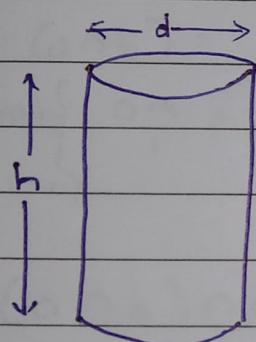
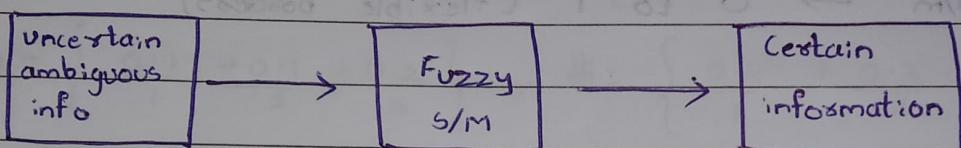
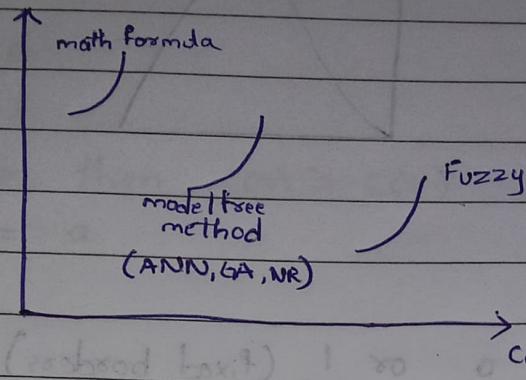
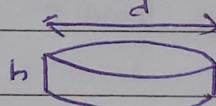


* Application Area

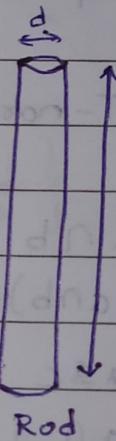
Precision



$$\frac{10}{12} = 0.8$$

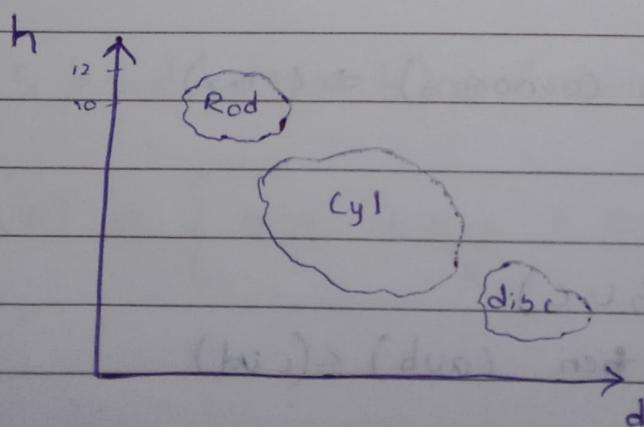


$$\frac{10}{2} = 5$$

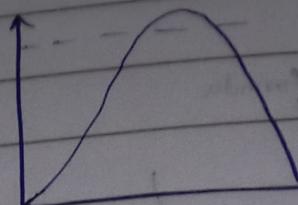
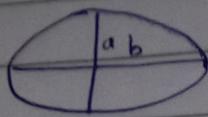


$$\frac{1}{16} = 0.1$$

$$d \approx h$$



* Ellipse



1) Classical Theory $\rightarrow 0 \text{ or } 1$ (fixed borders)

2) Fuzzy s/m $\rightarrow 0 \text{ to } 1$ (flexible borders)

* Properties of Fuzzy Set

1) T-norms (Triangular norms) $\Rightarrow (\min)$

1) $a \cap b = b \cap a$

2) $(a \cap b) \cap c = a \cap (b \cap c)$

3) $a \leq c \text{ & } b \leq d, (a \cap b) \leq (c \cap d)$

4) $a \cap 1 = a$

5) $a \cap 0 = 0$

2) S-Norms (Triangular co-norms) $\Rightarrow (\max)$

1) $a \cup b = b \cup a$

2) $(a \cup b) \cup c = a \cup (b \cup c)$

3) if $a \leq c \text{ & } b \leq d, \text{ then } (a \cup b) \leq (c \cup d)$

4) $a \cup 1 = 1$

5) $a \cup 0 = a$

3) C-norms (complimentary norms)

- 1) $C(1) = 0$
- 2) $C(0) = 1$
- 3) If $a < b$, then $C(a) > C(b)$
- 4) $C(C(a)) = a$

Ex. $A = \left\{ \frac{0.4}{1} + \frac{0.2}{2} + \frac{0.3}{3} \right\}$, $B = \left\{ \frac{0.5}{1} + \frac{0.6}{2} + \frac{0.2}{3} \right\}$
 $a = \left\{ \frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} \right\}$, $b = \left\{ \frac{0.4}{1} + \frac{0.3}{2} + \frac{0.1}{3} \right\}$

PROVE prop of fuzzy set

$$\Rightarrow (a \cap b) = (b \cap a)$$

$$(a \cap b) = \left\{ \underbrace{\frac{0.4}{1} + \frac{0.2}{2} + \frac{0.2}{3}}_{\text{min}} \right\} = (b \cap a) = \left\{ \frac{0.4}{1} + \frac{0.2}{2} + \frac{0.2}{3} \right\}$$

$$(a \cap b) \cap c = a \cap (b \cap c)$$

$$\left\{ \frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} \right\} = \left\{ \frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} \right\}$$

$$a \leq c \& b \leq d, (a \cap b) \leq (c \cap d)$$

$$(a \cap b) \leq \left\{ \frac{0.4}{1} + \frac{0.2}{2} + \frac{0.2}{3} \right\}$$

$$A/B = A \cap \bar{B}$$

$$B/A = B \cap \bar{A}$$

$$(A \cup B) = (\bar{A} \cap \bar{B})$$

2) $a = \left\{ \frac{0.6}{x_1} + \frac{0.4}{x_2} + \frac{0.6}{x_3} + \frac{0.5}{x_5} \right\} \in X$

