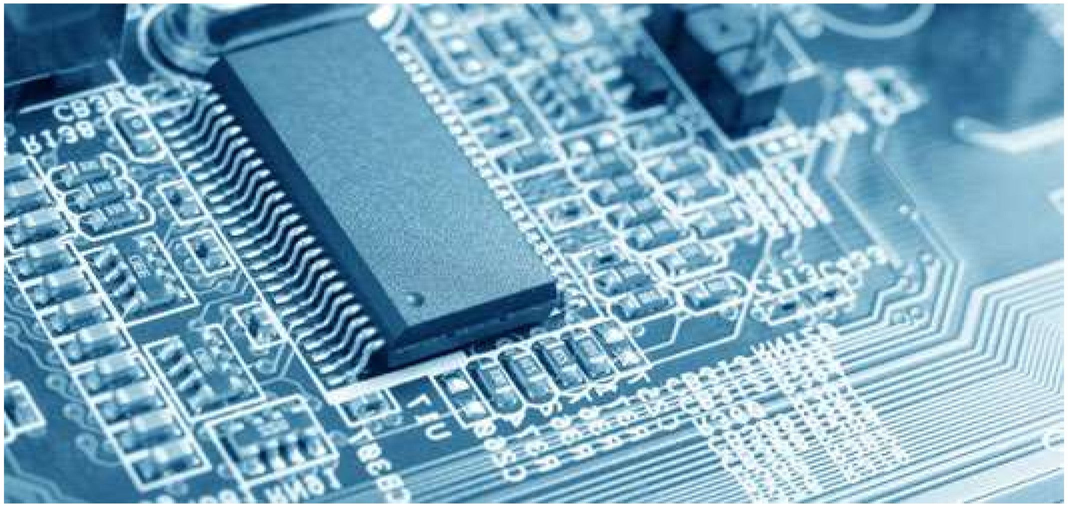
NAZARBAYEV UNIVERSITY
SCHOOL OF SCIENCE AND TECHNOLOGY



# ROBT206 – Microcontrollers with Lab

## Lecture 7 – Circuit Optimization

**30 January, 2018**

# Topics

| Today's Topics |
| --- |
| Circuit Optimization |
| Two Level Optimization |
| Map Manipulation → Karnaugh Maps |

# Circuit Optimization

- **Goal**: To obtain the simplest implementation for a given function
- Optimization is a more formal approach to simplification that is performed using a specific procedure or algorithm
- Optimization requires a cost criterion to measure the simplicity of a circuit

- Distinct cost criteria we will use:
  - Literal cost (L)
  - Gate input cost (G)
  - Gate input cost with NOTs (GN)

# Literal Cost

- Literal – a variable or it's complement
- Literal cost – the number of literal appearances in a Boolean expression corresponding to the logic circuit diagram
- Examples:
  - $F = BD + A\,B\,C + A\,\overline{C}\,\overline{D}$ $L = 8$
  - $F = BD + A\,\overline{B}C + A\,\overline{B}\overline{D} + AB\,\overline{C}$ $L =$
  - $F = (A + B)(A + D)(B + C + \overline{D})(\overline{C} + \overline{B} + D)$ $L =$
  - Which solution is best?

# Gate Input Cost

- **Gate input** costs **-** the number of inputs to the gates in the implementation corresponding exactly to the given equation or equations. (**G - inverters not counted**, **GN - inverters counted**)
- For SOP and POS equations, it can be found from the equation(s) by finding the sum of:
  - all literal appearances
  - the number of terms excluding single literal terms,(G) and
  - optionally, the number of distinct complemented single literals (GN).
- Example:
  - $F = BD + A\overline{B}C + A\overline{C}\,\overline{D}$ G = 11, GN = 14
  - $F = BD + A\overline{B}C + A\overline{B}\,\overline{D} + AB\overline{C}$ G = , GN =
  - $F = (A + \overline{B})(A + D)(B + C + \overline{D})(\overline{B} + \overline{C} + D)$ G = , GN =
  - Which solution is best?

