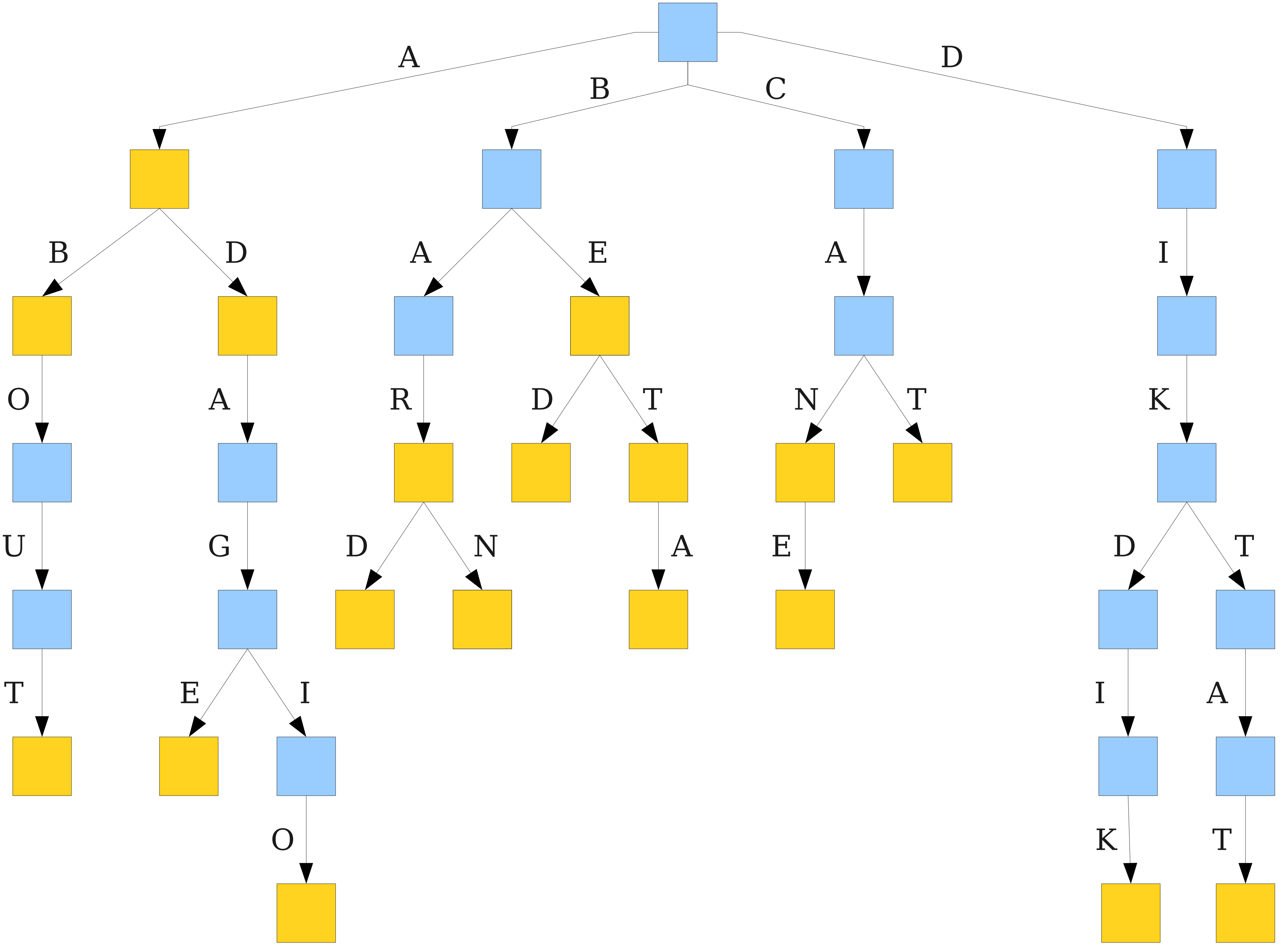
Suffix Trees

Outline for Today

- **Review from Last Time**
  - A quick refresher on tries.
- **Suffix Tries**
  - A simple data structure for string searching.
- **Suffix Trees**
  - A compact, powerful, and flexible data structure for string algorithms.
- **Generalized Suffix Trees**
  - An even more flexible data structure.

Review from Last Time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Tries

- A **trie** is a tree that stores a collection of strings over some alphabet Σ.
- Each node corresponds to a prefix of some string in the set.
  - Tries are sometimes called "prefix trees."
- If $|\Sigma| = O(1)$, all insertions, deletions, and lookups take time $O(|w|)$, where w is the string in question.
- Can also determine whether a string w is a prefix of some string in the trie in time $O(|w|)$ by walking the trie and returning whether we didn't fall off.

Aho-Corasick String Matching

- The **Aho-Corasick string matching algorithm** is an algorithm for finding all occurrences of a set of strings $P_1, \ldots, P_k$ inside a string T.
- Runtime is $\mathrm{O}(m + n + z)$, where
  - $m = |T|$,
  - $n = |P_1| + \ldots + |P_k|$
  - z is the number of matches.

Aho-Corasick String Matching

- The runtime of Aho-Corasick can be split apart into two pieces:
  - O(n) preprocessing time to build the matcher, and
  - O(m + z) time to find all matches.
- Useful in the case where the *patterns* are fixed, but the text might change.

Genomics Databases

- Many string algorithms these days pertain to computational genomics.
- Typically, have a *huge* database with many very large strings.
- More common problem: given a fixed string T to search and changing patterns $P_1, \ldots, P_k$, find all matches in T.
- **Question:** Can we instead preprocess T to make it easy to search for variable patterns?

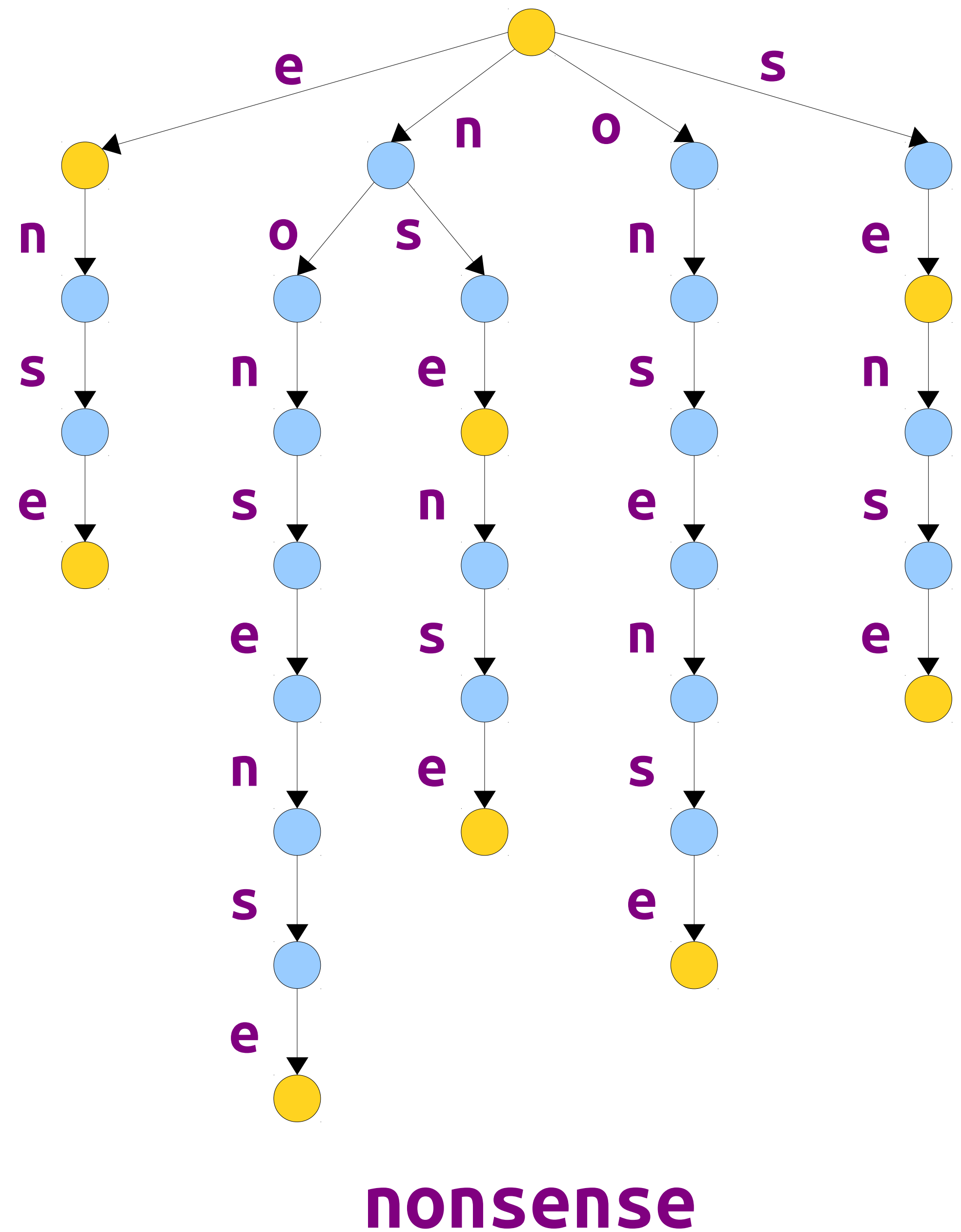
Suffix Tries

Substrings, Prefixes, and Suffixes

- **Recall:** If x is a substring of w, then x is a suffix of a prefix of w.
  - Write $w = \alpha x \omega$; then x is a suffix of αx.
- **Fact:** If x is a substring of w, then x is a prefix of a suffix of w.
  - Write $w = \alpha x \omega$; then x is a prefix of $x\omega$
- This second fact is of use because tries support efficient prefix searching.

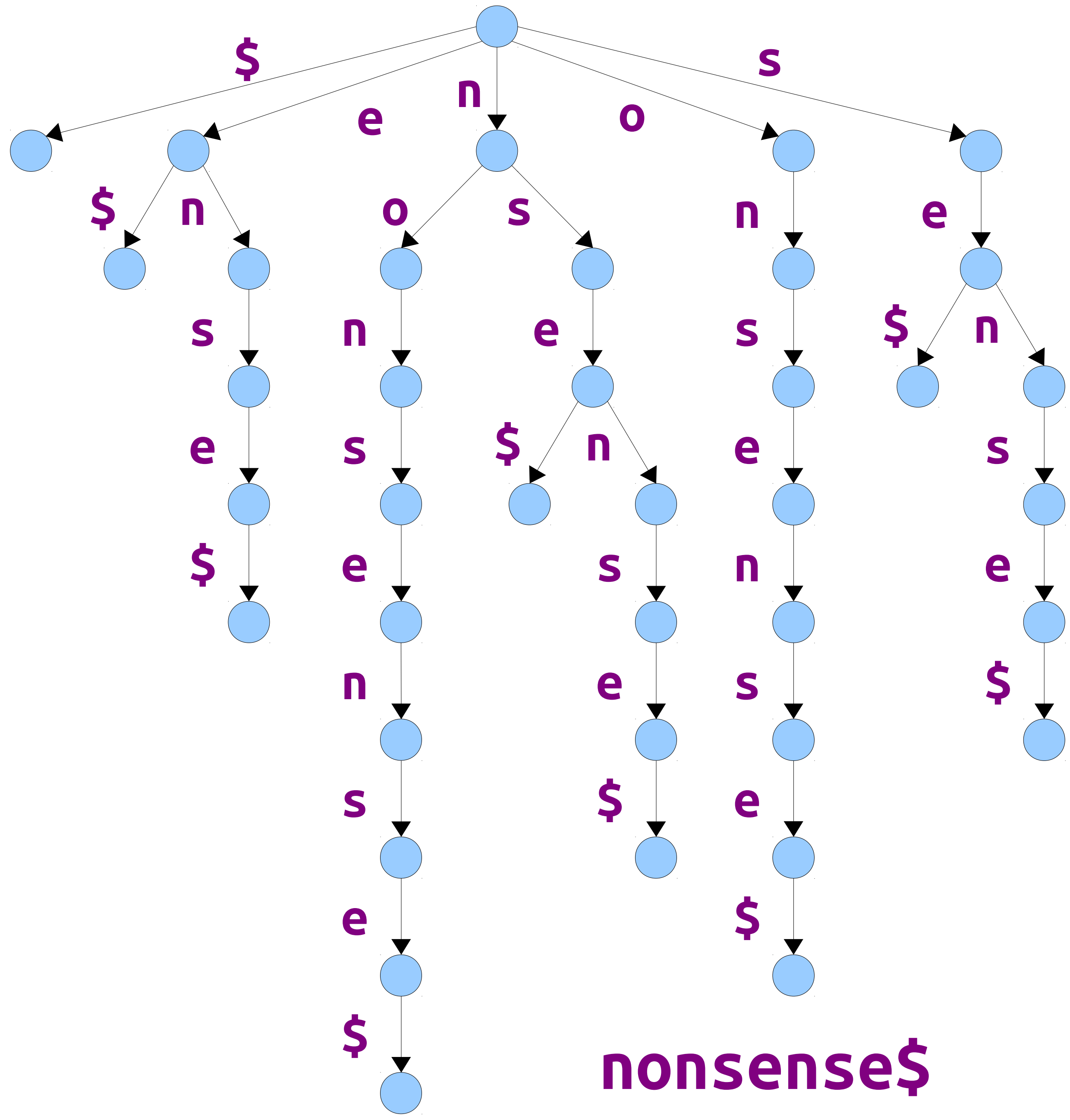
Suffix Tries

- A **suffix trie** of T is a trie of all the suffices of T.
- In time $O(n)$, can determine whether $P_1, \ldots, P_k$ exist in T by searching for each one in the trie.



A Typical Transform

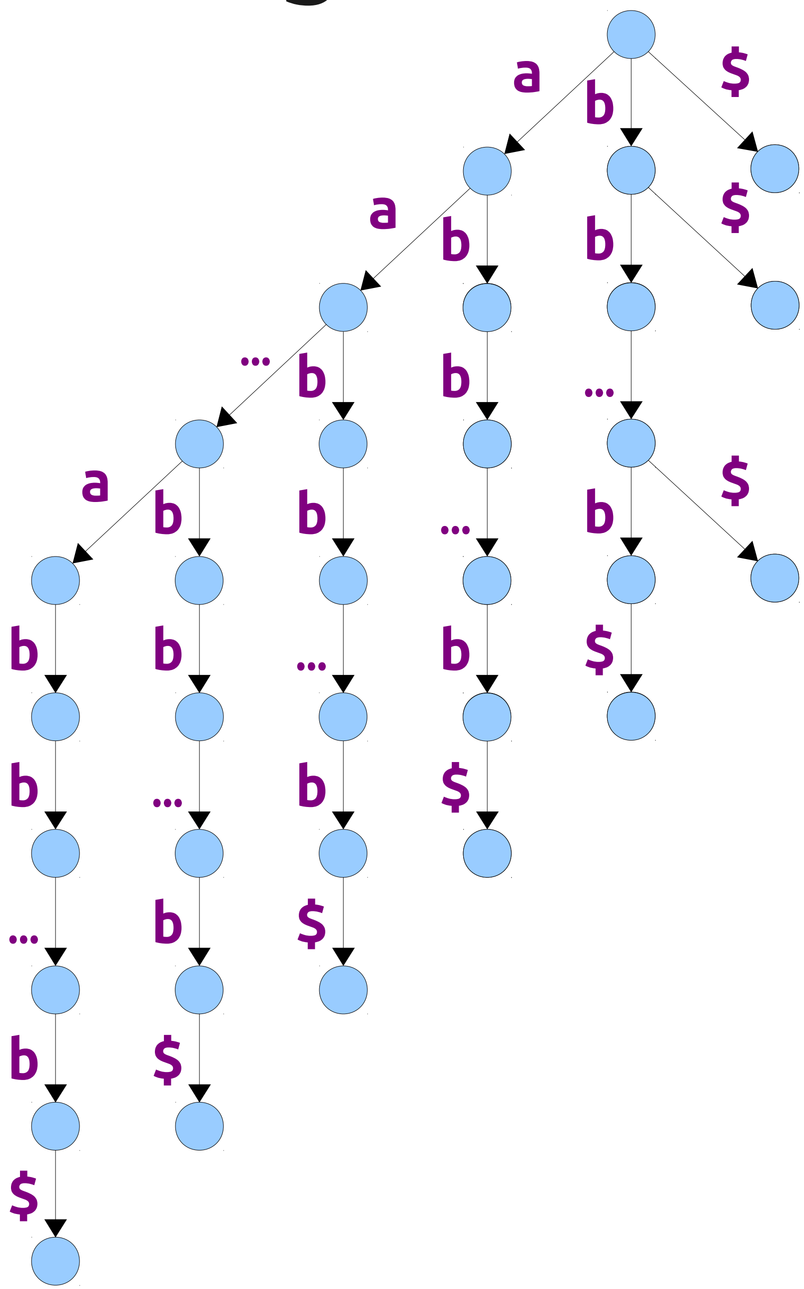
- Typically, we append some new character $\$ \notin \Sigma$ to the end of T, then construct the trie for $T\$$.
- Leaf nodes correspond to suffixes.
- Internal nodes correspond to prefixes of those suffixes.



Constructing Suffix Tries

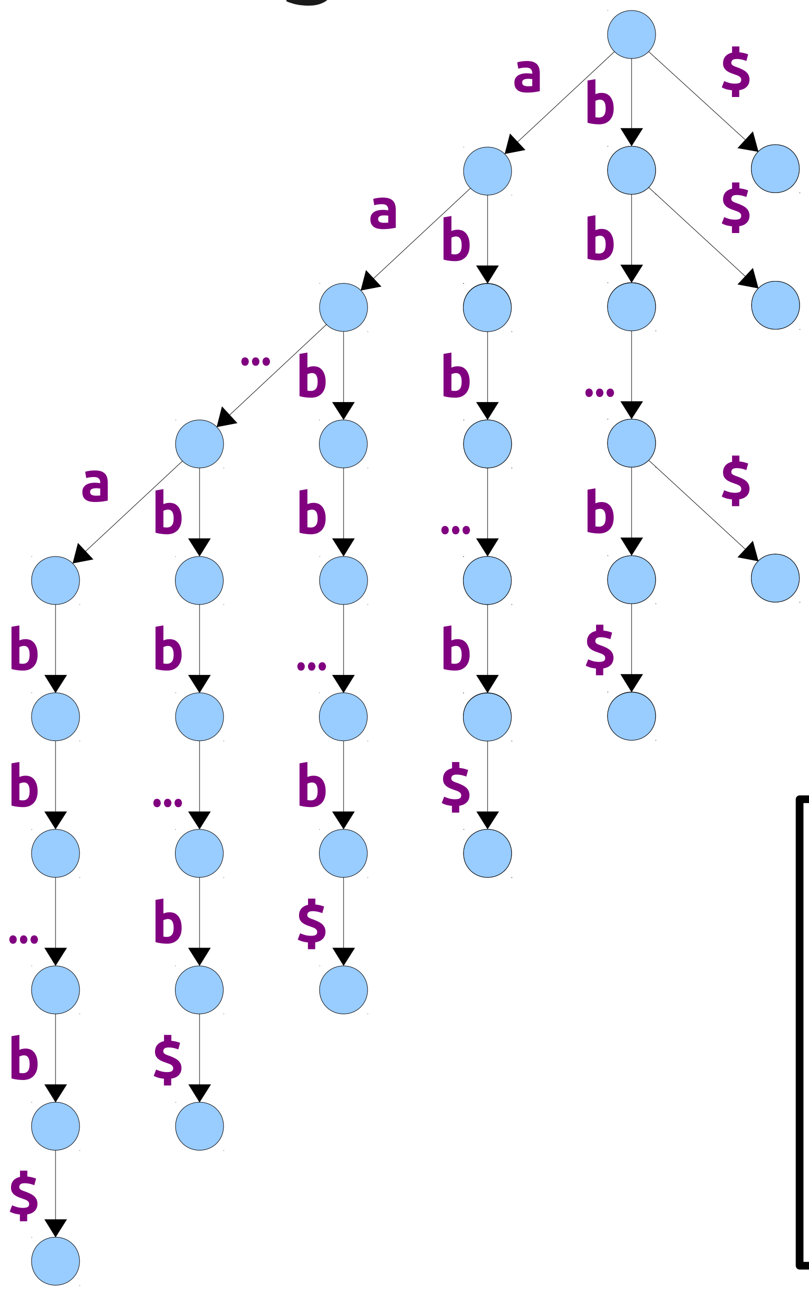
- Once we build a single suffix trie for string *T*, we can efficiently detect whether patterns match in time O(n).
- **Question:** How long does it take to construct a suffix trie?
- **Problem:** There's an $\Omega(m^2)$ lower bound on the worst-case complexity of *any* algorithm for building suffix tries.

A Degenerate Case



$a^m b^m \$$

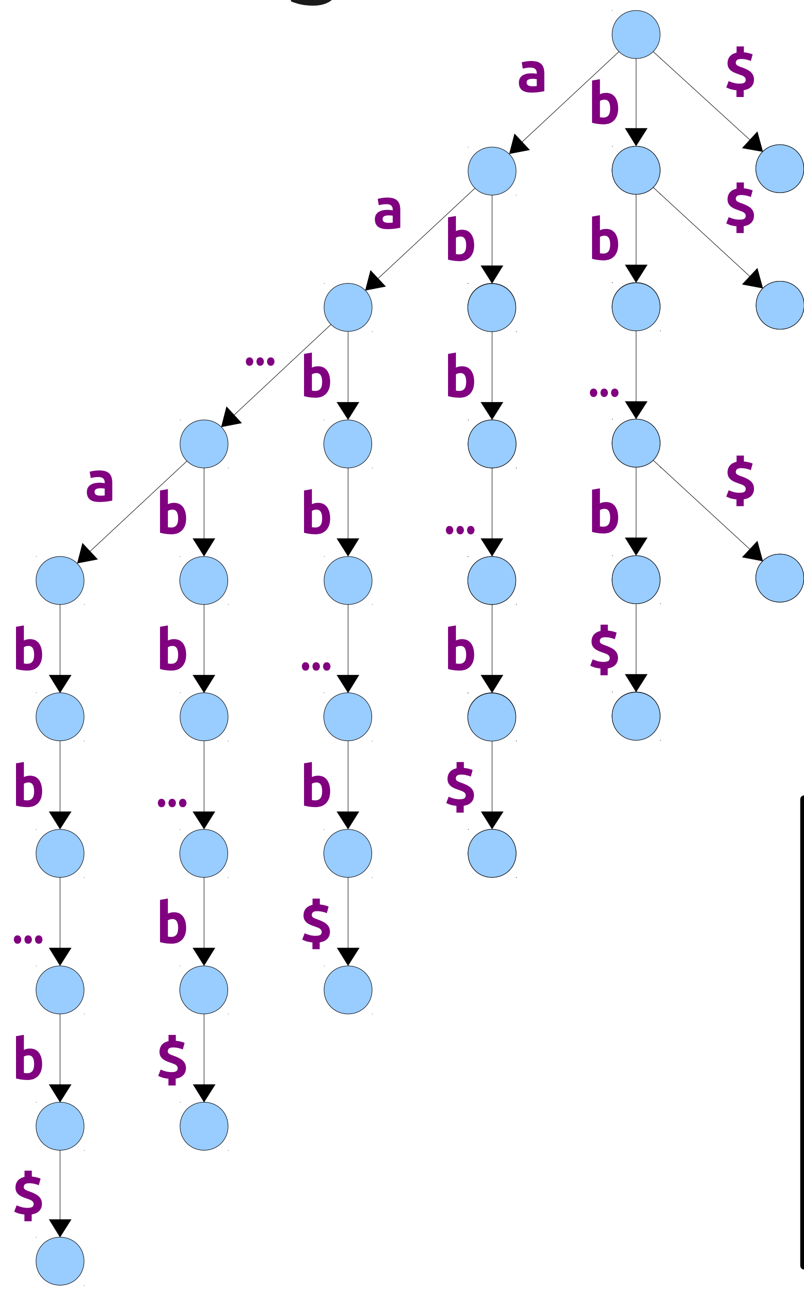
A Degenerate Case



a^m**b**^{m}**$**

There are $\Theta(m)$ copies of nodes chained together as **b**^{m}**$**.

A Degenerate Case



$a^m b^m \$$

There are $\Theta(m)$ copies of nodes chained together as $b^m\$$.

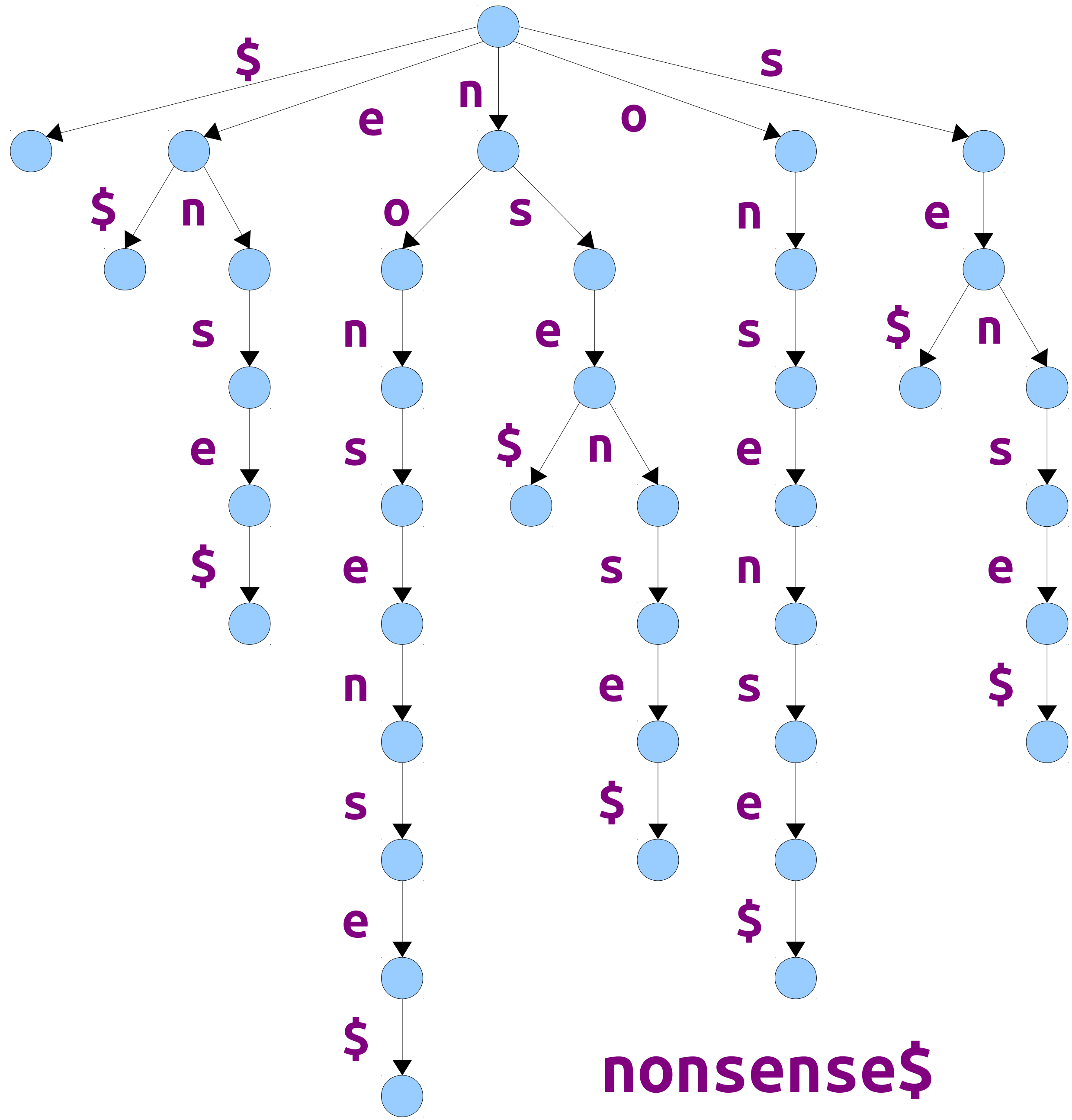
Space usage: $\Omega(m^2)$.

