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The Retail Rack Layout Problem

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Abstract

A retail store's layout affects a shopper's visual experience and correspondingly the time spent in the store, navigation through the aisles, and allocation of attention and money across departments and categories. We contend that alternate rack layouts allow for greater exposure of product locations, but which rack layouts maximize exposure of products to a shopper? To address this question, we introduce the retail rack layout problem (RRLP) of identifying the optimal single or multi-column rack orientations in a constrained space in order to maximize exposure. We propose a mixed-integer non-linear mathematical programming model for the RRLP. Given the complexity associated with this model, we propose a heuristic-based solution approach based on particle swarm optimization. Results indicate that maximum exposure of a layout is sensitive to shopper visual characteristics; orientation of racks in the column closest to the shopper is less than the degree of horizontal head movement exhibited by the shopper. Multiple competitive layout designs exist that offer similar exposure values for a given system configuration. Our findings provide quantitative evidence of the sensitivity of exposure to the characteristics of the shopper, confirming that there is no such thing as a one-size-fits-all approach to retail design.

Keywords
Retail layout; product exposure; particle swarm optimization

1. Introduction
The retail store provides a critical interface between retailers and shoppers. Marketers and retailers have observed that the store's layout influences how shoppers navigate through the store and interact with products [1], making it a critical trigger point in the shopping cycle that can significantly affect the attitude and behavior of shoppers [2,3]. Successful design of a retail facility's layout can, thus, be beneficial to both industry and shoppers alike.

Various models and approaches to solve the traditional facility layout problem (FLP) for manufacturing and warehousing facilities have been well studied [4,5]. Literature on retail facility layout, however, is fairly limited. Peters et al. [6] addressed the retail layout problem by developing a location assignment model to assign groups of products (i.e., departments) to locations in order to maximize impulse purchase revenue. Yapicioglu and Smith [7] focused on optimally designing a departmental block layout (departments and aisle space) within a racetrack structure that maximized the store's revenue. They then reformulated this problem as a bi-objective problem where the first objective maximized store revenue based on department layout and the second maximized the satisfaction of departmental adacencies [8]. Our contributions concentrate on the layout of the store at a finer level; i.e., optimizing the rack layout within a department.

Fundamental differences exist between the traditional FLP and the retail rack layout problem; (i) shopper travel paths are highly variable and unpredictable, (ii) department adjacencies are often violated on purpose (e.g., bread and milk) to increase travel paths and potential impulse purchase, (iii) shopper demand for products can be influenced by the layout, product placement, promotions, and stimuli, and (iv) the retailer's objective is often maximizing sales instead of minimizing material handling costs. These substantial departures from the traditional FLP, suggest that standard FLP metrics may be limited or even inappropriate for retail facility layout problems.

In this paper, we use exposure (introduced in [9]) as a potential measure of visibility. Exposure, defined as the possibility (not probability) that a rack location could be seen by traveling shopper, has been shown to be a predictor of product engagement, making it immediately relevant to the retail world as a precursor to product sales [10,11].

We propose an optimization model for the retail rack layout problem (RRLP), a special case of the FLP, to determine the optimal placement of rectangular racks within a constrained space in order to maximize the total rack exposure. Given the complexity associated with solving the mixed-integer, non-linear, mathematical programming model, we propose a heuristic-based solution approach from which we make several observations.

2. An Optimization Model for the Retail Rack Layout Problem (RRLP)

In the RRLP, we identify the optimal arrangement of *N* racks within *B* columns in order to maximize exposure (*E*) of the rack facings to a traveling shopper. We make the following assumptions:

- Shoppers travel along a straight pathway (e.g., the main aisle).
- They can turn their head and eyes, combined, no more than Φ from their direction of travel.
- Shoppers can discern recognizable words, logos, and packing up to a distance *DOV* away.

