Chapter 37 Homework (due 12/12/13)

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Problem 37.5

Young's double slit experiment is performed with 589 nm light in the distance of 2 m between the slits in the screen. The 10th interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.

Solution

The minimum condition of a double slit interference pattern is

$$d\sin\theta = (m - \tfrac{1}{2})\lambda \quad \text{where} \quad m = 1, 2, 3, \ldots$$

Using the small angle approximation, this is

$$d\sin\theta \cong d\tan\theta = d\frac{y}{L} = (m - \tfrac{1}{2})\lambda \quad \text{where} \quad m = 1, 2, 3, \ldots$$

For the current set of parameters, this is

$$d\frac{7.26 \text{ mm}}{2000 \text{ mm}} = (7 - \tfrac{1}{2})(589 \text{ nm}) \quad \Rightarrow \quad d = 1.0547 \text{ mm}$$

Problem 37.11

To radio antenna separated by 300 m simultaneously broadcast identical signals at the same wavelength. A car travels due north along a straight line at a position 1000 m from the center point between the antennas, and it's radio receives the signals.

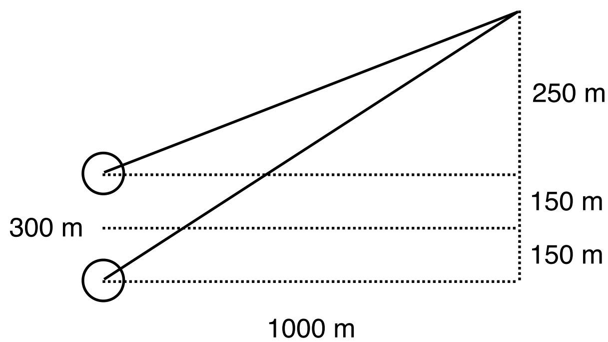
(a) If the car is at the position of the second maximum after he has travel a distance 400 m northward from the central maximum, what is the wavelength of the signal?

(b) How much farther must of car travel from this position to encounter the next minimum in reception?

Solution

(a) The condition for the maximum above is when the path length difference is 2 wavelengths.

$$\left|L_1 - L_2\right| = 2\lambda$$



The condition is

$$\sqrt{550^2 + 1000^2} - \sqrt{250^2 + 1000^2} = 1141.3 - 1030.8 = 110.52 = 2\lambda \quad \Rightarrow \quad \lambda = 55.262 \text{ m}$$

(b) The condition for the minima is when the path length difference is 2.5 wavelengths.

$$\sqrt{(550 + x)^2 + 1000^2} - \sqrt{(250 + x)^2 + 1000^2} = 2.5(55.262 \text{ m}) = 138.15 \text{ m}$$

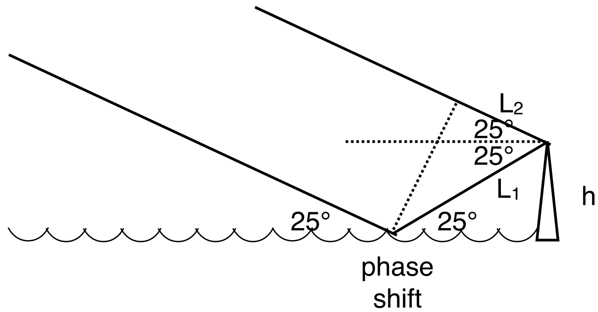
The solution to x is 123.36 m.

Problem 37.15

Radio waves of wavelength 125 m from the galaxy reach a radio telescope by two separate paths. When you could direct path to the receiver which is situated on the edge of the tall cliff by the ocean, and the second is my reflection off the water. As the galaxy rises in the east over the water, the first minimum of destructive interference occurs when the galaxy is at an angle of 25° above the horizon. Find the height of the radio telescope dish above the water.

Solution

The situation looks like this.