Correcting the Problem

- Because suffix tries may have $\Omega(m^2)$ nodes, all suffix trie algorithms must run in time $\Omega(m^2)$ in the worst-case.
- Can we reduce the number of nodes in the trie?

Patricia Tries

- A “silly” node in a trie is a node that has exactly one child.
- A **Patricia trie** (or **radix trie**) is a trie where all “silly” nodes are merged with their parents.



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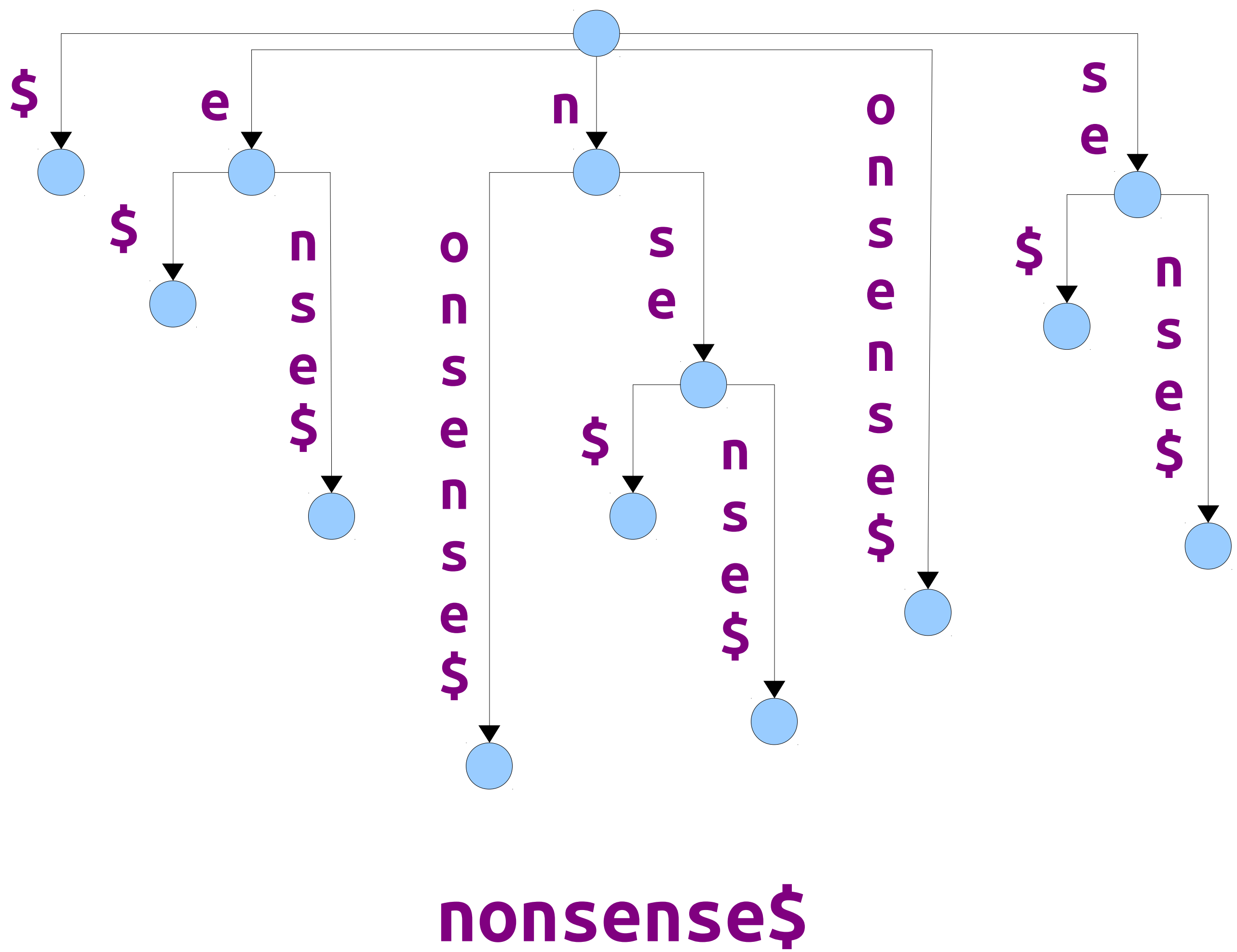
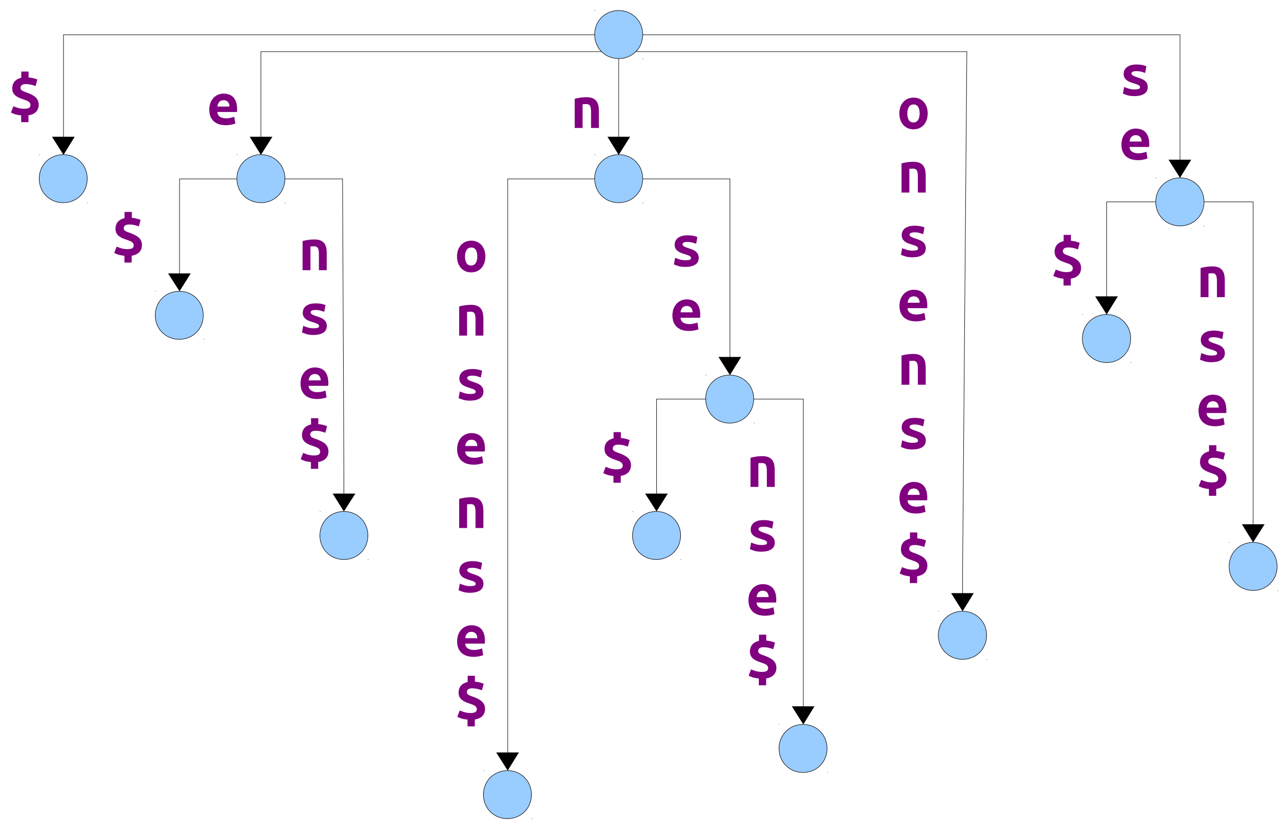


Suffix Trees

- A **suffix tree** for a string T is an Patricia trie of T\$ where each leaf is labeled with the index where the corresponding suffix starts in T\$.



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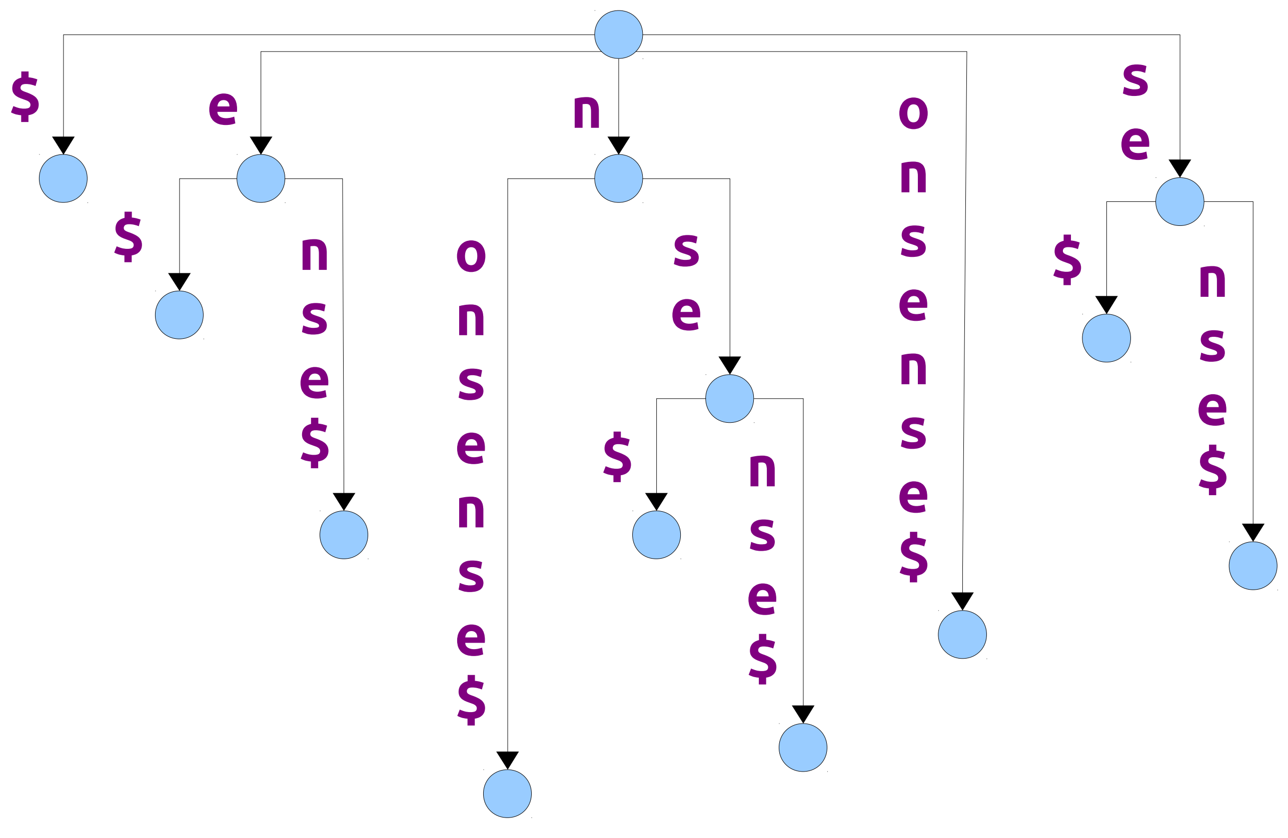

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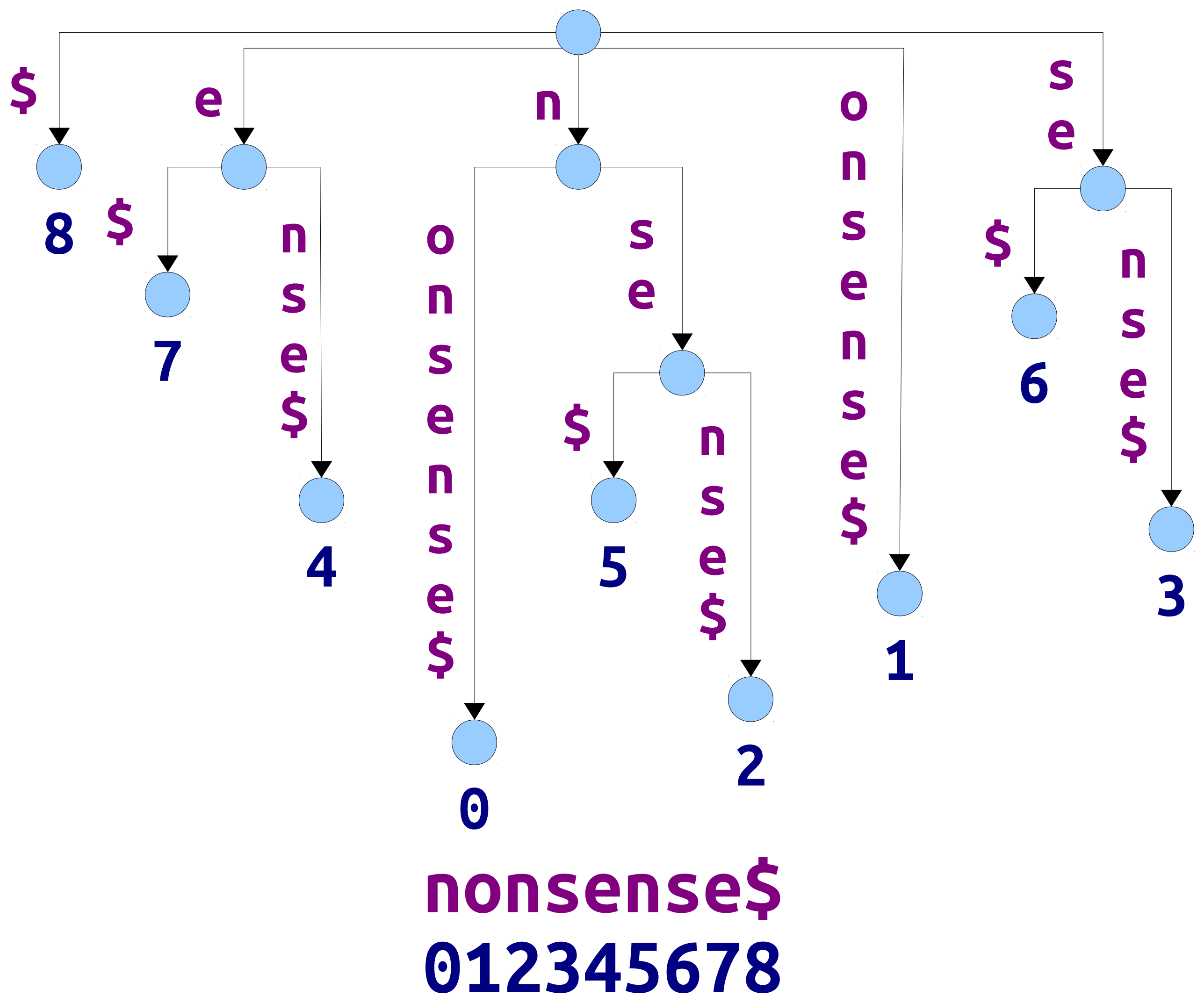
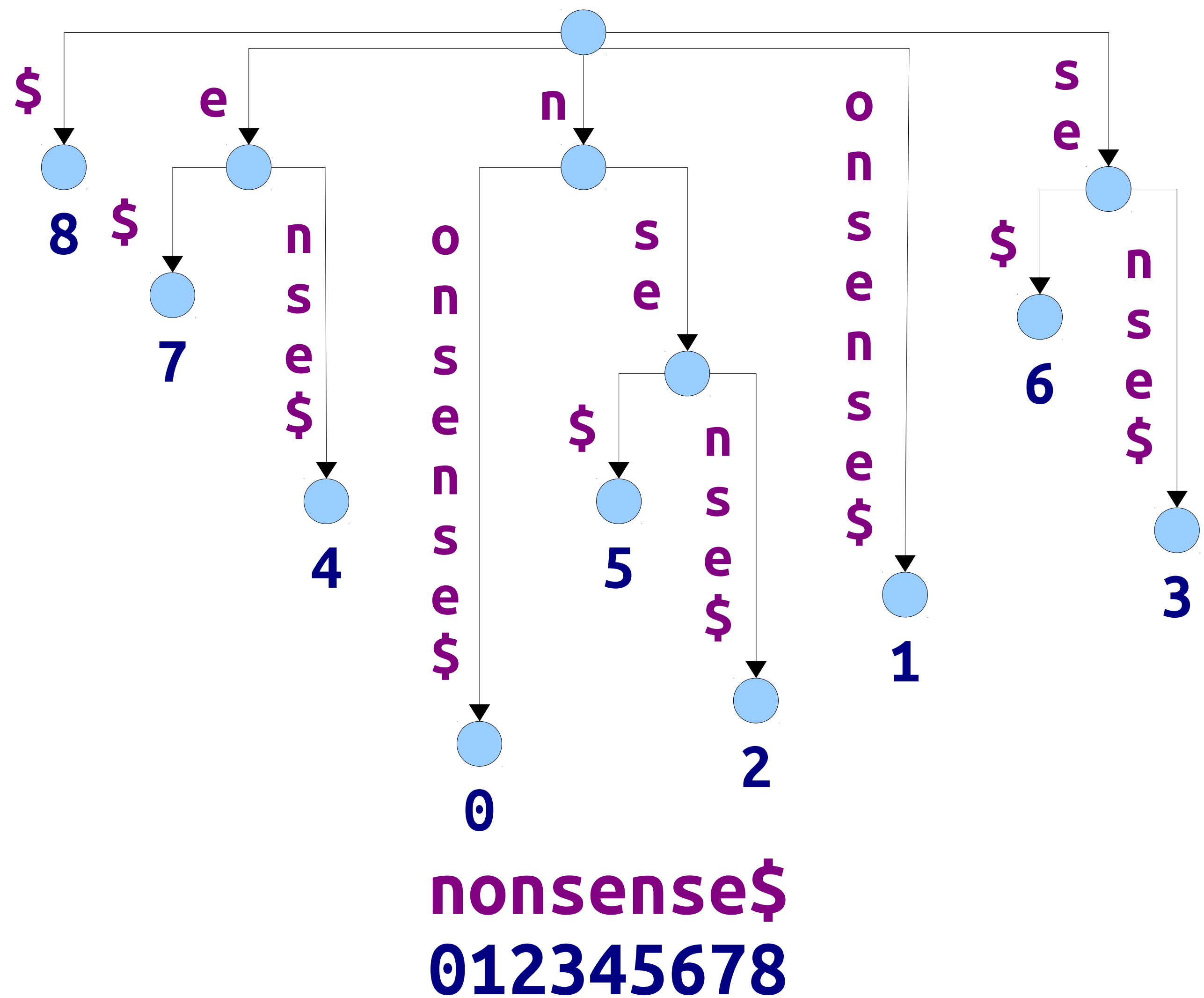


Properties of Suffix Trees

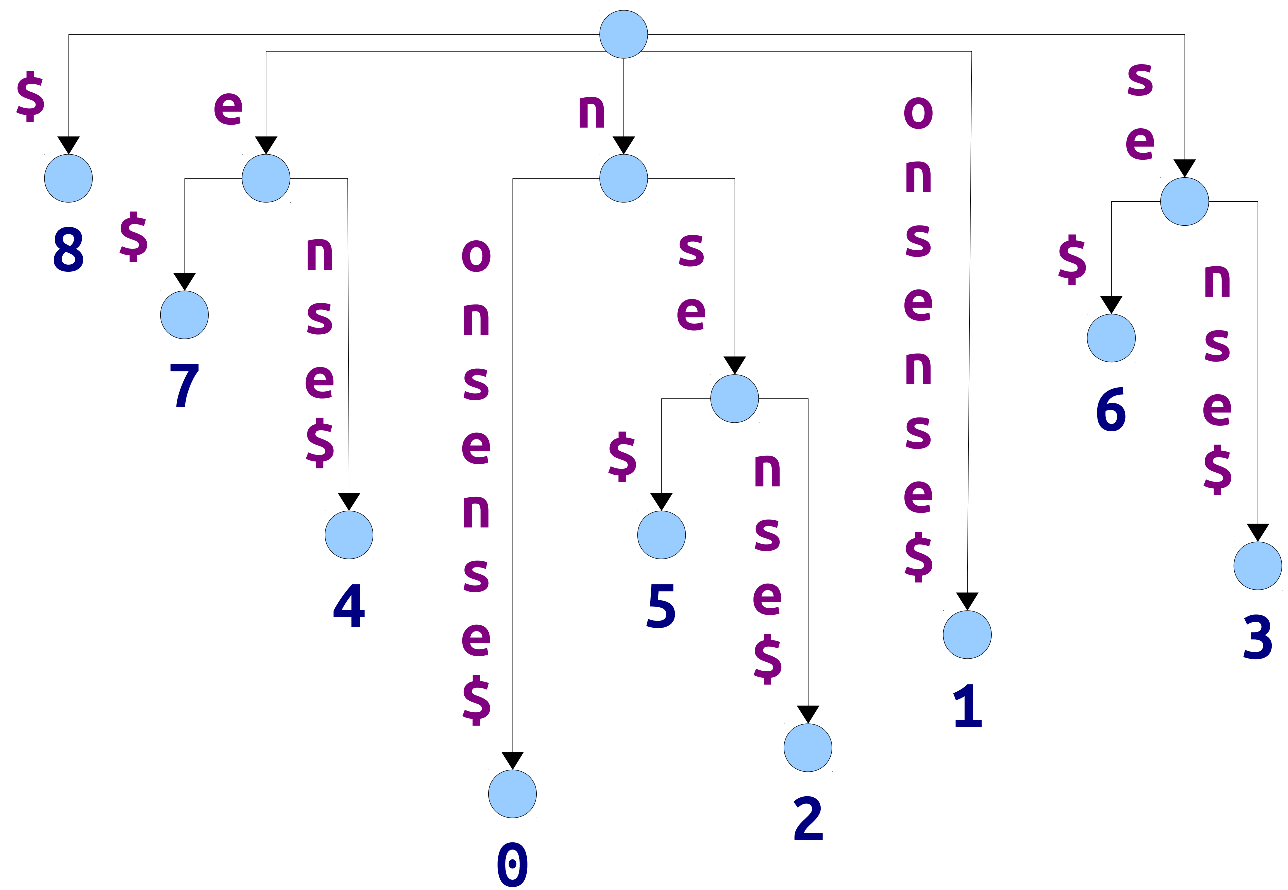
- If $|T| = m$, the suffix tree has exactly $m + 1$ leaf nodes.
- For any $T \neq \varepsilon$, all internal nodes in the suffix tree have at least two children.
- Number of nodes in a suffix tree is $\Theta(m)$.



Suffix Tree Representations

- Suffix trees may have $\Theta(m)$ nodes, but the labels on the edges can have size $\omega(1)$.
- This means that a naïve representation of a suffix tree may take $\omega(m)$ space.
- **Useful fact:** Each edge in a suffix tree is labeled with a consecutive range of characters from w.
- **Trick:** Represent each edge label α as a pair of integers [start, end] representing where in the string α appears.

