lab1

November 8, 2023

[294]:

```
import sys
print(sys.executable)
```

```
/Users/zeyuli/anaconda3/envs/ECE661_HW/bin/python
```

[295]:

```
import numpy as np
import matplotlib.pyplot as plt
from typing import Tuple

x1, y1 = np.array([ 1, -2, -1, -1,  1]), 7
x2, y2 = np.array([ 2, -1,  2,  0, -2]), 1
x3, y3 = np.array([-1,  0,  2,  2,  1]), 1

x_input = np.array([x1, x2, x3]).reshape(3, 5)
y_input = np.array([y1, y2, y3])

w0 = np.zeros(5).reshape(-1, 1)
mu = 0.02
n_epochs = 200
print(x_input, y_input)

LEGEND_size = 7
TITLE_size = 12
AXLABEL_size = 9
TICK_size = 8
```

```
[[ 1 -2 -1 -1  1]
 [ 2 -1  2  0 -2]
 [-1  0  2  2  1]] [7 1 1]
```

[296]:

```
def my_relu(a):
    return np.abs(a) * (a > 0)



def l1_norm_trimmed_update(in_x, in_y, w_prev, l1_reg_param, lr, thresh):
    """
    thresh: number of elements to keep unchanged when performing proximal l1
    """
```

```python
    ascend_idx = np.argsort(np.abs(w_prev.flatten()))
    weight_l1_proximal = l1_norm_proximal_update(in_x, in_y, w_prev,␣
↪l1_reg_param, lr,
                                                        which_weights=ascend_idx[0:
↪w_prev.shape[0] - thresh])
    return weight_l1_proximal


def l1_norm_proximal_update(in_x, in_y, w_prev, l1_reg_param, lr,␣
↪which_weights=None):
    """
    Computes weight using proximal gradient descent (l1-norm). threshold is␣
↪l1_reg_param*lr
    Args:
        in_x: x values (data)
        in_y: y values (target)
        w_prev: previous weight
        l1_reg_param: l1 regularisation (lambda)
        lr: learning rate, must be not None.
        which_weights: indices of which weights for which proximal update is␣
↪applied
    Note: threshold = lr * lambda should be in set{0.004, 0.01, 0.02, 0.04}
    Returns: updated weight
    """
    if which_weights is None:
        which_weights = np.array(range(len(w_prev.flatten())))
        # which_weights = [0,1,2,3,4] if w_prev.shape == (5, 1)
    weight_tilde = no_regularisation_update(in_x, in_y, w_prev, lr=lr) # main␣
↪objective
    weight_ret = np.copy(weight_tilde)

    threshold = lr * l1_reg_param
    assert threshold >= 0

    for iii in which_weights:  # only updates the weights specified in␣
↪which_weights
        this_weight = weight_tilde[iii, 0]
        weight_ret[iii, 0] = my_relu(np.abs(this_weight) - threshold) * np.
↪sign(this_weight)
    return weight_ret


def l1_norm_update(in_x, in_y, w_prev, l1_reg_param, lr):
    """
    Args:
        in_x: x values (data)
```

```
        in_y: y values (target)
        w_prev: previous weight
        l1_reg_param: l1 regularisation (lambda)
        lr: learning rate, must be not None
    Returns: updated weight
    """
    grad = no_regularisation_update(in_x, in_y, w_prev, l1_reg_param, lr=None)␣
↪+ \
           l1_reg_param * np.sign(w_prev)
    assert grad.shape == w_prev.shape
    return w_prev - lr * grad


def no_regularisation_update(in_x, in_y, w_prev, l1_reg_param=None, lr=None):
    """
    Args:
        in_x: x values (data)
        in_y: y values (target)
        w_prev: previous weight
        l1_reg_param: dummy
        lr: if None, it means some fn other than run_grad_descent called it; if␣
↪run_grad_descent called it,
        must be (not None)
    Returns: returns updated weight if lr is not None; returns gradient if lr␣
↪is None
    """
    n_r, n_c = in_x.shape
    grad = 0
    for inner in range(n_r):
            grad += 2 * (np.dot(in_x[inner, :], w_prev) - in_y[inner]) *␣
↪in_x[inner].reshape(-1, 1)
    if lr is not None:
        return w_prev - lr * grad
    else:
        return grad


def run_grad_descent(w_init, lr, in_x, in_y, epochs, w0_prune=False,␣
↪thresh=None, l1_reg_param=None,
                     grad_fn=no_regularisation_update) -> Tuple[np.array, np.
↪array, np.array]:
    """
    Args:
        w_init: w0 = np.zeros(5)
        lr: mu=0.02
        in_x: x1, x2, x3
```

```
        in_y: y1, y2, y3
        epochs: 200
        w0_prune: prune or not (lab 1 c). should be False in all except for lab␣
↪1c
        thresh: threshold for pruning or trimmed l1. should be True only in lab␣
↪1 c, lab 1 f
        l1_reg_param: lambda, used in update weight
        grad_fn: function for updating weight
    Returns: optimal weight (np.array), loss for each epoch (np.array), and␣
↪w_arr matrix to
    record the progression of each weight through time
    """
    w_prev = w_init
    l_arr = []
    w_arr = np.zeros((len(w_init), epochs))
    n_r, n_c = in_x.shape
    for i in range(epochs):
        if not w0_prune and thresh is not None:  # only for lab 1 f, l1-trimmed
            w_curr = grad_fn(in_x, in_y, w_prev, l1_reg_param, lr=lr,␣
↪thresh=thresh)
        else:
            w_curr = grad_fn(in_x, in_y, w_prev, l1_reg_param, lr=lr)

        if w0_prune and thresh is not None:  # only for lab 1 c, pruning
            w_curr_sorted_idx = np.argsort(np.abs(w_curr.flatten()))  # sort␣
↪ascending
            for inner in range(len(w_init) - thresh):
                # len(w_init) - thresh = 5 - 2 = 3, so three smallest weights␣
↪(absolute value) are
                # forced to zero
                w_curr[w_curr_sorted_idx[inner], 0] = 0  # zero-ing out␣
↪smallest elements
        l_curr = 0
        for inner in range(n_r):
            l_curr += (np.dot(in_x[inner, :], w_curr) - in_y[inner]) ** 2
        l_arr.append(l_curr)
        w_arr[:, i] = w_curr.flatten()
        w_prev = w_curr
    return w_prev, np.array(l_arr), w_arr


def plot_log_loss_vs_n_iters(num_epochs, loss_array, save_name_, save=False,
                             plt_axis=None):
    if plt_axis is None:
        fig, ax = plt.subplots(1, 1)
    else:
```

```
        fig, ax = None, plt_axis
    ax.semilogy(range(num_epochs), loss_array, label='Lowest loss=%g' % np.
↪min(loss_array))
    ax.set_xlabel('Number of Iterations', fontsize=AXLABEL_size)
    ax.set_ylabel('L (log10 scale)', fontsize=AXLABEL_size)
    ax.set_title('Loss vs Number of Iterations', fontsize=TITLE_size)
    ax.tick_params(axis='both', labelsize=TICK_size)
    ax.legend(loc='best', fontsize=LEGEND_size)
    if fig is not None:
        fig.tight_layout()
    if save:
        plt.savefig('%s.pdf' % save_name_, dpi=700, bbox_inches='tight')


def plot_weight_val_vs_n_iters(num_epochs, w_init, final_weights, weight_array,␣
↪save_name_, save=False,
                               plt_axis=None):
    if plt_axis is None:
        fig, ax = plt.subplots(1, 1)
    else:
        fig, ax = None, plt_axis
    line_styles = ['-', ':', '--', '-.', (0, (1, 1))]
    for iii in range(len(w_init)):
        ax.plot(range(num_epochs), weight_array[iii, :],␣
↪linestyle=line_styles[iii],
                linewidth=1., label='final value %.5f' % final_weights[iii])
    ax.legend(loc='best', fontsize=LEGEND_size)
    ax.tick_params(axis='both', labelsize=TICK_size)
    ax.set_xlabel('Number of Iterations', fontsize=AXLABEL_size)
    ax.set_ylabel('Value of Weights', fontsize=AXLABEL_size)
    ax.set_title('Weight Values vs Number of Iterations', fontsize=TITLE_size)
    if fig is not None:
        fig.tight_layout()
    if save:
        plt.savefig('%s.pdf' % save_name_, dpi=700, bbox_inches='tight')


def plot_many_w_vals_or_loss_vs_n_iters(num_epochs, w_init, l1_reg_params,␣
↪save_name_, save=False,
                                        loss_arr_lst=None, final_weights_list=None,␣
↪weight_array_list=None):
    """
    Args:
        num_epochs: number of iterations (200)
        w_init: initial weights
        final_weights_list: list of weights achieved by gradient descent by␣
↪default None
```

