Задача 5. Выполнил: А?. Д. Экзакин

1. $$\int d\vec{y}\, \frac{1}{(2\pi)^{l/2}} \sqrt{\det A^{-1}}\, \exp\left(-\frac{(\tilde{y}-y)^T A (\tilde{y}-y)}{2}\right) =$$

т.к. $SS^T = \mathbb{1}$

$$= \int d\vec{z}\, \frac{\overset{=1}{\det S}}{(2\pi)^{l/2} (\det A^{-1})^{1/2}} \exp\left(-\frac{z^T D z}{2}\right) =$$

$$= \int d\vec{z}\, \frac{1}{(2\pi)^{l/2} (\det A^{-1})^{\frac{1}{2}}} \exp\left(-\frac{1}{2} z_i^2 \lambda_i\right) =$$

$$= \frac{(2\pi)^{l/2}}{(2\pi)^{l/2}} \cdot \frac{\sqrt{\det A}}{\sqrt{\prod \lambda_i}} = 1$$

*здесь, в задании, опечатка.

2. $\langle\langle \tilde{y}_i \tilde{y}_j \rangle\rangle$ — ?

$$I(J) = \int d^l Y e^{-\frac{1}{2} Y^T A Y + J^T Y} = \int d^l Y e^{-\frac{1}{2}\{(Y+\xi)^T A (Y+\xi)\} + J^T Y}$$

$$+ \frac{1}{2}\xi^T A \xi + \xi^T A Y = \left| \xi^T = -J^T A^{-1} \right| = \int d^l Y e^{-\frac{1}{2} Y^T A Y + \frac{1}{2}\xi^T A \xi}$$

$$\Updownarrow$$

$$\xi = -A^{-T} J \quad ;(A^T = A)$$

$\frac{1}{(2\pi)^{l/2} (\det A^{-1})^{1/2}} I(y) \to I(y)$.

$$\Rightarrow \quad I(J) = 1 \cdot e^{+\frac{1}{2} J^T A^{-1} A A^{-T} J} = e^{\frac{1}{2} J^T \underset{\equiv \Sigma}{A^{-1}} J};$$

$$\partial_i \partial_j I(J)\Big|_{J=0} = \langle\langle \tilde{y}_i, \tilde{y}_j \rangle\rangle = \partial_i \partial_j \exp\left(\frac{1}{2} J_k J_l \delta_{kl}\right) =$$

$$= \partial_j \left(J_l \delta_{il} \exp(\dots)\right) = (\delta_{ij} + J_l \delta_{jl}) \exp(\dots) \Big|_{J=0}$$

$$= \delta_{ij} = \langle\langle \tilde{y}_i, \tilde{y}_j \rangle\rangle$$

3. $\langle\langle w_i, w_j \rangle\rangle \equiv W_{ij}$ $\qquad w = \underbrace{(X^T X^{-1}) X^T}_{\equiv Q} y$

$$W_{ij} = \langle\langle w w^T \rangle\rangle_{ij} = \langle\langle Q y y^T Q^T \rangle\rangle_{ij} =$$

$$= Q \langle\langle y y^T \rangle\rangle Q^T = Q \Sigma Q^T$$

$\hookrightarrow$ $\Delta w_\alpha = \sqrt{\sigma_{ij} q_{i\alpha} q_{j\alpha}}$ $\qquad$ Q.shape = (F, l)

F < l.

4. $A = \mathrm{diag}(A_1 \dots A_l) \Rightarrow \Sigma = \mathrm{diag}(\sigma_1, \dots, \sigma_l)$

$\hookrightarrow$ $\sqrt{W_{\alpha\alpha}} = \sqrt{Q \Sigma Q^T}_{\alpha\alpha} = \sqrt{\sigma_{ii} q_{i\alpha} q_{i\alpha}} = \sqrt{\sum_i \sigma_i q_{i\alpha}^2}$

$\quad \shortparallel$

Δw_α

Тогда $A_i = \frac{1}{S_i^2}$ для моделирования погрешности.