COMP4421 (Fall 2018)

Assignment #2

Name: CHEUNG, Daniel   Student ID: 20423088
E-mail: dcheungaa@connect.ust.hk

I. Exercises

1) Solution:

We know $A \circ B = (A \ominus B) \oplus B$, and $A \cdot B = (A \oplus B) \ominus B$.

We know $(A \oplus B)^c = A^c \ominus \hat{B}$ and $(A \ominus B)^c = A^c \oplus \hat{B}$.

$$\begin{aligned} \text{LHS} &= (A \cdot B)^c \\ &= \big((A \oplus B) \ominus B\big)^c \\ &= \big((A \oplus B)^c \oplus \hat{B}\big) \\ &= \Big(\big(A^c \ominus \hat{B}\big) \oplus \hat{B}\Big) \\ &= \big(A^c \circ \hat{B}\big) \\ &= \text{RHS} \end{aligned}$$

2) Solution:

A =

0	0	0	0	0	0	0	0
0	0	0	0	0	**1**	**1**	0
0	0	0	0	**1**	**1**	**1**	0
0	0	0	0	**1**	**1**	0	0
0	0	**1**	**1**	0	0	0	0
0	**1**	**1**	**1**	0	0	0	0
0	**1**	**1**	0	0	0	0	0
0	0	0	0	0	0	0	0

<1> B =

1	**1**	**1**
1	**1**	**1**
1	**1**	**1**

<2> B =

0	**1**	0
1	**1**	**1**
0	**1**	0

$<1>\hat{B} = <1>B$

$<2>\hat{B} = <2>B$

$A \oplus <1>B =$

0	0	0	0	**1**	**1**	**1**	**1**
0	0	0	**1**	**1**	**1**	**1**	**1**
0	0	0	**1**	**1**	**1**	**1**	**1**
0	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1	**1**	**1**	**1**	**1**	**1**	**1**	0
1	**1**	**1**	**1**	**1**	0	0	0
1	**1**	**1**	**1**	**1**	0	0	0
1	**1**	**1**	**1**	0	0	0	0

$A \ominus <1>B =$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$A \oplus \text{<2>} B =$

