STREAM 3D project

D. Cerroni, L. Formaggia, A. Scotti and P. Zunino

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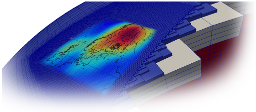
Politecnico di Milano

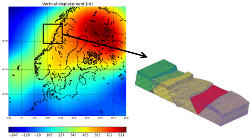
Overview

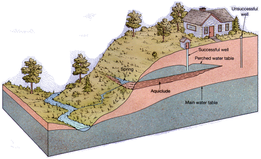
Introduction

Key processes of sedimentary basin evolution:

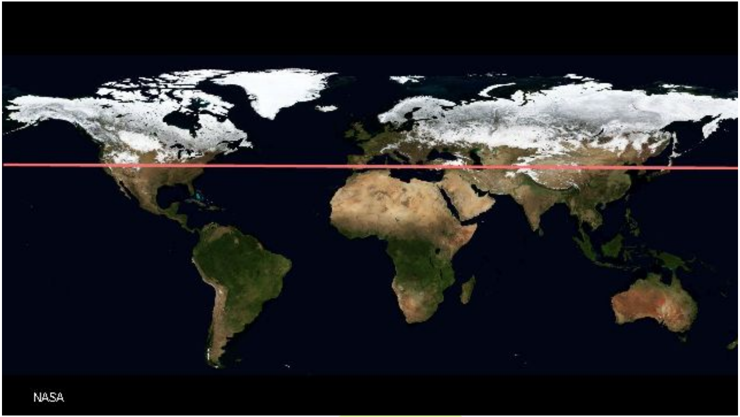
- geomechanics and dynamic evolution of stress and deformations;
- transport of dissolved chemicals;
- geochemical reactive processes.







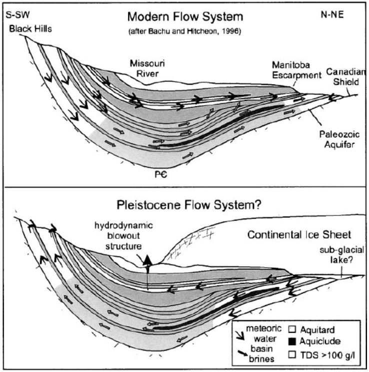
Background and Motivations



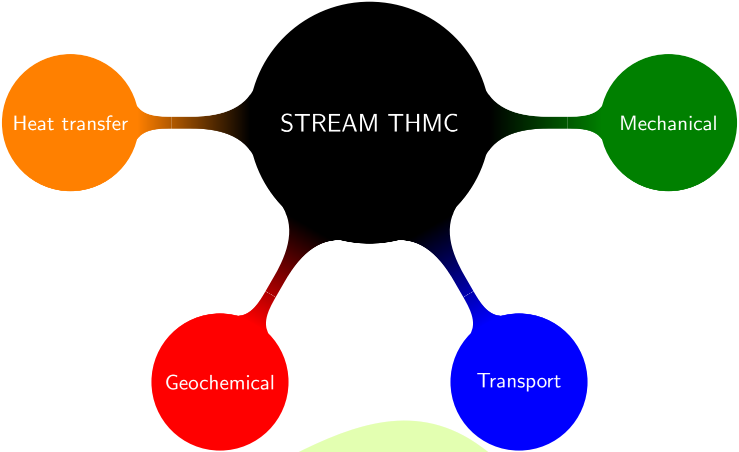
Background and Motivations

The effects of glaciations on the subsurface:

- the mechanical compaction due to the load of ice sheets;
- the deformation of the lithosphere by isostasy;
- the subglacial meltwater;
- the generation of permafrost.

