Topics

0. General Math Conventions

> $a, b, c, \ldots$ are used for constants

> $i, j, k, \ldots$ are used for enumeration

> n is used for quantity

> $x, y, z, \ldots$ are used for variables

1. Set Theory

1.1 Set Notation

Set of indices

> $I = \{0, 1, 2, 3, 4\}$

Set of countries

> $V = \{\text{``}America\text{''}, \text{``}Brazil\text{''}, \text{``}China\text{''}, \text{``}Denmark\text{''}, \text{``}Egypt\text{''}\}$

Element in a Set

> $i_1 \in I$

Set representation of a Python List

$$L = \{(i_1, v_1), (i_2, v_2), (i_3, v_3), (i_4, v_4), (i_5, v_5)\}$$

$$L = \{(i_k, v_j) \mid i_k \in I \text{ and } v_j \in V\}$$

or colon

$$L = \{(i_k, v_j) \ : \ i_k \in I \text{ and } v_j \in V\}$$

Cardinality of the set L

$$|L| = 5$$

1.2 Basic Set Operations

Given

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{4, 5, 6, 7, 8, 9, 10\}$$

Union of Two Sets

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

Intersection of Two Sets

$$A \cap B = \{4, 5\}$$

Union with Set Complement

$$A \cup B' = \{1, 2, 3, 4, 5\} = A$$

Intersection with Set Complement

$$A \cap B' = \{\} = \emptyset$$

1.3 More Set Operations

Given

$$A = \{1, 2, 3\}$$

$$B = \{4, 5, 6\}$$

$$C = \{7, 8, 9\}$$

Union of Multiple Sets

$$D = A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

or

$$D = \bigcup_{i=1}^{n} S_i$$

Intersection of Multiple Sets

$$E = \bigcap_{i=1}^{n} S_i = \{\} = \emptyset$$

where

$$S_1 = A, S_2 = B, S_3 = C$$
$$S = \{S_1, S_2, S_3\}$$

and

$$n = 3 = |S|$$

Propositional Calculus

2.1 Table of Symbols

Symbol	Name
$\wedge$	And

Symbol	Name
$\vee$	Or
$\neg$	Not
$\rightarrow$	Implies

2.2 Glossary of Words

Proposition
A statement that can only either be true of false

Argument
A collection of propositions

Premise
A clause of argument that can affect the conclusion

Conclusion
The truth value of an argument

2.3 Basic of Logic

p	q	p $\wedge$ q	p $\vee$ q	$\neg(p \wedge q)$
T	F	F	T	T

> p = "Fire is hot"
> q = "Japan is in South America"

> $p \wedge q$ = "Fire is hot and Japan is in South America"

> $p \vee q$ = "Fire is hot or Japan is in South America"

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

2.3 Inference

> $S_1 =$ "If you drive above speed limit, then you'll commit a crime"
> $S_2 =$ "If you commit a crime, then you'll be fined"

> $p =$ "You drive above the speed limit"
> $q =$ "You commit a crime"
> $r =$ "You are fined"

Rule of Inference: Syllogism

> $p \to q$
> $q \to r$
> $\therefore p \to r$

Rule of Inference: Modus Tollens

> $\neg q =$ "You DON'T commit a crime"
> $p \to q =$ "If you drive above the speed limit, then you commit a crime"
> $\therefore \neg p =$ "You DON'T drive above the speed limit"

2.4 Quantifiers

$\forall$ "For all"
$\exists$ "There exists"

Example 1

> $p(x) = x$ is greater than 5 and less than 10
> $T_p = \{x | x \in \mathbb{N}, \quad p(x) \text{ is true}\}$

> $T_p = \{6, 7, 8, 9\}$

Example 2

> $S_1 = \{3, 4, 5, 6, 8, 12, 14, 20\}$

> $p(x) = x \text{ ``is a multiple of'' } 2$
> $(\forall x \in S_1) p(x)$