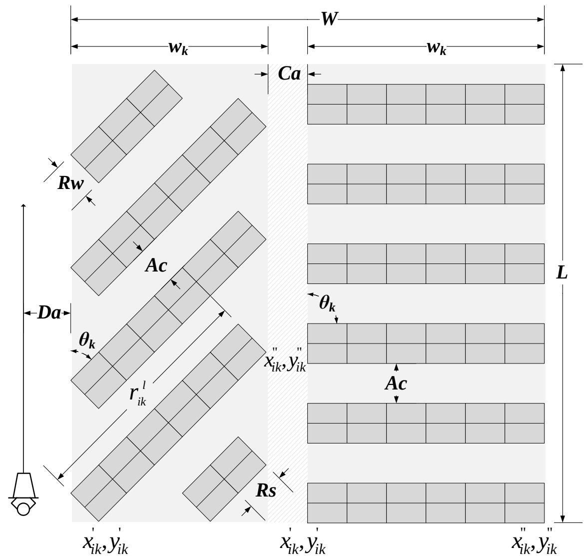


Figure 1: Example 2-column rack layout (b = 2, n_1 = 5, θ_1 = 45°, n_2 = 6, θ_2 = 90°)

In our model, a rack is composed of multiple rack segments, arranged end to end to compose rows of rectangular racks, all of which are identical in width and length. We assume a rack segment to be 2 ft. wide x 4 ft. long placed back-to-back (forming a 4 ft. x 4 ft. segment), which is in line with what we observed at most retail stores in our geographical region. A column of racks can be broadly defined as an arrangement of racks that all align along the x or y-axis, have the same orientation (θ) and are separated by a minimum spacing (*Ac*) to allow adequate room for shoppers to travel. Table 1 lists all the parameters and decision variables used in our model, and Figure 1 illustrates those specific to the layout.

Table 1: Variables used in our model

Parameters		Decision Variables	
W	width of the section (ft)	r^l_{ik}	no. of rack segments in rack i (ft) in column k
L	length of the section (ft)	w_k	width of column k (ft)
Ac	width of the shopping aisle (ft) between racks in column k	c_k	1 indicates column k exists, 0 otherwise; where $k = 1 \ldots B$
Rs	length of a rack segment (ft)	θ_k	rack orientation measured clockwise from primary direction of travel for column k
Rw	width of a rack's end cap display (ft)	v_{ik}	1 indicates rack i in column k exists, 0 otherwise, where $i = 1 \ldots N$
Ca	width of the cross-aisle between columns	x'_{ik} (y'_{ik})	x (y)-coordinate of corner between face I and II
Am	width of the main aisle	x''_{ik} (y'''_{ik})	x (y)-coordinate of corner between face II and III
Da	perpendicular distance of closest corner to the shopper's directional path	θ'_k	adjusted rack orientation; where $\theta'_k = t_k(180 - \theta_k) + (1 - t_k)\theta_k$
α (1-α)	percentage of traffic flowing in the primary (opposite) direction of travel	t_k	1 indicates θ_k is greater than 90°, 0 otherwise
Φ	shopper's angular limit of vision corresponding to half of FoR	t'_k	0 indicates θ_k = 0°, 1 otherwise
DOV	shopper's depth of focused vision, radius of FoR	t''_k	0 indicates θ_k = 90°, 1 otherwise
I	required amount of rack display in terms of total rack perimeter (ft)	z^x_{ijk}	1 indicates rack j is to the right of rack i in column k, 0 otherwise
M	large number	z^y_{ijk}	1 indicates rack j is above rack i in column k, 0 otherwise

We propose the following optimization model for the RRLP.