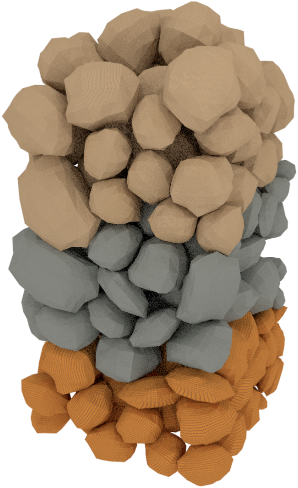


Conceptual


STREAM THMC
Heat transfer
Mechanical
Geochemical
Transport

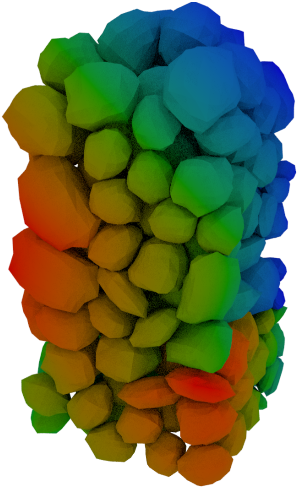
Mathematical model

Poromechanics



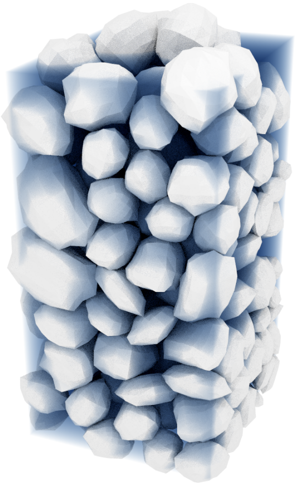
$$
\begin{aligned}
-\nabla \cdot (2\mu\varepsilon(\mathbf{u}) + \nabla \cdot \mathbf{u}) + \alpha \nabla p &= \rho \mathbf{g}\,, \\
\partial_t \left(\frac{p}{M} + \alpha \nabla \cdot \mathbf{u} \right) + \nabla \cdot \mathbf{u_d} &= S_f\,, \\
\mathbf{K}^{-1} \mathbf{u_d} + \nabla p &= \rho_f \mathbf{g}\,,
\end{aligned}
$$

Temperature Dynamics



$$C_T \frac{\partial T}{\partial t} - K_T \nabla^2 T + \left(\phi \rho_l c_l \mathbf{v_l} + (1-\phi)\rho_s c_s \mathbf{v_s}\right) \cdot \nabla T = Q\,.$$

Chemical transport



$$C_T \frac{\partial C}{\partial t} + \mathbf{u_D} \cdot \nabla C - D\nabla^2 T = Q_c \,.$$

Ice fraction

Porosity

$$\theta_l = S_i \phi_i$$

Ice fraction

$$S_i = \begin{cases} 1 & T > T_L\,, \\ (1 - S_{lres}) \exp\left[-\left(\dfrac{T - T_L}{w}\right)^2\right] + S_{lres} & T_L > T > T_{lres}\,, \\ S_{lres} & T < T_{lres}\,. \end{cases}$$

Constitutive law

$$\mu = 1.002\,10^{-3}\,(1 + \alpha_1(T) + \alpha_2(C_M))$$

$$\rho_l = \rho_0(1 + \beta_1(T) + \beta_2(C_M))\,.$$

Permeability

$$k = k_p K;\; k_p = \begin{cases} 1 & T > T_L\,, \\ \left(\dfrac{k_{min} - 1}{T_l}\right) T + 1 & T_L > T > T_{lres}\,, \\ k_{min} & T < T_{lres}\,. \end{cases}$$

Coupled model

Poromechanics - Temperature Dynamics - Chemical transport

$$
\begin{aligned}
-\nabla \cdot (2\mu\varepsilon(\mathbf{u}) + \lambda \nabla \cdot \mathbf{u}) + \alpha \nabla p = \rho \mathbf{g}\,, \\
\partial_t \left(\frac{p}{M} + \alpha \nabla \cdot \mathbf{u} \right) + \nabla \cdot \mathbf{u_d} = S_f\,, \\
\mathbf{K}^{-1}\mathbf{u_d} + \nabla p = \rho_f \mathbf{g}\,.
\end{aligned}
$$

$$C_D \frac{\partial C}{\partial t} + \mathbf{u_D} \cdot \nabla C - D\nabla^2 C = Q_c\,.$$

$$C_T \frac{\partial T}{\partial t} + (\phi \rho_l c_l \mathbf{v_l} + (1-\phi)\rho_s c_s \mathbf{v_s}) \cdot \nabla T - K_T \nabla^2 T = Q\,.$$