$$b = \left\{ \frac{0.3}{x_1} + \frac{0.2}{x_3} + \frac{0.4}{x_4} + \frac{1}{x_5} \right\} \in X$$

$$X = [x_1, x_2, x_3, x_4, x_5]$$

Find $A \cap B$, $A \cup \bar{B}$, $\bar{A} \cap B$, A/B , B/A , demorgans law

$$A \cap B = \left\{ \frac{0.3}{x_1} + \frac{0}{x_2} + \frac{0.2}{x_3} + \frac{0}{x_4} + \frac{0.5}{x_5} \right\}$$

$$\bar{B} = \left\{ \frac{0.7}{x_1} + \frac{1}{x_2} + \frac{0.8}{x_3} + \frac{0.6}{x_4} + \frac{0}{x_5} \right\}$$

$$A \cup \bar{B} = \left\{ \frac{0.6}{x_1} + \frac{0.4}{x_2} + \frac{0.6}{x_3} + \frac{0.4}{x_4} + \frac{1}{x_5} \right\}$$

$$A \cup \bar{B} = \left\{ \frac{0.7}{x_1} + \frac{1}{x_2} + \frac{0.8}{x_3} + \frac{0.6}{x_4} + \frac{0.5}{x_5} \right\}$$

$$\bar{A} \cap B = \left\{ \frac{0.3}{x_1} + \frac{0}{x_2} + \frac{0.2}{x_3} + \frac{0.4}{x_4} + \frac{0.5}{x_5} \right\}$$

$$\bar{A} = \left\{ \frac{0.4}{x_1} + \frac{0.6}{x_2} + \frac{0.4}{x_3} + \frac{1}{x_4} + \frac{0.5}{x_5} \right\}$$

* Zadch Implication

T-norm $\Rightarrow \min(a, b)$

S-norm $\Rightarrow \max(a, b)$

C-norm $\Rightarrow (1-a), (1-b)$

* Younger's Family Implication

T-norms

$$Y_q(a, b) = 1 - \min\left(1, ((1-a)^q + (1-b)^q)\right)$$

Assume $q=1$

$$A = \left\{ \frac{0}{1}, \frac{0.2}{2}, \frac{0.6}{3}, \frac{1}{4} \right\} \in \alpha$$

$$B = \left\{ \frac{0.9}{1}, \frac{0.2}{2}, \frac{1}{3}, \frac{0.7}{4} \right\} \in \alpha$$

Find $A \cup B$, $A \cap B$ using Zadch, Younger's

$$A \cap B = \left\{ \frac{0}{1}, \frac{0.2}{2}, \frac{0.6}{3}, \frac{0.7}{4} \right\}$$

$$\begin{aligned} Y(1, 1) &= 1 - \min(1, ((1-0)^1 + (1-0)^1)) \\ &= 1 - \min(1, 1 \cdot 1) \\ &= 0 \end{aligned}$$

$$\begin{aligned} Y(2, 2) &= 1 - \min(1, ((1-0.2)^1 + (1-0.2)^1)) \\ &= 1 - \min(1, 1 \cdot 6) \\ &= 0 \end{aligned}$$

$$\begin{aligned} Y(3, 3) &= 1 - \min(1, ((1-0.6)^1 + (1-1)^1)) \\ &= 1 - \min(1, 0 \cdot 4) \\ &= 0.6 \end{aligned}$$

$$\begin{aligned}
 y(4,4) &= 1 - \min(1, ((1-1)^1 + (1-0.7)^1)) \\
 &= 1 - \min(1, 0.3) \\
 &= 0.7
 \end{aligned}$$

$$A \cap B = \left\{ \frac{0}{1} + \frac{0}{2} + \frac{0.6}{3} + \frac{0.7}{4} \right\}$$

* Dubois & Prordan Implementation

$$T_\alpha(a, b) = \frac{a \cdot b}{\max(a, b, \alpha)} \quad \text{assume } \alpha \neq 1$$

$$A = \left\{ \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.9}{3} + \frac{1}{4} \right\} \in \mathbb{X}$$

$$B = \left\{ \frac{0.9}{1} + \frac{0.5}{2} + \frac{0.1}{3} + \frac{0.7}{4} \right\} \in \mathbb{X}$$

$$\overline{\alpha}(1,1) = \frac{1.8}{\max(0.2, 0.9, 1)} = 1.8$$

$$\overline{\alpha}(2,2) = \frac{0.2}{\max(0.4, 0.5, 1)} = 0.2$$

$$\overline{\alpha}(3,3) = \frac{0.9}{\max(0.9, 0.1, 1)} = 0.9$$

$$\overline{\alpha}(4,4) = \frac{0.7}{\max(1, 0.7, 1)} = 0.7$$

$$A \cup B = \left\{ \frac{0.1 \cdot 1.8}{1} + \frac{2}{2} + \frac{0.9}{3} + \frac{0.7}{4} \right\}$$

Zadeh method:

$$A \cup B = \left\{ \frac{0.1}{1} + \frac{0.5}{2} + \frac{0.9}{3} + \frac{1}{4} \right\}$$

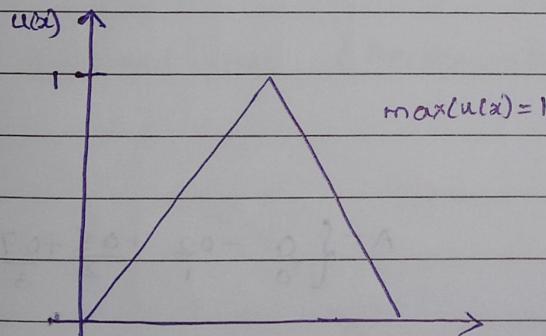
Yager's method:

$$\begin{aligned} Y_1(1,1) &= 1 - \min(1, (1-0.2)' + (1-0.9)') \\ &= 1 - \min(1, 0.9) \\ &= 0.1 \end{aligned}$$