Maximize: $$E_{Total} = \alpha E_{\alpha} + (1-\alpha)E_{1-\alpha} \tag{1}$$

Subject to:

$$\begin{aligned} E_{\alpha} &= f\left(\theta_k, Da, Ac, r^l_{ik}, r^l_{jk}, Rw, \Phi, DOV\right) \\ E_{1-\alpha} &= f\left(180° - \theta_j, (Am - Da), Ac, r^l_{ik}, r^l_{jk}, Rw, \Phi, DOV\right) \end{aligned} \tag{2}$$

$$\sum_{k=1}^{b} c_k w_k + Ca\left(\sum_{k=1}^{b} c_k - 1\right) \le W \tag{3}$$

$$c_k \le w_k \quad \forall k \tag{4}$$

$$c_k \ge v_{ik} \quad \forall i,k \tag{5}$$

$$v_{ik} M \ge r^l_{ik} \quad \forall i,k \tag{6}$$

$$x''_{ik} = x'_{ik} + r^l_{ik} Rs \sin\theta'_k \quad \forall i,k \tag{7}$$

$$y''_{ik} = y'_{ik} - (2t_k - 1) r^l_{ik} Rs \cos\theta'_k \quad \forall i,k \tag{8}$$

$$v_{ik}(1-t_k) Rw \cos\theta'_k \le x'_{ik} \quad \forall i,k \tag{9}$$

$$(1-t_k)x''_{ik} + t_k\left(x''_{ik} + Rw\cos\theta'_k\right) \le w_k \quad \forall i,k \tag{10}$$

$$(1-t_k)y''_{ik} + t_k y'_{ik} + Rw\sin\theta'_k \le L \quad \forall i,k \tag{11}$$

$$x'_{ik} + \frac{t''_k Rw + (1-t''_k)Rsr^l_{ik} + Ac}{\cos\theta'_k + (1-t''_k)} \le x'_{jk} + M\left(1 - z^x_{ijk}\right) \quad \forall k,i,j \quad i \ne j = 1..n \tag{12}$$

$$y'_{ik} + \frac{t'_k Rw + (1-t'_k)Rsr^l_{ik} + Ac}{\sin\theta'_k + (1-t'_k)} \le y'_{jk} + M\left(1 - z^y_{ijk}\right) \quad \forall k,i,j \quad i \ne j \tag{13}$$

$$z^x_{ijk}, z^y_{ijk} \le v_{ik} \quad \forall i,k \tag{14}$$

$$z^x_{ijk} + z^x_{jik} + z^y_{ijk} + z^y_{jik} \ge v_{ik} v_{jk} \quad \forall k,i,j \quad i \ne j \tag{15}$$

$$\theta_k M\left(2t'_k - 1\right) \ge t'_k \quad \forall k \tag{16}$$

$$\left(90 - \theta'_k\right) M\left(2t''_k - 1\right) \ge t''_k \quad \forall k \tag{17}$$

$$\begin{gathered} 0 \le \theta_k < 180° \\ w_k, x'_{ik}, x''_{ik}, y'_{ik}, y''_{ik} \ge 0 \\ r^l_{ik} \in \left\{0, Z^+\right\} \\ c_k, v_{ik}, z^x_{ijk}, z^y_{ijk}, t_k, t'_k, t''_k \in \{0,1\} \end{gathered} \tag{18}$$

This optimization model first solves a design problem by identifying the number of rack-columns (and their associated widths) that exist within the store sub-section, and then determining the maximum number of racks that can be feasibly placed. We only need to identify the length of each (4ft wide) rack in each column associated with the rack orientations (θ_k) for each column k to be solved by the model.

