

A theoretical and experimental research on detection efficiency of HPGe detector for disc source with heterogeneous distribution

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Abstract If we treat some heterogeneous distributed media including heterogeneous distributed radioactive sources, radioactive polluted field, “hot point/hot area” and “hot particle” as uniform distributed media, directly affects the accuracy of the detection efficiency calibrated, introduced the great metrical deviation. A Standard Normal distribution disc source was simulated by a point source experiment. The detection efficiency of heterogeneous distributed disc source is defined as multiply efficiency function of point source by the probability density function. And then we carried out process of the mathematical integrality and normalization. A description of the analysis and experimental results of this method are presented. The theoretical and experimental calculation of detection efficiency of HPGe detector for heterogeneous distribution disc source is consistent with each other.

Keywords Heterogeneous distribution · Detection efficiency · Hot particle · HPGe detector

Introduction

Detection efficiency is among the most important parameters for determining the operating characteristics of a spectrometer for radioactivity measurement [1]. In a case where the same efficiency is achieved, higher accuracy can reduce measurement uncertainty, thereby improving the utilization of the spectrometer. However, HPGe detectors often have some difficult in calibrating the full energy peak efficiency [2]. Therefore, improving the calibration of HPGe is important. The detection efficiency curve depends not only on a detection system but also on the radioactive material and the media. The simplest method is to use a closed point source standard placed at a distance from the detector to determine the full energy peak efficiency of cylindrical volume sources [3].

Large amounts of low and intermediate level radioactive waste including unknown quantity of ^{235}U , ^{239}Pu and other special nuclear material will be produced, piled up with the nuclear power industry development, decommission of nuclear facilities. The representative samples are hardly to obtain to carry out non-destructive assay-NDA for the heterogeneous distribution of ^{235}U and ^{239}Pu nuclear material. The ideal analytical technology is tomographic gamma scanner-TGS [4–6], the distribution of radionuclide composition and activity can be accurately measured, which do not need to change the structure of physics and chemistry. In the field of environmental radioactive measurement, we will deal with three types of source: volume source, disc source and point source. Volume source and disc source are not all uniform distribution, such as heterogeneous distributed radioactive source, “hot particle”, radioactive polluted field or “hot point/hot area”.

The first precondition is that the radioactive material and the media of object distribute uniformly, however, this will

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lead to great deviation of detection efficiency for the heterogeneous distributed source based on research. The theoretical calculation and experimental work for heterogeneous distribution source should be carried out, because it is the key factor of influence. Thus, investigating the parameters of different distribution is necessary to maximize the accuracy of the HPGe detector. In this paper, we use point source to simulation heterogenesis activity planar disc sources. A new calculation method about heterogeneous distribution source was developed by us after introducing the probability density function of the disc's activity distribution. The theoretical calculation and experimental measurement to the detection efficiency of HPGe detector for heterogeneous distributed disc source are shown in this paper.

The theoretical calculation method

The detection efficiency of HPGe detector is mainly determined by the factor of ray energy, dimension of disc source, distance, the attenuation of ray, the coincidence sum effect of ray, the dead time of measurement and the uniform of disc source. The proper parameters and methods can be found for the former six factors. For examples, the self-attenuation corrections can be calculated by LabSOCS Simulations [7]; The efficiency transfer (ET) method can also use a point source to calibrate the efficiency of the disc source, avoiding coincidence summing correction if exist, especially at close distance [8]. But the deviation caused by heterogeneity has not given by theories and experiments.

Monte Carlo simulation and LabSOCS software can also be used to calibrate efficiency of disc source [9–12], initially developed to complement the experimental calibration procedures. They can also be used to simulate heterogeneous distributed sources, but there is no an effective experimental method to simulate them.

