), \min(0.3, 0.3) \right]$$

$$= \max [0.1, 0.3, 0.3]$$

$$= 0.3$$

$$s_2 w_1 = \max \left[ \min(0.5, 0.1), \min(0.5, 0.6), \min(0.3, 1) \right]$$

$$= \max [0.1, 0.5, 0.3]$$

$$= 0.5$$

$$s_2 w_2 = \max \left[ \min(0.5, 0.1), \min(0.5, 0.5), \min(0.3, 0.3) \right]$$

$$= \max [0.1, 0.5, 0.3]$$

$$= 0.5$$

Problem

$$U_B = \left\{ \frac{0.5}{60} + \frac{0.7}{40} + \frac{1}{20} \right\}$$

$$U_T = \left\{ \frac{0.9}{10} + \frac{0.7}{8} + \frac{0.5}{6} \right\}$$

$$U_U = \left\{ \frac{1}{0.9} + \frac{0.8}{0.8} + \frac{0.6}{0.7} + \frac{0.4}{0.6} \right\}$$

$$R = BXT = \begin{matrix} & 10 & 8 & 6 \\ \gamma \times C & \begin{bmatrix} 60 & 0.5 & 0.5 & 0.5 \\ 40 & 0.7 & 0.7 & 0.7 \\ 30 & 0.9 & 0.7 & 0.8 \end{bmatrix} \end{matrix}$$

$$S = TXU = \begin{matrix} & 0.9 & 0.8 & 0.7 & 0.6 \\ T & \begin{bmatrix} 10 & 0.9 & 0.8 & 0.6 & 0.4 \\ 8 & 0.7 & 0.7 & 0.6 & 0.4 \\ 6 & 0.5 & 0.5 & 0.5 & 0.4 \end{bmatrix} \end{matrix}$$

I:-  $w = R.S$  using max-min composition.  
 $= (BXT). (TXU)$

$$W = BXU$$

M1:

	0.9	0.8	0.7	0.6
60	0.5	0.5	0.5	0.4
40	0.7	0.7	0.6	0.4
20	0.9	0.8	0.6	0.4

$$(60, 0.9) = \max[\min(0.5, 0.9), \min(0.5, 0.8), \min(0.5, 0.7)] \\ = \max[0.5, 0.5, 0.5] \\ = 0.5$$

$$(60, 0.8) = \max[\min(0.8, 0.9), \min(0.7, 0.8), \min(0.6, 0.8)] \\ = \max[0.5, 0.5, 0.5] \\ = 0.5$$

$$(60, 0.7) = \max[\min(0.5, 0.9), \min(0.5, 0.8), \min(0.5, 0.7)] \\ = \max[0.5, 0.5, 0.5] \\ = 0.5$$

$$(60, 0.6) = \max[\min(0.5, 0.9), \min(0.5, 0.8), \min(0.5, 0.7)] \\ = \max[0.4, 0.4, 0.4] \\ = 0.4$$

$$(40, 0.9) = \max[\min(0.7, 0.9), \min(0.7, 0.8), \min(0.7, 0.7)] \\ = \max[0.7, 0.7, 0.7] \\ = 0.7$$

$$(40, 0.8) = \max[\min(0.7, 0.9), \min(0.7, 0.8), \min(0.7, 0.7)] \\ = \max[0.7, 0.7, 0.7] \\ = 0.7$$

$$(40, 0.7) = \max[\min(0.7, 0.9), \min(0.7, 0.8), \min(0.7, 0.7)] \\ = \max[0.6, 0.6, 0.6] \\ = 0.6$$

(60, 10)

II: using max - product composition.

$w =$	0.9	0.8	0.7	0.6
60	0.45	0.40	0.30	0.2
40	0.63	0.56	0.42	0.32
20	0.81	0.72	0.54	0.38