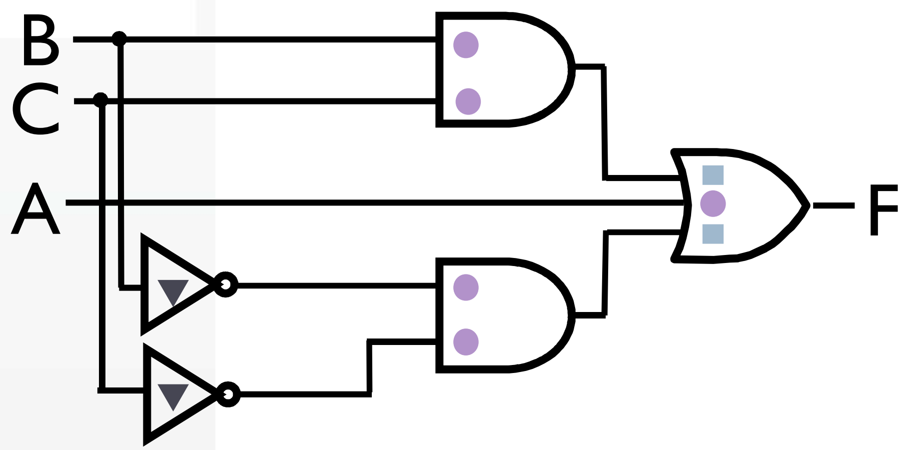
# Cost Criteria

- **Example 1:**
- $F = A + B\,C + \overline{B}\;\overline{C}$

**L = 5**

**G = L + 2 = 7**

**GN = G + 2 = 9**



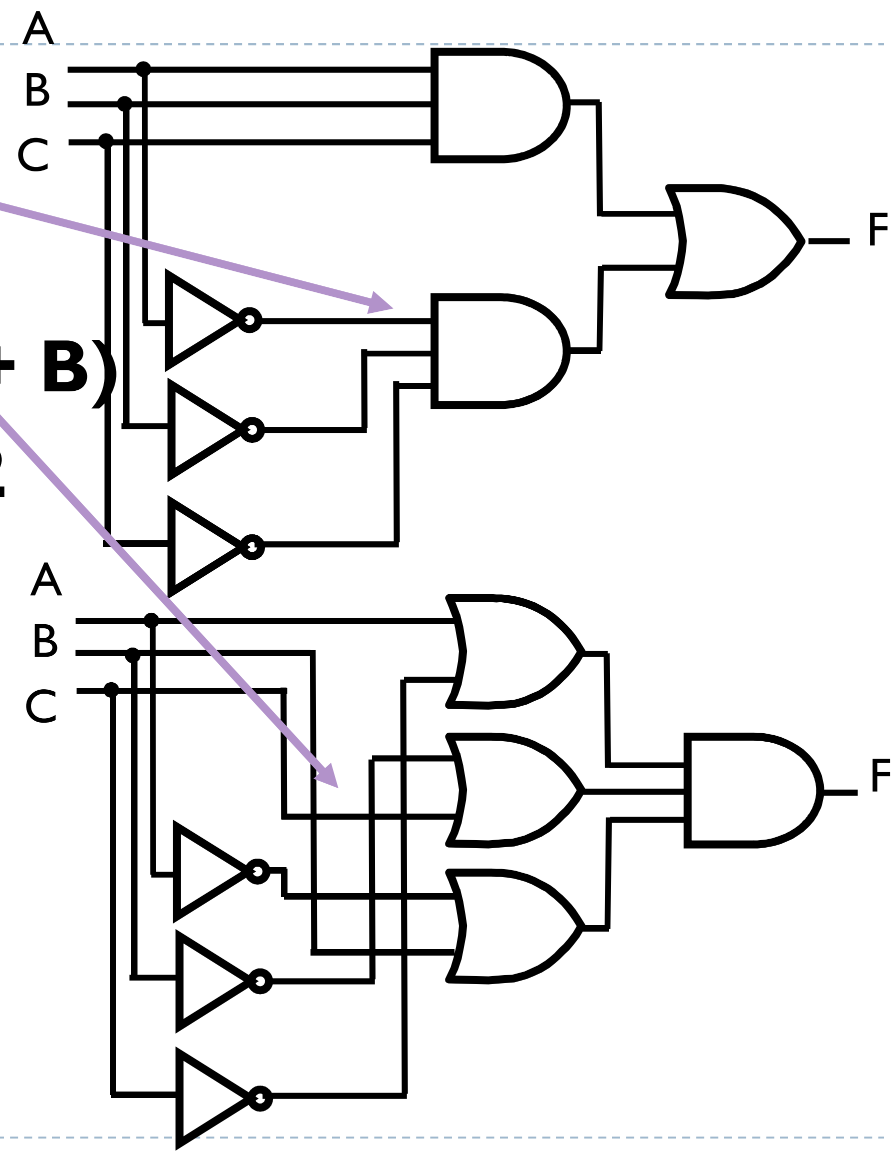

- L (literal count) counts the AND inputs and the single literal OR input.
- G (gate input count) adds the remaining OR gate inputs
- GN (gate input count with NOTs) adds the inverter inputs

