

STREAM 3D project

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Politecnico di Milano

Overview

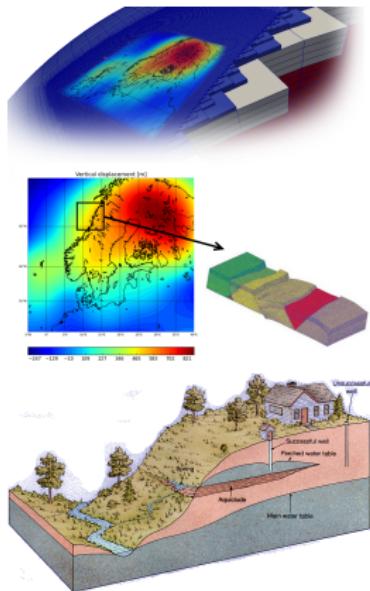
- 1 Introduction
- 2 Mathematical model
- 3 1D prototype
- 4 Numerical Platform
- 5 Conclusion

Introduction

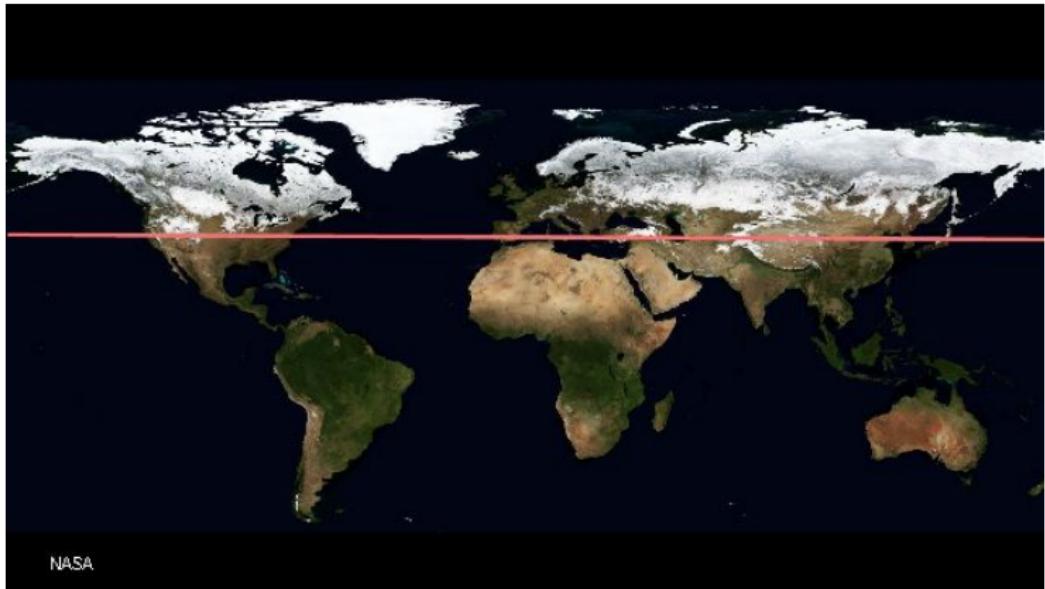
Background and Motivations

Key processes of sedimentary basin evolution:

- geomechanics and dynamic evolution of stress and deformations;
- transport of dissolved chemicals;
- geochemical reactive processes.



Background and Motivations

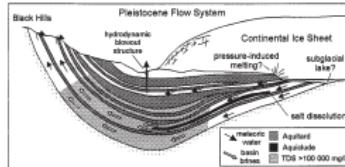


NASA

Background and Motivations

The effects of glaciations on the subsurface:

- the mechanical compaction due to the load of ice sheets;
- the deformation of the lithosphere by isostasy;
- the subglacial meltwater;
- the generation of permafrost.



