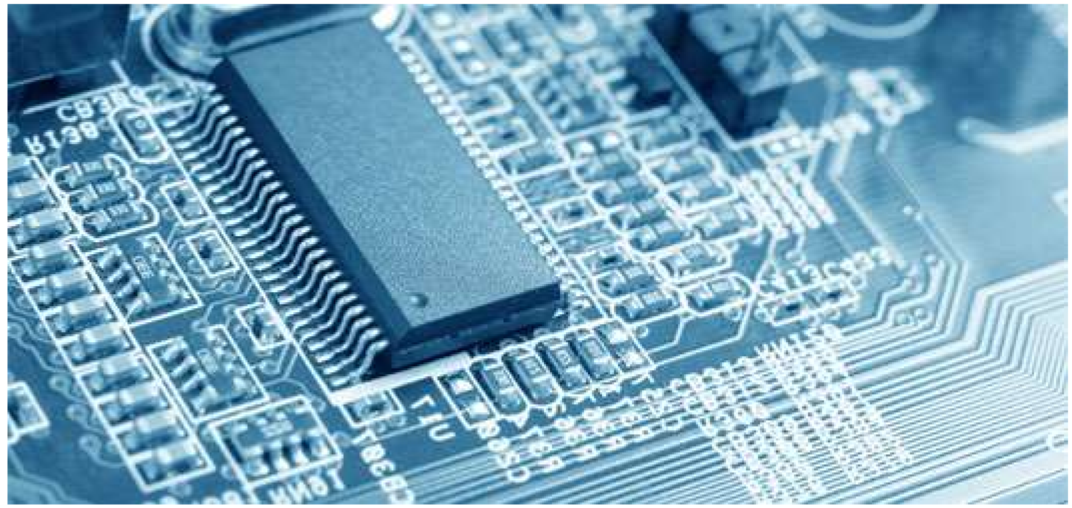
NAZARBAYEV UNIVERSITY
SCHOOL OF SCIENCE AND TECHNOLOGY



ROBT206 – Microcontrollers with Lab

Lecture 14 – Sequential Circuit Analysis

6 March, 2018

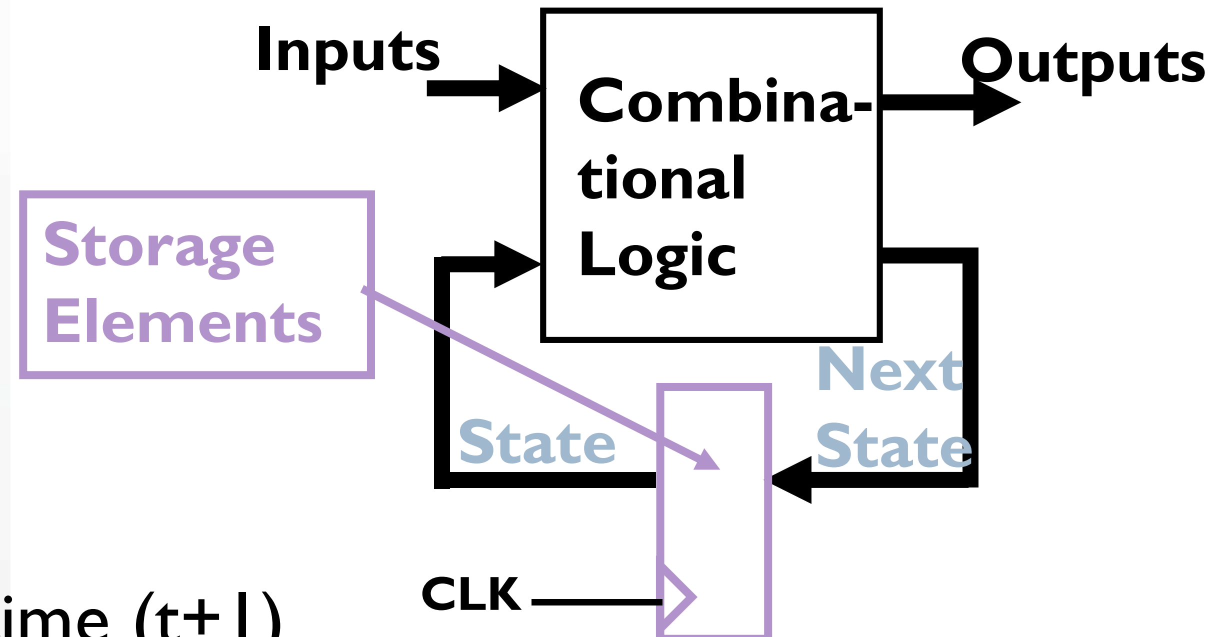
Topics

Today's Topics

- Sequential circuit analysis
  - State tables
  - State diagrams
  - Equivalent states
  - Moore and Mealy Models

Sequential Circuit Analysis

- General Model
  - Current State at time (t) is stored in an array of flip-flops.



