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Fibonacci numbers

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Definition

The Fibonacci sequence is defined as follows:

$$F_0 = 0,$$
$$F_1 = 1,$$
$$F_n = F_{n-1} + F_{n-2}.$$

The first few of its members:

$$0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ ,\ 55,\ 89,\ \ldots$$

History

These numbers introduced in 1202 by Leonardo Fibonacci (Leonardo Fibonacci) (also known as Leonardo of Pisa (Leonardo Pisano)). However, thanks to the 19th century mathematician Luca (Lucas) the name of "Fibonacci numbers" became common.

However, the Indian mathematicians mentioned number of this sequence even earlier: Gopal (Gopala) until 1135, Hemachandra (Hemachandra) - in 1150

Fibonacci numbers in nature

Fibonacci himself mentioned these numbers in connection with this task: "A man planted a pair of rabbits in a pen surrounded on all sides by a wall. How many pairs of rabbits per year can produce a pair of this, if you know that every month, starting from the second, each pair rabbits gives birth to a pair? ". The solution to this problem and will be the number of sequences, now called in his honor. However, the situation described by Fibonacci - more mind game than real nature.

Indian mathematicians Gopala and Hemachandra mentioned this sequence number in relation to the number of rhythmic patterns resulting from the alternation of long and short syllables in verse, or the strong and weak beats in the music. The number of such patterns having generally $n$shares power F_n.

Fibonacci numbers appear in the work of Kepler in 1611, which reflected on the numbers found in nature (the work "On the hexagonal flakes").

An interesting example of plants - yarrow, in which the number of stems (and hence the flowers) is always a Fibonacci number. The reason for this is simple: as originally with a single stem, this stem is then divided by two, and then branches from the main stalk another, then the first two stems branch again, and then all the stems, but the last two, branch, and so on. Thus, each stalk after his appearance "skips" one branch, and then begins to divide at each level of branches, which results in a Fibonacci number.

Generally speaking, many colors (eg, lilies), the number of petals is a way or another Fibonacci number.

Also botanically known phenomenon of 'phyllotaxis'. As an example, the location of sunflower seeds: if you look down on their location, you can see two simultaneous series of spirals (like overlapping): some are twisted clockwise, the other - against. It turns out that the number of these spirals is roughly equal to two consecutive Fibonacci numbers: 34 and 55 or 89 and 144 Similar facts are true for some other colors, as well as pine cones, broccoli, pineapple, etc.

For many plants (according to some sources, 90% of them) are true and an interesting fact. Consider any sheet, and will descend downwardly until, until we reach the sheet disposed on the stem in the same way (i.e., directed exactly in the same way). Along the way, we assume that all the leaves that fall to us (ie, located at an altitude between the start and end sheet), but arranged differently. Numbering them, we will gradually make the turns around the stem (as the leaves are arranged on the stem in a spiral). Depending on whether the windings perform clockwise or counterclockwise will receive a different number of turns. But it turns out that the number of turns, committed us clockwise the number of turns, committed anti-clockwise, and the number of leaves encountered form 3 consecutive Fibonacci numbers.

However, it should be noted that there are plants for which The above calculations give the number of all other sequences, so you can not say that the phenomenon of phyllotaxis is the law - it is rather entertaining trend.

Properties

Fibonacci numbers have many interesting mathematical properties.

Here are just a few of them:

- Value for Cassini:

$$F_{n+1} F_{n-1} - F_n^2 = (-1)^n.$$

- Rule "addition":

$$F_{n+k} = F_k F_{n+1} + F_{k-1} F_n.$$

- From the previous equation at $k = n$follows:

  $$F_{2n} = F_n (F_{n+1} + F_{n-1}).$$

- From the previous equality by induction we can show that

  F_{nk}always divisible F_n.
- Converse is also true to the previous statement:

  if $F_m$fold F_n, the $m$fold n.
- GCD-equality:

  $$\gcd(F_m, F_n) = F_{\gcd(m,n)}.$$

- With respect to the Euclidean algorithm Fibonacci numbers have the remarkable property that they are the worst input data for this algorithm (see. "Theorem Lame" in Euclid's algorithm).

Fibonacci number system

Zeckendorf theorem asserts that every positive integer $n$can be uniquely represented as a sum of Fibonacci numbers:

$$N = F_{k_1} + F_{k_2} + \ldots + F_{k_r}$$

where $k_1 \geq k_2 + 2, k_2 \geq k_3 + 2, \cdots, k_r \geq 2$(ie, can not be used in the recording of two adjacent Fibonacci numbers).

It follows that any number can be written uniquely in **the Fibonacci value** , for example:

$$9 = 8 + 1 = F_6 + F_1 = (10001)_F,$$
$$6 = 5 + 1 = F_5 + F_1 = (1001)_F,$$
$$19 = 13 + 5 + 1 = F_7 + F_5 + F_1 = (101001)_F,$$

And in any number can not go two units in a row.

