Chapter 36 Homework (due 12/03/13)

36.14

36.19

36.23

36.47

36.56

36.63

36.64

36.75

Problem 36.14

The dentist uses a spherical mirror to examine a tooth. The tooth is 1 cm in front of the mirror, and an image is formed 10 cm behind the mirror.

(a) Determined the mirror's radius of curvature.

(b) Determined the magnification of the image.

Solution

(a) To do the above, the mirror must be concave. The object distance to the mirror is 1 cm. The image distance is –10 cm. The focal length of the mirror is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{1 \text{ cm}}+\frac{1}{-10 \text{ cm}}=\frac{1}{f} \quad \Rightarrow \quad f=\frac{10}{9} \text{ cm}=1.11 \text{ cm}$$

The radius of curvature is twice the focal length which is 20/9 cm = 2.22 cm.

(b) The magnification is

$$M=-\frac{q}{p}=-\frac{-10 \text{ cm}}{1 \text{ cm}}=+10$$

Problem 36.19

An object 10 cm tall is placed at the zero mark of a meter stick. A spherical mirror located at some point on the meter stick creates an image of the object that is upright, 4 cm tall, and located at the 42 cm mark of the meter stick.

(a) Is the mirror convex or concave?

(b) Where is the mirror?

(c) What is the mirror's focal length?

Solution

(a) Here is what the situation looks like.

This is what a convex mirror does.

(b) The focal length is negative. Let the position of the mirror be x. The object distance is x. The image distance is 42–x and negative.

$$p = x \quad \text{and} \quad q = x - 42$$

The magnification is +0.4.

$$M = -\frac{q}{p} = +0.4 \quad \Rightarrow \quad q = -0.4p$$

Finally,

$$x - 42 = -0.4x \quad \Rightarrow \quad 1.4x = 42 \quad \Rightarrow \quad x = 30 \text{ cm}$$

(c) The focal length is

$$\frac{1}{30 \text{ cm}} + \frac{1}{-12 \text{ cm}} = \frac{1}{f} = \frac{1.2}{36 \text{ cm}} - \frac{3}{36 \text{ cm}} = -\frac{1.8}{36 \text{ cm}} \quad \Rightarrow \quad f = -20 \text{ cm}$$

Problem 36.23

A spherical mirror is to be used to form an image five times the size of an object on the screen located 5 m from the object.

(a) Is the mirror required concave or convex?

(b) What is the required radius of curvature of the mirror?

(c) Where should the mirror be positioned relative to the object?

Solution

(a) A real image from a mirror means the mirror is concave.

(b) Let the object distance be x. The image distance must be x + 5 as the image distance must be larger due to nature of the concave mirror.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{x}+\frac{1}{x+5\text{ m}}=\frac{1}{f}$$

The magnification is negative since a concave mirror produces inverted images.

$$M=-\frac{q}{p}=-\frac{x+5\text{ m}}{x}=-5 \quad \Rightarrow \quad x+5=5x \quad \Rightarrow \quad x=\frac{5}{4}\text{ m}$$

$$\frac{4}{5}+\frac{1}{\frac{5}{4}+5}=\frac{4}{5}+\frac{4}{25}=\frac{24}{25}=\frac{1}{f} \quad \Rightarrow \quad f=\frac{25}{24}$$

The radius of curvature is 2f = 50/24 m = 2.0833 m.

(c) The position of the mirror is p = x = 1.25 m.

Problem 36.47

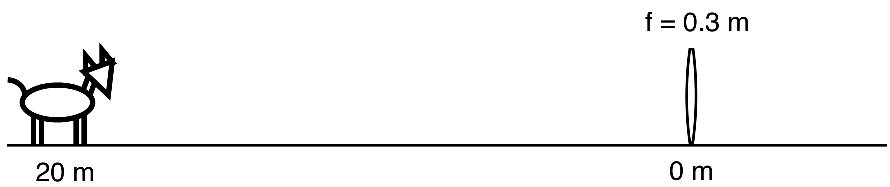
And antelope is at a distance of 20 m from a converging lens of focal length 30 cm. The lens forms an image of the animal. If the antelope runs away from lens at a speed of 5 m/s,

(a) How fast is the image move?

(b) Does the image move towards or away from the lens?

Solution

(a) Here is the situation.



