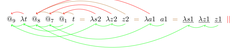
Notation:
|| denotes pause;
← denotes substitution;
→ bounds lambdas with corresponding arguments;
→ are pointers to last unfinished application within one run (between two neighbor '——');
⇒ are pointers to last unfinished application from one run to another one (pointer across some '——');
⇢ are binder pointers (invariant: for (BVar) it points to the corresponding (Lam) that bounds it; otherwise it point to the parent with respect to tree structure);
elements of traversal that will appear in normalized term are underlined.

Example p zero

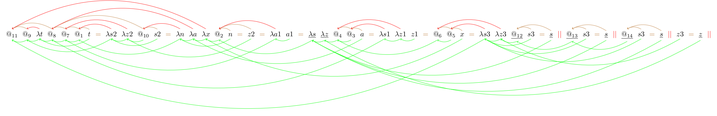
Input term: (λt.((((t(θ$_1$(λn.λa.λr.n(θ$_2$(λs.λz.(a(θ$_3$s)(θ$_4$((r(θ$_5$s)(θ$_6$z))))))(θ$_7$(λa1.a1)))(θ$_8$(λs1.λz1.z1)))))θ$_9$(λs2.λz2.z2))



Normal form: λs1.λz1.z1

Example p one three

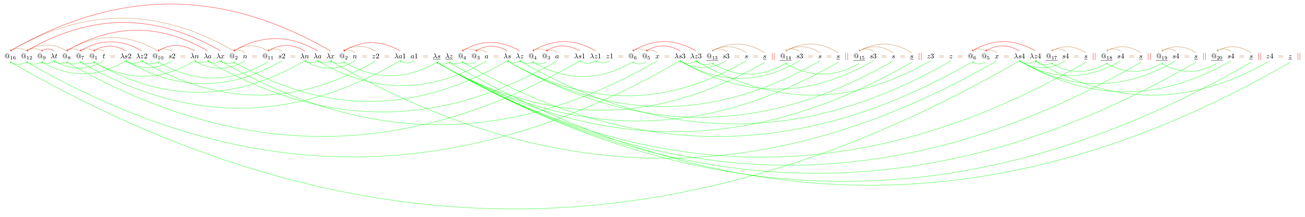
Input term: ((λt.(((t(θ$_{1}$(λn.λa.λx.nθ$_{2}$(λs.λz.(sθ$_{9}$s)θ$_{4}$((xθ$_{5}$s)θ$_{6}$z))))θ$_{7}$(λo1.o1))θ$_{8}$(λs1.λz1.z1)))θ$_{9}$(λs2.λz2.sθ$_{10}$z2))θ$_{11}$(λs3.λz3.s3θ$_{12}$(s3θ$_{13}$(s3θ$_{14}$z3))))



Normal form: λs.λz.sθ(sθ(sθ$_{2}$))

Example p two three four

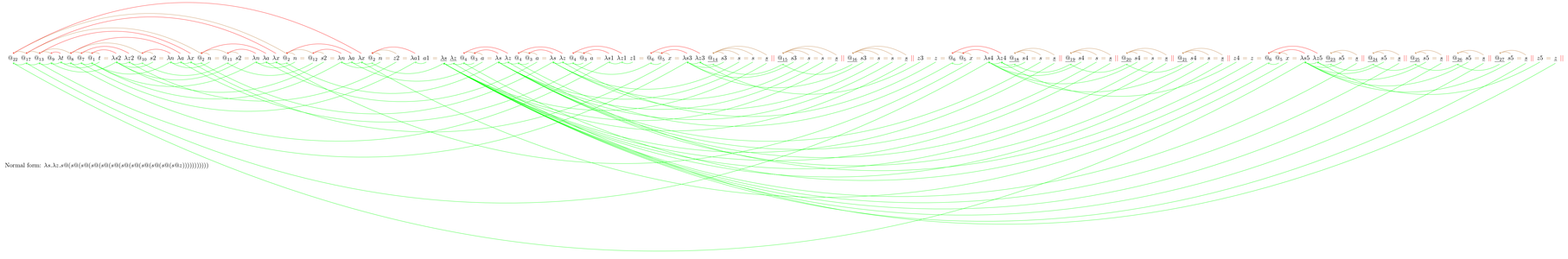
Input term: (((λf.(((θ$_{1}$(λn.λo.λr.n θ$_{2}$(λs.λz.(o θ$_{3}$s)θ$_{4}$((r θ$_{5}$s)θ$_{6}$z))))θ$_{7}$(λa1.a1))θ$_{8}$(λv1.λz1.z1)))θ$_{9}$(λx2.λz2.x2θ$_{10}$(x2θ$_{11}$z2)))θ$_{12}$(λv3.λz3.s30$_{13}$(s30$_{14}$(s30$_{15}$z3))))θ$_{16}$(λs4.λz4.s40$_{17}$(s40$_{18}$(s40$_{19}$(s40$_{20}$z4)))))



Normal form: λs.λz.s0((s0((s0((s0((s0((s0(z)))))))))))