# Cost Criteria

- **Example 2:**
- **$F = A\,B\,C + \bar{A}\,\bar{B}\,\bar{C}$**

  **$L = 6 \quad G = 8 \quad GN = 11$**
- **$F = (A + \bar{C})(\bar{B} + C)(\bar{A} + B)$**

  **$L = 6 \quad G = 9 \quad GN = 12$**
- <u>Same</u> function and <u>same</u> literal cost
- But first circuit has <u>better</u> gate input count and <u>better</u> gate input count with NOTs
- Select it!



# Boolean Function Optimization

- Minimizing the gate input (or literal) cost of a (a set of) Boolean equation(s) reduces circuit cost.
- We choose gate input cost.
- **Boolean Algebra** and **graphical techniques** are tools to minimize cost criteria values.
- Graphical technique using Karnaugh maps (K-maps)

# Karnaugh Maps (K-map)

- A K-map is a collection of squares
  - Each square represents a minterm
  - The collection of squares is a graphical representation of a Boolean function
  - Adjacent squares differ in the value of one variable
  - Alternative algebraic expressions for the same function are derived by recognizing patterns of squares
- The K-map can be viewed as a reorganized version of the truth table

| | $y = 0$ | $y = 1$ |
|---|---|---|
| $x = 0$ | $m_0 = \overline{x}\,\overline{y}$ | $m_1 = \overline{x}\,y$ |
| $x = 1$ | $m_2 = x\,\overline{y}$ | $m_3 = x\,y$ |

# Some Uses of K-Maps

- Provide a means for:
  - Finding optimum or near optimum
    - SOP and POS standard forms, and
    - two-level AND/OR and OR/AND circuit implementations

    for functions with small numbers of variables
  - Visualizing concepts related to manipulating Boolean expressions, and
  - Demonstrating concepts used by computer-aided design programs to simplify large circuits

# Two Variable Maps

- A 2-variable Karnaugh Map:
  - Note that minterm $m_0$ and minterm $m_1$ are "adjacent" and differ in the value of the variable y
  - Similarly, minterm $m_0$ and minterm $m_2$ differ in the x variable.
  - Also, $m_1$ and $m_3$ differ in the x variable as well.
  - Finally, $m_2$ and $m_3$ differ in the value of the variable y

| | y = 0 | y = 1 |
|---|---|---|
| x = 0 | $m_0 = \bar{x}\,\bar{y}$ | $m_1 = \bar{x}\,y$ |
| x = 1 | $m_2 = x\,\bar{y}$ | $m_3 = x\,y$ |

# K-Map and Truth Tables

- The K-Map is just a different form of the truth table.
- Example – Two variable function:
  - We choose a, b, c and d from the set {0,1} to implement a particular function, F(x, y).

**Function Table**

| Input Values (x,y) | Function Value F(x,y) |
|---|---|
| 0 0 | a |
| 0 1 | b |
| 1 0 | c |
| 1 1 | d |

**K-Map**

| | y = 0 | y = 1 |
|---|---|---|
| x = 0 | a | b |
| x = 1 | c | d |

# K-Map Function Representation

- Example: F(x, y) = x

| F = x | y = 0 | y = 1 |
|---|---|---|
| x = 0 | 0 | 0 |
| x = 1 | 1 | 1 |

- For function F(x, y), the two adjacent cells containing 1's can be combined using the Minimization Theorem:

$$F(x, y) = x\,\overline{y} + x\,y = x$$

# K-Map Function Representation

- Example: $G(x, y) = x + y$

| $G = x+y$ | $y = 0$ | $y = 1$ |
|---|---|---|
| $x = 0$ | 0 | 1 |
| $x = 1$ | 1 | 1 |