Max products

$$\begin{aligned}
 (60, 0.9) &= \max[(0.5, 0.9), (0.5, 0.7), (0.5, 0.5)] \\
 &= \max[0.45, 0.35, 0.35] \\
 &= 0.45
 \end{aligned}$$

$$\begin{aligned}
 (60, 0.8) &= \max[(0.5, 0.8), (0.5, 0.7), (0.5, 0.5)] \\
 &= \max[0.40, 0.35, 0.35] \\
 &= 0.40
 \end{aligned}$$

$$\begin{aligned}
 (60, 0.7) &= \max[(0.5, 0.6), (0.5, 0.6), (0.5, 0.5)] \\
 &= \max[0.30, 0.30, 0.35] \\
 &= 0.30
 \end{aligned}$$

$$\begin{aligned}
 (60, 0.6) &= \max[(0.5, 0.4), (0.5, 0.4), (0.5, 0.4)] \\
 &= 0.2
 \end{aligned}$$

$$1. \text{ Let } A = \left\{ \frac{0.3}{1} + \frac{1}{2} + \frac{0.7}{4} \right\}$$

$$B = \left\{ \frac{0.5}{1} + \frac{1}{3} \right\}$$

Find,

algebraic product ( $A \times B$ ) using max-min composition.

$$\Rightarrow A \times B = C = \left\{ \frac{1 \times 1}{1 \times 1} + \frac{1 \times 2}{1 \times 2} + \frac{2 \times 1}{2 \times 1} + \frac{2 \times 2}{2 \times 2} + \frac{4 \times 1}{4 \times 1} + \frac{4 \times 2}{4 \times 2} \right\}$$

$$C = \max \left\{ \min(0.3, 0.5) + \min(0.3, 1), \min(1, 0.5) + \min(1, 1), \min(6 \times 0.5, \min(6 \times 1)) \right\}$$

$$C = \max \left\{ \frac{0.3}{1} + \frac{0.3}{2} + \frac{0.5}{3} + \frac{1}{4} + \frac{0.5}{4} + \frac{0.7}{8} \right\}$$

$$= \left\{ \frac{0.3}{1} + \frac{\max(0.3, 0.5)}{2} + \frac{\max(1, 0.5)}{4} + \frac{0.7}{8} \right\}$$

$$= \left\{ \frac{0.3}{1} + \frac{0.5}{3} + \frac{1}{4} + \frac{0.7}{8} \right\}$$

$$2. \quad A = \left\{ \frac{0.6}{1} + \frac{1}{2} + \frac{0.8}{3} \right\} \in u = \{1, 2, 3\}$$

$$B = \left\{ \frac{0.8}{5} + \frac{1}{6} + \frac{0.7}{7} \right\}$$

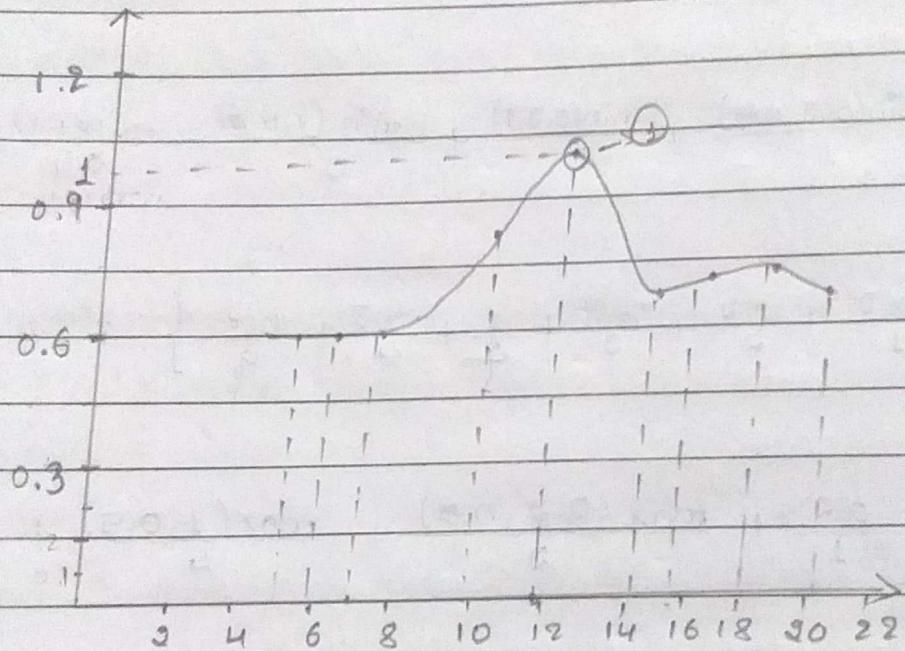
Find the product of approx two & approx six on domain V using max-min comp.