It is easy to get and usually adding one to the number in the Fibonacci value: if the least significant digit is 0, then it is replaced by 1, and if it is equal to 1 (ie, in the end there is a 01), then 01 then 10 is replaced by the "fix" record sequentially correcting all 011 by 100 As a result, the linear time is obtained by recording a new number.

Translation numbers in the Fibonacci number system with a simple "greedy" algorithm: just iterate through the Fibonacci numbers from high to low, and if for some $F_k \leq n$, it $F_k$is included in the record number n, and we take away F_k from n, and continue to search.

The formula for the n-th Fibonacci number

Formula by radicals

There is a wonderful formula, called by the name of the French mathematician Binet (Binet), although it was known to him Moivre (Moivre):

$$F_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}.$$

This formula is easy to prove by induction, but you can bring it by the concept of forming or using the solution of the functional equation.

Immediately you will notice that the second term is always less than 1 in absolute value, and furthermore, decreases very rapidly (exponentially). This implies that the value of the first term gives the "almost" value F_n. This can be written simply as:

$$F_n = \left[\frac{\left(\frac{1+\sqrt{5}}{2}\right)^n}{\sqrt{5}} \right],$$

where the square brackets denote rounding to the nearest integer.

However, for practical use in the calculation of these formulas hardly suitable, because they require very high precision work with fractional numbers.

Matrix formula for the Fibonacci numbers

It is easy to prove the following matrix equation:

$$\begin{pmatrix} F_{n-2} & F_{n-1} \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} F_{n-1} & F_n \end{pmatrix}.$$

But then, denoting

$$P \equiv \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix},$$

obtain

$$\begin{pmatrix} F_0 & F_1 \end{pmatrix} \cdot P^n = \begin{pmatrix} F_n & F_{n+1} \end{pmatrix}.$$

Thus, to find nth Fibonacci number necessary to build a matrix $P$of power n.

Remembering that the construction of the matrix in the $n$degree of th can be accomplished $O(\log n)$(see. binary exponentiation), it turns out that $n$the number of Fibonacci retracement can be easily calculated for $O(\log n)$c using only integer arithmetic.

Periodicity of the Fibonacci sequence modulo

Consider the Fibonacci sequence $F_i$modulo some p. We prove that it is periodic, and moreover the period begins with $F_1 = 1$(ie preperiod contains only F_0).

We prove this by contradiction. Consider pairs of Fibonacci numbers taken modulo p:

Since modulo $p$may be only different pairs, there exists the sequence of at least two of the same pair. This already means that the sequence is periodic.

We now choose among all such identical pairs of two identical pairs with the lowest numbers. Let this pair with some of the rooms and . We will prove that . Indeed, otherwise they will have to previous couple and that, by the property of the Fibonacci numbers, will also be equal to each other. However, this contradicts the fact that we chose the matching pairs with the lowest numbers, as required.

Literature

- Ronald Graham, Donald Knuth, and Oren Patashnik. **Concrete Mathematics** [1998]

3 • Ответить • Поделиться ›

Ivan Nikulin ➦ Guest • 2 года назад

f[0]=f[1]=1;
for (int i=2; i<=n; i++)
f[i]=f[i-1]+f[i-2];
Здесь не написано только это, насколько я вижу. Ссылка на возведение в степень дается, жадный алгоритм реализовать видимо не всем дано :) Чего вам еще не хватает?

P.S. Кстати, задача на перевод числа в Фиббоначиеву систему счисления: www.e-olimp.com/problems/1378/

4 • Ответить • Поделиться ›

Samandar Ravshanov • 2 года назад

Кому интересно на питоне можно реализовать такими способами:

```
def fibonacci(mnum):
"фунция выводит всех чисел Фибоначчи до определенного"
    a,b, fiblist = 0, 1, []
    while a < mnum:
        fiblist.append(a)
        a, b = b, a+b
    return fiblist

---------------------------------------
def fib(n):
"рекурсивная функция"
    if n < 3:
        return 1
    return fib(n-1) + fib(n-2)
```

2 • Ответить • Поделиться ›

Louise ➦ Samandar Ravshanov • 2 года назад

Второй вариант взорвётся уже при небольших n.

9 • Ответить • Поделиться ›

Sonych KO ➦ Samandar Ravshanov • 3 месяца назад

или как-то так на генераторах

```
def fib():
a, b = 0, 1
while True:
yield a
a, b = b, a+b
```

• Ответить • Поделиться ›

Владислав • 7 месяцев назад

Хотелось бы реализацию логарифмического алгоритма.

• Ответить • Поделиться ›

Luka • 9 месяцев назад

f(n+1)*f(n-1) - f(n)*f(n) = (-1)^n

Википедия: f(n+1)*f(n+2) - f(n)*f(n+3) = (-1)^n

Неувязочка... (в статье формула не верна)

• Ответить • Поделиться ›

ASD → Luka • 4 месяца назад

оу формула не верна ...

• Ответить • Поделиться ›