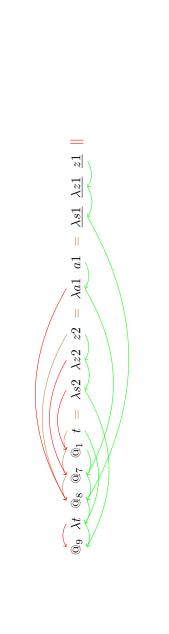
Notation:

|| denotes puase;
= denotes substitution;
= bounds lambdas with corresponding arguments;

→ bounds lambdas with corresponding arguments;

→ are pointers to last unfinished application;

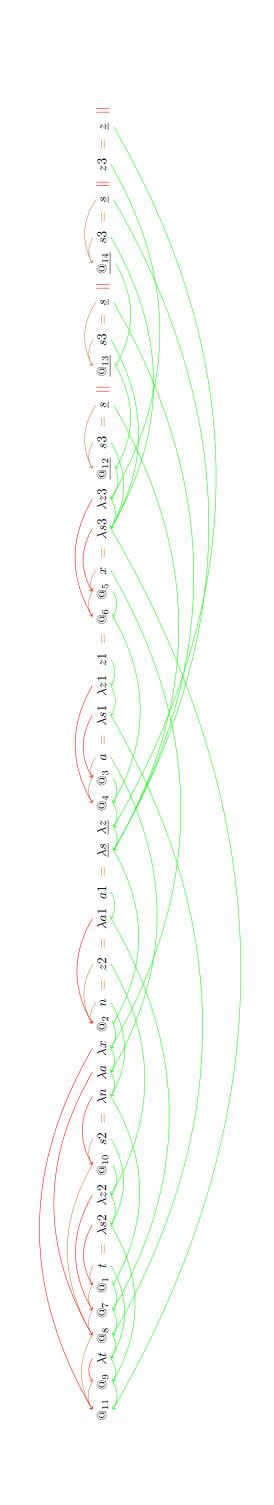
→ are binder pointers (invariant: for (BVar) it points to the corresponding (Lam) that bounds it; otherwise it point to the parent with respect to tree structure); elements of traversal that will appear in normalized term are <u>underlined</u>.