The objective of the model is to maximize E for a section that is located on one side of the pathway. As shoppers travel bidirectionally along the aisle, we estimate E of the left or right side of the pathway from each direction separately and then take the weighted average based on the traffic flow parameter, α. The main decision (or system design) variables include the width of column k (w_k), the orientation of racks in column k (θ_k), the number of fixed-length rack sections for each rack r^l_{ik} (which determine the rack-length), and the location of each rack's lower, interior corner (x'_{ik}, y'_{ik}). Constraints (2) estimate E of a store sub-section from both α and 1-α directions of shopper traffic. Given that E for a general rack layout is not expressed in closed-form, we use the algorithm presented in [9] to estimate both E_α and $E_{1-\alpha}$. This algorithm accounts for the human's field of view (both depth of vision and the angular limit of the forward line of sight) and obstructions. Constraint (3) ensures that the available width of the section is not exceeded and is utilized as either available space for racks within each column or as a cross-aisle between columns. Constraint (4) requires a column's width to equal 0 when it does not exist, while Constraints (5) and (6) prevent racks from existing in a column that does not exist. The remaining constraints are similar to Montreuil's MIP model for a traditional FLP in the continuous space [12]. Essentially, Constraints (7) and (8) calculate the x and y coordinates of the lower, outside corner point (corner between Face II and III) based on the coordinates of the lower, interior corner point (located between Face I and II, see Figure 1). Constraints (9) - (11) ensure that rack i is defined within the bounds of column j, while Constraints (12) - (15) ensure that rack j is sufficiently east (right) or north (above) of rack i so that minimum shopping aisle distance is maintained. Constraints (16) and (17) allow the model to adapt to all rack orientations including 0° and 90° orientations. Constraints (18) indicate bounds on the decision variables.

Considering the complexity associated with the above optimization model (nonlinear constraints and exposure not in closed form), we propose a heuristic-based solution approach using particle swarm optimization (PSO) to solve the RRLP. Our proposed PSO accepts a set of initial feasible solutions generated using a straightforward greedy approach before improving them. It uses two key subroutines for every particle at every iteration: (i) layout design, which builds a feasible arrangement of racks in each column for a given solution (first used to generate initial feasible solutions and then in improving these solutions) and (ii) exposure evaluation, which estimates E for this feasible rack layout by simulating a shopper walking through the system. We then use personal and neighborhood best solutions to update the particles after each iteration [13].

Our proposed PSO-based solution approach uses a swarm of 20 particles (in line with Clerc and Kennedy [14]) to search the solution space and determine the number of columns (k) in a store sub-section, their associated widths (w_k), and the corresponding rack orientations (θ_k). We represent a solution as a string of real numbers, which consists of 5 elements. The first 3 elements represent the rack angle (θ_k) for each possible column (k) and the following 2 elements represent the column width ratios (width of column k / available space). Feasible solutions will satisfy $0° \leq \theta_k < 180°$ and $0 \leq$ width ratio ≤ 1 for each column. Solutions that are not feasible are not evaluated and not considered as either personal or local best. The performance of the PSO was verified on small problem instances (results not shown due to space limitation).

3. Experimental Results

We consider a 44 ft x 44 ft section and assume bidirectional shopper traffic (α=0.5) walking in the middle of the main aisle, 5 ft away from the nearest point on a rack. We allow up to three columns of racks assuming shopping aisle and cross-aisle widths are 4 ft each. We evaluate two levels of head and eye rotation: small movement, Φ=45º (to one side of the forward line of sight), and large movement, Φ=90º. We also evaluate the shopper's depth of focused vision (*DOV*) at two levels, 25 ft and 65 ft. Combined, these two shopper characteristics define the limits of potential exposure, which we refer to as the Field of Regard (*FoR*). We model two scenarios with *FoR*, one that partially penetrates the 44 ft x 44 ft design section (i.e., *DOV*=25 ft) and the other that fully (or nearly-fully) penetrates it (i.e., *DOV*=65 ft), for both levels of Φ (Figure 2).

