Appendix to “Evaluating measurement invariance in categorical data latent variable models with the EPC-interest”

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ABSTRACT

This online appendix accompanies the Political Analysis paper “Evaluating measurement invariance in categorical data latent variable models with the EPC-interest”. For replication materials, see Oberski (2015).

A. COUNTRIES INCLUDED IN THE STUDY

ISO3 code	Country name	ISO3 code	Country name
DZA	Algeria	MAR	Morocco
ARM	Armenia	NLD	Netherlands, The
AUS	Australia	NZL	New Zealand
AZE	Azerbaijan	NGA	Nigeria
BLR	Belarus	PAK	Pakistan
CHL	Chile	PER	Peru
CHN	China	PHL	Philippines
COL	Colombia	POL	Poland
CYP	Cyprus	QAT	Qatar
ECU	Ecuador	RUS	Russian Federation
EGY	Egypt	RWA	Rwanda
EST	Estonia	SGP	Singapore
DEU	Germany	SVN	Slovenia
GHA	Ghana	ESP	Spain
IRQ	Iraq	SWE	Sweden
JPN	Japan	TTO	Trinidad and Tobago
JOR	Jordan	TUN	Tunisia
KAZ	Kazakhstan	TUR	Turkey
KOR	Korea, Republic of	UKR	Ukraine
KGZ	Kyrgyzstan	USA	United States
LBN	Lebanon	URY	Uruguay
LBY	Libya	UZB	Uzbekista
MYS	Malaysia	YEM	Yemen
MEX	Mexico	ZWE	Zimbabwe

B. LATENT GOLD CHOICE INPUT FOR THE FULL INVARIANCE MODEL

The input below fits the full invariance model described in the paper, setting the possible violations of invariance to zero (0). The option "score test" in the output section (only available in Latent GOLD or Latent GOLD Choice ≥ 5) is then used to obtain the EPC-interest values. Output and data for this example can be obtained from the online appendix at `http://`.

```
options

   maxthreads=all;
   algorithm
      tolerance=1e-008 emtolerance=0.01
      emiterations=450 nriterations=70 ;
   startvalues
      seed=0 sets=30 tolerance=1e-005 iterations=50;
   bayes
      categorical=0 variances=0 latent=0 poisson=0;
   missing  excludeall;

   // NOTE: The option "scoretest" for output is used to obtain
   //        the EPC-interest. This will also produce the score test ("MI")
   //        and EPC-self for the measurement invariance restriction

   output
      parameters=effect  betaopts=wl standarderrors profile
      probmeans=posterior
      frequencies bivariateresiduals estimatedvalues=regression
      predictionstatistics setprofile setprobmeans
      iterationdetails scoretest ;

    // There are several ways of modeling ranking data using LG or LGChoice.
    //    The most computationally efficient is to use the so-called "3-file"
    //    setup in LGChoice employed here (see LGChoice manual).

choice = 3
   alternatives 'inglehart_wvs6_long.alt' quote = single
   id=alt
   choicesets 'inglehart_wvs6_long.set' quote = single
   id=set;

variables
   groupid country;
   caseid id;
   choicesetid set ;
   dependent value ranking;
   independent NY_GDP_PCAP_CD, SG_GEN_PARL_ZS;
   attribute int1 nominal, int2 nominal, int3 nominal;
   latent
      GClass  group nominal 3,
      Class nominal 3;

equations
   // Group class intercept
   GClass <- 1 ;

   // Parameters of interest are logistic regression coefficients of
```

```
//        NY_GDP_PCAP_CD and SG_GEN_PARL_ZS.
Class <- 1 + GClass + NY_GDP_PCAP_CD + SG_GEN_PARL_ZS;

// Below, sets of possible violations of measurement invariance have been
//        explicitly restricted to equal zero using "(0)". This will
//        produce EPC-interest, EPC-self, and Score test output.
value <- int1 + int2 + int3 +
   int1 Class + int2 Class + int3 Class +
   (0) int1 GClass + (0) int2 GClass + (0) int3 GClass +
   (0) int1 Class  GClass +
   (0) int2 Class GClass +
   (0) int3 Class GClass ;
```

Table 1: Log-likelihood, number of parameters, and Bayesian Information Criterion (BIC) for models with different numbers of classes for the (post)materialism (within-country) and country group (between-country) latent class variables.

