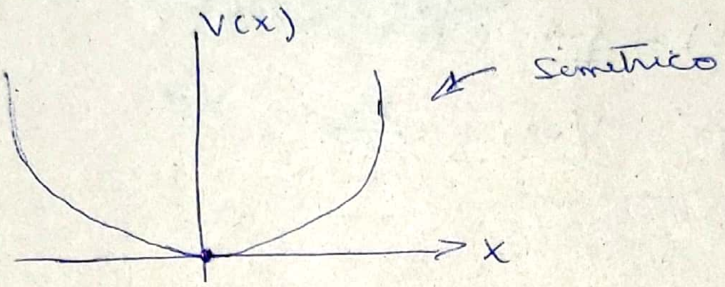
Examen final:

Metodos numericos 1

Machado Perez Franz Danylo:

1) a) $E = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 + V(x)$



$x(0) = a$ (amplitud)

$\frac{dx}{dt}(0) = 0$

→ pasamos a ~~calcular~~ hallar una expresión para la velocidad

$\rightarrow \frac{1}{2}\left(\frac{dx}{dt}\right)^2 m = E - V(x)$

$\rightarrow \left(\frac{dx}{dt}\right)^2 = \frac{2(E-V(x))}{m}$

entonces sacando raíz cuadrada

$$\frac{dx}{dt} = \pm\sqrt{\frac{2}{m}(E-V(x))}$$

$$\rightarrow \frac{dx}{\sqrt{\frac{2}{m}(E-V(x))}} = dt$$

$$\sqrt{\frac{m}{2(E-V(x))}} = dt$$

como en $t=0$; $x=a$ y $t=T/4$; $x=0$

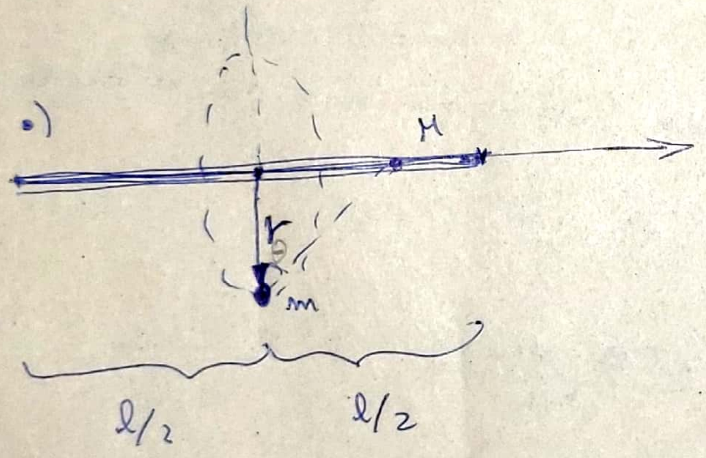
$$\int_0^{T/4} dt = \int_0^{a} \sqrt{\frac{m}{2(E-V(x))}}\, dx$$

$$\rightarrow T = 4 \int_0^{a} \sqrt{\frac{m}{2(E-V(x))}}\, dx$$

•) $E(x,t) = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 + V(x)$

$E(a,0) = V(a)$; $\frac{dx}{dt}(0) = 0$

2) •)



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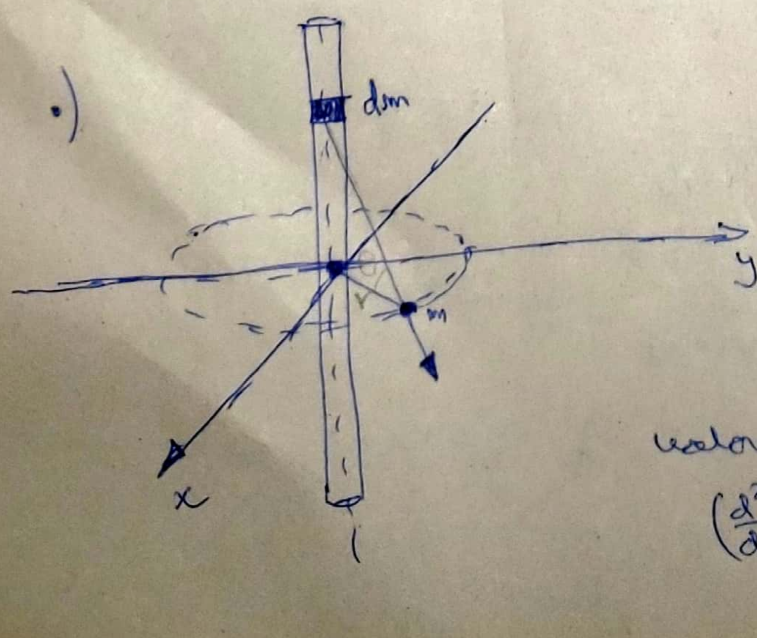
•)



$\rightarrow$ como $F = \frac{G\, m\, dM}{r'^2}$

una vez se calcula el modulo de F usamos

$m a = F$

de allí se desprende los valores de

$\left(\frac{d^2x}{dt^2}\right)$ y $\left(\frac{d^2y}{dt^2}\right)$

$$\Rightarrow dF = \frac{G\, m\, dM}{r'^2} \quad ; \quad dM = \left(\frac{M}{L}\right) dz$$

$$dF = \frac{\frac{G\, m\, M\, dz}{L}}{r'^2} = \frac{G\, m\, M}{L} \frac{dz}{r^2 + z^2}$$

$$\Rightarrow \quad F = \frac{G m M}{L} \int_{-L/2}^{L/2} \frac{dz}{r^2 + z^2}$$

$$\Rightarrow F = G m . M \cdot \frac{1}{r\sqrt{r^2 + (L/2)^2}}$$

$\Rightarrow$ como $F = ma$

$$\Rightarrow \quad F_x = m\, a_x$$

$$F_y = m a_y$$

$$\Rightarrow \left(\frac{d^2x}{dt^2}\right) = -GM \frac{x}{r^2\sqrt{r^2 + L^2/4}}$$

$$\left(\frac{d^2 y}{dt^2}\right) = -GM \frac{y}{r^2\sqrt{r^2 + L^2/4}}$$