Some authors also have developed methods to obtain the detection efficiency of volume or disc source by a point source calibrated at a point when the activity distribution of the sources is uniform [13–15]. For the uniform disc source, we assign S to the area of disc source, ε_d to the full-energy peak efficiency of disc source, $\varepsilon_p(r)$ to the full-energy peak efficiency function of point source. Then, a general integral formula is represented by the following Eq. (1):

$$\varepsilon_d = \frac{1}{S} \iint \varepsilon_p(r) ds = \frac{2\pi}{S} \int_0^R r \cdot \varepsilon_p(r) dr, \quad (1)$$

where p represents the point source, and R is the radius ($R = 3.75$ cm). The $\varepsilon_p(r)$ is found as a function of r . In this case, the semi-empirical formula of $\varepsilon_p(r)$ is preferred [16].

$$\varepsilon_p(r) = \exp(a \cdot r^2 + b \cdot r + c), \quad (2)$$

where a , b and c represent the fitted parameters, which can be given by the least-squares fit to experimental data. For the disc source, we assign f_d to the full-energy peak average rate of disc source cps/cm². Thus, a general integral formula is represented by the following Eq. (3):

$$f_d = \frac{1}{S} \iint f_p(r) ds = \frac{2\pi}{S} \int_0^R r \cdot f_p(r) dr, \quad (3)$$

where $f_p(r)$ is the full-energy peak rate (cps) in each point, found as a function of r . In this case, the semi-empirical formula of $f_p(r)$ is expressed as the following Eq. (4).

$$f_p(r) = \exp(lr^2 + mr + n), \quad (4)$$

where l , m and n represent the fitted parameters, which can be given by the least-squares fit to experimental data. For the heterogeneous distributed disc source, we assign A_d to the average activity of disc source Bq/cm². Thus, a general integral formula is represented by the following Eq. (5):

$$A_d = \frac{1}{S} \iint A_p(r) ds = \frac{2\pi}{S} \int_0^R r \cdot A_p(r) dr. \quad (5)$$

The $A_p(r)$ is the activity (Bq) found as a function of r . In this case, the semi-empirical formula of $A_p(r)$ is preferred.

$$A_p(r) = \exp(er^2 + dr + q), \quad (6)$$

where e , d and q represent the fitted parameters, which can be given by the least-squares fit to experimental data. A lot of sources are heterogeneous, but, the radioactive nuclides usually obey some probability density functions. Therefore, ε_d and $\varepsilon_p(r)$ satisfy the Eq. (8) after introducing the probability density function of the disc's activity distribution.

$$\begin{aligned} \varepsilon_d &= \frac{\frac{1}{S} \iint \varepsilon_p(r) \cdot \varphi(r) ds}{\frac{1}{S} \iint \varphi(r) ds} = \frac{\frac{2\pi}{S} \int_0^R r \cdot \varepsilon_p(r) \cdot \varphi(r) dr}{\frac{2\pi}{S} \int_0^R r \cdot \varphi(r) dr} \\ &= \frac{\int_0^R r \cdot \varepsilon_p(r) \cdot \varphi(r) dr}{\int_0^R r \cdot \varphi(r) dr}, \end{aligned} \quad (7)$$

namely:

$$\varepsilon_d = \frac{\int_0^R r \cdot \varepsilon_p(r) \cdot \varphi(r) dr}{\int_0^R r \cdot \varphi(r) dr}, \quad (8)$$

where $\varphi(r)$ represents the probability density function of the disc's activity distribution which is found as a function of r . The physical meanings of Eq. (8) is that the detection efficiency of disc source is defined as multiply $\varepsilon_p(r)$ by $\varphi(r)$. And then we carried out process of the mathematical integrality and normalization.