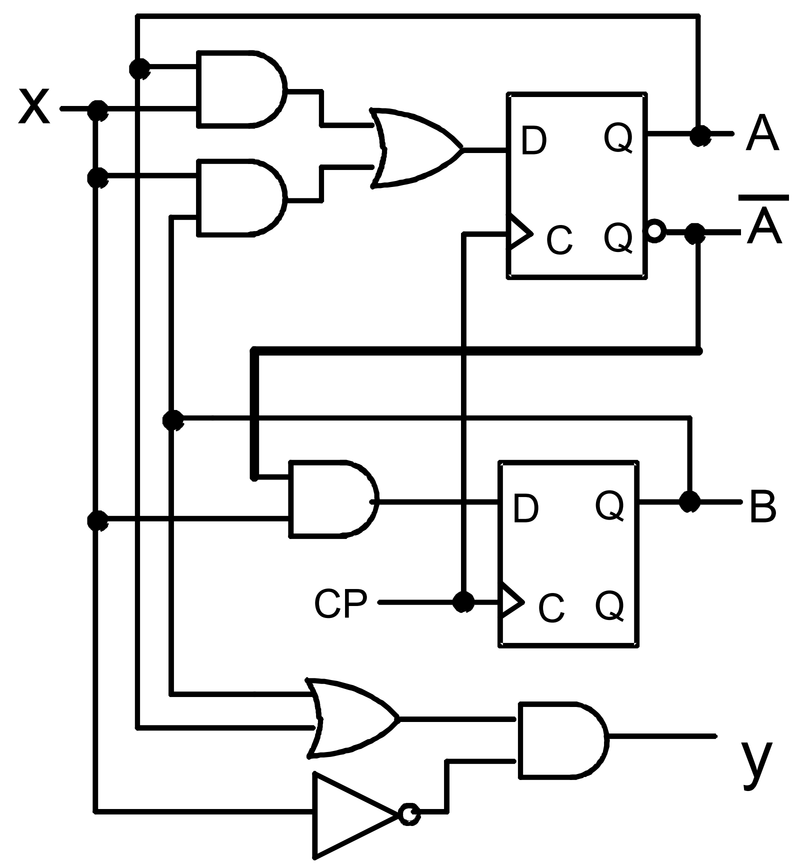
  - Next State at time (t+1) is a Boolean function of state and inputs.
  - Outputs at time (t) are a Boolean function of State (t) and (sometimes) Inputs (t).

Example 1

- Input: x(t)
- Output: y(t)
- State: (A(t), B(t))

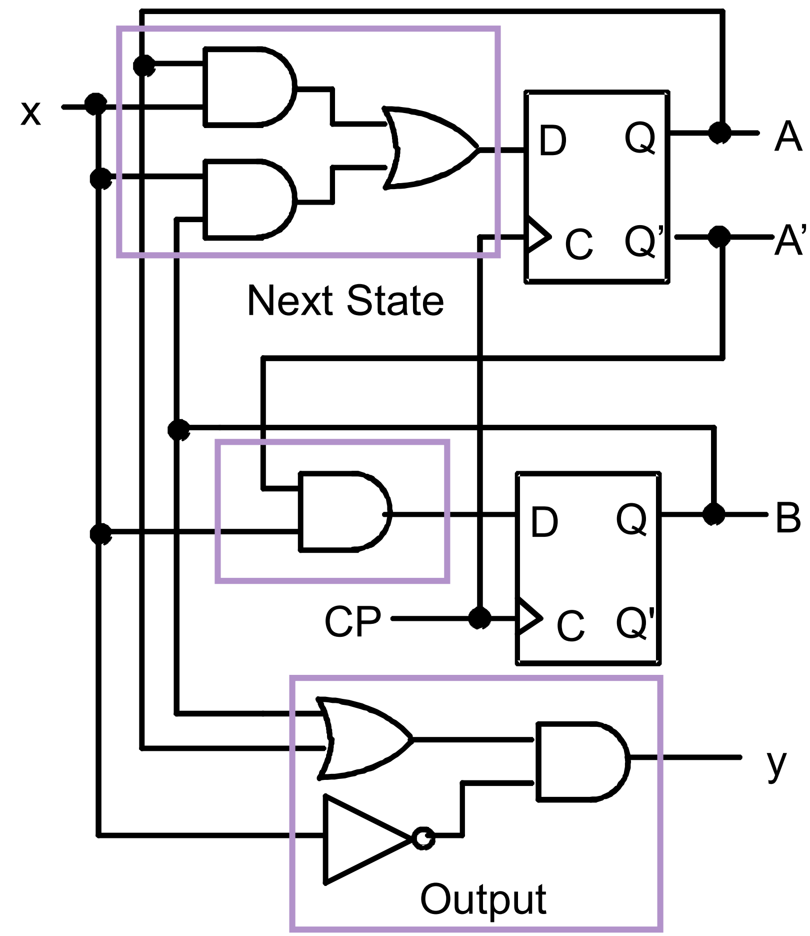
- What is the Output Function?

- What is the
- Next State Function?