For a minimum to occur, the path length difference is half a wavelength. The path length L_1 is

$$\sin 25° = \frac{h}{L_1}$$

The path length L_2 is

$$\cos 50° = \frac{L_2}{L_1} \quad \Rightarrow \quad L_2 = L_1 \cos 50°$$

The optical path length L_1 has an extra half wavelength.

$$L_1 - \frac{1}{2}\lambda - L_2 = \frac{1}{2}\lambda \quad \Rightarrow \quad L_1 - L_2 = \lambda \quad \Rightarrow \quad L_1 - L_1 \cos 50° = \lambda \quad \Rightarrow \quad L_1(1 - \cos 50°) = 125 \text{ m}$$

$$L_1 = \frac{125 \text{ m}}{(1 - \cos 50°)} = 349.93 \text{ m}$$

This means the height is

$$h = L_1 \sin 25° = (349.93 \text{ m}) \sin 25° = 147.89 \text{ m}$$

Problem 37.25

The intensity on the screen for a certain point in it doubles the interference pattern is 64% of the maximum value.

(a) What minimum phase difference between sources produces this result?

(b) Express this phase difference as a path difference for 486.1 nm light.

Solution

(a) With the small angle approximation, the intensity function looks like this.

$$I = I_{\max}\cos^2\left(\frac{d}{\lambda}\frac{x}{L}\pi\right)$$

At 64% of maximum, the minimum phase is

$$\cos^2\left(\frac{\phi}{2}\right) = 0.64 \quad \Rightarrow \quad \frac{\phi}{2} = 0.64350\ rad \quad \Rightarrow \quad \phi = 1.2870\ rad$$

(b) At 486.1 nm, the above represents

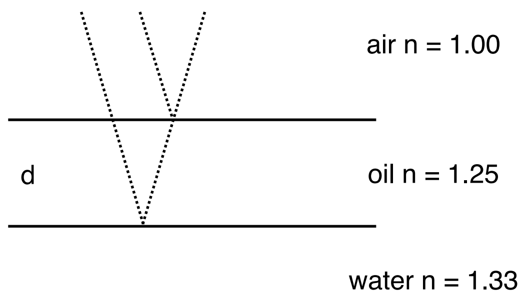
$$\phi = 1.2870\ rad\left(\frac{1\ wavelength}{2\pi\ rad}\right)\left(\frac{486.1\ nm}{1\ wavelength}\right) = 99.569\ nm$$

Problem 37.29

A thin-film of oil n = 1.25 is located on smooth, wet pavement. When viewed perpendicular to the pavement, the film reflects most strongly red light at 640 nm and reflects no green light at 512 nm. How thickest is the oil film?

Solution

Here is the situation.



Both rays go through a phase shift so there is no net phase shift. The extra distance of the longer path is 2d and its optical path length is 2dn. For the red light, the constructive interference condition is

$$2dn = m\lambda_{red}$$

For the green light, the destructive interference condition is

$$2dn = (m - \tfrac{1}{2})\lambda_{green}$$

We assume that these two features occur in the same "rainbow" so that the order is the same. This means the difference between these two equations is

$$(m - \tfrac{1}{2})\lambda_{green} - m\lambda_{red} = 0 \quad \Rightarrow \quad m(\lambda_{green} - \lambda_{red}) = +\tfrac{1}{2}\lambda_{green} \quad \Rightarrow \quad m = \frac{\lambda_{green}}{2(\lambda_{green} - \lambda_{red})}$$

$$m = \frac{\lambda_{green}}{2(\lambda_{green} - \lambda_{red})} = \frac{512\ nm}{2(640\ nm - 512\ nm)} = 2$$

At this order, the thickness must be

$$2dn = 2\lambda_{red} \quad \Rightarrow \quad d = \frac{\lambda_{red}}{n} = \frac{640\ nm}{1.25} = 512\ nm$$

Problem 37.32

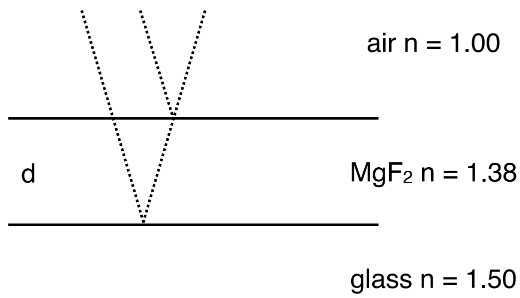
A film of magnesium fluoride n = 1.38 having thickness 1×10^{-5} cm is used to coat a camera lens.