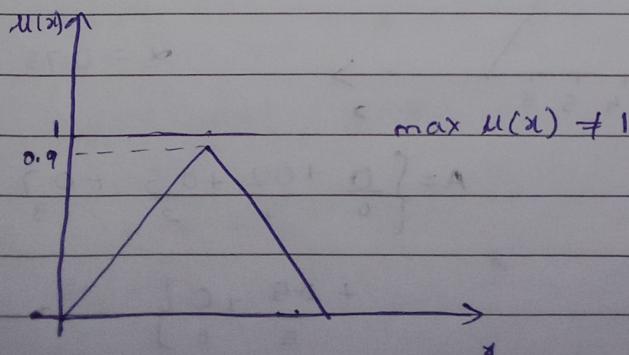
$$\begin{aligned} Y(2,2) &= 1 - \min(1, (1-0.4)' + (1-0.5)') \\ &= 1 - \min(1, 0.9) \\ &= 0.1 \end{aligned}$$

* Types of fuzzy function

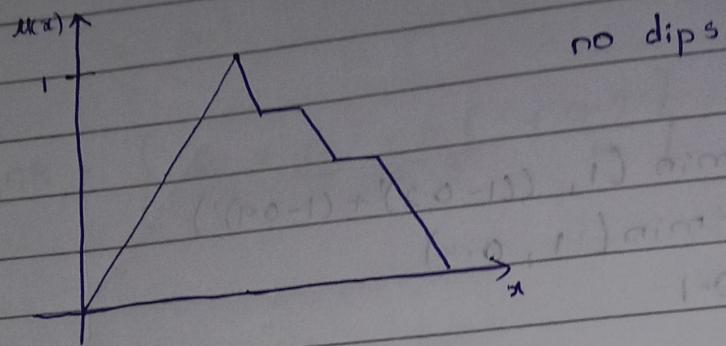
1) Normal fuzzy set



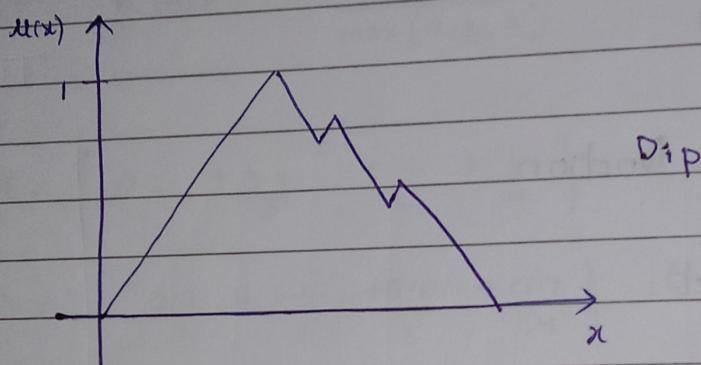
2) Sub-normal fuzzy set



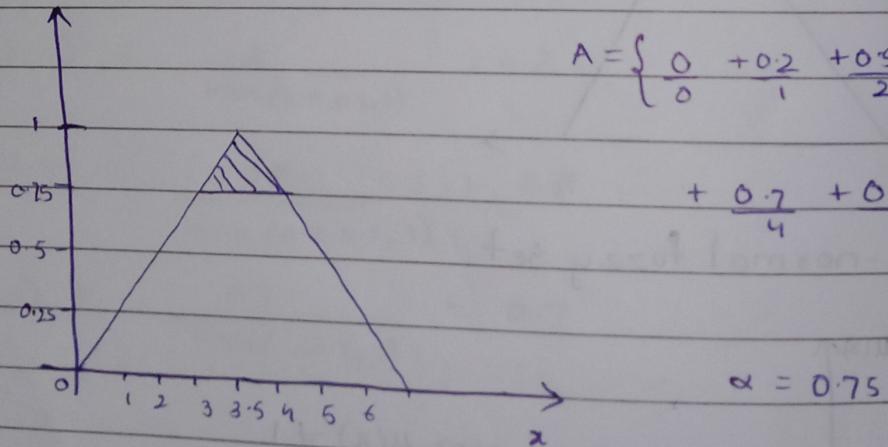
3) Convex fuzzy set



4) Non-convex fuzzy set



* λ -cut or α -cut

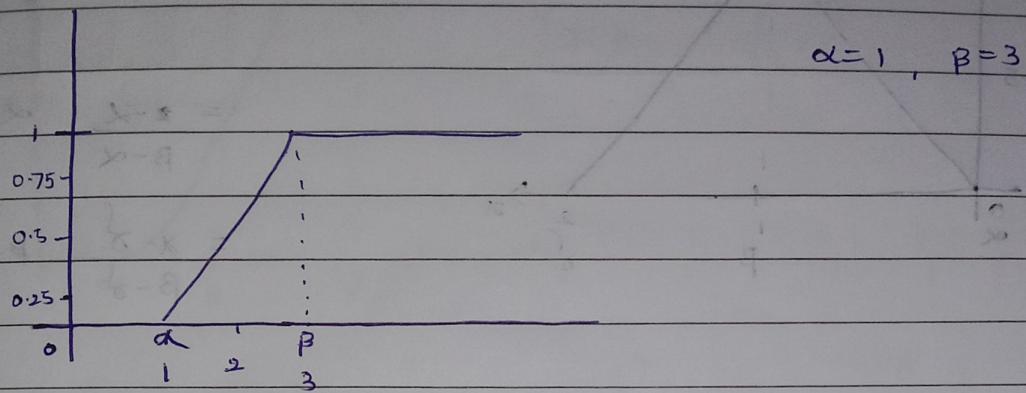


$$A = \left\{ \frac{0}{0} \frac{+0.2}{1} \frac{+0.5}{2} \frac{+0.7}{3} \frac{+0.7}{3.5} \frac{+0.7}{4} \right.$$