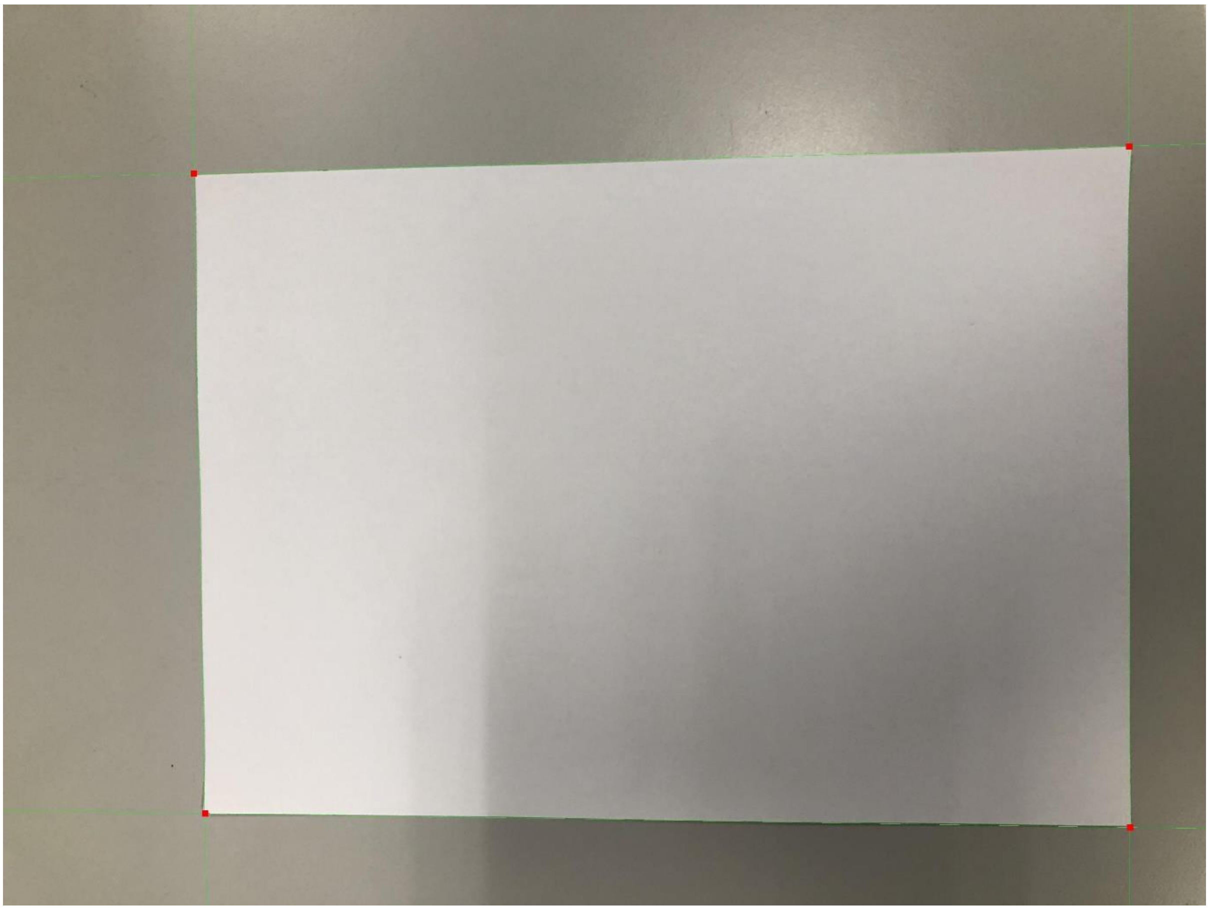
0	0	0	0	0	**1**	**1**	0
0	0	0	0	**1**	**1**	**1**	**1**
0	0	0	**1**	**1**	**1**	**1**	**1**
0	0	**1**	**1**	**1**	**1**	**1**	0
0	**1**	**1**	**1**	**1**	**1**	0	0
1	**1**	**1**	**1**	**1**	0	0	0
1	**1**	**1**	**1**	0	0	0	0
0	**1**	**1**	0	0	0	0	0

