Partial Evaluation and Normalisation by Traversals

Work in progress by:

- **Daniil Berezun**
  Saint Petersburg State University

- **Neil D. Jones**
  DIKU, University of Copenhagen (prof. emeritus)

INTRODUCTION

The much-studied game semantics for PCF can be thought of as a PCF interpreter.

Ong [?] shows that

a λ-expression M can be evaluated (i.e. normalised) by the algorithm that constructs a traversal of *M*.

A *traversal* is a sequence of

- subexpressions of *M*. This is a finite set, whose elements we will call tokens (think: M = program, tokens = program points)
- Any token may have a back pointers.

With this approach to normalisation: there is *no need for β-reduction, environments, "thunks" or "closures"* to do the evaluation

START POINT

- A view of the Oxford normalisation procedure (ONP for short): It is

  an interpreter for λ-expressions

- ONP builds a set of traversal $\mathfrak{Trav}(M)$
  - Let $tr = t_0 \bullet t_1 \bullet \cdots \bullet t_n \in \mathfrak{Trav}(M)$ where t_i is a token
  - Syntax-directed inference rules:

    based on syntax of the end-token t_n
  - Action: add 0, 1 or more extensions of tr to $\mathfrak{Trav}(M)$. For each,
    * Add a new token t', yielding $tr \bullet t'$
    * Add a back pointer from t'

Data types:

- $tr \in \mathit{Tr} = \mathit{Item}^*$
- $\mathit{Item} = \mathit{subexpression}(M) \times \mathit{Tr}$

SOME ONP CHARACTERISTICS

Oxford normalization procedure

- Applies to **simply-typed** λ-expressions
- Begins by translating M into η-**long** form
- Realises the compete head linear reduction of M
- **Correctness:** by game semantics and categories, using M's types
- ONP uses no β-reduction: all is based on subexpressions of M
- While running, ONP does not use the types of M at all

Goals of this research:

- Extend ONP to UNP, for the *untyped* lambda calculus
- Partially evaluate a normaliser with respect to "static" input M.
  Use this to compile λ-calculus into a *low-level language*.

PARTIAL EVALUATION, BRIEFLY

A partial evaluator is a program specialiser. Defining property of $spec$:

$$\forall p \in Programs \,.\, \forall s, d \in Data \,.\, [\![[\![spec]\!](p, s)]\!](d) = [\![p]\!](s, d)$$

- **Program speedup by precomputation.**
- **Given program p and static data s, $spec$ builds a residual program**

$$p_s \stackrel{def}{=} [\![spec]\!]\; p\; s$$

- **Staging transformation:**
  - $[\![p]\!](s, d)$ **is a 1 stage computation**
  - $[\![[\![spec]\!](p, s)]\!](d)$ **is a 2 stage computation**
- **Applications: compiling, and compiler generation**

  An old idea: Semantics directed compiler generation

WHY PARTIALLY EVALUATE NP

1. The $spec$ equation for a normaliser program $\boxed{\textbf{NP} : \ \Lambda \rightarrow Traversals}$

$$\boxed{\forall M \in \Lambda \,.\, [\![\, [\![spec]\!](\textbf{NP}, M)]\!]() = [\![\textbf{NP}]\!](M)}$$

2. λ-calculus tradition: M is self-contained.

   So why break normalisation into 2 stages?

   (a) The specialised output $\textbf{NP}_M = [\![spec]\!](\textbf{NP}, M)$ can be in a much simplier language than λ-calculus.
   Our candidate is some low-level language, LLL.

   (b) 2 stages will be natural for *semantics-directed compiler generation*.
   LLL can be an intermediate language to express semantics:

   - $\textbf{NP}_1 \ = \ [\![spec]\!] \ \textbf{NP} \ M \quad : \ \Lambda \rightarrow LLL$
   - $\textbf{NP}_2 \ = \ [\![\_]\!] \quad : \ LLL \rightarrow Traversals$ a semantic function

HOW TO PARTIALLY EVALUATE ONP

1. Annotate parts of normalization procedure as either static or dynamic. Variables ranging over
   (a) tokens are static, (subexpressions of M; finitely many);
   (b) back pointers are dynamic; (unboundedly many)
   (c) so the traversal is dynamic too.

2. Computations in normalization proicedure NP are either unfolded or residualised (runtime code is generated to do them later)
   - Perform fully static computations at partial evauation time.
   - Operations to build or test a traversal: generate residual code.

THE RESIDUAL PROGRAM ONP$_M$ $= [\![spec]\!]$ ONP M

ONP is not quite structurally inductive but it is semi-compositional:

Any recursive ONP call has a substructure of M as argument.

Consequences:

- The partial evaluator can do, at specialisation time,

  all of the ONP operations that depend only on M

- ONP$_M$ performs no operations at all on lambda expressions

- A specialised program ONP$_M$ contains “residual code”:
  - operations to extend the traversal
  - operations to follow back pointers

- Subexpressions of M will appear, but are only used as tokens:

  Tokens are indivisible: used as labels and for equality comparisons

STATUS: OUR WORK ON SIMPLY-TYPED λ-calculus

1. **We have one version of ONP in** HASKELL **and another in** SCHEME
2. HASKELL **version includes: typing; conversion to eta-long form; the traversal algorithm itself; and construction of the normalised term.**
3. SCHEME **version: nearly ready to apply automatic partial evaluation.**

   Plan: use the UNMIX **partial evaluator (Sergei Romanenko).**
4. **The LLL output size is only linearly larger than** M**,** $|p_M| = O(|M|)$
5. **We have also have a handwritten a *generating extension* of ONP.**

$$\text{If } p_M = [\![\text{ONP-gen}]\!]^{scheme}(M) \text{ then } \forall M \,.\, [\![M]\!]^{\Lambda} = [\![p_M]\!]^{LLL}$$

Now: LLL = is a tiny subset of SCHEME**, so the output** p_M **is a** SCHEME **program.**

MORE WORK FOR SIMPLY-TYPED λ-calculus

Next steps:

- **Produce a generating extension, automatically, by specialising the specialiser to a Λ-traverser, using UNMIX**
- **Redefine LLL formally as a clean stand-alone subset of HASKELL**
- **Use HASKELL supercompiler**
- **Extend existing approach to programs with input dynamic data**

STATUS: OUR WORK ON THE UNTYPED λ-calculus

1. UNP is a normaliser for $\Lambda^{untyped}$.

2. UNP has been done in HASKELL and works on a variety of examples.

3. Some traversal items may have two back pointers, in comparison: ONP uses only one.

4. As ONP, UNP is also defined semi-compositionally by recursion on syntax of λ-expression M.

5. By specialising UNP, an arbitrary untyped λ-expression can be translated to ***LLL***.

6. Correctness proof: pending.

TOWARDS SEPARATING PROGRAMS FROM DATA IN Λ

Also we have a one more direction for research:

1. **An idea is to regard a computation of λ-expression M on input d as a two-player game between the LLL-codes for M and d.**
2. **An interesting example in this case a is usual λ-calculus definition of function $mult$ on Church numerals.**
3. **Amaizing fact is that Loops come from out of nowhere:**
   - **Neither `mul` nor the data contain loops;**
   - **But `mul` function is compiled into an LLL-program with two nested loops;**
   - **We also expect that we can do the computation entirely without back pointers.**
4. **Right now we are trying to express such program-data games in a *communicating* version of *LLL*.**

REFERENCES