(Post)materialism (X) classes, $\|\{G\}\| = 1$				Country group (G) classes, $\|\{X\}\| = 3$			
#Classes	Log-lik	#Par	BIC(L^2)	#Classes	Log-lik	#Par	BIC
1	-460512.7	9	-10346.1	1	-447646.2	29	895616.3
2	-449836.9	19	-31586.2	2	-444754.8	32	889867.0
3	-447646.2	29	-35855.8	3	-443216.6	35	886824.1
4	-446211.1	39	-38614.4	4	-442734.2	38	885892.8
5	-445246.3	49	-40432.4	5	-442436.5	41	885330.9
6	-444776.4	59	-41260.3	6	-442110.5	44	884712.3
7	-444384.0	69	-41933.4	7	-441946.2	47	884417.3

C. MODEL SELECTION FOR THE EXAMPLE APPLICATION

In choosing the number of classes for the (post)materialism (within-country) and country group (between-country) latent class variables, we follow the advice of Lukočienė, Varriale and Vermunt (2010) to first fix the number of country-group classes to unity and choose a number of within-country classes, subsequently fixing the number of within-country classes to this chosen number and determining the number of country-group (between-country) classes. The left-hand side of Table 1 shows the log-likelihoods, number of parameters and Bayesian Information Criterion (BIC) values based on the L^2 for the model with one country-group class and an increasing number of (post)materialism classes. It can be seen that the BIC, which penalizes model complexity, decreases with each additional latent (post)materialism class. In fact, the BIC does not stop decreasing even when incrementing the number of classes to 14 (not shown in Table 1 for brevity).

In the literature on (post)materialism (e.g. Inglehart and Welzel, 2010), the number of (post)materialism classes is typically fixed to three: "postmaterialist", "materialist", and "mixed". Clearly, using the WVS ranking tasks and imposing full invariance, many more qualitative (post)materialism classes can be distinguished than the traditional three classes. This corresponds to findings by Moors and Vermunt (2007); however, these authors also argued that "one can safely interpret the results (...) if adding another class does not result in important changes of the latent class weights for the other classes" (p. 637). While this is a somewhat subjective criterion, the three-class solution found in the data does correspond to the theoretical "postmaterialist", "materialist", and "mixed" classes, whose parameters appear to change little in the models with a greater number of classes. Moreover, the greatest reduction in BIC seen in Table 1 takes place when moving from a one-class to a two-class model, with relatively small improvements after three or more classes. We therefore follow the theoretical literature in selecting the three-class model.

While selecting the number of country-group classes, we find that the BIC improves little after three classes (right-hand side of Table 1), so that our initial full invariance model has three (post)materialism (within-country) and three country group (between-country) classes.

Table 2: Estimated log-utilities under the final model. In each row, the highest log-utility has been printed in **bold face** to facilitate interpretation of the classes.

		Class 1	Class 2	Class 3
	Class label	"Materialist"	"Postmater."	"Mixed"
	Class size	0.569	0.213	0.218
	(s.e.)	(0.0114)	(0.0179)	(0.0280)
Set A				
M	1. Economic growth	**2.1102**	0.4837	0.4156
M	2. Strong defense	**-0.5285**	-1.4984	-0.9249
P	3. More say	-0.5519	**1.4683**	0.4643
P	4. More beauty	-1.0298	-0.4536	**0.0449**
Set B				
M	1. Order in the nation	**1.0016**	-0.5898	0.0435
P	2. More say	-0.4592	**0.6902**	-0.2763
M	3. Rising prices	**0.4281**	-0.2269	0.3719
P	4. Freedom of speech	-0.9705	**0.1266**	-0.1390
Set C				
M	1. Stable economy	**2.0086**	0.0789	0.1715
P	2. Humane society	-0.7919	**0.4450**	-0.0943
P	3. Ideas	-1.1402	**-0.0593**	-0.4550
M	4. Fight crime	-0.0765	-0.4646	**0.3778**

D. WVS RANKING DATA MODEL ESTIMATES

Table 2 shows the sizes of the three (post)materialism classes (third row) as well as the "attribute parameters", i.e. each class's average log-utility. Thus, when reading each row horizontally, the class with the highest log-utility represents respondents who value that object highest. For example, priorities A.1, A.2, B.1, B.3, and C1 have the highest log-utilities in Class 1. Since all of these priorities are "materialist" (labeled "M" in Table 2), we also labeled Class 1 "materialist". A caveat with this label is that the materialist priorities that are most strongly related to this class also happen to be the first item in each set, so that a primacy effect may play a role here as well. Class 2 is labeled "postmaterialist" because it has the highest log-utilities for all of the postmaterialist priorities (labeled "P" in Table 2), with the exception of A.4. Preferences in the third class appear to be for the most part in-between those of Classes 1 and 2. At the same time, however, this class has the highest log-utilities for A.4 (a "postmaterialist" object) and C.4 (a "materialist" object). For this reason we apply the label "mixed" to Class 3.

