Physics 41 Chapter 37 HW Solutions Spring 2013

1. Coherent light of wavelength 650 nm is sent through two parallel slits and an interference pattern in formed on a screen at a distance L=2.50 m behind the slits. Each slit is a = 0.700 μm wide. Their centers are d= 2.80 μm apart.

a) Derive from scratch the equation for the position of constructive interference fringes on the screen above the center of the central bright fringe in terms of the variables L, d, n, λ and θ.

b) Find the angle to the 3rd bright fringe.

c) Find the intensity of light on the screen at the center of the 3rd bright fringe, expressed as a fraction of the light intensity Imax at the center of the pattern.

See class case for key

2. A laser beam (λ = 632.8 nm) is incident on two slits 0.200 mm apart. How far apart are the bright interference fringes on a screen 5.00 m away from the double slits?

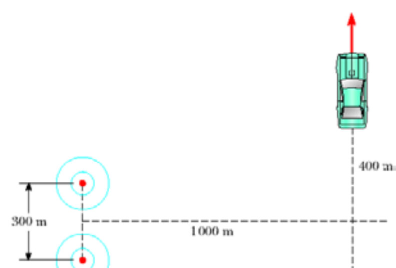
$$\Delta y_{\text{bright}} = \frac{\lambda L}{d} = \frac{(632.8\times10^{-9})(5.00)}{2.00\times10^{-4}}\text{ m} = \boxed{1.58\text{ cm}}$$

3 A Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.500 mm, and the interference pattern on a screen 3.30 m away shows the first side maximum 3.40 mm from the center of the pattern. What is the wavelength?

Solution: $y_{\text{bright}} = \frac{\lambda L}{d} m$ For $m = 1$,

$$\lambda = \frac{yd}{L} = \frac{(3.40\times10^{-3}\text{ m})(5.00\times10^{-4}\text{ m})}{3.30\text{ m}} = \boxed{515\text{ nm}}$$

4. Two radio antennas separated by 300 m as shown in Figure simultaneously broadcast identical signals at the same wavelength. A radio in a car traveling due north receives the signals. (a) If the car is at the position of the second maximum, what is the wavelength of the signals? (b) How much farther must the car travel to encounter the next minimum in reception? (*Note:* Do not use the small-angle approximation in this problem.)



Solution: Note, with the conditions given, the small angle approximation **does not work well**. That is, $\sin\theta$, $\tan\theta$, and θ are significantly different. We treat the interference as a Fraunhofer pattern.

(a)At the $m = 2$ maximum, $\tan\theta = \frac{400\text{ m}}{1\,000\text{ m}} = 0.400$

$$\theta = 21.8°$$

So $\lambda = \frac{d\sin\theta}{m} = \frac{(300\text{ m})\sin 21.8°}{2} = \boxed{55.7\text{ m}}$

(b) The next minimum encountered is the $m = 2$ minimum, and at that point, $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$

which becomes $$d\sin\theta = \frac{5}{2}\lambda$$

or $$\sin\theta = \frac{5}{2}\frac{\lambda}{d} = \frac{5}{2}\left(\frac{55.7\text{ m}}{300\text{ m}}\right) = 0.464$$

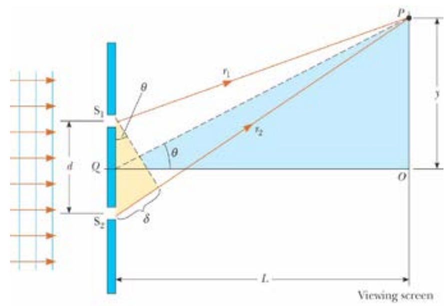
and $\theta = 27.7°$ so $$y = (1\,000\text{ m})\tan 27.7° = 524\text{ m}$$

Therefore, the car must travel an additional $\boxed{124\text{ m}}$.