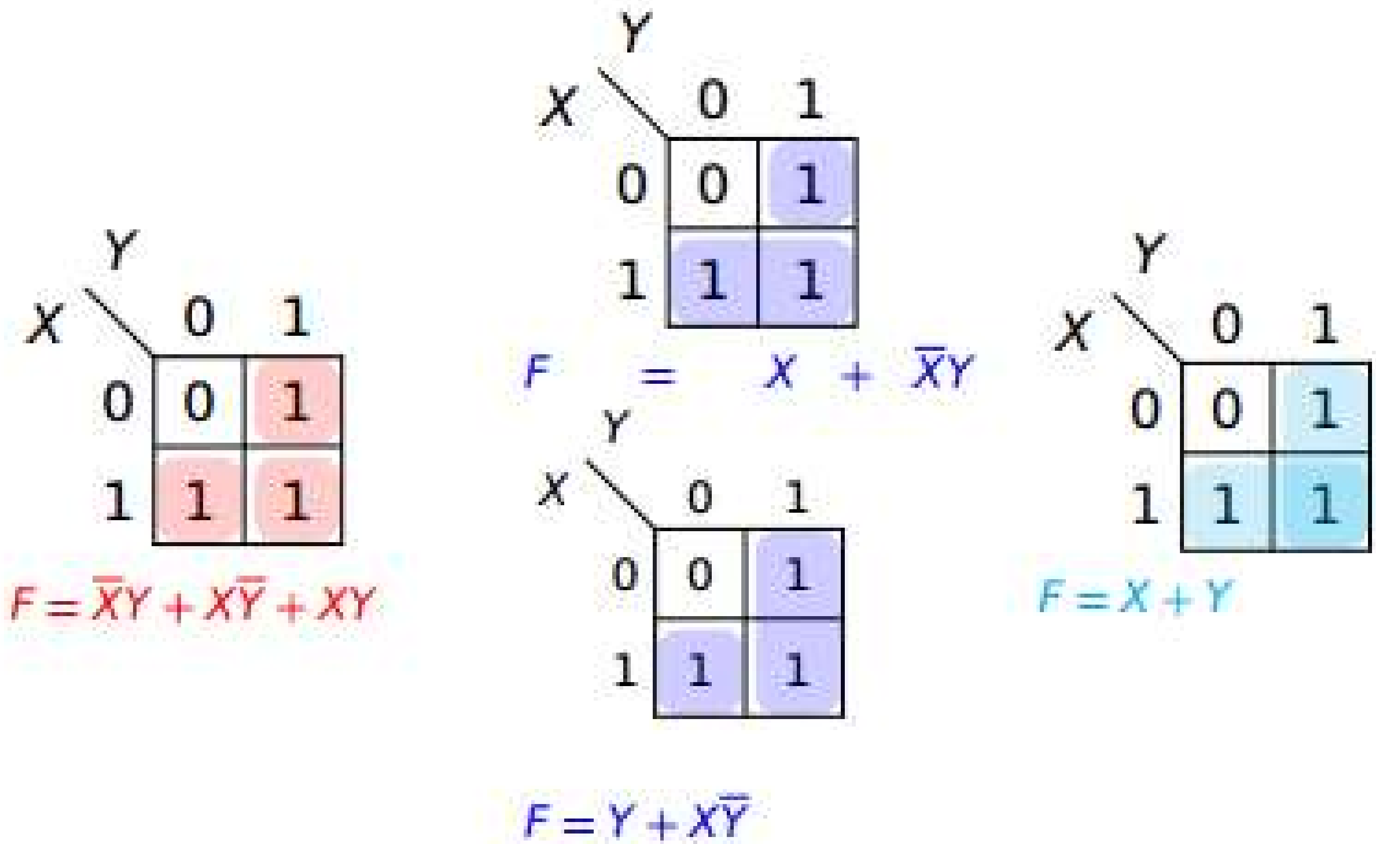
- For $G(x, y)$, two pairs of adjacent cells containing 1's can be combined using the Minimization Theorem:

$$G(x, y) = (x\,\overline{y} + x\,y) + (xy + \overline{x}\,y) = x + y$$

**Duplicate xy**

# K-Map Function Representation

When two cells share an edge and both are 1, those two terms can be combined to form a single, simpler term.



# Three Variable Maps

- A three-variable K-map:

| | yz=00 | yz=01 | yz=11 | yz=10 |
|---|---|---|---|---|
| x=0 | $m_0$ | $m_1$ | $m_3$ | $m_2$ |
| x=1 | $m_4$ | $m_5$ | $m_7$ | $m_6$ |