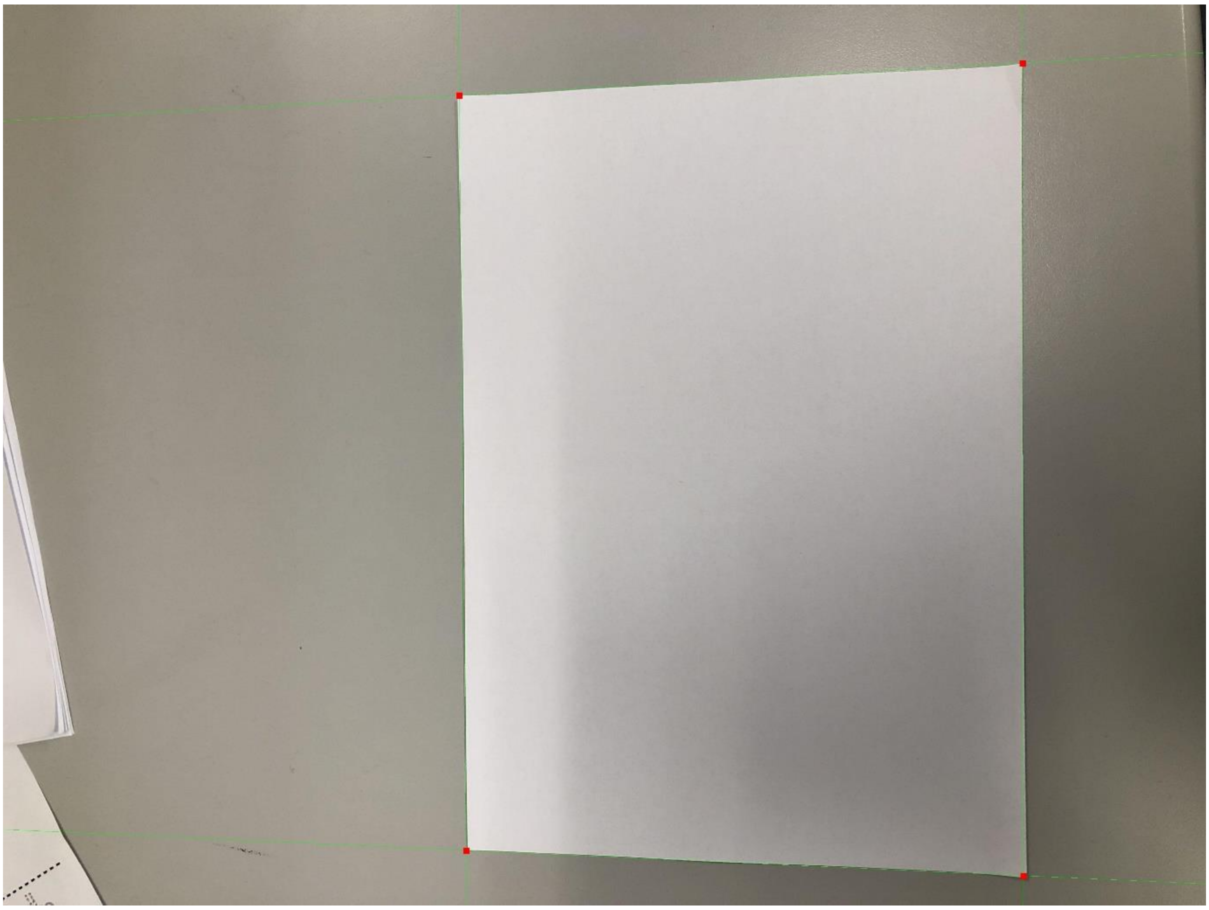
$A \ominus \text{<2>} B =$

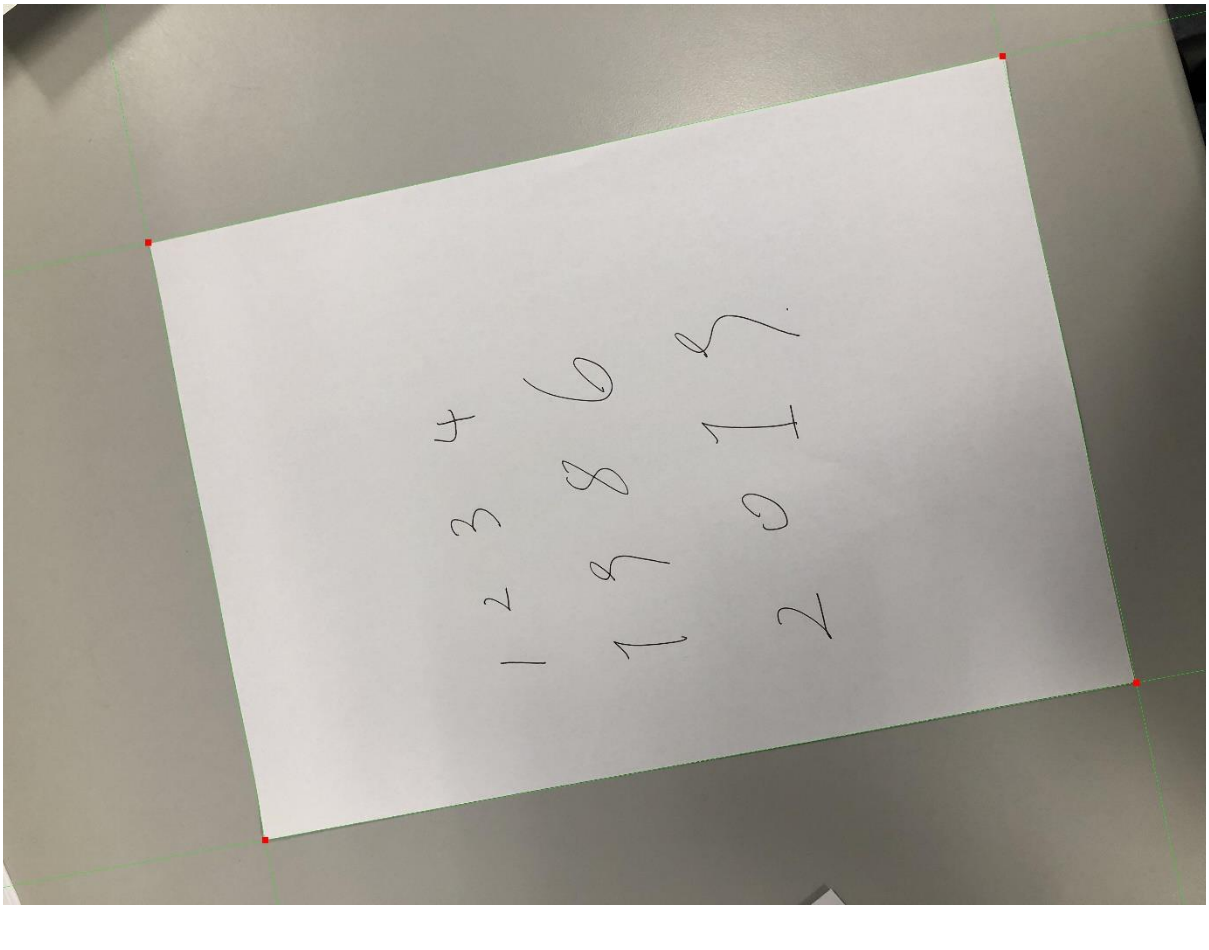
0	0	0	0	0	0	0	0
0	0	0	0	0	**1**	0	0
0	0	0	0	**1**	**1**	**1**	0
0	0	0	0	0	**1**	0	0
0	0	**1**	0	0	0	0	0
0	**1**	**1**	**1**	0	0	0	0
0	0	**1**	0	0	0	0	0
0	0	0	0	0	0	0	0

