COMP4901L Assignment4 Writeup

> WONG Yuk Chun (ycwongal@connect.ust.hk, 20419764)

1.1 Lambertian alberdo

By conservation of energy,

$$\begin{aligned}
0 \le \int_{\Omega_{out}} f(\hat{\omega}_{in}, \hat{\omega}_{out}) \cos\theta_{out} d\hat{\omega}_{out} &\le 1 \\
\int_{\Omega_{out}} \frac{\rho}{\pi} \cos\theta_{out} d\hat{\omega}_{out} &\le 1 \\
\int_{-\pi}^{\pi} \int_{0}^{\frac{\pi}{2}} \frac{\rho}{\pi} \cos\theta_i \sin\theta_i d\theta_i d\phi_i &\le 1 \\
\int_{-\pi}^{\pi} \int_{0}^{\frac{\pi}{2}} \frac{\rho}{2\pi} \sin 2\theta d\theta d\phi &\le 1 \\
\int_{-\pi}^{\pi} \frac{\rho}{2\pi} [-\frac{1}{2} \cos 2\theta]_0^{\frac{\pi}{2}} d\phi &\le 1 \\
\int_{-\pi}^{\pi} \frac{\rho}{2\pi} d\phi &\le 1 \\
\frac{\rho}{2\pi} [\phi]_{-\pi}^{\pi} &\le 1 \\
\implies 0 \le \rho &\le 1
\end{aligned}$$

1.2 Foreshortening

1.2.1 Solid angle

The solid angle at X_1

$$d\omega = \frac{dA}{D^2}$$

The solid angle at X_2

$$d\omega = \frac{dA \cos\alpha}{D^2}$$

1.2.2 Irradiance ratio

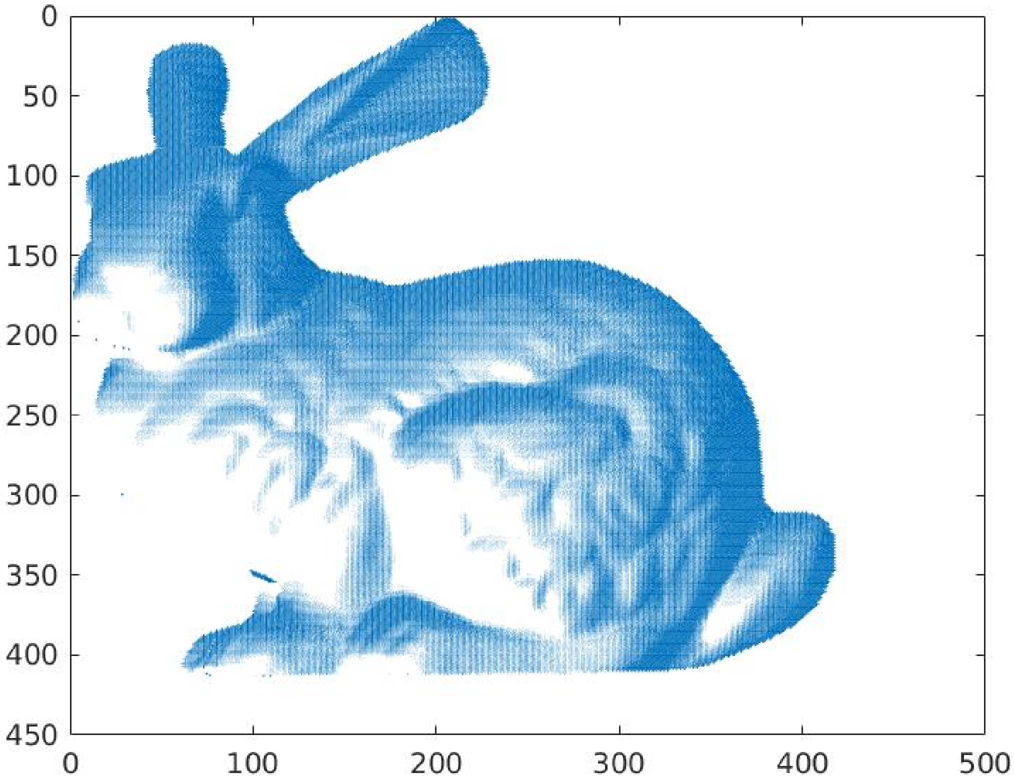
$$E(\omega) = \int_{\Omega} L(\omega_{in}) \cos\theta_{out} d\omega$$

