Consider the following transition system

$$\langle A\,[e_1\underline{@}e_2]\,;\ \Gamma;\ \Delta\rangle \longrightarrow \langle A\,[\underline{e_1}@e_2]\ ;\ \Gamma;\ e_2\bullet\Delta\rangle \qquad \text{[App]}$$

$$\langle A\,[\underline{\lambda x}.e]\,;\ \Gamma;\ \$\bullet\Delta\rangle \longrightarrow \langle A\,[\lambda x.\underline{e}]\,;\ \Gamma;\ \$\bullet\Delta\rangle \qquad \text{[Lam-Non-Elim]}$$

$$\langle A\,[\underline{\lambda x}.e]\,;\ \Gamma;\ B\bullet\Delta\rangle \longrightarrow \langle A\,[\lambda x.\underline{e}]\,;\ (x,B,B')\bullet\Gamma;\ \Delta\rangle,\ B\neq\$ \qquad \text{[Lam-Elim]}$$

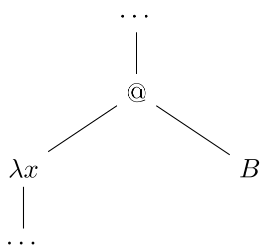
$$\langle A\,[\underline{x}]\,;\ (x,B,B')\bullet\Gamma;\ \Delta\rangle \longrightarrow \langle A\,[\underline{B}]\,;\ (x,B,B')\bullet\Gamma;\ \Delta\rangle \qquad \text{[BVar]}$$

Th 1. The above TS has following properties:
if$\langle A;\Gamma;\Delta\rangle \to \langle A';\Gamma';\Delta'\rangle$ then

1. either$\Gamma' = \Gamma$ or$\Gamma' = (x,B,B')\bullet\Gamma$ and(x,B') is a redex or it will become a redex after sequence of consecutive reductions that strongly follow Γ
   Denotation: let $\rightrightarrows$ denotes a sequential substition sequence of elements in Γ
2. $A \sim_\beta A'$

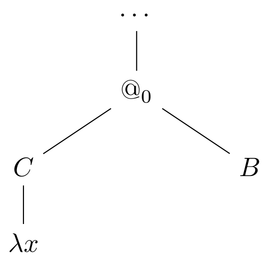
Proof .
■
First, note that all TS rules except $[Lam-Elim]$ do not change context Γ.
Second, also note that obviously the first element that will be added in Γ is a redex.
Third, note that when we extend context Γ with (x,B,B') we have one of the following situations:

- (x,B') is a redex itself. In other words, current term A is as follows:



And there is nothing to prove.

- (x,B') is not a redex. In other words, current term A is as follows:



Note that in this case subterm C contains only application nodes $@_i$ and lambda nodes λx_i. Moreover, from the TS rules it is easy to see that each lambda node λx_i is bound to some application node $@_i$ and vise versa. This bound nodes are well-bracketed sequence.
In other words, there is no λx_j that have no binders becase in this case it have to be bound to $@_0$ and there is no $@_1$ such that it is not bound because in this case our λx have to be bound to it insted of a_0. One can easly see it following the fact that Δ is a LIFO-stack.

Thus pairs $(y,\_,D')$ from Γ will eliminate all of these intermediate nodes and by definition $(x,\_,B')$ becomes a redex.

■
To prove the second statement note that only $[BVar]$ rule is of interest because all other TS rules do not change term itself.

In BVar-case we have a left side term $A[\underline{x}]$ and a right side term $A[\underline{B}]$ such that $(x,B,B')\in\Gamma$. Let $A[\underline{x}] \rightrightarrows A'[B']$ where A' is some new term. But in then $A[\underline{B}] \rightrightarrows A'[B']$ because $A[\underline{x}] == A[\underline{B}]$ up to the underlined subterm and $B \rightrightarrows B'$ by condition.

□