COMP4901L Assignment6 Writeup

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Part 1

Q1.1

a)

$$f(x, w) = \sigma(\sigma(x_1 w_1)w_2 + x_2)$$

$$\begin{aligned}\sigma(x) &= \frac{1}{1+e^{-x}} \\ \frac{d\sigma}{dx} &= \frac{e^{-x}}{(1+e^{-x})^2} \\ &= \frac{1+e^{-x}-1}{1+e^{-x}} \times \frac{1}{1+e^{-x}} \\ &= (1-\sigma(x))\sigma(x)\end{aligned}$$

$$\begin{aligned}\therefore \frac{\partial f}{\partial w_2} &= \frac{\partial f}{\partial \sigma(x_1 w_1)w_2 + x_2} \frac{\partial \sigma(x_1 w_1)w_2 + x_2}{\partial w_2} \\ &= (1-\sigma(\sigma(x_1 w_1)w_2 + x_2))\sigma(\sigma(x_1 w_1)w_2 + x_2)\sigma(x_1 w_1)\end{aligned}$$

$$\text{At the current step,}$$

$$\begin{aligned}\frac{\partial f}{\partial w_2} &= (1-\sigma(\sigma(1\cdot 0)0 + 0))\sigma(\sigma(1\cdot 0)0 + 0)\sigma(1\cdot 0) \\ &= 0.125\end{aligned}$$

b)

$$\text{L2 loss is defined as } (y_{\text{true}} - y_{\text{predicted}})^2$$

$$y_{\text{predicted}} = f(x, w) = \sigma(\sigma(1\cdot 0)0 + 0) = 0.5$$

$$\therefore \frac{\partial L}{\partial f} = 2(5 - 0.5)(-1) = -9$$

$$\begin{aligned}w_2 &:= w_2 - \eta \frac{\partial L}{\partial w_2} \\ &= 0 - 0.5 \frac{\partial L}{\partial f} \frac{\partial f}{\partial w_2} \\ &= 0 - 0.5 \times -9 \times 0.125 \\ &= 0.5625\end{aligned}$$

Q1.2

$$\text{Suppose a linear function: } g(x) = cx$$

$$\text{Then for the output of a 2-layer network}$$

$$\begin{aligned}c(\mathbf{W}_1 c(\mathbf{W}_2 \mathbf{x} + \mathbf{b}_2) + \mathbf{b}_1) &= c(c\mathbf{W}_1\mathbf{W}_2\mathbf{x} + c\mathbf{W}_1\mathbf{b}_2 + \mathbf{b}_1) \\ &= c(\mathbf{W}'\mathbf{x} + \mathbf{b}')\end{aligned}$$

The result corresponds to the result of a single layer network, just with a different set of weights and biases. As such, continuing on this logic, no matter how many number of hidden linear layers the network has, it will still be no different than with just 1 single linear layer.

Q1.3

This is because ReLUs are significantly faster than sigmoid, which is already quite fast. In contrast, ReLU ony need to do a single comparison to a value, whereas sigmoid needs to deal with e^x, divisions, subtractions and multiplications.

Q.1.4

Having weights as constant values would prevent the network from learning, because the gradient and propagation error would be the same. And as such, all the weight updates would subsequently also be the same.

Q1.5

a) AlexNet

Layer	Tensor	Weights	Biases	Parameters
Input Image	227x227x3	0	0	0
Conv-1 (n 96, size 11, stride 4, pad 0)	55x55x96	34848	96	34944
MaxPool-1 (size 3, stride 2)	27x27x96	0	0	0
Conv-2 (n 256, size 5, stride 1, pad 2)	27x27x256	614400	256	614656
MaxPool-2 (size 3, stride 2)	13x13x256	0	0	0
Conv-3 (n 384, size 3, stride 1, pad 1)	13x13x384	884736	384	885120
Conv-4 (n 384, size 3, stride 1, pad 1)	13x13x384	1327104	384	1327488
Conv-5 (n 256, size 3, stride 1, pad 1)	13x13x256	884736	256	884992
MaxPool-3 (size 3, stride 2)	6x6x256	0	0	0
FC-1 (size 4096)	4096x1	37748736	4096	37752832
FC-2 (size 4096)	4096x1	16777216	4096	16781312
FC-3 (size 1000)	1000x1	4096000	1000	4097000
Total				**62,378,344**

