Repaso

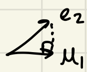
1) Método de ortogonalización de Grand - Schmidt

$A = [e_1, e_2, \dots, e_n]$ $\qquad$ $A \in \mathbb{R}^{m \times n}$

$c(A) = \text{span}(A) = \text{span}\{e_1, \dots, e_n\}$ $\quad$ li

dimensión del espacio

Base ortonormal $\qquad u_1 = \dfrac{e_1}{\|e_1\|}$



$w_2 = e_2 - \langle e_2, u_1 \rangle u_1 \rightarrow \langle u_1, w_2 \rangle = 0$

$u_2 = \dfrac{w_2}{\|w_2\|}$

$\vdots$

u_{n-1}

$w_n = e_n - \langle e_n, u_1 \rangle u_1 - \langle e_n, u_2 \rangle u_2 - \dots \langle e_n, u_{n-1} \rangle u_{n-1}$

$u_n = \dfrac{w_n}{\|w_n\|}$ $\qquad \{u_1, u_2, u_3, \dots u_n\}$

Factorización QR de A

Q: matriz ortogonal $(QQ^t = I)$

R: matriz triangular superior

$Q = [u_1, u_2, u_3, \dots u_n] \rightarrow$ son inuertibles

Como $\quad QR = A$

$Q^{-1}QR = Q^{-1}A \rightarrow \boxed{R = Q^t A}$

$R = Q^T A$

Regresión lineal

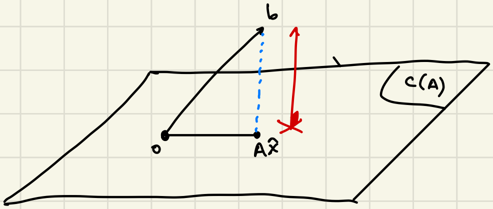
$A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, $x \in \mathbb{R}^n$ $(Ax = b)$

Encontrar $x \in \mathbb{R}^n$ tal que $\|Ax-b\| = 0 \iff \|Ax-b\|^2 = 0$

No se puede encontrar en general

$\exists\ \hat{x} \in \mathbb{R}^n$ tal que $\|A\hat{x}-b\| = \min\limits_{x\in\mathbb{R}^n} \|Ax-b\|$

Solución de mínimos cuadrados del sistema



$\langle b - A\hat{x}, e_j \rangle = 0 \quad \forall j$

$e_j^t (b - A\hat{x}) = 0$

$$\begin{bmatrix} e_1^t \\ e_2^t \\ \vdots \\ e_n^t \end{bmatrix} [b - A\hat{x}] = 0 \quad \rightarrow \quad A^t(b - A\hat{x}) = 0$$

$$A^t b = \underbrace{A^t A}_{} \hat{x}$$

es inversible

$\iff$ sus columnas son l.i

$$\Rightarrow \hat{x} = (A^tA)^{-1}(A^tb)$$

ECUACIONES NO LINEALES

1) Secante
2) Newton
3) falsa modificada
4) punto fijo

$$\begin{aligned} f_1(x_1, x_2, \dots, x_n) &= 0 \\ f_2(x_1, x_2, \dots, x_n) &= 0 \\ &\vdots \\ f_n(x_1, x_2, \dots, x_n) &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} &F(\underline{x}) = \underline{0} \\ &F(\underline{x}) = \underline{x} - G(\underline{x}) = 0 \end{aligned}$$

Resolver $G(\underline{x}) = \underline{x}$ problema del punto fijo sea la solución

x_0 aproximación inicial

$$G(\underline{x_0}) = \underline{x_1}$$
$$G(\underline{x_1}) = \underline{x_2}$$
$$\vdots$$

$\underline{x_n} \rightarrow \underline{x}^*$

Si $G : D \rightarrow D$ fuera lipschitziano $\rightarrow L < 1 \rightarrow$ contracción

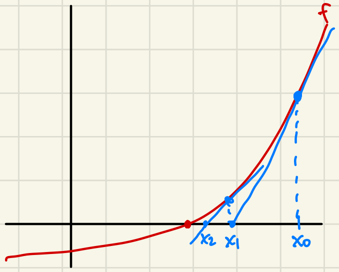
$\exists L < 1 : \quad \|G(x) - G(y)\| \leq L \|x - y\| \quad \forall \, x, y \in D$

$\Rightarrow \exists! \, x^* \in D$ tal que $G(x^*) = x^*$

Esquema numérico :
$$\begin{cases} x^{k+1} = G(x^k) \\ x^0 \in D \end{cases}$$

$x^k \rightarrow x^*$

Método de Newton



$x_n \rightarrow r$

Hallando los cortes de la recta con respecto al eje x

$$\begin{cases} x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} \\ x_0 \in \mathbb{R}_n \end{cases}$$

$$f_1(x_1, x_2, \dots, x_n) = 0$$
$$f_2(x_1, x_2, \dots, x_n) = 0$$
$$\vdots$$
$$f_n(x_1, x_2, \dots, x_n) = 0$$