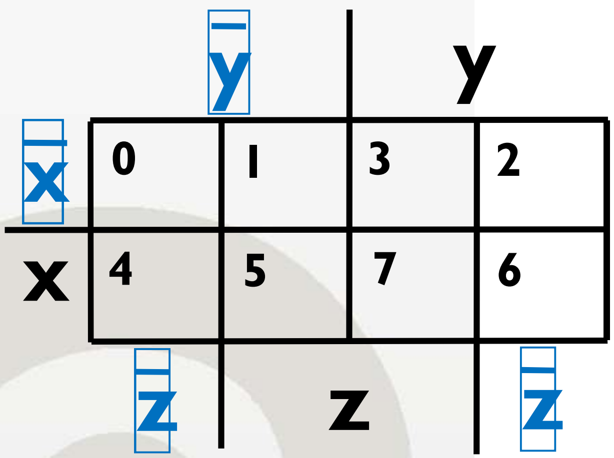
- Where each minterm corresponds to the product terms:

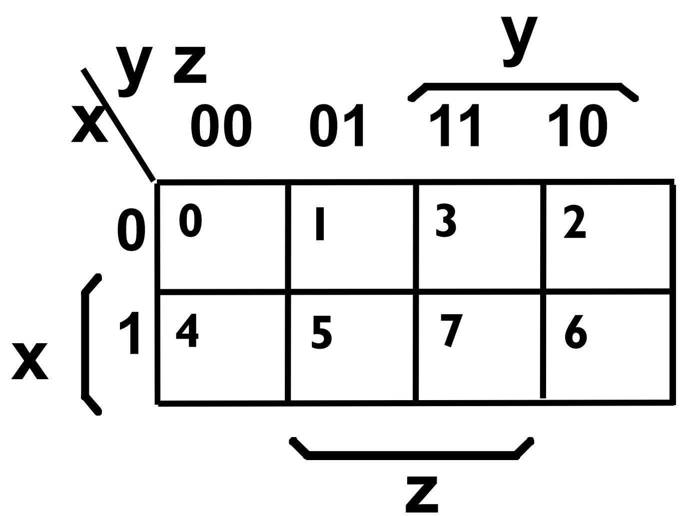
| | yz=00 | yz=01 | yz=11 | yz=10 |
|---|---|---|---|---|
| x=0 | $\overline{x}\,\overline{y}\,\overline{z}$ | $\overline{x}\,\overline{y}\,z$ | $\overline{x}\,y\,z$ | $\overline{x}\,y\,\overline{z}$ |
| x=1 | $x\,\overline{y}\,\overline{z}$ | $x\,\overline{y}\,z$ | $x\,y\,z$ | $x\,y\,\overline{z}$ |

- Note that if the binary value for an <u>index</u> differs in one bit position, the minterms are adjacent on the K-Map

# Alternative Map Labeling

- Map use largely involves:
  - Entering values into the map, and
  - Reading off product terms from the map.
- Alternate labelings are useful:

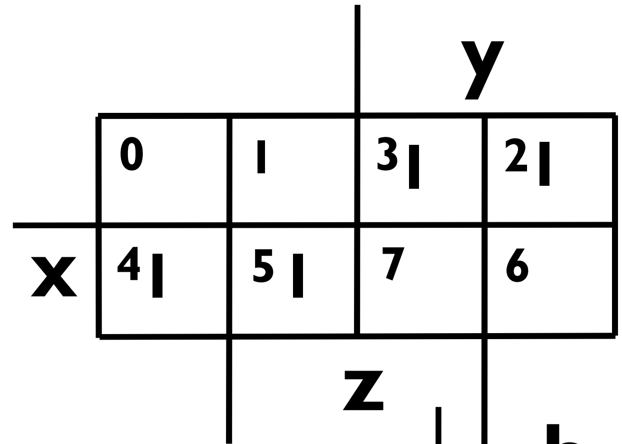


# Example Functions

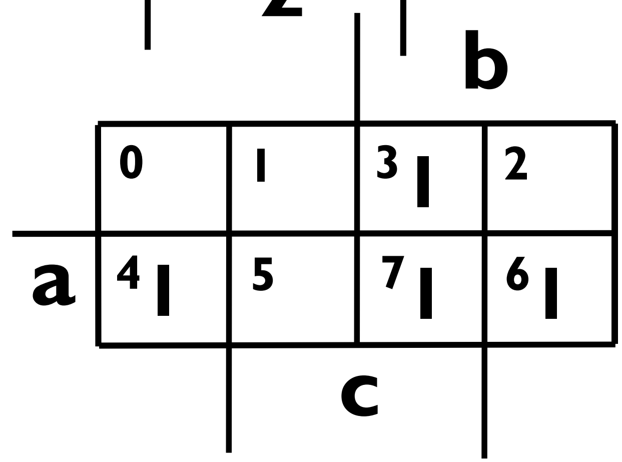
- By convention, we represent the minterms of F by a "1" in the map and leave the minterms of $\overline{F}$ blank
- Example:

$$F(x, y, z) = \Sigma m(2,3,4,5)$$




- Example:

$$G(a, b, c) = \Sigma m(3,4,6,7)$$




- <u>Learn</u> the locations of the 8 indices based on the variable order shown (x (a) - most significant and z (c) - least significant) on the map boundaries