Consider X_1 and X_2 as infinitesimal points, we have

$$\begin{aligned}
E(X_1) &= L \frac{dA}{D^2} \\
E(X_2) &= L \cos\alpha \frac{dA \cos\alpha}{D^2} \\
E(X_1)/E(X_2) &= \frac{1}{\cos^2\alpha}
\end{aligned}$$

1.3 Simple rendering

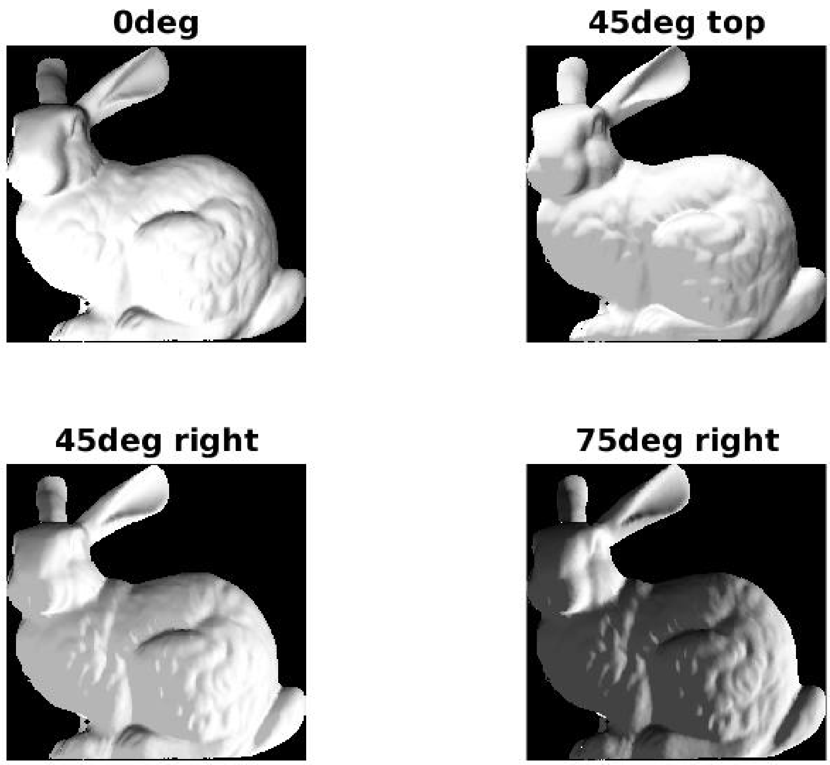
1.3.1 Normal visualization



1.3.2 light source at $\hat{s} = (0, 0, 1)$

The received intensity is albedo constant times dot product of light direction and normal direction time light intensity. Assume the light intensity is always 1, albedo equals 1(all light are reflected). Light direction is same as same as view port direction, the dot product of light direction and view port direction would only keep the z-component of the normal vector. Multiply all of these for all pixels, we have the z component of the normal map.

