Two Modified Otsu Image Segmentation Methods Based On Lognormal And Gamma Distribution Models

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***Abstract*—Otsu's method of image segmentation is one of the best methods for threshold selection. With Otsu's method an optimum threshold is found by maximizing the between-class variance; Otsu algorithm is based on the gray-level histogram which is estimated by a sum of Gaussian distributions. In some type of images, image data does not best fit in a Gaussian distribution model. The objective of this study is to develop and compare two modified versions of Otsu method, one is based on Log-normal distribution (Otsu-Lognormal), while the other is based on Gamma distribution (Otsu-Gamma); the maximum between-cluster variance is modified based on each model. The two proposed methods were applied on several images and promising experimental results were obtained. Evaluation of the resulting segmented images shows that both Otsu-Gamma method and Otsu-Lognormal yield better estimation of the optimal threshold than does the original Otsu method with Gaussian distribution (Otsu).**

***Index Terms*—Image Thresholding, Otsu Method, Log-normal Distribution, Gamma Distribution.**

I. Introduction

Image processing techniques can be classified into three layers: Image processing (low layer), image analysis (middle layer) and image understanding (high layer) [1]. Image segmentation is the initial step in the image analysis process and a very critical task. One can define image segmentation as a partitioning or clustering technique used for image analysis. In another word, it is a process of subdividing an image into its constituent regions or objects as part of the analysis process [2]. Image segmentation algorithms are generally based on one of two basic properties of intensity values, discontinuity and similarity. In the first type, the image is partitioned based on sudden changes in intensity (e.g gray level), while in the other type, the image is partitioned into regions that are considered to be similar based on certain criteria [2]. There is not a single image segmentation algorithm which can give the best result for every image [3]. According to the type of the given image -application- the most appropriate approach is to be chosen to achieve the best segmentation. One of these practical applications is skin detection which is very important for Medical Diagnostic Applications [4], [5] and Biometric Authentication and Verification Systems [6]. In such applications an accurate segmentation of skin images is a very essential issue. Actually, an accurate segmentation is not an easy goal to achieve; one reason is that segmenting an image into regions is a problem with a number of possible solutions, another reason is that in many cases the regions of interest are heterogeneous [7]; ambiguities arise and the necessary discerning information is not directly available. Several general-purpose methods and techniques have been developed for image segmentation, and since there is no general solution to the image segmentation problem, these techniques often have to be combined with domain knowledge in order to effectively solve an image segmentation problem for a problem domain [8].

The method proposed in this paper is aimed to develop a more accurate method for estimating an optimal thresholding by changing the underlying distribution to match that of data in different types of images. The thresholding techniques used in this work uses two modified versions of Otsu method, one is based on Log-normal distribution (Otsu-Lognormal), while the other is based on Gamma distribution (Otsu-Gamma). The proposed modified Otsu methods were evaluated on several test images and the performance of these two methods was compared against the original Otsu method which is based on Gaussian distribution. The Otsu-Gamma method yields better estimation of the optimal threshold than does both the modified Otsu method with Log-normal distribution (Otsu-Lognormal) and the original Otsu method with Gaussian distribution (Otsu). This paper is organized as follows: Section 2 introduces the segmentation process in general, and thresholding technique in particular; it also includes a brief introduction on Otsu method of thresholding. Since the two modified methods are based on Log-normal and Gamma distributions a brief definition of both distribution models is in sections 3 and 4; the proposed technique is presented in section 5; while section 6 is concerned with the experimental results carried out. Section 7 is about the conclusion of this study.