Suffix Tree Representations



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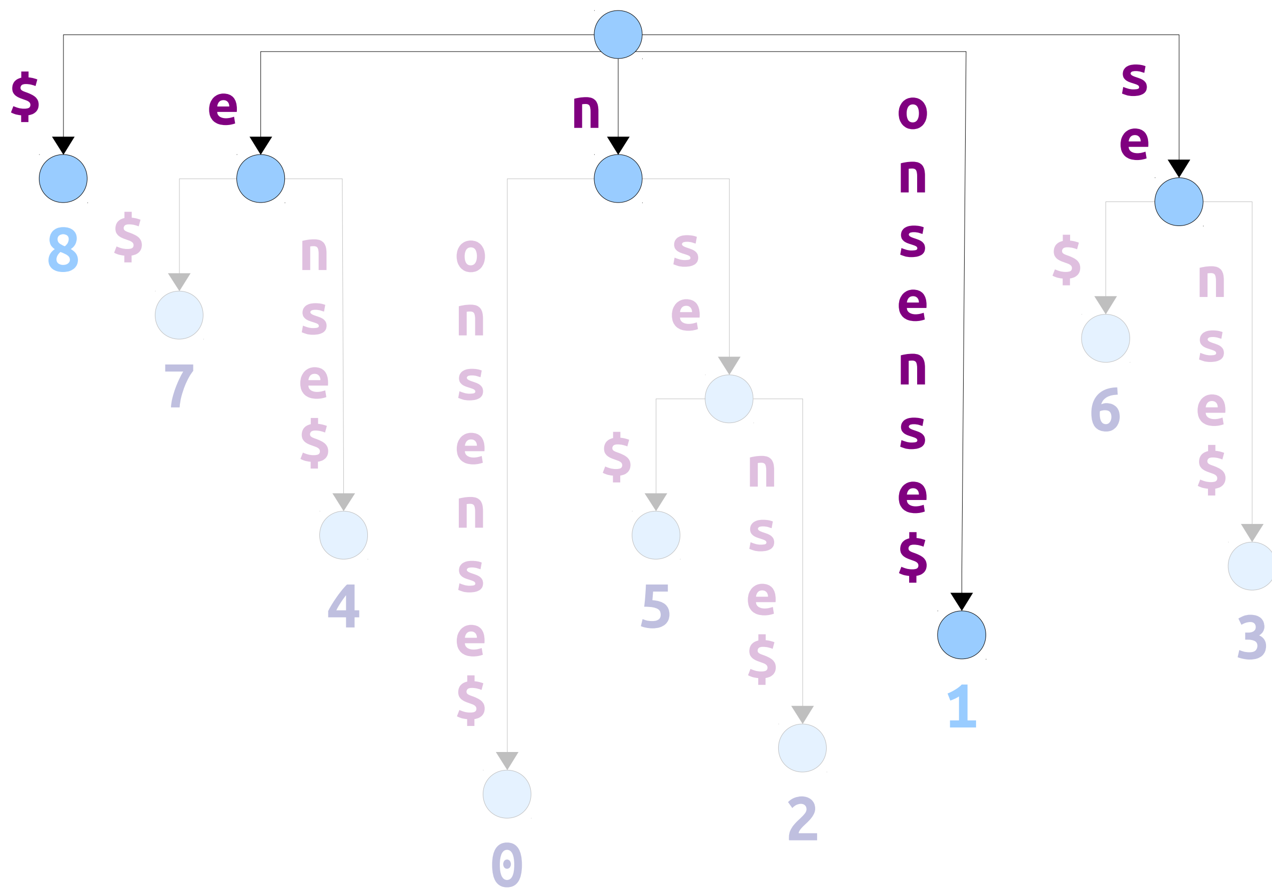
Suffix Tree Representations



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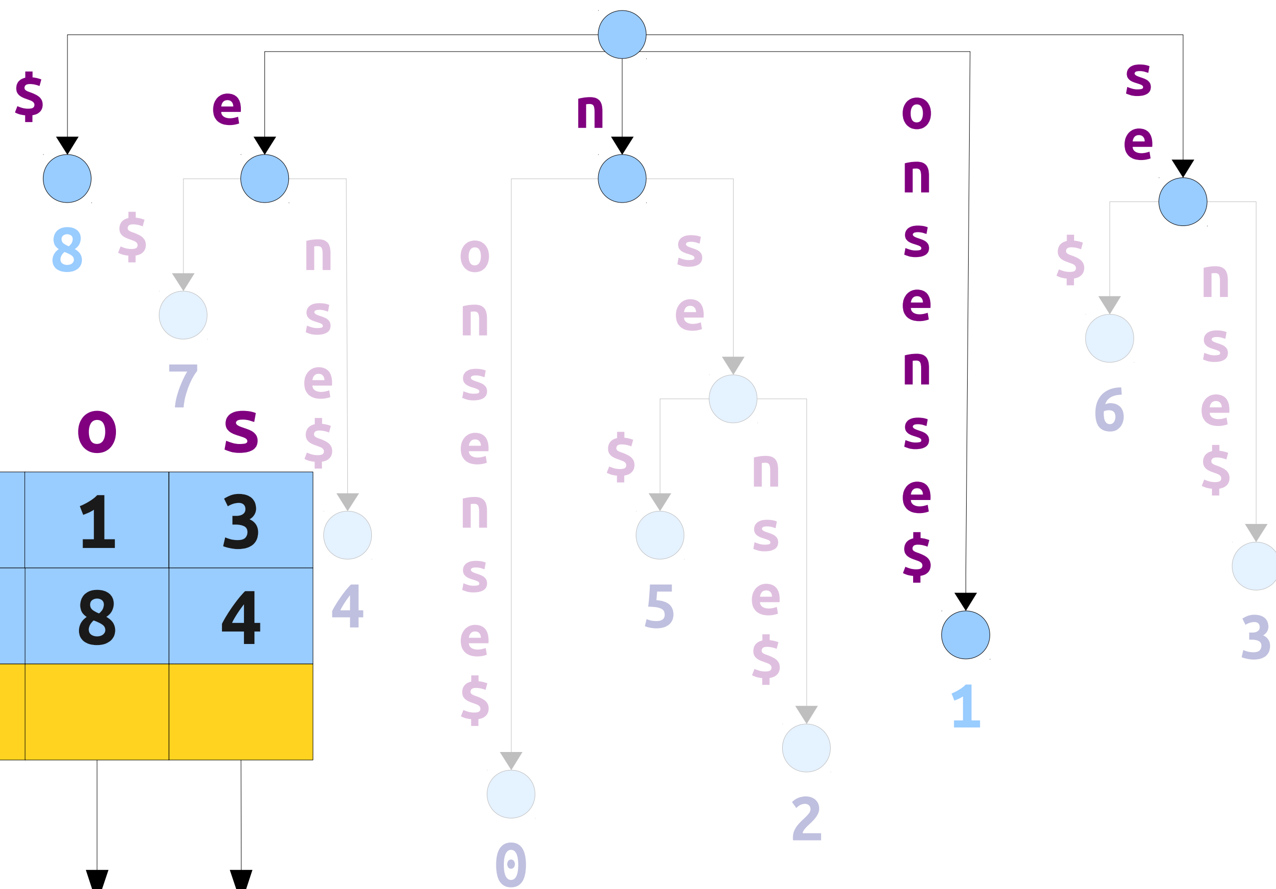
Suffix Tree Representations

	$	e	n	o	s
start	8	4	0	1	3
end	8	4	0	8	4
child					



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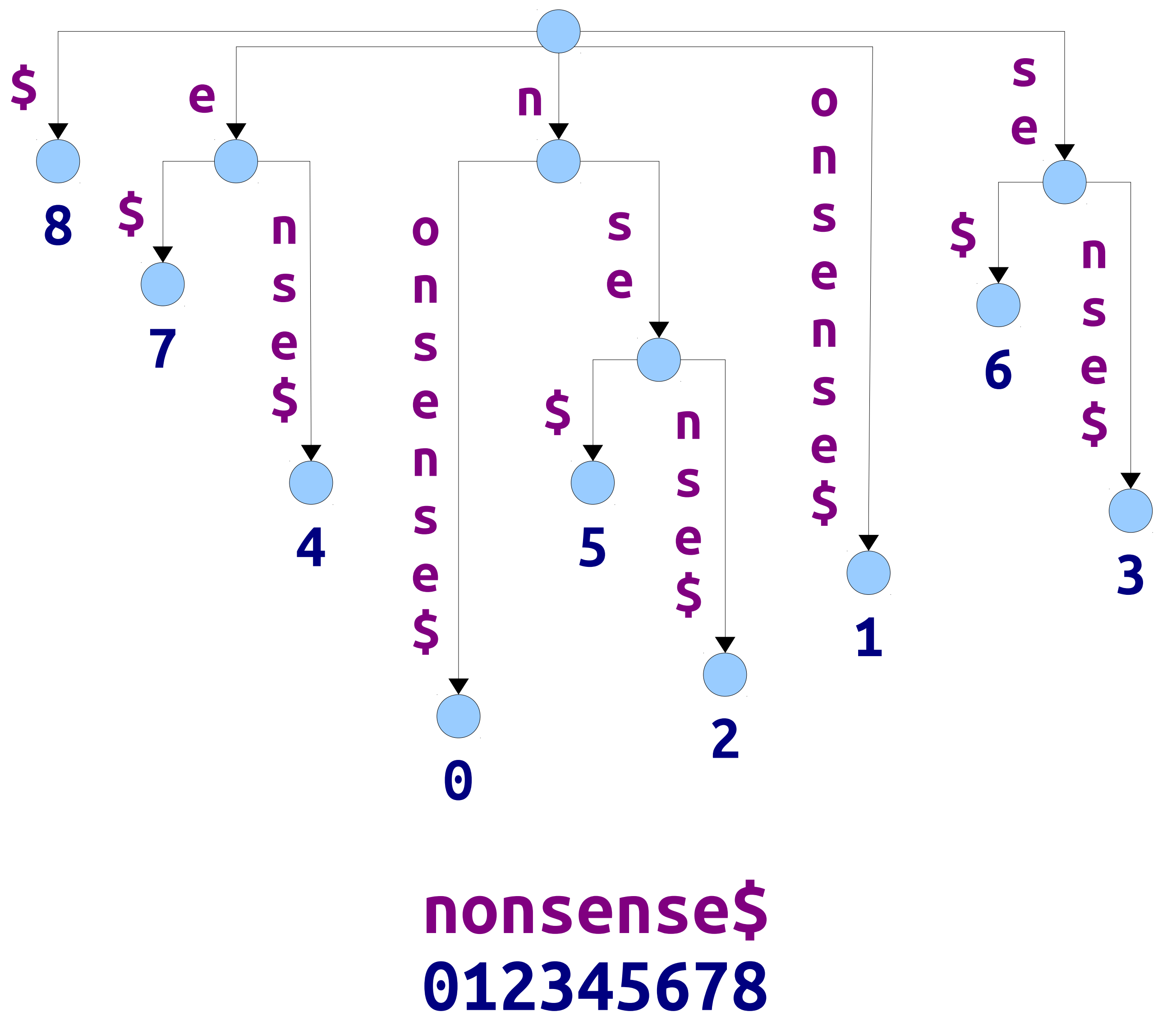
Building Suffix Trees

- Using this representation, suffix trees can be constructed using space $\Theta(m)$.
- **Claim:** There are $\Theta(m)$-time algorithms for building suffix trees.
- *These algorithms are not trivial*. We'll discuss one of them next time.

An Application: String Matching

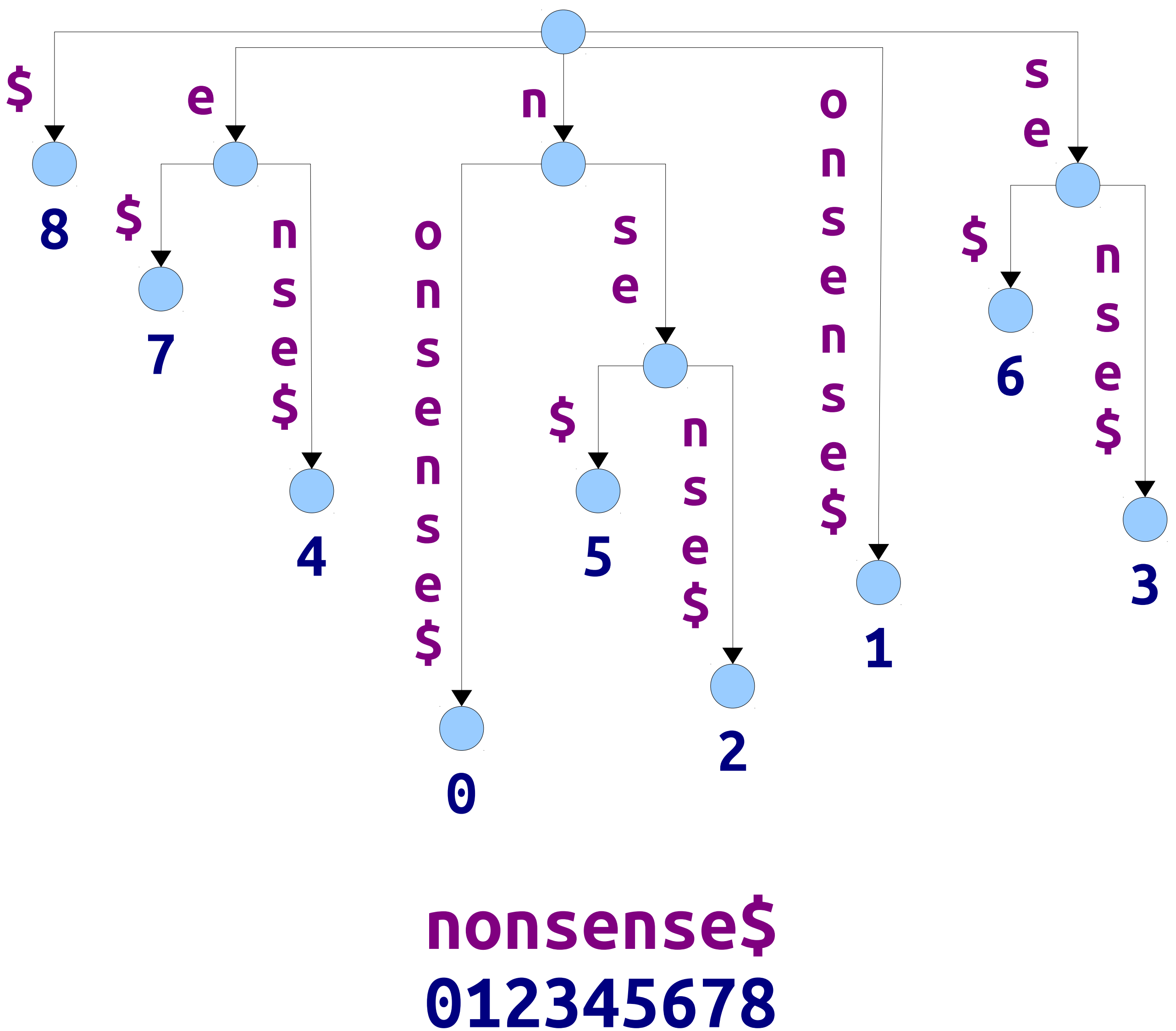
String Matching

- Given a suffix tree, can search to see if a pattern P exists in time $O(n)$.
- Gives an $O(m + n)$ string-matching algorithm.
- T can be preprocessed in time $O(m)$ to efficiently support binary string matching queries.



String Matching

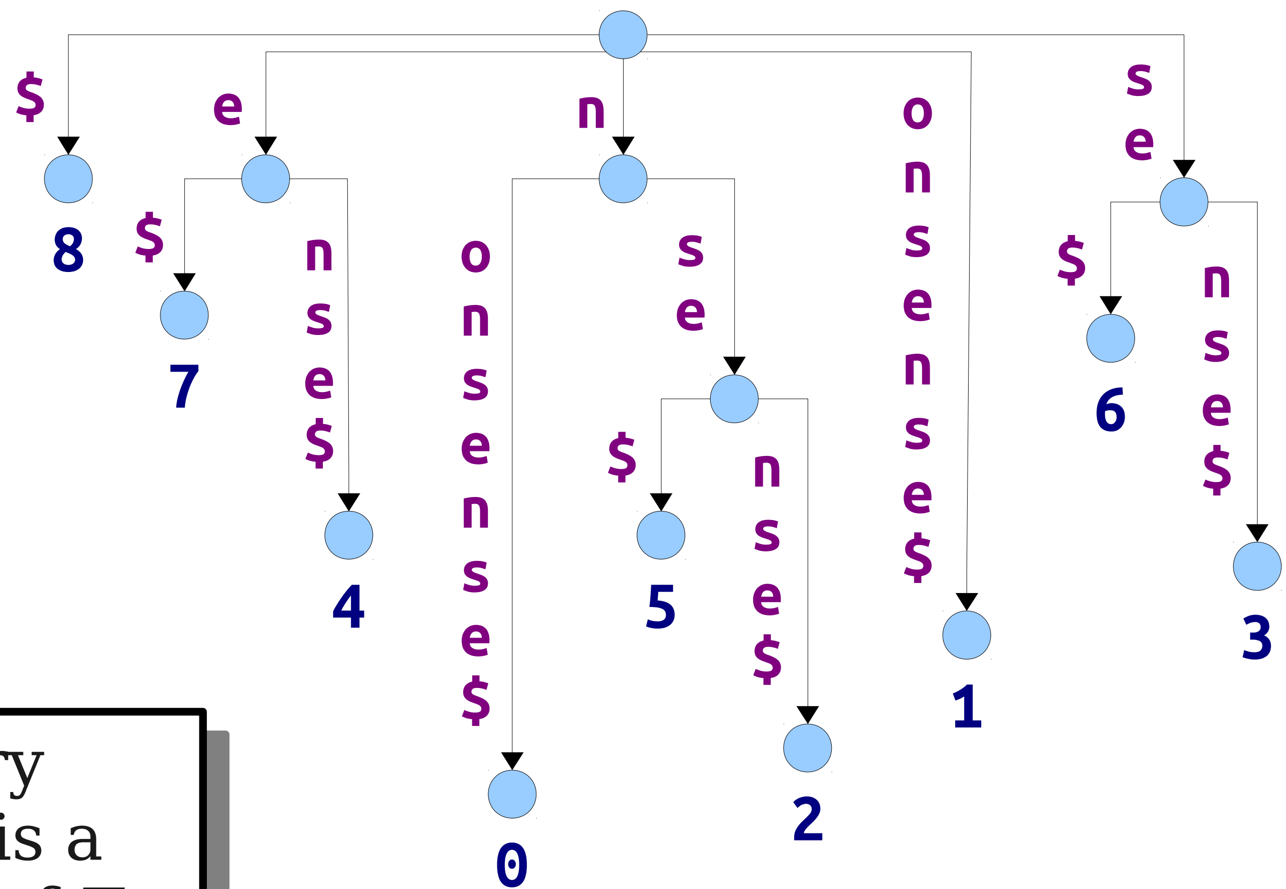
- **Claim:** After spending $O(m)$ time preprocessing $T\$$, can find all matches of a string P in time $O(n + z)$, where z is the number of matches.



String Matching

- **Claim:** After spending O(m) time preprocessing T\$, can find all matches of a string P in time O($n + z$), where z is the number of matches.

> **Observation 1:** Every occurrence of P in T is a prefix of some suffix of T.



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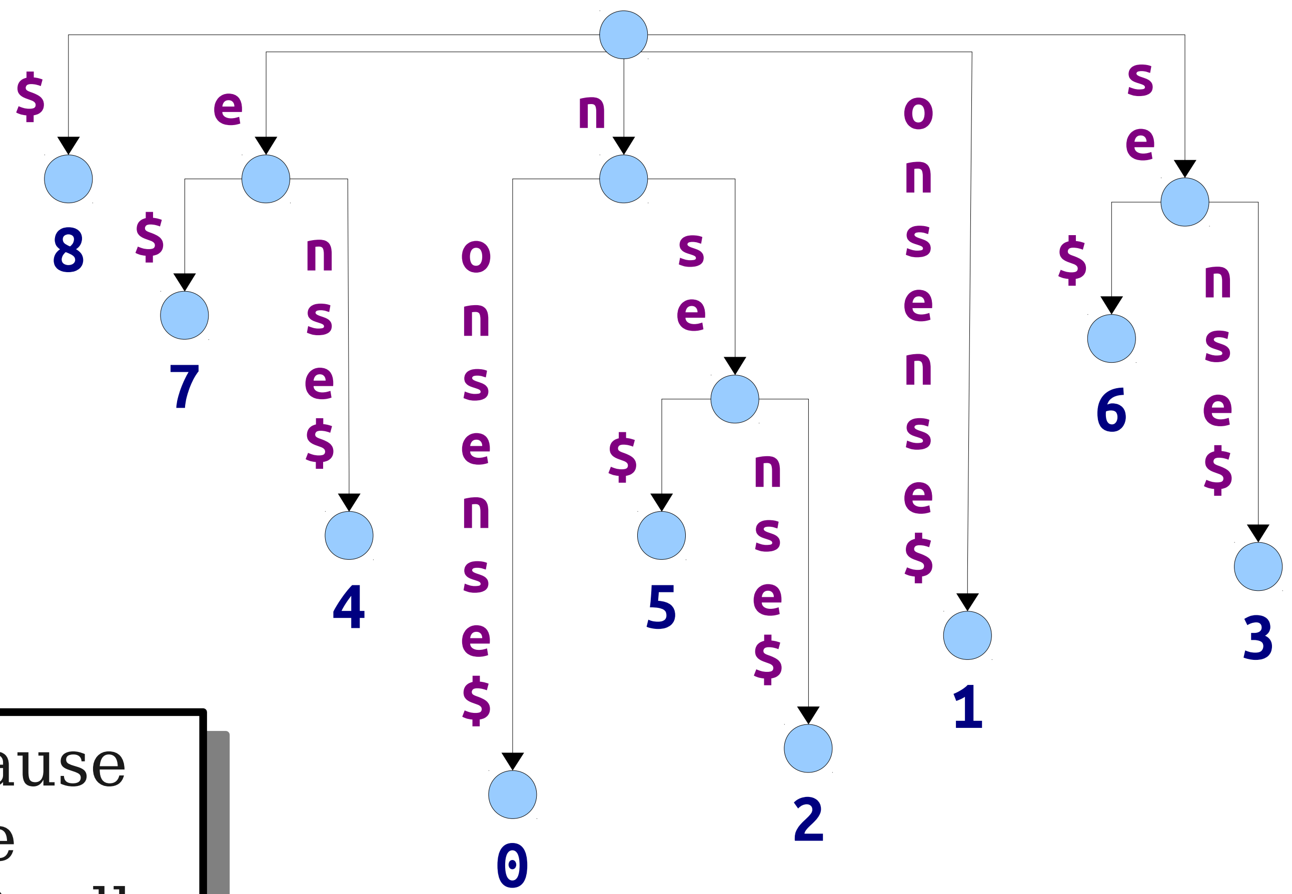

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> **Observation 2:** Because the prefix is the same each time (namely, P), all those suffixes will be in the same subtree.

String Matching

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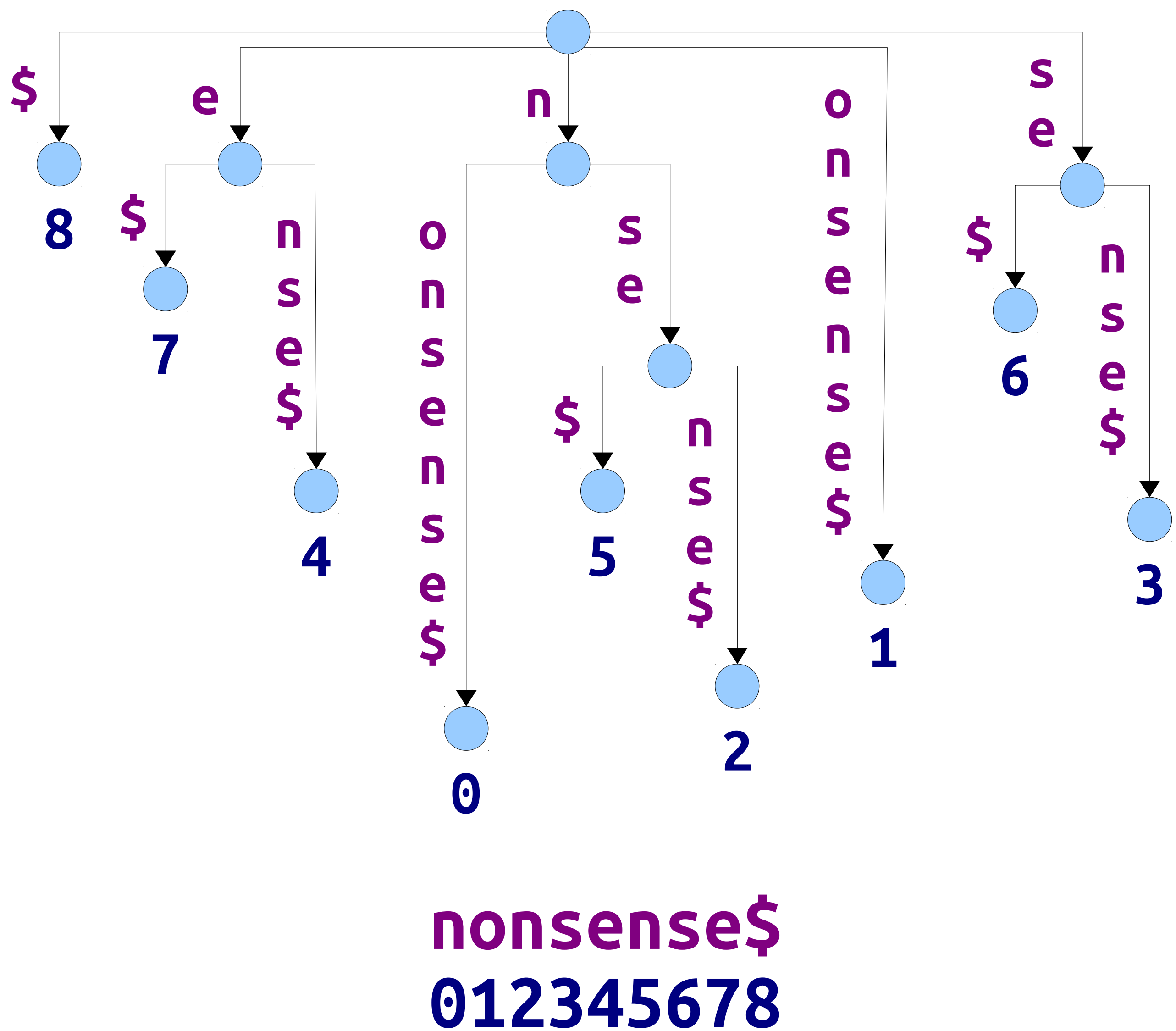

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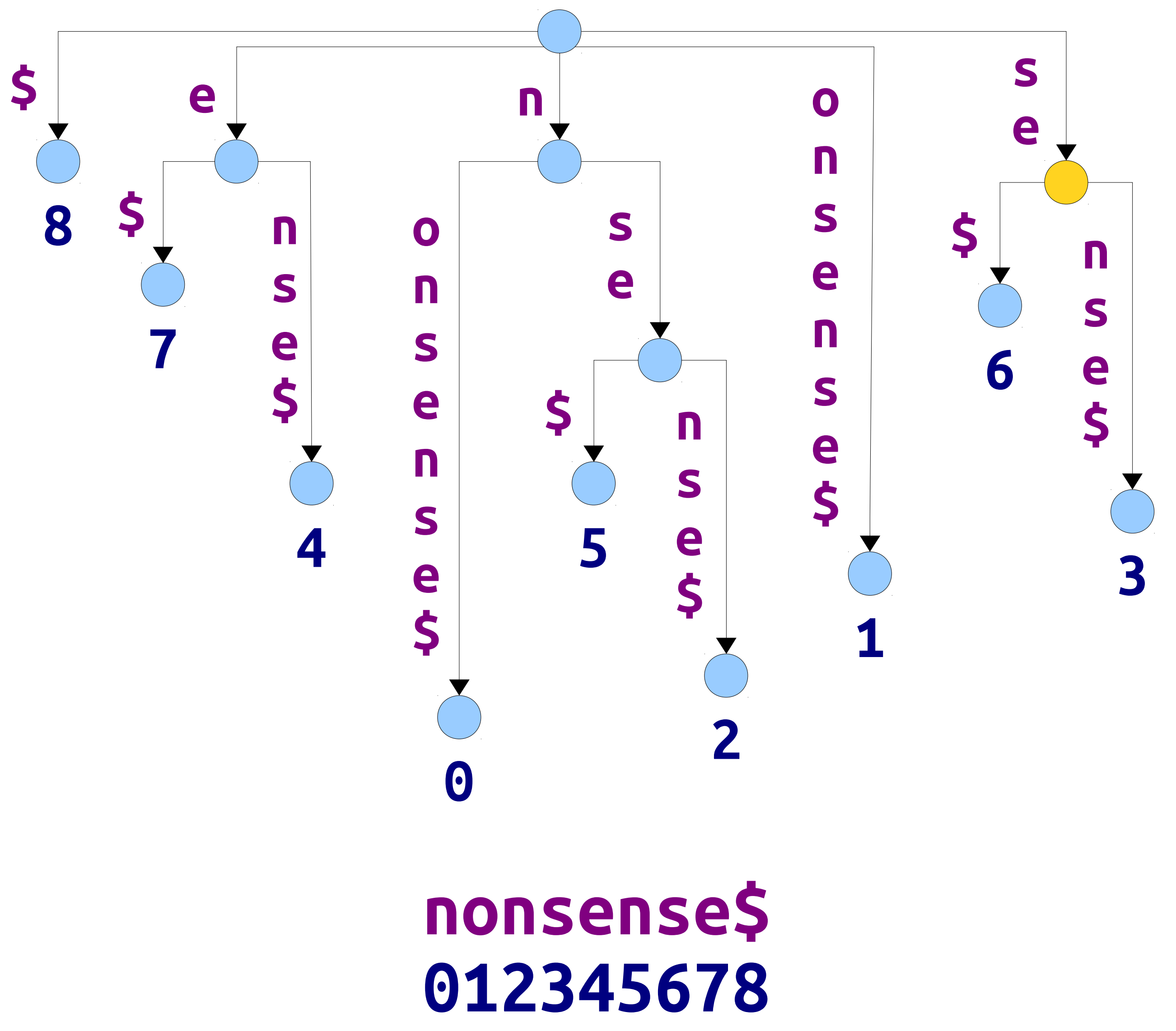

