Example plus 3 2 square

 $3(s2@_{14}(s2@_{15}z2)))$ $\sim 22.82@_{13}$ $z3.s3@_{10}(s3@_{11}z3)))@_{12}(\lambda s2.\lambda$ Input term: $((\lambda n.\lambda x.((n@_1(\lambda r.\lambda a.r@_2(r@_3a)))@_4(\lambda n1.\lambda s1.\lambda z1.(n1@_5s1)@_6(s1@_7z1)))@_8x)@_9(\lambda s3.\lambda.N)$ Normal form: $\lambda s1.\lambda z1.s1@(s1@(s1@(s1@(s1@(s1@(s1@(s1@(s1@z1))))))$

Example plus x n square . Input term: $(\lambda n.\lambda x.((n@_1(\lambda r.\lambda a.r@_2(r@_3a)))@_4(\lambda n1.\lambda s1.\lambda z1.(n1@_5s1)@_6(s1@_7z1)))@_8x)$ Normal form: $\lambda n.\lambda x.((n@(\lambda r.\lambda a.r@(r@a)))@(\lambda n1.\lambda s1.\lambda z1.(n1@s1)@(s1@z1)))@x$

Example ack three

2.s2@(s2@(s2@(s2@(s2@z2)))))Example ack times . In part to the form: $(\lambda m.(m@_1(\lambda g.\lambda n.(n@_2g)@_3(g@_4(\lambda s1.\lambda z1.s1@_5z1))))@_6(\lambda n2.\lambda s2.\lambda z2.(n2@_7s2)@_8(s2@_9z2)))@_{10}(\lambda s.\lambda z.s@_{11}(s@_{12}(s@_{13}z)))$ Normal form: $\lambda n.(n@(\lambda n.(n@(\lambda n2.\lambda s2.\lambda z2.\lambda z2.(n2@s2)@(s2@z2)))@(\lambda s2.\lambda z2.s2@(s2@(s2@(s2@(s2@z2))))@(\lambda s2.\lambda z2.s2@(s2@(s2@(s2@z2))))@(\lambda s2.\lambda z2.s2@(s2@(s2@z2))))$

Example p zero . Input term: $(\lambda t.((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))))@_7(\lambda a1.a1))@_8(\lambda s1.\lambda z1.z1)))@_9(\lambda s2.\lambda z2.z2)$ Normal form: $\lambda s1.\lambda z1.z1$

12(s3@13(s3@14z3)))s3 $s2@_{10}z2))@_{11}(\lambda s3.\lambda$ $z1)))@₉(\lambda s2.\lambda z2.$ (a1))@ $(\lambda s1$ Example p one three . Input term: $((\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))))@_7(\lambda a.\lambda a.\lambda z.s@(s@(s@z))))$ Normal form: $\lambda s.\lambda z.s@(s@(s@z))$

Example p two three four

 $z4.s4@_{17}(s4@_{18}(s4@_{19}(s4@_{20}z4))))$ $\lambda z 3.s 3@_{13}(s 3@_{14}(s 3@_{15}z 3))))@_{16}(\lambda s 4.\lambda s$ $z2)))@_{12}(\lambda s3$ $z2.s2@_{10}(s2@_{11}$ Example p two three rour . Input term: $(((\lambda t.(((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_5s)@_6z)))))@_7(\lambda a.\lambda a.\lambda z.s@(s@(s@(s@(s@(s@(s@(s@(s@(sa(s))))))))$ Normal form: $\lambda s.\lambda z.s@(s@(s@(s@(s@(s@(s@(s@(s@(s@(sa(s)))))))$

 $\lambda s5.\lambda z5.s5@_{23}(s5@_{24}(s5@_{25}(s5@_{26}(s5@_{27}z5)))))$ $@_{13}(\lambda s3.\lambda z3.s3@_{14}(s3@_{15}(s3@_{16}z3)))) @_{17}(\lambda s4.\lambda z4.s4@_{18}(s4@_{19}(s4@_{20}(s4@_{21}z4))))) @_{22}(s3.\lambda s3.\lambda s3.\lambda s3.s3))) \\$ $z2.s2@_{10}(s2@_{11}(s2@_{12}z2))))$ $z1)))@9(\lambda s2.\lambda$ (a1) $@_{8}(\lambda s1)$

 $.s2@_{10}$ $\lambda z 2.$ $z1)))@_{9}(\lambda s2)$ $\frac{1}{2}$ \prec (a1))@ $(\lambda s1$ (5s) (6c) (5s) (7aExample p one . Input term: $(\lambda t.((t@_1(\lambda n.\lambda a.\lambda x.n@_2(\lambda s.\lambda z.(a@_3s)@_4((x@_1\lambda s.\lambda z.(x@_s)@_2)))))$ Normal form: $\lambda x.\lambda s.\lambda z.(x@_s)@_2$