Example p three three four five

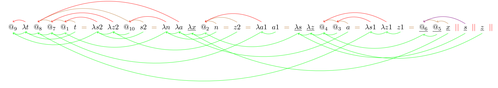
Input term: ((((λf.(((t0$_{1}$(λn.λa.λx.n0$_{2}$(λs.λz.(s0$_{3}$s)0$_{4}$((x0$_{5}$s)0$_{6}$)))0$_{7}$(λa1.a1))0$_{8}$(λa1.λz1.z1)))0$_{9}$(λx2.λz2.s20$_{10}$(s20$_{11}$(s20$_{12}$z2))))0$_{13}$(λs3.λz3.s30$_{14}$(s30$_{15}$(s30$_{16}$z3))))0$_{17}$(λs4.λz4.s40$_{18}$(s40$_{19}$(s40$_{20}$(s40$_{21}$z4))))))0$_{22}$(λs5.λz5.s50$_{23}$(s50$_{24}$(s50$_{25}$(s50$_{26}$(s50$_{27}$z5))))))



Normal form: λs.λz.s0((s0((s0((s0((s0((s0((s0((s0((s0((s0z)))))))))))))))))

Example p one

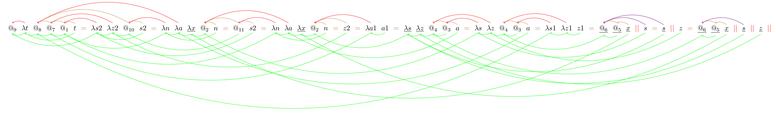
Input term: (λt.((((t0$_{1}$(λn.λa.λr.n0$_{2}$(λs.λz.(a0$_{3}$s)0$_{4}$((r0$_{5}$s)0$_{4}$z))))0$_{2}$(λa1.a1))0$_{6}$(λs1.λz1.z1)))0$_{6}$(λs2.λz2.s20$_{10}$z2)



Normal form: λr.λs.λz.(r0s)0z

Example p two

Input term: (λt.((((t0$_1$(λn.λa.λr.n0$_2$(λs.λz.(s0$_3$s)0$_4$((r0$_5$s)0$_6$z))))0$_7$(λa1.a1))0$_8$(λs1.λz1.z1)))0$_9$(λs2.λz2.s20$_{10}$(s20$_{11}$z2))



Normal form: λs.λz.λs.λz.(s0s)0((s0s)0z)

Example ex_1

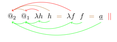
Input term: $(g 0 (\lambda n.n))$

Normal form: $g 0 \lambda n.n$

∞

Example ex_2

Input term: $((\lambda h.h)\theta_1(\lambda f.f))\theta_2 a$



Normal form: a

Example ex_3

Input term: $((\lambda h.h\,0\,i_1\,a)\,0\,i_2\,(\lambda f.f))$

Normal form: a

Example ex_4

Input term: $\lambda f.\lambda g.(g@_1 f)@_2 g$

Normal form: $\lambda f.\lambda g.(g@f)@g$

Example ex_4'

Input term: $\lambda f.\lambda g.(g\,0_1\,(\lambda z.z))\,0_2\,g$

Normal form: $\lambda f.\lambda g.(g 0 (\lambda z.z))\,0\,g$

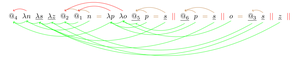
Example ex_5

Input term: $\lambda y.\lambda f.(y@_1 f)@_2 y$

Normal form: $\lambda y.\lambda f.(y@f)@y$

Example succ two

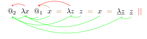
Input term: (λn.λs.λz.(n@1 s)@2(s@1 z))@4(λp.λs.p@1(p@1 s))



Normal form: λs.λz.s@(s@(s@1 z))

Example ex_9

Input term: $(\lambda x.x\theta_1 x)\theta_2(\lambda z.z)$



Normal form: $\lambda z.z$

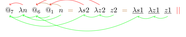
Example ex_11

Input term: $(\lambda x.\lambda y.x\theta_1(x\theta_2 y))\theta_3(\lambda p.q)$

Normal form: $\lambda y.q$

Example ex_f0

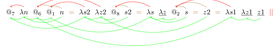
Input term: $(\lambda n.(n\,0_1\,(\lambda s.\lambda z.s\,0_2\,(s\,0_2\,((n\,0_4\,s)\,0_{\uparrow 2}))))\,0_6)(\lambda s1.\lambda z1.z1))\,0_7(\lambda s2.\lambda z2.z2)$



Normal form: $\lambda s1.\lambda z1.z1$

Example ex_f1

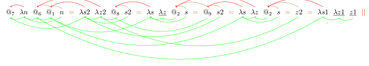
Input term: (λn.(n 0$_1$ (λs.λz.s 0$_2$ (s 0$_2$ ((n 0$_4$ s) 0$_{3}$z)))) 0$_6$ (λs1.λz1.z1)) 0$_7$ (λx2.λz2.x2 0$_8$ z2)



