03-09-2020

Fuzzy : Something which is not clear.

Fuzzy logic : A logic which is used to solve any problem with ambiguity and uncertainity.

It's a degree of truth.

It is a human language base system.

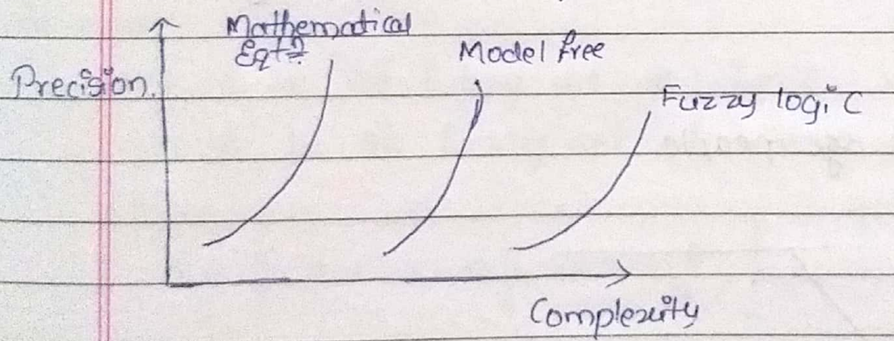
digital system

Is it cold — Yes / 1
Is it cold — No / 0

Fuzzy system

Is it cold — Very much / 0.9
Is it cold — little / 0.25
Is it cold — very less / 0.1

When we use fuzzy logic :



Operations on classical sets

Union : $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Intersection $A \cap B = \{x \mid x \in A \ \& \ x \in B\}$

Complement

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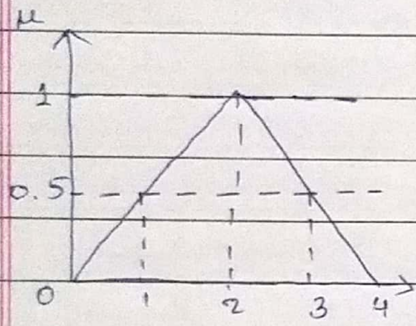
Operation on Fuzzy system

Union (max) : $\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \vee \mu_{\underset{\sim}{B}}(x)$

Intersection (min) $\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(x)$

Complement : $\mu_{\bar{A}}(x) = 1 - \mu_{\underset{\sim}{A}}(x)$

Representation of fuzzy sets.



$\therefore A = \left\{ \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \dots \frac{\mu_A(x_n)}{x_n} \right\}$

$\mu_A(x_1) \rightarrow$ degree of membership.

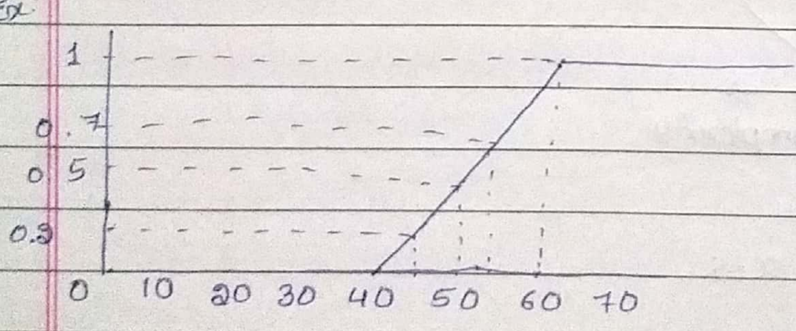
$x_1 \dots x_n \Rightarrow$ universe of discourse

$+ \Rightarrow$ seperation symbol

here, A =

Fuzzy sets for old age people.

Ex:



Age: $\left\{ \frac{0}{40} + \frac{0.2}{45} + \frac{0.5}{50} + \frac{0.7}{55} + \frac{1}{60} + \frac{1}{65} \dots \right\}$

Problems:

Ex: 1 $\underset{\sim}{A} = \left\{ \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$ & $B = \left\{ \frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}$

* Complement, $\bar{\underset{\sim}{A}} = \left\{ \frac{1}{1} + \frac{0}{2} + \frac{0.5}{3} + \frac{0.7}{4} + \frac{0.8}{5} \right\}$

$\mu_{\bar{A}}(x) = 1 - \mu_{\underset{\sim}{A}}(x)$

$\bar{\underset{\sim}{B}} = \left\{ \frac{1}{1} + \frac{0.5}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.6}{5} \right\}$

* Union (max). $\underset{\sim}{A} \cup \underset{\sim}{B} = \left\{ \frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5} \right\}$