Mathematical model



Poromechanics

$$-\nabla \cdot (2\mu\varepsilon(\mathbf{u}) + \nabla \cdot \mathbf{u}) + \alpha \nabla p = \rho \mathbf{g},$$

$$\partial_t \left(\frac{p}{M} + \alpha \nabla \cdot \mathbf{u} \right) + \nabla \cdot \mathbf{u}_d = S_f,$$

$$\mathbf{K}^{-1} \mathbf{u}_d + \nabla p = \rho_f \mathbf{g},$$

Temperature Dynamics

$$C_T \frac{\partial T}{\partial t} + (\phi \rho_l c_l \mathbf{v}_l + (1 - \phi) \rho_s c_s \mathbf{v}_s) \cdot \nabla T - K_T \nabla^2 T = Q.$$

Chemical transport

$$C_T \frac{\partial C}{\partial t} + \mathbf{u_D} \cdot \nabla C - D \nabla^2 T = Q_c .$$

Coupled model

Poromechanics - Temperature Dynamics - Chemical transport

$$-\nabla \cdot (2\mu\varepsilon(\mathbf{u}) + \nabla \cdot \mathbf{u}) + \alpha\nabla p = \rho\mathbf{g},$$

$$\partial_t \left(\frac{p}{M} + \alpha\nabla \cdot \mathbf{u} \right) + \nabla \cdot \mathbf{u}_d = S_f,$$

$$\mathbf{K}^{-1}\mathbf{u}_d + \nabla p = \rho_f\mathbf{g},$$

$$C_T \frac{\partial T}{\partial t} + (\phi\rho_l c_l \mathbf{v}_l + (1-\phi)\rho_s c_s \mathbf{v}_s) \cdot \nabla T - K_T \nabla^2 T = Q.$$

$$C_T \frac{\partial C}{\partial t} + \mathbf{u}_D \cdot \nabla C - D \nabla^2 C = Q_c.$$

Constitutive law

$$\mu = 1.002 \cdot 10^{-3} (1 + \alpha_1(T) + \alpha_2(C_M))$$

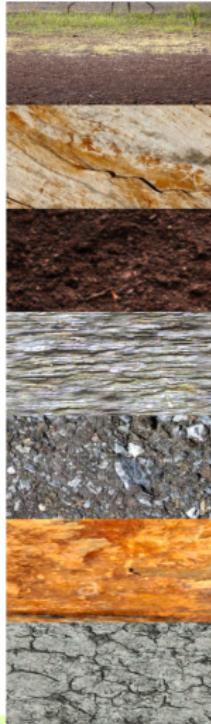
$$\rho_l = \rho_0 (1 + \beta_1(T) + \beta_2(C_M)) .$$

Ice fraction

$$\theta_l = S_i \phi_i$$
$$S_i = \begin{cases} 1 & T > T_L, \\ (1 - S_{lres}) \exp \left[- \left(\frac{T - T_L}{w} \right)^2 \right] + S_{lres} & T_L > T > T_{lres}, \\ S_{lres} & T < T_{lres}. \end{cases}$$

1D prototype





Materials	(Pa^{-1})	$K (m^2)$	$h (\text{m})$
7 Aquifer	10^{-9}	10^{-12}	825
6 Shale	$5 \cdot 10^{-8}$	10^{-16}	400
5 Aquifer	10^{-9}	10^{-13}	825
4 Shale	10^{-8}	10^{-17}	500
3 Aquifer	$5 \cdot 10^{-10}$	10^{-14}	825
2 Shale	$5 \cdot 10^{-9}$	10^{-18}	800
1 Aquifer	$5 \cdot 10^{-10}$	10^{-14}	825

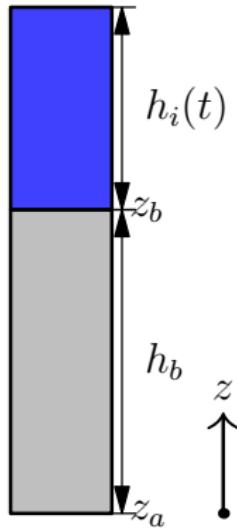
Set up

 Bense, V. F., and M. A. Person. (2008)

**Transient hydrodynamics within intercratonic
sedimentary basins during glacial cycles.**

Journal of Geophysical Research 113.F4 .

