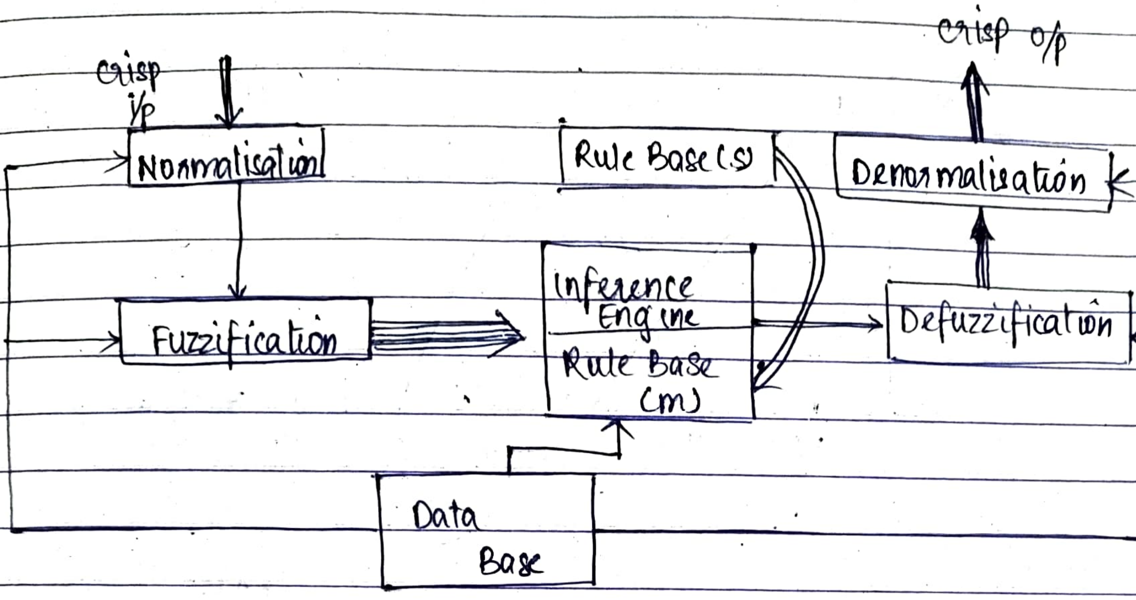
M = meaning
S = symbol

14/12/21

Fuzzy Knowledge Base controller (FKBC)



⟹ computational flow

⟶ Data flow

Normalisation: Normalisation is done to scale the values. Scaling can be up-scaling or down scaling.

Ex: speed = [1000, 2000, 3000, 4000].

To normalise the value, divide the above value with the largest value in the above domain

ie. speed $= \left[\frac{1000}{4000}, \frac{2000}{4000}, \frac{3000}{4000}, \frac{4000}{4000}\right]$

$= [0.25, 0.5, 0.75, 1]$

Ex: $A = [0.001, 0.002, 0.003]$

Multiply by 100

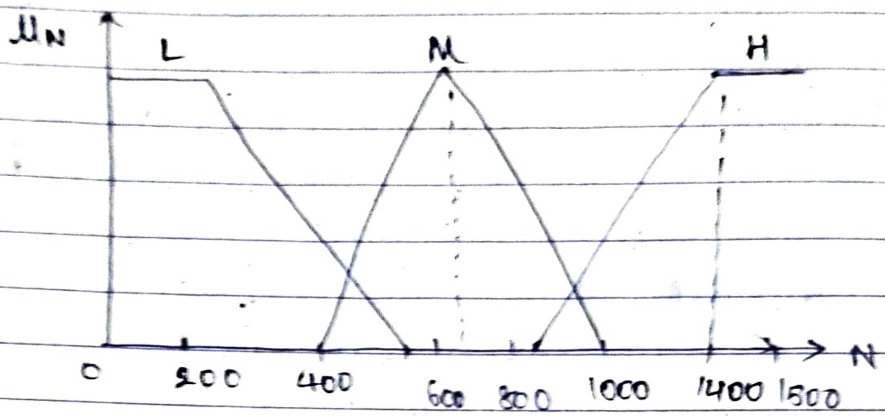
$\therefore A = [0.1, 0.2, 0.3]$

Fuzzification: It is used to convert crisp value into fuzzy value.

Ex: speed (N) = [0 to 1800 rpm]

Fuzzification process has to conducted through all the i/p and o/p variables.