The position of the image as a function of the position of the object is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{q}=\frac{1}{f}-\frac{1}{p} \quad \Rightarrow \quad q=\frac{pf}{p-f}$$

The velocity of the image is

$$\dot{q}=\frac{\dot{p}f(p-f)-pf(\dot{p})}{(p-f)^2}=\frac{f\dot{p}p-f^2\dot{p}-f\dot{p}p}{(p-f)^2}=-\dot{p}\frac{f^2}{(p-f)^2}$$

Running away from the lens is an increasing object distance.

$$\dot{q}=-\dot{p}\frac{f^2}{(p-f)^2}=-(5\ \mathrm{m/s})\frac{(0.3\ \mathrm{m})^2}{(20\ \mathrm{m}-0.3\ \mathrm{m})^2}=-0.0011595\ \mathrm{m/s}=-1.16\ \mathrm{mm/s}$$

(b) Since this velocity is negative, the image is moving toward from the lens.

Problem 36.56

The refracting telescope has a 1 meter diameter objective lens of focal length 20 m. Assuming it is used with that eyepiece focal length 2.5 cm.

(a) Determine the magnification of Mars as seen through this telescope.

(b) Are the Martian polar caps right side up or upside down?

Solution

(a) The magnification is

$$m = -\frac{f_o}{f_e} = -\frac{20 \text{ m}}{0.025 \text{ m}} = -800$$

(b) Since the magnification is negative, the image will be upside down.

Problem 36.63

The lens and mirror are separated by 1 m and have focal lengths of 80 cm and –50 cm. An object is placed 1 m left of the lens.

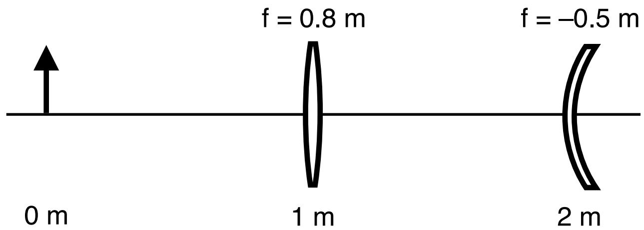
(a) Locate the final image, formed by light that has gone through the lens twice.

(b) Determine the overall magnification of the image.

(c) State whether the image is upright or inverted.

Solution

(a) Here is the situation.



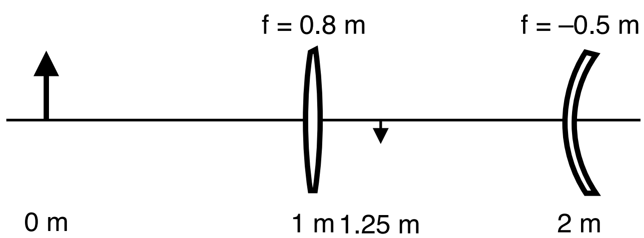
The first process is the object going through the converging lens.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{1\text{ m}}+\frac{1}{q}=\frac{1}{0.8\text{ m}} \quad \Rightarrow \quad \frac{1}{q}=\frac{1}{0.8\text{ m}}-\frac{1}{1\text{ m}} \quad \Rightarrow \quad q=0.25\text{ m}$$

This position is 1.25 m. The magnification is

$$M=-\frac{q}{p}=-\frac{0.25\text{ m}}{1\text{ m}}=-0.25$$

This is the situation now.



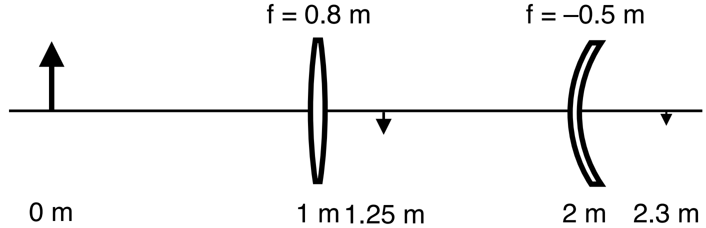
The second process is the image bouncing off of the convex mirror.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{0.75\text{ m}}+\frac{1}{q}=\frac{1}{-0.5\text{ m}} \quad \Rightarrow \quad \frac{1}{q}=-\frac{1}{0.5\text{ m}}-\frac{1}{0.75\text{ m}} \quad \Rightarrow \quad q=-0.3\text{ m}$$

This position is 2.3 m. The magnification is

$$M=-\frac{q}{p}=-\frac{-0.3\text{ m}}{0.75\text{ m}}=+0.40$$

This is the situation now.