* Intersection (min). $\underset{\sim}{A} \cap \underset{\sim}{B} = \left\{ \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5} \right\}$

09-09-2020

Let A be the fuzzy set of universe X

B be the fuzzy set of universe X

$$A = \left\{ \frac{0.1}{1} + \frac{0.4}{2} + \frac{0.8}{3} \right\}$$

$$B = \left\{ \frac{0.2}{1} + \frac{0.3}{2} + \frac{0.9}{3} \right\}$$

Find,

$A \cup B$, $A \cap B$, $\bar{A}$ & $\bar{B}$

$\Rightarrow$ $\mu(A \cup B)(x_1) = \max(\mu_A(x_1)) = \cancel{\max}\ \mu_B(x_2))$

$\mu_{(A \cup B)}(1.1) = \max(0.1, 0.2)$

$= 0.2$

$\mu_{(A \cup B)}(2.2) = \max(0.4, 0.3)$

$= 0.4$

$\mu_{(A \cup B)}(3.3) = 0.9$

$\mu(A \cup B) = \left\{ \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.9}{3} \right\}$

* $A \cap B = \left\{ \frac{0.1}{1} + \frac{0.3}{2} + \frac{0.9}{3} \right\}$

* $\overline{A} = \left\{ \frac{0.9}{1} + \frac{0.6}{2} + \frac{0.2}{3} \right\}$ $\qquad \overline{A} = 1 - A$

$\overline{B} = \left\{ \frac{0.8}{1} + \frac{0.7}{2} + \frac{0.1}{3} \right\}$

* As per the classical theory:

Axiom of the excluded middle $\quad A \cup \overline{A} = X \quad$ (X = universal)

Axiom of the contradiction $\quad A \cap \overline{A} = \emptyset$

Demorgan's principles:

$\overline{\tilde{A} \cap \tilde{B}} = \overline{\tilde{A}} \cup \overline{\tilde{B}}$

$\overline{\tilde{A} \cup \tilde{B}} = \overline{\tilde{A}} \cap \overline{\tilde{B}}$

* As per the fuzzy theory:

The excluded middle & contradiction axioms, extended for fuzzy sets, expressed by,

$\tilde{A} \cup \overline{\tilde{A}} \neq X$

$\tilde{A} \cap \overline{\tilde{A}} \neq \emptyset$

Demorgan's principles:

$\overline{\tilde{A} \cap \tilde{B}} = \overline{\tilde{A}} \cup \overline{\tilde{B}}$

$\overline{\tilde{A} \cup \tilde{B}} = \overline{\tilde{A}} \cap \overline{\tilde{B}}$

Same as the classical theory.

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* Difference:

$\underset{\sim}{A} \mid \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}}$

$\underset{\sim}{B} \mid \underset{\sim}{A} = \underset{\sim}{B} \cap \overline{\underset{\sim}{A}}$

Problem: $\underset{\sim}{A} = \left\{ \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$

$\underset{\sim}{B} = \left\{ \frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}$

→ Complement

$\overline{\underset{\sim}{A}} = \left\{ \frac{1}{1} + \frac{0}{2} + \frac{0.5}{3} + \frac{0.7}{4} + \frac{0.8}{5} \right\}$

$\overline{\underset{\sim}{B}} = \left\{ \frac{1}{1} + \frac{0.5}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.6}{5} \right\}$

Union: $\underset{\sim}{A} \cup \underset{\sim}{B} =$

Intersection: $\underset{\sim}{A} \cap \underset{\sim}{B} =$

Difference:

$\underset{\sim}{A} \mid \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}} = \left\{ \frac{0.5}{2} \right.$

11/09/2020

Properties of fuzzy

1. T-Norms
2. S-Norms
3. C-norms.

1. T-norms :: (Triangular norm) ($\wedge$)

* $a \wedge b = b \wedge a$

let, $a = \left\{ \frac{0.1}{1} + \frac{0.2}{2} \right\}$ $b = \left\{ \frac{0.3}{1} + \frac{0.4}{2} \right\}$

$a \wedge b = \left\{ \frac{0.1}{1} + \frac{0.2}{2} \right\} \rightarrow (1)$

$b \wedge a = \left\{ \frac{0.1}{1} + \frac{0.2}{2} \right\} \rightarrow (2)$

from (1) & (2)

$a \wedge b = b \wedge a$

2* $(a \wedge b) \wedge c = a \wedge (b \wedge c)$

let $c = \left\{ \frac{0.3}{1} + \frac{0.9}{2} \right\}$

$(a \cap b) \cap c = \left\{ \frac{0.1}{1} + \frac{0.2}{2} \right\} \cap \left\{ \frac{0.3}{1} + \frac{0.9}{2} \right\}$