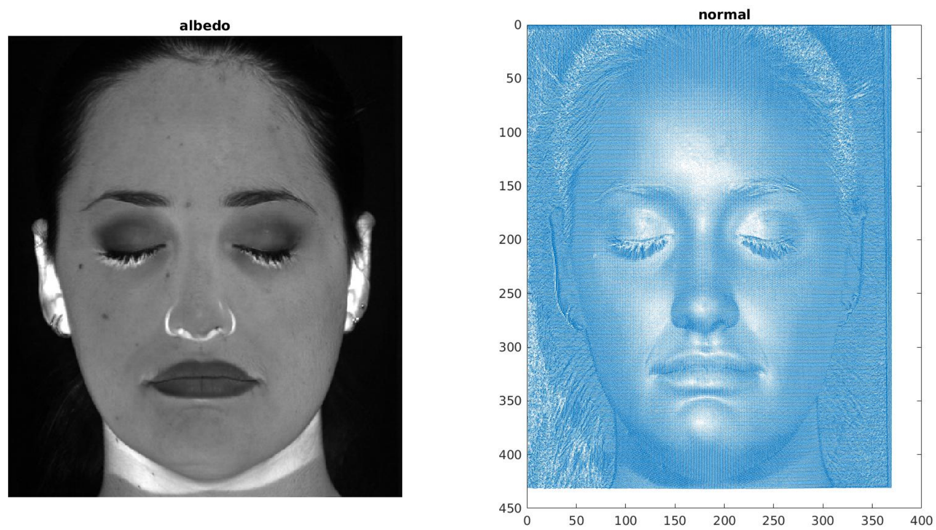
1.3.3 light source at 45deg up, 45 deg right, 75deg right



Some part of the rabbit should be blocked since the path to light is blocked by other part of the rabbit, aka shadow. Transparency and specularity are ignored. Distance attenuation is ignored.