E. DERIVATION OF THE EPC-INTEREST

In deriving the EPC-interest, the key concept is considering the likelihood not only as a function of the free parameters of the model, but also as a function of the parameters that are fixed to obtain the full invariance model. Collecting the free parameters in a vector $\boldsymbol{\theta}$ and the fixed parameters in a vector $\boldsymbol{\psi}$, we assume the likelihood can be written as an explicit function of both sets of parameters, $L(\boldsymbol{\theta}, \boldsymbol{\psi})$. The maximum-likelihood estimates $\hat{\boldsymbol{\theta}}$ of the free parameters can then be seen as obtained under the full invariance model that sets $\boldsymbol{\psi} = 0$, i.e. $\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} L(\boldsymbol{\theta}, \boldsymbol{\psi} = 0)$. Further, define the parameters of substantive interest as $\boldsymbol{\pi} := \mathbf{P}\boldsymbol{\theta}$, where $\mathbf{P}$ is typically a logical (0/1) selection matrix, although any linear function of the free parameters $\boldsymbol{\theta}$ may be taken. Interest then focuses on the likely value these free parameters $\boldsymbol{\pi}$ would take if the fixed $\boldsymbol{\psi}$ parameters were freed in an alternative model, $\hat{\boldsymbol{\pi}}_a = \mathbf{P} \arg\max_{\boldsymbol{\theta}, \boldsymbol{\psi}} L(\boldsymbol{\theta}, \boldsymbol{\psi})$.

We now show how these changes in the parameters of interest as a consequence of freeing the fixed parameters $\boldsymbol{\psi}$ can be estimated without fitting the alternative model. Let the Hessian $\hat{\mathbf{H}}_{\mathbf{ab}}$ be the matrix of second derivatives of the likelihood with respect to vectors $\mathbf{a}$ and $\mathbf{b}$, evaluated at the maximum likelihood solution of the full invariance model, $\hat{\mathbf{H}}_{\mathbf{ab}} := (\partial^2 L/\partial \mathbf{a} \partial \mathbf{b}')|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}$. The expected change in the parameters of interest is then measured by the EPC-interest,

$$\text{EPC-interest} = \hat{\boldsymbol{\pi}}_a - \hat{\boldsymbol{\pi}} = \mathbf{P}\hat{\mathbf{H}}_{\boldsymbol{\theta\theta}}^{-1}\hat{\mathbf{H}}_{\boldsymbol{\theta\psi}}\mathbf{D}^{-1}\left[\left.\frac{\partial L(\boldsymbol{\theta},\boldsymbol{\psi})}{\partial \boldsymbol{\psi}}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}\right] + O(\boldsymbol{\delta}'\boldsymbol{\delta}), \tag{1}$$

where $\mathbf{D} := \hat{\mathbf{H}}_{\boldsymbol{\psi\psi}} - \hat{\mathbf{H}}_{\boldsymbol{\theta\psi}}'\hat{\mathbf{H}}_{\boldsymbol{\theta\theta}}^{-1}\hat{\mathbf{H}}_{\boldsymbol{\theta\psi}}$ and the deviation from the true values is $\boldsymbol{\delta} := \boldsymbol{\vartheta} - \hat{\boldsymbol{\vartheta}}$, with $\boldsymbol{\vartheta}$ collecting the free and fixed parameters in a vector, $\boldsymbol{\vartheta} := (\boldsymbol{\theta}', \boldsymbol{\psi}')'$. Note that, apart from the order of approximation term $O(\boldsymbol{\delta}'\boldsymbol{\delta})$, Equation 1 contains only terms that can be calculated after fitting the invariance model. Thus, it is not necessary to fit the alternative model to obtain the EPC-interest.