$= \left\{ \frac{0.1}{1} + \frac{0.2}{2} \right\}$

$(b \cap c) = \left\{ \frac{0.3}{1} + \frac{0.4}{2} \right\}$

$a \cap b \cap c = \left\{ \frac{0.1}{1} + \frac{0.2}{2} \right\} \cap \left\{ \frac{0.3}{1} + \frac{0.4}{2} \right\}$

$= \left\{ \frac{0.1}{1} + \frac{0.2}{2} \right\}$

* 3) Ex: $a = \{0.1\}$; $c = \{0.3\}$
$b = \{0.6\}$ $d = \{0.9\}$

$\therefore a \wedge b = \{0.1\}$
$c \wedge d = \{0.3\}$

$(a \wedge b) < (c \wedge d)$

* 4) $a = \left\{ \frac{0.1}{1} + \frac{0.2}{2} \right\}$

$a \wedge 1 = \left\{ \frac{0.1}{1} + \frac{0.2}{2} \right\} \wedge \left\{ \frac{1}{1} + \frac{1}{1} \right\}$

$= \left\{ \frac{0.1}{1} + \frac{0.2}{2} \right\}$

i.e $= a$
$\therefore a \wedge 1 = a$

2. S-Norms: (max) (V)
* $a \vee b = b \vee a$
* $(a \vee b) \vee c = a \vee (b \vee c)$
* $a \le c$ & $b \le d \Rightarrow a \vee b \le c \vee d$.
* $a \vee 0 = a$
* $a \vee 1 = 1$

3. C norms compliment:
* $c(0) = 1$
* $a < b \Rightarrow c(a) > c(b)$
* $c(c(a)) = a$.

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* Yager family implication : (T-norm)

$y_{q(a,b)} = 1 - \min[1, ((1-a)^q + (1-b)^q)^{1/q}]$

where $q \geq 0$.

Problem: let $A = \left\{ \frac{1}{2} + \frac{0.8}{3} + \frac{0.7}{4} \right\}$

$B = \left\{ \frac{0.6}{2} + \frac{0.9}{3} + \frac{0.1}{4} \right\}$

let $q = 1$.

$y_1(a_1, b_1) = 1 - \min[1, ((1-a_1)^1 + (1-b_1)^1)^{1/1}]$

$y_1(1, 0.6) = 1 - \min[1, ((1-1)^1 + (1-0.6)^1)^1]$

$= 1 - \min[1, 0.4]$

$= 1 - 0.4$

$= 0.6.$

$y_2(a_2, b_2) = 1 - \min[1, ((1-a_2)^1 + (1-b_2)^1)^{1/1}]$

$= 1 - \min[1 - (0.2 + 0.1)]$

$= 1 - \min[1, 0.3]$

$= 1 - 0.3$

$= 0.7$

$y_3(a_3, b_3) = 1 - \min[1 - ((1-a_3)^1 + (1-b_3)^1)^{1/1}]$

$= 1 - \min[1 - ((1-0.7) + (1-0.1))^1]$

$= 1 - \min[1, (0.3 + 0.9)]$

$= 1 - \min[1, 1.2]$

$= 1 - 1$

$= 0.$

$y(A, B) = \left\{ \frac{0.6}{2} + \frac{0.7}{3} + \frac{0}{4} \right\}$

* Dubois & Prade family implication :- (T-norms)

$$\sigma_\alpha(a,b) = \frac{a.b}{\max(a,b,\alpha)}$$

Ex :- $A = \left\{\frac{1}{2} + \frac{0.8}{3} + \frac{0.7}{4}\right\}$, $B = \left\{\frac{0.6}{2} + \frac{0.9}{3} + \frac{0.1}{4}\right\}$

let, $\alpha = 1$

$$\sigma_1(1, 0.6) = \frac{1 \times 0.6}{\max(1, 0.6, 1)} = 0.6$$

$$\sigma_2(0.8, 0.9) = \frac{0.8 \times 0.9}{\max(0.8, 0.9, 1)} = \frac{0.72}{1} = 0.72$$