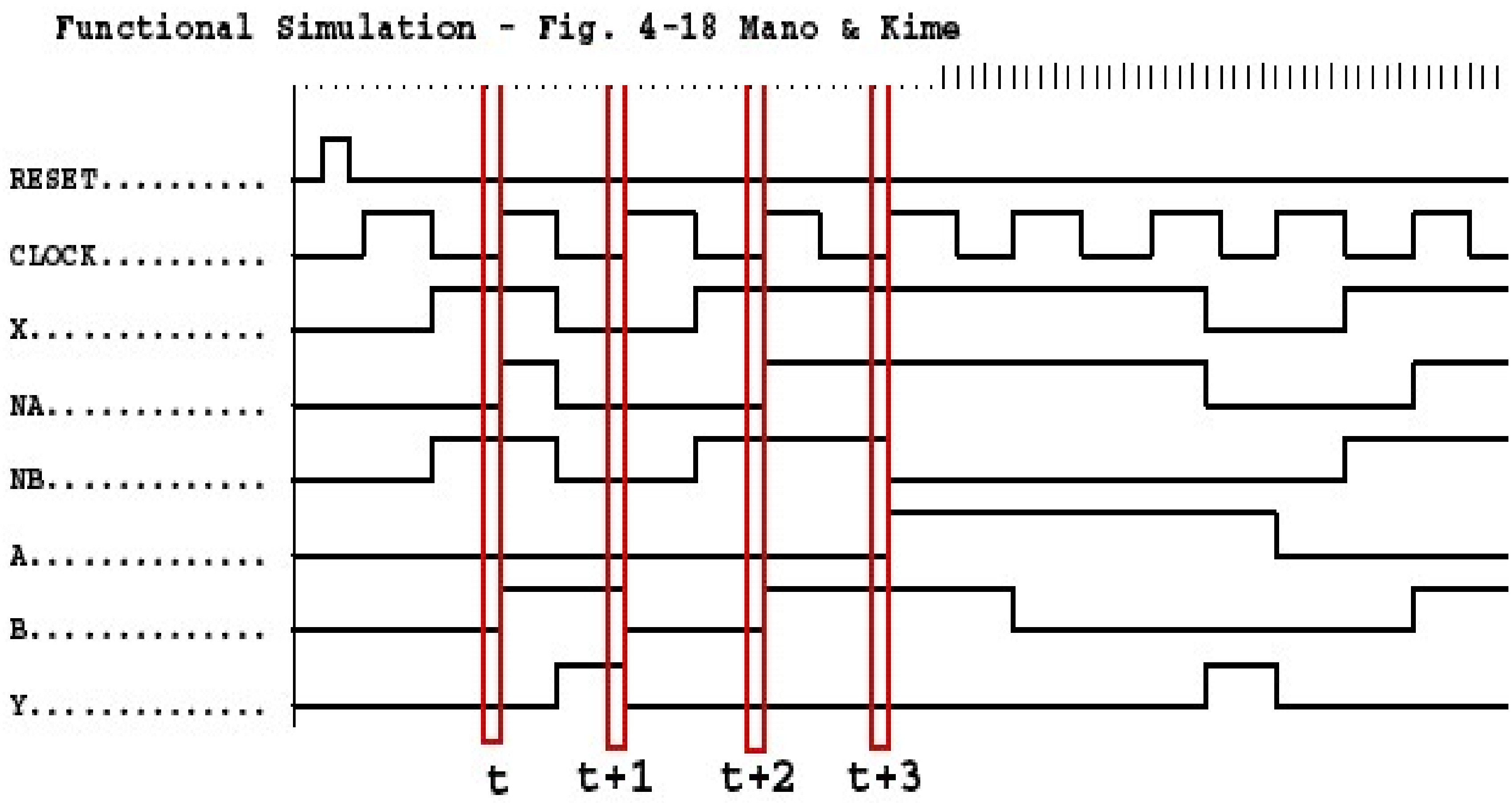
Example 1 (continued)

- Boolean equations for the functions:
  - $A(t+1) = A(t)x(t) + B(t)x(t)$
  - $B(t+1) = \overline{A}(t)x(t)$
  - $y(t) = \overline{x}(t)(B(t) + A(t))$



Example 1 (continued)

- Where in time are inputs, outputs and states defined?



State Table Characteristics

- *State table* – a multiple variable table with the following four sections:
  - ***Present State*** – the values of the state variables for each allowed state.
  - ***Input*** – the input combinations allowed.
  - ***Next-state*** – the value of the state at time (t+1) based on the **present state** and the **input**.
  - ***Output*** – the value of the output as a function of the **present state** and (sometimes) the **input**.
- From the viewpoint of a truth table:
  - the inputs are Input, Present State
  - and the outputs are Output, Next State

Example 1: State Table

- The state table can be filled in using the next state and output equations:

$$A(t+1) = A(t)x(t) + B(t)x(t)$$

$$B(t+1) = \overline{A}(t)x(t)$$

$$y(t) = \overline{x}(t)(B(t) + A(t))$$

Present State		Input	Next State		Output
A(t)	B(t)	x(t)	A(t+1)	B(t+1)	y(t)
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Example 1: Alternate State Table