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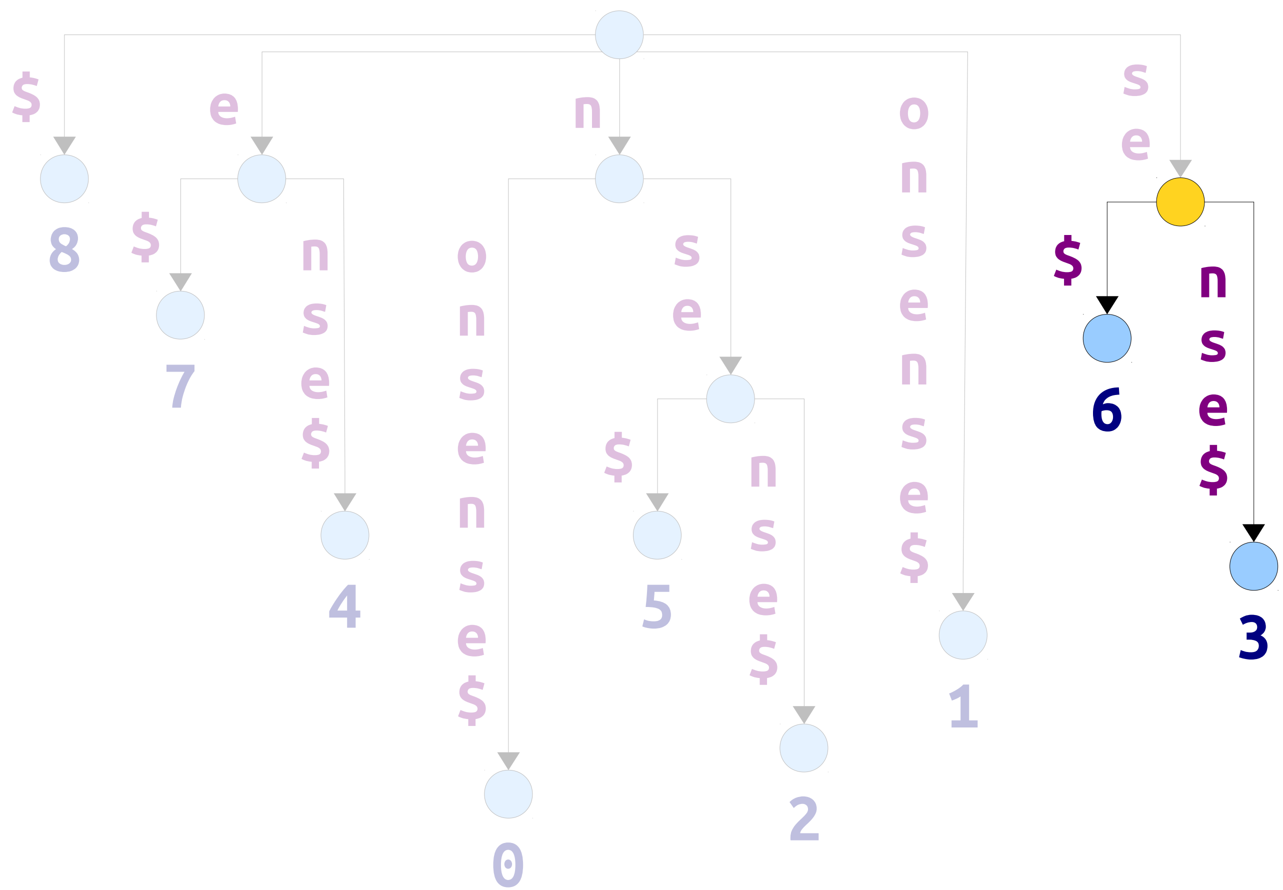

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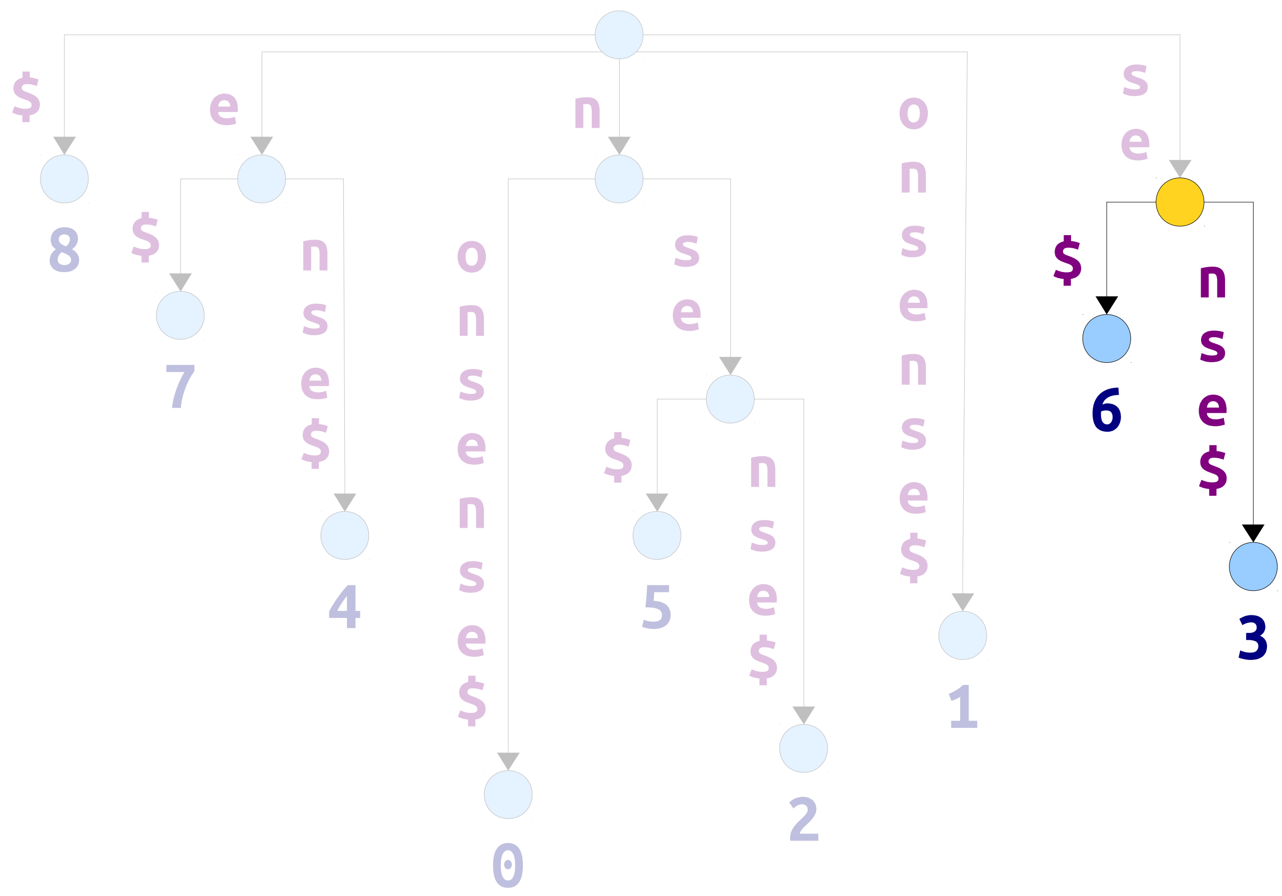

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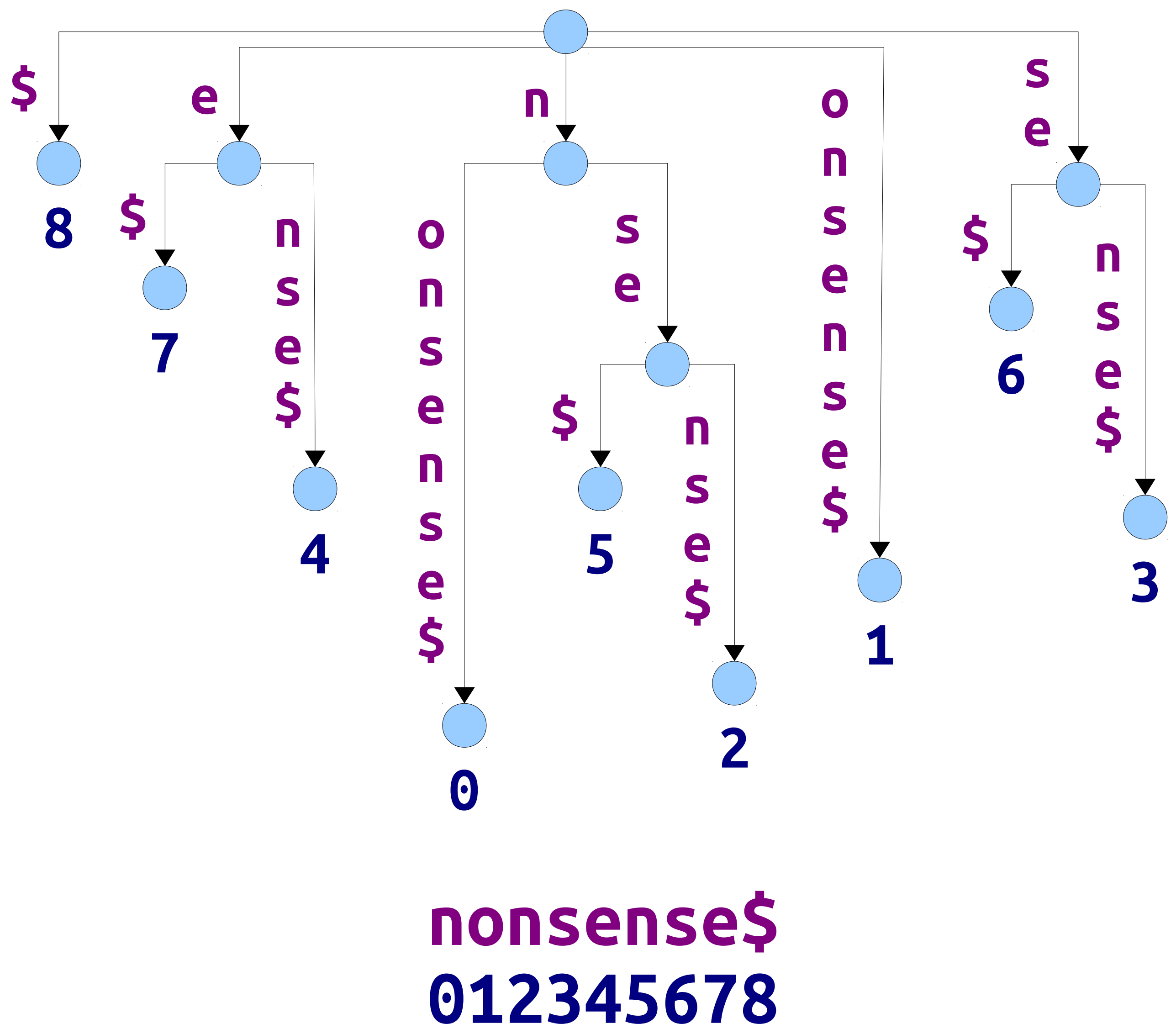

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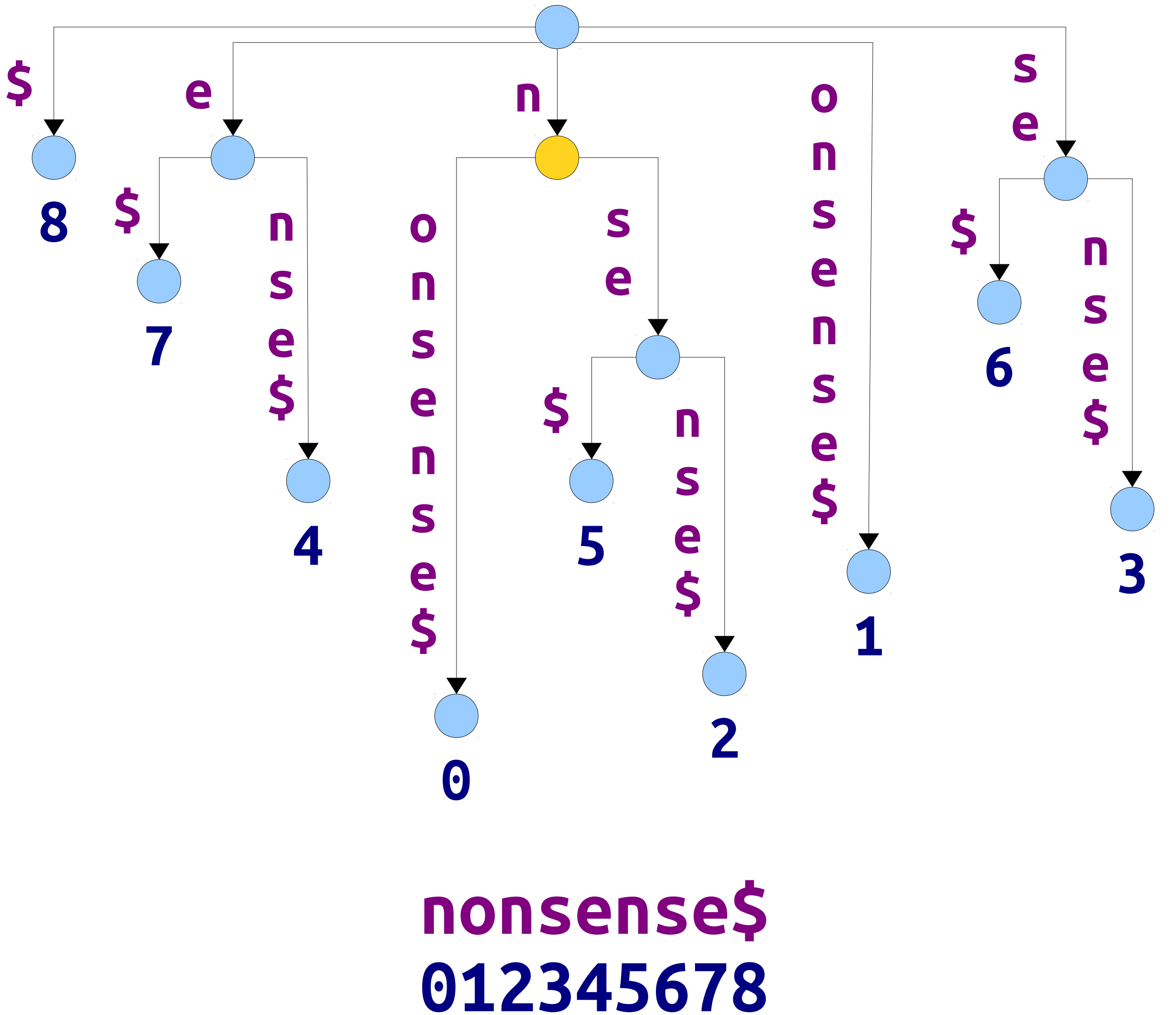

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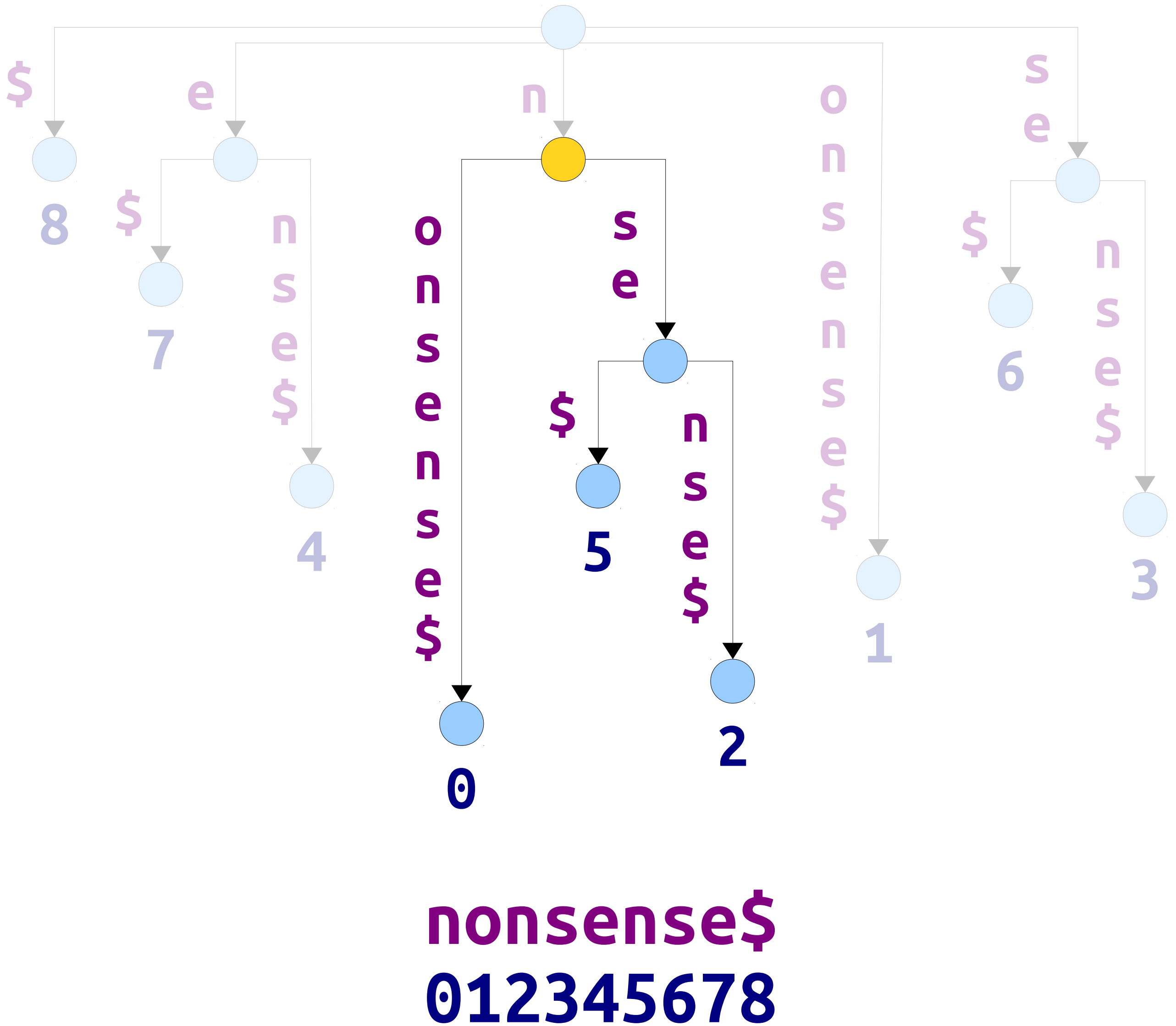

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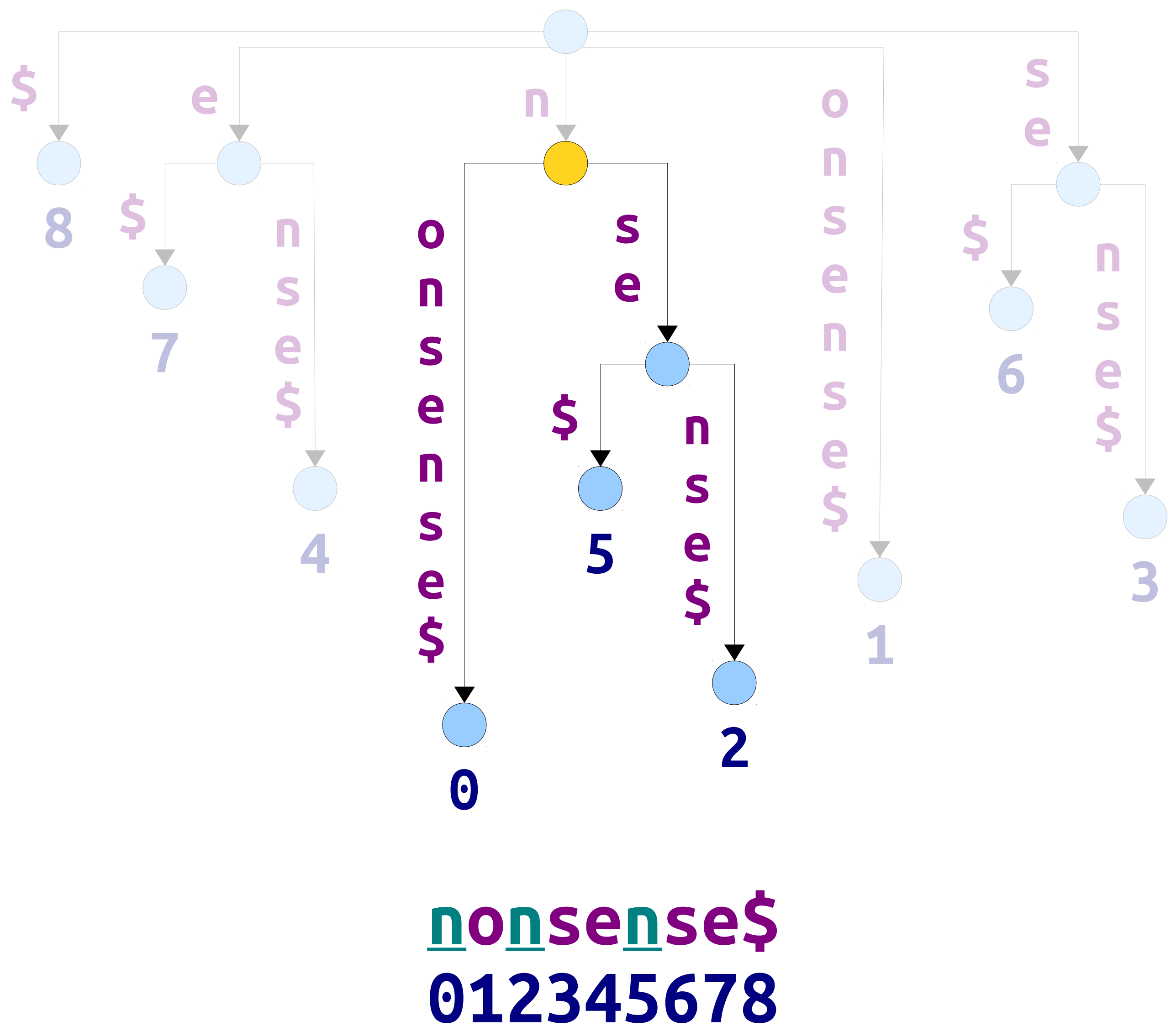

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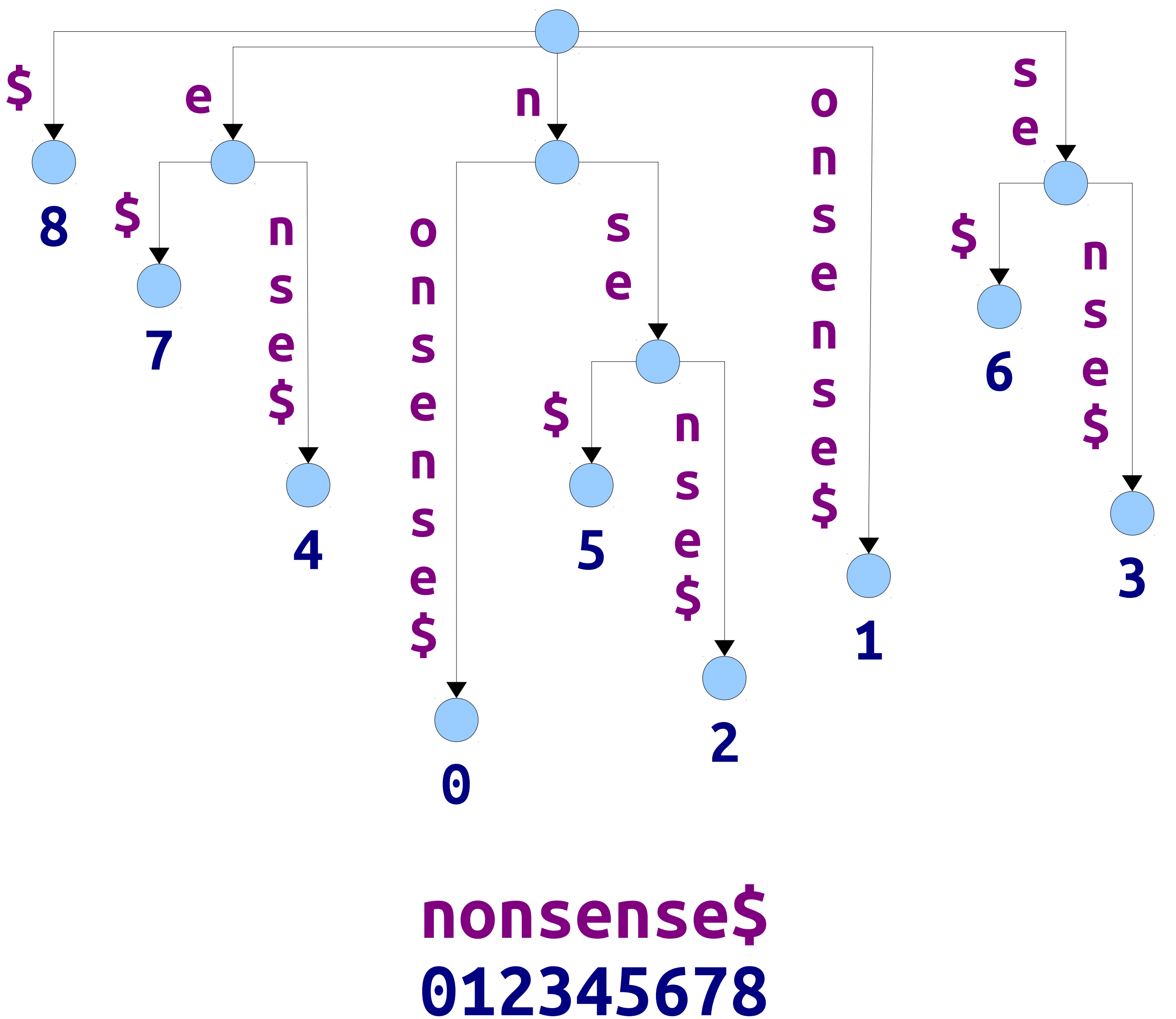

