CHAPTER 37- Interference of Light Waves

1-) A laser beam (λ=632.8 nm) is incident on two slits 0.200 mm apart. How far apart are the bright interference fringes on a screen 5.00 m away from the double slits?

$$\Delta y_{bright} = \frac{\lambda L}{d} = \frac{(632{,}8.10^{-9}).5}{2.10^{-4}} = 1{,}58cm$$

2-) If Young's double-slit experiment were performed under water,how would the observed interference pattern be affected?

Underwater, the wavelength of the light would decrease, $\lambda_{WATER} = \frac{\lambda_{AIR}}{n_{WATER}}$ Since the positions of bright and dark bands are proportional to λ, according to

$$\delta = dsin\theta_{bright} = m\lambda$$

$$\delta = dsin\theta_{dark} = \left(m + \frac{1}{2}\right)\lambda$$

the underwater fringe separations will decrease.

3-) Two radio antennas separated by 300 m as shown in Figure P37.3 simultaneously broadcast identical signals at the same wavelength. A radio in a car traveling due North receives the signals. If the car is at the position of the second maximum, what is the wavelength of the signals? (*Note:* Do not use the small-angle approximation in this problem.)

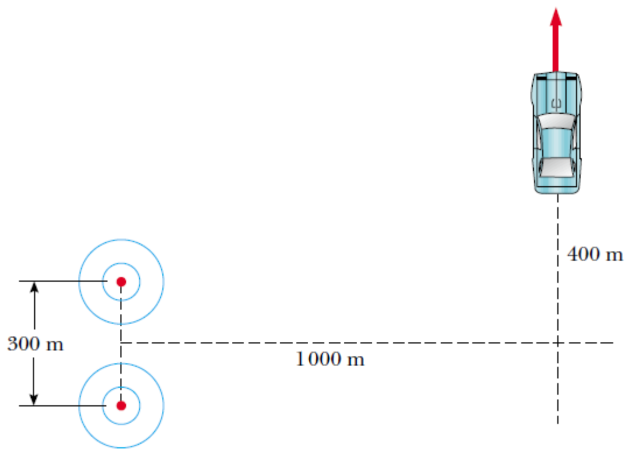

Figure P37.3

$$tan\theta = \frac{400}{1000} = 0.4$$
$$\theta = 21.8^0$$

Second maximum => m=2

$$\lambda = \frac{dsin\theta}{m} = \frac{300m.sin21.8^0}{2} = 55.7m$$

(In this problem we did not approximate sinθ to tanθ)

4-) Light with wavelength 442 nm passes through a double-slit system that has a slit separation *d* = 0.400 mm. Determine how far away a screen must be placed in order that a dark fringe appear directly opposite both slits, with just one bright fringe between them.

The dark fringes are located at

$$y_{dark} = \frac{\lambda L}{d}\left(m \pm \frac{1}{2}\right) \qquad m = \pm 1, \pm 2, \pm 3, ..$$

Taking $m = 0$ and $y = 0.200$ mm in the above equation,

$$L \approx \frac{2dy}{\lambda} = \frac{2(0{,}4.10^{-3}).(0{,}2.10^{-3})}{442.10^{-9}} = 0.362\ m = 36.2\ cm$$

Geometric optics incorrectly predicts bright regions opposite the slits and darkness in between. But, as this example shows, interference can produce just the opposite.

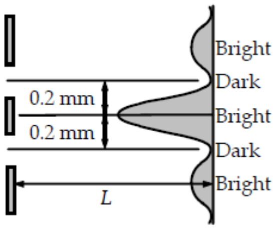

FIG. P37.7

5-) Two slits are separated by 0.320 mm. A beam of 500-nm light strikes the slits, producing an interference pattern. Determine the number of maxima observed in the angular range -30.0° < θ< 30.0°.

At 30.0° , $d \sin\theta = m\lambda$

$\left(3.20 \cdot 10^{-4}\ \text{m}\right)\sin 30.0° = m\left(500 \cdot 10^{-9}\ \text{m}\right)$

so m=320.

There are 320 maxima to the right, 320 to the left, and one for $m = 0$ straight ahead. There are 641 maxima.