Coupled model

Poromechanics - Temperature Dynamics - Chemical transport

$$\begin{aligned} -\nabla \cdot (2\mu\varepsilon(\mathbf{u}) + \lambda\nabla \cdot \mathbf{u}) + \alpha\nabla p = \rho\mathbf{g}\,, \\ \partial_t\left(\frac{p}{M} + \alpha\nabla \cdot \mathbf{u}\right) + \nabla \cdot \mathbf{u_d} = S_f\,, \\ \mathbf{K}^{-1}\mathbf{u_d} + \nabla p = \rho_f\mathbf{g}\,. \end{aligned}$$

$$C_D\frac{\partial C}{\partial t} + \mathbf{u_D} \cdot \nabla C - D\nabla^2 C = Q_c\,.$$

$$C_T\frac{\partial T}{\partial t} + (\phi\rho_l c_l\mathbf{v_l} + (1-\phi)\rho_s c_s\mathbf{v_s}) \cdot \nabla T - K_T\nabla^2 T = Q\,.$$

1D prototype

STREAM 1D THHC (Matlab prototype) Frozen Geco

Momentum

$$\phi(u^l - u^s) = -\frac{K}{\mu^l}\left(\frac{dp^l}{dz} - \rho^l \mathbf{g}\right)$$

Geomechanics

$$\frac{d\phi_M}{dt} = -\beta(\phi_0 - \phi_f)\exp(-\beta\sigma)\frac{d\sigma}{dt}$$

Heat transfer

$$C_t\frac{dT}{dt} + \rho c\mathbf{u}_d \cdot \frac{dT}{dz} - K\frac{dT^2}{dz^2} = 0$$

Chemical Transport

$$\phi\rho_l\frac{\partial C}{\partial t} + \rho_l\mathbf{u}_d \cdot \frac{dC}{dz} = \phi\rho_l D\frac{dC^2}{dz^2}$$

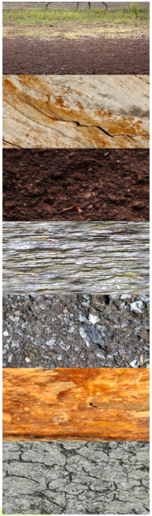
u^l liquid velocity

u^s solid velocity

u_d Darcy velocity

σ axial effective stress

- Mono dimensional domain
- Transient simulation
- Ice formation
- Complex constitutive models

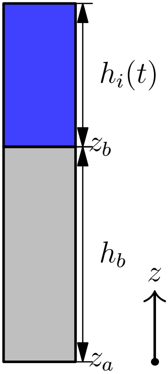


Materials	(Pa^{-1})	K (m^2)	h (m)
7 Aquifer	10^{-9}	10^{-12}	825
6 Shale	$5\,10^{-8}$	10^{-16} 400	
5 Aquifer	10^{-9}	10^{-13}	825
4 Shale	10^{-8}	10^{-17}	500
3 Aquifer	$5\,10^{-10}$	10^{-14}	825
2 Shale	$5\,10^{-9}$	10^{-18}	800
1 Aquifer	$5\,10^{-10}$	10^{-14}	825

Set up

Bense, V. F., and M. A. Person. (2008)
Transient hydrodynamics within intercratonic sedimentary basins during glacial cycles.
Journal of Geophysical Research 113.F4 .