- 2-dimensional table that matches well to a K-map. Present state rows and input columns in Gray code order.
  - $A(t+1) = A(t)x(t) + B(t)x(t)$
  - $B(t+1) = \overline{A}(t)x(t)$
  - $y(t) = \overline{x}(t)(B(t) + A(t))$

Present State	Next State		Output	
	$x(t)=0$	$x(t)=1$	$x(t)=0$	$x(t)=1$
$A(t)\ B(t)$	$A(t+1)B(t+1)$	$A(t+1)B(t+1)$	$y(t)$	$y(t)$
0 0	0 0	0 1	0	0
0 1	0 0	1 1	1	0
1 0	0 0	1 0	1	0
1 1	0 0	1 0	1	0

State Diagrams

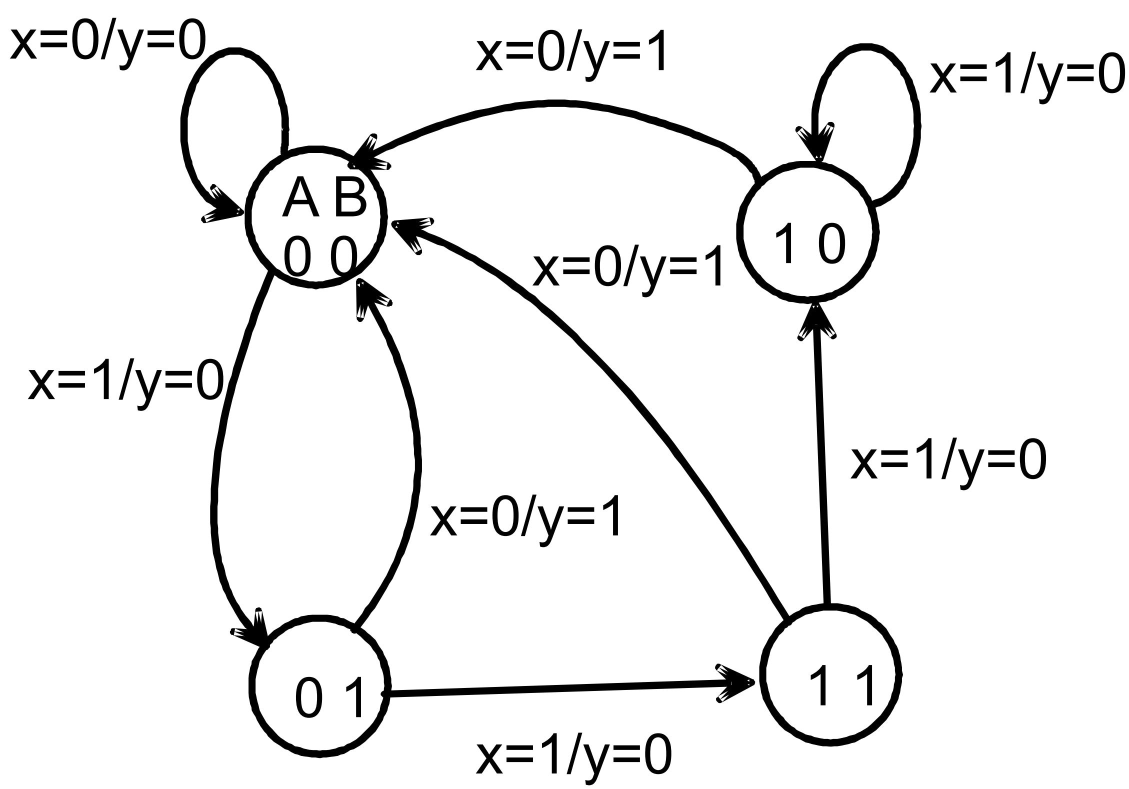
- The sequential circuit function can be represented in graphical form as a <u>state diagram</u> with the following components:
  - A <u>circle</u> with the state name in it for each state
  - A <u>directed arc</u> from the <u>Present State</u> to the <u>Next State</u> for each <u>state transition</u>
  - A label on each <u>directed arc</u> with the <u>Input</u> values which causes the <u>state transition</u>, and
  - A label:
    - On each <u>circle</u> with the <u>output</u> value produced, or
    - On each <u>directed arc</u> with the <u>output</u> value produced.

State Diagrams

- Label form:
  - On circle with output included:
    - state/output
    - Moore type output depends only on state
  - On directed arc with the output included:
    - input/output
    - Mealy type output depends on state and input

Example 1: State Diagram

- Which type?
- Diagram gets confusing for large circuits
- For small circuits, usually easier to understand than the state table