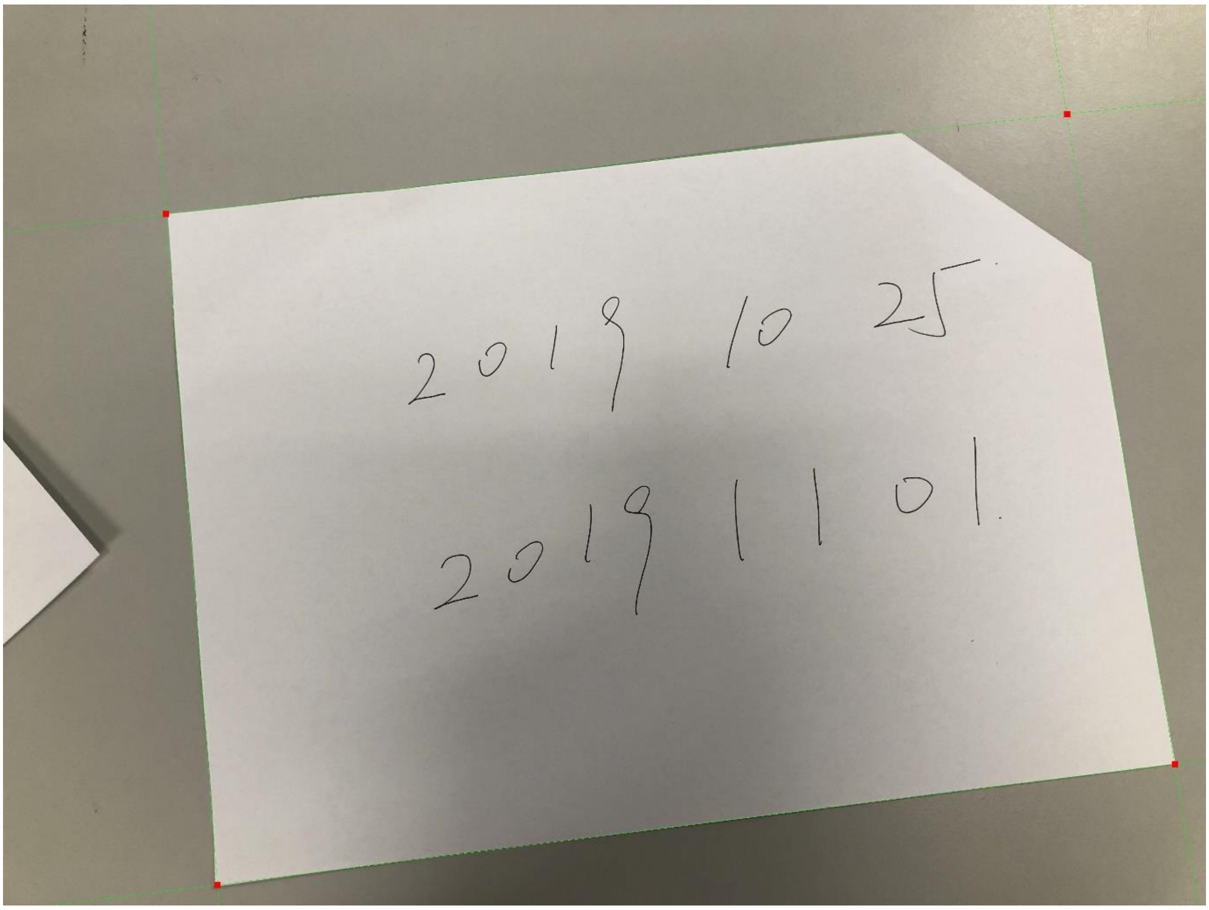
II. PROGRAMMING TASKS

1) Hough Transform






1 2 3 4
1 9 8 6
2 0 1 9



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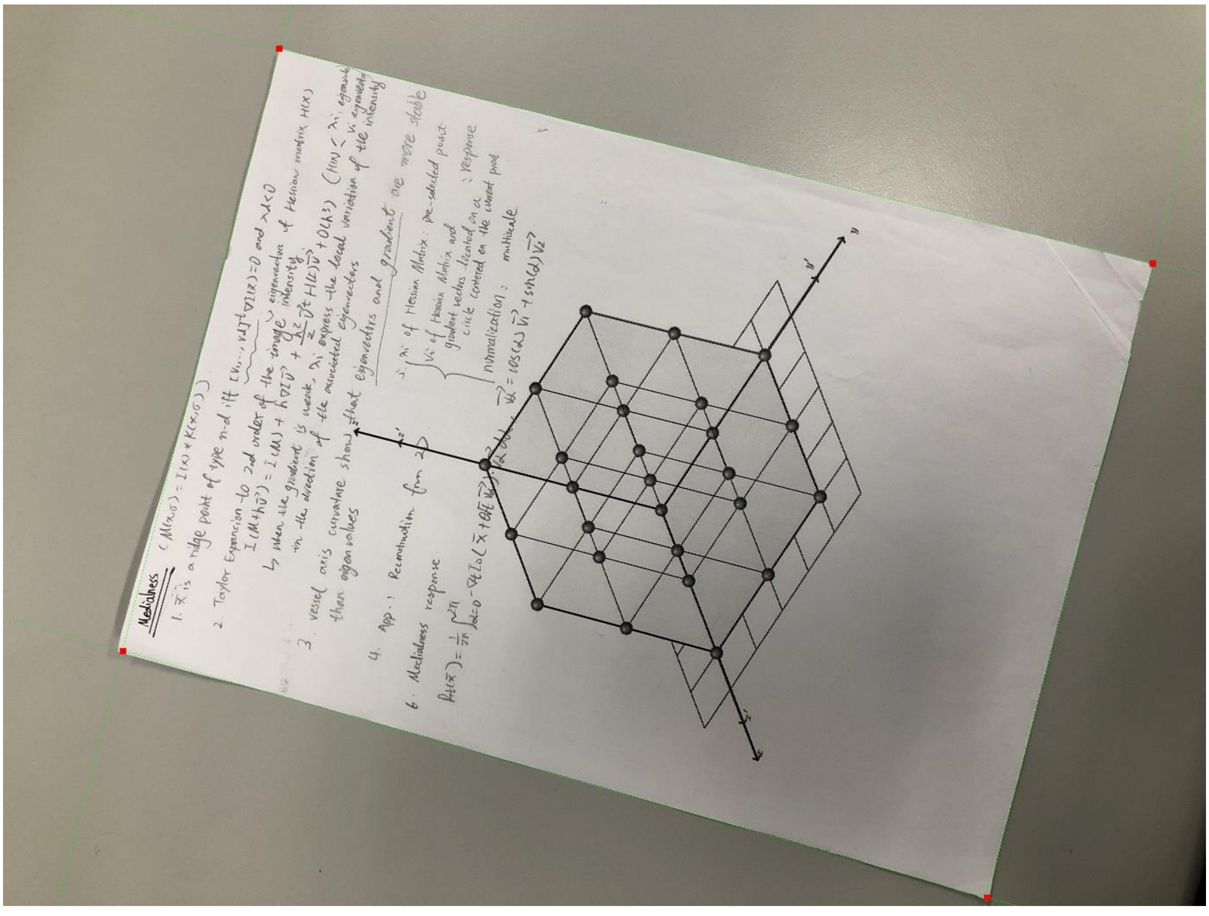