Set up



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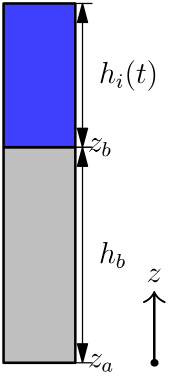

$$h_b(t=0) = 5000m$$
$$D = 10^{-10}m^2/s$$
$$C(t=0) = -\frac{(0.4-0.1)}{h_b}z + 0.1$$
$$C(z_a,t) = 0, \partial C/\partial z|_{z_b} = 0$$
$$dt = 0.3y, \quad \Delta t = 10y$$
$$h_s = 500m, 2000m, \frac{2000}{\Delta t}t$$

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$$\Delta t_{tot} = 32Ky \qquad \Delta t_0 = 0.1y \qquad \Delta t_{max} = 0.1Ky$$
$$h_b(t=0) = 5000m \qquad D = 10^{-10} m^2/s \qquad \theta_l = S_l \phi$$

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Transient hydrodynamics within intercratonic sedimentary basins during glacial cycles.
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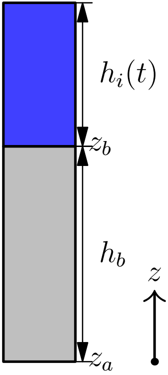
Test case



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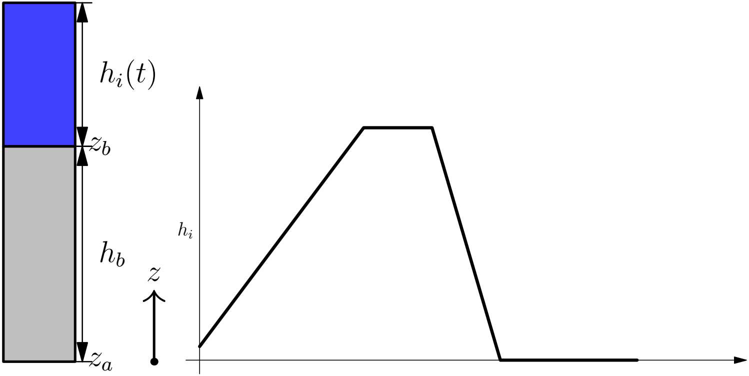

$\Delta t_{tot} = 32Ky \qquad \Delta t_0 = 0.1y \qquad \Delta t_{max} = 0.1Ky$

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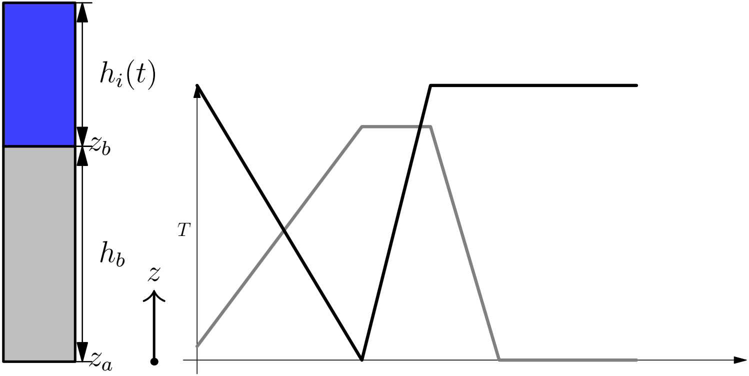