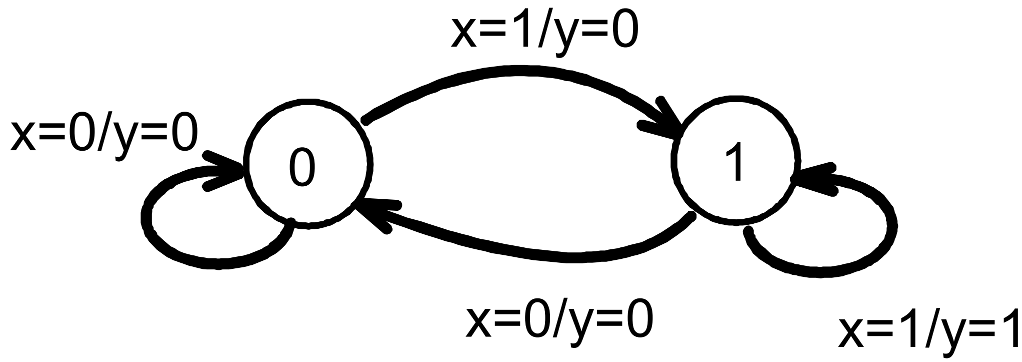
Moore and Mealy Models

- Sequential Circuits or Sequential Machines are also called ***Finite State Machines*** (FSMs). Two formal models exist:

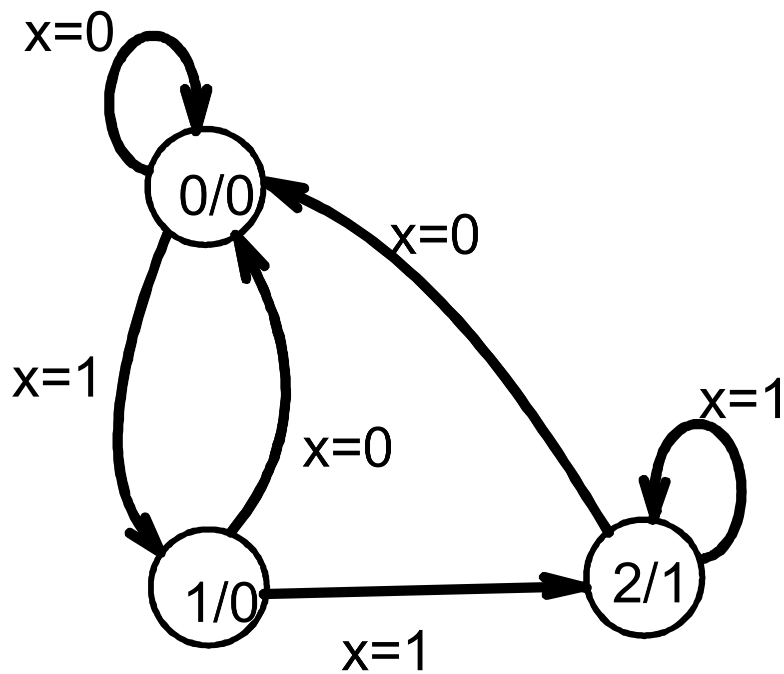
  - Moore Model
    - Named after E.F. Moore
    - Outputs are a function ONLY of states
    - Usually specified on the states.
  - Mealy Model
    - Named after G. Mealy
    - Outputs are a function of inputs AND states
    - Usually specified on the state transition arcs.

Moore and Mealy Example Diagrams

- Mealy Model State Diagram maps **<u>inputs and state</u>** to **<u>outputs</u>**



- Moore Model State Diagram maps **<u>states</u>** to **<u>outputs</u>**



Moore and Mealy Example Tables

- Moore Model state table maps state to outputs

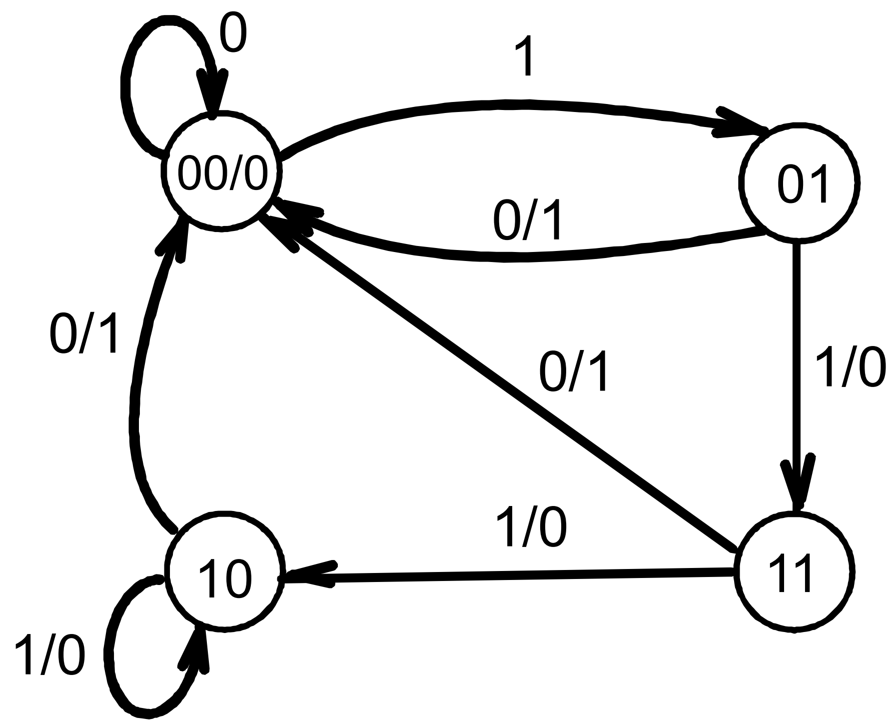
Present State	Next State x=0	Next State x=1	Output
0	0	1	0
1	0	2	0
2	0	2	1