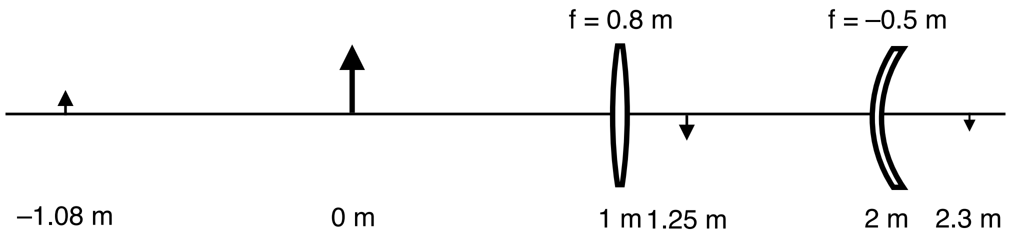
The final process is the image going through the converging lens again.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad\Rightarrow\quad \frac{1}{1.3\text{ m}}+\frac{1}{q}=\frac{1}{0.8\text{ m}} \quad\Rightarrow\quad \frac{1}{q}=\frac{1}{0.8\text{ m}}-\frac{1}{1.3\text{ m}} \quad\Rightarrow\quad q=2.08\text{ m}$$

The final position is –1.08 m. The magnification is

$$M=-\frac{q}{p}=-\frac{2.08\text{ m}}{1.3\text{ m}}=-1.6$$

This is the result.



(b) The total magnification is

$$M=-0.25\cdot +0.40\cdot -1.6=+0.15360$$

(c) The image is upright since the magnification is positive.

Problem 36.64

Two converging lenses having focal lengths of f_1 = 10 cm and f_2 = 20 cm are placed a distance d = 50 cm apart. The image due to light passing through both lenses is to be located between the lenses at the position x = 31 cm.

(a) At what value of p should the object be positioned to the left of the first lens?

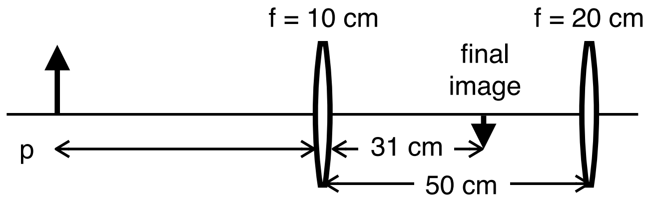
(b) What is the magnification of the final image?

(c) Is the final image upright or inverted?

(d) Is the final image real of virtual?

Solution

(a) Here is the situation.

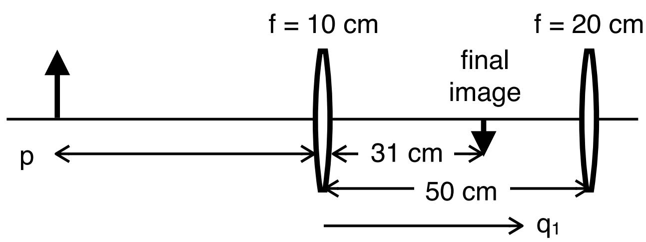


The first process is described by

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{p}+\frac{1}{q_1}=\frac{1}{10 \text{ cm}} \quad \Rightarrow \quad \frac{1}{q_1}=\frac{1}{10 \text{ cm}}-\frac{1}{p}=\frac{p-10}{10p}$$

The magnification is

$$M=-\frac{q}{p}=-\frac{10p}{(p-10)}\frac{1}{p}=-\frac{10}{p-10}$$



The second process is described by (assuming the first image is between the lenses)

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{50 \text{ cm}-q_1}+\frac{1}{-19 \text{ cm}}=\frac{1}{20 \text{ cm}} \quad \Rightarrow \quad \frac{1}{50 \text{ cm}-q_1}=\frac{1}{20}+\frac{1}{19} \quad \Rightarrow \quad 50-q_1=9.7436$$

$$q_1=40.256 \text{ cm}$$

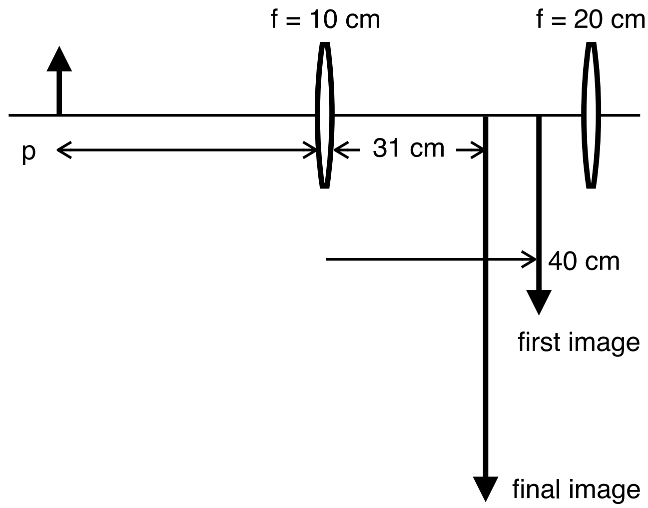
This is possible. The magnification is

$$M=-\frac{q}{p}=-\frac{-19 \text{ cm}}{9.7436 \text{ cm}}=+1.95$$

The value of p is

$$q_1=\frac{10p}{p-10}=40.256 \quad \Rightarrow \quad 10p=40.256p-402.56 \quad \Rightarrow \quad 30.256p=402.56 \quad \Rightarrow \quad p=13.305 \text{ cm}$$

The situation looks like this.