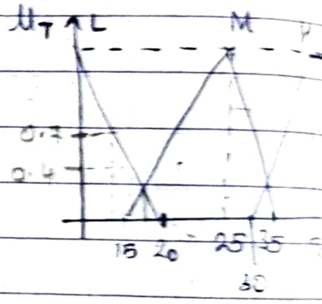
[0-500] [4000-1000] [9000-1500]
Low (L) Medium (M) High (H)



15/12/21

Fuzzification:

Ex: Temp [0 to 50°C]
[0-20] [15-35] [30-50]
L M H



→ Data base and Rule Base

(i) Data Base:

All the ranges of i/p and o/p variables scaling function used for normalisation and denormalisation, fuzzification and defuzzification data is stored in this data base.

(a) Choice of data base:

(i) choice of membership functions
(ii) choice of scaling function used for normalisation and denormalisation

(ii) Rule Base:

a) Choice of Rule base:

i. Choice of i/p and o/p variables

ii. Choice of content of rule anticedent and rule consequent.

iii. Choice of term sets for anticident and consequent

iv. Derivation of set of rules.

Ex: Washing Machine:

i/p : (i) Dirt level (D) : [L M H] (0 to 100%)

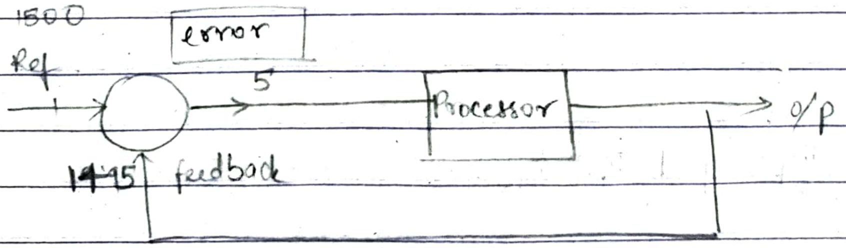
(ii) [illegible] of cloth (Wt) [L M H] (0 to 7kg)

o/p : Time for washing (T) [L M H] (0 to 60min

Dirt \ Wt	L	M	H
L	L	L	M
M	L	M	M
H	M	H	H

21/12/21

→ Choice of variable and content of rule:



P, PI, PD, PID

I/P : (i) error 'e'
(ii) change in error Δe or $\dot{e}$
(iii) sum of error δe

O/P : (i) control o/p 'u'
(ii) change in control O/P Δu or $\dot{u}$

→ P type of FKBC : (Proportional type)

$u = K_p \cdot e$
$K_p \rightarrow$ Proportional constant
$e \rightarrow$ error
$u \rightarrow$ control o/p

if e is ____ then u is ____
Ex : if e is PS then u is M

PS = +ve small
M = medium

→ P-D type of FKBC

$u = K_p \cdot e + K_D \cdot \Delta e$

if e is ____ & Δe is ____ then u is ____

Ex: if e is PM & Δe is PS then u is S

PM = +ve medium
PS = +ve small
S = small

→ P-I type of FKBC : (Proportional integral type)

$u = K_p \cdot e + K_I \cdot \int e \, dt$
$\Delta u = K_p \cdot \Delta e + K_I \cdot e$

if Δe is ____ & e is ____, then Δu is ____

Ex: if Δe is PL & e is PM, then Δu is M

PL = +ve large
PM = +ve medium
M = medium

→ PID type of FKBC:

$$u = K_P \cdot e + K_D \cdot \dot{e} + K_I \int e\,dt$$

if e is ____ & Δe is ____ & δe is ____ then u is ____.

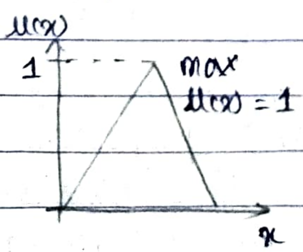
eg: if e is PL & Δe is PM & δe is PM then u is M

→ Data Base:

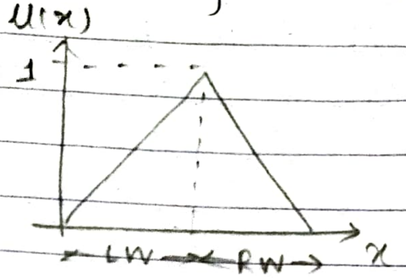
1. Choice of membership function
2. Choice of scaling function