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II. IMAGE SEGMENTATION

Image segmentation has been a major research topic for many image-processing researchers resulting a variety of methods and algorithms developed and improved for image segmentation ranging from general segmentation methods to tailored for certain image type or a certain application. Each method has a different approach in defining what characterizes a good segmentation and uses a different technique to find the optimal segmentation. There are different methods for generating image objects. Some are fully automated (unsupervised methods) while others are semi-automatic (supervised methods) [9]. In an unsupervised segmentation method, pixels are grouped automatically according to some criteria, while in a supervised one, the user has an influence on the segmentation by interventions made through the process by the user in order to guide the segmentation. Image thresholding is considered one of the main approaches of segmentation and entropy based thresholding is one broad type of thresholding techniques. In the following sub-section A, a brief introduction on thresholding techniques is presented; while sub-section B discusses the Otsu thresholding technique.

A. Thresholding Technique

Image thresholding is considered the easiest way to segment an image. Although, it seems simple the problem of choosing a good and an accurate threshold value is a difficult task. Thresholding is most commonly used for separating objects from the background [10]. The most widely used thresholding technique uses the gray level histogram. When an image, f(x,y), is composed of dark objects on a light background, then the foreground is clearly distinguishable from the background and this case the image histogram will be bimodal so that the threshold value will lie in the valley of the histogram ensuring that the objects can be extracted by comparing pixel values with a threshold T. I any pixel (x,y) for which $f(x,y) \geq T$ is considered as belonging to the object class, otherwise, it belongs to the background class [2]. Unfortunately, this is not the case in most images. This has lead some researchers to develop techniques to transform the image histogram into a bimodal form. In addition to histogram based thresholding techniques, several techniques have been proposed, trying to find a way of choosing the best value of threshold (T) that will result in an accurate segmentation. Entropy based thresholding is one broad types of thresholding techniques, which is most commonly used for separating objects from the background [10].

B. The Otsu's Method

Let the pixels of a given picture be represented in L gray levels $[1;2;...;L]$.The number of pixels at level i is denoted by n_i and the total number of pixels by $N = n_l + n_2 + .. + n_L$. ; The probability distribution [11], [12]:

$$p_i = \frac{n_i}{N}; p_i \geq 0, \sum_{i=1}^{L} p_i = 1 \tag{1}$$

now when we classify the pixels into two classes C_o and C_1 (background and objects) by a threshold at level t ,C_o denotes pixels with grey levels $[1,2,..,t]$, and C_1 denotes pixels with levels $[t+1,...,L]$. Then the probability of class occurrence and class mean respectively , are [11], [12] :

$$\omega_0 = p_r(C_0) = \sum_{i=1}^{t} p_i = \omega(t) \tag{2}$$

$$\omega_1 = p_r(C_1) = \sum_{i=t+1}^{L} p_i = 1 - \omega(t) \tag{3}$$

and

$$\mu_0 = \sum_{i=1}^{t} i p_r(i \backslash C_0) = \sum_{i=1}^{t} i \frac{p_i}{\omega_0} = \frac{\mu(t)}{\omega(t)} \tag{4}$$

$$\mu_1 = \sum_{i=t+1}^{L} i p_r(i \backslash C_1) = \sum_{i=t+1}^{L} i \frac{p_i}{\omega_1} = \frac{\mu_T - \mu(t)}{1 - \omega(t)} \tag{5}$$

where ω_t and μ_t are the 0^{th} and $1^s t$ order cumulative moments of the histogram up to t^{th} level , and are defined as follow:

$$\omega(t) = \sum_{i=1}^{t} p_i \tag{6}$$

and

$$\mu(t) = \sum_{i=1}^{t} i p_i \tag{7}$$

and μ_T is the total mean level of the original image which is computed as :

$$\mu_T = \mu(L) = \sum_{i=1}^{L} i p_i \tag{8}$$

For any choice of k ; the following relation is valid :

$$\mu_0 \omega_0 + \mu_1 \omega_1 = \mu_T; \omega_0 + \omega_1 = 1 \tag{9}$$

The class variances and total variance are given by:

$$\sigma_0^2 = \sum_{i=1}^{t} (i - \mu_0)^2 p_r(i \backslash C_0) = \sum_{i=1}^{t} (i - \mu_0)^2 \frac{p_i}{\omega_0} \tag{10}$$

$$\sigma_1^2 = \sum_{i=t+1}^{L} (i - \mu_1)^2 p_r(i \backslash C_1) = \sum_{i=t+1}^{L} (i - \mu_1)^2 \frac{p_i}{\omega_1} \tag{11}$$

