THE UNIVERSITY OF MICHIGAN
Department of Civil & Environmental Engineering

Computational Methods for Engineers

CEE/ENG 303 - Winter 2003

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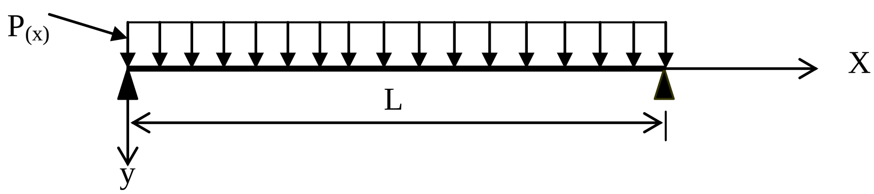
Project

Part One

Part one is for writing a code and checking its performance against a case in which an easy analytical solution is available.
You are asked to compute the deflection of a beam that is supported at both ends as shown in the figure below. The beam supports a uniform load $P_{(x)}$=10 MN/m along its length, L, of 50m. The modulus of elasticity for the beam is E=26000 MN/m^2. The moment of inertia, I, for the beam is constant and equal to 31.25m^4. The basic beam equation, which correlates deflection to moment, is a second order O.D.E. given by:

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} \quad \text{where,} \quad M = (\frac{pL}{2})x - \frac{p}{2}x^2$$

"y" is the deflection of the beam along the x axis. The two ends are restrained against vertical motion, however they are free to rotate. Hence, $y_{(0)} = y_{(L)} = 0$



Write a computer program that solves for the deflection of the beam, using a centered difference approximation of the second order differential operator in the beam-deflection equation.

$$y_{i-1} - 2y_i + y_{i+1} = -\frac{M_i}{E_i I_i}(\Delta x)^2$$

This F.D. discretization creates a tridiagonal system of linear equations. You should use the Thomas Algorithm to solve it. To check the proper operation of your code check your numerical results versus the maximum deflection of the beam at the center which is equal to $y_{\max} = \frac{5}{384} \cdot \frac{pL^4}{EI}$

Start with a delta x value of 12.5m and change it within a do loop until the numerical value for the maximum deflection is within 1% of the analytical value. In your report mention how many segments do you need to reach this level of accuracy.

Part Two

We want to reduce the amount of steel being used in the beam. We decide to divide the beam into three equal sectors. The two end sectors are going to have a new moment of inertia which is equal to half of what is given in part one. You are hired to determine the moment of inertia of the middle sector so that the maximum deflection remains within 1% of the maximum analytical value of deflection **in part one**. Modify your code in part one to accomplish this task by using 0.1 meter for dx.

Part 3:

Finding the Analytical Solution

Analytical solution is a fourth order polynomial obtained from four successive integration of the basic beam equation:

$$\frac{d^4 y}{dx^4} = \frac{P_{(x)}}{E_{(x)} I_{(x)}}$$

Decide on using four of the following interdependent conditions to determine the constants of integration:

$$y_{(0)} = y_{(L)} = 0,$$

$$M_{(0)} = M_{(L)} = 0,$$

$$\theta_{(L/2)} = 0$$

$$Q_{(L/2)} = 0$$

$$Q_{(0)} = Q_{(L)} = \frac{PL}{2}$$

Y is deflection, Theta is angle of rotation (the first derivative of y), M is moment (the second derivative of y), Q is the shear force (the third derivative of .y), and P is the load (the fourth derivative of y).

Part 4:

Manual Numerical Solution

You have to find the numerical solution of this beam problem manually (hand calculation).
Divide the beam into six segments. Number the nodal point deflections as Y0 to Y6. Develop the system of linear equations. Note that two of your 5 equations are trivial due to geometric symmetry of the problem. So, end up really solving only for three equations.
Show all of your operations. Calculate the numerical relative percent error versus the exact analytical solution.

Part 5:

Graphics

Draw the straight beam at the center of the viewport. Choose your x coordinate scale so that the length of the beam would cover approximately 90% of the viewport. Then draw the analytical solution. Use a scaling factor for the y direction so that the beam deflection would be magnified 100 times. Draw the deflected beam according to the numerical calculations with the lines connecting the segment nodes. At each node draw a tiny circle so that nodal points can be seen in your graphical presentation.

Draw your numerical results, in different colors, for 2, 3, 4, and 48 segmented beam. At the lower corner of the viewport add information of the beam such as the E, I and Max Analytical Deflection of the beam.

On the right hand side corner of the viewport add the max deflection of the beam for 2, 3, 4, and 48 segmented beam in addition to relative percent error for each of the calculations.

<u>Part 6:</u>

<u>Additional Graphics and Additional Computations</u>

Extra Credit:

a) Draw two small solid (filled) triangles as the beam supports at the two ends. (5%)
b) Show the distributed force P(x) as vertical arrows downward and show its magnitude. (10%)
c) Draw the non-deflected beam as a thicker solid line. (5%)
d) Draw the results for Shear stress, Moment, and the angular deflection of the beam. Use the analytical equations developed in class for this and draw some arrows downward or upward as appropriate or needed for each parameter's positive or negative values. (20%)
e) Use a non-constant load distribution as a function of x by calling this load function in your program for calculation of each segment's load and the reaction force in the end supports for the beam and solve the problem numerically. Note that you may need to use some of the skills you have learned in your numerical integration homework. (35%)
f) Add a graphical section for drawing the free body diagram for any arbitrary section x with showing the effects of moments with curved arrows. (15%)
g) Provide a good powerpoint file for the problem, your program and the results. (10%)