Set up

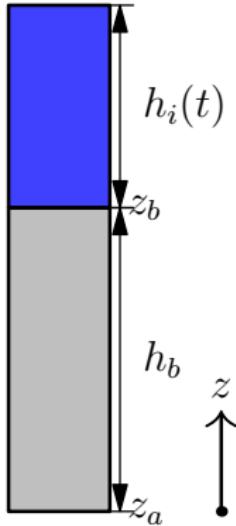


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Set up



$$h_b(t = 0) = 5000m$$

$$D = 10^{-10} m^2/s$$

$$C(t = 0) = -\frac{(0.4 - 0.1)}{h_b} z + 0.1$$

$$C(z_a, t) = 0, \partial C / \partial z|_{z_b} = 0$$

$$dt = 0.3y, \quad \Delta t = 10y$$

$$h_s = 500m, 2000m, \frac{2000}{\Delta t}t$$



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Test case

$$\Delta t_{tot} = 32Ky \quad \Delta t_0 = 0.1y \quad \Delta t_{max} = 0.1Ky$$
$$h_b(t=0) = 5000m \quad D = 10^{-10}m^2/s \quad \theta_l = S_l\phi$$

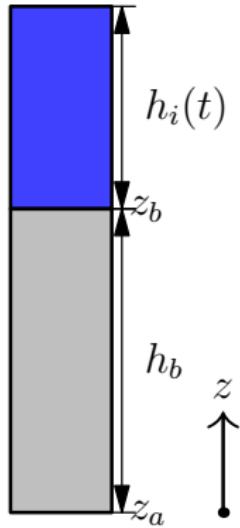


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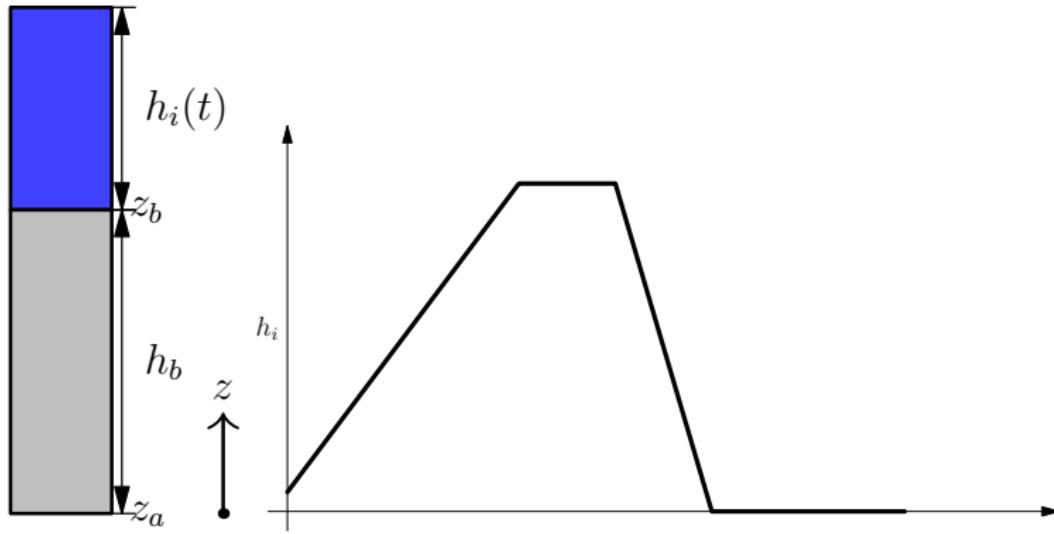


