# Example ack three

2.s2@(s2@(s2@(s2@(s2@z2)))))Input term:  $(\lambda m.(m@_1(\lambda g.\lambda n.(n@_2g)@_3(g@_4(\lambda s1.\lambda z1.s1@_5z1))))@_6(\lambda n2.\lambda s2.\lambda z2.(n2@_7s2)@_8(s2@_9z2)))@_{10}(\lambda s.\lambda z.s@_{11}(s@_{12}(s@_{13}z)))$  Normal form:  $\lambda n.(n@(\lambda n.(n@(\lambda n.\lambda z.\lambda z2.\lambda z2.\lambda z2.(n2@_sz)@(s2@_zz))))@(\lambda s2.\lambda z2.s2@(s2@(s2@(s2@(s2@(s2@(z2)))))@(\lambda s2.\lambda z2.s2@(s2@(s2@(s2@(z2)))))@(\lambda s2.\lambda z2.s2@(s2@(s2@(s2@(z2)))))$ 

Example p zero . Input term:  $(\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.z2)$  Normal form:  $\lambda s1.\lambda z1.z1$ 

 $z3.s3@_{12}(s3@_{13}(s3@_{14}z3)))$  $z1)))@_{9}(\lambda s2.\lambda z2.s2@_{10}z2))@_{11}(\lambda s3.\lambda z)$ 21: **₹** .a1))@ $8(\lambda s1$ Input term:  $((\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))))@_7(\lambda a.\lambda a.\lambda z.s@(s@(s@z))))$  Normal form:  $\lambda s.\lambda z.s@(s@(s@z))$ 

 $\lambda z 3.s3@_{13}(s3@_{14}(s3@_{15}z3))))@_{16}(\lambda s 4.\lambda z 4.s4@_{17}(s4@_{18}(s4@_{19}(s4@_{20}z4))))\\$ Example p two three four . In the proof of the proof of

 $\lambda s5.\lambda z5.s5@_{23}(s5@_{24}(s5@_{25}(s5@_{26}(s5@_{27}z5)))))$  $@_{13}(\lambda s3.\lambda z3.s3@_{14}(s3@_{15}(s3@_{16}z3)))) @_{17}(\lambda s4.\lambda z4.s4@_{18}(s4@_{19}(s4@_{20}(s4@_{21}z4))))) @_{22}(s3.\lambda s3.\lambda s3.s3@_{14}(s3@_{16}z3)))) @_{17}(\lambda s4.\lambda s4.\lambda s4.s4@_{18}(s4@_{19}(s4@_{20}(s4@_{21}z4))))) @_{22}(s3.\lambda s3.s3.s3)) @_{22}(s3.\lambda s3.s3.s3) \\$  $s2@_{10}(s2@_{11}(s2@_{12}z2))))$ % 22 .  $z1)))@9(\lambda s2.\lambda$ 2 

 $(\lambda z_1.z_1)))@_9(\lambda s_2.\lambda z_2.s_2@_{10}z_2)$ Example p one . Input term:  $(\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))))@_7(\lambda a1.a1))@_8(\lambda s1.a1))$ Normal form:  $\lambda x.\lambda s.\lambda z.(x@s)@z$ 

Example p two . Input term:  $(\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.s2@_{10}(s2@_{11}z2))$  Normal form:  $\lambda x.\lambda x.\lambda x.\lambda s.\lambda z.(x@s)@((x@s)@z)$ 

**Example ex\_1** . Input term:  $(g@_1(\lambda n.n))$  Normal form:  $g@\lambda n.n$ 

**Example ex\_2** . Input term:  $((\lambda h.h)@_1(\lambda f.f))@_2a$  Normal form: a

## Example ex\_3 . Input term: $((\lambda h.h@_1a)@_2(\lambda f.f))$ Input term: $((\lambda h)$ Normal form: a

 $\lambda f.\lambda y.(y@_1f)@_2y$ Input term:  $\lambda f.\lambda y.(y@_1f)@_2y$ Normal form:  $\lambda f.\lambda y.(y@f)@y$ Example ex\_4