Medialness ($M(x,\sigma) = I(x) * K(x,\sigma)$)
1. $\bar{x}$ is a ridge point of type n-d iff $[v_1, \dots, v_d]^t \nabla I(x) = 0$ and $\lambda_d < 0$
eigenvectors of Hessian matrix $H(X)$
2. Taylor Expansion to 2nd order of the image intensity
$I(M + h\vec{v}) = I(M) + h \nabla I \vec{v} + \frac{h^2}{2} \vec{v}^t H(L) \vec{v} + O(h^3)$ ($H(x)$ λ_i: eigenvalues, v_i eigenvectors)
when the gradient is weak, λ_i express the local variation of the intensity in the direction of the associated eigenvectors
3. vessel axis curvature show that eigenvectors and gradient are more stable than eigen values
4. App.: Reconstruction from 2D
5. λ_i of Hessian Matrix: pre-selected point
v_i of Hessian Matrix and gradient vectors located on a circle centered on the current point: response
normalization: multiscale
6. Medialness response
$R_t(\bar{x}) = \frac{1}{2\pi} \int_{\alpha=0}^{2\pi} -\nabla_t I_\sigma(\bar{x} + \theta \vec{v_\alpha}) \cdot \vec{v_\alpha} \, d\alpha$, $\vec{v_\alpha} = \cos(\alpha) \vec{v_1} + \sin(\alpha) \vec{v_2}$

Medialness

$(M(x,\sigma) = I(x) * K(x,\sigma))$

1. $\bar{x}$ is a ridge point of type n-d iff $\underbrace{[v_1, \dots, v_d]^t}_{\text{eigenvectors of Hessian matrix } H(x)} \nabla I(\bar{x}) = 0$ and $\lambda_d < 0$

2. Taylor Expansion to 2nd order of the $\underbrace{\text{image}}_{\text{intensity}}$

   $I(M + h\vec{v}) = I(M) + h \nabla I \vec{v} + \frac{h^2}{2} \vec{v}^t H(I) \vec{v} + O(h^3)$ ($H(x)$ ⟨ λ_i: eigenvalue, v_i: eigenvector)

   ↳ when the gradient is weak, λ_i express the local variation of the intensity in the direction of the associated eigenvectors

3. vessel axis curvature shows that eigenvectors and gradient are more stable than eigenvalues

4. App.: Reconstruction from 2D

5. λ_i of Hessian Matrix: pre-selected point

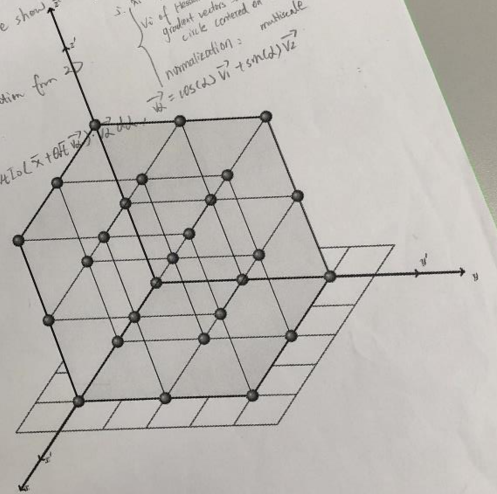
   v_i of Hessian Matrix and gradient vectors located on a circle centered on the current point: response