In order to evaluate the goodness of the threshold(at level k), within-class variance and between-class variance are used as measures of class separability, which are defined respectively as :

$$\sigma_w^2 = \omega_0 \sigma_0^2 + \omega_1 \sigma_1^2 \tag{12}$$

$$\sigma_B^2 = \omega_0 (\mu_0 - \mu_T)^2 + \omega_1 (\mu_1 - \mu_T)^2 = \omega_0 \omega_1 (\mu_1 - \mu_0)^2 \tag{13}$$

and the following relations hold :

$$\eta = \frac{\sigma_B^2}{\sigma_T^2} \tag{14}$$

$$\sigma_T^2 = \sigma_B^2 + \sigma_W^2 \tag{15}$$

The problem then is reduced to just maximize one of these criterion measures; It is noticed that σ_B^2 is dependent on $1^s t$ order statistics while σ_W^2 depends on the 2^{nd} order statics; therefore , σ_B^2 is the simplest measure with respect to t; Thus $\eta(t)$ is adopted as the criterion measure to chose the best threshold value; that threshold (t) is selected in a sequential search by using the following functions [11], [12]:

$$\eta(t) = \frac{\sigma_B^2(t)}{\sigma_T^2} \quad (16)$$

$$\sigma_B^2(t) = \frac{[\mu_T\omega(t) - \mu(t)]^2}{\omega(t)[1-\omega(t)]} \quad (17)$$

since σ_T^2 is not a function of threshold (t) ; then the optimal threshold should be the one that maximizes value of $\sigma_B^2(t)$;thus the optimal value threshold (t^*) is found with the following equation [11], [12]

$$\sigma_B^2(t^*) = max\sigma_B^2(t); 1 \le t < L \quad (18)$$

III. The log-normal distribution

In probability theory, a log-normal distribution is a probability distribution of a random variable whose logarithm is normally distributed. If Y is a random variable with a normal distribution, then $X = exp(Y)$ has a log-normal distribution; likewise, if X is log-normally distributed, then $Y = \log(X)$ is normally distributed. In the case of a digital image whose histogram $h(g)$ is assumed to be modeled as log-normal , then it can be defined as [6] :

$$h(g) = \sum_{i=1}^{m} p_i.f(x, \mu_i, \sigma_i), g > 0 \quad (19)$$

where $(p_i.f(x, \mu_i, \sigma_i))$ is the i^{th} mode in the histogram, p_i is the probability of this i^{th} mode, and m is the number of modes in that histogram. The probability density function (pdf) of the log-normal distribution is defined as :

$$F(g, \mu, \sigma) = \frac{1}{g\sigma\sqrt{2\pi}}.e^{\frac{1}{2}(\frac{lng-\mu}{\sigma})^2} \quad (20)$$

where g : pixel's intensity level (e.g grey level); σ : standard deviation ; μ : mean;

Now, if the data in an image is assumed to be modeled by a Log-normal distribution, then, one can see the image $I(x, y)$ is composed by two Log-normal distributions. Therefore,μ_{0Log} and μ_{1Log} can be estimated as follows [6]:

$$\mu_{0Log}^2(t) = \frac{\sum_{i=1}^{t-1} h(i) \cdot \log(i)}{\sum_{i=1}^{t-1} h(i)} \quad (21)$$

$$\mu_{1Log}^2(t) = \frac{\sum_{i=t}^{L} h(i) \cdot log(i)}{\sum_{i=t}^{L} h(i)} \quad (22)$$