$\longrightarrow$ $\underbrace{F(x) = 0}_{\text{sistema no lineal}}$, $x \in \mathbb{R}^n$

$$x^{k+1} = x^k - JF(x^k)^{-1} \cdot F(x^k)$$

$$\Delta x^k = x^{k+1} - x^k = -JF(x^k)^{-1} F(x^k)$$

sistema $\longleftarrow$ $\begin{cases} JF(x^k) \cdot \Delta x^k = -F(x^k) \\ x^0 \in \mathbb{R}^n \end{cases}$

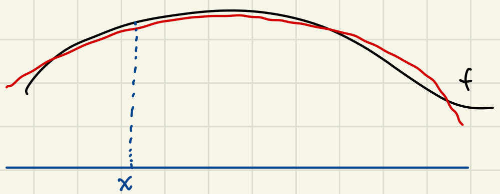
$\rightarrow$ obtenemos Δx^k y x^k $\rightarrow$ $\underbrace{x^{k+1}}_{\text{actualizar}} = x^k + \Delta x^k$

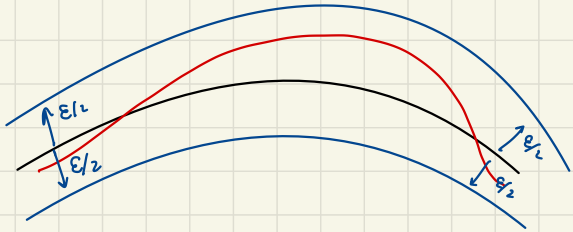
Interpolación polinomial

$f \in C[a,b] \rightarrow \exists\, p$ de $\overset{\text{polinomio}}{\text{interpolación}}$ de un grado suficientemente grande tq

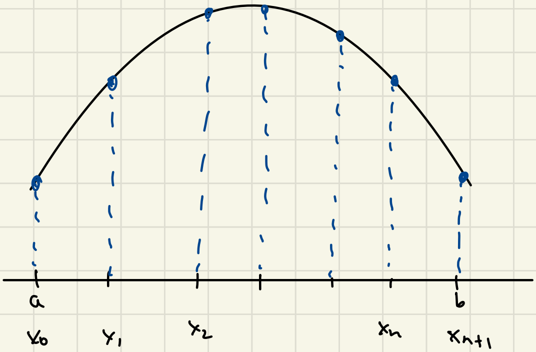
$$\| f - p \| < \underbrace{\varepsilon}_{\text{franja alrededor de } f}$$

$$| f(x) - p(x) |$$





$$\| f - p \| = \max_{x \in [a,b]} | f(x) - p(x) |$$



$p_{n+1}(x_i) = f(x_i)$

$p_n(x_i) = f(x_i)$

$0 \leq i \leq n$

$p_{n+1}(x) = p_n(x) + q_x$

$p_{n+1}(x_1) = p_n(x_1) + q(x_1)$

$q_1(x) = 0, \; q(x_2) = 0, \; \dots \; q(x_n) = 0$

$$p_{n+1}(\underset{\uparrow}{x}^{x_{n+1}}) = p_n(x) + c(x - x_0)(x - x_1)\dots(x - x_n)(x - x_{n+1})$$

FORMA DE INTERPOLACIÓN DE NEWTON GRADO MAXIMO es N+1

$$\Rightarrow p_{n+1}(\overset{x_{n+1}}{x}) = c_0 + c_1(x - x_0) + c_2(x - x_0)(x - x_1) + \dots \; c_{n+1}(x - x_0)\dots(x - x_n)(x - x_{n+1})$$

FORMA DE INTERPOLACIÓN DE LAGRANGE

$\{ \ell_j^{n+1} \}$ $\quad \ell_j^{n+1}(x_i) = \begin{cases} 1 & j = i \\ 0 & j \neq i \end{cases}$

$$\ell_j^n(x) = \frac{(x - x_0)(x - x_1)\dots(x - x_{j-1})(x - x_{j+1})\dots(x - x_{n+1})}{(x_j - x_0)(x_j - x_1)\dots(x_j - x_{j-1})(x_j - x_{j+1})\dots(x_j - x_{n+1})}$$

$\ell_j(x_j) = 1$

$$p(x) = \sum_{k=0}^{n+1} c_k \, \ell_k(x)$$

$$f(x_j) = p(x_j) = c_j \underbrace{\ell_j(x_j)}_{=1} \Rightarrow c_j = f(x_j)$$

$$\boxed{p(x) = \sum_{j=0}^{n+1} f(x_j)\, \ell_j(x)}$$

De la forma de Newton

$$\boxed{C_n = \frac{f(x_{n+1}) - p(x_{n+1})}{(x_{n+1}-x_0)(x_{n+1}-x_1)\cdots(x_{n+1}-x_n)}}$$

método de diferencias divididas

$$C_{n+1} = f[x_0, x_1, \ldots, x_{n+1}] = \frac{f[x_1, x_2, \ldots x_{n+1}] - f[x_0, \ldots x_n]}{x_{n+1} - x_0}$$

$$f[x_i, x_{i+1}, \ldots, x_{i+k-1}, x_{i+k}] = \frac{f[x_{i+1}, x_{i+2}, \ldots, x_{i+k}] - f[x_i, x_{i+1}, \ldots x_{i+k-1}]}{x_{i+k} - x_i}$$