**Example ex\_4'** . Input term:  $\lambda f.\lambda y.(y@_1(\lambda z.z))@_2y$  Normal form:  $\lambda f.\lambda y.(y@\lambda z.z)@y$ 

Input term:  $\lambda y.\lambda f.(y@_1f)@_2y$ Normal form:  $\lambda y.\lambda f.(y@f)@y$ Example ex\_5

Example succ two . Input term:  $(\lambda n.\lambda s.\lambda z.(n@_1s)@_2(s@_3z))@_4(\lambda p.\lambda o.y.$  Normal form:  $\lambda s.\lambda z.s@(s@(s@(s@z))$ 

 $(\lambda x.x@_1x)@_2(\lambda z.z)$ **Example ex\_9** Input term:  $(\lambda x.x \otimes$  Normal form:  $\lambda z.z$ 

Input term:  $(\lambda x.\lambda y.x@_1(x@_2y))@_3(\lambda p.q)$ Normal form:  $\lambda y.q$ Example ex\_11

Example ex\_f0 . Input term:  $(\lambda n.(n@_1(\lambda s.\lambda z.s@_2(s@_3((n@_4s)@_5z))))@_6(\lambda s1.\lambda z1.z1))@_7(\lambda s2.\lambda z2.z2)$  Normal form:  $\lambda s1.\lambda z1.z1$ 

Example ex\_f1 . Input term:  $(\lambda n.(n@_1(\lambda s.\lambda z.s@_2(s@_3((n@_4s)@_5z))))@_6(\lambda s1.\lambda z1.z1))@_7(\lambda s2.\lambda z2.s2@_8z2)$  Normal form:  $\lambda z.\lambda z1.z1$ 

Example ex\_f2 . Input term:  $(\lambda n.(n@_1(\lambda s.\lambda z.s@_2(s@_3((n@_4s)@_5z)))))@_6(\lambda s1.\lambda z1.z1))@_7(\lambda s2.\lambda z2.s2@_8(s2@_9z2))$  Normal form:  $\lambda z.\lambda z1.z1$ 

**Example ex\_LO1** . Input term:  $(\lambda f.\lambda x.f@_1(x@_2((f@_3x)@_4x)))@_5(\lambda a.\lambda b.a)$  Normal form:  $\lambda x.\lambda b.x@x$ 

**Example ex\_LO2** . Input term:  $(\lambda x.(x@_1x)@_2x)@_3(\lambda a.\lambda b.a)$  Normal form:  $\lambda a.\lambda b.a$ 

**Example ex\_LO3** . Input term:  $(\lambda x.x@_1(\lambda y.y))@_2(\lambda a.\lambda b.b)$  Normal form:  $\lambda b.b$ 

**Example ex\_LO4** . Input term:  $(\lambda x.x@_1(\lambda y.y))@_2(\lambda a.\lambda b.a)$  Normal form:  $\lambda b.\lambda y.y$ 

**Example ex\_1** . Input term:  $(g@_1(\lambda n.n))$  Normal form:  $g@\lambda n.n$ 

Example NPR .  $| (\lambda h. \lambda z. ((h@_1(\lambda x. ((h@_2(\lambda q. x))@_3a))) @_4(z@_5a))) @_6(\lambda f. \lambda y. f@_7((g@_8(\lambda b. b))@_9y))) @_{10}(g@_{11}(\lambda n. n))) \\$  | Normal form:  $(g@\lambda b. b)@((g@\lambda n. n)@a)$ 

Example mut three two . Input term:  $((\lambda m.\lambda n.\lambda s.\lambda z.(m@_1(n@_2s))@_3z)@_4(\lambda a.\lambda q.a@_5(a@_6(a@_7q))))@_8(\lambda d.\lambda e.d@_9(d@_{10}e))$  Normal form:  $\lambda s.\lambda z.s@(s@(s@(s@(s@(s@z)))))$