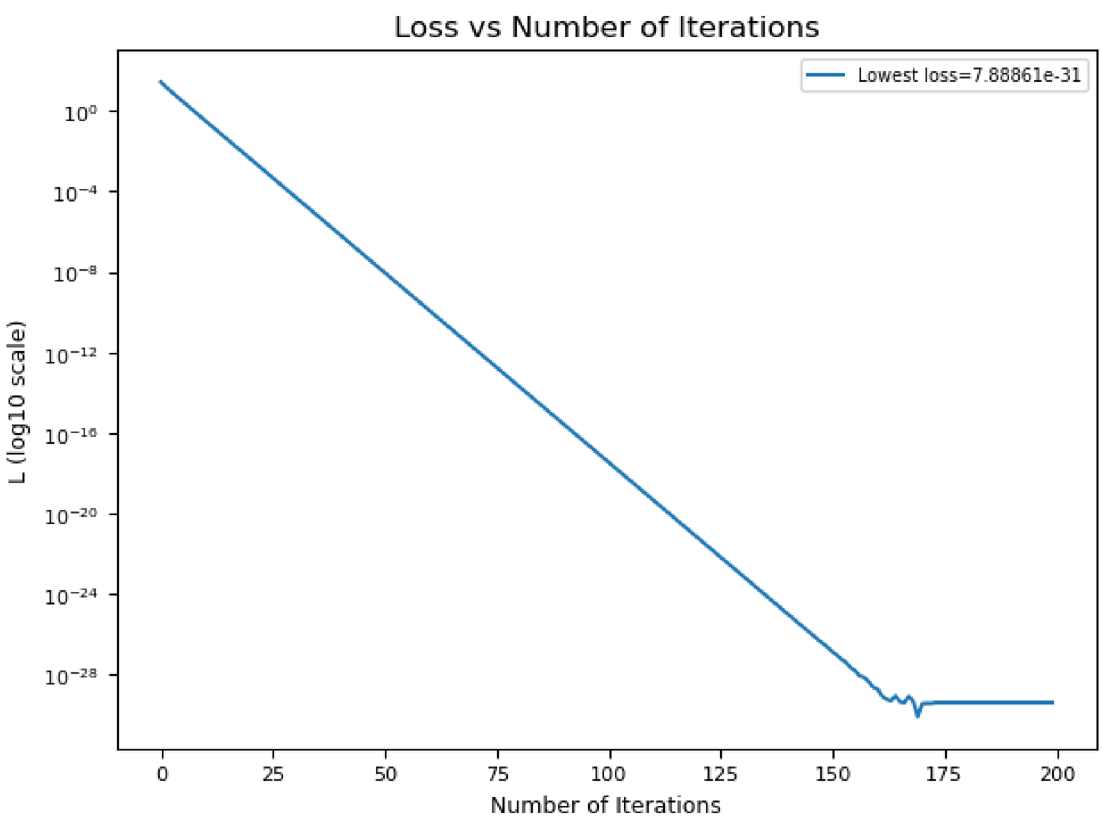
```
        weight_array_list: list of np array that describe weight progression by␣
↪default None
        loss_arr_lst: list of loss arrays (len(loss_arr_lst) corresponds to␣
↪len(l1_reg_params))
        by default, loss_arr_lst=None. If not None,
        l1_reg_params: reg params (lambda)
        save_name_: name of file
        save: save or not
    Either (final_weights_list and weight_array_list are not None) or␣
↪(loss_arr_lst is not None)
    Returns: None
    """
    assert (final_weights_list is not None and weight_array_list is not None)␣
↪or \
        loss_arr_lst is not None, \
        "Either (final_weights_list and weight_array_list are not None) or␣
↪(loss_arr_lst is not None)"
    assert not (final_weights_list is not None and weight_array_list is not␣
↪None and
        loss_arr_lst is not None), "all three are not None."
    fig, ax = plt.subplots(2, 2, figsize=(10, 8))
    ax_flat = ax.flatten()
    for iii, axis in enumerate(ax_flat):
        if loss_arr_lst is None:
            plot_weight_val_vs_n_iters(num_epochs, w_init,␣
↪final_weights=final_weights_list[iii],
                                       weight_array=weight_array_list[iii],␣
↪save_name_=None, save=False,
                                       plt_axis=axis)
        else:
             plot_log_loss_vs_n_iters(num_epochs, loss_array=loss_arr_lst[iii],␣
↪save_name_=None,
                                      save=False, plt_axis=axis)
        ax_flat[iii].set_title("lambda=%g" % l1_reg_params[iii])
    if fig is not None:
        fig.tight_layout()
    if save_name_ is not None and save:
        plt.savefig('%s.pdf' % save_name_, dpi=700, bbox_inches='tight')
    return
```