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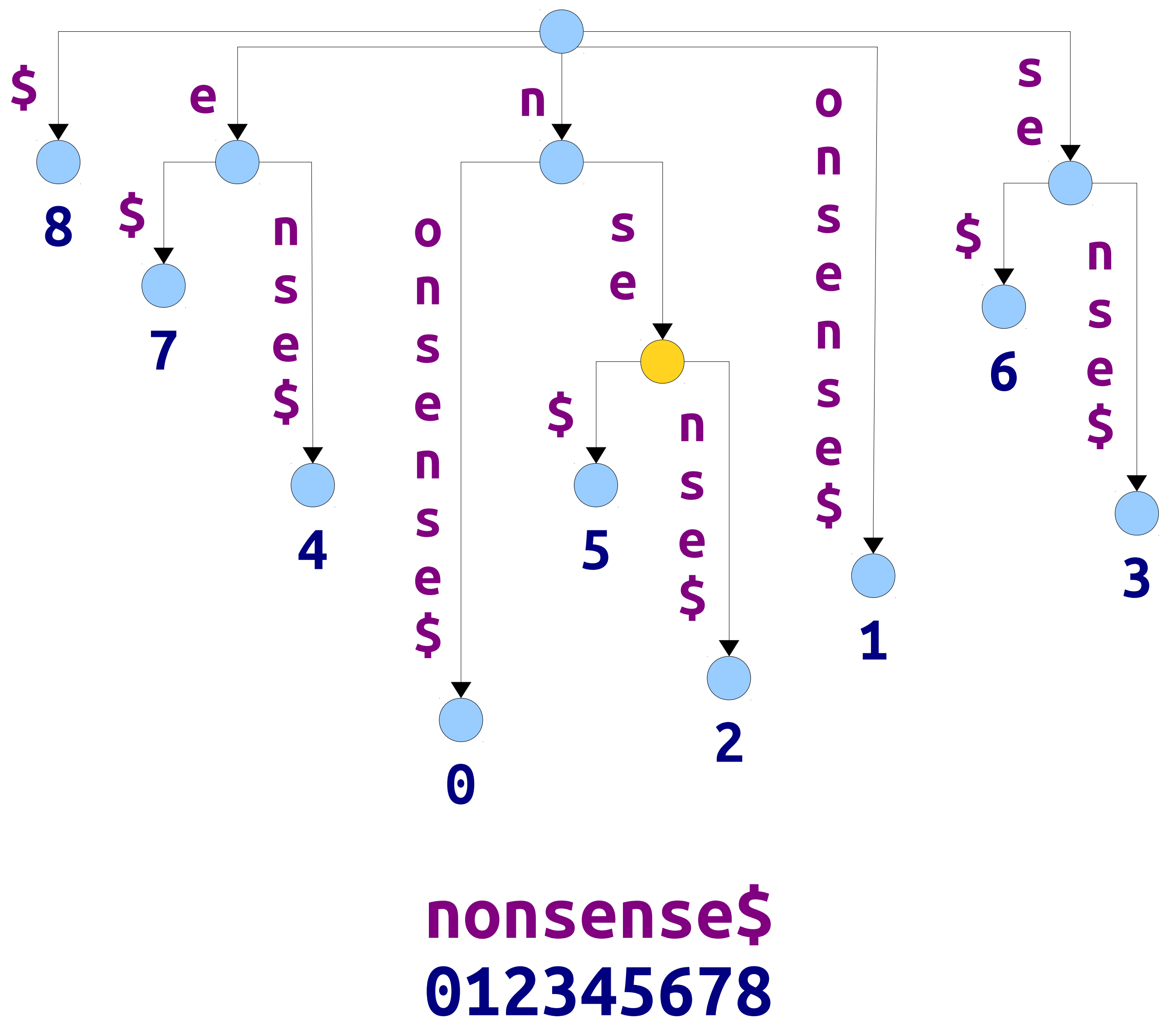

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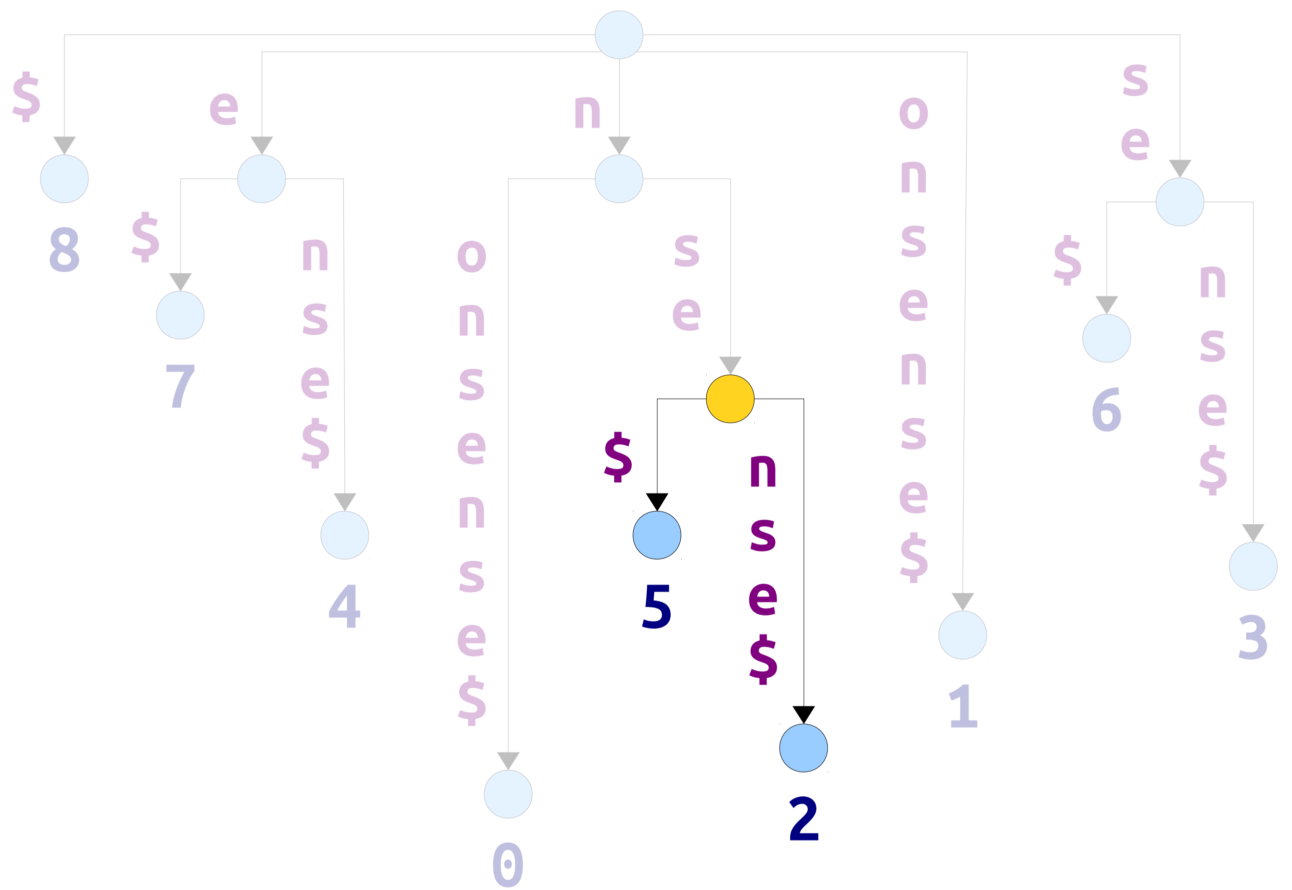
- **Claim:** After spending $O(m)$ time preprocessing $T\$$, can find all matches of a string P in time $O(n + z)$, where z is the number of matches.



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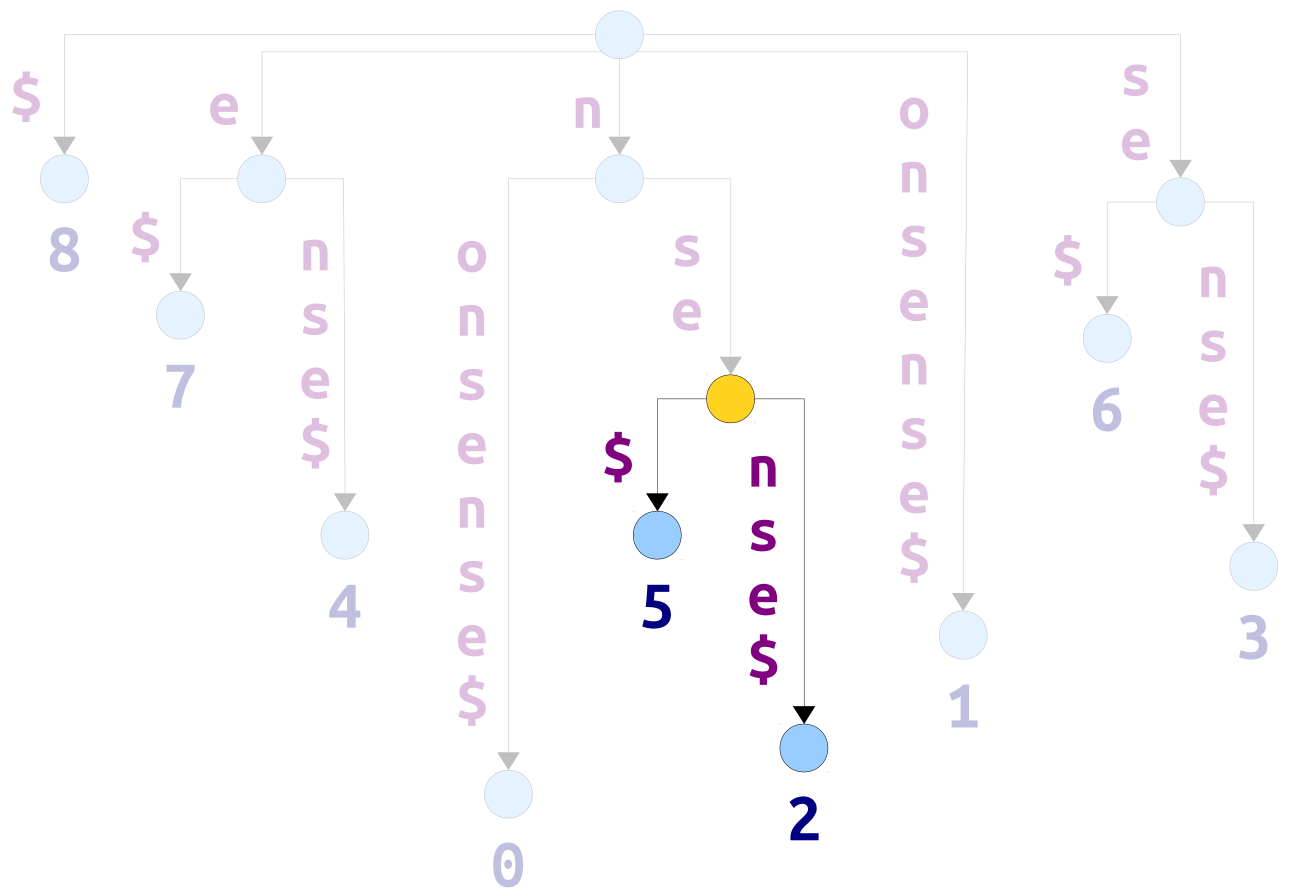

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Finding All Matches

- To find all matches of string P, start by searching the tree for P.
- If the search falls off the tree, report no matches.
- Otherwise, let v be the node at which the search stops, or the endpoint of the edge where it stops if it ends in the middle of an edge.
- Do a DFS and report all leaf numbers found. The indices reported this way give back all positions at which P occurs.

Finding All Matches

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If the search falls off the tree, report no matches.

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Finding All Matches

- To find all matches of string P, start by searching the tree for P.
- If the search falls off the tree, report no matches.
- Otherwise, let v be the node at which the search stops, or the endpoint o[illegible] where it stops if it ends in the r[illegible] edge.

How fast is this step?

- Do a DFS and report all leaf numbers found. The indices reported this way give back all positions at which P occurs.

Claim: The DFS to find all leaves in the subtree corresponding to prefix P takes time O(z), where z is the number of matches.

Proof: If the DFS reports z matches, it must have visited z different leaf nodes.

Since each internal node of a suffix tree has at least two children, the total number of internal nodes visited during the DFS is at most z – 1.

During the DFS, we don't need to actually match the characters on the edges. We just follow the edges, which takes time O(1).

Therefore, the DFS visits at most O(z) nodes and edges and spends O(1) time per node or edge, so the total runtime is O(z). ■

Reverse Aho-Corasick

- Given patterns P_1, … P_k of total length n, suffix trees can find all matches of those patterns in time $O(m + n + z)$.
  - Search for all matches of each P_i; total time across all searches is $O(n + z)$.
- Acts as a “reverse” Aho-Corasick:
  - Aho-Corasick preprocesses the *patterns* in time $O(n)$, then spends $O(m + z)$ time per tested string.
  - Suffix trees preprocess the *string* in time $O(m)$, then spends $O(n + z)$ time per set of tested patterns.

Another Application:

Longest Repeated Substring

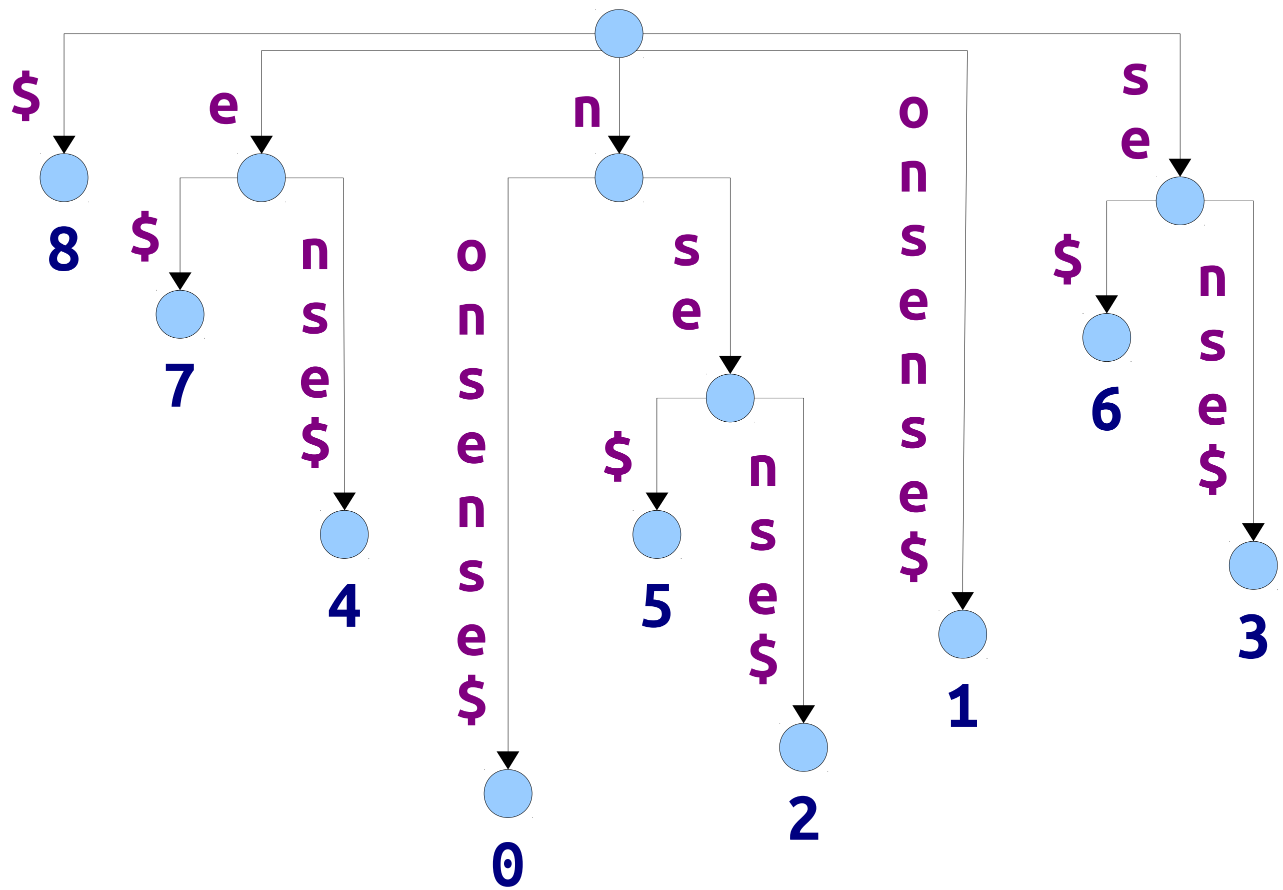
Longest Repeated Substring

- Consider the following problem:

  Given a string T, find the longest substring w of T that appears in at least two different positions.

- Applications to computational biology: more than half of the human genome is formed from repeated DNA sequences!

Longest Repeated Substring



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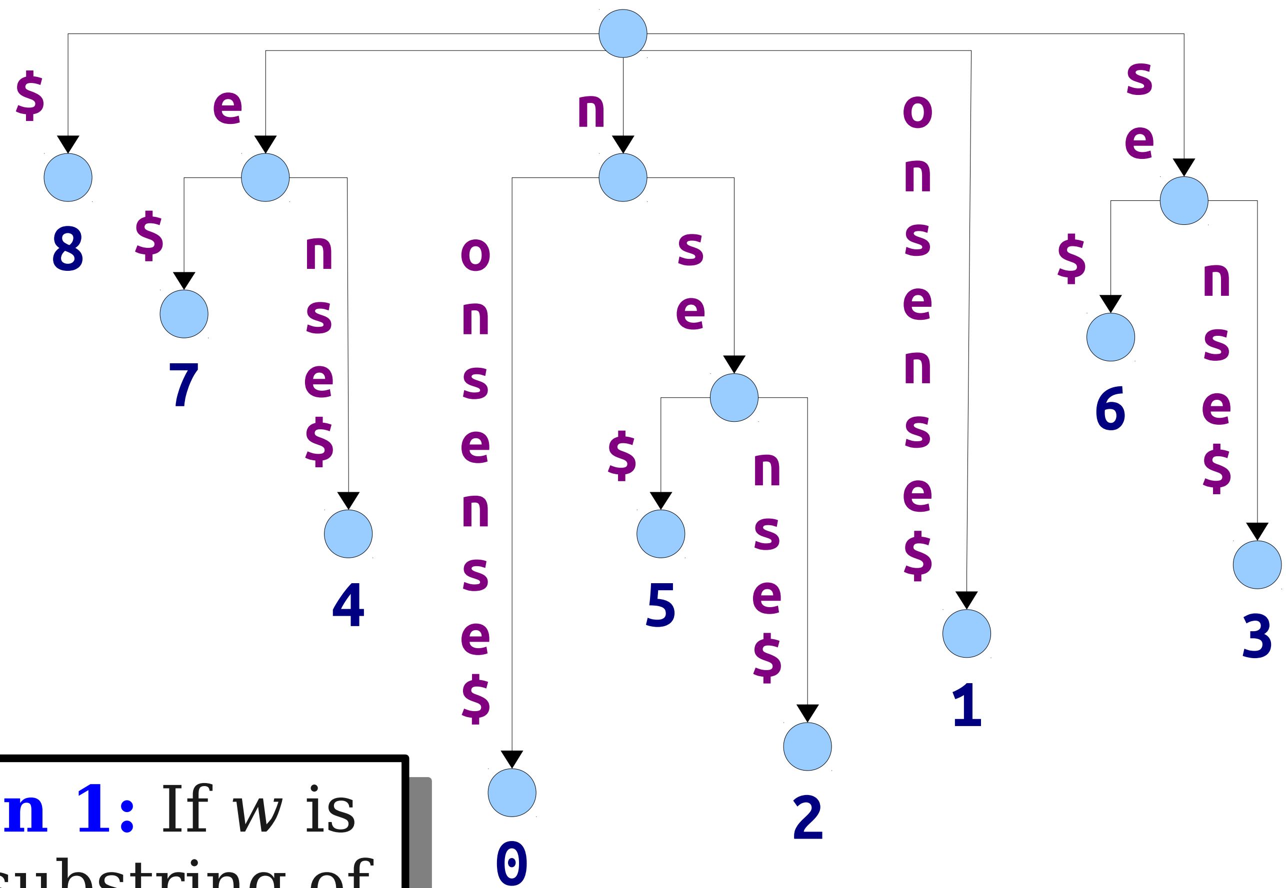
Longest Repeated Substring



Observation 1: If *w* is a repeated substring of *T*, it must be a prefix of at least two different suffixes.

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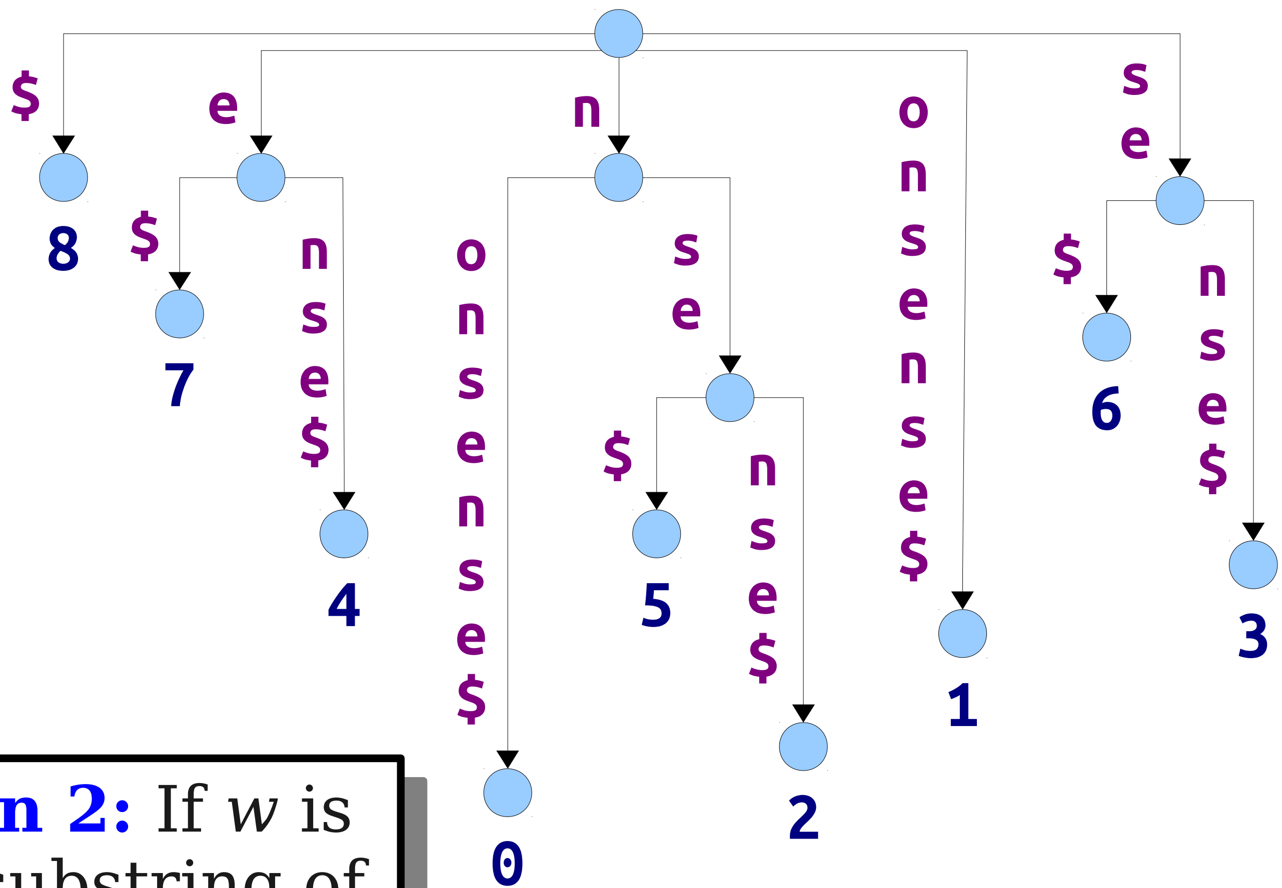
Longest Repeated Substring



Observation 2: If *w* is a repeated substring of *T*, it must correspond to a prefix of a path to an internal node.

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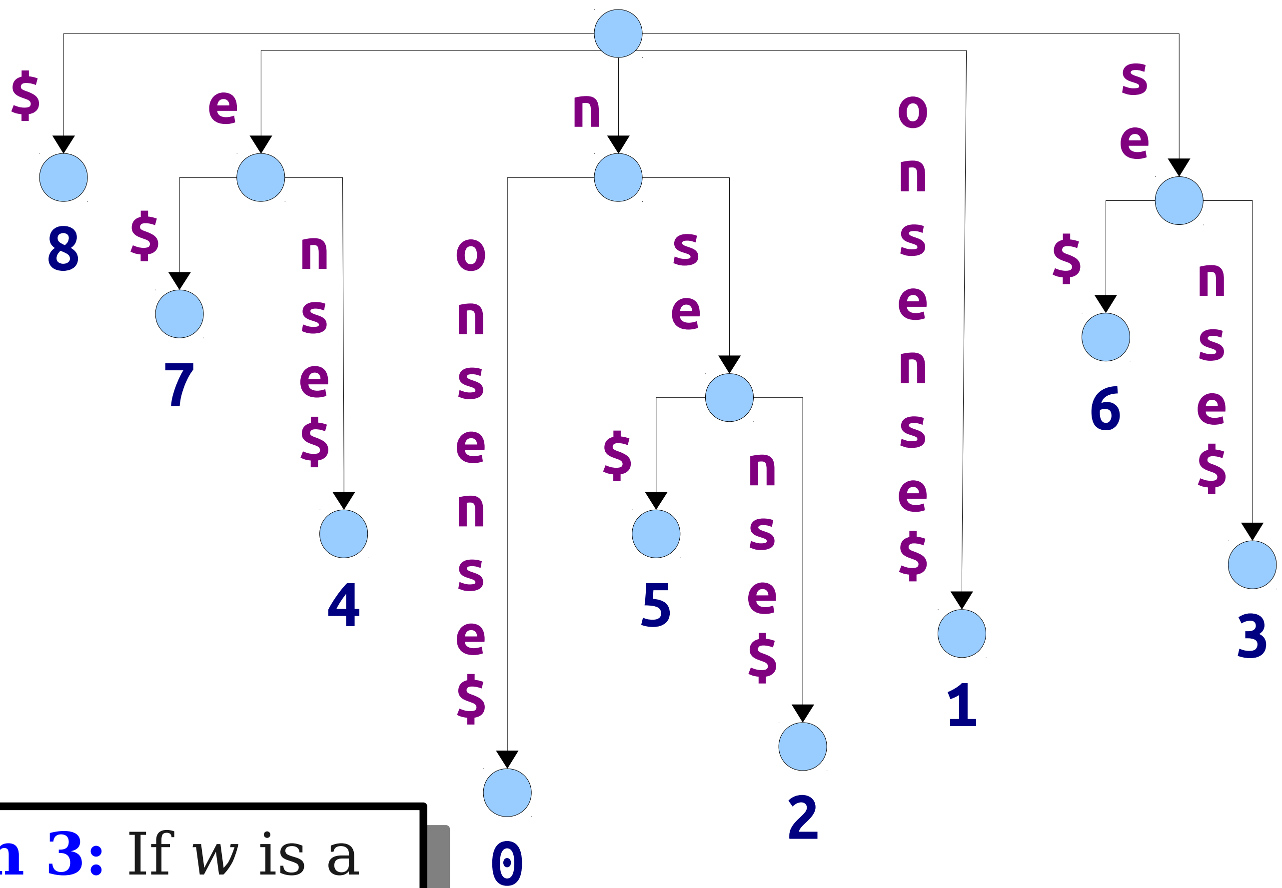
Longest Repeated Substring



Observation 3: If *w* is a longest repeated substring, it corresponds to a full path to an internal node.