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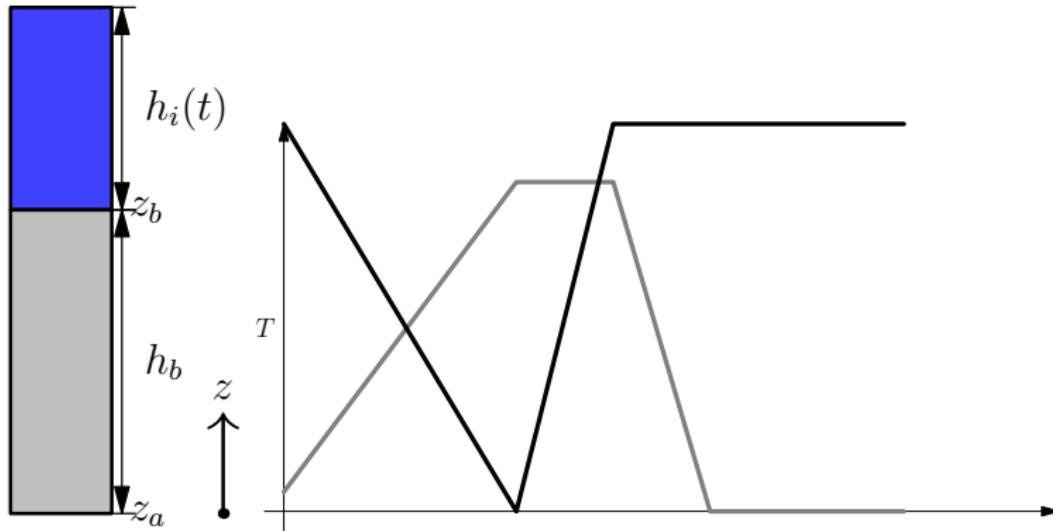


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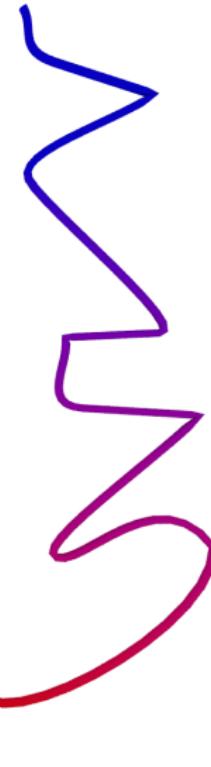
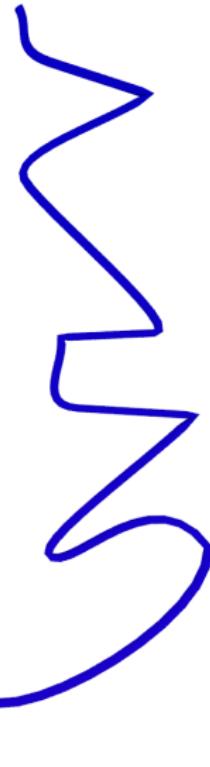