b) VGG-16

Layer	Tensor	Weights	Biases	Parameters
Input Image	224x224x3	0	0	0
conv3-64	224x224x64	1728	64	1792
conv3-64	224x224x64	36864	64	36928
maxpool (size 2, stride 2)	112x112x64	0	0	0
conv3-128	112x112x128	73728	128	73856
conv3-128	112x112x128	147459	128	147584
maxpool	56x56x128	0	0	0
conv3-256	56x56x256	294912	256	295168
conv3-256	56x56x256	589824	256	590080
conv3-256	56x56x256	589824	256	590080
maxpool	28x28x256	0	0	0
conv3-512	28x28x512	1179648	512	1180160
conv3-512	28x28x512	2359296	512	2359808
conv3-512	28x28x512	2359296	512	2359808
maxpool	14x14x512	0	0	0
conv3-512	14x14x512	2359296	512	2359808
conv3-512	14x14x512	2359296	512	2359808
conv3-512	14x14x512	2359296	512	2359808
maxpool	7x7x512	0	0	0
FC-4096	4096x1	102760448	4096	102764544
FC-4096	4096x1	16777216	4096	16781312
FC-1000	1000x1	4096000	1000	4097000
Total				**138,357,544**

c) GoogLeNet

From the paper,
$\#\text{ of parameters} = (2.7 + 112 + 159 + 380 + 364 + 437 + 463 + 580 + 840 + 1072 + 1388 + 1000)\text{K} = 6797.7\text{K}$

Nets ordered in desc. # of parameters: VGG16 > GoogLeNet > AlexNet

Comparing to VGG16, GoogLeNet has most of its parameters in its inception blocks, but VGG16 has most in its fully connected layers. By using instead, an average pool then a linear layer, not much more parameters are added to the model.

Part 4

Accuracy: 0.974

confusion matrix:

```
51    0    0    0    0    0    0    0    0    0
0    52    1    0    0    0    0    0    0    0
0     0   41    1    0    0    0    0    1    0
0     0    0   61    0    1    0    1    1    0
0     0    0    0   46    0    1    1    0    0
0     0    0    0    0   45    0    1    0    0
0     0    0    0    0    0   41    0    0    0
0     0    0    0    0    0    0   47    0    2
0     0    0    1    0    0    0    0   59    0
0     0    0    0    0    0    0    1    0   44
```

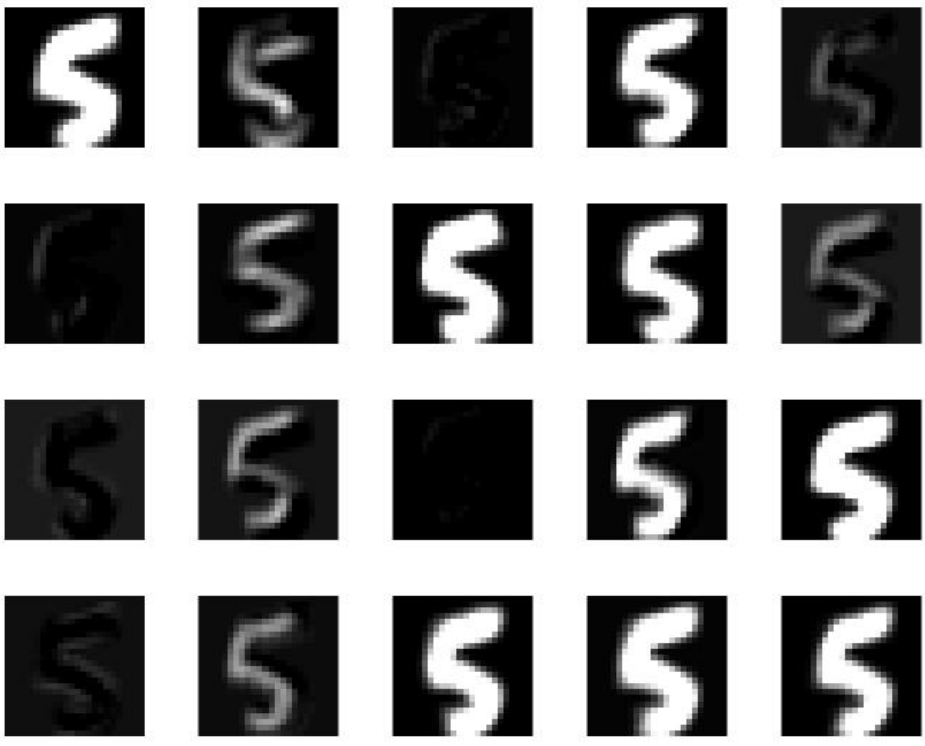
7 and 9 are the most easily confused digit. If we sqeueeze the loop of 9, it looks like 7. Or in some style of 7 there is verticle stroke at top left and horizontal line in middle and we intersect both line, it looks like 9.

3 and 8 are also easily confused. If the start and end of 3 is closer to the middle, it looks like 8.