Different disc sources have different probability density function of $\varphi(r)$. In this paper, we selected some certain distribution function to make an in-depth study of this

theory. Therefore, the discussion is carried out about the Standard Normal distribution disc source. The Standard Normal distribution is expressed as Eq. (9).

$$\varphi(r) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{r^2}{2}\right\}. \quad (9)$$

The Eqs. (9) and (2) are integrated into Eq. (8), which can be expressed as another form of Eq. (8).

$$\begin{aligned} \varepsilon_d &= \frac{\frac{1}{\sqrt{2\pi}} \int_0^R r \cdot \exp(a \cdot r^2 + b \cdot r + c) \exp\left(-\frac{r^2}{2}\right) dr}{\frac{1}{\sqrt{2\pi}} \int_0^R r \cdot \exp\left\{-\frac{r^2}{2}\right\} dr} \\ &= \frac{\int_0^R r \cdot \exp\left(a \cdot r^2 + b \cdot r + c - \frac{r^2}{2}\right) dr}{\int_0^R r \cdot \exp\left\{-\frac{r^2}{2}\right\} dr} \end{aligned} \quad (10)$$

Experimental simulations

Sources and apparatus

The Ultra-Low Background γ spectrum system produced by CANBERRA Company was used as the measurement instruments. It comprises a BE3830 detector with nominal diameter of 70.0 mm and nominal length of 30.0 mm, relative efficiency of 35.6 %. The HPGe detector has full width at half maximum of 1.79 keV for ^{60}Co gamma energy transition at 1332.5 keV. It is connected to a multi-channel spectrometer (DSA1000). The software Genie-2000 was used for acquisition and analysis of spectral data. A reference ^{137}Cs (661.6 keV) point source was used in the experiment, certified by China Institute of Atomic Energy (CIAE). The activity of point source was 844 Bq with uncertainty of 1.9 % ($k = 2$).

Experiment

The experiment was carried out by using one point source. The measured information consisted of the full energy peak area. The counting time for each measured spectra was large enough to ensure a statistical accuracy better than 1%. The full energy peak rates were measured at a specific source-detector distance of 1.0 cm with different radium varied from 0.0 to 3.8 cm ($r = 0.0, 0.76, 1.52, 2.28, 3.04$ and 3.8). The net count rates were obtained through the Genie-2000 gamma spectrum analysis. The data is shown in Table 1 as follows. In this simulation, the attenuation in the disc matrix is not taken into account, because the height of the disc is negligible. If the height is not negligible, the correction method of the attenuation in the disc matrix can be found in Ref. [7] (Fig. 1).

$$y = -0.0355 \cdot x^2 - 0.0188 \cdot x - 3.53 \quad R^2 = 1.0 \quad (11)$$

Equation (11) shows the fitted parameters of $a = -0.0355$, $b = -0.0188$ and $c = -3.53$. The correlation

Table 1 The full-energy peak net count rates N/cps for uniform distribution

r (cm)	$f(r)$ (cps)	$\varepsilon(r) \times 10^{-2}$	$\text{Ln } \varepsilon(r)$
0	21.0	2.926	−3.53
0.76	20.5	2.851	−3.56
1.52	18.8	2.618	−3.64
2.28	16.8	2.333	−3.76
3.04	14.3	1.998	−3.91
3.8	11.8	1.636	−4.11

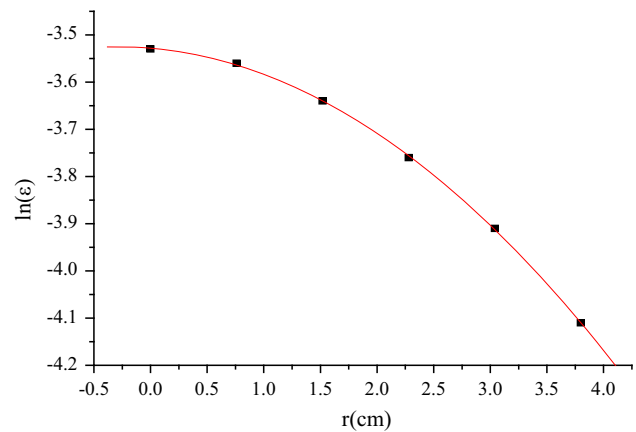


Fig. 1 Fitting curve of logarithmic efficiency of point source ($\ln(\varepsilon(r))$) versus different radius (r)