$$\Rightarrow \max C = \max \left\{ \min_{1 \times 5} (0.6, 0.8) + \min_{1 \times 6} (0.6, 0.1) + \min_{1 \times 7} (0.1, 0.8) + \min_{5 \times 5} (1, 1) \right.$$

$$\left. + \min_{2 \times 1} (0.1, 0.7) + \min_{3 \times 5} (0.8, 0.8) + \min_{3 \times 6} (0.8, 0.1) + \min_{3 \times 7} (0.1, 0.1) \right)$$

$$C = \max \left\{ \frac{0.6}{5}, \frac{0.6}{6}, \frac{0.6}{7}, \frac{0.8}{10}, \frac{1}{15}, \frac{0.7}{14}, \frac{0.8}{15}, \frac{0.8}{18}, \frac{0.7}{31} \right\}$$

$$C = \left\{ \frac{0.6}{5} + \frac{0.6}{6} + \frac{0.6}{7} + \frac{0.8}{10} + \frac{1}{15} + \frac{0.7}{14} + \frac{0.8}{15} + \frac{0.8}{18} + \frac{0.7}{31} \right\}$$



## UNIT-8 Theory of Approximate Reasoning

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Linguistic terms

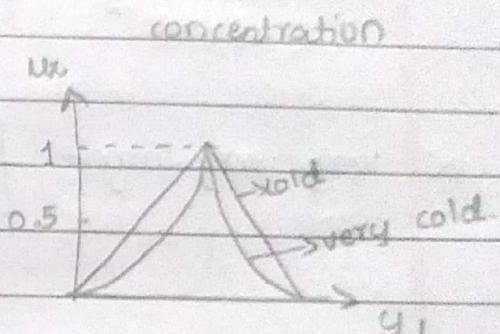
" [Adj added to intensive or moderate the system]

1. Very - concentrating
2. Very very - concentrating
3. Plus - concentrating
4. Slightly - diluting
5. Minus - diluting.

\*  $\alpha = \int_0^n [u_\alpha(y)]^4 \frac{dy}{y}$  for very.

Ex:- cold =  $\left\{ \frac{0.6}{50} + \frac{0.5}{51} \right\}$

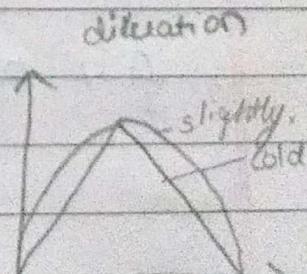
Very cold =  $\left\{ \frac{(0.6)^2}{50} + \frac{(0.5)^2}{51} \right\}$   
 $= \left\{ \frac{0.36}{50} + \frac{0.25}{51} \right\}$



\*  $\alpha = \int_0^n [u_\alpha(y)]^4 \frac{dy}{y}$  for very very.

\*  $\alpha = \int_0^n [u_\alpha(y)]^{0.5} \frac{dy}{y}$  for plus

\*  $\alpha = \int_0^n [u_\alpha(y)]^{0.5} \frac{dy}{y}$  for slightly



\*  $\alpha = \int_0^n [u_\alpha(y)]^{0.75} \frac{dy}{y}$  for minus.

Linguistic variables:

$$\text{size} = \{S, M, L, XL\}$$

$$\text{Temp} = \{L, M, H\}$$

Fuzzy rules:

1) If \_\_\_\_\_ then \_\_\_\_\_

2) If \_\_\_\_\_ then \_\_\_\_\_ else \_\_\_\_\_

1) If \_\_\_\_\_ then \_\_\_\_\_

⇒ If  $x$  is A then  $y$  is B.

rule antecedent      rule consequent.

Ex: If motor speed is low then increase voltage to medium.

If  $x$  is A then  $y$  is B

$$R = (A \times B) \cup (\bar{A} \times Y)$$

$\downarrow$   $\downarrow$   
 (cartesian) (union)  
 product                  product

2) If \_\_\_\_\_ then \_\_\_\_\_ else \_\_\_\_\_

∴ If hot climate then slow on AC else slow off AC

rule antecedent

rule

consequent.

