Métodos Numéricos
PC N°3
Parte Teórica

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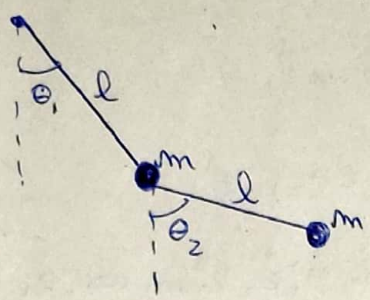
2)



•) $x_1 = l\, sen(\theta_1)$

$y_1 = -l \cos(\theta_1)$

$x_2 = l\, sen(\theta_1) + l\, sen(\theta_2)$

$y_2 = -l\cos(\theta_1) - l\cos(\theta_2)$

Eligiendo los coordenadas generalizadas $(\theta_1 \text{ y } \theta_2)$

$\rightarrow T = \frac{1}{2} m (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m (\dot{x}_2^2 + \dot{y}_2^2)$

$V = m g y_1 + m g y_2 = mg(y_1 + y_2)$

$\rightarrow$ reemplazando los valores de x_1, y_1, y_2, x_2 en T y V para obtener $T(\theta_1, \theta_2)$ y $V(\theta_1, \theta_2)$

$\rightarrow$ T: $\dot{x}_1 = l\cos(\theta_1) \cdot \dot{\theta}_1$

$\dot{y}_1 = l\, sen(\theta_1) \cdot \dot{\theta}_1$

$\dot{x}_2 = l\cos(\theta_1) \cdot \dot{\theta}_1 + l\cos(\theta_2)\dot{\theta}_2$

$\dot{y}_2 = l\, sen(\theta_1) \cdot \dot{\theta}_1 + l\, sen(\theta_2) \cdot \dot{\theta}_2$

$\rightarrow T = \frac{1}{2} m \left(l^2 \cos^2(\theta_1)\dot{\theta}_1^2 + l^2 sen^2(\theta_1) \cdot \dot{\theta}_1^2 + (l\cos(\theta_1)\dot{\theta}_1 + l\cos(\theta_2)\dot{\theta}_2)^2 \right.$

$\left. + (l\, sen(\theta_1) \cdot \dot{\theta}_1 + l\, sen(\theta_2)\dot{\theta}_2)^2 \right)$

$= \frac{1}{2} m \left(\dot{\theta}_2 l^2 + 2\dot{\theta}_1 \dot{\theta}_2 l^2 \cos(\theta_1 - \theta_2) + 2\dot{\theta}_1^2 l^2 \right)$

$\rightarrow T = \frac{1}{2} m l^2 \left(\dot{\theta}_2 + 2\dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) + 2\dot{\theta}_1^2 \right)$

$$V = -gml(\cos(\theta_2) + 2\cos(\theta_1))$$

$$= -gml(2\cos(\theta_1) + \cos(\theta_2))$$

→ como $\mathcal{L} = T - V$

→ $$\mathcal{L} = 2glm\cos(\theta_1) + glm\cos(\theta_2) + \dot{\theta}_1^2 l^2 m + \frac{1}{2}\dot{\theta}_2^2 l^2 m + \dot{\theta}_1\dot{\theta}_2 l^2 m\cos(\theta_1 - \theta_2)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} = 2\dot{\theta}_1 l^2 m + \dot{\theta}_2 l^2 m\cos(\theta_1 - \theta_2)$$

$$\frac{\partial \mathcal{L}}{\partial \theta_1} = -2glm\,\mathrm{sen}(\theta_1) - \dot{\theta}_1\dot{\theta}_2 l^2 m\,\mathrm{sen}(\theta_1 - \theta_2)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} = \dot{\theta}_2 l^2 m + \dot{\theta}_1 l^2 m\cos(\theta_1 - \theta_2)$$

$$\frac{\partial \mathcal{L}}{\partial \theta_2} = -glm\,\mathrm{sen}(\theta_2) + \dot{\theta}_1\dot{\theta}_2 l^2 m\,\mathrm{sen}(\theta_1 - \theta_2)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1}\right) = 2l^2 m\ddot{\theta}_1 + l^2 m\left(\ddot{\theta}_2\cos(\theta_1 - \theta_2) - \mathrm{sen}(\theta_1 - \theta_2)(\dot{\theta}_1 - \dot{\theta}_2)\cdot\dot{\theta}_2\right)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}\right) = \ddot{\theta}_2 l^2 m + l^2 m\left(\ddot{\theta}_1\cos(\theta_1 - \theta_2) - \mathrm{sen}(\theta_1 - \theta_2)(\dot{\theta}_1 - \dot{\theta}_2)\cdot\dot{\theta}_1\right)$$