(i) Choice of membership function:
- The system should have high computational efficiency.
- It should use the memory efficiently
- Should have uniform representation.
- The best choice is to always go with a triangular function

(a) Peak function:

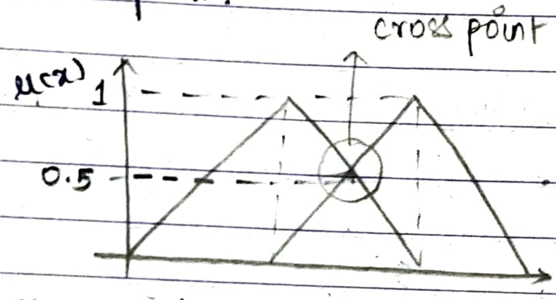


Choose a Δ function with a degree of maximum membership value = 1 b.) Left and Right width :



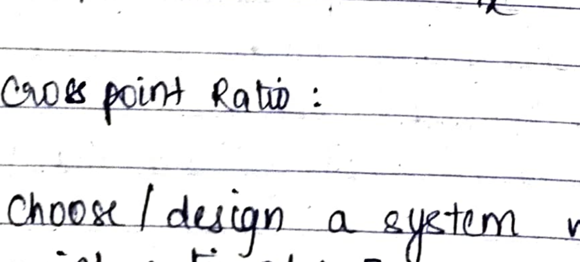
Choose a f^n whose leftwidth is equal to right width
(ie. LW = RW)

c.) Cross point :



Choose a function which has no. of cross points equal to 1

(ie. No. of cross point = 1)



d.) Cross point Ratio :

Choose / design a system which has a cross point ratio of 0.5

Hence, we need to choose a membership function such that it should be a triangular f^n with a peak value of 1, having LW = RW, having no. of cross points = 1 and the cross point ratio = 0.5

22/12/2021

(ii) Choice of scaling function:

There are 2 approaches for selecting the scaling function:

1. Heuristic method (simple, easy to use)
2. Analytical method (ie. formula based, complex, but more accurate)

Heuristic method is a trial and error based method.

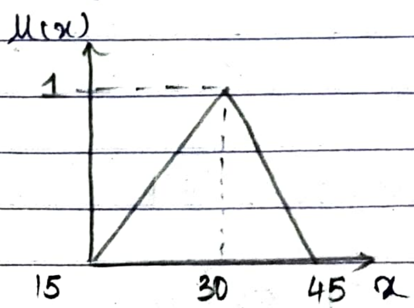
I: Select randomly the scaling f^n
II: Check the performance of the controller
III: If not satisfactory then go to step I
IV: If satisfatory select

Analytical method is a formula based method, where mathematic models must be available to evaluate the scaling factor

→ Defuzzification: (Fuzzy to crisp)

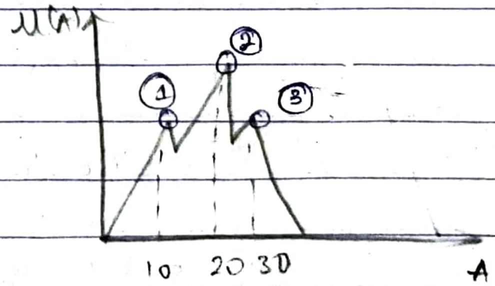
1. Height method
2. weighted average
3. Middle of maximum (MoM)
4. first of max
5. Last of max
6. Centre of gravity (CoG)

a.) Height Method:



$z^* = 30$

Here the crisp value associated with the peak value is the defuzzified value



$z^* = 20$

It is the highest peak

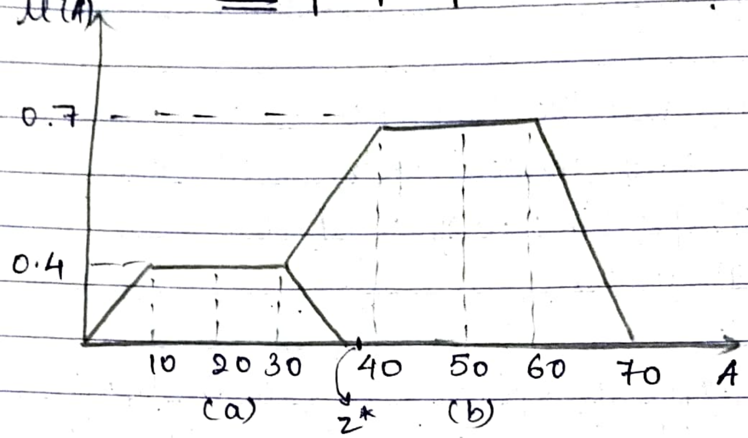
this f^n/ method is not applicable for flat functions