If  $x$  is A then  $y$  is B else is C

$$R = (A \times B) \cup (\bar{A} \times C)$$

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upto if then

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Problem

$$1. \quad X = [1, 0, 3, 4], \quad Y = [1, 0, 3, 4, 5, 6]$$

$$A = \text{medium uniqueness} = \left\{ \frac{0}{5} + \frac{1}{3} + \frac{0}{4} \right\}$$

$$A = \left\{ \frac{0}{1} + \frac{0}{5} + \frac{1}{3} + \frac{0}{4} \right\}$$

$$B = \text{Medium market size} = \left\{ \frac{0.4}{2} + \frac{1}{3} + \frac{0.8}{4} + \frac{0.3}{5} \right\}$$

$$B = \left\{ \frac{0}{1} + \frac{0.4}{2} + \frac{1}{3} + \frac{0.8}{4} + \frac{0.3}{5} + \frac{0}{6} \right\}$$

To find, if A then B  $\Rightarrow (A \times B) \cup (\bar{A} \times Y)$   
 universe

$$\bar{A} = \left\{ \frac{1}{1} + \frac{0.4}{2} + \frac{0}{3} + \frac{0.8}{4} \right\}$$

$$Y = \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \right\}$$

$$A \times B = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ R \setminus \{ & \begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0.4 & 0.6 & 0.6 & 0.3 & 0 \\ 3 & 0 & 0.4 & 1 & 0.8 & 0.3 & 0 \\ 4 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0 \end{array} \end{array}$$

$$\bar{A} \times Y = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ S \setminus \{ & \begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{array} \end{array}$$

If A then B  $\Rightarrow (A \times B) \cup (\bar{A} \times Y)$

$(A \times B) \cup (\bar{A} \times Y) =$

	1	2	3	4	5	6
1	1	1	1	1	1	1
2	0.4	0.4	0.6	0.6	0.4	0.4
3	0	0.4	1	0.8	0.3	0
4	0.8	0.8	0.8	0.8	0.8	0.8

If A then B else C  $\Rightarrow (A \times B) \cup (\bar{A} \times C)$

$$A = \left\{ \frac{1}{1} + \frac{0.4}{2} + \frac{0}{3} + \frac{0.8}{4} \right\}$$

$$C = \left\{ \frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.6}{4} + \frac{0.5}{5} + \frac{0.3}{6} \right\}$$

$\bar{A} \times C =$

	1	2	3	4	5	6
1	0.3	0.5	0.6	0.6	0.5	0.3
2	0.3	0.4	0.4	0.4	0.4	0.3
3	0	0	0	0	0	0
4	0.3	0.5	0.6	0.6	0.5	0.3

$(A \times B) \cup (\bar{A} \times C) =$

	1	2	3	4	5	6
1	0.3	0.5	0.6	0.6	0.5	0.3
2	0.3	0.4	0.6	0.6	0.4	0.3
3	0	0.4	1	0.8	0.3	0
4	0.3	0.5	0.6	0.6	0.5	0.3

$$x = \{1, 2, 3, 4\}$$

$$Y = \{1, 2, 3, 4, 5, 6\}$$

$$A = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{1}{3} \right\} = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{1}{3} + \frac{0}{4} \right\}$$

$$B = \left\{ \frac{0}{1} + \frac{0}{2} + \frac{1}{3} + \frac{1}{4} + \frac{0}{5} + \frac{0}{6} \right\}$$

$$\text{If } A \text{ then } B = (A \times B) \cup (\bar{A} \times Y)$$

$$\bar{A} = \left\{ \frac{1}{1} + \frac{0}{2} + \frac{0}{3} + \frac{1}{4} \right\}$$

$$Y = \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \right\}$$

$$A \times B = \{$$

	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	1	1	0	0
3	0	0	1	1	0	0
4	0	0	0	0	0	0

$$\bar{A} \times Y$$

	1	2	3	4	5	6
1	1	1	1	1	1	1
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	1	1	1	1	1	1

$$(A \times B) \cup (\bar{A} \times C) =$$

	1	2	3	4	5	6
1	1	1	1	1	1	1
2	0	0	1	1	0	0
3	0	0	1	1	0	0
4	1	1	1	1	1	1

$$\bar{A} = \{$$

$$C = \{$$

$$(A \times B) \cup (\bar{A} \times C) =$$

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$$5.11 \quad x = [0, 50, 100, 150, 200]$$

$$y = [0, 50, 100, 150, 200]$$

$$W = \text{weak stimulus} = \left\{ \frac{1}{0}, \frac{0.9}{50}, \frac{0.3}{100}, \frac{0}{150}, \frac{0}{200} \right\} \rightarrow x$$

$$S = \text{severe response} = \left\{ \frac{0}{0}, \frac{0}{50}, \frac{0.5}{100}, \frac{0.9}{150}, \frac{1}{200} \right\} \rightarrow y$$