[297]:
```
""" part b (3 pts) In Python, directly minimize the objective L without any␣
↪sparsity-inducing regularisation/constraint. Plot the value of log(L) vs.␣
↪#steps throughout the training, and use another figure to plot how the value␣
↪of each element in W is changing throughout the training. From your result,␣
↪is W converging to an optimal solution? Is W converging to a sparse solution?
"""
```

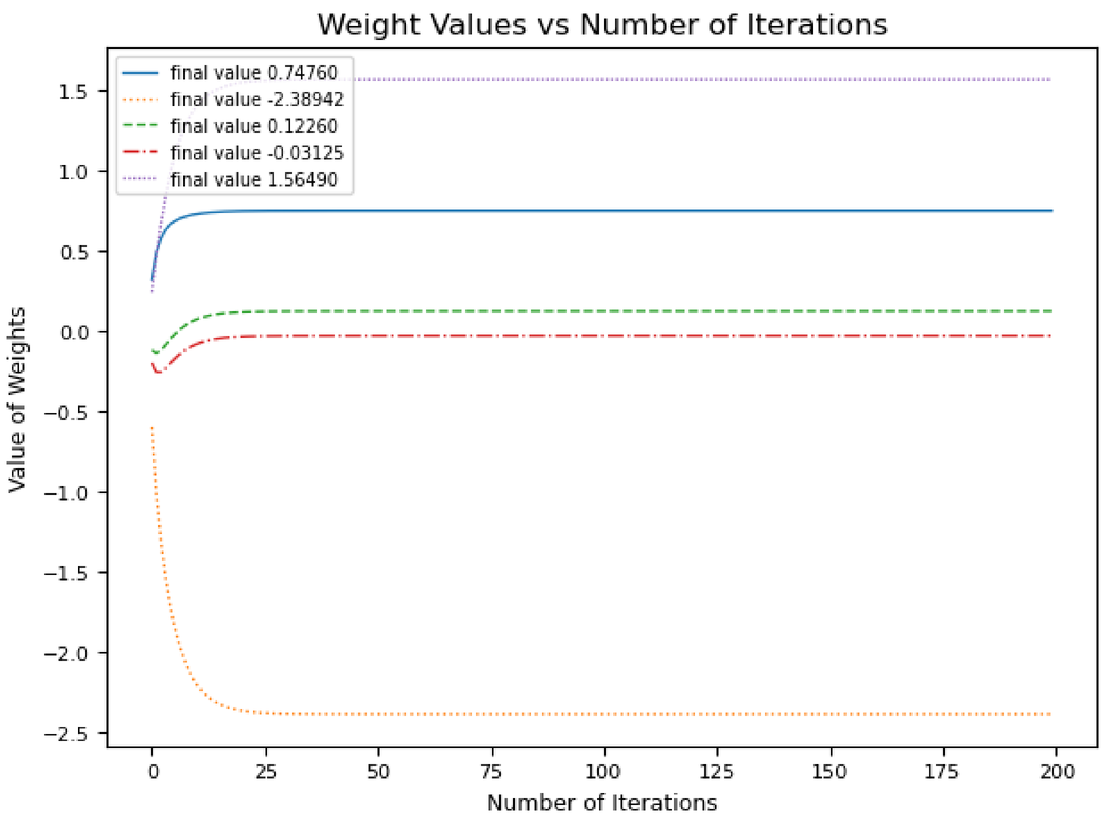
```
weight, loss_arr, weight_arr = run_grad_descent(w0, mu, x_input, y_input,␣
↪epochs=n_epochs)
print(weight.shape, loss_arr.shape, weight_arr.shape)
```

(5, 1) (200, 1) (5, 200)

[298]:
```
# part b
plot_log_loss_vs_n_iters(num_epochs=n_epochs, loss_array=loss_arr,␣
↪save_name_='lab1b_loss', save=True)
```