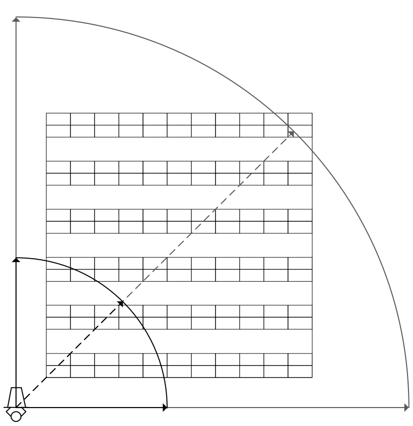
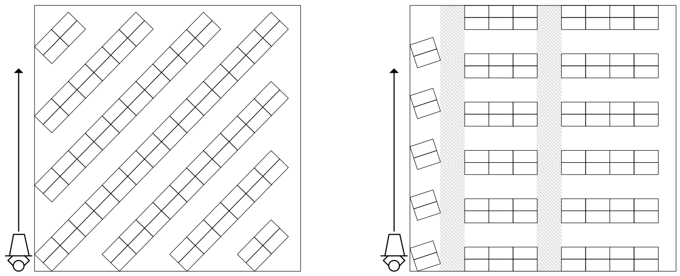


Figure 2: Penetration of the *FoR* (with example 90° racks) for Φ=45º (dotted line) and 90º (solid line) and *DOV*=25 ft (black line) and 65 ft (grey line).

Each layout generated by the PSO is associated with its corresponding display (total linear feet of rack perimeter that products may occupy) and E. Because the 90°-rack angle is the most common orientation in retail stores, we report a %-increase in E and %-loss in display compared to the equivalent 1-column 90°-rack layout (Figure 2). We ran all 4 combinations of Φ and *DOV*, and then analyzed all solutions whose E was within 1% of the maximum exposure found. Figures 3 and 4 illustrate our findings. Based on these results, we observe the following:

Obs. 1: When the FoR fully penetrates the design section, solutions that result in the highest exposure favor 1-column layouts for small shopper head turns and multiple-column layouts for large shopper head turns.

Best layouts are those that strategically place as many racks as possible within the *FoR* in order to maximize E. With *DOV*=65ft, the entire sub-section falls within the shopper's *FoR* as the shopper travels and all locations can potentially contribute to E. Our experiments suggest that when shoppers make small head movements, the best layouts increased E by 226.5% (with %-loss of display ranging 22.2%-23.6%) with respect to the traditional 90º layout and resulted exclusively in 1-column layouts.



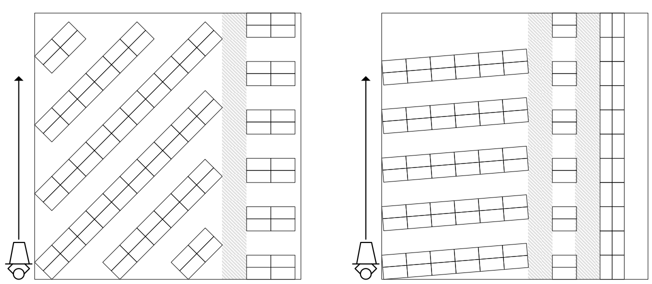
(a) θ_1=44.3º
for Φ = 45°
226.5% increase in E
23.6% loss of display

(b) (θ_1, θ_2, θ_3)= (72º, 90º, 90º)
for Φ=90°
20.5% increase in E
11.1% loss of display

Figure 3: Example 1- and 3-column designs for *DOV*=65 ft

Layouts with large head movements increase E by 20.5% (with %-loss of display ranging 8.3%-11.1%) and produce 3-column layouts. When we increase Φ from 45º to 90º, the *FoR* doubles in size. This increase in the size allows more opportunities for racks to be exposed over the course of the shopper's path (both in the ability to see around an obstructing corner and deeper reach of vision into the section), making multiple column solutions appealing. All 1-column designs were found to converge to 44.3° for Φ=45°, while all 3-column designs converged to (72°, 90°, 90°) when Φ=90°. We illustrate both of these layouts in Figure 3.