IV. The Gamma distribution

The gamma distribution, like the lognormal distribution, is an alternative to consider for data that seem to be highly skewed. Gamma distribution is more general than Gaussian and it can be encountering the limitations of Gaussian due to its ability to provide symmetric and non-symmetric histograms. For a Gamma distribution with parameters α and β, the probability density functions in homogeneous area is given by [13], [14]:

$$F(g, \mu, N) = \frac{2q}{\mu} \cdot \frac{N^N}{\Gamma(N)} \cdot (\frac{qx}{\mu})^{2N-1} \cdot e^{-N(\frac{qx}{\mu})^2} \quad (23)$$

where $q = \frac{\gamma(N+0.5)}{\sqrt{N}\cdot\gamma(N)}$, x is the intensity of the pixel, μ is the mean value of the distribution and N is the shape of distribution. The shape of the Gamma distribution could be symmetry or skewed to the right. If the data in an image is assumed to be modeled by a Gamma distribution, then, one can see the image $I(x, y)$ is composed by two Gamma distributions. Therefore,μ_{0Gamma} and μ_{1Gamma} can be estimated as follows [14]:

$$\mu_{0Gamma}^2(t) = \frac{\sum_{i=1}^{t-1} h(i) \cdot i^2 \cdot q^2}{\sum_{i=1}^{t-1} h(i)} \quad (24)$$

$$\mu_{1Gamma}^2(t) = \frac{\sum_{i=t}^{L} h(i) \cdot i^2 \cdot q^2}{\sum_{i=t}^{L} h(i)} \quad (25)$$

V. Proposed Modified Otsu Methods Based on Log-Normal and Gamma distributions

The Otsu method is used for finding the optimal threshold t^* by minimizing the value of σ_B^2 ; thus the optimal value threshold t^* has been previously defined in (18) as :

$$\sigma_B^2(t^*) = max\sigma_B^2(t); 1 \le t < L \quad (26)$$

Where $\sigma_B^2(t)$ is the between class variance, which is previously defined in (17) as :

$$\sigma_B^2(t) = \frac{[\mu_T\omega(t) - \mu(t)]^2}{\omega(t)[1-\omega(t)]} \quad (27)$$

As previously mentioned, according to the nature of the data in an image it can be assumed to best fit certain distribution model such as : Gaussian, Gamma and Log-normal distributions; the proposed modification of Otsu method is to compute the between class variance based on the values of μ_0 and μ_1 are estimated based on the adopted distribution model ; In case of proposed Otsu-Gamma, the optimal threshold is estimated using the sequential search function defined in (18) with the values of μ_0 and μ_1 defined with equations (24) and (25) respectively. On the other hand, when applying the Otsu-Lognormal method, the optimal threshold is estimated using the same sequential search function defined in (18) but with values of μ_0 and μ_1 defined with equations (21) and (22) respectively.

VI. Experimental results

The proposed method was implemented and applied on several images with different dimensions and gray levels. First the original Otsu method (Otsu-Gaussian) thresholding method is applied on 12 test images; the same images are segmented using the same method but under the assumption of Gamma distribution (Otsu-Gamma) and once again under the assumption of Lognormal distribution (Otsu-Lognormal). Figures(1,2 and 3) present a sample of the testing results; each figure represents a real image and the segmented images with Otsu, Otsu-Gamma and Otsu-Lognormal respectively. For the purpose of evaluating the performance of the proposed methods against Otsu-Gaussian, metrics of Image uniformity, and Inter-region contrast are used as performance measures [15], [16]. Table-1 lists for each test image the threshold value for each of the three tested methods and the value of each the evaluation metric, the methods are then ranked for each image according to the value of each metric, higher value means better threshold estimation. and an overall measure is computed by averaging both metrics. Table-2 shows that the proposed modified methods yield promising results and a better estimation of the optimal threshold value. This is demonstrated by the fact that in the average of performance metrics 58% of the tested images have better segmentation

TABLE I
EVALUATION OF PERFORMANCE MEASURES FOR ALL 3 METHODS : OTSU-LOGNORMAL , OTSU-GAMMA , OTSU-GAUSSIAN.