Table 2 The experimental data for the standard normal distribution

r (cm)	$\varphi(r)$	$\varphi(r)'$	$A(r)$ (Bq)	$f'(r)$ (cps)
0	0.3990	1367	1,153,400	28,716
0.76	0.2989	1024	864,083	20,963
1.52	0.1257	430	363,315	8,093
2.28	0.02966	102	85,731	1,702
3.04	0.003929	13	11,355	193
3.8	0.0002920	1.0	844	11.8

coefficient of $R^2 = 1.0$ is presented as well. The Eq. (2) is integrated into Eq. (1). And then we obtained the detection efficiency of 0.0221 through the integral Eq. (1). The Standard Normal distribution disc source cannot be made easily, but, we presented an experimental simulation by a point source and the Standard Normal distribution function. Firstly, we think that the nuclides along radius obey the Standard Normal distribution. And then the probability values of $\varphi(r)$ are calculated by Eq. (9), which are shown in Table 2; Secondly, normalization processing is carried out for $\varphi(r)$, thus, the normalization data of $\varphi(r)'$ are shown in Table 2; Finally, we multiplied 844(Bq) by $\varphi(r)'$ to obtain the Standard Normal distribution activity of $A(r)$, shown in Table 2. Then, the fitted parameters of e , d and q can be given by the least-squares fit to

experimental data of $A(r)$ and r . The Eq. (6) is integrated into Eq. (5). And then we get A_d through the integral Eq. (5). We multiplied the $f(r)$ (in Table 1) by $\varphi(r)'$ to get the Standard Normal distribution peak count rate $f_d(r)$, shown in Table 2. The fitted parameters of l , m and n can be given by the least-squares fit to experimental data of $f(r)$ and r . The Eq. (4) is integrated into Eq. (3). And then we get f_d through the integral Eq. (3)

$$y = -0.50 \cdot x^2 - 4.0 \times 10^{-14} \cdot x + 14.0 \quad R^2 = 1.0 \quad (12)$$

Equation (12) shows the fitted parameters of $e = -0.5$, $d = -4.0 \times 10^{-14}$ and $q = 14.0$. And then we get $A_d = 1.71 \times 10^5$ Bq/cm² through the integral Eq. (5) (Fig. 2).

$$y = -0.536 \cdot x^2 - 0.0188 \cdot x + 10.3 \quad R^2 = 1.0 \quad (13)$$

Equation (13) shows the fitted parameters of $l = -0.536$, $m = -0.0188$ and $n = 10.3$. And then we get $f_d = 3.85 \times 10^3$ cps/cm² through the integral Eq. (3). Finally, the detection efficiency is $\varepsilon = f_d / (A_d \cdot P_\gamma) = 3.85 \times 10^3 / (1.71 \times 10^5 \times 0.851) = 0.027$. The efficiency discrepancy of disc source between the Standard Normal distribution and uniform distributed is $\delta = (0.027 - 0.0221) / 0.0221 = 20\%$. We can see that an error of 20 % can be introduced when we treat Standard Normal distribution as uniform distribution. Therefore, the heterogeneous distributed source cannot be processed as uniform source (Fig. 3).

Applied methods

The application based on efficiency function of point source

Equation (10) can be used to calculate the detection efficiency of Standard Normal distribution disc source when

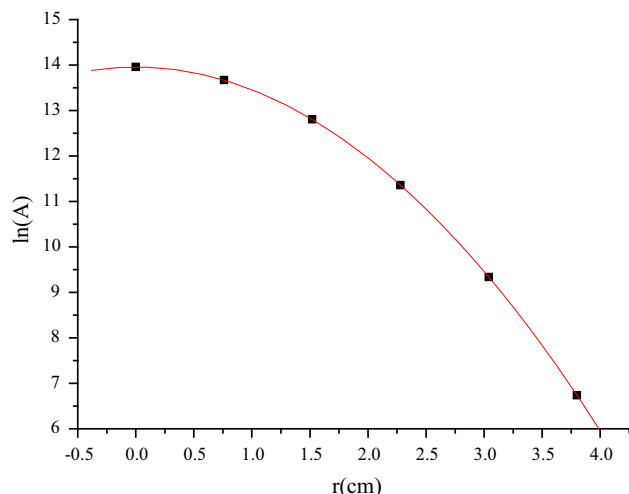


Fig. 2 Fitting curve of logarithmic activity of point source ($\ln(A(r))$) versus different radius (r)