In the structural equation modeling literature, the expected change in the fixed parameters $\boldsymbol{\psi}$ is commonly found and implemented in standard SEM software. This measure is commonly know as the "EPC", but to avoid confusion we term it "EPC-self" here. The EPC-self and EPC-interest both consider the impact of freeing restrictions, but differ in the target of this impact: the EPC-self evaluates the impact on the restriction itself, whereas the EPC-interest evaluates the impact on the parameters of interest. In spite of these differences, the two measures are related: this can be seen by recognizing that $-\mathbf{D}^{-1}\left[\left.\frac{\partial L(\boldsymbol{\theta},\boldsymbol{\psi})}{\partial \boldsymbol{\psi}}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}\right] = \text{EPC-self} \approx \boldsymbol{\psi} - \hat{\boldsymbol{\psi}}$ so that, from Equation 1,

$$\text{EPC-interest} = -\mathbf{P}\hat{\mathbf{H}}_{\boldsymbol{\theta\theta}}^{-1}\hat{\mathbf{H}}_{\boldsymbol{\theta\psi}}\,\text{EPC-self} \approx -\mathbf{P}\hat{\mathbf{H}}_{\boldsymbol{\theta\theta}}^{-1}\hat{\mathbf{H}}_{\boldsymbol{\theta\psi}}\left(\boldsymbol{\psi} - \hat{\boldsymbol{\psi}}\right) \tag{2}$$

Furthermore, since $\hat{\boldsymbol{\psi}}$ and $\hat{\boldsymbol{\theta}}$ are implicitly related by the fact that they are both solutions to the equation $\partial L/\partial \boldsymbol{\vartheta} = \mathbf{0}$, invoking the implicit function theorem yields $-\hat{\mathbf{H}}_{\boldsymbol{\theta\theta}}^{-1}\hat{\mathbf{H}}_{\boldsymbol{\theta\psi}} = \partial\boldsymbol{\theta}/\partial\boldsymbol{\psi}'$, so that

$$\text{EPC-interest} = \mathbf{P}\left(\frac{\partial \boldsymbol{\theta}}{\partial \boldsymbol{\psi}'}\right)\left(\boldsymbol{\psi} - \hat{\boldsymbol{\psi}}\right), \tag{3}$$

that is, the EPC-interest can be seen simply as the coefficient of a linear approximation to the relationship between the free and fixed parameters, multiplied by the change in the fixed parameters. This demonstrates the difference with the sensitivity analysis approach common in econometrics (Magnus and Vasnev, 2007, p. 168) in which only $\partial\boldsymbol{\theta}/\partial\boldsymbol{\psi}'$ is considered: the EPC-interest combines both the direction ($\partial\boldsymbol{\theta}/\partial\boldsymbol{\psi}'$) and the magnitude ($\boldsymbol{\psi} - \hat{\boldsymbol{\psi}}$) of the misspecification.

The derivation of the EPC-interest given in Equation 1 starts from the full invariance solution. We then find a hypothetical new maximum of the likelihood by setting the gradient of a Taylor expansion of the likelihood around the full invariance solution to zero:

$$\frac{\partial L(\boldsymbol{\theta},\boldsymbol{\psi})}{\partial \boldsymbol{\vartheta}} = \mathbf{0} = \begin{pmatrix} \frac{\partial L(\boldsymbol{\theta},\boldsymbol{\psi})}{\partial \boldsymbol{\theta}}|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} \\ \frac{\partial L(\boldsymbol{\theta},\boldsymbol{\psi})}{\partial \boldsymbol{\psi}}|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} \end{pmatrix} + \begin{pmatrix} \hat{\mathbf{H}}_{\boldsymbol{\theta\theta}} & \hat{\mathbf{H}}_{\boldsymbol{\psi\theta}} \\ \hat{\mathbf{H}}_{\boldsymbol{\psi\theta}} & \hat{\mathbf{H}}_{\boldsymbol{\psi\psi}} \end{pmatrix}\begin{pmatrix} \boldsymbol{\theta} - \hat{\boldsymbol{\theta}} \\ \boldsymbol{\psi} - \hat{\boldsymbol{\psi}} \end{pmatrix} + O(\boldsymbol{\delta}'\boldsymbol{\delta}). \tag{4}$$

A similar device was used to derive the so-called "modification index" or "score test" for the significance of the hypothesis $\boldsymbol{\psi} = \mathbf{0}$ by Sörbom (1989, p. 373). Equation 1 follows directly by noting that $(\partial L(\boldsymbol{\theta},\boldsymbol{\psi})/\partial\boldsymbol{\theta})|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} = \mathbf{0}$ and applying the standard linear algebra result on the inverse of a partitioned matrix $\left(\hat{\mathbf{H}}^{-1}\right)_{\boldsymbol{\theta\psi}} = -\hat{\mathbf{H}}_{\boldsymbol{\theta\theta}}^{-1}\hat{\mathbf{H}}_{\boldsymbol{\theta\psi}}\mathbf{D}^{-1}$ (e.g. Magnus and Neudecker, 2007, p. 12).

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