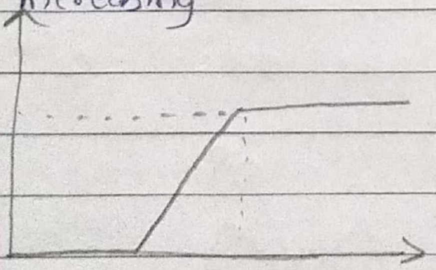
$$\sigma_3(0.7, 0.1) = \frac{0.7 \times 0.1}{\max(0.7, 0.1, 1)} = \frac{0.7}{1} = 0.7$$

$$\sigma(A, B) = \left\{\frac{0.6}{2} + \frac{0.72}{3} + \frac{0.7}{4}\right\}$$

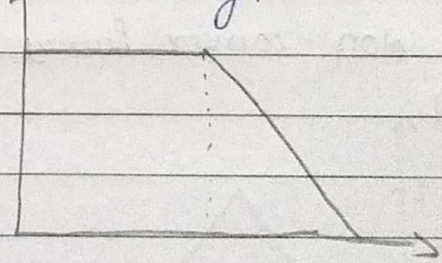
1. Overlapping [desired nature).

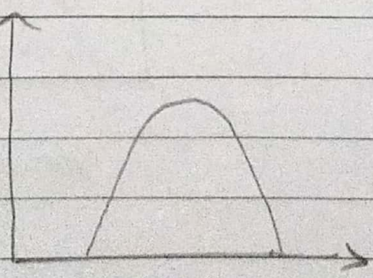
Fuzzy set types
- increasing Eg. old, hot, high etc
- decreasing Eg. young, cold, low etc.
- mixed.

increasing



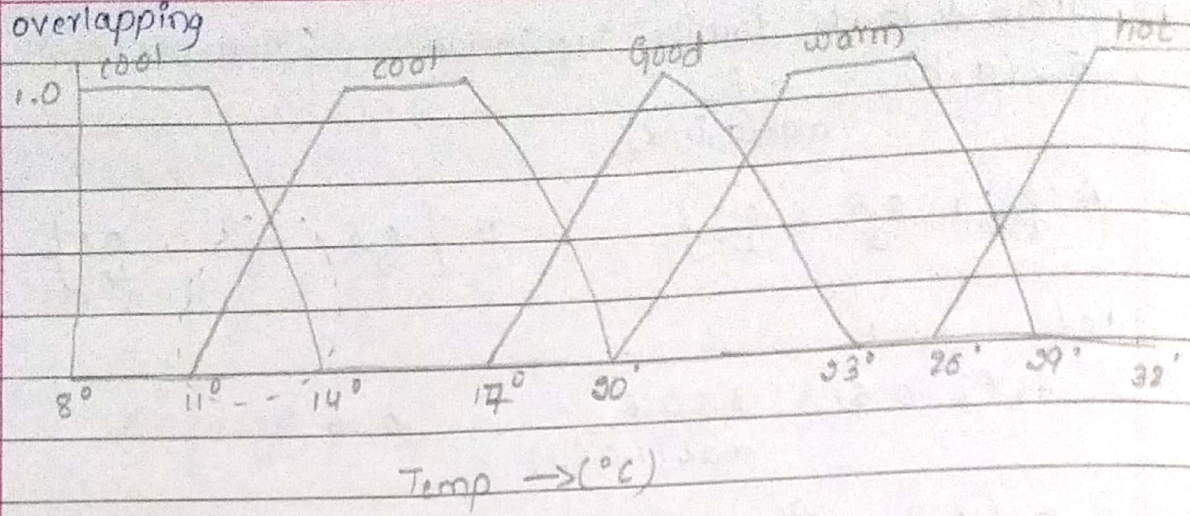
decreasing.



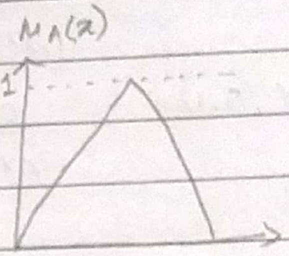


mixed.

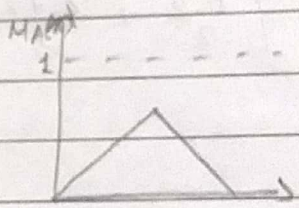
* overlapping



Types of fuzzy sets :



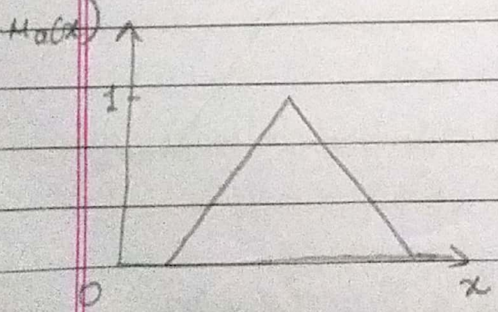
1. Normal fuzzy set
Degree of membership reached 1.