$$\left. + \frac{0.5}{5} \frac{+0.2}{6} \right\}$$

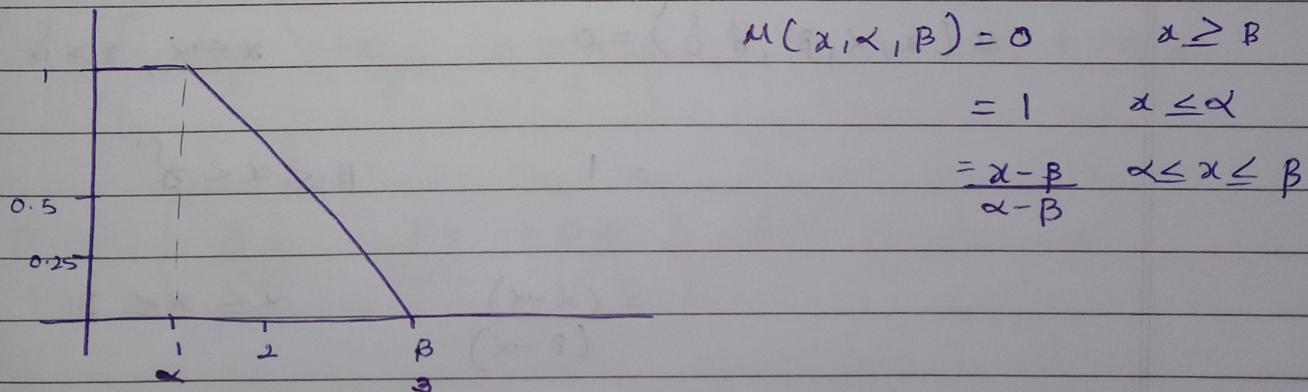
* Shapes of membership functⁿ

1) T- Function (Incremental fⁿ)



$$\begin{aligned}\Gamma(x, \alpha, \beta) &= 0 & x \leq \alpha \\ &= 1 & x \geq \beta \\ &= \frac{(x - \alpha)}{(\beta - \alpha)} & \alpha \leq x \leq \beta\end{aligned}$$

2) L- function : (Decremental)

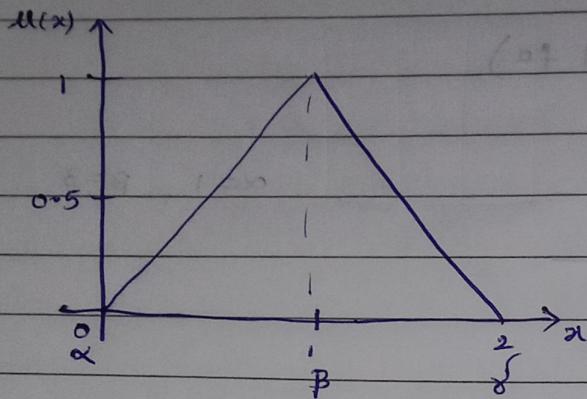


Let $\alpha = 1, \beta = 3$

$$x = \alpha \Rightarrow 1$$

$$x = \beta \Rightarrow 0$$

$$x = 2 \Rightarrow 0.5$$

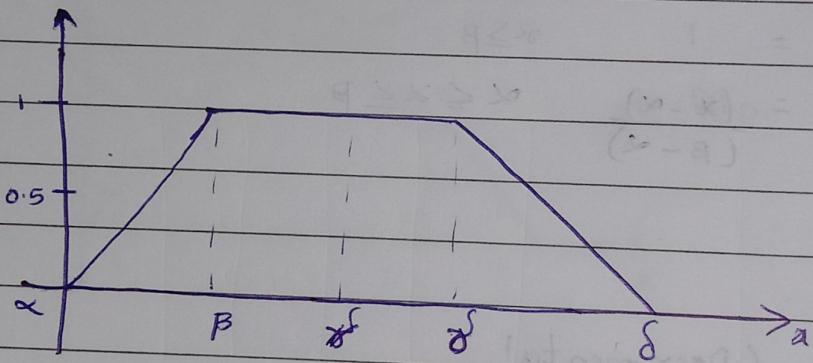
3) Λ -function

$$u(x, \alpha, \beta, \delta) = 0 \quad x \leq \alpha, x \geq \delta$$

$$= 1 \quad x = \beta$$

$$= \frac{x - \alpha}{\beta - \alpha} \quad \alpha \leq x \leq \beta$$

$$= \frac{\alpha - \delta}{\beta - \delta} \quad \beta \leq x \leq \delta$$

4) Π -function

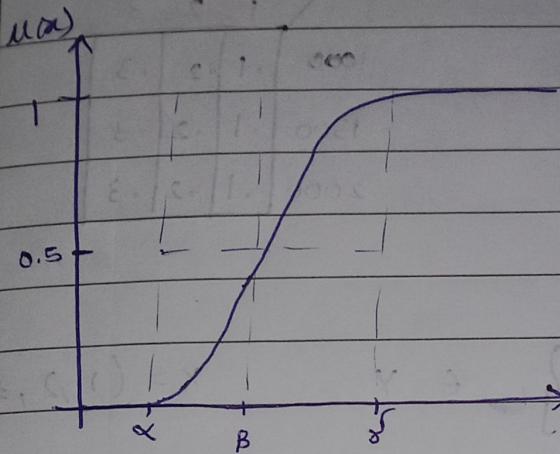
$$u(x, \alpha, \beta, \gamma, \delta) = 0 \quad x \leq \alpha, x \geq \delta$$

$$= 1 \quad \beta \leq x \leq \gamma$$

$$= \frac{(\alpha - x)}{(\beta - \alpha)} \quad \alpha \leq x \leq \beta$$

$$= \frac{(x - \delta)}{(\gamma - \delta)} \quad \gamma \leq x \leq \delta$$

5) S-function



$$u(x, \alpha, \beta, \gamma) = \begin{cases} 0 & x \leq \alpha \\ 1 & x \geq \gamma \end{cases}$$

$$= 2 \left(\frac{x - \alpha}{\gamma - \alpha} \right)^2 \quad \alpha \leq x \leq \beta$$

$$= 1 - 2 \left(\frac{x - \beta}{\gamma - \beta} \right) \quad \beta \leq x \leq \gamma$$