b.) weighted average method:

$$z^* = \frac{\sum \mu(z) \cdot z}{\sum \mu(z)}$$

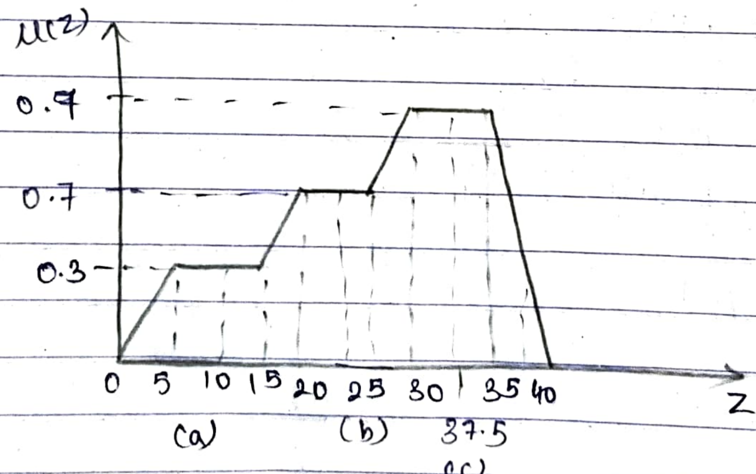
This method is applicable only for flat f^n

Ex 1:



$$z^* = \frac{(0.4 \times 20) + (0.7 \times 50)}{0.4 + 0.7} = \frac{35 + 8}{1.1} = 39.09$$

Ex 2:



find z^* using weighted average method

$$z^* = \frac{(0.3 \times 10) + (0.7 \times 25) + (0.9 \times 37.5)}{(0.3 + 0.7 + 0.9)}$$

$= 28.5$

c) Middle of max:

Only applicable for flat f^n.
This method finds z^* by taking the middle value of maximum flat function.

$\therefore$ Taking Ex: 1:

z^* would be: $z^* = \frac{40+60}{2} = \underline{\underline{50}}$

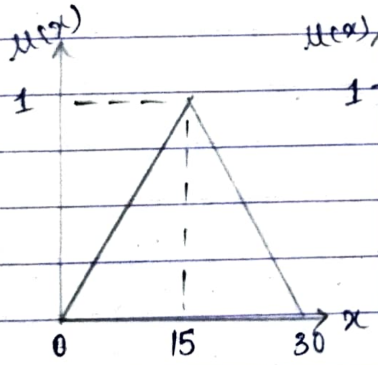
04/01/2022

d) First of max:

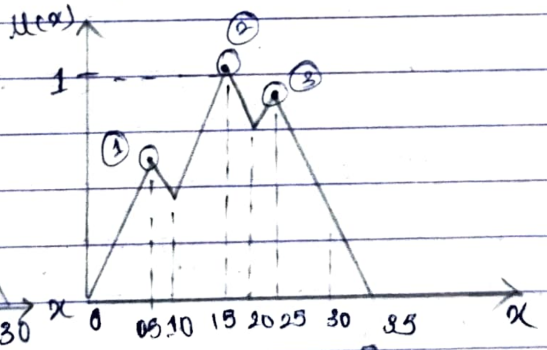
This is applicable for both flat and peak functions. It considers the first highest value as the defuzzified value.

e) Last of max:

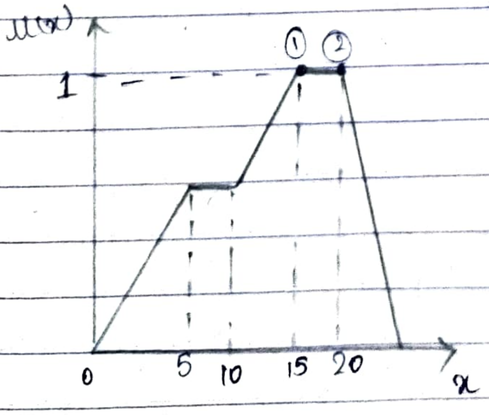
This function is also applicable for both flat and peak functi[ons].
It considers the last highest peak as the defuzzified value.



