

5-Day Diet Optimization Problem

Sets and Indices

I : set of food items (indexed by i); $D = \{0, 1, 2, 3, 4\}$: set of days (indexed by d)

Parameters

c_i : cost per serving (USD); f_i : fat (g); s_i : sodium (mg); p_i : protein (g); k_i : calories (kcal); b_i : fiber (g); u_i : sugar (g)

Decision Variables

$x_{i,d} \in \{0, 1, 2\}$: servings of food i on day d ; $y_{i,d} \in \{0, 1\}$: binary indicator for food i on day d

Objective Function

$$\min \sum_{i \in I} \sum_{d \in D} c_i \cdot x_{i,d} \quad (1)$$

Constraints

Daily nutritional constraints: For each day $d \in D$,

$$\sum_{i \in I} f_i \cdot x_{i,d} \leq 275 \quad (\text{Fat}) \quad \sum_{i \in I} s_i \cdot x_{i,d} \leq 6000 \quad (\text{Sodium}) \quad (2)$$

$$\sum_{i \in I} p_i \cdot x_{i,d} \geq 150 \quad (\text{Protein}) \quad \sum_{i \in I} k_i \cdot x_{i,d} \geq 10000 \quad (\text{Calories}) \quad (3)$$

$$\sum_{i \in I} b_i \cdot x_{i,d} \geq 75 \quad (\text{Fiber}) \quad \sum_{i \in I} u_i \cdot x_{i,d} \leq 100 \quad (\text{Sugar}) \quad (4)$$

Maximum items per day: For each day $d \in D$,

$$\sum_{i \in I} x_{i,d} \leq 30 \quad (5)$$

Linking constraints: For each food item $i \in I$ and day $d \in D$,

$$x_{i,d} \leq 2 \cdot y_{i,d} \quad x_{i,d} \geq y_{i,d} \quad (6)$$

Minimum variety per day: For each day $d \in D$,

$$\sum_{i \in I} y_{i,d} \geq 10 \quad (7)$$

Maximum frequency per food: For each food item $i \in I$,

$$\sum_{d \in D} y_{i,d} \leq 2 \quad (8)$$

Variable domains: $x_{i,d} \in \{0, 1, 2\} \forall i \in I, d \in D$; $y_{i,d} \in \{0, 1\} \forall i \in I, d \in D$