added: 10 Jun 2008 18:05
Edited: 24 Aug 2011 1:41

Gray code

Contents [hide]

Definition

Gray code is called a system of numbering negative numbers when the codes of two adjacent numbers differ in exactly one bit.

For example, for the numbers of length 3 bits, we have a sequence of Gray codes: $000, 001, 011, 010, 110, 111, 101, 100$. Eg $G(4) = 6$.

This code was invented by Frank Gray (Frank Gray) in 1953.

Finding the Gray code

Consider the number of bits $n$and the number of bits $G(n)$. Note that ith bit $G(n)$is equal to one only in the case where ith bit $n$equal to one and $i + 1$th bit is zero, or vice versa (ith bit is zero, and $i + 1$th is equal to unity). Thus, we have $G(n) = n \oplus (n >> 1)$:

```
int g (int n) {
        return n ^ (n >> 1);
}
```

Finding the inverse Gray code

Required by the Gray code $g$to restore the original number n.

We shall go on to junior high order bits (albeit the least significant bit is numbered 1, and the oldest - k). Obtain the following relations between the bits $n_i$of $n$bits and $g_i$number g:

$$n_k = g_k,$$
$$n_{k-1} = g_{k-1} \oplus n_k = g_k \oplus g_{k-1},$$
$$n_{k-2} = g_{k-2} \oplus n_{k-1} = g_k \oplus g_{k-1} \oplus g_{k-2},$$
$$n_{k-3} = g_{k-3} \oplus n_{k-2} = g_k \oplus g_{k-1} \oplus g_{k-2} \oplus g_{k-3},$$
$$\ldots$$

In the form of program code is the easiest way to record as follows:

```
int rev_g (int g) {
        int n = 0;
        for (; g; g>>=1)
                n ^= g;
        return n;
}
```

Applications

Gray codes have several applications in different areas, sometimes quite unexpected:

- n-bit Gray code corresponds to a Hamiltonian cycle on the nn-dimensional cube.
- In the art, Gray codes are used to **minimize errors** when converting the analog signals into digital signals (for example, sensors). In particular, the Gray code and are visible in connection with this application.
- Gray codes are used in solving the problem of **the Tower of Hanoi** .

  Let n- number of disks. Let's start with the Gray code of length n, consisting of all zeros (ie $G(0)$), and will move on Gray codes (from $G(i)$ proceeding to $G(i+1)$). With every $i$bit of this -th Gray code ith disk (and the most significant bit corresponds to the smallest size drive, and the most significant bit - the largest). Since at each step exactly one bit is changed, then we can understand the bit change $i$as moving ith disc. Note that for all drives except the smallest, at each step there is exactly one option course (except for the starting and final products). For the smallest disk always has two variations, however, there is the strategy of choice course, always leads to the answer: if $n$is odd, then the sequence of movements of the smallest drive is of the form $f \to t \to r \to f \to t \to r \to \ldots$(where f- starting rod t- the final rod r- the remainder of the rod), and if it $n$is even then $f \to r \to t \to f \to r \to t \to \ldots$.
- Gray codes also are used in the theory of **genetic algorithms** .

Problem in online judges

List of tasks that can be taken, using Gray codes:

- SGU # 249 **"Matrix"** [Difficulty: Medium]