1.4 Photometric stereo

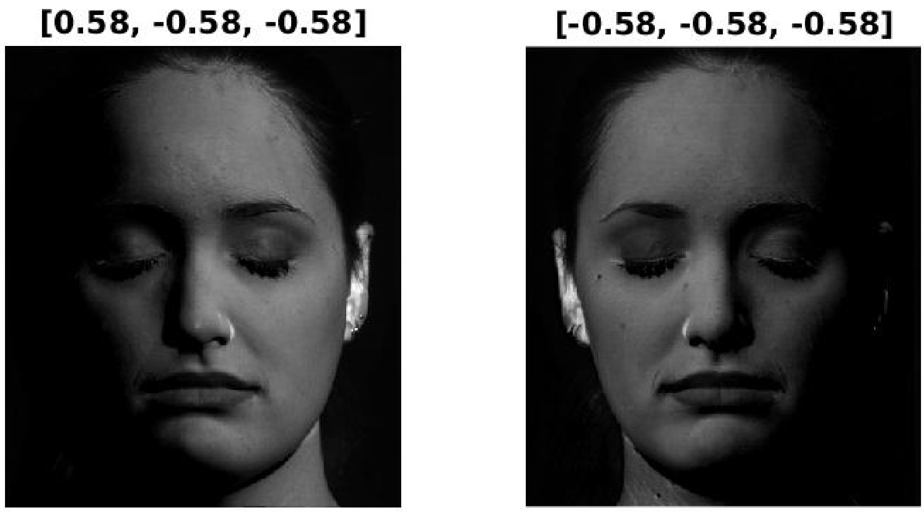
1.4.1 Solve for albedo and normal



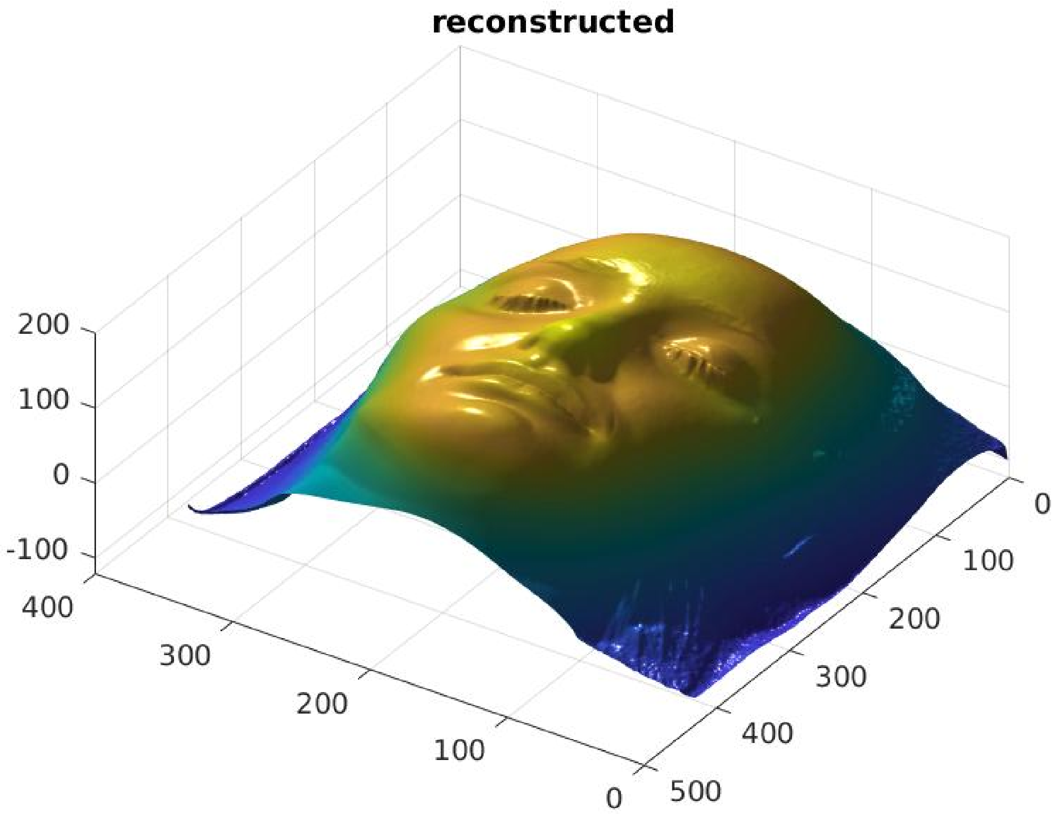
1.4.2 Nostrils error

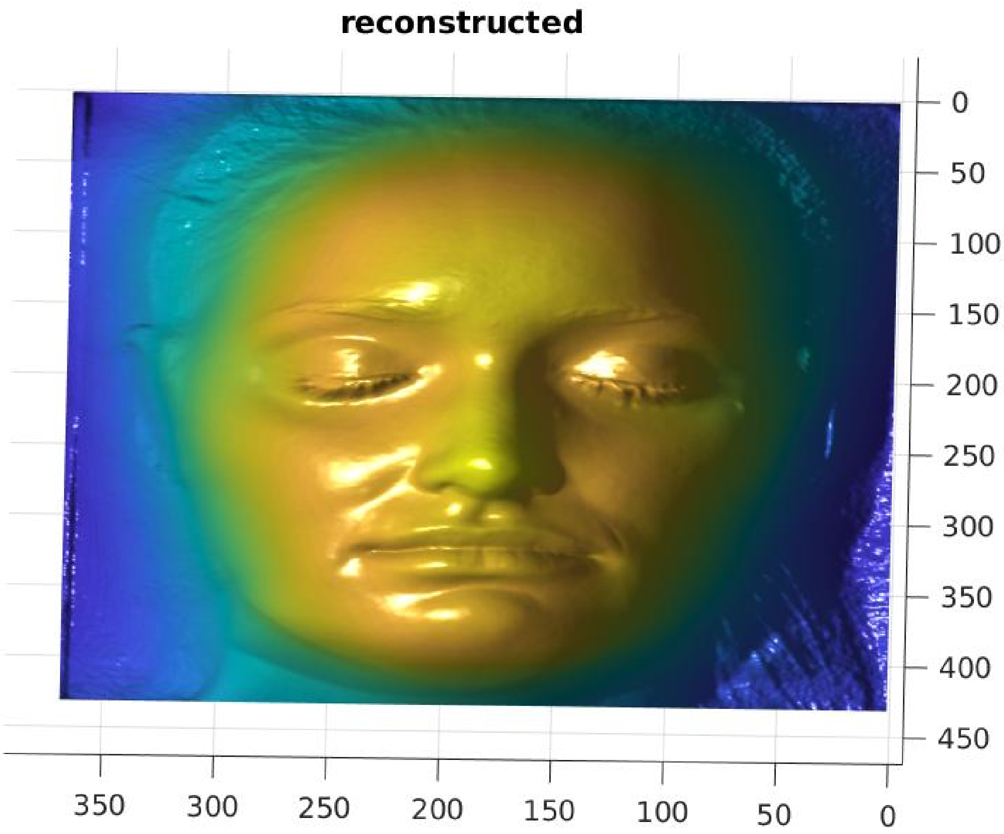
In using photometric stereo, we are making an assumption of no shadow, however nostrils have shadow. We can calculate albedo of nostrils using only photos that do not generate shadow around nostrils.