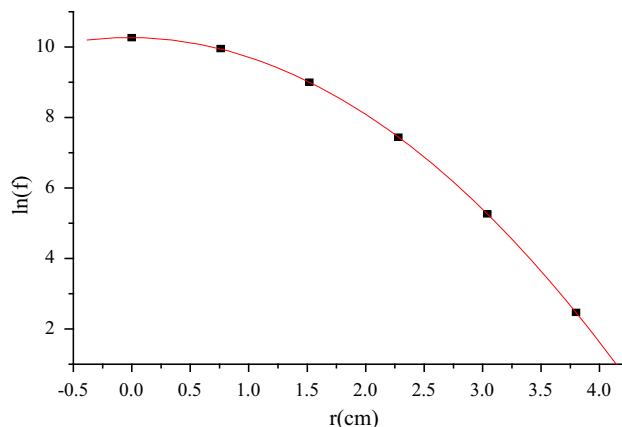


Fig. 3 Fitting curve of logarithmic peak count rate of point source ($\ln(f(r))$) versus different radius (r)

the efficiency function of point source is known. The fitted parameters of $a = -0.0355$, $b = -0.0188$ and $c = -3.53$ have obtained. And then we obtain the detection efficiency of 0.027 through the integral Eq. (10), which is consistent with result of Sect. 3.2.

The application based on total count rate

The relationship between $\varepsilon_p(r)$ and $f_p(r)$ can be expressed as following Eq. (14).

$$\varepsilon_p(r) = \frac{f_p(r)}{A(R) \cdot P_\gamma} \quad (14)$$

P_γ is the corresponding photo emission probability, which is 0.851. The $A_p(R)$ is the activity (Bq) at the edge of the disc source which is 844(Bq). Generally, we put the detector on the center of disc source at certain distance to obtain the total peak count rate f_d when the large Standard Normal distribution disc source is actually measured. The probability density function of $\varphi(r)'$ should be used in this situation. The Eq. (14) is integrated into Eq. (7). The deduced procession of Eq. (7) can be expressed as another form of Eq. (15).

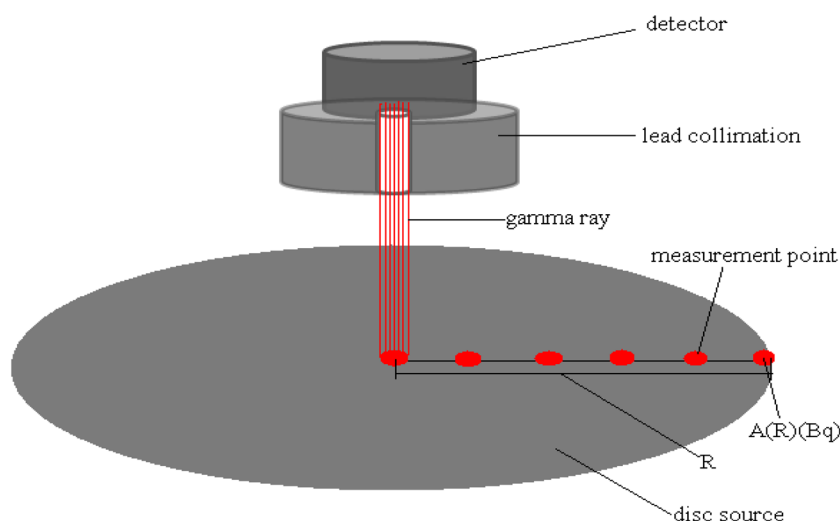
$$\varepsilon_d = \frac{\frac{1}{S} \iint f_p(r) \cdot \varphi(r)' ds}{A(R) \cdot P_\gamma \cdot \frac{1}{S} \iint \varphi(r)' ds} = \frac{\frac{2\pi}{S} \int_0^R r \cdot f_p(r) \cdot \varphi(r)' dr}{A(R) \cdot P_\gamma \cdot \frac{2\pi}{S} \int_0^R r \cdot \varphi(r)' dr} \quad (15)$$