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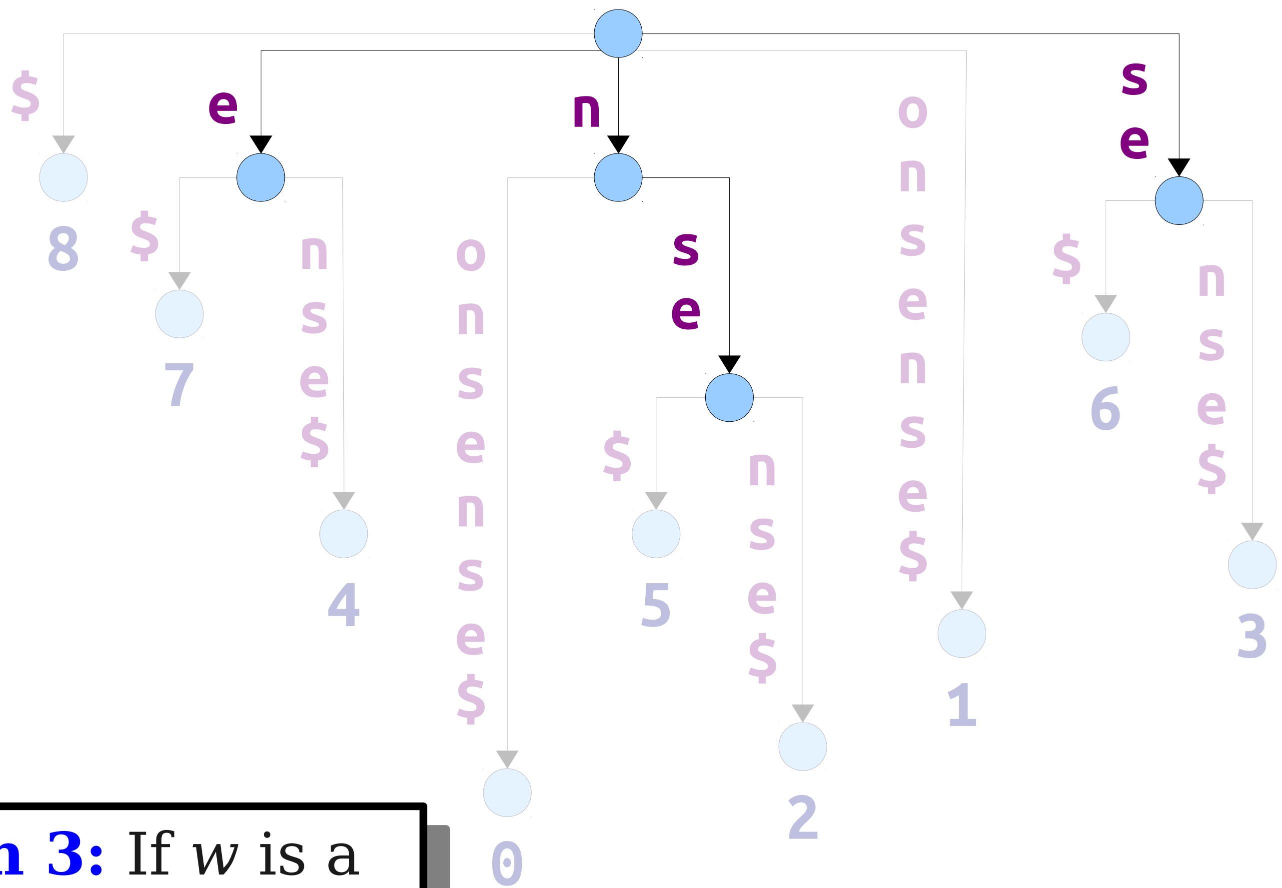
Longest Repeated Substring



Observation 3: If *w* is a longest repeated substring, it corresponds to a full path to an internal node.

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Longest Repeated Substring

- For each node v in a suffix tree, let $s(v)$ be the string that it corresponds to.
- The *string depth* of a node v is defined as $|s(v)|$, the length of the string v corresponds to.
- The longest repeated substring in T can be found by finding the internal node in T with the maximum string depth.

Longest Repeated Substring

- Here's an O(m)-time algorithm for solving the longest repeated substring problem:
  - Build the suffix tree for T in time O(m).
  - Run a DFS over T, tracking the string depth as you go, to find the internal node of maximum string depth.
  - Recover the string T corresponds to.
- **Good exercise:** How might you find the longest substring of T that repeats at least k times?

Challenge Problem:

Solve this problem in linear time without using suffix trees (or suffix arrays).

Time-Out For Announcements!

OH This Week

- I will be splitting my OH into two time slots this week:
  - Monday: 3:30PM – 4:45PM
  - Tuesday: 1:30PM – 2:30PM
- This is a temporary change; normal OH times resume next week.

PS4 Grading

- The TAs have not yet finished grading PS4.
  - Q3 is tough to grade!
- We'll have it ready by Wednesday.
- Solutions are available up front.

Final Project Logistics

- We've released a handout with some suggested data structures or techniques you might want to explore for the final project.
- We recommend trying to find a group of 2-3 people and finding some topics that look interesting.
- We'll release details about the formal final project proposal on Wednesday.

Your Questions

"How do functional data structures work, and what are some common ones?"

Check out Chris Okasaki's book ***Purely Functional Data Structures*** for an excellent exposition on the topic.

Some data structures like binomial heaps and red/black trees are actually *easier* to code up in a purely functional setting.

Some new structures (like **skew binomial random access lists**) need to be introduced in place of common structures like arrays.

“What's the best way to be prepared for the midterm?”

A few suggestions:

1. Make sure you understand the intuition behind the different data structures.

2. Make sure that you can solve all the homework problems, even if you're working in a pair.

3. Look over the readings for each class to get a better understanding of each topic.

Back to CS166!

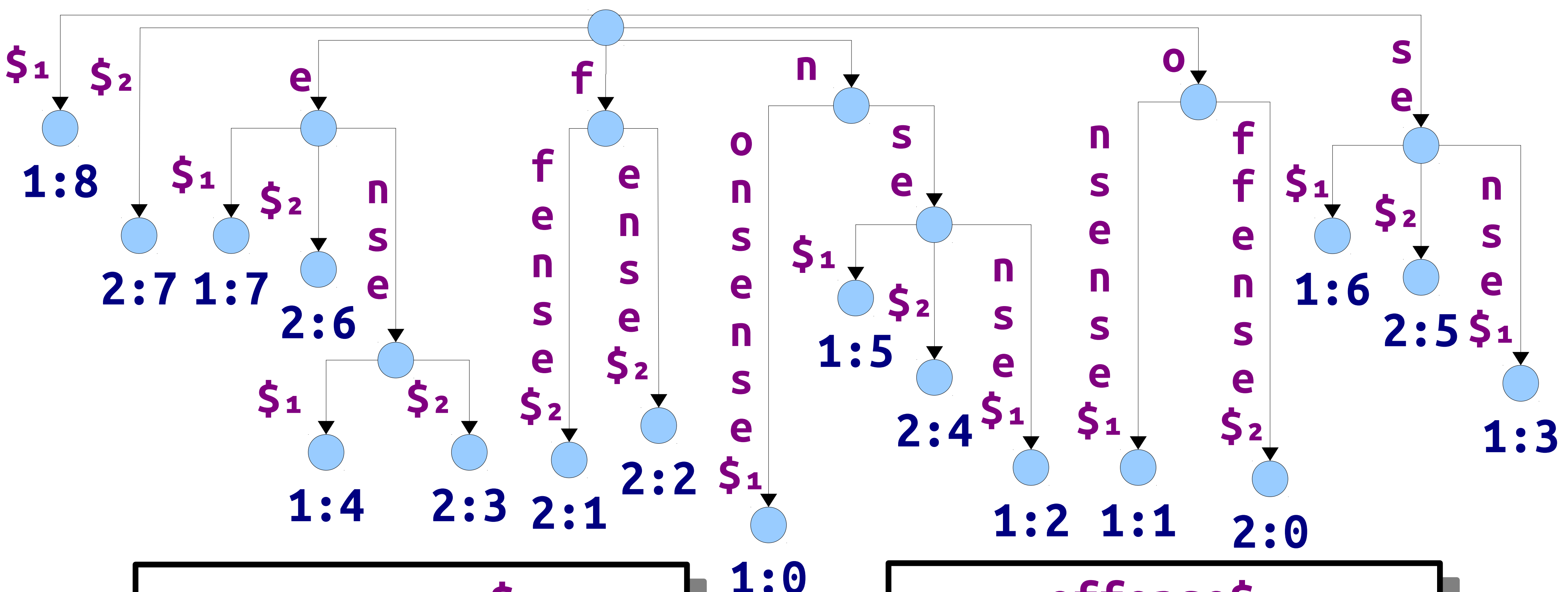
Generalized Suffix Trees

Suffix Trees for Multiple Strings

- Suffix trees store information about a single string and exports a huge amount of structural information about that string.
- However, many applications require information about the structure of multiple different strings.

Generalized Suffix Trees

- A **generalized suffix tree** for $T_1, \ldots, T_k$ is a Patricia trie of all suffixes of $T_1\$_1, \ldots, T_k\$_k$. Each T_i has a unique end marker.
- Leaves are tagged with ***i:j***, meaning "jth suffix of string T_i"



nonsense$\$_1$
012345678

offense$\$_2$
01234567

Generalized Suffix Trees

- **Claim:** A generalized suffix tree for strings $T_1, \ldots, T_k$ of total length m can be constructed in time $\Theta(m)$.
- Use a two-phase algorithm:
  - Construct a suffix tree for the single string $T_1\$_1T_2\$_2 \ldots T_k\$_k$ in time $\Theta(m)$.
    - This will end up with some invalid suffixes.
  - Do a DFS over the suffix tree and prune the invalid suffixes.
    - Runs in time $O(m)$ if implemented intelligently.

Applications of Generalized Suffix Trees

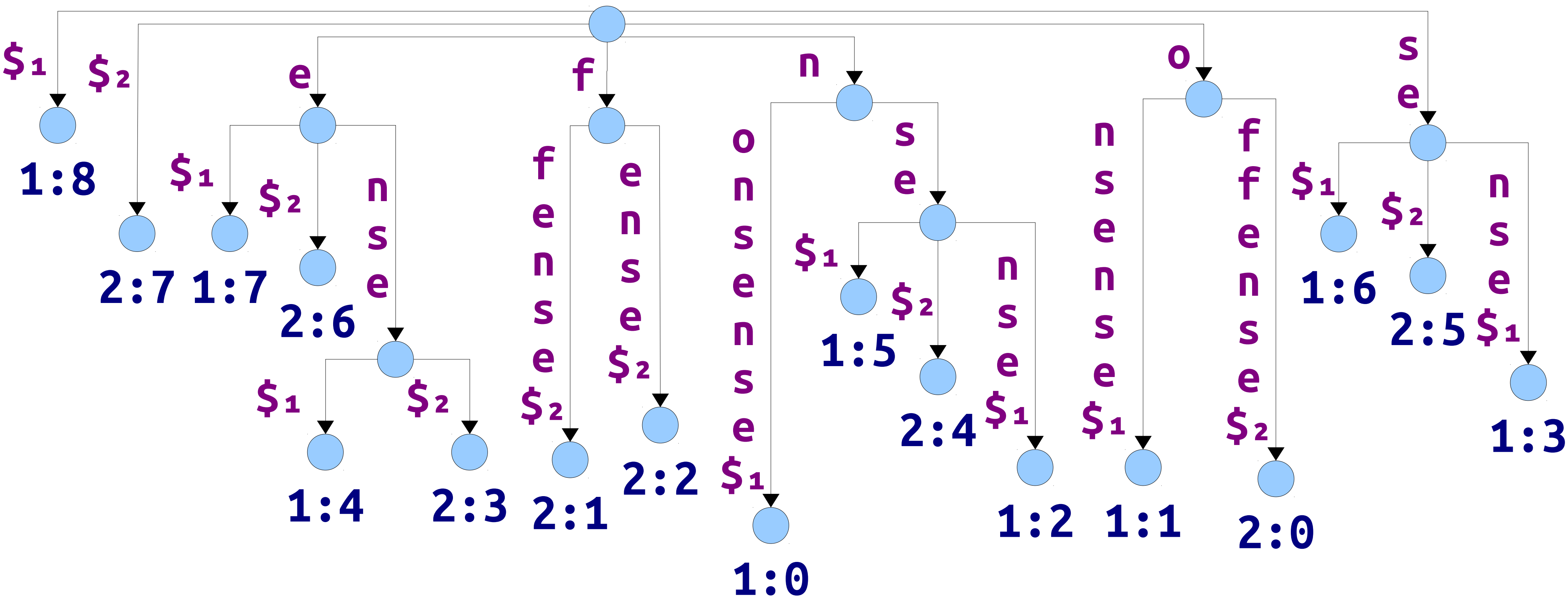
Longest Common Substring

- Consider the following problem:

  Given two strings T_1 and T_2, find the longest string w that is a substring of both T_1 and T_2.

- Can solve in time $O(|T_1| \cdot |T_2|)$ using dynamic programming.
- Can we do better?

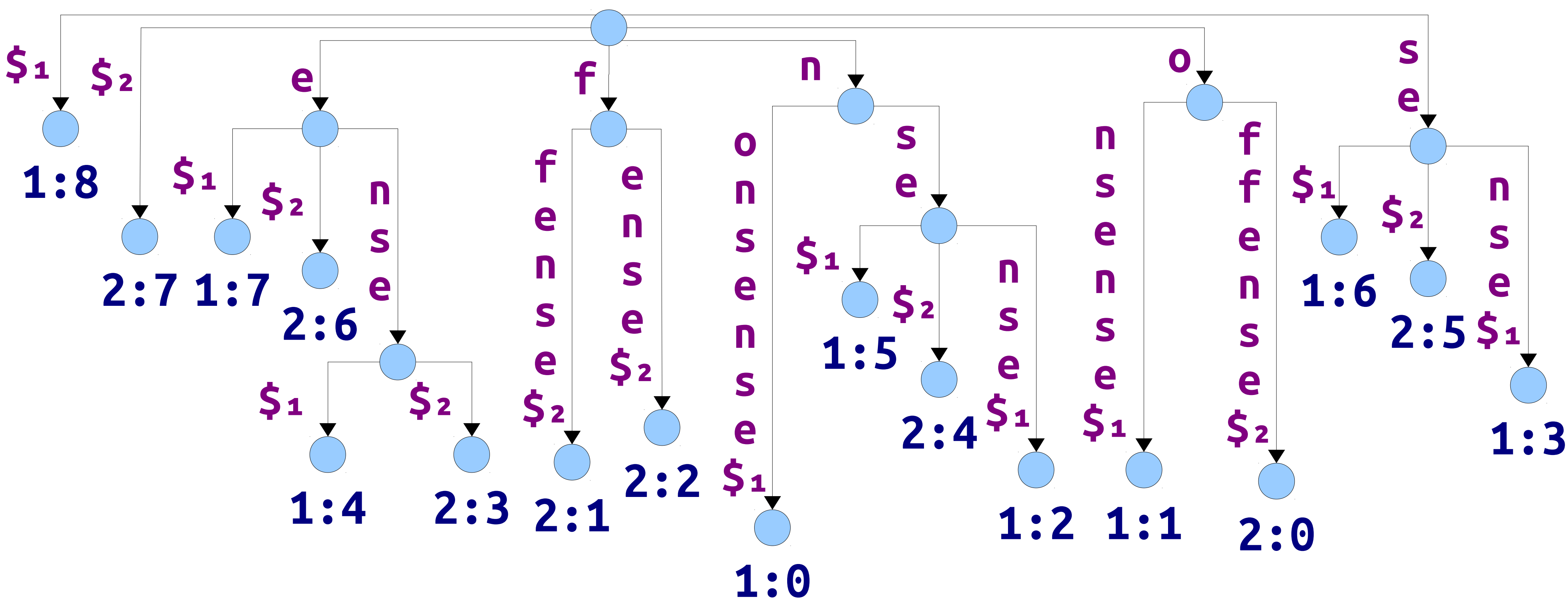
Longest Common Substring



nonsense$\$_1$
012345678

offense$\$_2$
01234567

Longest Common Substring

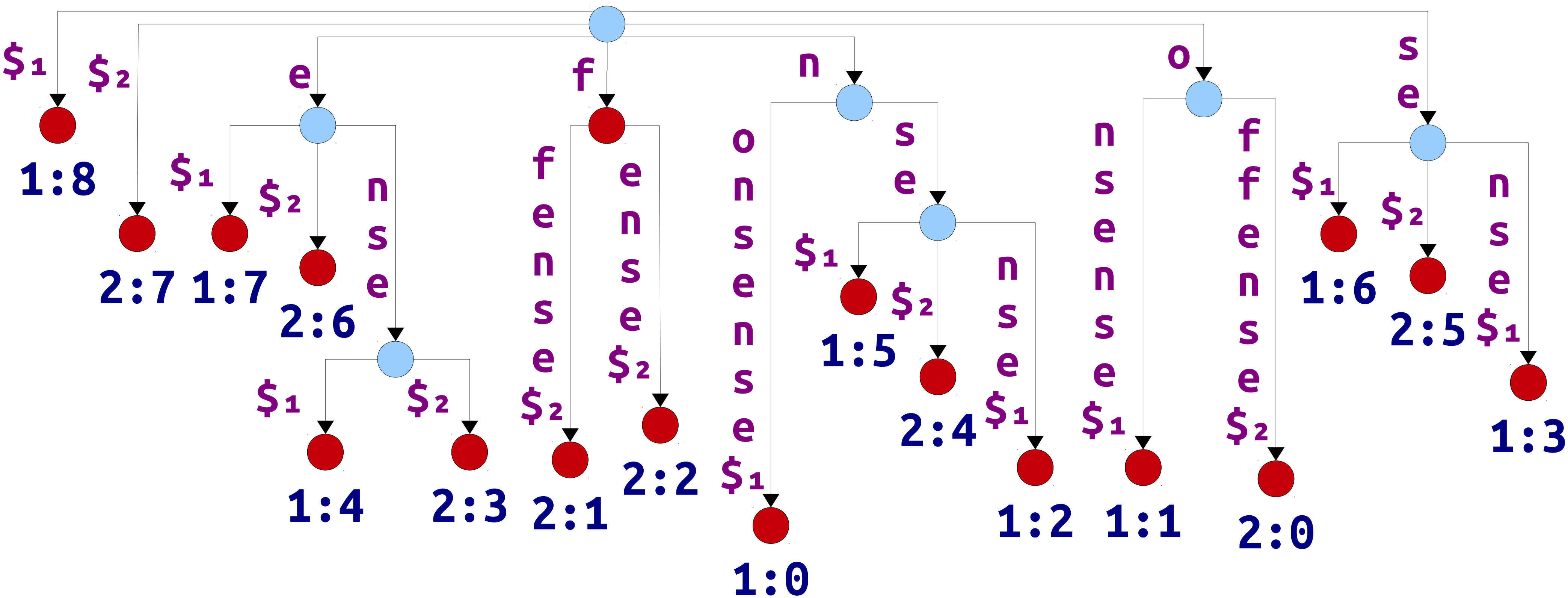


nonsense$\$_1$
012345678

Observation: Any common substring of T_1 and T_2 will be a prefix of a suffix of T_1 and a prefix of a suffix of T_2.

nse$\$_2$
4567

Longest Common Substring



nonsense$\$_1$
012345678

offense$\$_2$
01234567

Longest Common Substring

- Build a generalized suffix tree for T_1 and T_2 in time O(m).
- Annotate each internal node in the tree with whether that node has at least one leaf node from each of T_1 and T_2.
  - Takes time O(m) using DFS.
- Run a DFS over the tree to find the marked node with the highest string depth.
  - Takes time O(m) using DFS
- Overall time: **O(m)**.

Longest Common Extensions

Longest Common Extensions

- Given two strings T_1 and T_2 and start positions i and j, the **longest common extension** of T_1 and T_2, starting at positions i and j, is the length of the longest string w that appears at position i in T_1 and position j in T_2.
- We'll denote this value by $\mathrm{LCE}_{T_1,\, T_2}(i, j)$.
- Typically, T_1 and T_2 are fixed and multiple (i, j) queries are specified.

n	o	n	s	e	n	s	e
o	f	f	e	n	s	e	

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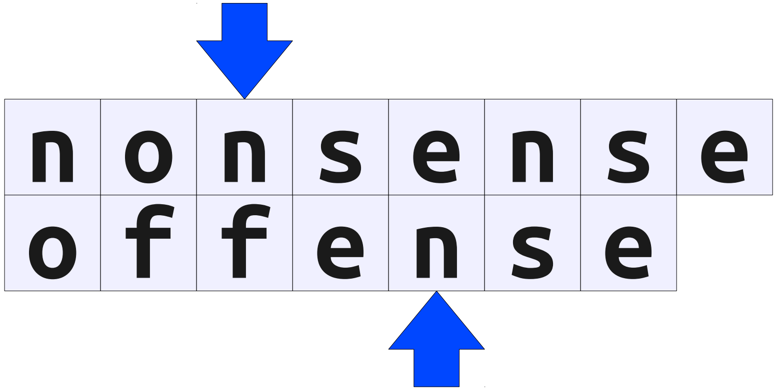

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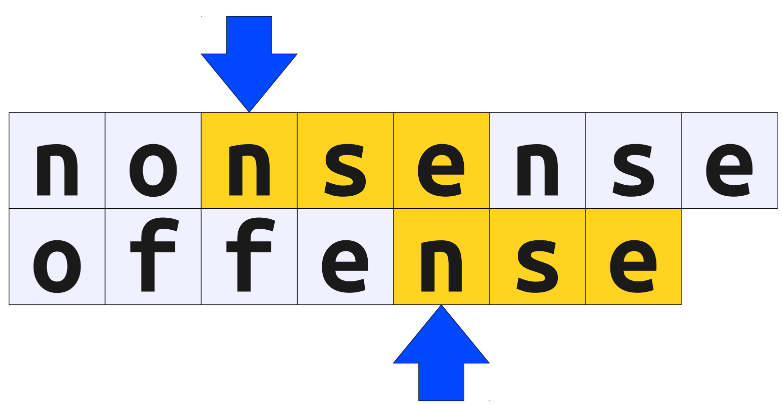

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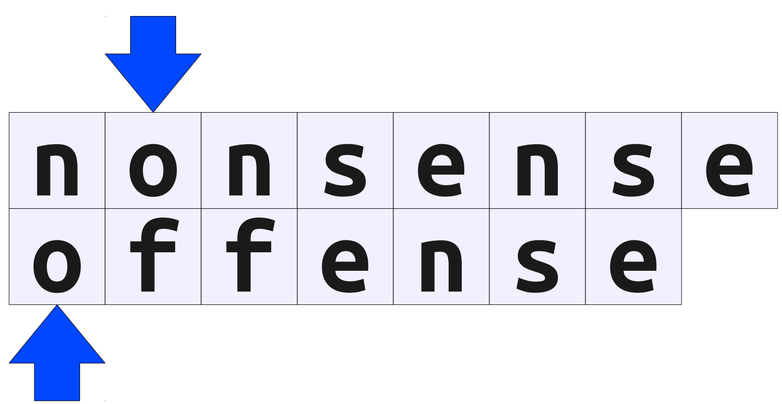

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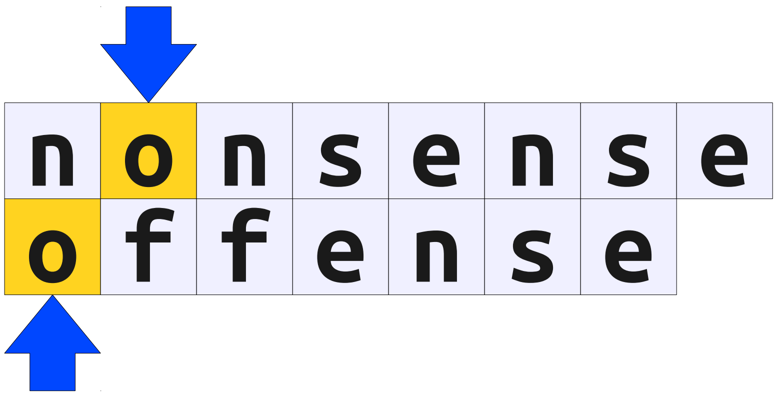

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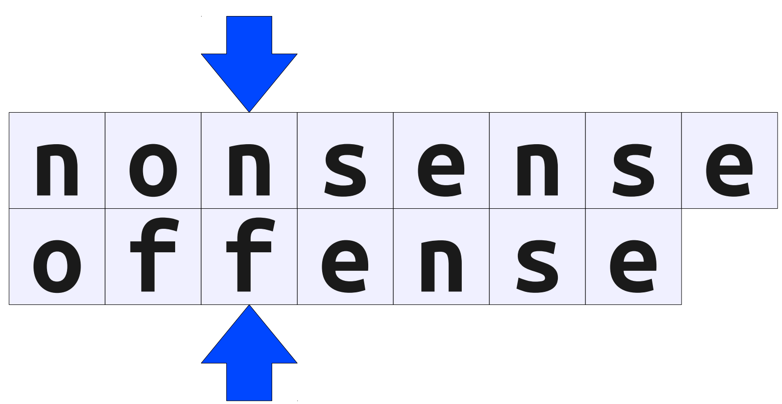
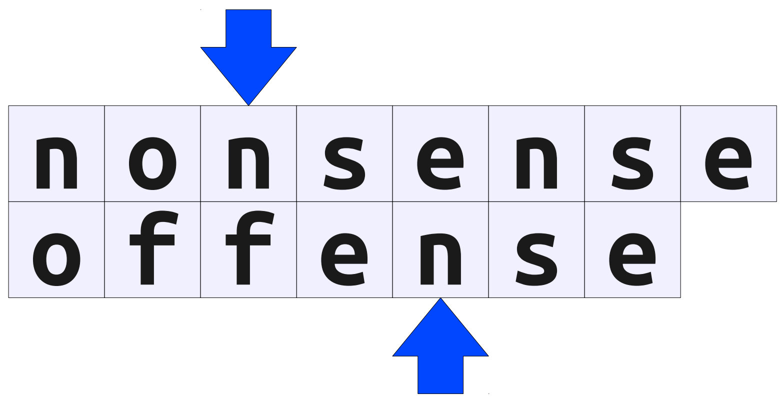


Longest Common Extensions

- **Observation:** $\text{LCE}_{T_1,\,T_2}(i, j)$ is the length of the longest common prefix of the suffixes of T_1 and T_2 starting at positions i and j.