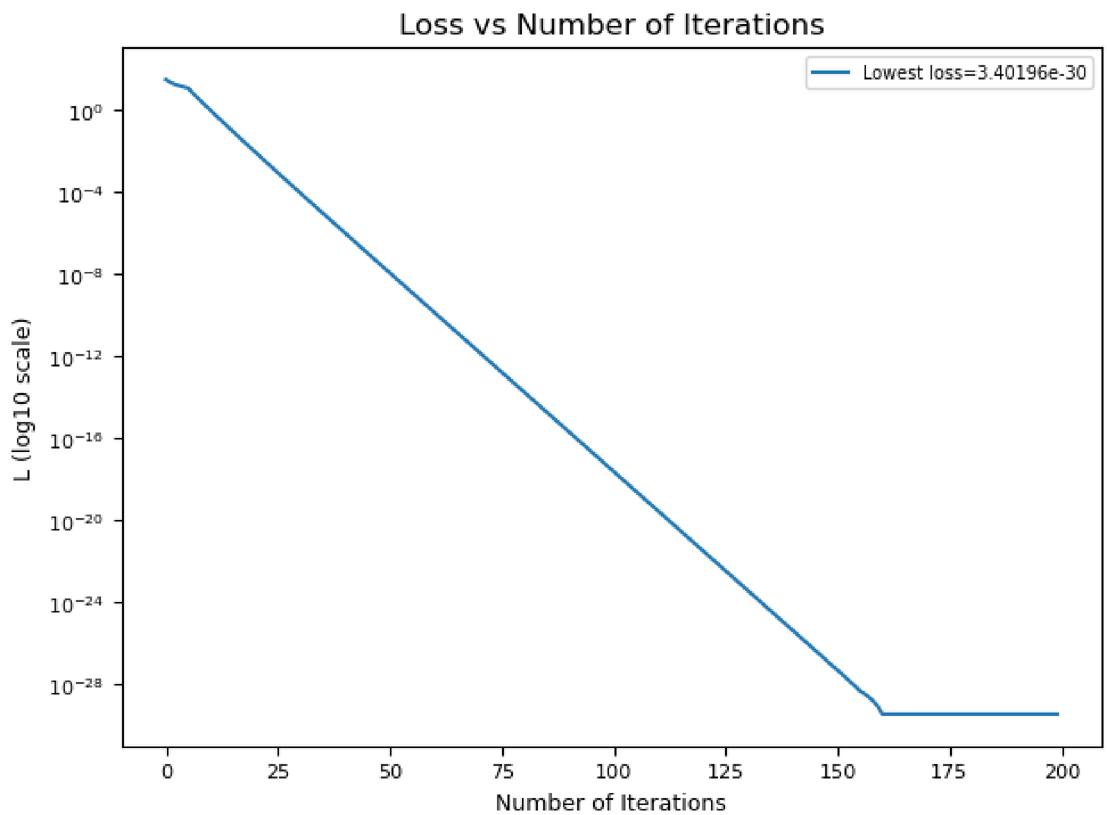


[299]:
```
# part b
plot_weight_val_vs_n_iters(num_epochs=n_epochs, w_init=w0, final_weights=weight,
                           weight_array=weight_arr, save_name_='lab1b_weights',␣
↪save=True)
```



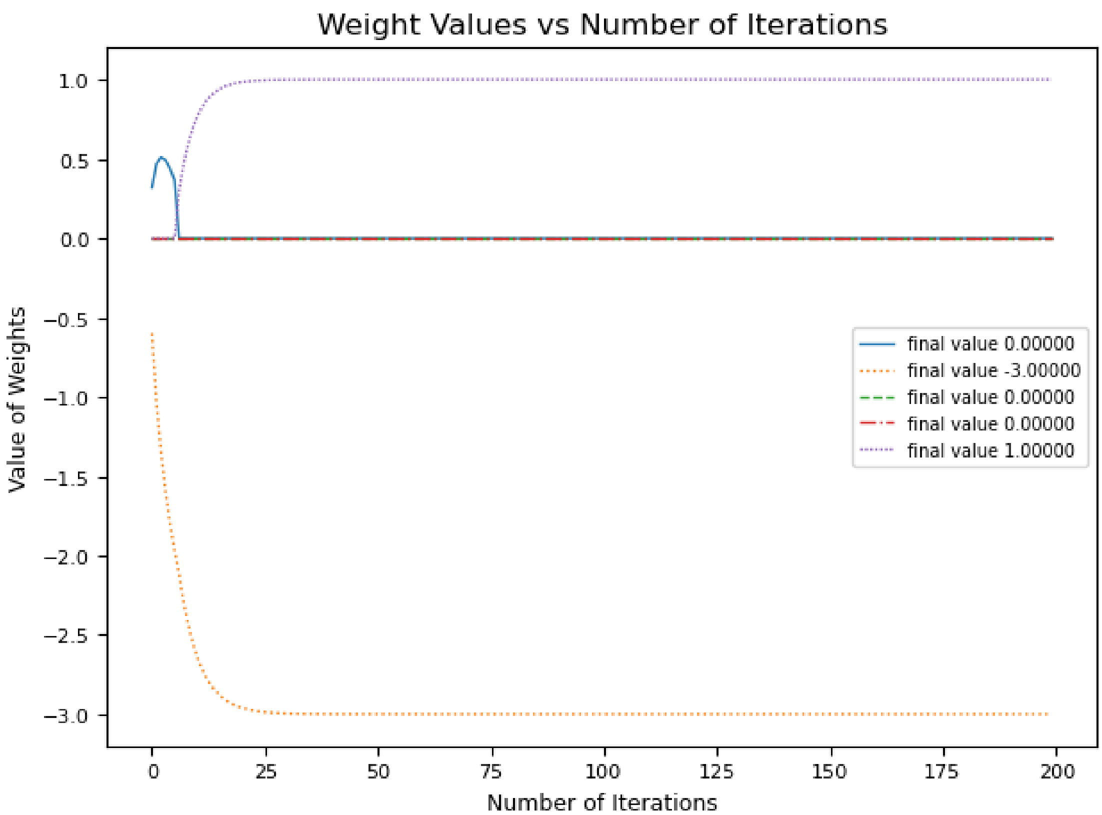
[300]:
```
"""part (c) (6 pts) Since we have the knowledge that the ground-truth weight␣
↪should have ||W||0   2, we can apply projected gradient descent to enforce␣
↪this sparse constraint. Redo the optimization process in (b), this time␣
↪prune the elements in W after every gradient descent step to ensure ||W^l||0␣
↪ 2. Plot the value of log(L) throughout the training, and use another figure␣
↪to plot the value of each element in W in each step. From your result, is W␣
↪converging to an optimal solution? Is W converging to a sparse solution?"""
weight_pruned, loss_arr_pruned, weight_arr_pruned = \
    run_grad_descent(w0, mu, x_input, y_input, n_epochs, w0_prune=True,␣
↪thresh=2)
```