6-) In a double-slit arrangement of Figure 37.5, d =0.150 mm, L= 140 cm, λ=643 nm, and y= 1.80 cm. (a) What is the path difference " for the rays from the two slits arriving at P ? (b) Express this path difference in terms of λ. (c) Does P correspond to a maximum, a minimum, or an intermediate condition?

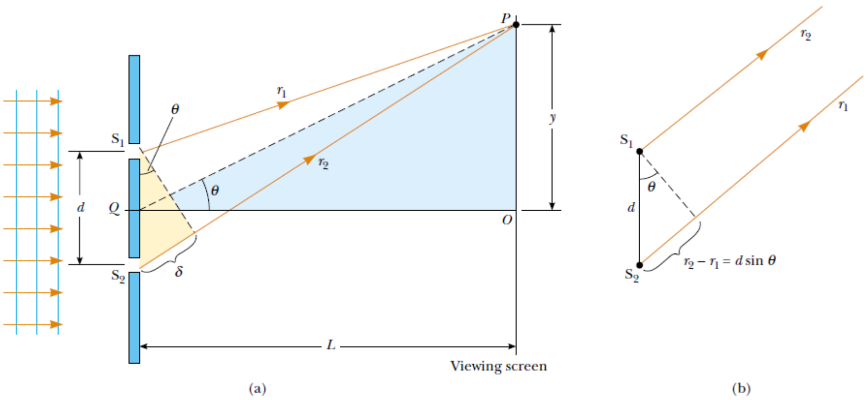

Figure 37.5 (a) Geometric construction for describing Young's double-slit experiment (not to scale). (b) When we assume that r_1 is parallel to r_2, the path difference between the two rays is $r_2 - r_1 = d \sin\theta$. For this approximation to be valid, it is essential that $L >> d$.

(a) The path difference $\delta = d \sin\theta$ and when $L >> y$

$\delta = yd/L$

$= (1{,}8.10^{-2}.1{,}5.10^{-4})/1{,}4 = 1{,}93\mu m$

(b) $\delta/\lambda = (1{,}93.10^{-6}/6{,}43.10^{-7}) = 3$

$\delta = 3\lambda$

(c) Point P will be a maximum since the path difference is an integer multiple of the wavelength.

7-) The intensity on the screen at a certain point in a doubleslit interference pattern is 64.0% of the maximum value. (a) What minimum phase difference (in radians) between sources produces this result? (b) Express this phase difference as a path difference for 486.1-nm light.

(a) $\frac{I}{I_{max}} = cos^2\left(\frac{\phi}{2}\right)$
Therefore, φ=1.29 rad.

(b) $\delta = \frac{\lambda\phi}{2\pi} = \frac{486nm.1{,}29rad}{2\pi} = 99{,}8nm$

8-) Monochromatic coherent light of amplitude E0 and angular frequency ω passes through three parallel slits each separated by a distance d from its neighbor. Show that the time-averaged intensity as a function of the angle θ is

$$I(\theta) = \left[1 + 2cos\left(\frac{2\pi dsin\theta}{\lambda}\right)\right]^2 I_{max}$$

The resultant amplitude is

$E_r = E_0$ sinωt + E_0 sin(ωt +Φ)+ E_0 (ω t + 2 Φ), where $\phi = \frac{2\pi}{\lambda}dsin\theta$
$E_r = E_0$ (sinωt+sinωt.cosΦ+cosωt.sinΦ+sinωt.cos2Φ+cosωt.sin2Φ)

Using the trigonometric relations of cos2Φ=2.$\cos^2$Φ-1, sin2Φ=2sinωt.cosΦ,

$E_r = E_0$.sinωt.(1+ cosΦ+2.$\cos^2$Φ-1)+ E_0.cosωt.(sinΦ+2sinΦcosΦ)
$E_r = E_0$ (1+2cosΦ)(sinωt.cosΦ+cosωt.sinΦ)= E_0 (1+2cosΦ)sin(ωt+Φ)

Then the intensity is

$$I \sim E_r^2 = E_0^2\ (1 + 2cos\Phi)^2\left(\frac{1}{2}\right)$$

Where the time average of $sin^2(\omega t + \Phi) = 1/2$
From one slit alone, we would get intensity $Imax \sim E_0^2\left(\frac{1}{2}\right)$ so

$$I(\theta) = \left[1 + 2cos\left(\frac{2\pi dsin\theta}{\lambda}\right)\right]^2 I_{max}$$