* 1) $A = \left\{ \frac{0.1}{1} + \frac{0.2}{2} + \frac{0.3}{3} \right\} \in \alpha$ Find $A \cup B$

$$B = \left\{ \frac{0.6}{1000} + \frac{0.4}{1500} + \frac{0.6}{2000} \right\} \in \gamma$$

\rightarrow $A \cup B$
(max)

	1	2	3
1000	.6	.6	.6
1500	.4	.4	.4
2000	.6	.6	.6

 $A \cap B$

	1	2	3
1000	.1	.2	.3
1500	.1	.2	.3
2000	.1	.2	.3

$$x = (1, 2, 3, 4)$$

$$A = \left\{ \frac{0.1}{1} + \frac{0.6}{2} + \frac{0.4}{4} \right\} \in X$$

$$B = \left\{ \frac{0.3}{1} + \frac{0.9}{2} + \frac{0.8}{4} \right\} \in Y$$

$$y = (1, 2, 3, 4)$$

 $A \cup B$

	1	2	3	4
1	0.3	0.6	0.3	0.4
2	0.9	0.9	0.9	0.9
3	0.1	0.6	0	0.4
4	0.8	0.8	0.8	0.8

 $A \cap B$

	1	2	3	4
1	0.1	0.3	0	0.3
2	0.1	0.6	0	0.4
3	0	0	0	0
4	0.1	0.6	0	0.4

You are asked to select & implement a technology for a numerical process. Computation throughput is directly related to clock speed assume that all implementations will be in the same family (CMOS). You are considering whether the design should be implemented using medium scale integration (MSI) with discrete parts, field programmable array (FPGA) or multichip module (MCM). A universe of potential clock frequency is $X = [1, 10, 20, 40, 80, 100] \text{ MHz}$ and define MSI, FPGA, MCM as fuzzy sets of clock frequency that should be implemented with each of these technologies.

Table I defines the membership values of the above fuzzy sets

Representing 3 fuzzy sets as $M_{SI} = M$, $F_{PGA} = F$, $M_{CM} = C$
 Find the following, ~~MUF~~, M_{NF} , \bar{F} , $C \cap \bar{F}$, $\overline{M_{NC}}$, M/F

MUF	M	F	C
check freq (MHz)	MIS	FPGA	MCM
1	1	0.3	0
10	0.9	1	0
20	0.4	1	0.5
40	0	0.5	0.7
80	0	0.2	1
100	0	0	1

$$\rightarrow M = \left\{ \frac{1}{1}, \frac{0.9}{10}, \frac{0.4}{20}, \frac{0}{40}, \frac{0}{80}, \frac{0}{100} \right\}$$

$$F = \left\{ \frac{0.3}{1}, \frac{1}{10}, \frac{1}{20}, \frac{0.5}{40}, \frac{0.2}{80}, \frac{0}{100} \right\}$$

$$C = \left\{ \frac{0}{1}, \frac{0}{10}, \frac{0.5}{20}, \frac{0.7}{40}, \frac{1}{80}, \frac{1}{100} \right\}$$

$$MUF = \left\{ \frac{1}{1}, \frac{0.9}{10}, \frac{1}{20}, \frac{0.5}{40}, \frac{0.2}{80}, \frac{0}{100} \right\}$$

$$M_{NF} = \left\{ \frac{0.3}{1}, \frac{0.7}{10}, \frac{0.4}{20}, \frac{0}{40}, \frac{0}{80}, \frac{0}{100} \right\}$$

$$\bar{F} = \left\{ \frac{0.7}{1}, \frac{0}{10}, \frac{0}{20}, \frac{0.5}{40}, \frac{0.8}{80}, \frac{1}{100} \right\}$$

$$C \cap \bar{F} = \left\{ \frac{0}{1}, \frac{0}{10}, \frac{0}{20}, \frac{0.5}{40}, \frac{0.8}{80}, \frac{1}{100} \right\}$$

$$M_{NC} = \left\{ \frac{0}{1}, \frac{0}{10}, \frac{0.4}{20}, \frac{0}{40}, \frac{0}{80}, \frac{0}{100} \right\}$$

$$\overline{M_{NC}} = \left\{ \frac{1}{1}, \frac{1}{10}, \frac{0.6}{20}, \frac{1}{40}, \frac{1}{80}, \frac{1}{100} \right\}$$

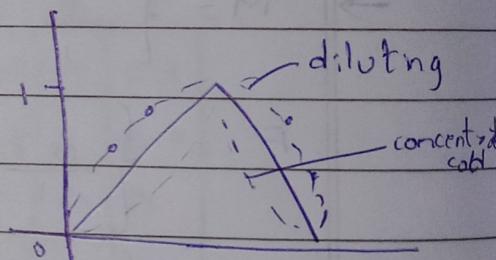
* linguistic variables:
are the values in terms of words or statement
ex. very small, large, good,

* linguistic hedges:
These are adjective associated with linguistic variables in order to concentrate or dilute the system
Very Verygood Slightly

$$1) \text{ Very } = \alpha = \int_0^1 \left[\frac{(u(x))^2}{x} \right]$$

$$2) \text{ Very Very } = \alpha = \int_0^1 \left[\frac{(u(x))^4}{x} \right]$$

$$3) \text{ Plus } = \alpha = \int_0^1 \left[\frac{(u(x))^{1.5}}{x} \right]$$



$$4) \text{ Slightly } = \alpha = \int_0^1 \left[\frac{(u(x))^{0.5}}{x} \right]$$

$$5) \text{ minus } = \alpha = \int_0^1 \left[\frac{(u(x))^{0.75}}{x} \right]$$

1, 2, 3 are used concentrate & 4, 5 to dilute

$$\text{Small} = \left\{ \frac{1}{1} + \frac{0.8}{2} + \frac{0.5}{3} + \frac{0.2}{4} \right\} \in X$$

$$\text{large} = \left\{ \frac{0.1}{1} + \frac{0.3}{2} + \frac{0.8}{3} + \frac{1}{4} \right\} \in X$$