[301]:
```
plot_log_loss_vs_n_iters(num_epochs=n_epochs, loss_array=loss_arr_pruned,␣
↪save_name_='lab1c_loss',
                         save=True)
```



[302]:

```
# part c
plot_weight_val_vs_n_iters(num_epochs=n_epochs, w_init=w0,␣
 ↪final_weights=weight_pruned,
                                weight_array=weight_arr_pruned,␣
 ↪save_name_='lab1c_weights', save=True)
```

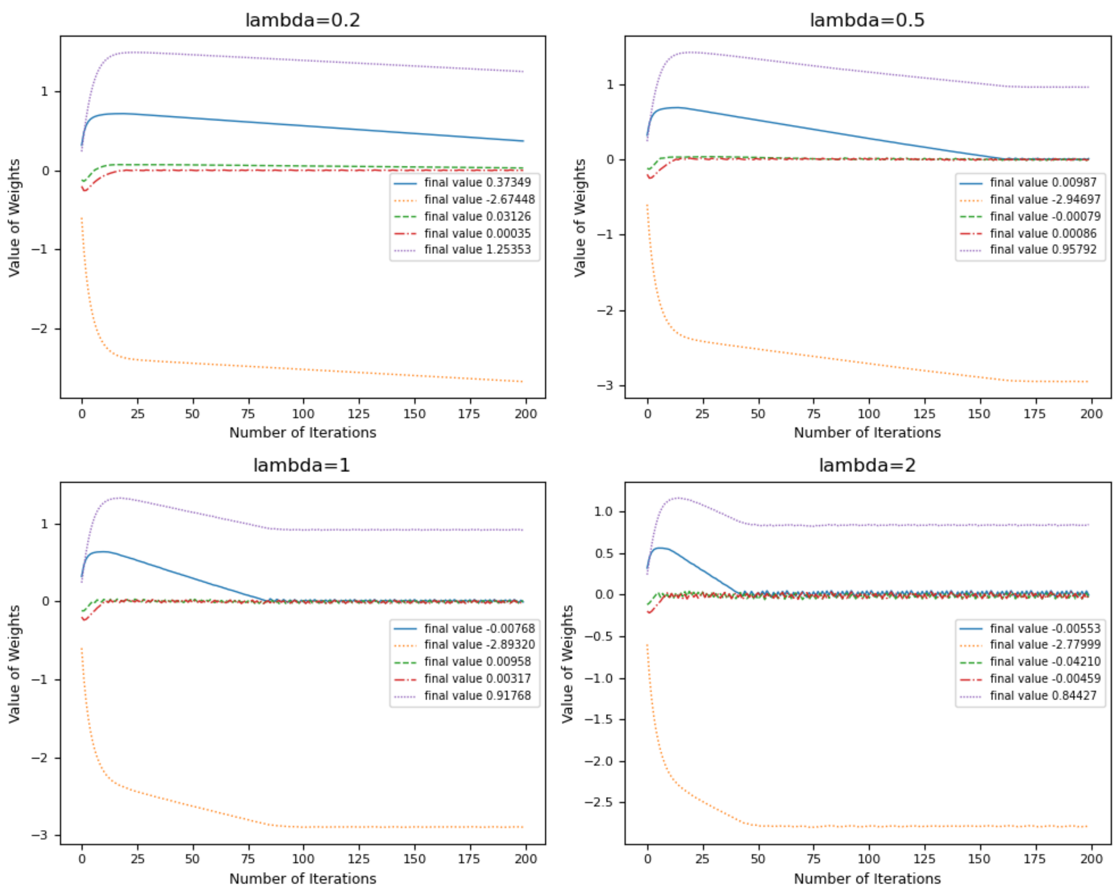


[303]:

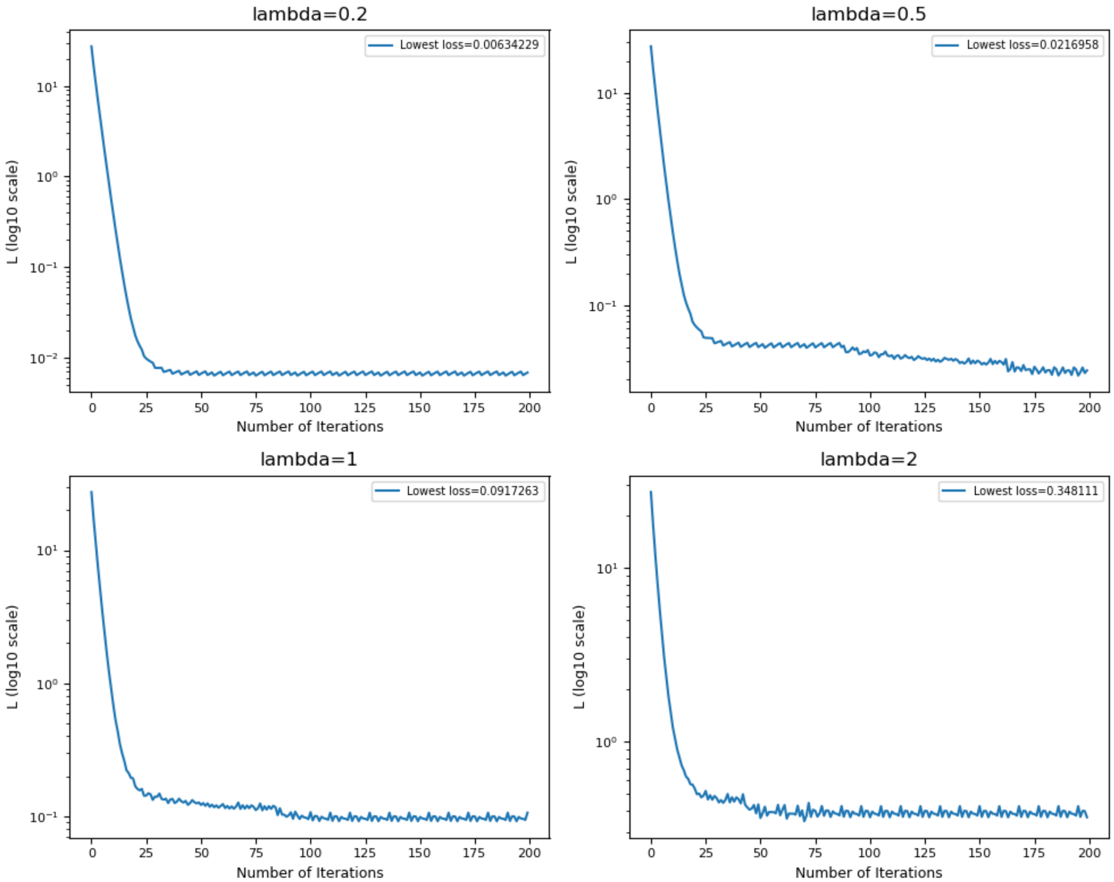
```
"""part (d) (5 pts) In this problem we apply l1 regularization to induce the␣
↪sparse solution. The minimization objective therefore changes to L +␣
↪lambda||W||_1. Please use full-batch gradient descent to minimize this␣
↪objective, with lambda = {0.2, 0.5, 1.0, 2.0} respectively. For each case,␣
↪plot the value of log(L) throughout the training, and use another figure to␣
↪plot the value of each element in W in each step. From your result, comment␣
↪on the convergence performance under different lambda."""

lambda_list = [0.2, 0.5, 1.0, 2.0]
weight_l1_list, loss_l1_list, weight_arr_l1_list = [], [], []
for idx, lll in enumerate(lambda_list):
    weight_l1, loss_arr_l1, weight_arr_l1 = run_grad_descent(w0, mu, x_input,␣
↪y_input, n_epochs,
                                                                w0_prune=False,␣
↪thresh=None,
                                                              ␣
↪l1_reg_param=lambda_list[idx],
                                                              ␣
↪grad_fn=l1_norm_update)
    weight_l1_list.append(weight_l1)
```

```
    loss_l1_list.append(loss_arr_l1)
    weight_arr_l1_list.append(weight_arr_l1)
```

[304]:
```
plot_many_w_vals_or_loss_vs_n_iters(num_epochs=n_epochs, w_init=w0,␣
↪final_weights_list=weight_l1_list,
                                    weight_array_list=weight_arr_l1_list,␣
↪loss_arr_lst=None,
                                    l1_reg_params=lambda_list,␣
↪save_name_='lab1d_weight', save=True)
```



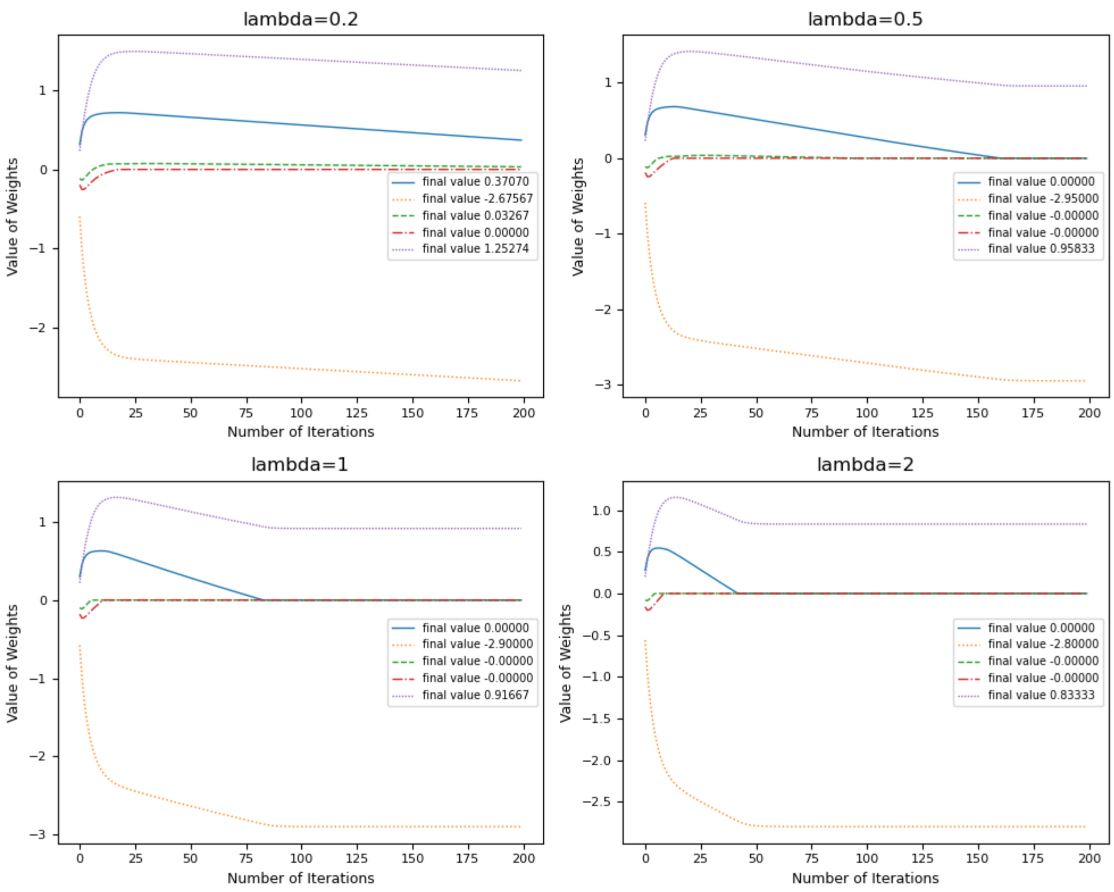
[305]:
```
plot_many_w_vals_or_loss_vs_n_iters(num_epochs=n_epochs, w_init=w0,␣
↪final_weights_list=None,
                                    weight_array_list=None,␣
↪loss_arr_lst=loss_l1_list,
                                    l1_reg_params=lambda_list,␣
↪save_name_='lab1d_loss', save=True)
```



