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Posted: 5 Nov 2008 20:17
Edited: 23 Aug 2010 22:24

# Kirchhoff matrix theorem. Finding the number of spanning trees

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Connected undirected graph given its adjacency matrix. Multiple edges in the graph are allowed. Required to count the number of different spanning trees of this graph.

The below formula belongs to Kirchhoff (Kirchhoff), who proved it in 1847

## Kirchhoff matrix theorem

Take the adjacency matrix of G, replace each element of this matrix is the opposite, and instead of a diagonal element of $A_{I, I}$ will deliver the degree of node i (if there are multiple edges, the vertex degree, they are taken into account with its multiplicity). Then, according to Theorem Kirchhoff matrix, all cofactors of this matrix are equal to each other, and equal to the number of spanning trees of the graph. For example, you can delete the last row and last column of the matrix, and the modulus of its determinant is equal to the desired quantity.

Determinant of the matrix can be found in O ($N^3$) using the Gauss method or the method of Kraut .

The proof of this theorem is quite difficult and is not presented here (see., For example, coming VB "dimer problem and the Kirchhoff theorem").

## Communication with the laws of Kirchhoff in the electrical circuit

Between the matrix and the Kirchhoff theorem Kirchhoff laws for circuit has a surprising connection.

It can be shown (as a consequence of Ohm's law and Kirchhoff's first law), that the resistance $R_{ij}$ between points i and j is equal to the circuit:

$$R_{ij} = | T^{(I, J)} | / | T^{J} |$$

where the matrix T is obtained from the matrix A *reverse* resistance of the conductor ($A_{ij}$ - the inverse number of the resistance of the conductor between two points i and j) transformation matrix described in Theorem Kirchhoff, and the notation T $^{(I)}$ denotes vychërkivanie row and column with the number i, and T $^{(I,\ J)}$ - vychërkivanie two rows and columns i and j.

Kirchhoff theorem gives this formula geometric meaning.



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У Вас здесь орфографическая ошибка: не "на диагонале", а "на диагонали".