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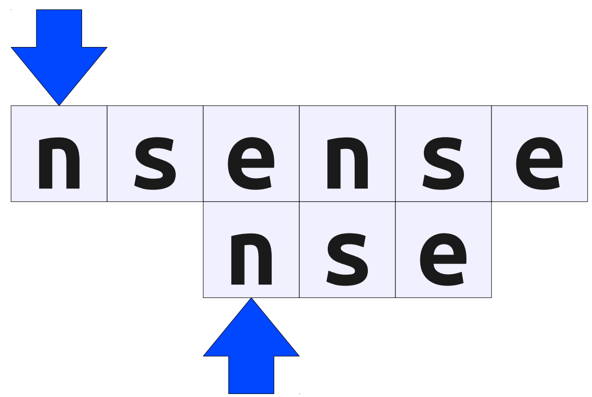

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n	s	e	n	s	e
n	s	e			

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n	s	e	n	s	e
n	s	e			

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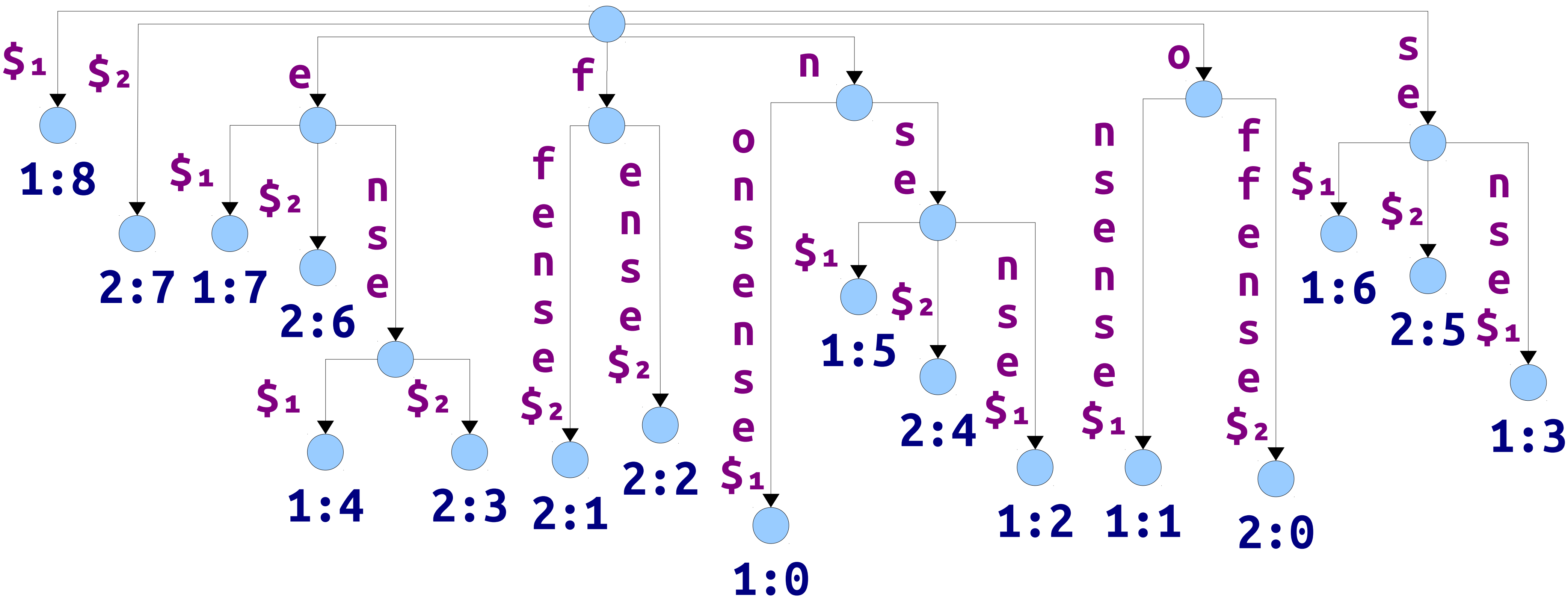
An Observation



nonsense$\$_1$
012345678

offense$\$_2$
01234567

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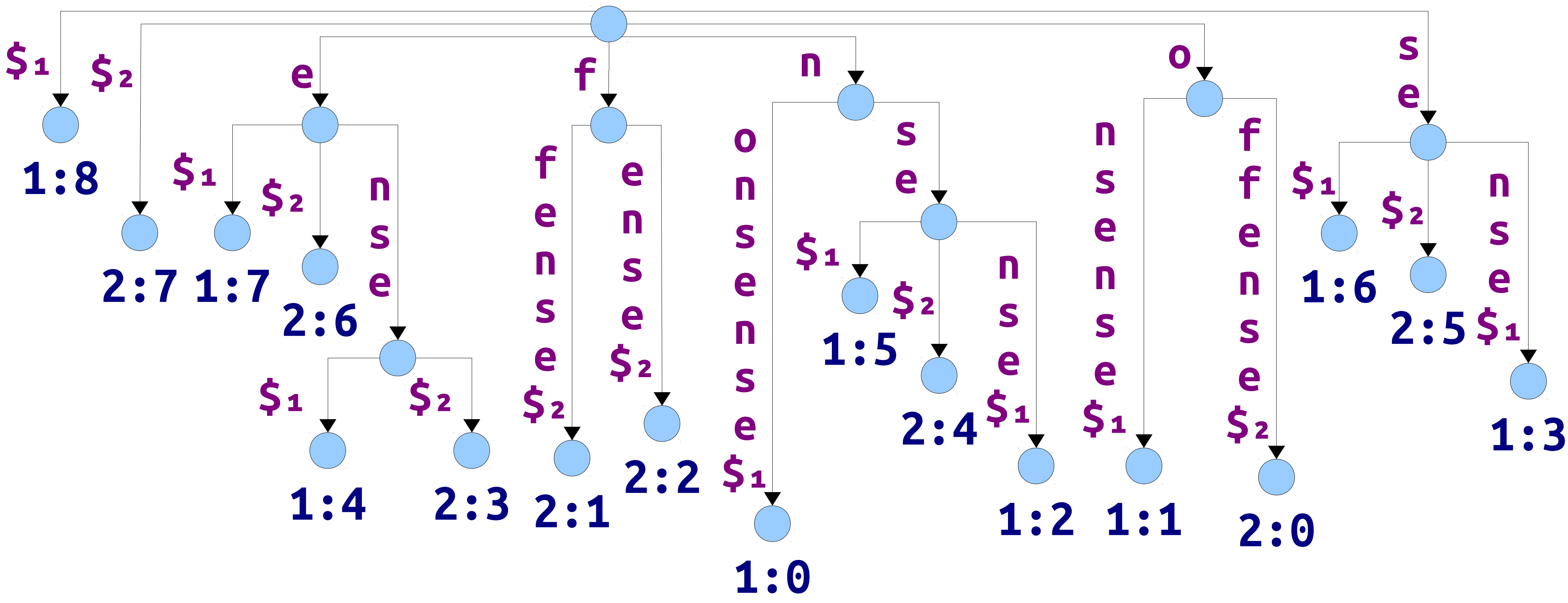

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01234567

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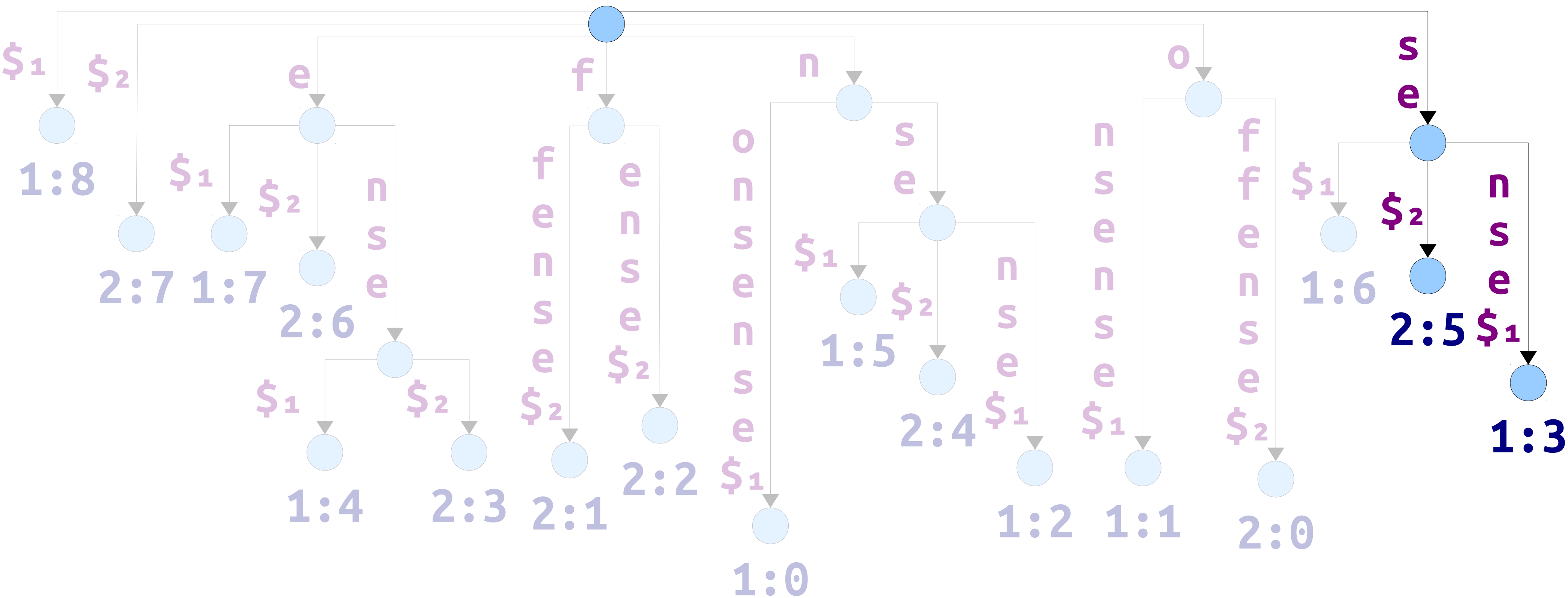

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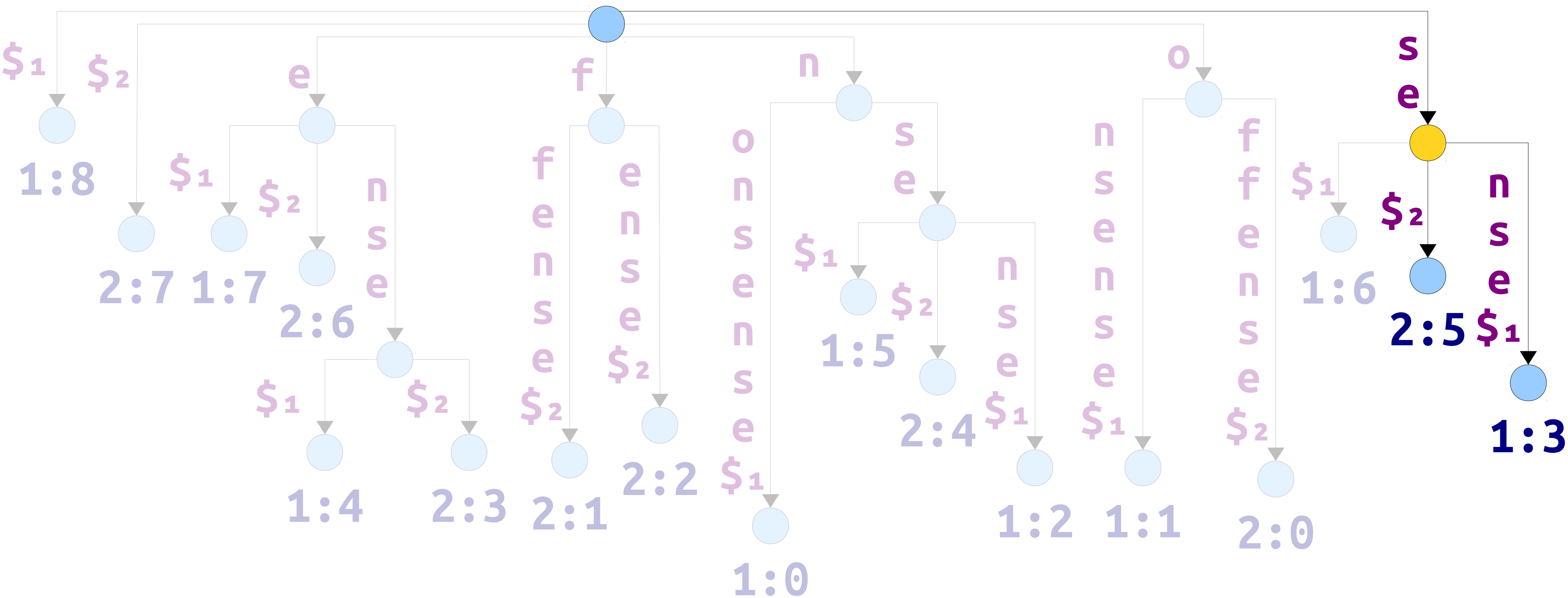

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012345678

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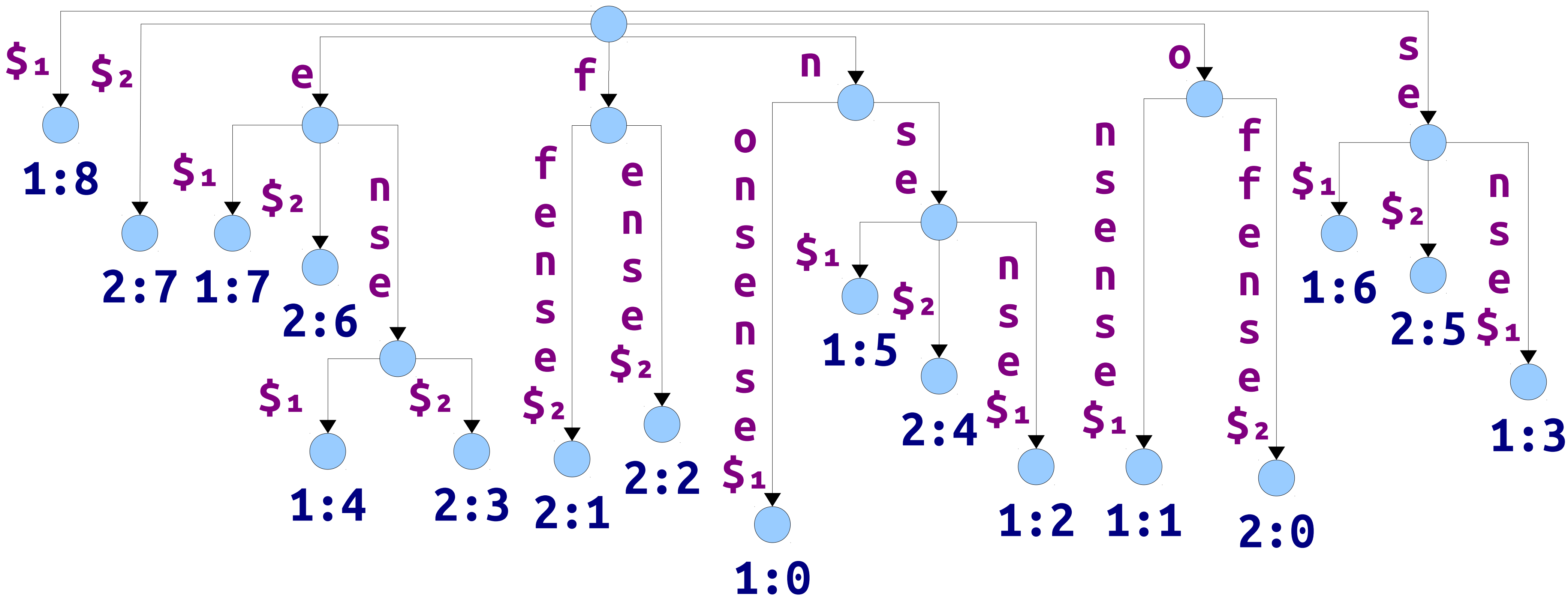

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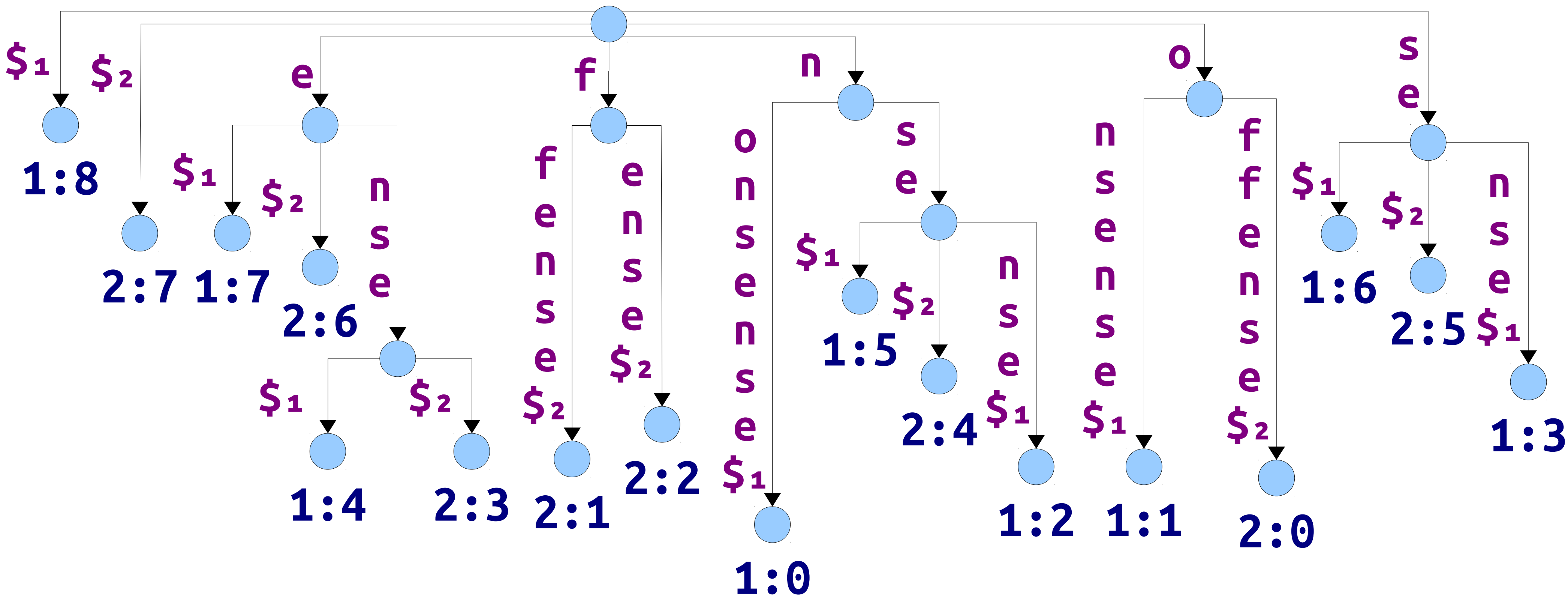

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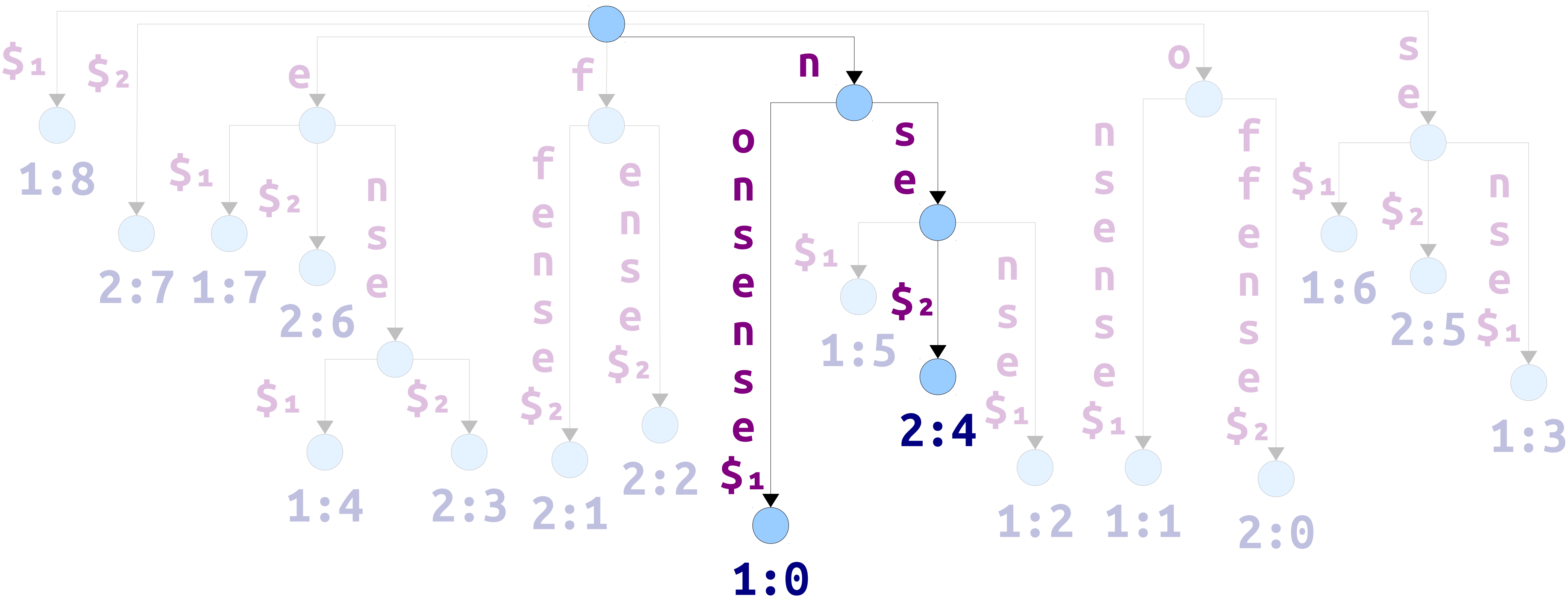

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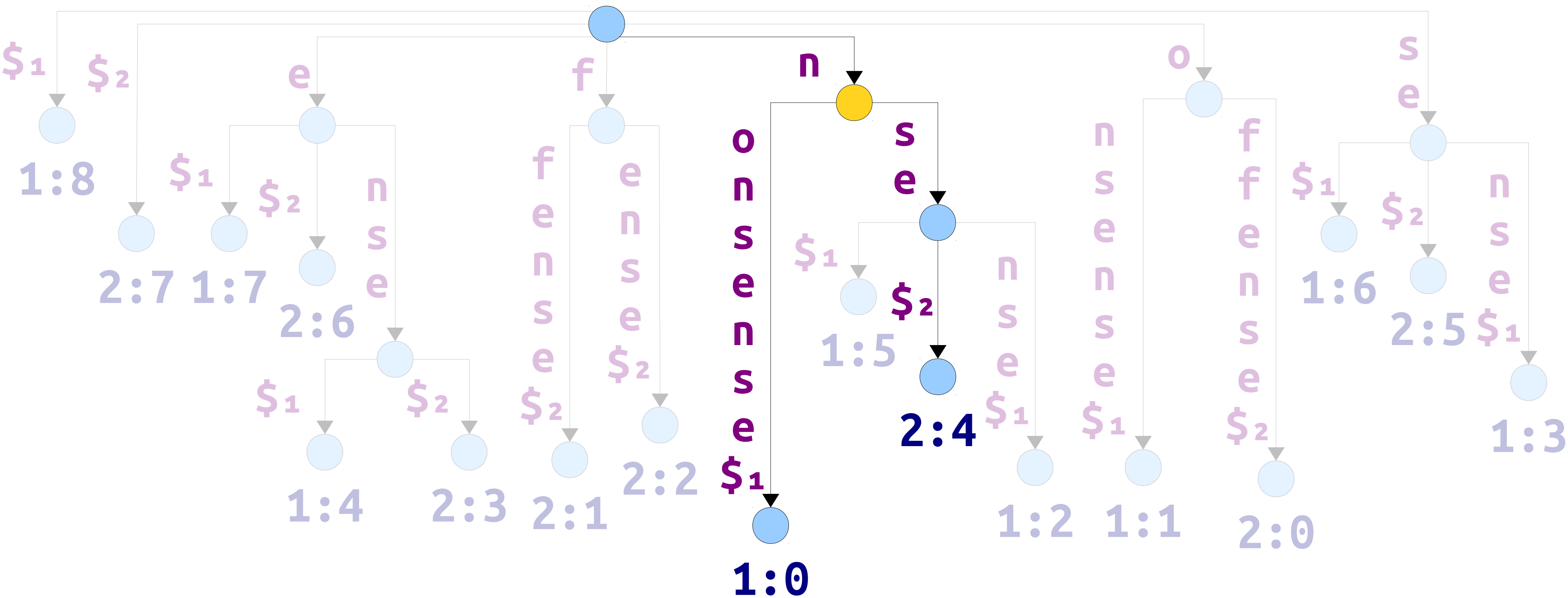


nonsense$_1$
012345678

offense$_2$
01234567

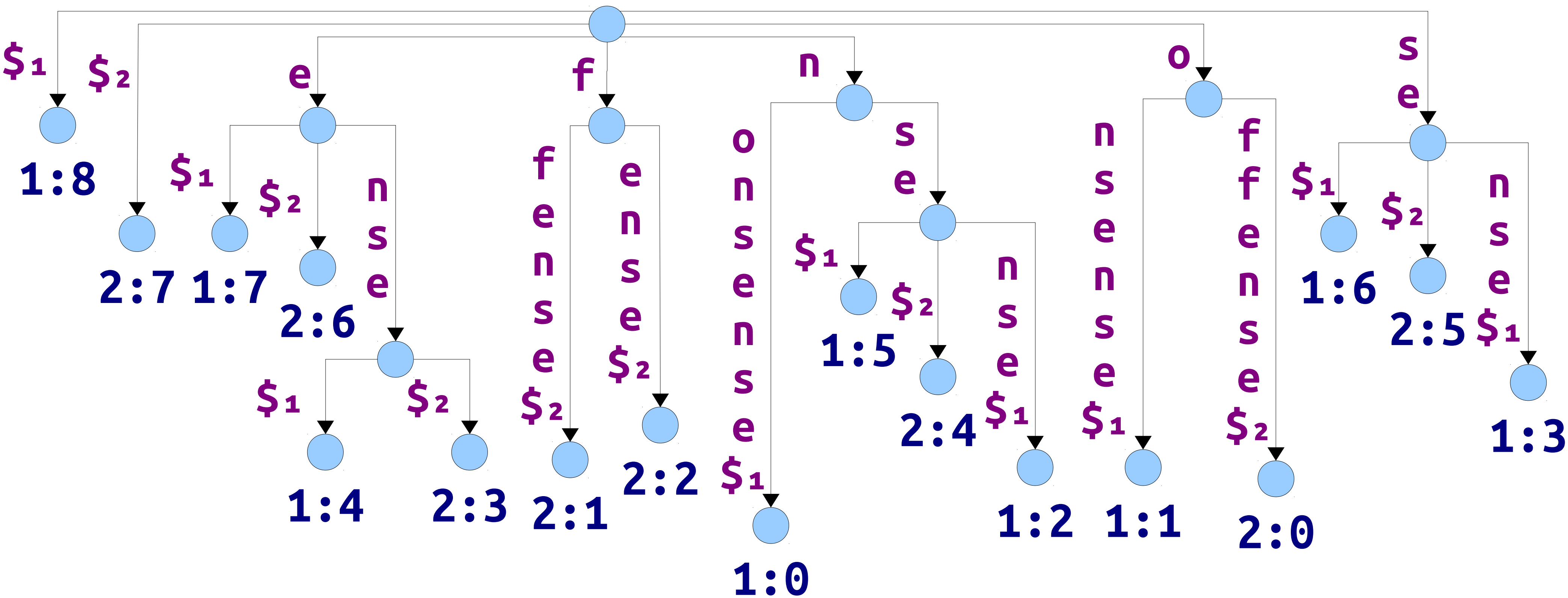
An Observation

- **Notation:** Let $S[i{:}]$ denote the suffix of string S starting at position i.
- **Claim:** $\mathrm{LCE}_{T_1, T_2}(i, j)$ is given by the string label of the LCA of $T_1[i{:}]$ and $T_2[j{:}]$ in the generalized suffix tree of T_1 and T_2.
- And hey… don't we have a way of computing these in time O(1)?

