COMP4901L Assignment7 Writeup

> WONG Yuk Chun (ycwongal, 20419764)
>
> CHEUNG Daniel (dcheungaa, 20423088)

Part 1 Theory Questions

Q1.1: Calculating the Jacobian

$$\text{Assuming}$$

$$\mathbf{W}(\mathbf{x};\mathbf{p}) = \begin{bmatrix} \mathbf{W}_x \\ \mathbf{W}_y \\ \mathbf{W}_1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1+p_1 & p_3 & p_5 \\ p_2 & 1+p_4 & p_6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

$$\text{and,}$$

$$\mathbf{p} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 \end{bmatrix}^T$$

$$\begin{aligned} \frac{\partial \mathbf{W}}{\partial \mathbf{p}} &= \begin{bmatrix} \frac{\partial \mathbf{W}_u}{\partial \mathbf{p}_1} & \dots & \frac{\partial \mathbf{W}_u}{\partial \mathbf{p}_6} \\ \frac{\partial \mathbf{W}_v}{\partial \mathbf{p}_1} & \dots & \frac{\partial \mathbf{W}_v}{\partial \mathbf{p}_6} \end{bmatrix} \\ &= \begin{bmatrix} u & 0 & v & 0 & 1 & 0 \\ 0 & u & 0 & v & 0 & 1 \end{bmatrix} \end{aligned}$$

$$L = \sum_{\mathbf{x}} [\mathbf{T}(x) - \mathbf{I}(\mathbf{W})]^2$$

$$\begin{aligned} \mathbf{J} = \frac{\partial L}{\partial \mathbf{p}} &= \frac{\partial L}{\partial \mathbf{I}(\mathbf{W})} \frac{\partial \mathbf{I}(\mathbf{W})}{\partial \mathbf{W}} \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \\ &= -2\sum_{\mathbf{x}} [\mathbf{T}(x) - \mathbf{I}(\mathbf{W})] \begin{bmatrix} \frac{\partial \mathbf{I}}{\partial u} & \frac{\partial \mathbf{I}}{\partial v} \end{bmatrix} \begin{bmatrix} u & 0 & v & 0 & 1 & 0 \\ 0 & u & 0 & v & 0 & 1 \end{bmatrix} \end{aligned}$$

Q1.2: Computational complexity

Initialization step

Evaluate $\nabla \mathbf{T}$, which the runtime is $O(n)$

Evaluate $\mathbf{J} = \frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ at $\mathbf{p} = \mathbf{0}$,

To compute $\nabla \mathbf{T} \frac{\partial \mathbf{W}}{\partial \mathbf{p}}$, since $\nabla \mathbf{T}$ is of dimension $(n \times 2)$ and $\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ is of dimension $(2 \times p)$, this multiplication will have computational cost $O(np)$.

To compute Hessian matrix $\mathbf{H} = \mathbf{J^T J}$, since $\mathbf{J}$ is of dimension $(n \times p)$, this multiplication will have computational cost $O(np^2)$

Thus the initialization cost will be $O(np + np^2) = O(np^2)$.

Incremental step

First compute $\mathbf{I(W(x;p))}$, which is $O(np)$.

Then compute $[\mathbf{I(W(x;p))} - \mathbf{T}]$, which is $O(n)$.

Then compute $\sum_{\mathbf{x}} (\nabla \mathbf{T} \frac{\partial \mathbf{W}}{\partial \mathbf{p}})^{\mathbf{T}} [\mathbf{I(W(x;p))} - \mathbf{T(x)}]$, which is $O(pn)$.

Then multiply previous result with inverse of Hessian to get $\Delta\mathbf{p}$, which is $O(p^3)$.

Then update the warp function, which is $O(p^2)$

Total $O(pn + p^3)$

Part 2 Lucas-Kanade Tracker

Q2.1

video in `lk_results/`. initial rects:

car: `[180 180 36 36]`

landing: `[552 126 20 20]`

Q2.3

video in `mb_results`. initial rects:

car: `[126    102    208    177]`

landing: `[440     79    118     58]`

Q2.4

Small templates on spots or corners generally work best for Lucas-Kanade. Anything that would not exibit the barber-pole problem. The tracker tends to breakdown when large motions occur or when the brightness levels suddenly changed because there would be big different everywhere inside the tracker so it difficult for the Jacobian to tell which direction is the steepest

Part 3

Q3.1X Robust LK

Video in `ilk_results`, initial rect:

car: `[158   144    92    66]`

landing: `[440     79    118     58]`

use `ec/ilk_demo.m` to run

Q3.2X Pyramid LK

Video in `plk_results`, initial rect:

car: `[188   175    85    34]`

landing: `[440     82     52     44]`

use `ec/plk_demo.m` to run

Best Videos

For the best outputs, the following methods are used for the clips in our `results/` folder.

- Car: Robust LK

- Landing: Pyramid LK