- Mealy Model state table maps inputs and state to outputs

Present State	Next State x=0	Next State x=1	Output x=0	Output x=1
0	0	1	0	0
1	0	1	0	1

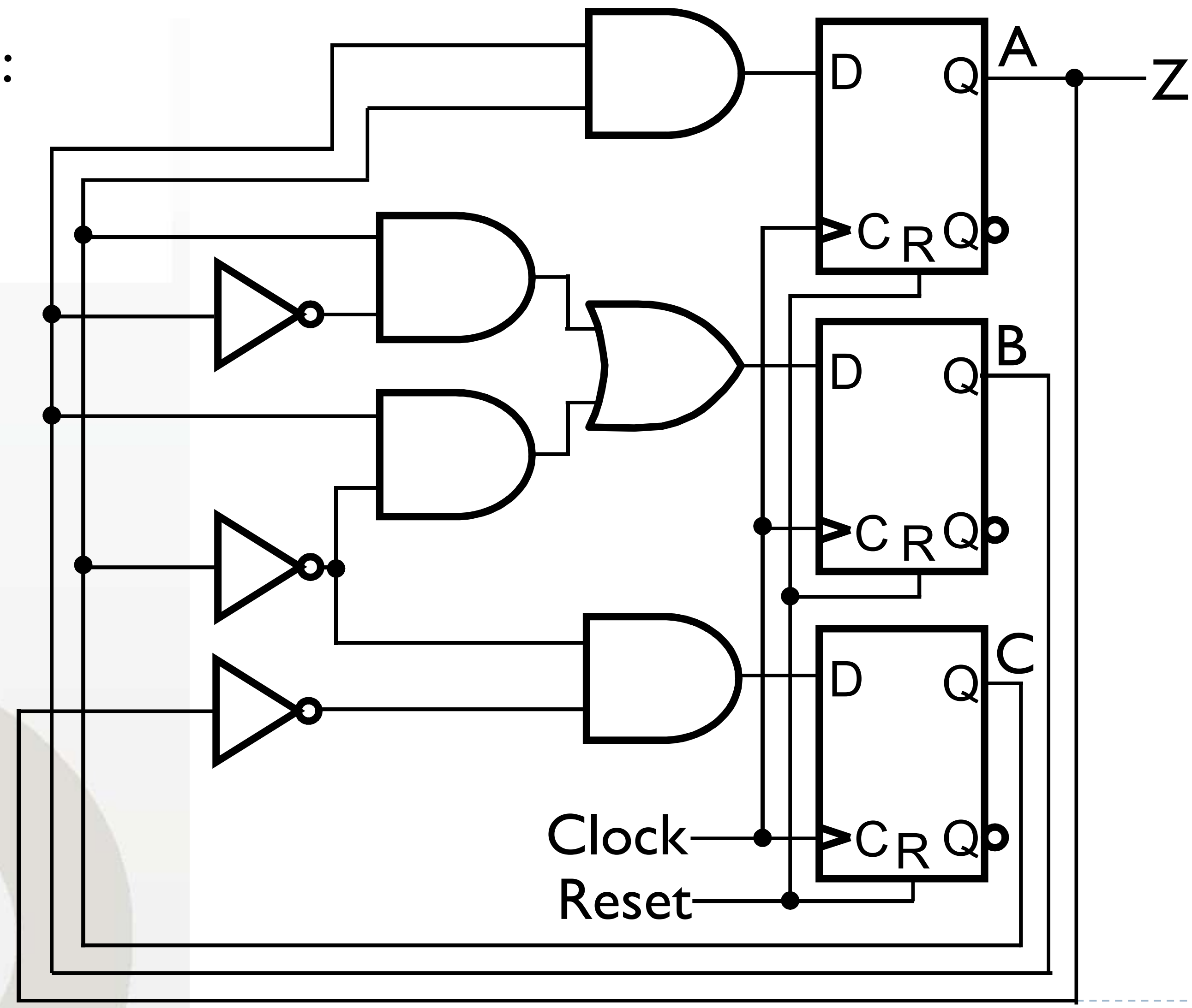
Mixed Moore and Mealy Outputs

- In real designs, some outputs may be Moore type and other outputs may be Mealy type.
- Example:
  - State 00: Moore
  - States 01, 10, and 11: Mealy
- Simplifies output specification



Example 2: Sequential Circuit Analysis

- Logic Diagram:



Example 2: Flip-Flop Input Equations

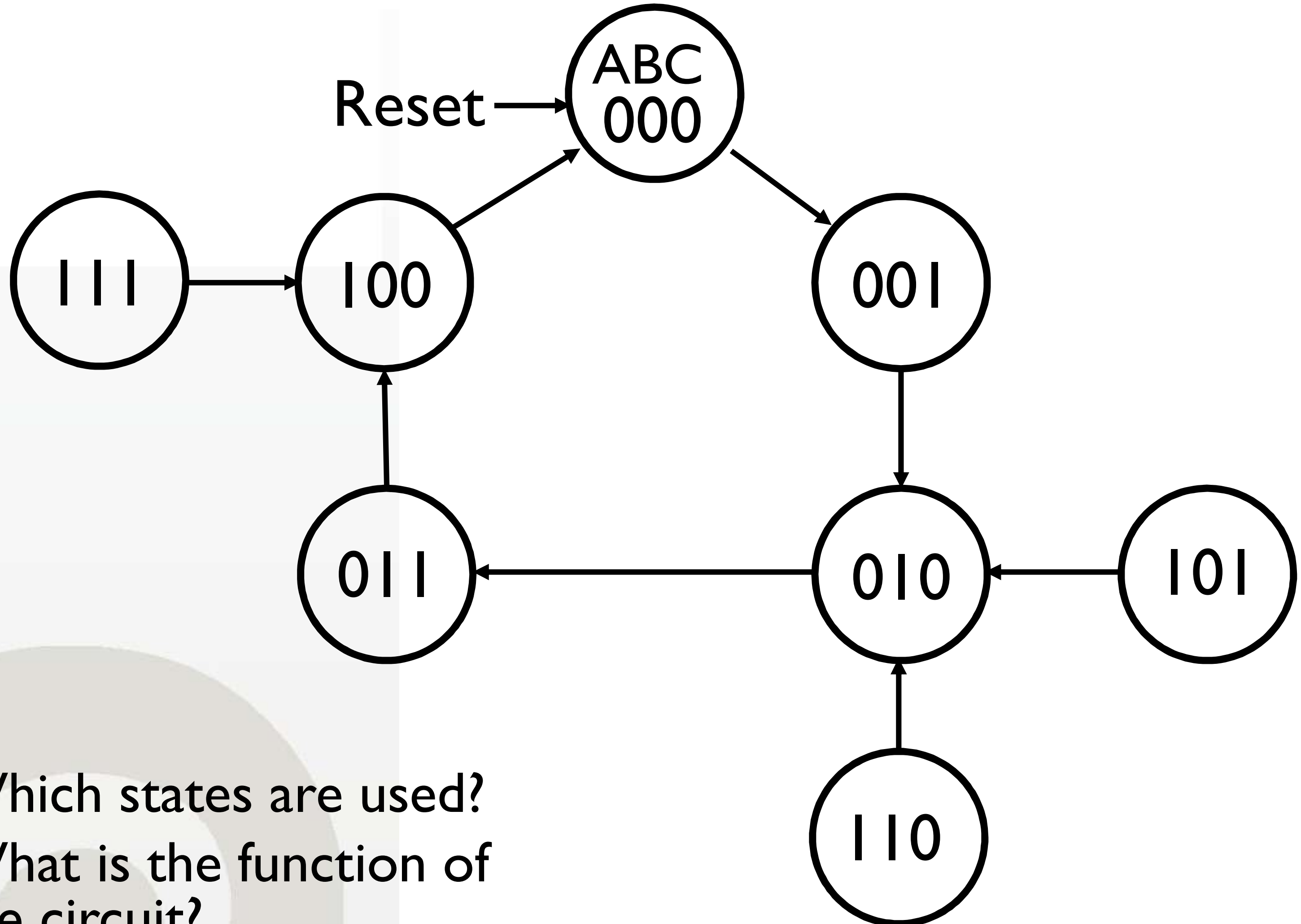
- Variables
  - Inputs: None
  - Outputs: Z
  - State Variables: A, B, C
- Initialization: Reset to (0,0,0)
- Equations
  - $A(t+1) =$ $\qquad\qquad$ $Z =$
  - $B(t+1) =$
  - $C(t+1) =$

Example 2: State Table

$X' = X(t+1)$

A B C	A'B'C'	Z
0 0 0		
0 0 1		
0 1 0		
0 1 1		
1 0 0		
1 0 1		
1 1 0		
1 1 1		

Example 2: State Diagram



- Which states are used?
- What is the function of the circuit?

Example 3: State Diagram

A sequential circuit with two D flip-flops A and B, two inputs X and Y, and one output Z is specified by the two following input equations:

$A(t+1) = \overline{X}A + XY$

$B(t+1) = XA + \overline{X}B$

$Z = XB$

a) Draw the logic diagram of the circuit
b) Derive the state table
c) Derive the state diagram