FOM = LOM = 15

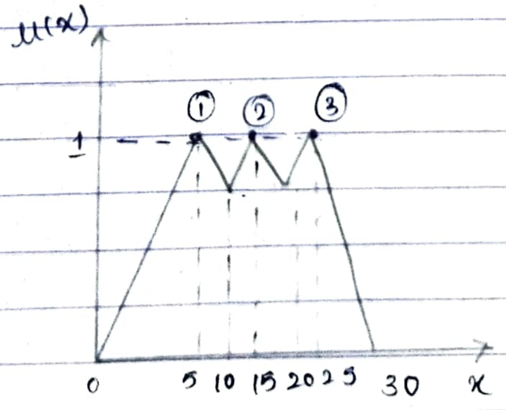


FOM = LOM = 15



FOM = 15

LOM = 20

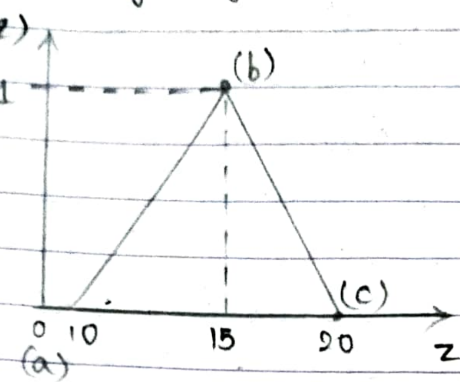


FOM = 5

LOM = 25

→ f) Centre of gravity (COG):

Ex 1:



for ab:

$y = mx + c$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{15-10} = \frac{1}{5} = 0.2$

Here, $y = 1, x = 15$

$1 = 0.2 \times 15 + c$

$1 = 3 + c$

$c = -2$

$\mu(z) = mz + c$

$\boxed{\mu(z_1) = 0.2z - 2}$ — (i)

for bc:

$y = mx + c$

here $y = 0, x = 20$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{20-15} = \frac{-1}{5} = -0.2$

$0 = -0.2 \times 20 + c$

$0 = -4 + c$

$\boxed{c = 4}$

$\mu(z) = mz + c$

$\boxed{\mu(z_2) = -0.2z + 4}$ — (ii)

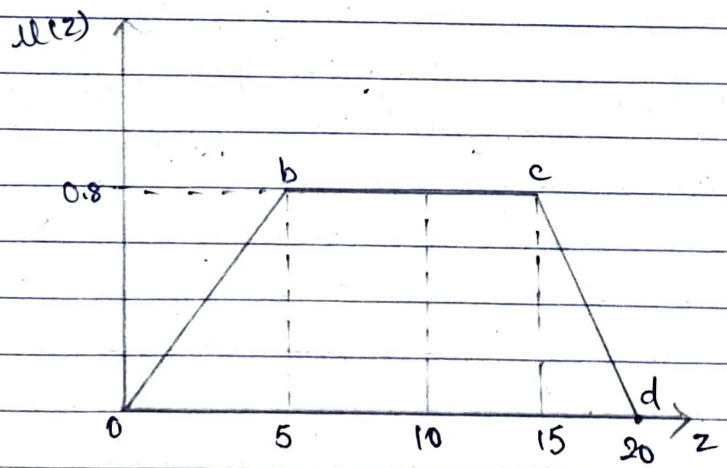
$$z^* = \frac{\int \mu(z)\cdot z\,dz}{\int \mu(z)\cdot dz} = \frac{\int_{10}^{20} \mu(z)\cdot z\,dz}{\int_{10}^{20} \mu(z)\,dz}$$

$$z^* = \frac{\int_{10}^{15} \mu(z_1)\,z\,dz + \int_{15}^{20} \mu(z_2)\cdot z\,dz}{\int_{10}^{15} \mu(z_1)\,dz + \int_{15}^{20} \mu(z_2)\cdot dz}$$

$$= \frac{\int_{10}^{15} (0.2z-2)\cdot z\,dz + \int_{15}^{20} (-0.2z+4)\,z\,dz}{\int_{10}^{15} (0.2z-2)\,dz + \int_{15}^{20} (-0.2z+4)\,dz}$$

$= \underline{\underline{15}}$ [Can get by substituting in calci]

Ex2:



FOM = 5
LOM = 15
~~W.A = 10~~
MOM = 10

COG: i) for ab : $y = mx + c$

$y = 0.8,\ x = 5$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.8 - 0}{5 - 0} = \underline{\underline{0.16}}$$

$0.8 = 0.16 \times 5 + C$

$\boxed{C = 0}$

$\mu(z) = mz + C$

$\boxed{\mu(z_1) = 0.16z}$ - (i)

(ii) for bc :

$\boxed{\mu(z_2) = 0.8}$ - (ii)

(iii) for cd:

$y = mx + C$

$y = 0, \ x = 20$

$m = \frac{0 - 0.8}{20 - 15} = \frac{-0.8}{5} = -0.16$

$0 = -0.16 \times 20 + C$

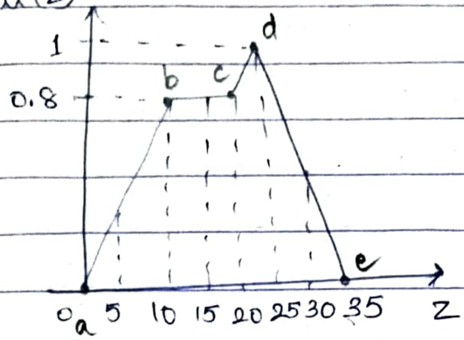
$\underline{\underline{C = 3.2}}$

$\boxed{\mu(z_3) = -0.16z + 3.2}$ - (iii)

$$z^* = \frac{\int_0^5 (0.16z) z\,dz \ \int_5^{15} 0.8z\,dz \ \int_{15}^{20} (-0.16z + 3.2) z\,dz}{\int_0^5 0.16z\,dz + \int_5^{15} 0.8\,dz + \int_{15}^{20} (-0.16z + 3.2)\,dz}$$

05/01/2022

Q3: $\mu(z)$



Find z^* with all the applicable methods:

(i) Height method:

$$z^* = 25$$

(ii) weighted avg. method

$$z^* = \frac{0.8 \times 15}{0.8} = \underline{\underline{15}}$$

(iii) Middle of max: $z^* = 15$

(iv) First of max = $z^* = 25$

(v) Last of max = $z^* = 25$

(vi) centre of gravity:

for ab:

$$y = mx + c$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.8 - 0}{10 - 0} = 0.08$$

Here $y = 0.8$, $x = 10$

$$0.8 = 0.08 \times 10 + c$$

$$\boxed{c = 0}$$

$$\boxed{\mu(z_1) = 0.08z} \quad - (i)$$

for bc: $\boxed{\mu(z_2) = 0.8}$ — (ii)

for cd:

$$y = mx + c$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0.8}{25 - 20} = 0.04$$

Here $y = 1$, $x = 25$

$$1 = 0.04 \times 25 + c$$

$$\boxed{c = 0}$$

$$\boxed{\mu(z_3) = 0.04z} \quad \text{— (iii)}$$

for de:

$$y = mx + c$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1}{35 - 25} = -0.1$$

Here $y = 0$, $x = 35$

$$0 = -0.1 \times 35 + c$$

$$c = 3.5$$

$$\therefore \boxed{\mu(z_4) = -0.1z + 3.5} \quad \text{— (IV)}$$

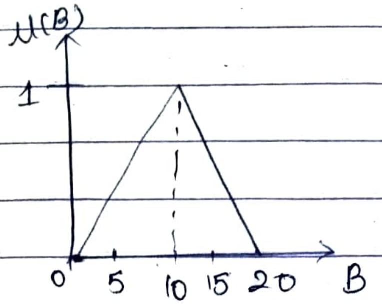
$$z^* = \frac{\sum \mu(z) z}{\sum \mu(z)}$$

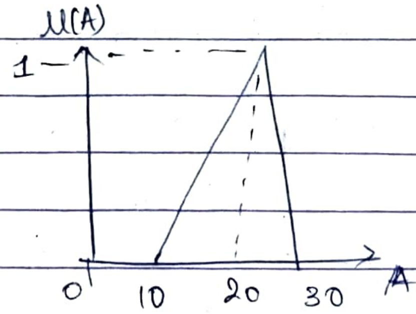
$$= \frac{\int_0^{10} (0.08z) z\, dz + \int_{10}^{20} (0.8) z\, dz + \int_{20}^{25} (0.04z) z\, dz + \int_{25}^{35} (-0.1z + 3.5) z\, dz}{\int_0^{10} 0.08z\, dz + \int_{10}^{20} 0.8\, dz + \int_{20}^{25} 0.04z\, dz + \int_{25}^{35} (-0.1z + 3.5)\, dz}$$