   normalization: multiscale

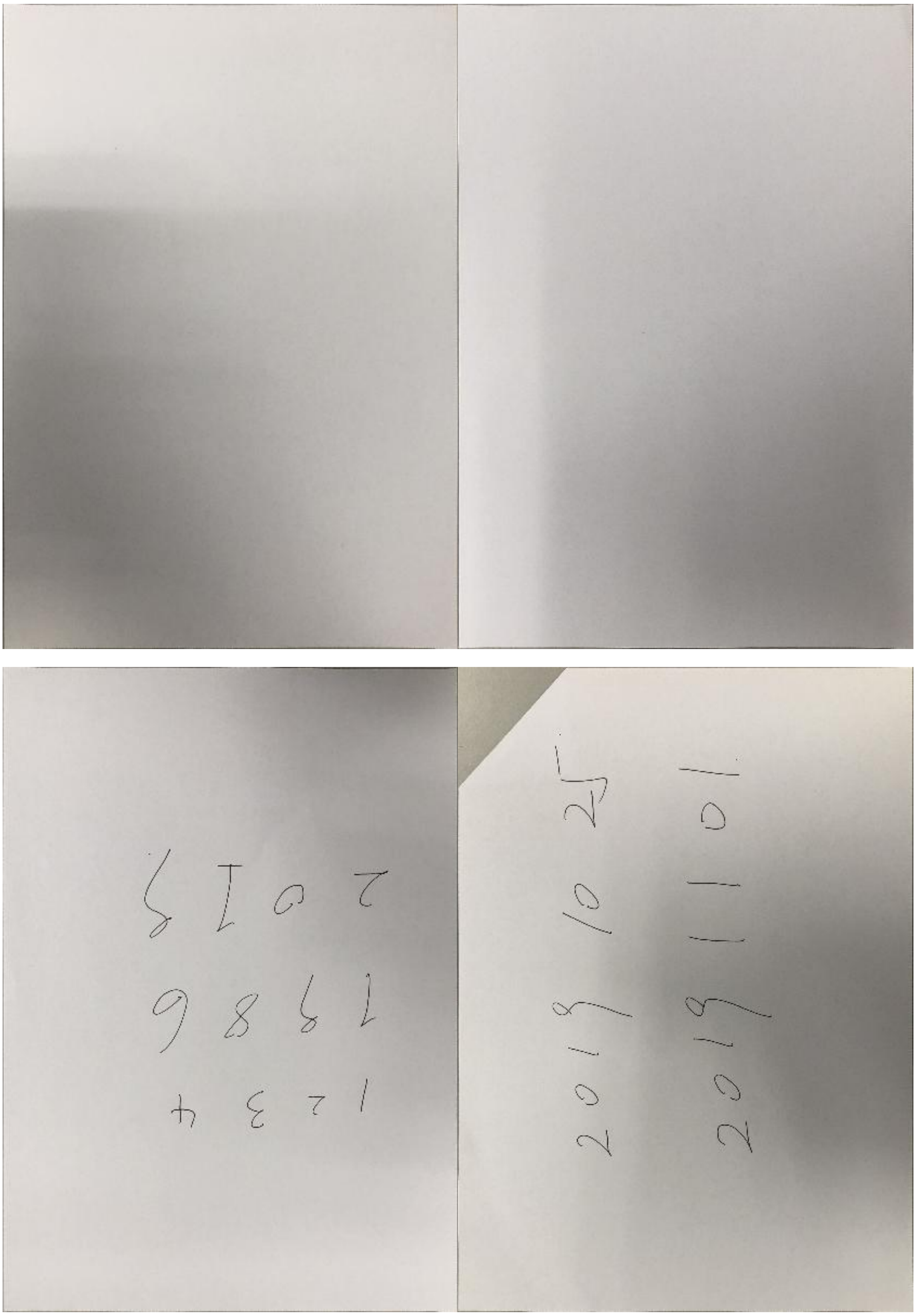
   $\vec{v_\alpha} = \cos(\alpha)\vec{v_1} + \sin(\alpha)\vec{v_2}$

