## 1 Labelled Transition System for Head Linear Reduction

## 1.1 Notes

State is a tuple  $\langle \lambda$ -term with underlined node, context, list of arguments  $\rangle$ , where

- $\lambda$ -term with underlined node is a usual lambda term with one underlined position;
- context  $\Gamma$  is an unordered list of pair (variable : term);
- list of arguments  $\Delta$  is an ordere list of  $\lambda$ -terms.

## 1.2 Rules

$$\dots (e_1 \underline{@} e_2) \dots; \ \Gamma; \ \Delta \longrightarrow \dots (e_1 \underline{@} e_2) \dots; \ \Gamma; \ e_2 : \Delta$$

$$\dots (\underline{\lambda x}.e_1)\dots; \ x:B,\ \Gamma;\ B,\ \Delta \longrightarrow \dots (\lambda x.\underline{e_1})\dots; \ x:B,\ \Gamma;\ \Delta$$

3. (Lam-non-elim)

$$\ldots(\underline{\lambda x}.e_1)\ldots;\ x:B,\ \Gamma;\ \$,\ \Delta\longrightarrow\ldots(\lambda x.\underline{e_1})\ldots;\ \Gamma;\ \$,\ \Delta$$

4. (BVar)

$$\dots \underline{x} \dots; \ x : B, \ \Gamma; \ \Delta \longrightarrow \dots \underline{B} \dots; \ x : B, \ \Gamma; \ \Delta$$

5. (FVar-pause-0)

$$\dots \underline{x} \dots ; (x : \underline{\ }) \notin \Gamma; B, \Delta \longrightarrow \dots \underline{B} \dots ; \Gamma; \$, \Delta$$

6. (FVar-pause-1)

$$\ldots \underline{x}\ldots;\ (x:\underline{\ })\not\in\Gamma;\ \$,\ B,\ \Delta\longrightarrow\ldots\underline{B}\ldots;\ \Gamma;\ \$,\ \Delta$$

7. (FVar-pause-2)

$$\dots \underline{x} \dots ; (x: ) \notin \Gamma; \$, \$, \Delta \longrightarrow \dots \underline{x} \dots ; \Gamma; \$, \Delta$$

8. (FVar–stuck)

$$\dots \underline{x} \dots; (x:\underline{\phantom{x}}) \notin \Gamma; \emptyset \longrightarrow \text{THE END}$$