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Test cases

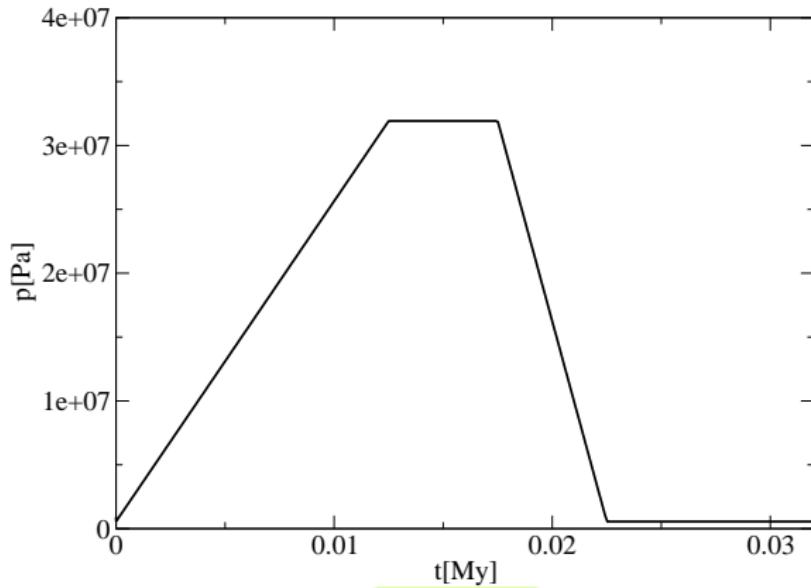


A

B

C

Ice evolution



Numerical Plattofrm





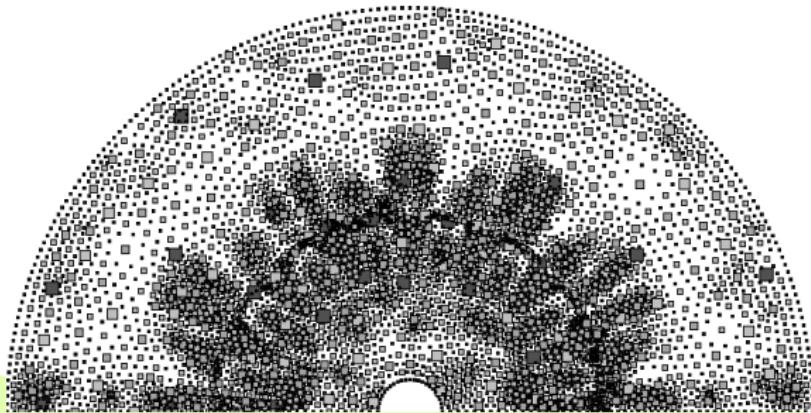
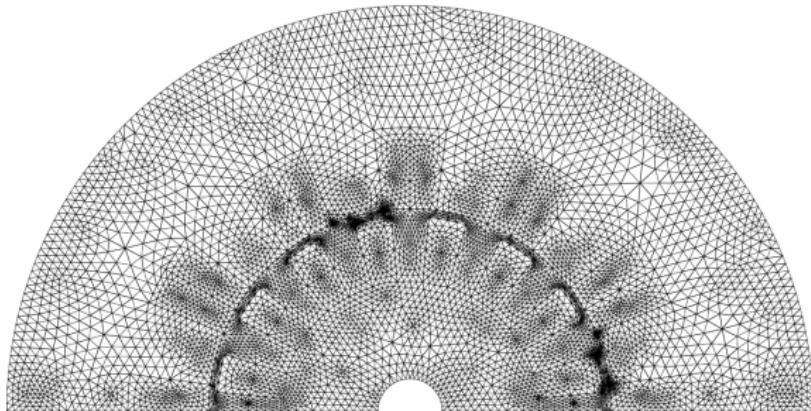
GetFem

- c++ finite element platform
- Generic assembly language
- Level-set and finite element cut by one or several level-set (Xfem)

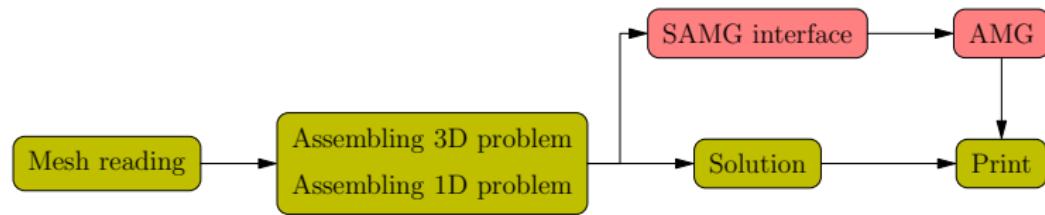
SAMG library Algebraic Multigrid Methods for Systems

- solution of large linear systems
- highly scalable
- easy to integrate

Geometric VS Algebraic



Integration



- Simple interface (CSR matrix)
- Same programming language

Conclusion