(a) What are the three longest wavelengths that are intensified in the reflected light?

(b) Are any of these wavelengths in the visible spectrum?

Solution

(a) Here is the situation.



At the given thickness, the longest wavelengths that are constructively reflected are

$$2dn = \lambda \quad \Rightarrow \quad \lambda = 2(1 \times 10^{-5}\ cm)(1.38) = 276\ nm$$

$$2dn = 2\lambda \quad \Rightarrow \quad \lambda = \frac{2(1 \times 10^{-5}\ cm)(1.38)}{2} = 138\ nm$$

$$2dn = 3\lambda \quad \Rightarrow \quad \lambda = \frac{2(1 \times 10^{-5}\ cm)(1.38)}{3} = 92\ nm$$

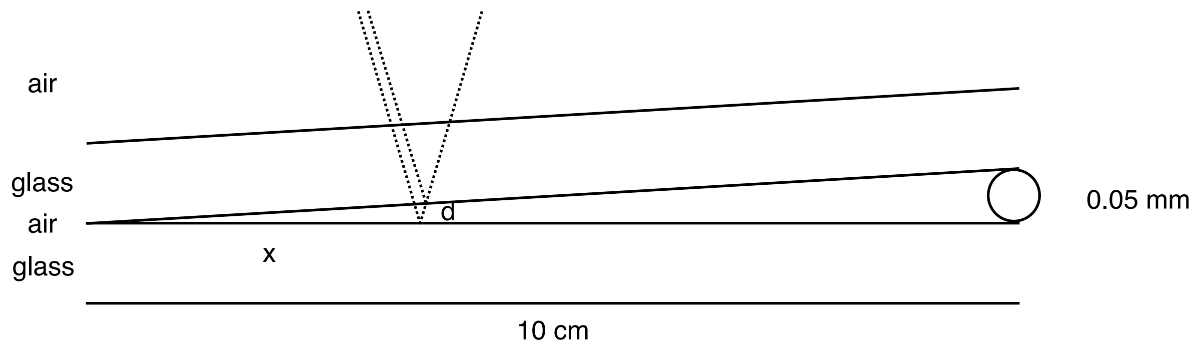
(b) None of the wavelengths is visible.

Problem 37.39

To glass plates 10 cm long are in contact at one end and separated at the other end by a thread with a diameter of 0.05 mm. Light containing the two wavelengths 400 nm and 600 nm is incident perpendicularly and viewed by reflection. At what distance from the contact point is next dark fringe?

Solution

Here is the situation.



A dark fringe from two wavelengths can be produced only if each wavelength is destructively interfering.

The dark fringes from the 400 nm light is given by the following thicknesses.

$$2d + \tfrac{1}{2}\lambda = (m + \tfrac{1}{2})\lambda \quad \Rightarrow \quad d = m\frac{\lambda}{2} = 200\ nm, 400\ nm, 600\ nm, ...$$

The dark fringes from the 600 nm light is given by the following thicknesses.

$$2d + \tfrac{1}{2}\lambda = (m + \tfrac{1}{2})\lambda \quad \Rightarrow \quad d = m\frac{\lambda}{2} = 300\ nm, 600\ nm, 900\ nm, ...$$

The first dark fringe they have in common is at d = 600 nm. The distance x at which this thickness is formed is

$$\tan\theta = \frac{0.05\ mm}{10\ cm} = \frac{d}{x} = \frac{600\ nm}{x} \quad \Rightarrow \quad x = \frac{(10\ cm)(600\ nm)}{0.05\ mm} = \frac{(100\ mm)(600 \times 10^{-6}\ mm)}{0.05\ mm} = 1.2\ mm$$