[306]:
```
"""
part (e) (6pts) Here we optimize the same objective as in (d), this time using␣
↪proximal gradient update. Recall that the proximal operator of the l1␣
↪regulariser is the soft thresholding function. Set the threshold in the soft␣
↪thresholding function to {0.004, 0.01, 0.02, 0.04} respectively. Plot the␣
↪value of log(L) throughout the training, and use another figure to plot the␣
↪value of each element in W in each step. Compare the convergence performance␣
↪with the results in (d). (Hint: Optimizing L + lambda||W||_1 using gradient␣
↪descent with learning rate mu should correspond to proximal gradient update␣
↪with threshold mu*lambda)"""

lambda_list = [0.2, 0.5, 1.0, 2.0]
weight_prox_list, loss_prox_list, weight_arr_prox_list = [], [], []
for idx, lll in enumerate(lambda_list):
    weight_prox, loss_arr_prox, weight_arr_prox = run_grad_descent(w0, mu,␣
↪x_input, y_input, n_epochs,
                                                                              ␣
↪w0_prune=False, thresh=None,
```

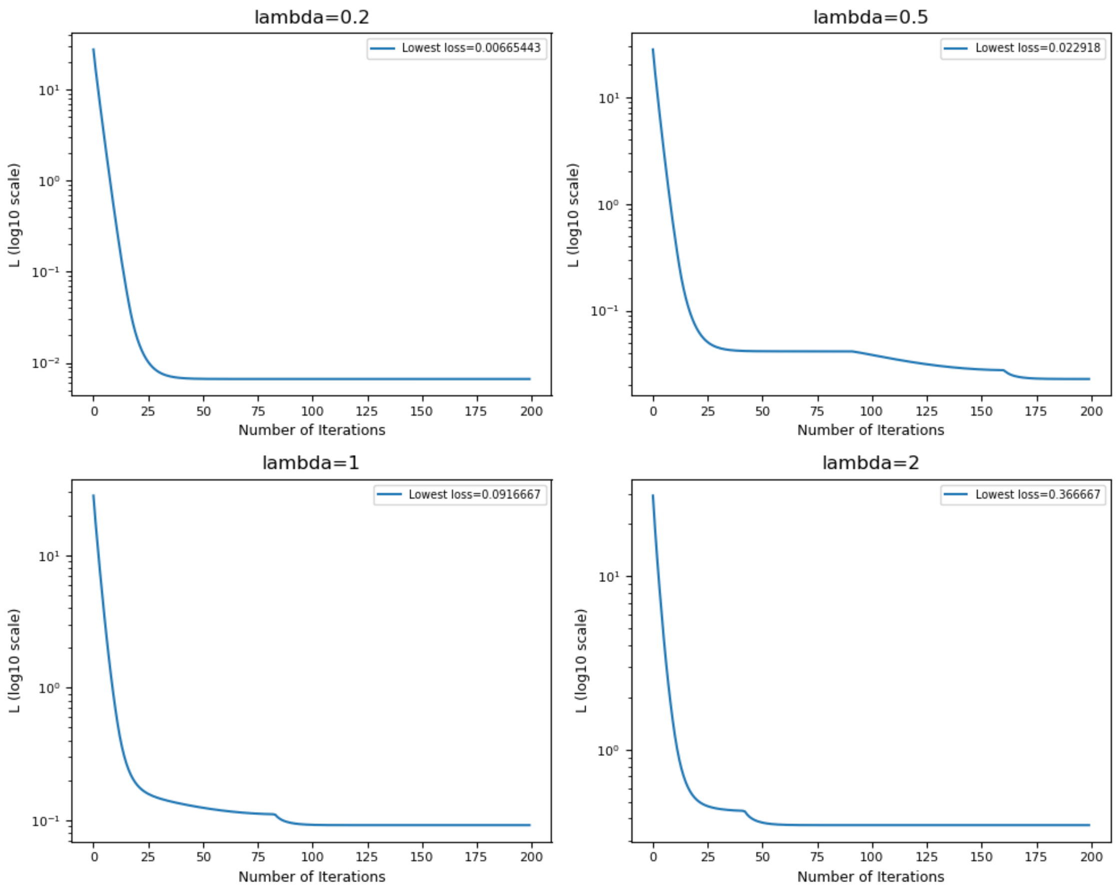
```
↪l1_reg_param=lll,                                                          ␣

↪grad_fn=l1_norm_proximal_update)
    weight_prox_list.append(weight_prox)
    loss_prox_list.append(loss_arr_prox)
    weight_arr_prox_list.append(weight_arr_prox)
```

[307]:
```
plot_many_w_vals_or_loss_vs_n_iters(num_epochs=n_epochs, w_init=w0,␣
↪final_weights_list=weight_prox_list,
                                    weight_array_list=weight_arr_prox_list,␣
↪loss_arr_lst=None,
                                    l1_reg_params=lambda_list,␣
↪save_name_='lab1e_weight', save=True)
```