If we considered Fresnel interference, we would more precisely find

(a) $\lambda = \frac{1}{2}\left(\sqrt{550^2 + 1\,000^2} - \sqrt{250^2 + 1\,000^2}\right) = 55.2\text{ m}$ and (b) 123 m

5. For a double slit setup, let $L = 1.20$ m and $d = 0.120$ mm and assume that the slit system is illuminated with monochromatic 500-nm light. Calculate the phase difference between the two wave fronts arriving at P when (a) $\theta = 0.500°$ and (b) $y = 5.00$ mm. (c) What is the value of θ for which the phase difference is 0.333 rad? (d) What is the value of θ for which the path difference is $\lambda/4$?



$$\phi = \frac{2\pi}{\lambda}d\sin\theta = \frac{2\pi}{\lambda}d\left(\frac{y}{L}\right)$$

(a) $$\phi = \frac{2\pi}{(5.00\times10^{-7}\text{ m})}(1.20\times10^{-4}\text{ m})\sin(0.500°) = \boxed{13.2\text{ rad}}$$

(b) $$\phi = \frac{2\pi}{(5.00\times10^{-7}\text{ m})}(1.20\times10^{-4}\text{ m})\left(\frac{5.00\times10^{-3}\text{ m}}{1.20\text{ m}}\right) = \boxed{6.28\text{ rad}}$$

(c) If $\phi = 0.333\text{ rad} = \dfrac{2\pi d\sin\theta}{\lambda}$ $$\theta = \sin^{-1}\left(\frac{\lambda\phi}{2\pi d}\right) = \sin^{-1}\left[\frac{(5.00\times10^{-7}\text{ m})(0.333\text{ rad})}{2\pi(1.20\times10^{-4}\text{ m})}\right]$$

$$\theta = \boxed{1.27\times10^{-2}\text{ deg}}$$

(d) If $d\sin\theta = \dfrac{\lambda}{4}$ $$\theta = \sin^{-1}\left(\frac{\lambda}{4d}\right) = \sin^{-1}\left[\frac{5\times10^{-7}\text{ m}}{4(1.20\times10^{-4}\text{ m})}\right]$$

$$\theta = \boxed{5.97\times10^{-2}\text{ deg}}$$

6. In a double-slit arrangement, $d = 0.150$ mm, $L = 140$ cm, $\lambda = 643$ nm, and $y = 1.80$ cm. (a) What is the path difference δ for the rays from the two slits arriving at P? (b) Express this path difference in terms of λ. (c) Does P correspond to a maximum, a minimum, or an intermediate condition?

***solution** (a) The path difference $\delta = d\sin\theta$ and when $L >> y$

$$\delta = \frac{yd}{L} = \frac{(1.80\times10^{-2}\text{ m})(1.50\times10^{-4}\text{ m})}{1.40\text{ m}} = 1.93\times10^{-6}\text{ m} = \boxed{1.93\ \mu\text{m}}$$

(b) $$\frac{\delta}{\lambda} = \frac{1.93\times10^{-6}\text{ m}}{6.43\times10^{-7}\text{ m}} = 3.00, \text{ or } \boxed{\delta = 3.00\lambda}$$

(c) Point P will be a $\boxed{\text{maximum}}$ because the path difference is an integer multiple of the wavelength.

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7. In a double slit setup let $L = 120$ cm and $d = 0.250$ cm. The slits are illuminated with coherent 600-nm light. Calculate the distance y above the central maximum for which the average intensity on the screen is 75.0% of the maximum.

Solution: $$I_{av} = I_{\max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)$$ For small θ, $$\sin\theta = \frac{y}{L}$$

and $$I_{av} = 0.750 I_{\max}$$

$$y = \frac{\lambda L}{\pi d}\cos^{-1}\sqrt{\frac{I_{av}}{I_{\max}}}$$

$$y = \frac{(6.00\times10^{-7})(1.20\text{ m})}{\pi(2.50\times10^{-3}\text{ m})}\cos^{-1}\sqrt{\frac{0.750 I_{\max}}{I_{\max}}} = \boxed{48.0\ \mu\text{m}}$$

8. A 2-slit arrangement with 60.3 μm separation between the slits is illuminated with 482.0 nm light. Assuming that a viewing screen is located 2.14 m from the slits, find the distance from the first dark fringe on one side of the central maximum to the second dark fringe on the other side.

Answer: 34.2 mm

9. The electric fields arriving at a point P from three coherent sources are described by $E_1 = E_0 \sin \omega t$, $E_2 = E_0 \sin(\omega t + \pi/4)$ and $E_3 = E_0 \sin(\omega t + \pi/2)$. Assume the resultant field is represented by $E_p = E_R \sin(\omega t + \alpha)$. The amplitude of the resultant wave at P is

Answer: $2.4E_0$.

10. Two slits are illuminated with red light (λ = 650 nm). The slits are 0.25 mm apart and the distance to the screen is 1.25 m. What fraction of the maximum intensity on the screen is the intensity measured at a distance 3.0 mm from the central maximum?

Answer. 0.94