Obs. 2: When the FoR partially penetrates the design section, regardless of head movement (Φ), competitive rack layouts exist with either 1-, 2-, and 3- columns.
When *DOV*=25 ft, orienting racks at an angle in column 1 (and sometimes column 2 depending on how much of it falls within the *FoR*) tends to increase E. The remaining area of the sub-section falls outside of the *FoR* and does not contribute to E. Hence, layouts may differ in their designs outside the limits of the *FoR*, but may still be competitive as long as they have a similar E. In our experiments, we noticed that the orientation of racks in the first column (θ_1) ranged 43.5°-44.8° for Φ=45° and ranged 77.8°-89.9° for Φ=90°, regardless of whether they were 1-, 2-, or 3-column solutions. The rack angles in the 2nd and 3rd columns range anywhere between 0º and 180º. Figure 4 provides an example 2- and 3-column layout.



(a) (θ_1, θ_2)=(44.8º, 90º)
for Φ = 45°
67.3% increase in E
16.7% loss of display

(b) (θ_1, θ_2, θ_3)=(85.3º, 90.0º, 0º)
for Φ=90°
17.6% increase in E
18.1% loss of display

Figure 4: Example 2- and 3-column designs for *DOV*=25 ft

Obs. 3: Rack angles for the first column (θ_1) tend to be ≤ Φ, independent of DOV.
At both *DOV* values, the rack angles in the first column that produce the highest amounts of E are less than or equal to their respective Φ. Notice that the orientation for θ_1 ranges between 43.5º and 44.8º when Φ=45º regardless of *DOV*. Similarly, a range of values <90º seem to dominate designs for large head movements. These findings are different than what was observed in a model with unconstrained space where 30º racks produced the most exposure over 90º [6]. Intuitively, E should increase as we allow the racks to be more angular. But this angularity increases loss in display, which then reduces the %-increase in E, eventually making the design worse than the equivalent 90º-layout. Clearly, the shopper's head and eye movements (horizontal scanning behavior) strongly influence the rack layout (at least those closest to the shopper), arguably more so than the shopper's *DOV*.

Obs. 4: When shoppers make small head movements (Φ=45º), angled-rack layouts can offer substantial improvements in exposure irrespective of rack length.
Notice that the solutions generated with small head movements result in the greatest increase in E; an increase of 226.5% was observed when *DOV*=65 ft and 67.3% was observed when *DOV*=25 ft. For small head movements, only a small percentage of racks are exposed when racks are oriented at 90º. Angled-rack layouts align better with the shopper's head movements, in turn increasing E dramatically. In contrast, solutions associated with large head movements result in small increases that were less than 21%. With large head movements, exposure of the 90º racks is much higher. So even though angled-rack layouts offer a further increase in E, the %-increase from 90º is limited and less impactful. For this reason, alternate rack layouts may be more desirable in settings associated with small

head movements; e.g., utilitarian shopper vs. leisure shopper, familiar vs. unfamiliar layout, or time-constrained vs. unconstrained shopping.

4. Conclusions and Future Work

The physical layout of the retail store is known to influence the attitude and behavior of shoppers and affect store performance. An important role of the store layout is to expose merchandise to customers in order to facilitate consideration and ultimately purchasing of the products. Key locations known to be highly visible to shoppers are considered extremely valuable to retailers and are sought after by manufacturers because they are known to increase sales and satisfy customers. In our work, we present an MIP model and solution procedure that allows store designers to create substantially better rack layout designs with respect to product exposure.

The main contribution of our work was the introduction of the RRLP that incorporates the shopper's *FoR* in identifying rack layouts that maximize exposure. In so doing, our approach sheds light on a component of retail design that had previously been ignored, the racks that display the products. Preliminary results indicate that layouts with acute- (and sometimes obtuse-) angled racks produce substantially larger amounts of exposure, as compared to their 90° counterparts, even with competitive amounts of display. Further, many designs appear to produce the same amount of exposure, suggesting that designers may have options when selecting a layout that fits their unique preferences and needs. With this paper we add to the limited amount of literature on retail facility layout and hope to motivate future work in retail design from within the research community. Evaluating exposure in 3D and considering newer racks designs would be an interesting future research avenue.

Acknowledgements

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