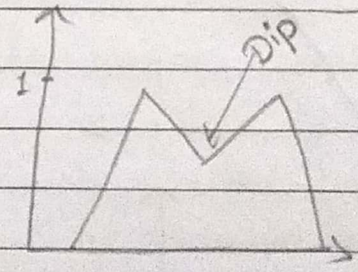
2. Subnormal fuzzy set.
Degree of membership is lower than 1.

3. Convex fuzzy set :
If there are no dips.

4. Non - convex fuzzy set :-



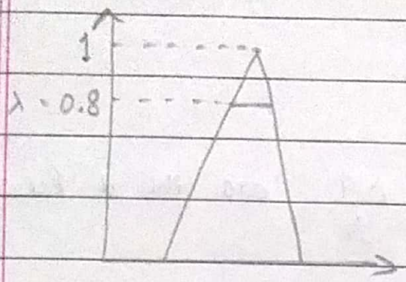
Convex fuzzy set



Non convex fuzzy set.

5. lamda cut or alpha cut.

If A is a fuzzy set, lambda cut is $A\lambda$ for which $\lambda \leq \mu_A(x) \leq 1$.

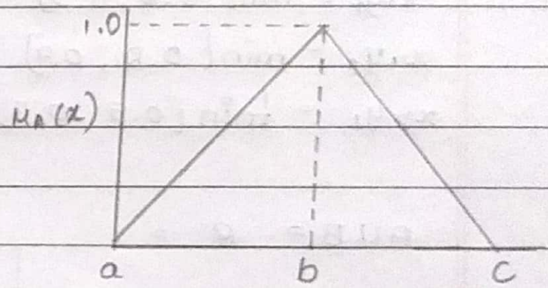


$A_\lambda \simeq A_\alpha$

Shapes of membership function:

1. Triangular fuction.

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \frac{c-x}{c-b} & \text{if } b \leq x \leq c \\ 0 & \text{if } x \geq c \end{cases}$$



24/09/2020

Relations

Union : $R \cup S \rightarrow \chi_{R \cup S}(x,y) : x_{R \cup S}(x,y)$

Intersection:

Complement:

Ex:- 1) Let $\frac{0.2}{x_1} + \frac{0.5}{x_2} + \frac{1}{x_3}$ & $B = \frac{0.3}{y_1} + \frac{0.9}{y_2}$ are the 2 fuzzy sets.

=> $A \cap B = R =$

$$\begin{array}{c|cc|} & y_1 & y_2 \\ \hline x_1 & 0.2 & 0.2 \\ x_2 & 0.3 & 0.5 \\ x_3 & 0.3 & 0.9 \end{array}$$

$x_1y_1 = \min[0.2, 0.3]$

$x_1y_2 = \min[0.2, 0.9]$

$x_2y_1 = \min[0.5, 0.3]$

$x_2y_2 = \min[0.5, 0.3]$

$x_3y_1 = \min[1, 0.3]$

$x_2y_2 = \min[1, 0.9]$

$A \cup B = R =$

$$\begin{array}{c|cc|} & y_1 & y_2 \\ \hline x_1 & 0.3 & 0.9 \\ x_2 & 0.5 & 0.9 \\ x_3 & 1 & 1 \end{array}$$

$x_1y_1 = \max[0.2, 0.3]$

$x_1y_2 = \max[0.2, 0.9]$

$x_2y_1 = \max[0.5, 0.3]$

$x_2y_2 = \max[0.5, 0.3]$

$x_3y_1 = \max[1, 0.3]$

$x_3y_2 = \max[1, 0.9]$

2] $A = \frac{0.2}{p_1} + \frac{0.6}{p_2} + \frac{0.5}{p_3} + \frac{0.9}{p_4}$

$B = \frac{0.4}{q_1} + \frac{0.7}{q_2} + \frac{0.8}{q_3}$

$\Rightarrow$

$$A \cup B = R = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{ccc} q_1 & q_2 & q_3 \end{array} \\ \begin{bmatrix} 0.4 & 0.7 & 0.8 \\ 0.6 & 0.7 & 0.8 \\ 0.5 & 0.7 & 0.8 \\ 0.9 & 0.9 & 0.9 \end{bmatrix} \end{array}$$

Cartesian product ["X"] :- Means minimisation.

for the above example :-

$$A \times B = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.4 & 0.6 & 0.6 \\ 0.4 & 0.5 & 0.5 \\ 0.4 & 0.7 & 0.8 \end{bmatrix}$$