(b) The final magnification is

$$M = -\frac{10}{p-10} \cdot +1.95 = -\frac{19.5}{13.305-10} = -5.8999$$

(c) The final image is upside down since the final magnification is negative.

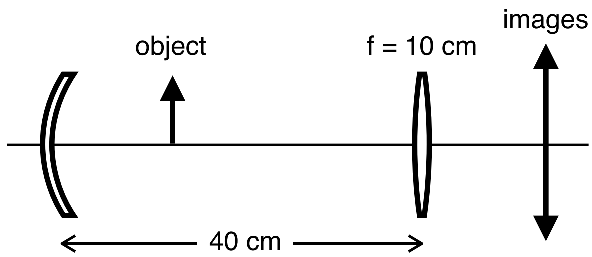
(d) The final image is virtual since the final image distance is negative.

Problem 36.75

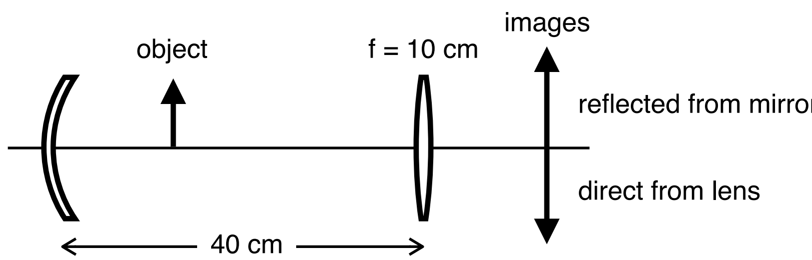
An observer to the right of the mirror-lens combination sees two real images that are the same size and the same location. One image is upright, and the other is inverted. Both images are 1.5 times larger than the object. The lens has a focal point of 10 cm. The lens and the mirror are separated by 40 cm. Determine the focal point of the mirror.

Solution

Here is the situation.



Since the images are real, the upside down one is the one that is produced directly from the converging lens.



What we know from this are

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{1}{10\ \text{cm}}$$

$$M=-\frac{q}{p}=-1.5 \quad\Rightarrow\quad q=1.5p$$

$$\frac{1}{p}+\frac{1}{1.5p}=\frac{1}{10\ \text{cm}} \quad\Rightarrow\quad \frac{1}{p}\left(1+\frac{1}{1.5}\right)=\frac{1}{p}\frac{5}{3}=\frac{1}{10} \quad\Rightarrow\quad p=\frac{50}{3}\ \text{cm}=16.667\ \text{cm}$$

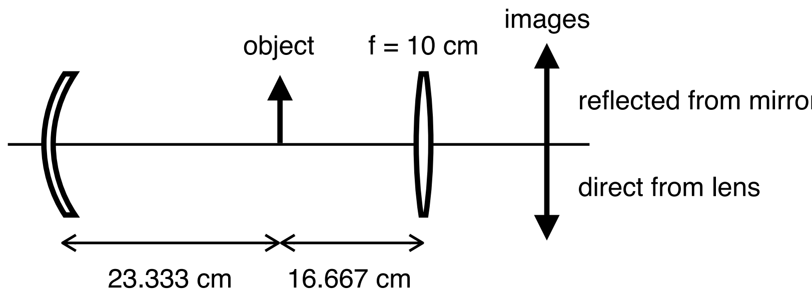
$$q=\frac{3}{2}\frac{50}{3}=25\ \text{cm}$$

The other image contains two processes. The reflection from the mirror is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{1}{40-16.667}+\frac{1}{q_1} \quad\Rightarrow\quad \frac{1}{f}=\frac{1}{23.333}+\frac{1}{q_1}$$

$$M_1=-\frac{q_1}{23.333}$$

The situation looks like this.



The passage through the lens is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{1}{40-16.667}+\frac{1}{q_1} \quad \Rightarrow \quad \frac{1}{f}=\frac{1}{23.333}+\frac{1}{q_1}$$

This image must be negative so that the final image is upright. Since the final image sizes are the same, the objects of the final images must the same size but opposite orientations. The location of the first image is also at the original object position.

$$q_1 = 23.333 \text{ cm}$$

$$\frac{1}{f}=\frac{1}{23.333}+\frac{1}{23.333} \quad \Rightarrow \quad f=\frac{23.333}{2}=11.667 \text{ cm}$$