1.4.3 Stereo recover



1.4.4 Integrate frankot





1.5 Dichromatic reflectance

1.5.1

Let $\vec{c} = (c_R(\vec{u}), c_G(\vec{u}), c_B(\vec{u}))$

$$\begin{aligned}\vec{C}(\vec{u}) &= \langle \vec{n}(\vec{u}), \vec{l}\rangle \int_\lambda \vec{c} I(\lambda) f(\lambda, \hat{\omega}_i, \hat{\omega}_o) d\lambda \\ &= \langle \vec{n}(\vec{u}), \vec{l}\rangle \int_\lambda \vec{c} I(\lambda)(f_d(\lambda) + f_s(\hat{\omega}_i, \hat{\omega}_o)) d\lambda \\ \text{and } \vec{C}(\vec{u}) &= \langle \vec{n}(\vec{u}), \vec{l}\rangle \vec{d}(\vec{u}) + g_s(\vec{u})\vec{s}\end{aligned}$$

By comparing like terms, we have

$$\begin{aligned}\vec{d}(\vec{u}) &= \int_\lambda (c_R(\vec{u}), c_G(\vec{u}), c_B(\vec{u})) I(\lambda) f_d(\lambda) d\lambda \\ g_s(\vec{u})\vec{s} &= \langle \vec{n}(\vec{u}), \vec{l}\rangle \int_\lambda \vec{c} I(\lambda) f_s(\hat{\omega}_i, \hat{\omega}_o) d\lambda\end{aligned}$$

Since $f_s(\hat{\omega}_i, \hat{\omega}_o)\langle \vec{n}(\vec{u}), \vec{l}\rangle$ is a function depends non-linearly on $\hat{n}(\vec{u})$, we can set that as g_s and have $\vec{s} = \int_\lambda (c_R(\vec{u}), c_G(\vec{u}), c_B(\vec{u})) I(\lambda) d\lambda$.

1.5.2

As $\overrightarrow{r_1}, \overrightarrow{r_2}$ orthogonal to $\vec{s}$, we have $\langle \overrightarrow{r_1}, \vec{s}\rangle = \langle \overrightarrow{r_2}, \vec{s}\rangle = 0$.

$$\begin{aligned}\langle \overrightarrow{r_1}, \hat{C}(\hat{u})\rangle &= \langle \overrightarrow{r_1}, \langle \vec{n}, \vec{l}\rangle \vec{d}\rangle + \langle \overrightarrow{r_1}, g\vec{s}\rangle = \langle \overrightarrow{r_1}, \langle \vec{n}, \vec{l}\rangle \vec{d}\rangle \\ \langle \overrightarrow{r_2}, \hat{C}(\hat{u})\rangle &= \langle \overrightarrow{r_2}, \langle \vec{n}, \vec{l}\rangle \vec{d}\rangle + \langle \overrightarrow{r_2}, g\vec{s}\rangle = \langle \overrightarrow{r_2}, \langle \vec{n}, \vec{l}\rangle \vec{d}\rangle\end{aligned}$$

Obviously both of them independent of $g\vec{s}$, that is specular term equals 0, then it is independent from $f_s(\hat{\omega}_i, \hat{\omega}_o)$.