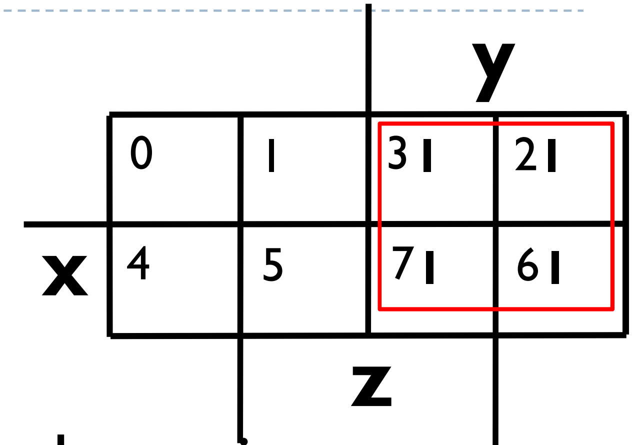
# Combining Squares

- By combining squares, we reduce number of literals in a product term, reducing the literal cost, thereby reducing the other two cost criteria
- On a 3-variable K-Map:
  - One square represents a minterm with three variables
  - Two adjacent squares represent a product term with two variables
  - Four "adjacent" terms represent a product term with one variable
  - Eight "adjacent" terms is the function of all ones (no variables) = 1.

# Example: Combining Squares

- Example: Let

$$F = \Sigma m(2,3,6,7)$$




- Applying the Minimization Theorem three times:

$$\begin{aligned} F(x,y,z) &= \bar{x}yz + xyz + \bar{x}y\bar{z} + xy\bar{z} \\ &= yz + y\bar{z} \\ &= y \end{aligned}$$

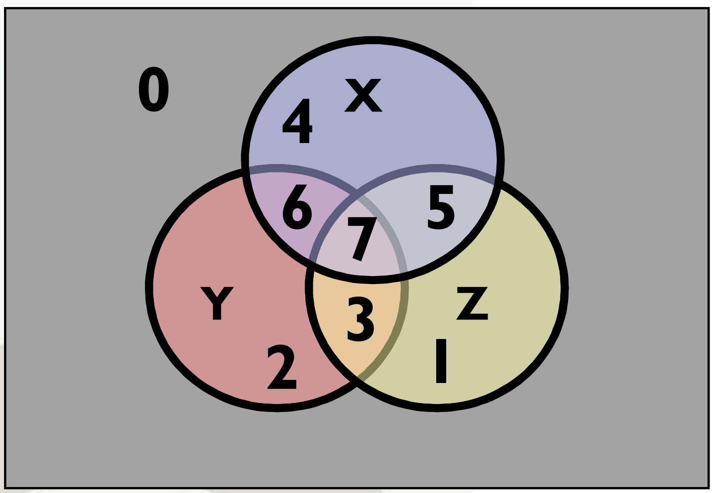
- Thus the four terms that form a 2 × 2 square correspond to the term "y".

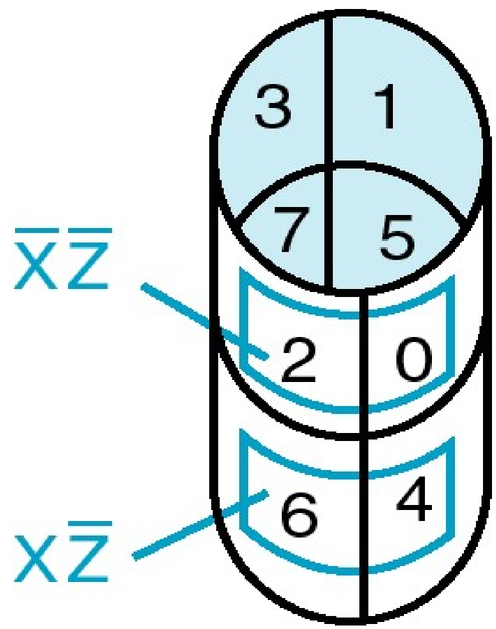
# Three-Variable Maps

- Reduced literal product terms for SOP standard forms correspond to <u>rectangles</u> on K-maps containing cell counts that are powers of 2.
- Rectangles of 2 cells represent 2 adjacent minterms; of 4 cells represent 4 minterms that form a "pairwise adjacent" ring.
- Rectangles can contain non-adjacent cells as illustrated by the "pairwise adjacent" ring above.

