

Example ack three

Input term: $(\lambda m.(m @_1 (\lambda g.\lambda n.(n @_2 g) @_3 (g @_4 (\lambda s1.\lambda z1.s1 @_5 z1)))) @_6 (\lambda n2.\lambda s2.(n2 @_7 s2) @_8 (s2 @_9 z2))) @_{10} (\lambda s.\lambda z.s @_{11} (s @_{12} (s @_{13} z)))$
Normal form: $\lambda n.(n @ (\lambda n.(n @ (\lambda n2.\lambda s2.\lambda z2.(n2 @ s2) @ (s2 @ s2)))) @ (\lambda s2.\lambda z2.s2 @ (s2 @ (s2 @ (s2 @ (s2 @ z2)))) @ (\lambda s2.\lambda z2.s2 @ (s2 @ (s2 @ z2))))$

Example p zero

Input term: $(\lambda t.(((t @_1 (\lambda n.\lambda a.\lambda x.n @_2 (\lambda s.\lambda z.(a @_3 s) @_4 ((x @_5 s) @_6 z)))) @_7 (\lambda a1.a1)) @_8 (\lambda s1.\lambda z1.z1))) @_9 (\lambda s2.\lambda z2.z2)$
Normal form: $\lambda s1.\lambda z1.z1$

Example p one three

Input term: $((\lambda t.(((t @_1 (\lambda n.\lambda a.\lambda x.n @_2 (\lambda s.\lambda z.(a @_3 s) @_4 ((x @_5 s) @_6 z)))) @_7 (\lambda a1.a1)) @_8 (\lambda s1.\lambda z1.z1))) @_9 (\lambda s2.\lambda z2.s2 @_{10} z2))) @_{11} (\lambda s3.\lambda z3.s3 @_{12} (s3 @_{13} (s3 @_{14} z3))))$
Normal form: $\lambda s.\lambda z.s @ (s @ (s @ (s @ (s @ z))))$

Example p two three four

Input term: $(((\lambda t.(((t @_1 (\lambda n.\lambda a.\lambda x.n @_2 (\lambda s.\lambda z.(a @_3 s) @_4 ((x @_5 s) @_6 z)))) @_7 (\lambda a1.a1)) @_8 (\lambda s1.\lambda z1.z1))) @_9 (\lambda s2.\lambda z2.s2 @_{10} (s2 @_{11} z2))) @_{12} (\lambda s3.\lambda z3.s3 @_{13} (s3 @_{14} (s3 @_{15} z3)))) @_{16} (\lambda s4.\lambda z4.s4 @_{17} (s4 @_{18} (s4 @_{19} (s4 @_{20} z4))))$
Normal form: $\lambda s.\lambda z.s @ (s @ (s @ (s @ (s @ (s @ z))))$

Example p three three four five

Input term: $(((((\lambda t.(((t @_1 (\lambda n.\lambda a.\lambda x.n @_2 (\lambda s.\lambda z.(a @_3 s) @_4 ((x @_5 s) @_6 z)))) @_7 (\lambda a1.a1)) @_8 (\lambda s1.\lambda z1.z1))) @_9 (\lambda s2.\lambda z2.s2 @_{10} (s2 @_{11} z2))) @_{13} (\lambda s3.\lambda z3.s3 @_{14} (s3 @_{15} z3)))) @_{17} (\lambda s4.\lambda z4.s4 @_{18} (s4 @_{19} (s4 @_{20} (s4 @_{21} z4)))) @_{22} (\lambda s5.\lambda z5.s5 @_{23} (s5 @_{24} (s5 @_{25} (s5 @_{26} (s5 @_{27} z5))))))$
Normal form: $\lambda s.\lambda z.s @ (s @ (s @ (s @ (s @ (s @ (s @ z))))$

Example p one

Input term: $(\lambda t.(((t @_1 (\lambda n.\lambda a.\lambda x.n @_2 (\lambda s.\lambda z.(a @_3 s) @_4 ((x @_5 s) @_6 z)))) @_7 (\lambda a1.a1)) @_8 (\lambda s1.\lambda z1.z1))) @_9 (\lambda s2.\lambda z2.s2 @_{10} z2)$
Normal form: $\lambda x.\lambda s.\lambda z.(x @ s) @ z$

Example p two

Input term: $(\lambda t.(((t @_1 (\lambda n.\lambda a.\lambda x.n @_2 (\lambda s.\lambda z.(a @_3 s) @_4 ((x @_5 s) @_6 z)))) @_7 (\lambda a1.a1)) @_8 (\lambda s1.\lambda z1.z1))) @_9 (\lambda s2.\lambda z2.s2 @_{10} (s2 @_{11} z2))$
Normal form: $\lambda x.\lambda x.\lambda s.\lambda z.(x @ s) @ ((x @ s) @ z)$

Example ex 1

Input term: $(g @_1 (\lambda n.n))$
Normal form: $g @ \lambda n.n$

Example ex 2

Input term: $((\lambda h.h) @_1 (\lambda f.f)) @_2 a$
Normal form: a

Example ex 3

Input term: $((\lambda h.h @_1 a) @_2 (\lambda f.f))$
Normal form: a

Example ex 4

Input term: $\lambda f.\lambda y.(y @_1 f) @_2 y$
Normal form: $\lambda f.\lambda y.(y @ f) @ y$

Example ex 4'

Input term: $\lambda f.\lambda y.(y @_1 (\lambda z.z)) @_2 y$
Normal form: $\lambda f.\lambda y.(y @ \lambda z.z) @ y$

Example ex 5

Input term: $\lambda y.\lambda f.(y @_1 f) @_2 y$
Normal form: $\lambda y.\lambda f.(y @ f) @ y$

Example succ two

Input term: $(\lambda n.\lambda s.\lambda z.(n @_1 s) @_2 (s @_3 z)) @_4 (\lambda p.\lambda o.p @_5 (p @_6 o))$
Normal form: $\lambda s.\lambda z.s @ (s @ (s @ z))$

Example ex 9

Input term: $(\lambda x.x @_1 x) @_2 (\lambda z.z)$
Normal form: $\lambda z.z$

Example ex 11

Input term: $(\lambda x.\lambda y.x @_1 (x @_2 y)) @_3 (\lambda p.q)$
Normal form: $\lambda y.q$