Normal form: λz.λz1.z1

Example ex_f2

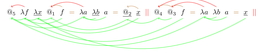
Input term: (λn.(n θ$_1$ (λs.λz.s θ$_2$ (s θ$_3$ ((n θ$_4$ s) θ$_5$ z)))) θ$_6$ (λs1.λz1.z1)) θ$_7$ (λs2.λz2.s2 θ$_8$ (s2 θ$_9$ z2))



Normal form: λz.λz1.z1

Example ex_LO1

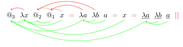
Input term: $(\lambda f.\lambda x.f\,@_1(x@_2((f@_3x)@_4x)))@_5(\lambda a.\lambda b.a)$



Normal form: $\lambda x.\lambda b.x@x$

Example ex_LO2

Input term: $(\lambda x.(x\,0_1\,x)\,0_2\,x)\,0_3\,(\lambda a.\lambda b.a)$



Normal form: $\lambda a.\lambda b.a$

Example ex_LO3

Input term: $(\lambda x.x\,\theta_1(\lambda y.y))\,\theta_2(\lambda a.\lambda b.b)$

Normal form: $\lambda b.b$

Example ex_LO4

Input term: $(\lambda x.x\,\theta_1(\lambda y.y))\,\theta_2(\lambda a.\lambda b.a)$

Normal form: $\lambda b.\lambda y.y$

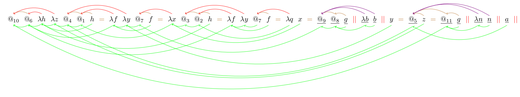
Example ex_1

Input term: $(g 0 (\lambda n.n))$

Normal form: $g 0 \lambda n.n$

Example NPR

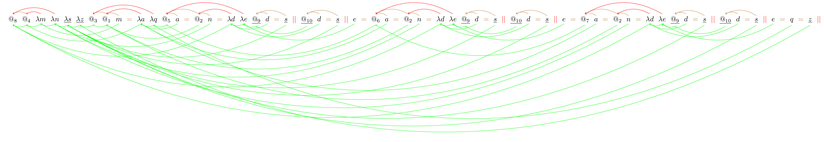
Input term: $((\lambda h.\lambda z.((h\theta_1(\lambda x.((h\theta_2(\lambda q.x))\theta_3 a)))\theta_4(=\theta_5 a)))\theta_6(\lambda f.\lambda y.f\,\theta_7((g\theta_8(\lambda b.b))\theta_9 y)))\theta_{10}(g\theta_{11}(\lambda n.n))$



Normal form: $(g\theta_1\lambda b.b)\theta((g\theta_1\lambda n.n)\,\theta a)$

Example mut three two

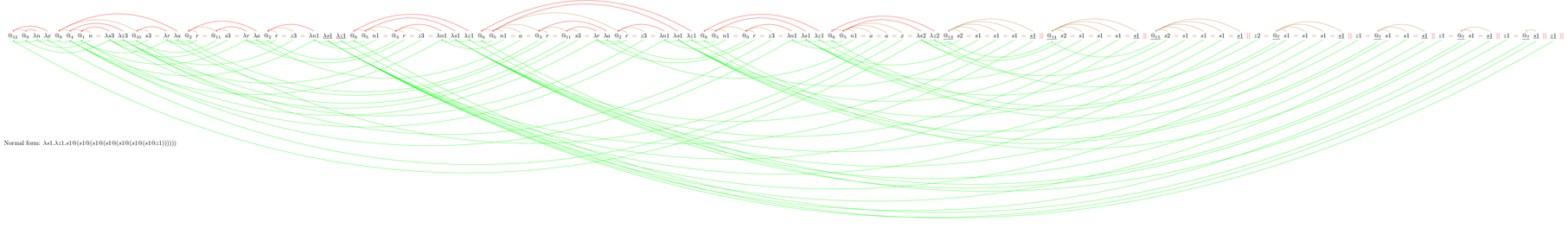
Input term: ((λm.λn.λs.λz.(m𝔅1(n𝔅2s))𝔅3z)𝔅4(λa.λq.a𝔅5(a𝔅6(a𝔅7q))))𝔅8(λd.λe.d𝔅9(d𝔅10e))



Normal form: λs.λz.s𝔅0(s𝔅0(s𝔅0(s𝔅0(s𝔅0(s𝔅0z)))))

Example plus 3 2^2

Input term: ((λn.λx.((n 0$_1$(λr.λa.r 0$_2$(r 0$_3$a 0)))0$_4$(λn1.λs1.λz1.(n1 0$_5$s1)0$_6$(s1 0$_7$z1)))0$_8$x)0$_9$(λs3.λz3.s3 0$_{10}$(s3 0$_{11}$z3)))0$_{12}$(λs2.λz2.s2 0$_{13}$(s2 0$_{14}$(s2 0$_{15}$z2)))



Normal form: λs1.λz1.s1 0(s1 0(s1 0(s1 0(s1 0(s1 0(s1 0(s1 0z1)))))))