$\Delta t_{tot} = 32Ky$ $\quad \Delta t_0 = 0.1y$ $\quad \Delta t_{max} = 0.1Ky$

$h_b(t=0) = 5000m$ $\quad D = 10^{-10} m^2/s$ $\quad \theta_l = S_l \phi$

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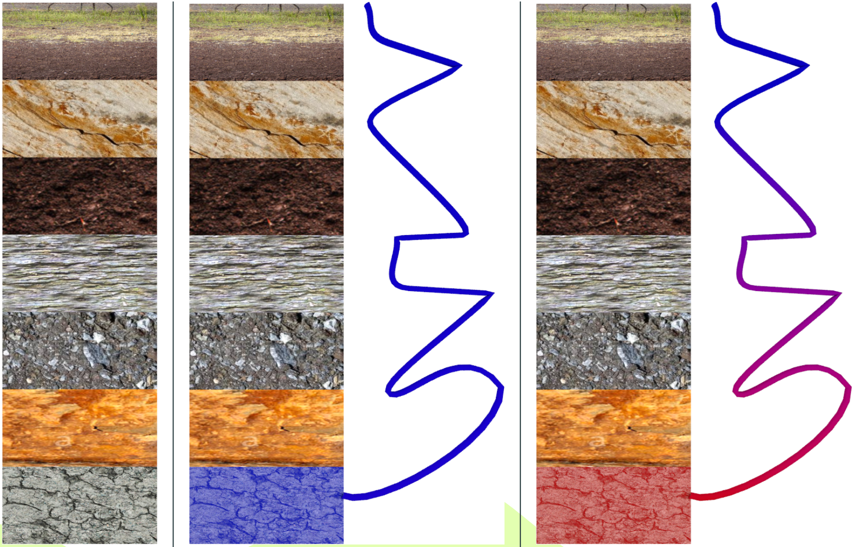
Test cases



A B C

Numerical Plattofrm

Numerical platform

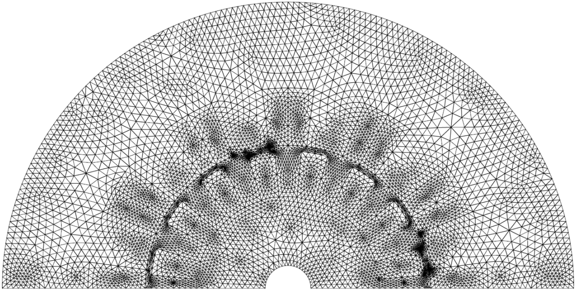
GetFem

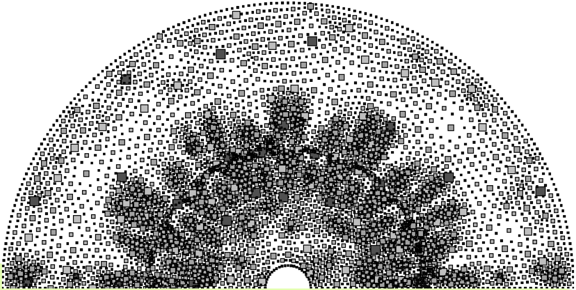
- c++ finite element platform
- Generic assembly language
- Level-set and finite element cut by one or several level-set (Xfem)

SAMG library
Algebraic Multigrid Methods for Systems

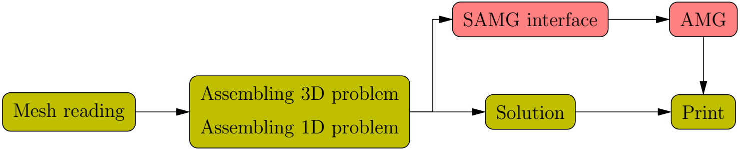
- solution of large linear systems
- higly scalable
- easy to integrate

Geometric VS Algebraic





Integration



- Simple interface (CSR matrix)
- Same programming language

Benchmark

$$
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\partial_t \left(\frac{p}{M} + \alpha \nabla \cdot \mathbf{u} \right) + \nabla \cdot \mathbf{u_d} &= S_f\,, \\
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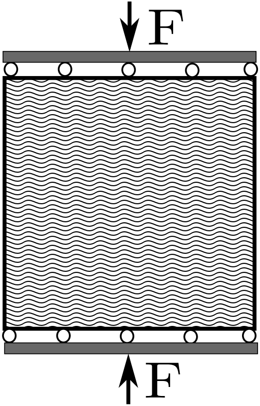
\end{aligned}
$$

Mandel benchmark



- Both, Jakub Wiktor, et al. *Robust fixed stress splitting for Biot's equations in heterogeneous media. Applied Mathematics Letters 2017.*
- Cui, L., et al. *Finite element analyses of anisotropic poroelasticity: a generalized Mandel's problem and an inclined*

Conclusion