6. Medialness response

   $R_t(\bar{x}) = \frac{1}{2\pi} \int_{\alpha=0}^{2\pi} -\nabla_t I_\sigma(\bar{x} + \theta t \vec{v_\alpha}) \cdot \vec{v_\alpha} \, d\alpha$



1) Image Warping



Medialness ($M(x, \sigma) = I(x) * K(x, \sigma)$)

1. $\bar{x}$ is a ridge point of type n-d iff $[v_1, \dots, v_d]^t \nabla I(x) = 0$ and $\lambda_d < 0$ — eigenvectors of Hessian matrix $H(x)$

2. Taylor Expansion to 2nd order of the image intensity

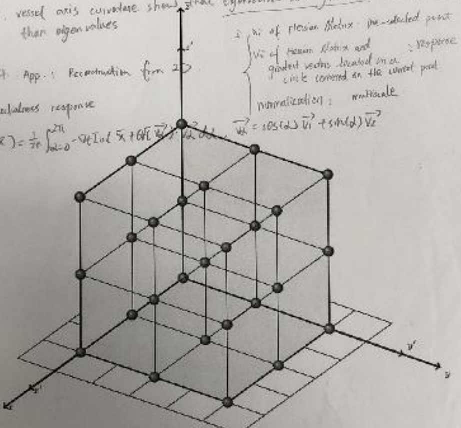
$$I(M + h\vec{v}) = I(M) + h \nabla I \vec{v} + \frac{h^2}{2} \vec{v}^t H(x) \vec{v} + O(h^3)$$

($H(x)$ < λ_i eigenvalues, v_i eigenvectors)

↳ when the gradient is weak, λ_i express the local variation of the intensity in the direction of the associated eigenvectors

3. vessel axis curvature shows that eigenvectors and gradient are more stable than eigenvalues

4. App. : Reconstruction from 2D



5. λ_i of Hessian Matrix : pre-selected point
   $\vec{v_i}$ of Hessian Matrix and gradient vectors located on a circle centered on the current point : response
   normalization : multiscale

6. Medialness response

$$R_t(\bar{x}) = \frac{1}{2\pi} \int_{\alpha=0}^{2\pi} -\nabla_t I_\sigma(\bar{x} + t\sigma \vec{v_\alpha}) \cdot \vec{v_\alpha} \, d\alpha, \quad \vec{v_\alpha} = \cos(\alpha) \vec{v_1} + \sin(\alpha) \vec{v_2}$$

Medialness ($M(x, \sigma) = I(x) * K(x, \sigma)$)

1. $\bar{x}$ is a ridge point of type n-d iff $[v_1, \dots, v_d]^t \nabla I(x) = 0$ and $\lambda_d < 0$ — eigenvectors of Hessian matrix $H(x)$

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($H(x)$ < λ_i eigenvalues, v_i eigenvectors)

↳ when the gradient is weak, λ_i express the local variation of the intensity in the direction of the associated eigenvectors

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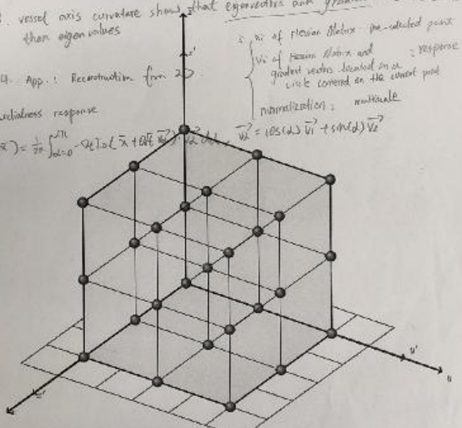
4. App. : Reconstruction from 2D



5. λ_i of Hessian Matrix : pre-selected point
   $\vec{v_i}$ of Hessian Matrix and gradient vectors located on a circle centered on the current point : response
   normalization : multiscale

6. Medialness response

$$R_t(\bar{x}) = \frac{1}{2\pi} \int_{\alpha=0}^{2\pi} -\nabla_t I_\sigma(\bar{x} + t\sigma \vec{v_\alpha}) \cdot \vec{v_\alpha} \, d\alpha, \quad \vec{v_\alpha} = \cos(\alpha) \vec{v_1} + \sin(\alpha) \vec{v_2}$$