[308]:
```
plot_many_w_vals_or_loss_vs_n_iters(num_epochs=n_epochs, w_init=w0,␣
↪final_weights_list=None,
                                    weight_array_list=None,␣
↪loss_arr_lst=loss_prox_list,
```

```
                                          l1_reg_params=lambda_list,␣
↪save_name_='lab1e_loss', save=True)
```



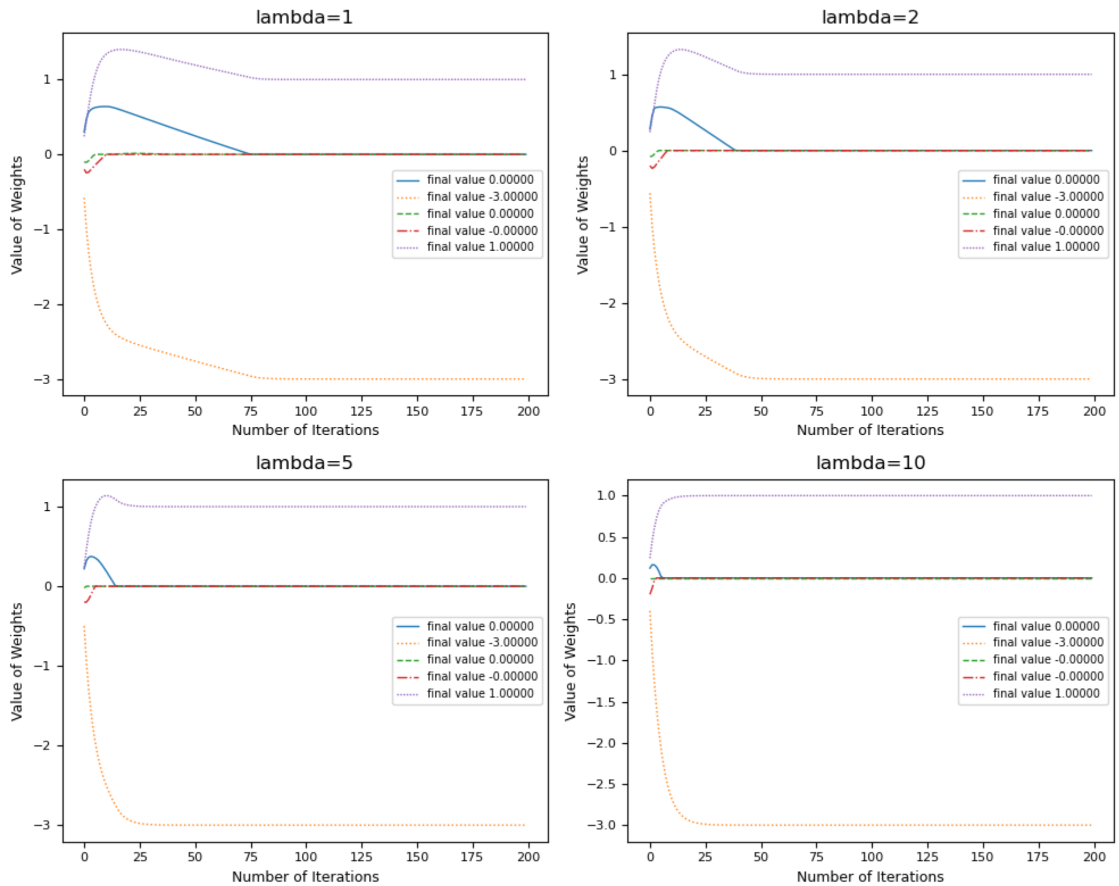
[309]:

```
"""(f) (6 pts) Trimmed l1 (Tl1) regulariser is proposed to solve the "bias"␣
↪problem of l1. For simplicity you may implement the T l1 regulariser as␣
↪applying a l1 regularization with strength   on the 3 elements of W with the␣
↪smallest absolute value, with no penalty on other elements. Minimize L + T␣
↪l1 (W ) using proximal gradient update with   = {1.0, 2.0, 5.0, 10.0}␣
↪(correspond the soft thresholding threshold {0.02, 0.04, 0.1, 0.2}). Plot␣
↪the value of log(L) throughout the training, and use another figure to plot␣
↪the value of each element in W in each step. Comment on the convergence␣
↪comparison of the Trimmed l1 and the l1. Also compare the behavior of the␣
↪early steps (e.g. first 20) between the Trimmed l1 and the iterative pruning.
↪"""
lambda_list_f = np.array([1.0, 2.0, 5.0, 10.0]) * 1

weight_trim_list, loss_trim_list, weight_arr_trim_list = [], [], []
for idx, lll in enumerate(lambda_list_f):
```

```
    w, l, w_array = run_grad_descent(w0, mu, x_input, y_input, n_epochs, w0_prune=False, thresh=2,
                                      l1_reg_param=lll, grad_fn=l1_norm_trimmed_update)
    weight_trim_list.append(w)
    loss_trim_list.append(l)
    weight_arr_trim_list.append(w_array)
```

[310]:

```
plot_many_w_vals_or_loss_vs_n_iters(num_epochs=n_epochs, w_init=w0, final_weights_list=weight_trim_list,
                                    weight_array_list=weight_arr_trim_list, loss_arr_lst=None,
                                    l1_reg_params=lambda_list_f, save_name_='lab1f_weight', save=True)
```



[311]:

```
plot_many_w_vals_or_loss_vs_n_iters(num_epochs=n_epochs, w_init=w0, final_weights_list=None,
                                    weight_array_list=None, loss_arr_lst=loss_trim_list,
```

```
                                    l1_reg_params=lambda_list_f,␣
↪save_name_='lab1f_loss', save=True)
```