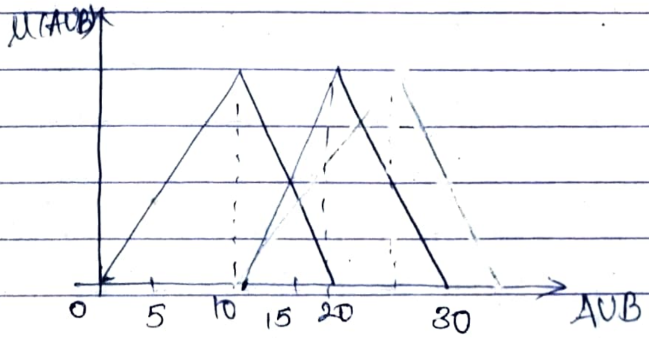
$$= \frac{26.6 + 120 + 101.6 + 141.6}{4 + 8 + 4.5 + 5}$$

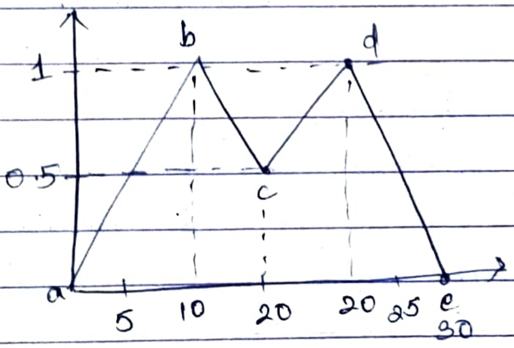
$$= \underline{\underline{18.13}}$$

Q4:









0.12+3.5)zdz

Height method, weighted avg. method, middle of max are not applicable

FOM = 10, LOM = 20

dz

COG:

for ab:

$y = mx + c$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{10-0} = 0.1$

Here, $y = 1$, $x = 10$

$1 = 0.1 \times 10 + c$

$\boxed{c = 0}$

$\therefore \boxed{\mu(z_1) = 0.1z}$ — (i)

for bc: $y = mx + c$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.5 - 1}{15 - 10} = \frac{-0.5}{5} = -0.1$

Here $y = 0.5$, $x = 15$

$0.5 = -0.1 \times 15 + c$

$\boxed{c = 2}$

$\boxed{\mu(z_2) = -0.1z + 2}$ — (ii)

for cd: $y = mx + c$

$m = \frac{1 - 0.5}{20 - 15} = 0.1$

Here $y = 1$, $x = 20$

$1 = 0.1 \times 20 + c$

$\boxed{c = -1}$

$\boxed{\mu(z_3) = 0.1z - 1}$ — (iii)

for de =

$$m = \frac{0-1}{30-20} = -0.1$$

Here $y = 0$, $x = 30$

$$0 = 30 \times -0.1 + C$$

$$0 = -3 + C$$

$$\boxed{C = 3}$$

$$\boxed{\mu(z_4) = -0.1z + 3} \quad \text{— (iv)}$$

$$z^* = \int_0^{10} (0.1z)\, z\,dz + \int_{10}^{15} z(-0.1z+2)\,dz + \int_{15}^{20} (0.1z-1)\,z\,dz + \int_{20}^{30} (-0.1z+3)\,z\,dz$$

$$\int_0^{10} (0.1z)\,dz + \int_{10}^{15} (-0.1z+2)\,dz + \int_{15}^{20} (0.1z-1)\,dz + \int_{20}^{30} (-0.1z+3)\,dz$$

$= 15$

12/01/22 UNIT 4 (Applications)

*) Washing Machine

i.) i/p and o/p :

i/p → 1.) Type of cloth (C) ⇒ [0 to 10]
2.) Dirt level (D) ⇒ [0 to 100%]
3.) weight of cloth (W) ⇒ (0 to 7kg)

o/p → i) water qty (wt) ⇒ [0 to 50lt]
ii) Time for washing (T) ⇒ [0 to 60 min]
iii) speed (N) ⇒ [0 to 1500rpm]

ii) Normalisation not needed.
iii) Fuzzification :- for simplicity we'll be selecting 2 i/p and 1 o/p.