Image	Method	Threshold	Uniformity	Rank	Inter-Region contrast	Rank	Avg: $\frac{(UN+RC)}{2}$	Rank
Einstein.bmp	OTSU-Log Normal	66	0.96827	1	0.46405	2	0.71616	2
	OTSU-Gamma	65	0.96817	2	0.46656	1	0.71737	1
	OTSU-Gaussian	90	0.96527	3	0.37661	3	0.67094	3
bacteria.bmp	OTSU-Log Normal	58	0.96684	2	0.47335	2	0.7201	2
	OTSU-Gamma	53	0.96696	1	0.49602	1	0.73149	1
	OTSU-Gaussian	98	0.96546	3	0.2602	3	0.61283	3
boats.bmp	OTSU-Log Normal	54	0.95762	1	0.74822	2	0.85292	2
	OTSU-Gamma	52	0.95706	2	0.75343	1	0.85525	1
	OTSU-Gaussian	105	0.9518	3	0.59049	3	0.77115	3
brain.bmp	OTSU-Log Normal	9	0.96371	3	0.96642	1	0.96507	1
	OTSU-Gamma	25	0.97402	2	0.87566	2	0.92484	2
	OTSU-Gaussian	46	0.97661	1	0.78347	3	0.88004	3
cameraman.bmp	OTSU-Log Normal	45	0.96367	2	0.80806	2	0.88587	2
	OTSU-Gamma	42	0.96242	3	0.81273	1	0.88758	1
	OTSU-Gaussian	88	0.96479	1	0.73479	3	0.84979	3
columbia.bmp	OTSU-Log Normal	53	0.92801	3	0.54603	1	0.73702	1
	OTSU-Gamma	106	0.94444	1	0.48466	3	0.71455	3
	OTSU-Gaussian	100	0.94382	2	0.4872	2	0.71551	2
face.bmp	OTSU-Log Normal	27	0.9191	3	0.81621	1	0.86766	1
	OTSU-Gamma	91	0.9384	1	0.59626	3	0.76733	3
	OTSU-Gaussian	90	0.93817	2	0.59772	2	0.76795	2
house.bmp	OTSU-Log Normal	101	0.96363	3	0.38886	1	0.67625	3
	OTSU-Gamma	114	0.96815	2	0.38867	2	0.67841	1
	OTSU-Gaussian	117	0.96848	1	0.38828	3	0.67838	2
lena.bmp	OTSU-Log Normal	59	0.89781	3	0.69654	2	0.79718	1
	OTSU-Gamma	66	0.90008	2	0.67666	1	0.78837	2
	OTSU-Gaussian	130	0.90625	1	0.47129	3	0.68877	3
rice.bmp	OTSU-Log Normal	113	0.9586	3	0.30925	3	0.63393	3
	OTSU-Gamma	131	0.96488	1	0.31526	1	0.64007	1
	OTSU-Gaussian	125	0.96332	2	0.31381	2	0.63857	2
shadow.bmp	OTSU-Log Normal	119	0.93672	3	0.35238	1	0.64455	1
	OTSU-Gamma	147	0.9465	2	0.31916	2	0.63283	2
	OTSU-Gaussian	153	0.94807	1	0.31532	3	0.6317	3
shot1.bmp	OTSU-Log Normal	105	0.93653	3	0.42921	2	0.68287	2
	OTSU-Gamma	91	0.93813	2	0.45474	1	0.69644	1
	OTSU-Gaussian	173	0.95874	1	0.24992	3	0.60433	3
tire.bmp	OTSU-Log Normal	23	0.88831	3	0.83628	1	0.8623	1
	OTSU-Gamma	82	0.91995	2	0.74422	2	0.83209	2
	OTSU-Gaussian	84	0.92048	1	0.74152	3	0.831	3

results with proposed modefied method Otsu-Gamma, while the rest 42% the best results were also acheived with the other modefied method Otsu-Lognormal.