→ reemplazando en

$$\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1}\right) - \frac{\partial \mathcal{L}}{\partial \theta_1} = 0$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}\right) - \frac{\partial \mathcal{L}}{\partial \theta_2} = 0$$

$$\bullet)\ l\left(\dot\theta_2^2\, l m\, \mathrm{sen}(\theta_1-\theta_2) + 2m(g\, \mathrm{sen}\,\theta_1 + l\ddot\theta_1)\right)$$
$$+ l(l m\ddot\theta_2)\cos(\theta_1-\theta_2) = 0$$

$$\bullet)\ l m\left(g\, \mathrm{sen}(\theta_2) - \dot\theta_1^2\, l\, \mathrm{sen}(\theta_1-\theta_2)\right)$$
$$+ l m\left(l\ddot\theta_1 \cos(\theta_1-\theta_2) + l\ddot\theta_2\right) = 0$$

Considerando $\omega_1 = \dot\theta_1$ y $\omega_2 = \dot\theta_2$

$$\bullet)\ l\left(\omega_2^2\, l m\, \mathrm{sen}(\theta_1-\theta_2) + 2m(g\, \mathrm{sen}(\theta_1) + l\dot\omega_1)\right)$$
$$+ l^2 m\dot\omega_2 \cos(\theta_1-\theta_2) = 0$$

$$\bullet)\ l m\left(g\, \mathrm{sen}(\theta_2) - \omega_1^2\, l\, \mathrm{sen}(\theta_1-\theta_2)\right)$$
$$+ l m\left(l\dot\omega_1 \cos(\theta_1-\theta_2) + l\dot\omega_2\right) = 0$$

$$\bullet)\ \omega_2^2\, l m\, \mathrm{sen}(\theta_1-\theta_2) + 2mg\, \mathrm{sen}(\theta_1) = -l m\dot\omega_2 \cos(\theta_1-\theta_2) - 2ml\dot\omega_1$$
$$g\, \mathrm{sen}(\theta_2) - \omega_1^2\, l\, \mathrm{sen}(\theta_1-\theta_2) = -l\dot\omega_1 \cos(\theta_1-\theta_2) - l\dot\omega_2$$

$$\bullet)\ \omega_2^2\, l\, \mathrm{sen}(\theta_1-\theta_2) + 2g\, \mathrm{sen}(\theta_1) = -l\dot\omega_2 \cos(\theta_1-\theta_2) - 2l\dot\omega_1 \quad \ldots(1)$$
$$g\, \mathrm{sen}(\theta_2) - \omega_1^2\, l\, \mathrm{sen}(\theta_1-\theta_2) = -l\dot\omega_1 \cos(\theta_1-\theta_2) - l\dot\omega_2 \quad \ldots(2)$$

$\bullet$ $(1)/\left(-\cos(\theta_1-\theta_2)\right) + (2)$

$$\bullet)\ \frac{\omega^2\, l\, \mathrm{sen}(\theta_1-\theta_2) + 2g\, \mathrm{sen}(\theta_1)}{(-\cos(\theta_1-\theta_2))} + g\, \mathrm{sen}(\theta_2) - \omega_1^2\, l\, \mathrm{sen}(\theta_1-\theta_2)$$
$$= \left(\frac{2l}{\cos(\theta_1-\theta_2)} - l\cos(\theta_1-\theta_2)\right)\dot\omega_1$$

$$2 \cdot (2) \Big/ \left(-\cos(\theta_1-\theta_2)\right) + (1)$$

$$\rightarrow \frac{2\left(g \operatorname{sen}(\theta_2) - \omega_1^2 \, l \operatorname{sen}(\theta_1-\theta_2)\right)}{\left(-\cos(\theta_1-\theta_2)\right)} + \omega_2^2 \, l \operatorname{sen}(\theta_1-\theta_2) + 2g \operatorname{sen}(\theta_1)$$

$$= \left(\frac{2l}{\cos(\theta_1-\theta_2)} - l\cos(\theta_1-\theta_2)\right) \dot{\omega}_1$$

$\rightarrow$ ordenando obtenemos

$$\dot{\omega}_1 = -\left(\frac{\omega_1^2 \operatorname{sen}(2\theta_1 - 2\theta_2) + 2\omega_2^2 \operatorname{sen}(\theta_1-\theta_2) + \frac{g}{l}\left(\operatorname{sen}(\theta_1 - 2\theta_2) + 3\operatorname{sen}(\theta_1)\right)}{3 - \cos(2\theta_1 - 2\theta_2)} \right)$$

$$\dot{\omega}_2 = \left(\frac{4\omega_1^2 \operatorname{sen}(\theta_1-\theta_2) + \omega_2^2 \operatorname{sen}(2\theta_1 - 2\theta_2) + 2\left(\frac{g}{l}\right)\left(\operatorname{sen}(2\theta_1-\theta_2) - \operatorname{sen}(\theta_2)\right)}{3 - \cos(2\theta_1 - 2\theta_2)} \right)$$