

Example plus 3 2 square

Input term: $((\lambda n.\lambda x.((n@_1(\lambda r.\lambda a.r@_2(r@_3a)))@_4(\lambda n1.\lambda s1.\lambda z1.(n1@_5s1)@_6(s1@_7z1)))@_8x)@_9(\lambda s3.\lambda z3.s3@_{10}(s3@_{11}z3)))@_{12}(\lambda s2.\lambda z2.s2@_{13}(s2@_{14}(s2@_{15}z2))))$
Normal form: $\lambda s1.\lambda z1.s1@_{16}(s1@_{17}(s1@_{18}(s1@_{19}(s1@_{20}(s1@_{21}(s1@_{22}(s1@_{23}(s1@_{24}(s1@_{25}(s1@_{26}(s1@_{27}z5))))))))))$

Example plus x n square

Input term: $(\lambda n.\lambda x.((n@_1(\lambda r.\lambda a.r@_2(r@_3a)))@_4(\lambda n1.\lambda s1.\lambda z1.(n1@_5s1)@_6(s1@_7z1)))@_8x)$
Normal form: $\lambda n.\lambda x.((n@_1(\lambda r.\lambda a.r@_2(r@_3a)))@_4(\lambda n1.\lambda s1.\lambda z1.(n1@_5s1)@_6(s1@_7z1)))@_8x$

Example ack three

Input term: $(\lambda m.(m@_1(\lambda g.\lambda n.(n@_2g)@_3(g@_4(\lambda s1.\lambda z1.s1@_5z1)))@_6(\lambda n2.\lambda s2.\lambda z2.(n2@_7s2)@_8(s2@_9z2)))@_{10}(\lambda s.\lambda z.s@_{11}(s@_{12}(s@_{13}z))))$
Normal form: $\lambda n.(n@_1(\lambda n.(n@_1(\lambda n2.\lambda s2.\lambda z2.(n2@_2s2)@_3(s2@_4z2)))@_4(\lambda s2.\lambda z2.s2@_5(s2@_6z2)))@_6(\lambda s2.\lambda z2.s2@_7(s2@_8z2)))@_9(\lambda s2.\lambda z2.s2@_{10}(s2@_{11}z2))))$

Example p zero

Input term: $(\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.z2))$
Normal form: $\lambda s1.\lambda z1.z1$

Example p one three

Input term: $((\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.s2@_{10}z2)))@_{11}(\lambda s3.\lambda z3.s3@_{12}(s3@_{13}(s3@_{14}z3))))$
Normal form: $\lambda s.\lambda z.s@_{15}(s@_{16}(s@_{17}(s@_{18}(s@_{19}(s@_{20}z))))))$

Example p two three four

Input term: $((((\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.s2@_{10}(s2@_{11}z2)))@_{12}(\lambda s3.\lambda z3.s3@_{13}(s3@_{14}(s3@_{15}z3)))@_{16}(\lambda s4.\lambda z4.s4@_{17}(s4@_{18}(s4@_{19}(s4@_{20}z4))))@_{22}(\lambda s5.\lambda z5.s5@_{23}(s5@_{24}(s5@_{25}(s5@_{26}(s5@_{27}z5))))))$
Normal form: $\lambda s.\lambda z.s@_{28}(s@_{29}(s@_{30}(s@_{31}(s@_{32}(s@_{33}(s@_{34}(s@_{35}(s@_{36}(s@_{37}(s@_{38}(s@_{39}z))))))))))$

Example p three three four five

Input term: $(((((\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.s2@_{10}(s2@_{11}z2)))@_{13}(\lambda s3.\lambda z3.s3@_{14}(s3@_{15}(s3@_{16}z3)))@_{17}(\lambda s4.\lambda z4.s4@_{18}(s4@_{19}(s4@_{20}(s4@_{21}z4))))@_{22}(\lambda s5.\lambda z5.s5@_{23}(s5@_{24}(s5@_{25}(s5@_{26}(s5@_{27}z5))))))$
Normal form: $\lambda s.\lambda z.s@_{30}(s@_{31}(s@_{32}(s@_{33}(s@_{34}(s@_{35}(s@_{36}(s@_{37}(s@_{38}(s@_{39}(s@_{40}(s@_{41}z))))))))))$

Example p one

Input term: $(\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.s2@_{10}z2))$
Normal form: $\lambda x.\lambda s.\lambda z.(x@_5s)@_z$

Example p two

Input term: $(\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.s2@_{10}(s2@_{11}z2))$
Normal form: $\lambda x.\lambda x.\lambda s.\lambda z.(x@_5s)@_z((x@_5s)@_z)$

Example ex 1

Input term: $(g@_1(\lambda n.n))$
Normal form: $g@_{\lambda n.n}$

Example ex 2

Input term: $((\lambda h.h)@_1(\lambda f.f))@_2a$
Normal form: a

Example ex 3

Input term: $((\lambda h.h@_1a)@_2(\lambda f.f))$
Normal form: a

Example ex 4

Input term: $\lambda f.\lambda y.(y@_1f)@_2y$
Normal form: $\lambda f.\lambda y.(y@_1f)@_y$

Example ex 4'

Input term: $\lambda f.\lambda y.(y@_1(\lambda z.z))@_2y$
Normal form: $\lambda f.\lambda y.(y@_{\lambda z.z})@_y$

Example ex 5

Input term: $\lambda y.\lambda f.(y@_1f)@_2y$
Normal form: $\lambda y.\lambda f.(y@_1f)@_y$

Example succ two

Input term: $(\lambda n.\lambda s.\lambda z.(n@_1s)@_2(s@_3z))@_4(\lambda p.\lambda o.p@_5(p@_6o))$
Normal form: $\lambda s.\lambda z.s@_7(s@_8(s@_9z))$