VII. CONCLUSION

Image segmentation is an essential tool in many application of image processing. Having a better and more efficient segmentation is a critical issue in those applications. This paper proposed two modified versions of Otsu thresholding technique by adopting the Lognormal and Gamma distribution models. From the evaluation of the resulting images, we conclude that the proposed modified methods yield better images compared to the original Otsu method based on Gaussian distribution.

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TABLE II
EVALUATION OF PERFORMANCE MEASURES FOR ALL 3 METHODS. R/M = RANK/METHOD. OLOG = OTSU-LOGNORMAL, OGAM = OTSU-GAMMA. OGAU = OTSU-GAUSSIAN.

Performance Metric	R/M	OLog	OGam	OGau
UN : Uniformity	1st	17%	33%	50%
	2nd	17%	58%	25%
	3rd	67%	8%	25%
RC : Inter-Region Contrast	1st	42%	58%	0%
	2nd	50%	25%	25%
	3rd	8%	17%	75%
Average : $\frac{UN+RC}{2}$	1st	42%	58%	0%
	2nd	42%	25%	33%
	3rd	17%	17%	67%

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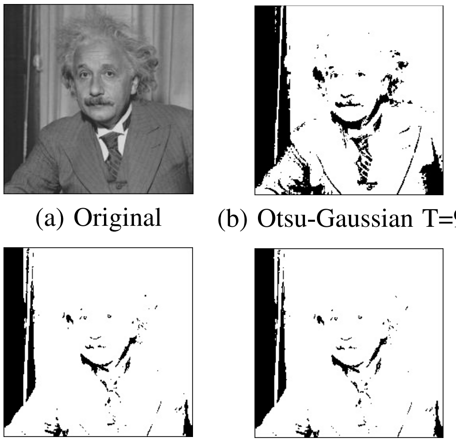
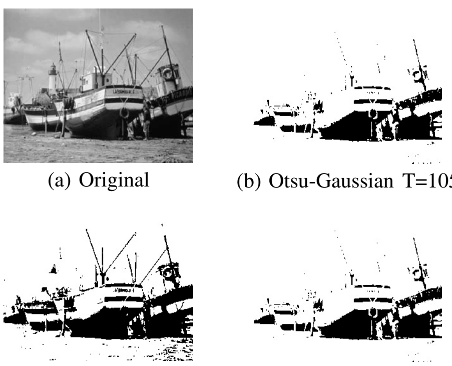


(a) Original (b) Otsu-Gaussian T=90

(c) Otsu-Lognormal T=66 (d) Otsu-Gamma T=65

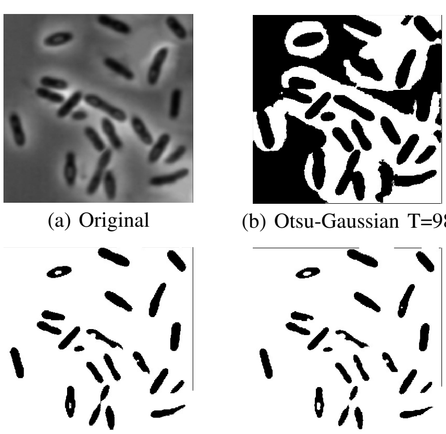
Fig. 1. Original and Segmented Images of (Einstein.bmp)



(a) Original (b) Otsu-Gaussian T=105

(c) Otsu-Lognormal T=54 (d) Otsu-Gamma T=52

Fig. 2. Original and Segmented Images of (Boats.bmp)



(a) Original (b) Otsu-Gaussian T=98

(c) Otsu-Lognormal T=58 (d) Otsu-Gamma T=53

Fig. 3. Original and Segmented Images of (Bacteria.bmp)

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