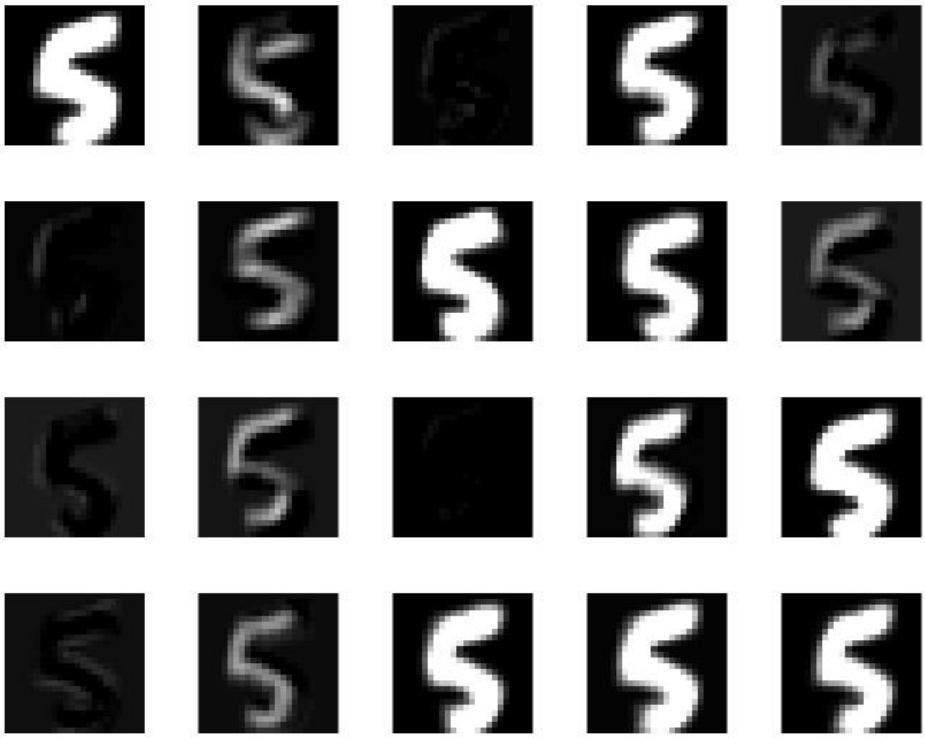
Part 5

Layer 1 (original image)

Layer 2



Layer 3



Some filters' output are blank which mean they are not activated by this digit or their weights are zero

Some filters' output are almost the same with other filter output

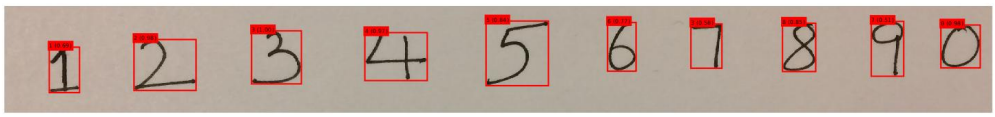
Some filters' output are the blur version of the original image

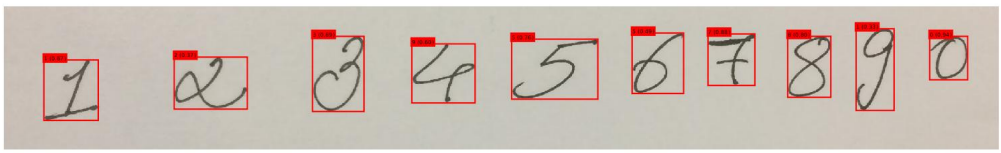
Some filters are extracting the vertical edges, some are extracting the horizontal edges

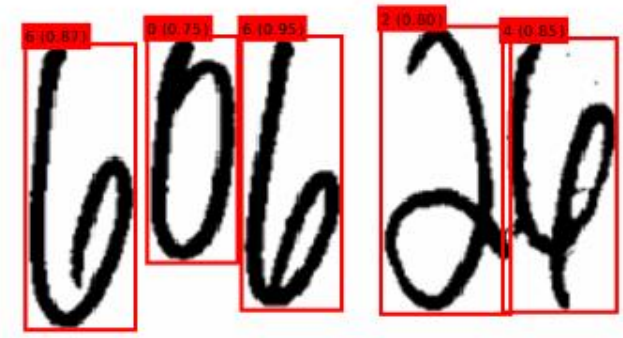
Some filters make the digit thicker, some make the digit thinner

Layer 3 is RELU layer which makes all negative value in layer 2 zero, but negative value are displayed as black, so layer2 and layer3 display the same.

Part 6







```
image1.JPG contains 10 numbers:  1 2 3 4 5 6 3 8 7 0
image2.JPG contains 10 numbers:  1 2 3 9 5 5 7 8 1 0
image3.png contains 5 numbers:  6 0 6 2 4
image4.jpg contains 54 numbers:  1 0 7 1 1 6 3 2 6 1 4 6 3 4 1 4 2 2 0 5 4 4 1 1 1
1 4 9 0 2 5 5 1 7 7 4 4 1 7 4 2 9 1 4 3 4 2 2 9 7 4 4 1 1
```

Some digits in image4 are disconnected, so misdetection happened.