If "weak stimulus" then not severe response"

If  $\max x$  is  $w$  the  $y$  is  $\bar{s}$

$$\Rightarrow (w \times \bar{s}) \cup (\bar{w} \times y)$$

$$\bar{w} = \left\{ \frac{0}{0}, \frac{0.1}{50}, \frac{0.7}{100}, \frac{1}{150}, \frac{1}{200} \right\}$$

$$y = \left\{ \frac{1}{0}, \frac{1}{50}, \frac{1}{100}, \frac{1}{150}, \frac{1}{200} \right\}$$

$$\bar{s} = \left\{ \frac{1}{0}, \frac{1}{50}, \frac{0.5}{100}, \frac{0.1}{150}, \frac{0}{200} \right\}$$

$$(w \times \bar{s}) = \begin{array}{c|ccccc} & 0 & 50 & 100 & 150 & 200 \\ \hline 0 & 1 & 1 & 0.5 & 0.1 & 0 \\ 50 & 0.9 & 0.9 & 0.5 & 0.1 & 0 \\ 100 & 0.3 & 0.3 & 0.3 & 0.1 & 0 \\ 150 & 0 & 0 & 0 & 0 & 0 \\ 200 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$(\bar{w} \times y) = \begin{array}{c|ccccc} & 0 & 50 & 100 & 150 & 200 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 50 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 100 & 0.7 & 0.7 & 0.7 & 0.7 & 0.7 \\ 150 & 1 & 1 & 1 & 1 & 1 \\ 200 & 1 & 1 & 1 & 1 & 1 \end{array}$$

$$(\bar{w} \times y) = \begin{array}{c|ccccc} & 0 & 50 & 100 & 150 & 200 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 50 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 100 & 0.7 & 0.7 & 0.7 & 0.7 & 0.7 \\ 150 & 1 & 1 & 1 & 1 & 1 \\ 200 & 1 & 1 & 1 & 1 & 1 \end{array}$$

$$R = (W \times 5) \cup (W \times 4)$$

	0	50	100	150	200
0	1	1	0.5	0.1	0
50	0.9	0.9	0.5	0.1	0.1
100	0.7	0.7	0.7	0.7	0.7
150	1	1	1	1	1
200	1	1	1	1	1

\* If a new antecedent is introduced.

$$M = \left\{ \frac{0}{50}, \frac{0.4}{100} + \frac{1}{100} + \frac{0.4}{150} + \frac{0}{200} \right\}$$

Then find the consequent  $P'$  using max-min composition.

$$\Rightarrow P' = M \cdot R = \begin{bmatrix} 0 & 0.4 & 1 & 0.4 & 0 \end{bmatrix}_{1 \times 5} \begin{bmatrix} 1 & 1 & 0.5 & 0.1 & 0 \\ 0.9 & 0.9 & 0.5 & 0.1 & 0.1 \\ 0.7 & 0.7 & 0.7 & 0.7 & 0.7 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}_{5 \times 5}$$

$$\begin{aligned} &= \max[\min(0, 1), \min(0.4, 0.9), \min(1, 0.7), \min(0.4, 1), \min(0, 0.1)] \\ &= \max[0, 0.4, 0.7, 0.4, 0] \\ &= 0.7 \end{aligned}$$

$$\begin{aligned} &= \max[\min(0, 1), \min(0.4, 0.9), \min(1, 0.7), \min(0.4, 1), \min(0, 0.1)] \\ &= 0 \max(0, 0.4, 0.7, 0.4, 0) \\ &= 0.7 \end{aligned}$$

$$\begin{aligned} &= \max(0, 0.4, 0.7, 0.4, 0) \\ &= 0.7 \end{aligned}$$

$$P' = [0.7 \ 0.7 \ 0.7 \ 0.7 \ 0.7]$$

"Not measurable response."