5-Day Diet Optimization Problem

Sets and Indices

I: set of food items (indexed by i); $D = \{0, 1, 2, 3, 4, 5, 6\}$: set of days (indexed by d)

Parameters

c_i: cost per serving (USD); f_i: fat (g); s_i: sodium (mg); p_i: protein (g); k_i: calories (kcal); b_i: fiber (g); u_i: sugar (g)

Decision Variables

$x_{i,d} \in \{0, 1, 2\}$: servings of food i on day d; $y_{i,d} \in \{0, 1\}$: binary indicator for food i on day d

Objective Function

$$\min \sum_{i \in I} \sum_{d \in D} c_i \cdot x_{i,d} \tag{1}$$

Constraints

Daily nutritional constraints: For each day $d \in D$,

$$\sum_{i \in I} f_i \cdot x_{i,d} \leq 275 \quad \text{(Fat)} \qquad \sum_{i \in I} s_i \cdot x_{i,d} \leq 6000 \quad \text{(Sodium)} \tag{2}$$

$$\sum_{i \in I} p_i \cdot x_{i,d} \geq 150 \quad \text{(Protein)} \qquad \sum_{i \in I} k_i \cdot x_{i,d} \geq 10000 \quad \text{(Calories)} \tag{3}$$

$$\tag{4}$$

Maximum items per day: For each day $d \in D$,

$$\sum_{i \in I} x_{i,d} \leq 30 \tag{5}$$

Linking constraints: For each food item $i \in I$ and day $d \in D$,

$$x_{i,d} \leq 2 \cdot y_{i,d} \qquad x_{i,d} \geq y_{i,d} \tag{6}$$

Minimum variety per day: For each day $d \in D$,

$$\sum_{i \in I} y_{i,d} \geq 10 \tag{7}$$

Maximum frequency per food: For each food item $i \in I$,

$$\sum_{d \in D} y_{i,d} \leq 2 \tag{8}$$

Variable domains: $x_{i,d} \in \{0, 1, 2\} \ \forall i \in I, d \in D$; $y_{i,d} \in \{0, 1\} \ \forall i \in I, d \in D$