# Three-Variable Maps

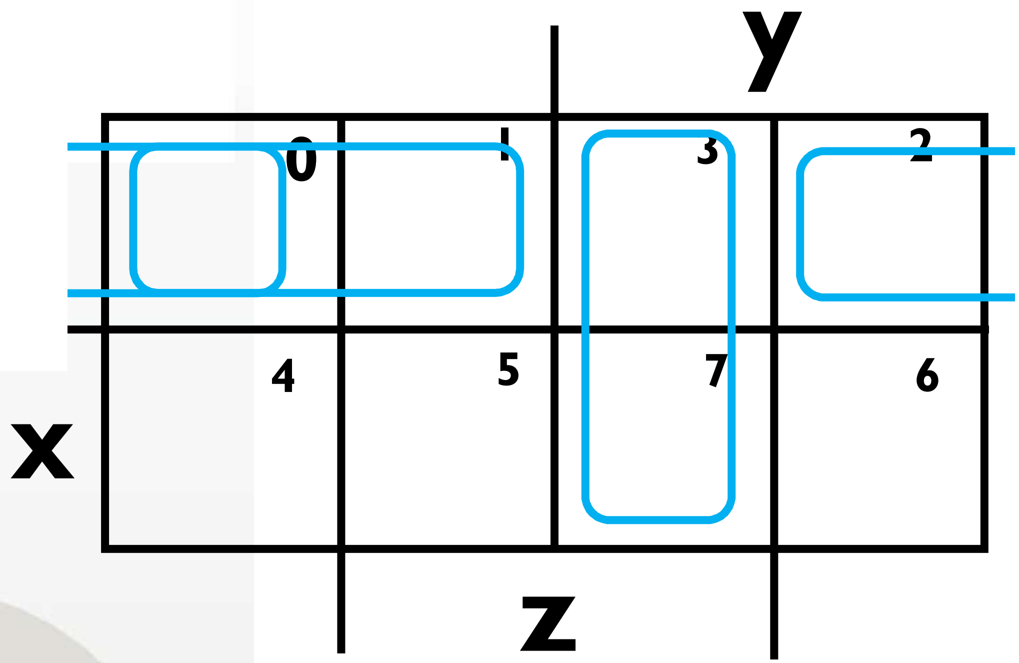
- **Topological warps of 3-variable K-maps that show *all* adjacencies:**
  - Venn Diagram
  - Cylinder



# Three-Variable Maps

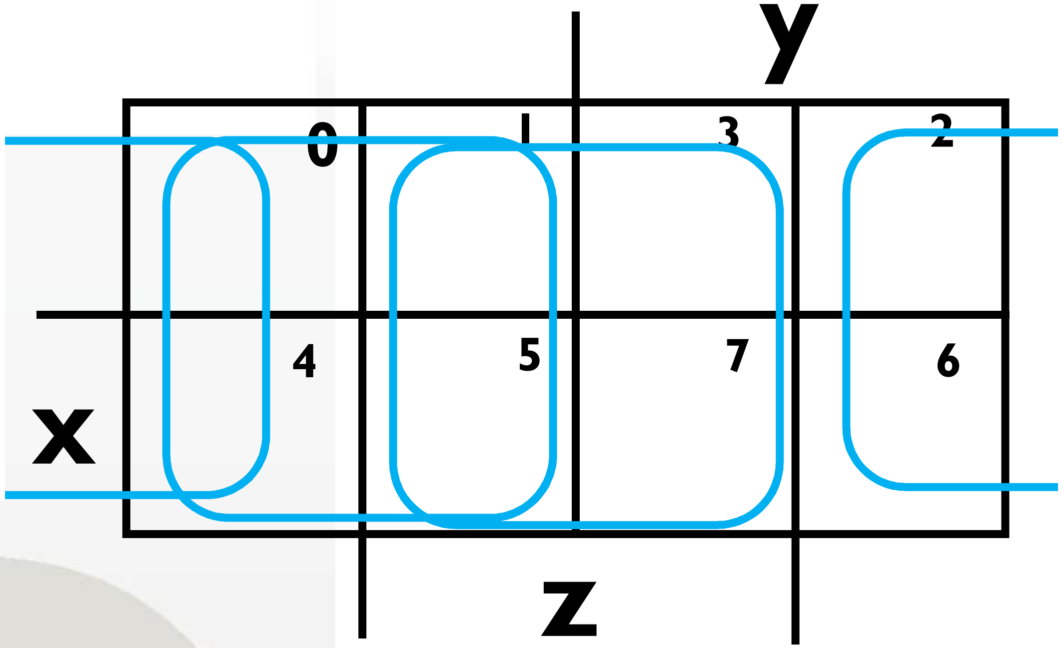
- **Example Shapes of 2-cell Rectangles:**