Computing LCE's

- Given two strings T_1 and T_2, construct a generalized suffix tree for T_1 and T_2 in time $O(m)$.
- Construct an LCA data structure for the generalized suffix tree in time $O(m)$.
  - Use Fischer-Heun plus an Euler tour of the nodes in the tree.
- Can now query for the node representing the LCE in time $O(1)$.

One Last Detail



nonsense$\$_1$
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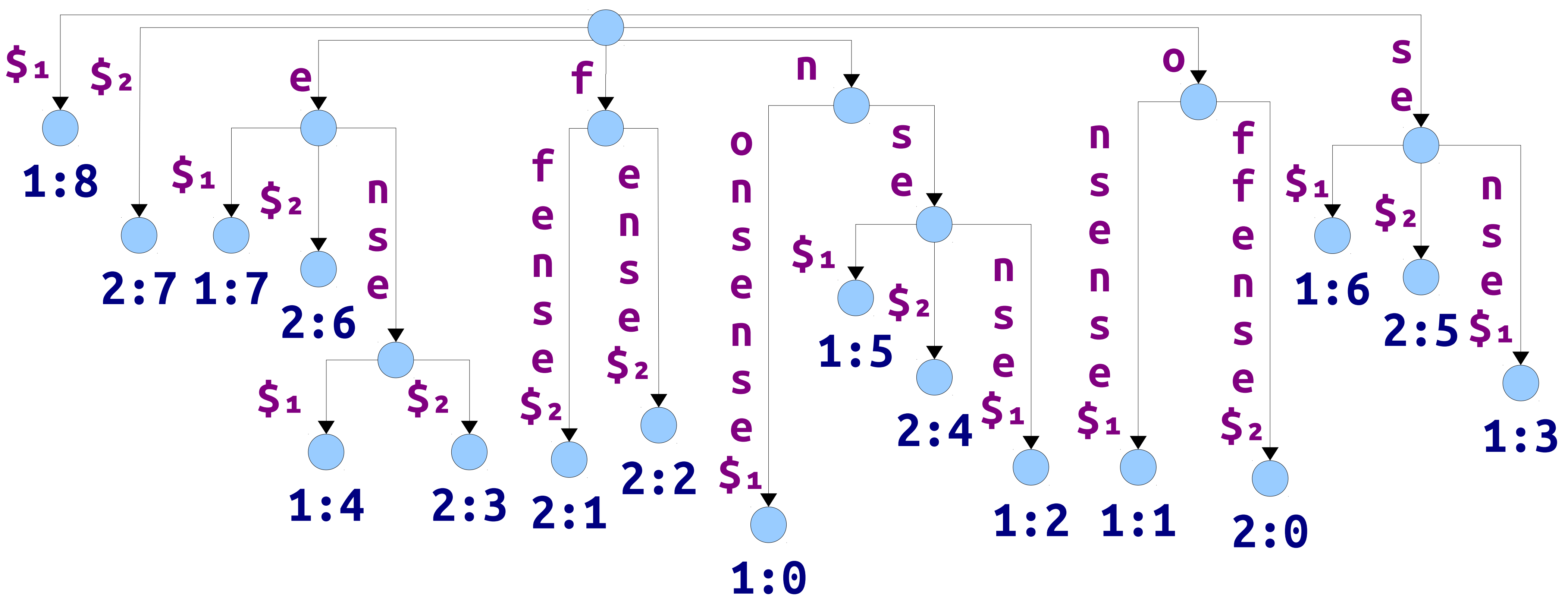

nonsense$\$_1$
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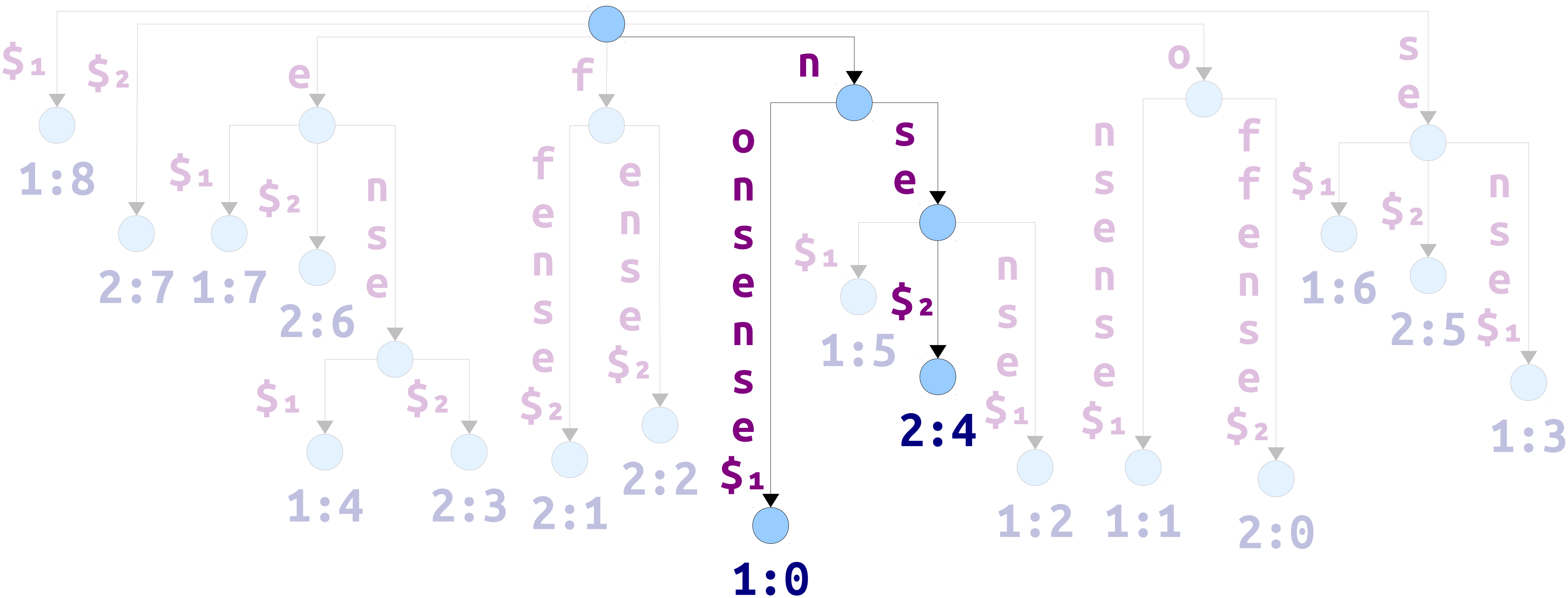

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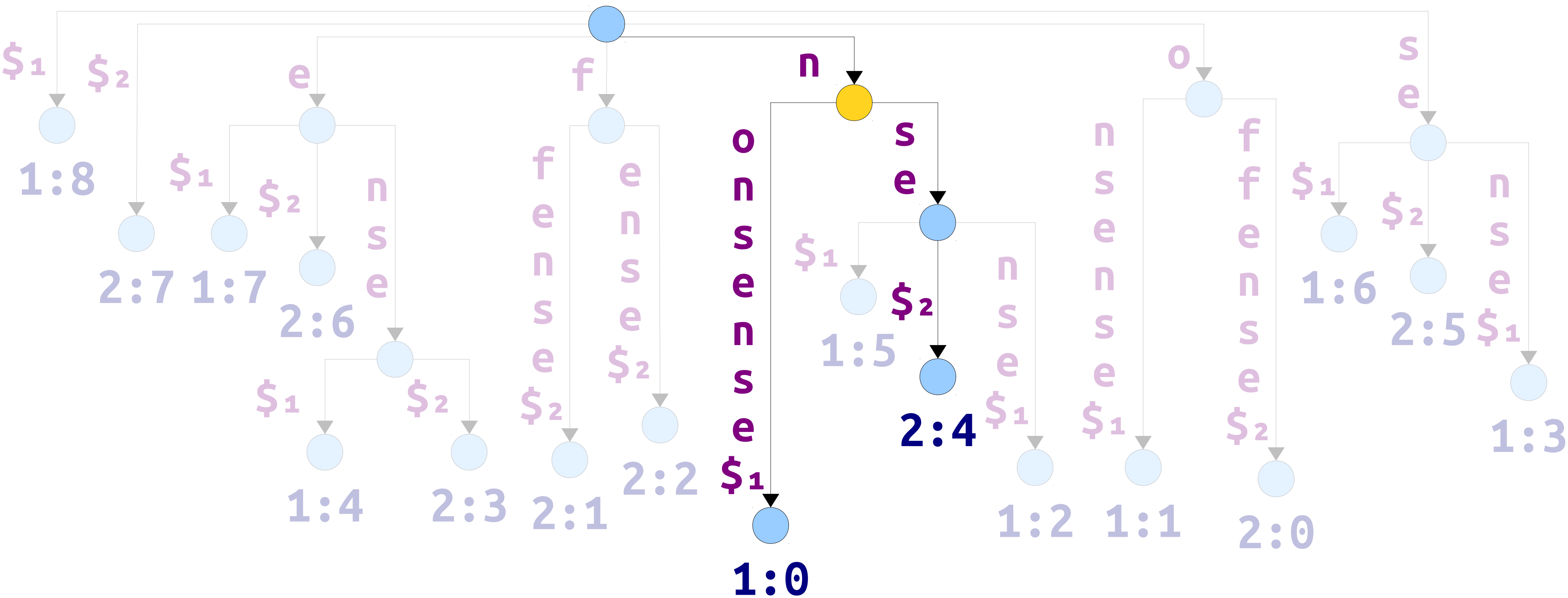

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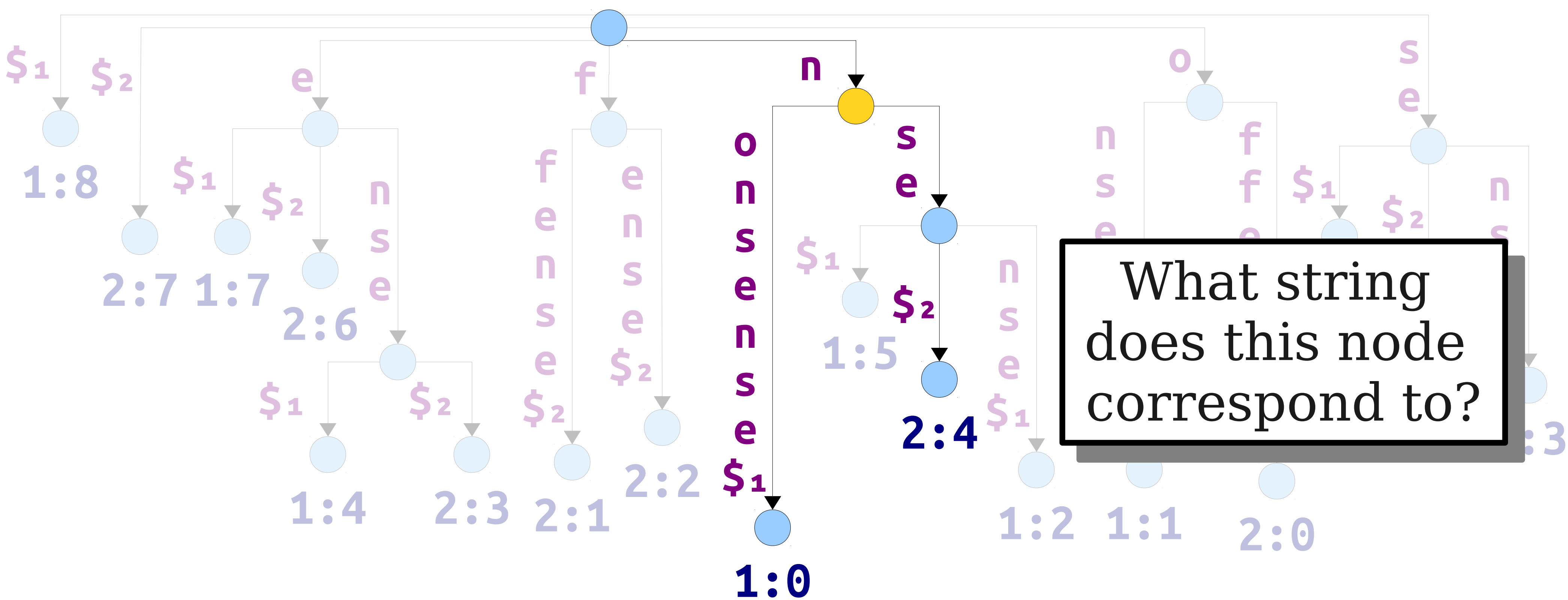


What string does this node correspond to?

nonsense$_1$
012345678

offense$_2$
01234567

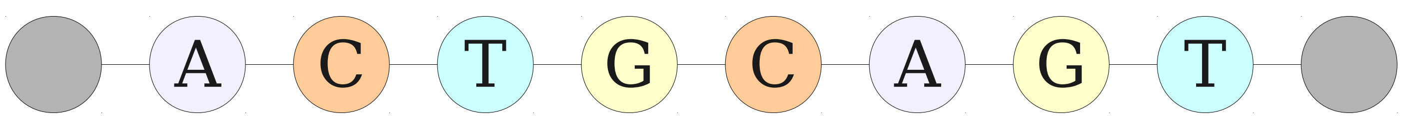
The Overall Construction

- Using an $O(m)$-time DFS, annotate each node in the suffix tree with its string depth.
- To compute LCE:
  - Find the leaves corresponding to $T_1[i:]$ and $T_2[j:]$.
  - Find their LCA; let its string depth be d.
  - Report $T_1[i:i + d - 1]$ or $T_2[j:j + d - 1]$.
- Overall, requires $O(m)$ preprocessing time to support $O(1)$ query time.

An Application: Longest Palindromic Substring

Palindromes

- A **palindrome** is a string that's the same forwards and backwards.
- A **palindromic substring** of a string T is a substring of T that's a palindrome.
- Surprisingly, of great importance in computational biology.



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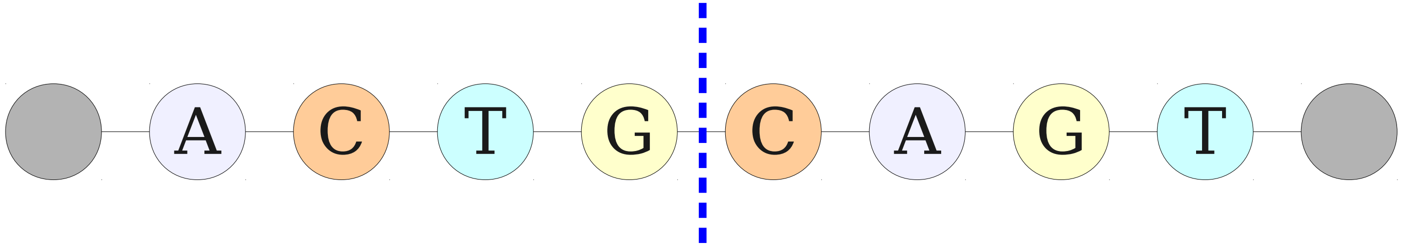

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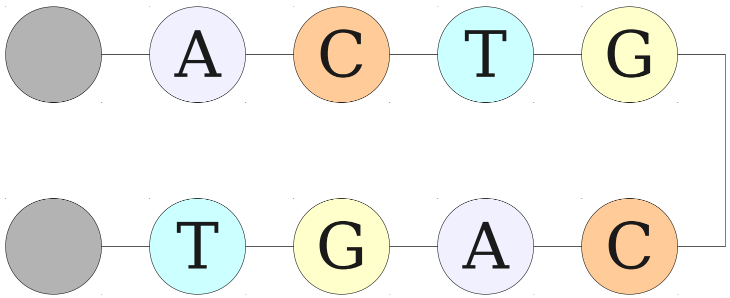

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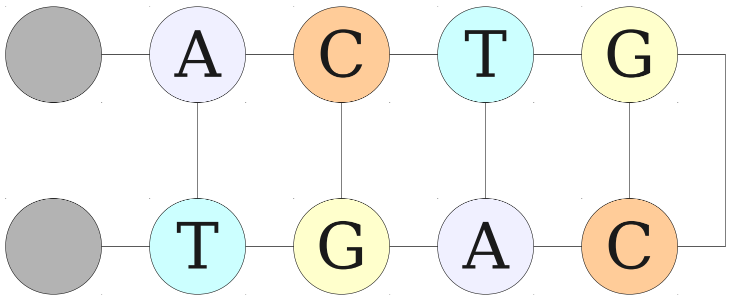


Longest Palindromic Substring

- The **longest palindromic substring** problem is the following:

  Given a string T, find the longest substring of T that is a palindrome.

- How might we solve this problem?

An Initial Idea

- To deal with the issues of strings going forwards and backwards, start off by forming T and T^R, the reverse of T.
- **Initial Idea:** Find the longest common substring of T and T^R.
- Unfortunately, this doesn't work:
  - $T =$ **abbccbbabccbba**
  - $T^R =$ **abbccbabbccbba**
  - Longest common substring: **abbccb**

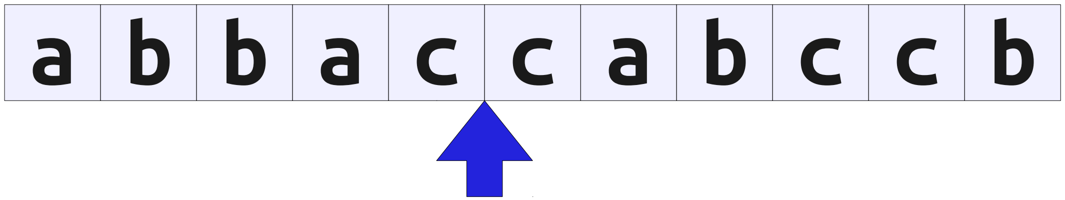
Palindrome Centers and Radii

- For now, let's focus on even-length palindromes.
- An even-length palindrome substring ww^R of a string T has a *center* and *radius*:
  - **Center:** The spot between the duplicated center character.
  - **Radius:** The length of the string going out in each direction.
- **Idea:** For each center, find the largest corresponding radius.

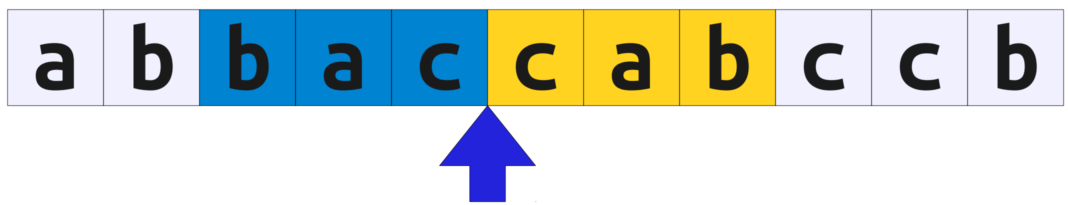
Palindrome Centers and Radii

a	b	b	a	c	c	a	b	c	c	b

Palindrome Centers and Radii



Palindrome Centers and Radii



Palindrome Centers and Radii

a	b	b	a	c	c	a	b	c	c	b

Palindrome Centers and Radii

a	b	b	a	c	c	a	b	c	c	b

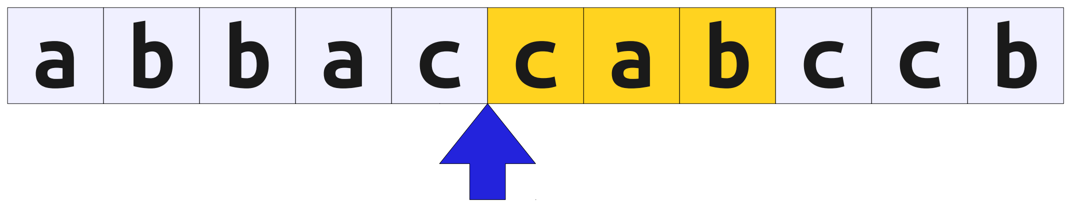
b	c	c	b	a	c	c	a	b	b	a

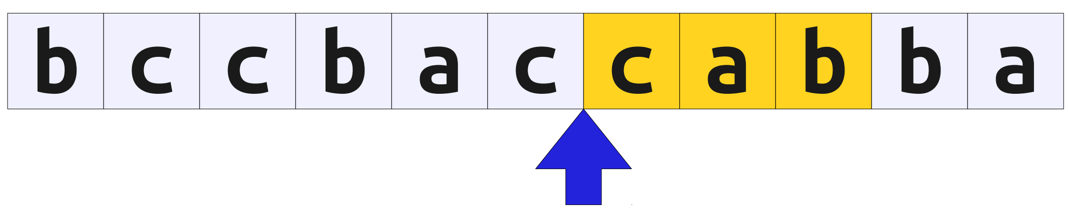
Palindrome Centers and Radii

a	b	b	a	c	c	a	b	c	c	b

b	c	c	b	a	c	c	a	b	b	a

Palindrome Centers and Radii





Palindrome Centers and Radii

a	b	b	a	c	c	a	b	c	c	b

b	c	c	b	a	c	c	a	b	b	a

Palindrome Centers and Radii

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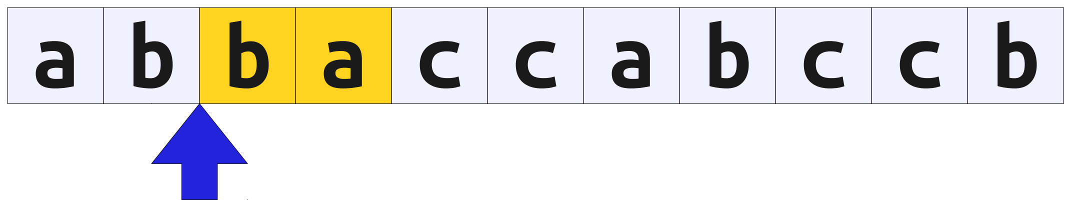
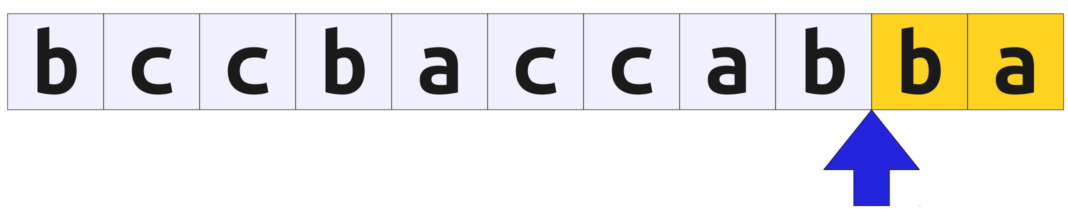
Palindrome Centers and Radii





Palindrome Centers and Radii

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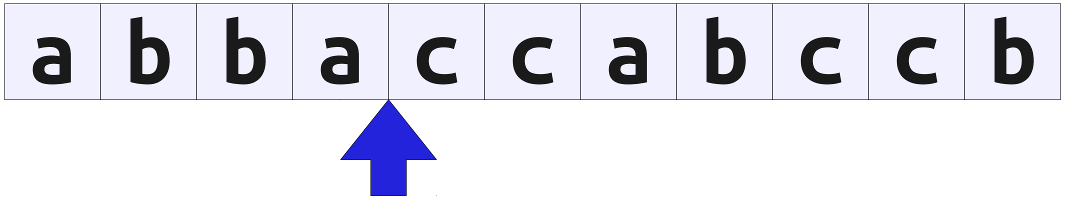
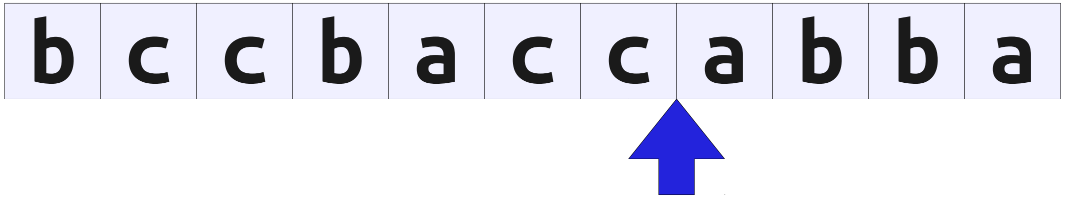
Palindrome Centers and Radii





An Algorithm

- In time O(m), construct T^R.
- Preprocess T and T^R in time O(m) to support LCE queries.
- For each spot between two characters in T, find the longest palindrome centered at that location by executing LCE queries on the corresponding locations in T and T^R.
  - Each query takes time O(1) if it just reports the length.
  - Total time: O(m).
- Report the longest string found this way.
- Total time: **O(m)**.

Suffix Trees: The Catch

Space Usage

- Suffix trees are memory hogs.
- Suppose $\Sigma = \{A, C, G, T, \$\}$.
- Each internal node needs 15 machine words: for each character, words for the start/end index and a child pointer.
- This is still $O(m)$, but it's a huge hidden constant.

Combating Space Usage

- In 1990, Udi Manber and Gene Myers introduced the **suffix array** as a space-efficient alternative to suffix trees.
- Requires one word per character; typically, an extra word is stored as well (details Wednesday)
- Can't support all operations permitted by suffix trees, but has much better performance.
- Curious? Details are next time!

Next Time

- **Suffix Arrays**
  - A space-efficient alternative to suffix trees.
- **LCP Arrays**
  - A useful auxiliary data structure for speeding up suffix arrays.
- **Constructing Suffix Trees**
  - How on earth do you build suffix trees in time O(m)?
- **Constructing Suffix Arrays**
  - Start by building suffix arrays in time O(m)…
- **Constructing LCP Arrays**
  - … and adding in LCP arrays in time O(m).