\$2 $z1)))@9(\lambda s2.\lambda$ 7 ~

Example ex_1 . Input term: $(g@_1(\lambda n.n))$ Normal form: $g@\lambda n.n$

Example ex_2

 $((\lambda h.h)@_1(\lambda f.f))@_2a$ Input term: (λh) Normal form: a

Example ex_3 . Input term: $((\lambda h.h@_1a)@_2(\lambda f.f))$ Normal form: a

Example ex_4

Input term: $\lambda f.\lambda y.(y@_1f)@_2y$ Normal form: $\lambda f.\lambda y.(y@f)@y$

Example ex_4'

Input term: $\lambda f.\lambda y.(y@_1(\lambda z.z))@_2y$ Normal form: $\lambda f.\lambda y.(y@\lambda z.z)@y$

Example ex_5 . Input term: $\lambda y.\lambda f.(y@_1f)@_2y$ Normal form: $\lambda y.\lambda f.(y@f)@y$

Example succ two . Input term: $(\lambda n.\lambda s.\lambda z.(n@_1s)@_2(s@_3z))@_4(\lambda p.\lambda o.p@_5(p@_6o))$ Normal form: $\lambda s.\lambda z.s@(s@(s@z))$

Example succ two

Example ex_9 . Input term: $(\lambda x.x@_1x)@_2(\lambda z.z)$ Normal form: $\lambda z.z$

Example ex_11 . Input term: $(\lambda x.\lambda y.x@_1(x@_2y))@_3(\lambda p.q)$ Normal form: $\lambda y.q$

Example ex_f0 . Input term: $(\lambda n.(n@_1(\lambda s.\lambda z.s@_2(s@_3((n@_4s)@_5z))))@_6(\lambda s1.\lambda z1.z1))@_7(\lambda s2.\lambda z2.z2)$ Normal form: $\lambda s1.\lambda z1.z1$

Example ex_f1 . Input term: $(\lambda n.(n@_1(\lambda s.\lambda z.s@_2(s@_3((n@_4s)@_5z))))@_6(\lambda s1.\lambda z1.z1))@_7(\lambda s2.\lambda z2.s2@_8z2)$ Normal form: $\lambda z.\lambda z1.z1$

Example ex_f2 . Input term: $(\lambda n.(n@_1(\lambda s.\lambda z.s@_2(s@_3((n@_4s)@_5z)))))@_6(\lambda s1.\lambda z1.z1))@_7(\lambda s2.\lambda z2.s2@_8(s2@_9z2))$ Normal form: $\lambda z.\lambda z1.z1$

Example ex_LO1 . Input term: $(\lambda f.\lambda x.f@_1(x@_2((f@_3x)@_4x)))@_5(\lambda a.\lambda b.a)$ Normal form: $\lambda x.\lambda b.x@x$

Example ex_LO2 . Input term: $(\lambda x.(x@_1x)@_2x)@_3(\lambda a.\lambda b.a)$ Normal form: $\lambda a.\lambda b.a$

Example ex_LO3 . Input term: $(\lambda x.x@_1(\lambda y.y))@_2(\lambda a.\lambda b.b)$ Normal form: $\lambda b.b$

Example ex_LO4 . Input term: $(\lambda x.x@_1(\lambda y.y))@_2(\lambda a.\lambda b.a)$ Normal form: $\lambda b.\lambda y.y$

Example ex_1 . Input term: $(g@_1(\lambda n.n))$ Normal form: $g@\lambda n.n$

Example NPR . Input term: $((\lambda h.\lambda z.((h@_1(\lambda x.((h@_2(\lambda q.x)))@_3a)))@_4(z@_5a)))@_6(\lambda f.\lambda y.f@_7((g@_8(\lambda b.b))@_9y)))@_{10}(g@_{11}(\lambda n.n)))$ Normal form: $(g@\lambda b.b)@((g@\lambda n.n)@a)$

Example mut three two . Input term: $((\lambda m.\lambda n.\lambda s.\lambda z.(m@_1(n@_2s))@_3z)@_4(\lambda a.\lambda q.a@_5(a@_6(a@_7q))))@_8(\lambda d.\lambda e.d@_9(d@_{10}e))$ Normal form: $\lambda s.\lambda z.s@(s@(s@(s@(s@(s@(s@(s@(s))))))$

 $3(s2@_{14}(s2@_{15}z2)))$