## 1 Labelled Transition System for repeated Head Linear Reduction

## 1.1 Notes

State is a tuple  $\langle \lambda$ -term with underlined node, context, list of arguments  $\rangle$ , where

- $\lambda$ -term (a tree; by considering  $\lambda$ -term as a tree it becomes possible to cross arguments out of tree (... without term)) with underlined node is a usual lambda term with one underlined position;
- context  $\Gamma$  is an unordered list of pair (variable : term);
- list of arguments  $\Delta$  is an ordere list of  $\lambda$ -terms. (one can also think about  $\Delta$  as unordered list of pointers to the corresponding subtree)

## 1.2 Rules

$$\langle A [e_1@e_2]; \; \Gamma; \; \Delta \rangle \longrightarrow \langle A [e_1@e_2]; \; \Gamma; \; e_2 \bullet \Delta \rangle \qquad [App]$$

$$\langle A [\underline{\lambda x}.e]; \; \Gamma; \; B \bullet \Delta \rangle \longrightarrow \langle A_{\mathcal{K}}[\lambda x.e]; \; x : B, \; \Gamma; \; \Delta \rangle \qquad [Lam-Elim]$$

$$\langle A [\underline{\lambda x}.e]; \; \Gamma; \; \$ \bullet \Delta \rangle \longrightarrow \langle A [\lambda x.e]; \; \Gamma; \; \Delta \rangle \qquad [Lam-Non-Elim]$$

$$\langle A [\underline{x}]; \; x : B, \; \Gamma; \; \Delta \rangle \longrightarrow \langle A [\underline{B}]; \; x : B, \; \Gamma; \; \Delta \rangle \qquad [BVar]$$

$$\langle A [\underline{x}]; \; x : B, \; \Gamma; \; \Delta \rangle \longrightarrow \langle A [\underline{B}]; \; \Gamma; \; \$ \bullet \Delta \rangle, \; x \notin dom \; \Gamma \qquad [FVar-Pause-0]$$

$$\langle A [\underline{M} [\underline{x}] @B]; \; \Gamma; \; \$ \bullet B \bullet \Delta \rangle \longrightarrow \langle A [\underline{M} @\underline{B}]; \; \Gamma; \; \$ \bullet \Delta \rangle, \; x \notin dom \; \Gamma \qquad [FVar-Pause-1]$$

$$\langle A [\underline{x}]; \; \Gamma; \; \$ \bullet \$ \bullet \Delta \rangle \longrightarrow \langle A [\underline{x}]; \; \Gamma; \; \$ \bullet \Delta \rangle, \; x \notin dom \; \Gamma \qquad [FVar-Pause-2]$$