Find: Very Very Small and Not very large $\approx \text{AVB}$

$$A \cap B = \left\{ \frac{0.1}{1} + \frac{0.3}{2} + \frac{0.5}{3} + \frac{0.2}{4} \right\}$$

$$\text{VVS} = \left\{ \frac{0.1}{1} + \underline{0.4096} + \underline{0.0625} + \underline{0.0016} \right\}$$

$$\text{VL} = \left\{ \frac{0.01}{1} + \frac{0.09}{2} + \frac{0.64}{3} + \frac{1}{4} \right\}$$

$$\text{NVL} = \left\{ \frac{0.99}{1} + \frac{0.91}{2} + \frac{0.36}{3} + \frac{0}{4} \right\}$$

$$= \left\{ \frac{0.99}{1} + \frac{0.4096}{2} + \frac{0.0625}{3} + \frac{0}{4} \right\}$$

$$* \text{Cold} = \left\{ \frac{0.1}{10} + \frac{0.2}{20} + \frac{0.6}{30} \right\} \in X$$

$$\text{Hot} = \left\{ \frac{0.4}{10} + \frac{0.1}{20} + \frac{1}{30} \right\} \in X$$

Find "Not very cold or slightly Hot"

$$\text{VL} = \left\{ \frac{0.01}{10} + \frac{0.04}{20} + \frac{0.36}{30} \right\} \quad \text{SH} = \left\{ \frac{0.632}{10} + \frac{0.316}{20} + \frac{1}{30} \right\}$$

$$\text{NVL} = \left\{ \frac{0.99}{10} + \frac{0.96}{20} + \frac{0.64}{30} \right\}$$

$$\text{NVL SH} = \left\{ \frac{0.99}{10} + \frac{0.96}{20} + \frac{1}{30} \right\}$$

EX
EY

	10	20	30
10	0.99	0.99	0.1
20	0.96	0.96	0.06
30	0.64	0.64	1

* Cylindrical Extension:

* Projection: Projection operation converts a relation to a fuzzy set or fuzzy set to a single crisp value

$$R = \begin{matrix} & y_1 & y_2 & y_3 \\ x_1 & [0.1 & 0.2 & 0.4] \\ x_2 & [0.5 & 0.2 & 0.6] \\ x_3 & [0.8 & 0.3 & 0.1] \end{matrix}$$

Proj on x domain

$$\text{Max } [(x_1, y_1) (x_2, y_2) (x_3, y_3)]$$

$$\text{Max } [0.1 \quad 0.2 \quad 0.4]$$

$$A = \left\{ \frac{0.4}{x_1} + \frac{0.6}{x_2} + \frac{1}{x_3} \right\} = \text{tall}$$

$$B = \left\{ \frac{0.8}{y_1} + \frac{0.3}{y_2} + \frac{1}{y_3} \right\}_{0.0}^{1.0} = 1V$$

$$\left\{ \frac{0.9}{0.8} + \frac{0.9}{0.8} + \frac{0.9}{0.1} \right\} = 1V$$

$$\left\{ \frac{1}{0.8} + \frac{2P}{0.8} + \frac{PP}{0.1} \right\} = 1V$$

cylindrical extension is used to convert single crisp value into a fuzzy or a fuzzy set into a relation

$$A = \left\{ \frac{0.1}{x_1} + \frac{0.2}{x_2} + \frac{0.3}{x_3} \right\} \in X$$

$$Y = \{y_1, y_2, y_3, y_4\}$$

A on domain x

	y_1	y_2	y_3	y_4			
x_1	0.1	0.1	0.1	0.1	0	0.01	0.02
x_2	0.2	0.2	0.2	0.2	0	0.02	0.03
x_3	0.3	0.3	0.3	0.3	0	0.03	0.04

* composition

1) Max-Min composition

2) Max-product "

3) Min-max "

4) Max-max "

(5) Min-min ("join") $(x_1, y_1) \text{ and } (x_2, y_2) \rightarrow (0.001, 0.01)$
 $(1.0, 1.0, 1.0, 1.0) \times 0.01 =$

$$1) A = \left\{ \frac{0.1}{10} + \frac{0.2}{20} + \frac{0.3}{30} \right\} \in X$$

$$B = \left\{ \frac{0.6}{100} + \frac{0.5}{200} + \frac{0.6}{300} + \frac{0.4}{500} \right\} \in Y$$

$$C = \left\{ \frac{0}{1000} + \frac{0.5}{1500} + \frac{1}{2000} \right\} \in Z$$

* Cartesian Product: $(X) \rightarrow (min)$

$$\underset{(x)}{A \times B} = A \cap B$$

$$T = R \cdot S$$

$$R = A \times B$$

$$S = B \times C$$

	100	200	300	400	500
10	0.1	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2	0.2
30	0.3	0.3	0.3	0.3	0.3

	1000	1500	2000
100	0	0.5	0.6
200	0	0.5	0.5
300	0	0.5	0.6
500	0	0.4	0.4

Find $T = R \cdot S$ using max min composition

	1000	1500	2000
100	0	0.1	0.1
200	0	0.2	0.2
300	0	0.3	0.3
500	0	0.1	0.1

$$R \cdot S$$

$$A \otimes B \quad B \otimes C$$

$$\cancel{A \otimes C}$$

$$\begin{aligned}
 T(10, 1500) &= \max(\min(0.1, 0.5), \min(0.1, 0.5), \min(0.1, 0.5)) \\
 &= \max(0.1, 0.1, 0.1, 0.1) \\
 &= 0.1 \times \{0.1 + 0.5 + 0.1\} = A
 \end{aligned}$$

$$\begin{aligned}
 T(20, 1000) &\Rightarrow \max(\min(0.2, 0)) \\
 &= \max(0.2, 0.0, 0.0, 0.0) = 0 \\
 T(20, 1500) &= \max(\min(0.2, 0.5))
 \end{aligned}$$