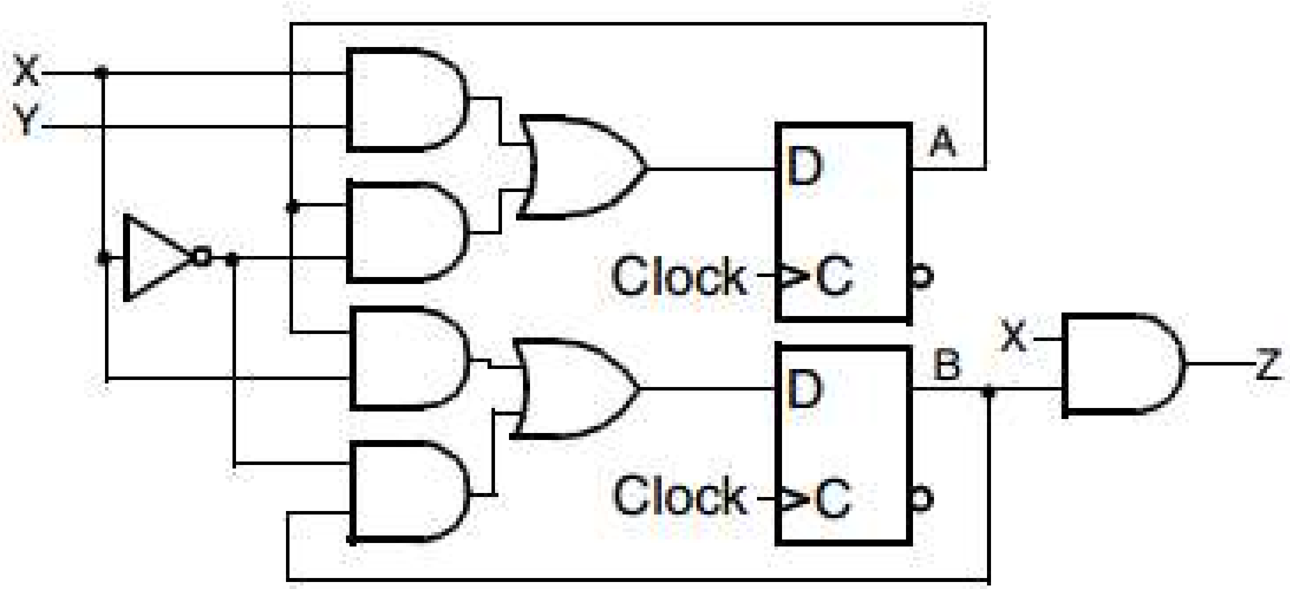
Example 3: State Diagram

$A(t+1) = \overline{X}A + XY$

$B(t+1) = XA + \overline{X}B$

$Z = XB$

a) The logic diagram of the circuit



Example 3: State Diagram

$A(t+1) = \overline{X}A + XY$

$B(t+1) = XA + \overline{X}B$

$Z = XB$

b) The state table

Present state		Inputs		Next state		Output
A	*B*	*X*	*Y*	*A*	*B*	*Z*
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	0	1	1	1	0	0
0	1	0	0	0	1	0
0	1	0	1	0	1	0
0	1	1	0	0	0	1
0	1	1	1	1	0	1
1	0	0	0	1	0	0
1	0	0	1	1	0	0
1	0	1	0	0	1	0
1	0	1	1	1	1	0
1	1	0	0	1	1	0
1	1	0	1	1	1	0
1	1	1	0	0	1	1
1	1	1	1	1	1	1

Example 3: State Diagram

$A(t+1) = \overline{X}A + XY$

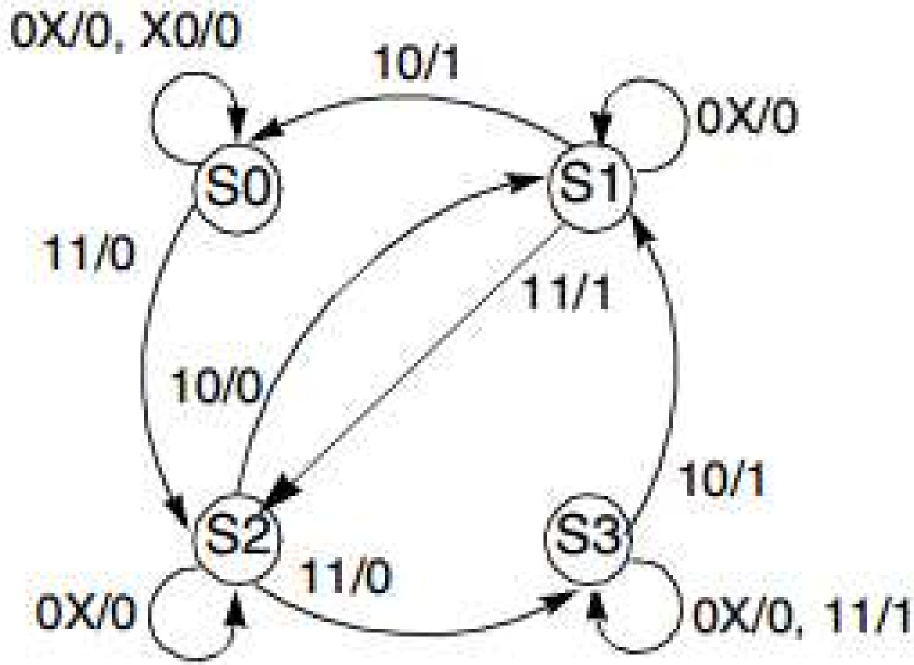
$B(t+1) = XA + \overline{X}B$

$Z = XB$

c) The state diagram

S0 - 00
S1 - 01
S2 - 10
S3 - 11

Format: XY/Z (X = unspecified)



Any Questions?