$\vec{d}$ is independent from $\vec{n}$ and the inner product is linear, so both component depend linearly on the surface normal.

1.5.3

$$
\begin{aligned}
J(\vec{u}) &= \sqrt{(\langle\overrightarrow{r_1}, \langle\vec{n}(\vec{u}), \vec{l}\rangle\vec{d}\rangle)^2 + (\langle\overrightarrow{r_1}, \langle\vec{n}(\vec{u}), \vec{l}\rangle\vec{d}\rangle)^2} \\
&= |\langle\vec{n}(\vec{u}), \vec{l}\rangle| \sqrt{\langle\overrightarrow{r_1}, \vec{d}\rangle^2 + \langle\vec{r}_2, \vec{d}\rangle^2} \\
&= \langle\vec{n}(\vec{u}), \vec{l}\rangle \sqrt{\langle\overrightarrow{r_1}, \vec{d}\rangle^2 + \langle\vec{r}_2, \vec{d}\rangle^2}
\end{aligned}
$$

Since original 2-channel image already independent from specular term, its gray scale image must be independent from the specular term.

Since $\langle\vec{n}(\vec{u}), \vec{l}\rangle$ must be positive or it is intersecting from the bottom side of the surface which is impossible. Then it depends linearly on the normal.

1.5.4

This is useful since the specularity are removed, which specularity introduce some view angle specific term to the picture that is not good for multiple photos computer vision processing. After that whole picture only depends linearly on surface normal and can produce better result for stereo reconstruction.

1.6 Color metamers

1.6.1

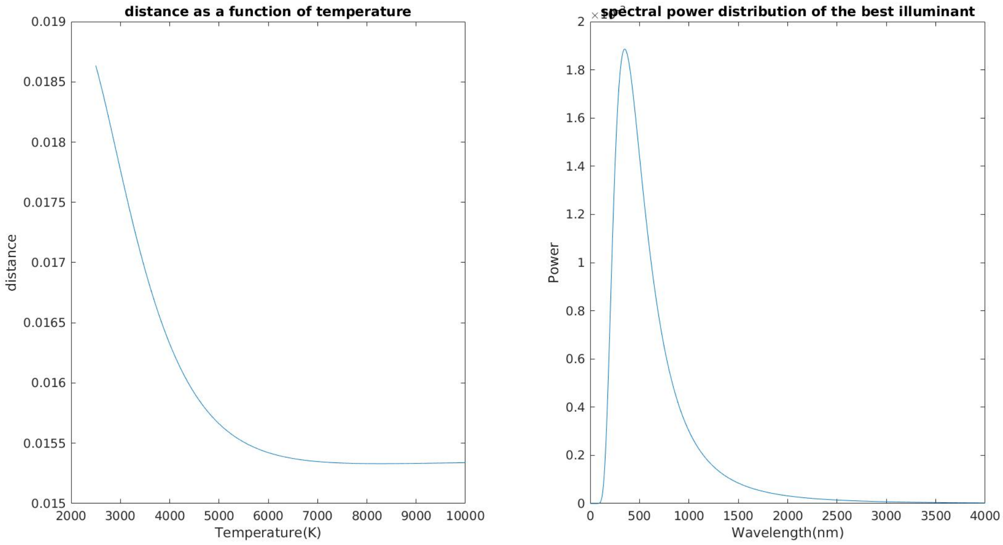
$$
\vec{C}_f = \int_\lambda \vec{C}(\lambda) f(\lambda) l(\lambda) d\lambda = R(\vec{f} \odot \vec{l}) = L_f \vec{l}
$$

i^{th} column of $(L_f)_i = R(:, i) f(i)$.

1.6.2

$$\left\| \vec{C}_f - \vec{C}_g \right\| = \left\| L_f \vec{l} - L_g \vec{l} \right\|$$

1.6.4



The best temperature is 8300K, precision 50K.