- Lets read the product terms for the rectangles shown

# Three-Variable Maps

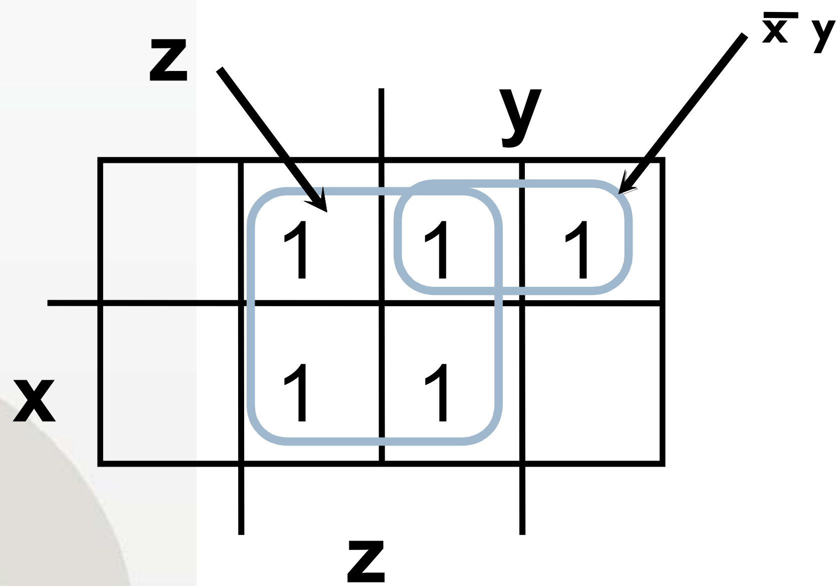
**Example Shapes of 4-cell Rectangles:**




- Read off the product terms for the rectangles shown

# Three Variable Maps

- K-Maps can be used to simplify Boolean functions by systematic methods. Terms are selected to cover the "1s"in the map.
- Example: Simplify $F(x, y, z) = \Sigma m(1,2,3,5,7)$




$F(x, y, z) = z + \bar{x}y$

# Three-Variable Map Simplification

- Use a K-map to find an optimum SOP equation for

$$F(X, Y, Z) = \Sigma m(0,2,4,5,6)$$

| | | | y | y |
|---|---|---|---|---|
| | 0 | 1 | 3 | 2 |
| x | 4 | 5 | 7 | 6 |
| | | z | z | |

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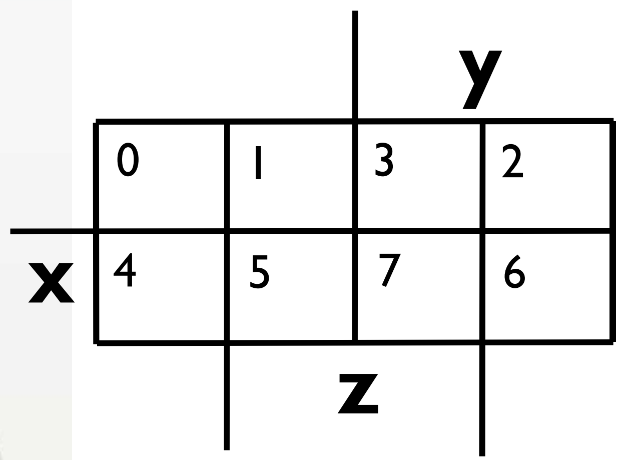
- Use a K-map to find an optimum SOP equation for

$$F(X,Y,Z) = \Sigma m(0,1,2,4,6,7)$$



# Three-Variable Map Simplification

- Use a K-map to find an optimum SOP equation for

$$F(X,Y,Z) = \Sigma m(1,3,4,5,6)$$

| | | | y | y |
|---|---|---|---|---|
| | 0 | 1 | 3 | 2 |
| x | 4 | 5 | 7 | 6 |
| | | z | z | |

# Any Questions?