$$2 \cdot 0 = 0$$

$$2 \cdot 0 = 0$$

$$2 \cdot 0 = 0$$

(since $x < 0$: Huber activation)

$$80A - 8KA$$

$$1) A = \left\{ \frac{0.6}{x_1} + \frac{0.5}{x_2} + \frac{0.8}{x_3} + \frac{0.2}{x_5} \right\} \text{ G } x$$

$$B = \left\{ \frac{1}{y_1} + \frac{2}{y_2} + \frac{3}{y_3} \right\} \text{ G } y$$

$$C = \left\{ \frac{0.8}{z_1} + \frac{1}{z_2} + \frac{0.3}{z_3} + \frac{1}{z_5} \right\} \text{ G } z$$

where $x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]$

$$Y = [y_1 \ y_2 \ y_3] \quad (3 \times 3) \text{ matrix} \quad X^T = [1 \ 2 \ 3 \ 4 \ 5]$$

$$Z = [z_1 \ z_2 \ z_3 \ z_4 \ z_5] \quad (5 \times 5) \text{ matrix}$$

find $T = R \cdot S$ using max min composition:

$$R = A \times B$$

$$S = B \times C$$

$R = A \times B$	y_1	y_2	y_3
x_1	0.8	0.6	0.6
x_2	0.5	0.5	0.5
x_3	0.8	0.8	0.8
x_4	0	0	0
x_5	0.2	0.2	0.2

$S = B \times C$	z_1	z_2	z_3	z_4	z_5
y_1	0.8	1	0.3	0	1
y_2	0.8	1	0.3	0	1
y_3	0.8	1	0.3	0	1

$$R \cdot S$$

$$(A \times B) \quad (B \times C)$$

$$T = R \cdot S$$

$$= (A \times B) \quad (B \times C)$$

$$= A \times C$$

$T =$	z_1	z_2	z_3	z_4	z_5
x_1	0.6	0.6	0.3	0	0.6
x_2	0.5	0.5	0.3	0	0.5
x_3	0.8	0.8	0.3	0	0.8
x_4	0	0	0	0	0
x_5	0.2	0.2	0.2	0	0.2

$$(x_1, z_1) = \max(\min(0.6, 0.8), \min(0.6, 0.8), \min(0.6, 0.8))$$

$$= \max(0.6, 0.6, 0.6)$$

$$= 0.6$$

$\therefore A = R$

E	d	M	R
6.0	6.0	6.0	6.0
2.0	2.0	2.0	2.0
8.0	8.0	8.0	8.0
0	0	0	0
5.0	5.0	5.0	5.0

E	d	M	S	R
1	0	8.0	1	8.0
1	0	6.0	1	8.0
1	0	6.0	1	8.0

$$(3 \times 8) (8 \times 1) = 24$$

and has staff, including a di "bagus keo erahmet"

hydrolytic and hydroxyl transferases

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en bâtonnage et — tout au pâté n°

$$\text{es ist } (A \times A) \cup (A \times A) = A$$

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3 min solo 811 modt Aax II (II)

so botanique est le passé et

$$(3 \times \bar{A}) \cup (8 \times A) = 2$$

14. Tropidurus reticulatus as well as also A. sius exed

sep. taurideos del año

begin for each figure we will have one section of the file.

* Rules

Theory of approximate Reasoning

Rules in Fuzzy

i) If — Then —

If x is A then y is B

where A & B are fuzzy sets on universe x & y respectively.
ex. If tomatoes are red then tomatoes are ripe
here "tomatoes are red" is an antecedent whereas
"tomatoes are ripe" is a consequent. There can't be a
consequent without an antecedent.

ex. If ~~motor speed is low~~ ^{anticident} then ~~increase the voltage~~ ^{consequent}

In fuzzy If — then — is represented as

$$R = (A \times B) \cup (\bar{A} \times C) \text{ where}$$

\downarrow
Cart. Product

Universe

ii) If x is A then y is B else y is C.

In fuzzy it is represented as

$$R = (A \times B) \cup (\bar{A} \times C)$$

here x is A acts as an antecedent for consequent y is B
also for consequent y is C

ex. If tomatoes are red they are ripe else not ripe

$$A = \left\{ \frac{0}{1} + \frac{1}{2} + \frac{1}{3} + \frac{0}{4} \right\} \in X$$

$$B = \left\{ \frac{0}{1} + \frac{0}{2} + \frac{1}{3} + \frac{1}{4} + \frac{0}{5} + \frac{0}{6} \right\} \in Y$$

Find if x is A then y is B

$$\text{where } x = [1, 2, 3, 4]$$

$$y = [1, 2, 3, 4, 5, 6]$$

$$\rightarrow y = \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \right\}$$

$A \times B$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	0	0	0	0	0	0
x_2	0	0	1	1	0	0
x_3	0	0	1	1	0	0
x_4	0	0	0	0	0	0

$\bar{A} \times Y$

	1	2	3	4	5	6
1	1	1	1	1	1	1
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	1	1	1	1	1	1

$(A \times B) \cup (\bar{A} \times Y)$

1	1	1	1	1	1	1
0	0	1	1	0	0	0
0	0	1	1	0	0	0
1	1	1	1	1	1	1

if μ and λ are 1, then

$$[\mu \otimes, \lambda \otimes] = \lambda \otimes \mu$$

$$[\delta \otimes, \lambda \otimes] = \lambda \otimes$$

$$\{f_1 \otimes f_2 + f_3 \otimes f_4\} = k \otimes$$

$$f_1 \otimes f_2 + f_3 \otimes f_4 \in \mathcal{A} \otimes$$

$$0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1$$

$$(\mathcal{A} \otimes) \cup (\mathcal{B} \otimes)$$

$$1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1$$

→ $X = [1, 2, 3, 4] \Rightarrow$ uniqueness
 $Y = [1, 2, 3, 4, 5, 6] \Rightarrow$ Market size

$$A = \text{Medium uniqueness} = \left\{ \frac{0.6}{2} + \frac{1}{3} + \frac{0.2}{4} \right\}$$

$$B = \text{Medium market size} = \left\{ \frac{0.4}{2} + \frac{1}{3} + \frac{0.8}{4} + \frac{0.3}{5} \right\}$$

$$C = \text{diff used market sizes} = \left\{ \frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.6}{4} + \frac{0.5}{5} + \frac{0.3}{6} \right\}$$

$$\rightarrow R = (A \times B) \cup (\bar{A} \times Y)$$

	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0.4	0.6	0.6	0.3	0
3	0	0.4	1.08	0.3	0	0
4	0	0.2	0.2	0.2	0.2	0

$$\bar{A} = \left\{ \frac{1}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.6}{4} \right\} \quad Y = \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \right\}$$