In this paper, the probability density function $\varphi(r)'$ can be given by the least-squares fit to experimental data of $\varphi(r)'$ and r in Table 2, which can be expressed as Eq. (16).

$$\varphi(r)' = \exp(-0.5 \cdot r^2 + 7.22) \quad (16)$$

$$f_d = \frac{2\pi}{S} \int_0^R r \cdot f_p(r) \cdot \varphi(r)' dr \quad (17)$$

Fig. 4 Activity distribution measurement. There are six measurement points. The gamma rays from these points can only be detected after they are collimated by the lead collimation. The activity distribution of disc source is heterogeneous. The activity of measurement points which is at the edge of disc source is defined as $A(R)(\text{Bq})$



The Eqs. (16) and (17) are integrated into Eq. (15), which can be expressed as another form of Eq. (18).

$$\varepsilon_d = \frac{f_d}{A(R) \cdot P_\gamma \cdot \frac{2\pi}{S} \int_0^R r \cdot \exp(-0.5 \cdot r^2 + 7.22) dr} \quad (18)$$

We obtain $f_d = 3,854$ cps through the integral Eq. (17). In the end, we can get detection efficiency of 0.028 through the calculation of Eq. (18), which is consistent with result of Sect. 3.2, indicating the credibility of this method.

Results and discussion

In my opinion, there are two methods to obtain the activity distribution and $A(R)(\text{Bq})$; one is collimation method, such as in situ HPGe γ spectrometry, shown in Fig. 4. A collimated ray beam will produce the full energy peak rates in the detector when the detector is located at measurement point. Another is the sampling method. The sample can be gotten at measurement point, shown in Fig. 4, and then measured by laboratory HPGe detector or mass spectrometer. And then, the parameters of activity distribution can be given by the least-squares fit to these experimental data. $A(R)(\text{Bq})$ can also be given by these two methods. If we know the activity distribution previously, the detection efficiency will be more accurately, and the uncertainty of activity concentration analysis for radioactive nuclide will be improved.

On the other hand, If the total activity (or point activity) and probability density function of source are known, we can ascertain range and boundary of contaminated site in radioactivity pollution survey when using on-site γ spectrometer system to scan radioactive polluted field or search for “hot point/hot area” and “hot particle”. The theory

applied in fixing boundary of contaminated site may be carried out in-depth study. Firstly, the distribution must be known. Therefore, the probability density function for all kind sources should be carried out to study; Secondly, for the application based on total count rate, f_d must be obtained after we put the detector on the center of disc source at some distance. And then the activity $A(R)$ at the edge of disc source is obtained by the measurement. The detection efficiency can be calculated when these two conditions are satisfied.

Conclusions

This paper presents a method to correct the detection efficiency of HPGe detectors when an heterogeneous disk source is used. The technical results can be applied to fix detection efficiency of large disc source accurately in our actual work, improving the uncertainty of activity concentration analysis for radioactive nuclide. This method is based on using an empirical formula that is obtained from fitting a quadratic formula. We show that the HPGe efficiency can be calculated correctly for a Standard Normal distribution disc source. By performing an experimental simulation, we show the influence of the heterogeneity of a disc source in the efficiency computation. In conclusion, the results for two applied technology are consistent with each other. Appropriate theoretical and experimental method will be selected according to actually applied situation. We think that the theory and experimental method in this paper will be applied in study of radionuclide migration, “hot particle”, radioactive polluted field or “hot point/hot area” and the heterogeneous distribution of ^{235}U and ^{239}Pu nuclear material.

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