Normal form:  $\lambda s1.\lambda z1.z1$ 

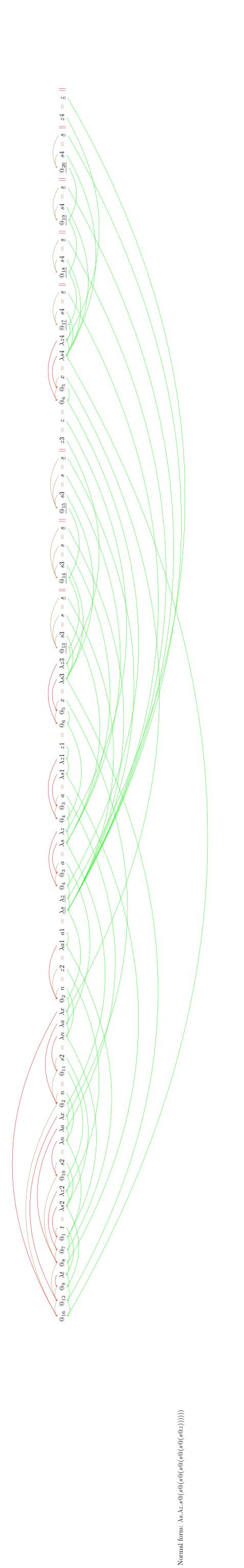
2

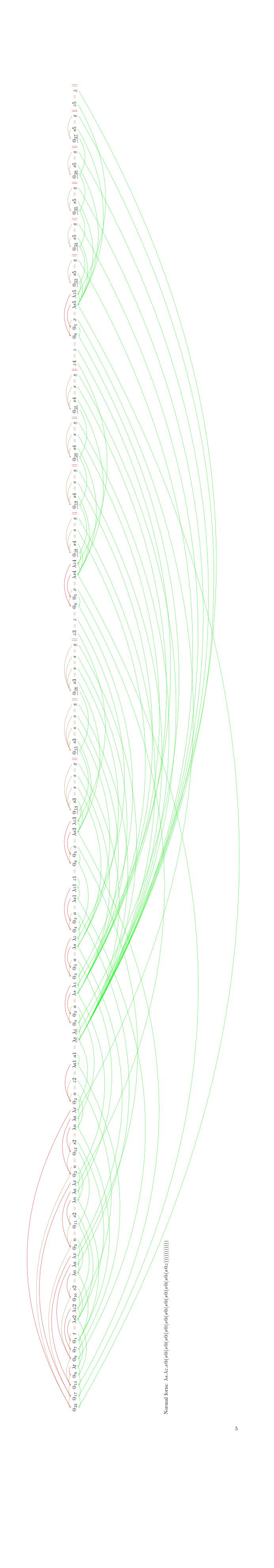
Example p zero Input term:  $(\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.z2)$ 



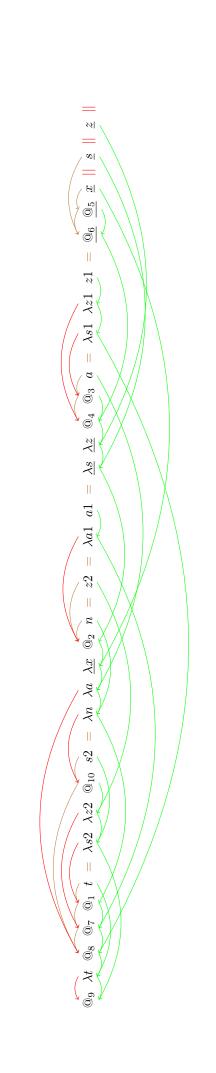
Example p one three Input term:  $((\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.s2@_{10}z2))@_{11}(\lambda s3.\lambda z3.s3@_{12}(s3@_{13}(s3@_{14}z3)))$ 

Normal form:  $\lambda s.\lambda z.s@(s@(s@z))$ 



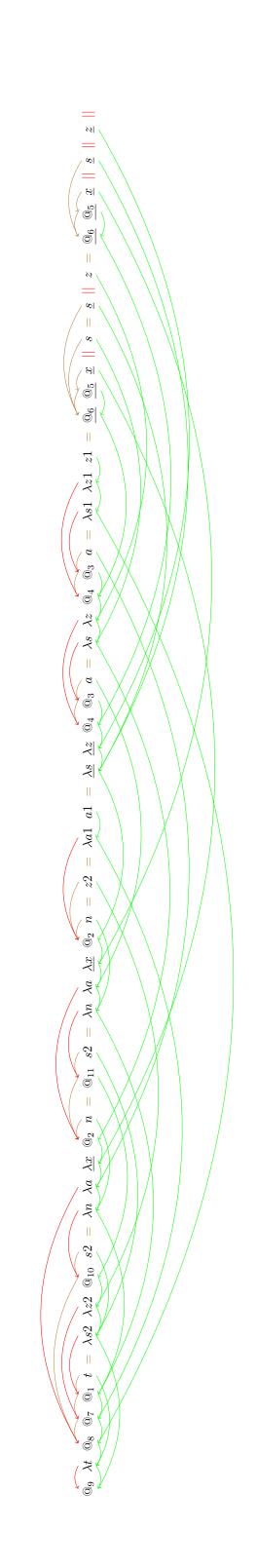


Example p three three four five



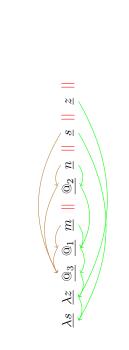
Normal form:  $\lambda x.\lambda s.\lambda z.(x@s)@z$ 

Example p one Input term:  $(\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.s2@_{10}z2)$ 



Normal form:  $\lambda x.\lambda x.\lambda s.\lambda z.(x@s)@((x@s)@z)$ 

Example p two Input term:  $(\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.s2@_{10}(s2@_{11}z2))$ 



Normal form:  $\lambda s.\lambda z.(m@(n@s))@z$ 

Example ex mult free m n Input term:  $\lambda s.\lambda z.(m@_1(n@_2s))@_3z$