	1	2	3	4	5	6
1	1	1	1	1	1	1
2	0.4	0.4	0.4	0.4	0.4	0.4
3	0	0	0	0	0	0
4	0.8	0.8	0.8	0.8	0.8	0.8

	1	2	3	4	5	6
1	1	1	1	1	1	1
2	0.4	0.4	0.6	0.6	0.4	0.4
3	0	0.4	1	0.8	0.3	0
4	0.8	0.8	0.8	0.8	0.8	0.8

also find the logical conditional implication in the compound form i.e "If medium uniqueness then medium market size else defused market size." If A then B else C.

- a. continuing with problem no 1, suppose that the fuzzy relation just develop i.e R describe the inventions commercial potential. We wish to know what market size to be associated with the uniqueness score of "almost high uniqueness", i.e with the new antecedent A' find the consequent B' using max-min composition.

$$A' = \left\{ \frac{0.5}{1} + \frac{1}{2} + \frac{0.3}{3} + \frac{0}{4} \right\}$$

$$B' = A' \cdot R$$

$$= [0.5 \ 1 \ 0.3 \ 0] \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0.4 & 0.4 & 0.6 & 0.6 & 0.4 & 0.4 \\ 0 & 0.4 & 1 & 0.8 & 0.3 & 0 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{bmatrix}$$

$$= \left[\frac{0.5}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.6}{4} + \frac{0.5}{5} + \frac{0.5}{6} \right]$$

$$Q \quad w = \text{weak stimulus} = \left\{ \frac{1}{0} + \frac{0.9}{50} + \frac{0.3}{100} + \frac{0.01}{150} \right\}_{200} c \times$$

$$M = \{\text{medium stimulus}\} = \left\{ \frac{0}{0} + \frac{0.4}{50} + \frac{1}{100} + \frac{0.4}{150} + \frac{0}{200} \right\}_{200} c \times$$

$$S = \text{Severe Response} = \left\{ \frac{0}{0} + \frac{1}{50} + \frac{0.5}{100} + \frac{0.9}{150} + \frac{1}{200} \right\}_{200} c \times$$

Find "if weak stimulus then Not severe response"

$$\rightarrow R = (w \times \bar{s}) \cup (\bar{w} \times y)$$

$$\bar{s} = \left\{ \frac{1}{0} + \frac{1}{50} + \frac{0.5}{100} + \frac{0.1}{150} + \frac{0}{200} \right\}_{200}$$

$w \times \bar{s}$	0	50	100	150	200
0	1	1	0.5	0.1	0
50	0.9	0.9	0.5	0.1	0
100	0.3	0.3	0.3	0.1	0
150	0	0	0	0	0
200	0	0	0	0	0

$\bar{w} \times y$	0	50	100	150	200
0	0	0	0	0	0
50	0.1	0.1	0.1	0.1	0.1
100	0.7	0.7	0.7	0.7	0.7
150	1	1	1	1	1
200	1	1	1	1	1

$$(w \times \bar{s}) \cup (\bar{w} \times y) =$$

	0	50	100	150	200
0	1	1	0.5	0.1	0
50	0.9	0.9	0.5	0.1	0.1
100	0.7	0.7	0.7	0.7	0.7
150	1	1	1	1	1
200	1	1	1	1	1

Q.) Now using the new antecedent for the input $M = \text{medium stimulus}$. Find the consequent on the universe of to relate the approximately to the new stimulus M using max-min composition, i.e. Find $M \cdot R$

$$\rightarrow [0 \ 0 \ 0 \ 1 \ 0.4 \ 0] : \begin{bmatrix} 1 & 0.5 & 0.1 & 0 \\ 0.9 & 0.4 & 0.5 & 0.1 & 0.1 \\ 0.7 & 0.7 & 0.7 & 0.7 & 0.7 \\ 1 & 1 & 1 & 1 & 1 \\ 0 + 10 & 12 & 11 & 11 & 11 \end{bmatrix}$$

$$= \left[\frac{0.7}{0} + \frac{0.7}{50} + \frac{0.7}{100} + \frac{0.7}{150} + \frac{0.7}{200} \right]$$

Q. Suppose we have distillation process where objective is to separate the components of mixture in the input Scheme, the process is pictured in the fig. The relation b/w the input variable, temp, output variable, distillation fractions is not precise but human operator of this process has developed an intuitive understanding of the Relation. The universe of each of variable is $X = [\text{temp} = \{160, 165, 170, 175, 180, 185, 190, 195\}]$

$$Y = \text{distillation fraction} = \{77, 80, 83, 86, 89, 92, 95, 98\}$$

$$A = \text{temp of i/p stem} = \left\{ \frac{0}{175} + \frac{0.7}{180} + \frac{1}{185} + \frac{0.4}{190} \right\}$$

$$B = \text{separation of mixture is good} = \left\{ \frac{0}{89} + \frac{0.5}{92} + \frac{0.8}{95} + \frac{1}{98} \right\}$$

a) Find "if temp is hot then separation of mixture is good"

b) Let $A' = \left\{ \frac{1}{170}, \frac{0.8}{175}, \frac{0.5}{180}, \frac{0.2}{185} \right\}$

for the new rule "if A' then B'", find B' using max-product

